TRANSACTIONS

OF THE

AMERICAN SOCIETY

OF

CIVIL ENGINEERS

(INSTITUTED 1852)

VOLUME 123

1958

Edited by the Executive Secretary, under the direction of the Committee on Publications.

Reprints from this publication, which is copyrighted, may be made on condition that the full title of paper, name of author, and page reference (or paper number) are given.

NEW YORK
PUBLISHED BY THE SOCIETY
1958

Transportation
Libr..

TA
1
.A526
V.123

Copyright 1958 by the AMERICAN SOCIETY OF CIVIL ENGINEERS

NOTE.—The Society is not responsible for any statement made or opinion expressed in its publications.

CONTENTS

No. Page

2907 STABILIZATION OF MATERIALS BY COMPACTION
BY WILLARD J. TURNBULL AND CHARLES R. FOSTER 1

Discussion:
By Gerald A. Leonards 16
John A. Focht, Jr. 19
Edmund J. Zegarra 20
Kazuo B. Hirashima 23
Willard J. Turnbull and Charles R. Foster 25

2908 TRENDS IN HYDRAULIC GATE DESIGN
BY DOW A. BUZZELL 27

Discussion:
By Abraham Streiff 33
George R. Latham 35
William G. H. Holt 36
Kandaswami S. Chetty 36
William Gustave Weber 38
Dow A. Buzzell 40

2909 TRANSITION PROFILES IN NONUNIFORM CHANNELS
BY FRANCIS F. ESCOFFIER 43

Discussion:
By Achille Lazard 57
Léon J. Tison 58
Francis F. Escoffier 63

2910 MINIMUM-WEIGHT DESIGN OF A PORTAL FRAME
BY WILLIAM PRAGER 66

Discussion:
By Robert L. Ketter 75
William Prager 76

2911 DEVELOPMENTS IN SEPTIC TANK SYSTEMS
BY JOHN E. KIKER, JR. 77

2912 COST ALLOCATION FOR MULTIPLE-PURPOSE WATER PROJECTS
BY NEWCOMB B. BENNETT, JR. 85

Discussion:
By Frederick L. Hotes 93
Eugene W. Weber 95
Wendell E. Johnson and Charles A. Cocks 96
Newcomb B. Bennett, Jr. 99

2913 DESIGN OF STABLE CANALS AND CHANNELS IN ERODIBLE MATERIAL
BY PETE W. TERRELL AND WHITNEY M. BORLAND 101

2914 SOIL LYSIMETERS IN WASTE WATER RECLAMATION STUDIES
BY GERALD T. ORLOB AND ROBERT G. BUTLER 116

Discussion:
By Ralph Stone 137
Gerald T. Orlob and Robert G. Butler 137

2915 WAVE RUN-UP ON SHORE STRUCTURES
BY THORNDIKE SAVILLE, JR. 139

2916 EXPERIENCES WITH LOESS AS FOUNDATION MATERIAL
BY WILLIAM A. CLEVENGER 151

Discussion:
By Harry R. Cedergren 170
Ralph B. Peck and Herbert O. Ireland 171
William A. Clevenger 179

No. PAGE

2917 A FLOODWAY FOR HOUSTON, TEX.
BY KENNETH HEAGY 181

2918 SEDIMENT-TRANSPORT MECHANICS IN STABLE-CHANNEL DESIGN
BY EMMETT M. LAURSEN 195

DISCUSSION:
By SAM SHULITS 204
EMMETT M. LAURSEN 206

2919 MORRO BAY STEAM ELECTRIC PLANT
BY J. GEORGE THON AND GORDON L. COLTRIN 207

DISCUSSION:
By GLENN B. WOODRUFF 233
JACOB FELD 233
WILLIAM W. MOORE 234
GEORGE L. JORDY 235
J. GEORGE THON AND GORDON L. COLTRIN 237

2920 DEVELOPING PORT FACILITIES ON HOUSTON'S SHIP CHANNEL
BY FRANK H. NEWNAM, JR 239

DISCUSSION:
By ERSEL G. LANTZ 258
AUSTIN E. BRANT, JR 260
FRANK H. NEWNAM, JR 263

2921 DYNAMIC STRESSES IN CONTINUOUS PLATE-GIRDER BRIDGES
BY ROY C. EDGERTON AND GORDON W. BEECROFT 266

DISCUSSION:
By ROBERT K. L. WEN 287
ROY C. EDGERTON AND GORDON W. BEECROFT 290

2922 THE VISCOUS SUBLAYER ALONG A SMOOTH BOUNDARY
BY HANS A. EINSTEIN AND HUON LI 293

DISCUSSION:
By EDWARD SILBERMAN 314
NICOLS N. AMBRASEYS 315
HANS A. EINSTEIN AND HUON LI 316

2923 MOMENT-DISTRIBUTION CONSTANTS FROM MODELS
BY OTAKAR ONDRA 318

DISCUSSION:
By JOHN C. MURPHY 344
THOMAS D. Y. FOK 345
FRANK J. CAIN AND GERALD LUCK 350
OTAKAR ONDRA 353

2924 TRANSPORATATION PLANNING: A SYMPOSIUM
FOREWORD 360
THE PORT, A FOCAL POINT
BY ROGER H. GILMAN 361
THE AIRPORT, A NATIONAL FACILITY
BY WILFRED M. POST, JR 370
PROSPECTS FOR COORDINATION
BY BURTON W. MARSH 377

2925 EVAPORATION FROM FREE WATER SURFACES AT HIGH ALTITUDES
BY HARRY F. BLANEY 385

DISCUSSION:
By CARL ROHWER 396
KENNETH M. TURNER 397
IRVIN M. INGERSON AND JOHN W. SHANNON 401
HARRY F. BLANEY 402

No. PAGE

2926 AN IMPROVED DILUTION METHOD FOR FLOW MEASUREMENTS
BY WILLIAM A. CAWLEY AND JACK W. WOODS........................ 405

2927 ANCHORAGES FOR LARGE TAINTER GATES
BY ALEXANDER H. KENIGSBERG........................ 409

DISCUSSION:
By Stephen Wearne........................ 417
Alexander H. Kenigsberg........................ 417

2928 EFFECTS OF STORAGE IMPOUNDMENTS ON WATER QUALITY
BY MILO A. CHURCHILL........................ 419

2929 CONTROL OF HIGHWAY ACCESS: A SYMPOSIUM

FOREWORD
BY CURTIS J. HOOPER........................ 466

EXPERIENCES IN NEW YORK
BY BERTRAM D. TALLAMY........................ 467

ECONOMIC EFFECTS OF THE GULF FREEWAY
BY DEWITT C. GREER........................ 472

EXPERIENCES IN INDIANA
BY CARL E. VOGELGESANG........................ 477

EXPERIENCES IN LOS ANGELES
BY HUGO H. WINTER........................ 486

USER BENEFITS IN CALIFORNIA
BY RALPH A. MOYER........................ 499

2930 PLASTICS: ENGINEERING MATERIALS
BY C. HOWARD ADAMS........................ 511

2931 MECHANICS OF STREAMS WITH MOVABLE BEDS OF FINE SAND
BY NORMAN H. BROOKS........................ 526

DISCUSSION:
By Thomas Blench........................ 550
James R. Barton........................ 550
Hans A. Einstein and Ning Chien........................ 553
Enos J. Carlson and M. Gamal Mostafa........................ 563
Hsin-Kuan Liu........................ 565
Pin-Nam Lin........................ 569
Norman H. Brooks........................ 572

2932 MUNICIPAL ORDINANCES FOR INDUSTRIAL WASTES
BY JULIAN R. FLEMING........................ 595

2933 SIMPLIFICATION OF DESIGN BY ULTIMATE STRENGTH PROCEDURES
BY PHIL M. FERGUSON........................ 602

DISCUSSION:
By Tung Au........................ 629
Herman S. Schick........................ 630
Zdeněk Sobotka........................ 633
Milan Spanovich........................ 637
Sylvester M. Ulicny........................ 637
Phil M. Ferguson........................ 639

2934 MECHANISM OF REAERATION IN NATURAL STREAMS
BY DONALD J. O'CONNOR AND WILLIAM E. DOBBINS........................ 641

DISCUSSION:
By Thomas R. Camp........................ 667
Alexander N. Diachishin........................ 672
Aale Pasveer........................ 677
Myrton C. Rand........................ 679
Donald J. O'Connor and William E. Dobbins........................ 679

No. PAGE

2935 OPEN-CHANNEL FLOW AT SMALL REYNOLDS NUMBERS
BY LORENZ G. STRAUB, EDWARD SILBERMAN, AND HERBERT C. NELSON........ 685

DISCUSSION:

BY WALLACE M. LANSFORD AND JAMES M. ROBERTSON........ 707
CHESLEY J. POSEY........ 712
LORENZ G. STRAUB, EDWARD SILBERMAN, AND HERBERT C. NELSON........ 713

2936 PILE TESTS, LOW-SILL STRUCTURE, OLD RIVER, LOUISIANA
BY CHARLES I. MANSUR AND ROBERT I. KAUFMAN........ 715

DISCUSSION:

BY YOSHICHIKA NISHIDA........ 744
STANLEY F. GIZIENSKI........ 744
CHARLES I. MANSUR AND ROBERT I. KAUFMAN........ 746

2937 DESIGN OF VENTURI FLUMES IN CIRCULAR CONDUITS
BY EDWIN A. WELLS, JR., AND HAROLD B. GOTAAS........ 749

DISCUSSION:

BY KEENO FRASCHINA........ 772
EDWIN A. WELLS, JR., AND HAROLD B. GOTAAS........ 774

2938 IMPORTANCE OF GROUND WATER IN THE NATIONAL ECONOMY
BY ALBERT G. FIEDLER........ 776

2939 EARTHQUAKE RESISTANCE OF ROCK-FILL DAMS
BY RAY W. CLOUGH AND DAVID PIRTZ........ 792

DISCUSSION:

BY NICOLS N. AMBRASEYS........ 811
JOHN V. SPIELMAN........ 813
RAY W. CLOUGH AND DAVID PIRTZ........ 814

2940 HARRISON COUNTY (MISSISSIPPI) ARTIFICIAL BEACH
BY FRANCIS F. ESCOFFIER........ 817

2941 MOMENTS IN FLAT SLABS
BY MARK W. HUGGINS AND WATONE L. LIN........ 824

DISCUSSION:

BY JAMES CHINN........ 842
MARK W. HUGGINS AND WATONE L. LIN........ 844

2942 RELATIONSHIP OF IRRIGATION TO PUBLIC HEALTH
BY LLOYD E. MYERS, JR........ 846

2943 SHORESIDE FACILITIES FOR SPECIAL-PURPOSE SHIPS
BY HOWARD J. MARSDEN........ 851

2944 OPERATION PROBLEMS ON CONTROLLED-ACCESS HIGHWAYS
BY CHARLES M. NOBLE........ 858

2945 LABORATORY AND IN-SITU PERMEABILITY OF SAND
BY CHARLES I. MANSUR........ 868

2946 A FLOW CONTROLLER FOR OPEN OR CLOSED CONDUITS
BY VICTOR L. STREETER........ 883

2947 HYDRAULIC MODEL STUDY OF HYPERION SEWER INTERCHANGE
BY ALFRED C. INGERSOLL AND HAJIME TANAKA........ 908

2948 BRIDGE CLEARANCES: A SYMPOSIUM
FOREWORD........ 926

POLICIES AND PRACTICE
BY EUGENE W. WEBER........ 927

THE INTEREST OF THE BUREAU OF PUBLIC ROADS
BY WALTER KURYLO........ 932

No. PAGE

NEED FOR A REALISTIC APPROACH
BY WILLIAM E. CLEARY.......... 942

THE OPERATOR'S VIEW
BY NICKLES L. CARUTHERS.......... 946

PROBLEMS IN NORTHEASTERN UNITED STATES
BY ERHARD E. DITTBRENNER.......... 949

2949 BEARING-RATIO EFFECT ON STRENGTH OF RIVETED JOINTS
BY JONATHAN JONES.......... 964

2950 THE APALACHICOLA RIVER BASIN PROJECT
BY CLEMENT P. LINDNER.......... 973

2951 SHELLS OF DOUBLE CURVATURE
BY ALFRED L. PARME.......... 989

Discussion:
By Tung Au.......... 1014
W. Watters Pagon.......... 1015
Santi P. Banerjee.......... 1016
Mario G. Salvadori.......... 1018
Alfred L. Parme.......... 1021

2952 BUREAU OF PUBLIC ROADS EXPERIENCES IN HIGHWAY SURVEYS
BY WILLIAM T. PRYOR.......... 1026

2953 A PRESSURE-LINE CONCEPT FOR INELASTIC BENDING
BY FRANK BARON.......... 1039

Discussion:
By John A. Hribar.......... 1047
Frank Baron.......... 1047

2954 SOIL MODULUS FOR LATERALLY LOADED PILES
BY BRAMLETTE McCLELLAND AND JOHN A. FOCHT, JR.......... 1049

Discussion:
By Ralph F. Reuss.......... 1064
Ralph B. Peck, Melvin T. Davisson, and Vello Hansen.......... 1065
Raymond Lundgren.......... 1069
Lymon C. Reese.......... 1071
Eugene A. Ripperger.......... 1074
Hudson Matlock.......... 1077
Bramlette McClelland and John A. Focht, Jr.......... 1081

2955 BEHAVIOR OF RIVETED TRUSS-TYPE CONNECTIONS
BY EUGENE CHESSON, JR., AND WILLIAM H. MUNSE.......... 1087

Discussion:
By Arthur J. Francis.......... 1127
Eugene Chesson, Jr., and William H. Munse.......... 1128

2956 OLD RIVER DIVERSION CONTROL: A SYMPOSIUM

FOREWORD.......... 1130

THE GENERAL PROBLEM
BY JOHN R. HARDIN.......... 1131

HYDRAULIC REQUIREMENTS
BY EUGENE A. GRAVES.......... 1142

FOUNDATION DESIGN
BY WILLARD J. TURNBULL AND WOODLAND G. SHOCKLEY.......... 1160

STRUCTURES REQUIRED
BY NORMAN R. MOORE.......... 1172

2957 TRUSS ANALYSIS BY STIFFNESS CONSIDERATIONS
BY HAROLD C. MARTIN.......... 1182

No. PAGE

2958 MOBILE (ALA.) HARBOR AND SHIP CHANNEL
BY HAROLD E. BISBORT 1195

2959 SURFACE WATER RESOURCES
BY JOSEPH V. B. WELLS 1206

2960 LOAD DISTRIBUTION IN HIGHWAY BRIDGE DECKS
BY ARNOLD W. HENDRY AND LESLIE G. JAEGER 1214

DISCUSSION:
BY PRANAB KUMAR CHAUDHURI 1245
ARNOLD W. HENDRY AND LESLIE G. JAEGER 1246

2961 FRICTION MEASUREMENTS IN THE APALACHIA TUNNEL
BY REX A. ELDER 1249

2962 THE ENGINEER'S PLACE IN THE SECOND CENTURY OF TECHNOLOGY: ANNUAL ADDRESS AT THE SUMMER CONVENTION, PORTLAND, ORE., JUNE 25, 1958
BY LOUIS R. HOWSON, PRESIDENT 1275

MEMOIRS OF DECEASED MEMBERS: ABSTRACTS 1283

SUBJECT INDEX 1312

AUTHOR INDEX 1352

ERRATA

Transactions, Vol. 119, 1954

Page 815, Eq. 13 should be changed to

$$S = \frac{(1 - 2\,M_n)^4}{24\,i} - \frac{M^2_n}{3}(3 - 2\,M_n) + \frac{2\,M^3_n}{3}(1 - M_n) \quad \dots . (13a)$$

Transactions, Vol. 122, 1957

Page 1165, line 6 of Mr. Beedle's closure, the reference to Figs. 6 and 7 should be changed to Figs. 3, 8, and 9.

Page 1166, lines 1 and 2, the reference to Fig. 8 should read Figs. 10(*b*) and 11, and the references to Figs. 14(*a*) and 14(*b*) should read Figs. 17(*a*) and 17(*b*).

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2907

STABILIZATION OF MATERIALS BY COMPACTION

By Willard J. Turnbull,[1] M. ASCE, and Charles R. Foster,[2] A. M. ASCE

With Discussion by Messrs. Gerald A. Leonards; John A. Focht, Jr.; Edmund J. Zegarra; Kazuo B. Hirashima; and Willard J. Turnbull and Charles R. Foster

Synopsis

A soil can often be stabilized by compaction at the proper water content. This paper shows the relationship between strength and both water content and density for cohesive soils, and it describes the variations in the density and strength produced by variations in rolling and lift thicknesses in field compaction.

Introduction

The term "soil stabilization" is usually defined as the method by which soil structures of desired bearing capacities are produced. Most definitions are amplified by the statement that the desired bearing capacities are produced by the addition of other substances. Soil stabilization processes that include the addition of other substances require the processing and mixing of the soil and, in most cases, the adjustment of the mixture to a desired water content followed by compaction. In many instances the desired bearing capacity can be obtained by the latter two processes—water-content adjustment and compaction—without the addition of another substance; therefore, it seems logical to consider compaction as a means of soil stabilization.

The attainment of a high degree of strength is usually associated with a high degree of compaction. This simple association is not entirely correct because the attainment of a high degree of strength, or any desired degree of

---

Note.—Published, essentially as printed here, in April, 1956, in the Journal of the Soil Mechanics and Foundations Division, as *Proceedings Paper 934*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Chief, Soils Div., Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss.

[2] Chief, Flexible Pavement Branch, Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss.

strength in cohesive soils, is contingent on the presence of the proper water content and density, and both these properties must be considered in all cases.

VARIATION OF STRENGTH WITH MOISTURE AND DENSITY

The necessity for considering both water content and density in studying the strength of compacted soils is well illustrated by the laboratory test for the California Bearing Ratio (CBR). Fig. 1(*a*) shows the water content-density relationships, and Fig. 1(*b*) shows the water content-CBR relationships that are developed when a group of samples of a cohesive soil are compacted at various water contents and compactive efforts and subsequently tested for their CBR-value. The procedure of using more than one compactive effort yields a range of densities for each water content. The results shown in Figs. 1(*a*) and 1(*b*) are for a lean clay, classified as ML-CL according to the Unified

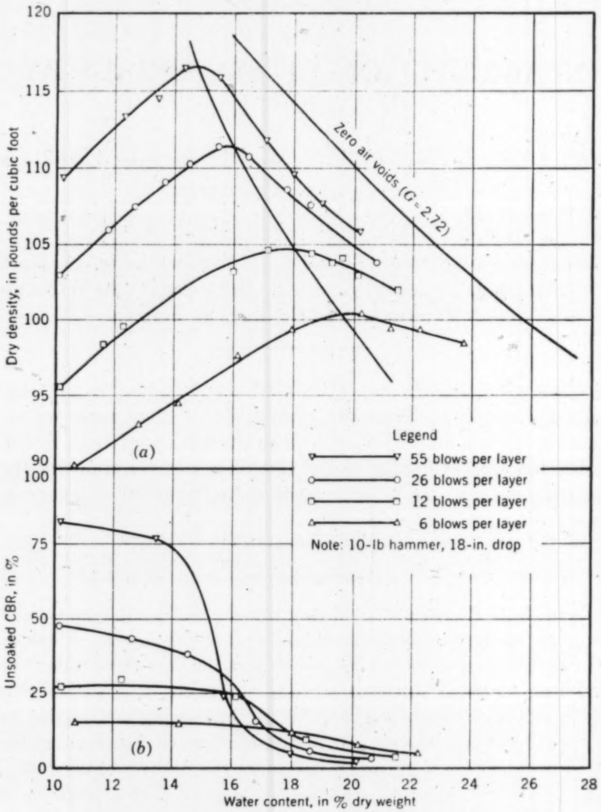


FIG. 1.—MOLDING WATER CONTENT VERSUS DENSITY AND CBR (LABORATORY COMPACTION)

Soil Classification System of the United States Department of the Army. The liquid limit is approximately 38, and the plasticity index is approximately 13. The soil was obtained from the zone of weathered loess native to the grounds of the Waterways Experiment Station, Corps of Engineers, United States Department of the Army, at Vicksburg (Miss.) As shown in Fig. 1(*a*), for any single compactive effort, the density increases as the water content increases until an optimum water content is reached beyond which the density decreases with increasing water content. Also, as the compactive effort increases (additional blows), the maximum density increases and the optimum water content decreases. The optimum water contents can be connected generally with a smooth curve having approximately the same shape as the curve of zero air voids.

Fig. 1(*b*) shows that, as the water content increases, the CBR-value generally decreases. The curves in Fig. 1(*b*) were developed from CBR-tests performed on the samples compacted for the moisture-density relationships. Each point in Fig. 1(*b*) has a corresponding point in Fig. 1(*a*).

The moisture-density-CBR relationships shown in Fig. 1 can be plotted to show the relationship between the CBR and the density for equal water contents (Fig. 2). The plot in Fig. 2(*a*) was made by reading density values from Fig. 1(*a*) at given water contents and corresponding CBR-values from Fig. 1(*b*). These values are plotted in Fig. 2(*a*) and are joined by smooth curves. For the lower water contents, the CBR increases with an increase in density. For the intermediate water contents, the CBR increases with increases in density to a certain value of density, at which point further increases in density produce decreases in the CBR. Generally, the CBR decreases when the moisture and density conditions are such that they plot above and to the right of the line of optimums shown in Fig. 1(*a*).

Curves of unconfined compressive strength versus water content and density show the same pattern when plotted in the manner just described. Triaxial test results will also show the same pattern if the deviator stress is used at a low percentage of strain; these results do not show the reduction in strength at the higher densities if the maximum value or ultimate value of the deviator stress is used. Unconfined and triaxial test results have been presented[3]; however, only the CBR-test results are examined herein.

The decrease in CBR with an increase in density has also been observed (under certain circumstances) in the field test sections at the Waterways Experiment Station. The most striking occurrence was in a series of test sections constructed to test the life of steel landing mat under a heavy airplane-wheel load. The subgrade was the same as the soil described in the foregoing. The test sections were intended to have a CBR of 15%, and the first test section, constructed at an average water content of 16.9%, yielded CBR-values in the desired range. However, as traffic was applied the CBR decreased to considerably less than the desired value of 15%. The test was not entirely successful because failures developed too early. In the second test section the soil was placed at a water content of 14.7%, and the subgrade CBR

---

[3] "Soil Compaction Investigation," *Technical Memorandum 3-271*, Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss.

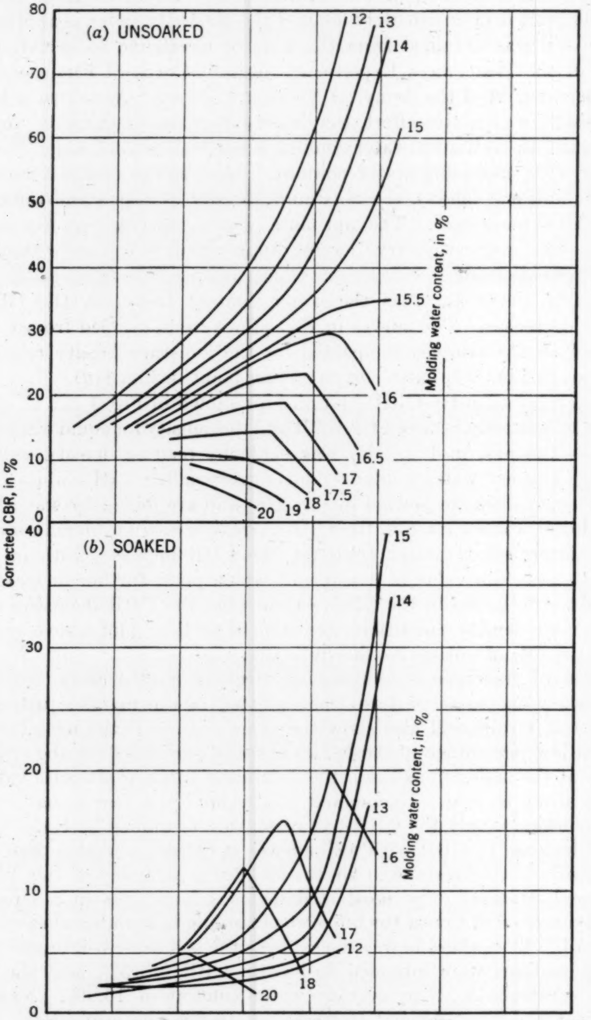


FIG. 2.—EFFECT OF COMPACTION ON CBR (LABORATORY COMPACTION)

increased during traffic to more than 15%. Again the test was not entirely successful because no failures developed during traffic. In the third test section the average water content was 15.2%. The CBR during traffic was generally in the desired range, and the tests were successful because part of the track failed and part of it did not. The reduction in strength that occurs under certain conditions in the laboratory and in the field tests has been described elsewhere.[4]

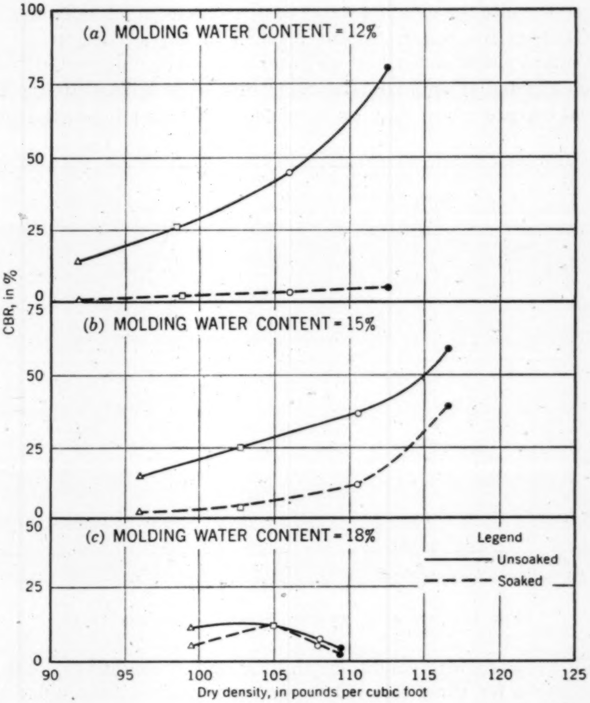


FIG. 3.—EFFECT OF SOAKING ON STRENGTH

The CBR-test results shown in Fig. 2(*a*) were made on samples in the as-molded condition. Therefore, the pattern of results shown in Fig. 2(*a*) represents conditions immediately after compaction. The effect of increases in moisture after compaction can be studied by compacting specimens at a range of water contents and compactive efforts, and by subjecting the specimens to a soaking procedure before making the CBR-tests. The Corps uses a four-day soaking period to estimate the most severe condition likely to occur in nature (excluding frost action). The samples are confined during the soaking period by a surcharge equivalent to the weight of the pavement and base that will be above the material when construction is completed.

---

[4] "Reduction in Soil Strength with Increase in Density," by Charles R. Foster, *Transactions*, ASCE, Vol. 120, 1955, p. 803.

Typically, the soaking tests produce a large reduction in the CBR of materials compacted dry of optimum, a smaller change for those materials compacted at approximately optimum, and only a slight change for those compacted wet of optimum. Fig. 2(*b*) shows the pattern of the CBR versus density for equal molding water contents. The term "molding" should be emphasized because the molding water content has as much influence, or more, on the CBR as the water content reached in the soaking test. The low CBR for samples compacted both dry and wet and the higher CBR-values for samples compacted at approximately optimum are well illustrated. The effect of the soaking procedure may be demonstrated better by comparing results of tests for the as-molded conditions and the soaked conditions. Fig. 3 shows this comparison for three selected water contents and exemplifies the trend described previously.

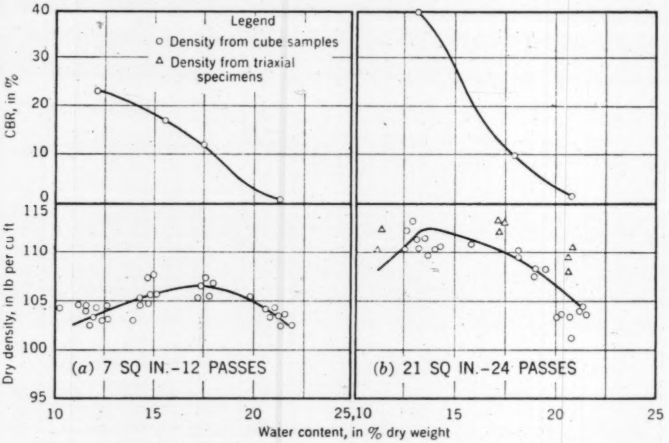


Fig. 4.—Typical Data from Field Compaction Tests

Field Compaction

The soil compaction investigation conducted at the Waterways Experiment Station included field-test sections to study certain variables. Data are presented herein to show the effect of (*a*) repeated passes of the rollers, (*b*) variation in the size of feet on a sheepsfoot roller, (*c*) variations in tire pressure on a rubber-tired roller, and (*d*) variations in lift thickness. All data are for conditions immediately following compaction.

The test sections for the different studies were generally similar. All were made of the lean clay described herein. Soil for the sections was processed to the desired water content and spread over the test sections to the necessary thickness. Each test section contained from four items to six items, each at different water contents ranging from dry of optimum to wet of optimum. As each lift was spread, it was compacted with full-scale construction equipment to the intended number of passes. All lifts were 6 in. thick after compaction except in the tests to determine the effect of lift thickness. The sections were generally built three lifts high. Following construction, in-place CBR-tests, water-content tests, and density tests were made within the middle lift.

Sheepsfoot Roller.—In the study of variation in size of sheepsfoot-roller feet, the roller was operated with 7-sq-in. feet, 14-sq-in. feet, and 21-sq-in. feet. The two larger sizes were obtained by welding plates on the normal sheepsfoot-roller feet. The roller was a dual-drum oscillating type. Each drum was 60 in. in diameter and 66 in. long, and was equipped with 120 feet (30 rows with 4 feet per row). The shanks of the feet were nominally 7 in. long but were slightly longer for the larger foot size because of the plates welded to the feet. In each case the roller was loaded to give a nominal contact pressure of 250 lb per sq in. (assuming that only one row of feet was in contact with the soil). The gross weight for the roller, including both drums, was 14,000 lb, 28,000 lb, and 42,000 lb for the 7-sq-in. feet, the 14-sq-in. feet, and the 21-sq-in. feet, respectively. The roller was pulled by a 34,500-lb crawler-type tractor operating at an average speed of 3 miles per hr. Separate lanes were provided in which the roller was operated to allow for 6 passes, 12 passes, and 24 passes.

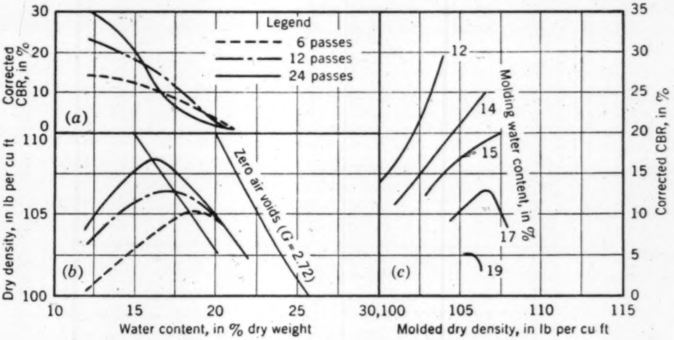


FIG. 5.—CBR, DENSITY, AND WATER-CONTENT DATA (FIELD COMPACTION)

Fig. 4 shows typical plots of the data collected in the study. The plots are for 12 passes of the 7-sq-in. roller and 24 passes of the 21-sq-in. roller. The CBR-values are the average of five tests; the density values are the results of individual tests. Similar plots were prepared for each of the variables (nine plots each for CBR and for density). Fig. 4(*a*) illustrates the best of the nine plots from the standpoint of scattering of data, and Fig. 4(*b*) illustrates an average plot. The curves of density were first drawn to fit the data on the individual plots and were then adjusted to provide the best fit to the entire group (9 plots). Fig. 5 shows the adjusted curves for a 250-lb-per-sq-in. sheepfoot roller with a foot area of 7 sq in. Fig. 5(*c*) shows curves of density versus CBR for equal water contents.

An analysis of the density and water-content data plotted in Fig. 5 shows that an increase in the number of passes or an increase in foot size had the same effect as an increase in the compactive effort in the laboratory test. As the number of passes or the foot size was increased, the maximum density increased and the optimum water content decreased. Fig. 6 presents plots of maximum density and optimum water content versus the number of passes; separate curves are shown for each different foot size.

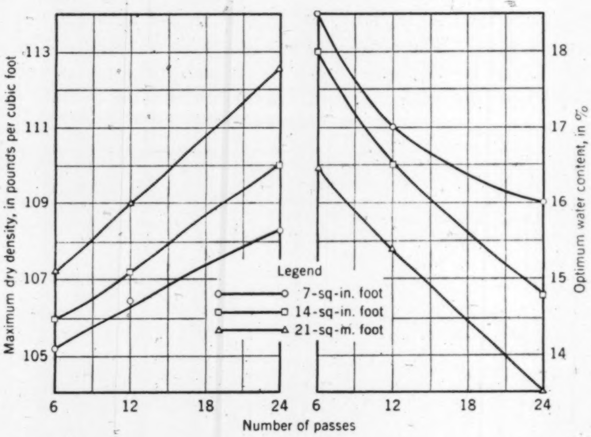


FIG. 6.—EFFECT OF NUMBER OF PASSES AND FOOT SIZE ON MAXIMUM DENSITY AND OPTIMUM WATER CONTENT

The combined effect of changes in passes and foot size can be shown by using the combined factor, E, termed the index of compactive effort, in which

$$E = \frac{\text{foot size} \times \text{number of passes}}{42} \qquad (1)$$

The smallest value of the product of foot size times the number of passes equals 42(7-sq-in. foot times 6 passes); therefore, the constant 42 is used so that the smallest value becomes unity. Fig. 7 shows plots of maximum density and optimum water content versus the combined factor, E; good relationships were found for the combined factor.

In Fig. 8 CBR-curves versus density for equal water-content values are plotted. Water-content values, w, were selected to illustrate conditions on the dry side of optimum, at approximately optimum, and on the wet side of optimum. The curves show that the size of the foot had little effect on the CBR-value obtained.

Rubber-Tired Roller.—The field compaction tests with the rubber-tired roller followed the same general pattern described in the foregoing. The roller consisted of a load box of the trailer type mounted on two sets of dual wheels arranged so that all four wheels were abreast. Each set was mounted on a walking beam, which was free to oscillate from side to side to insure equal load on all wheels. The wheels were equally spaced with a clear width between them that was almost equal to the width of the tire print. There were two assemblies for the wheels, one for pressures of as much as 90 lb per sq in. and the other for pressures of as much as 150 lb per sq in. The former assembly used 18:00 × 24 tires, and the latter used 16:00 × 21 tires. Tests were made with the tires inflated to 50 lb per sq in., 90 lb per sq in., and 150 lb per sq in. It was desired to maintain the tire-contact area for each test as nearly the same as possible; therefore, the load was varied. The gross roller weight was 63,500 lb, 100,000 lb, and 125,000 lb, respectively, for the inflation pressures of 50 lb per sq in., 90 lb per sq in., and 150 lb per sq in. The roller was pulled by the same tractor used in the sheeps-foot-roller tests, and separate lanes were provided in which the roller was operated for 4, 8, and 16 coverages. A coverage is defined as one application of a tire print to the entire area being compacted; two passes of the roller produced one coverage.

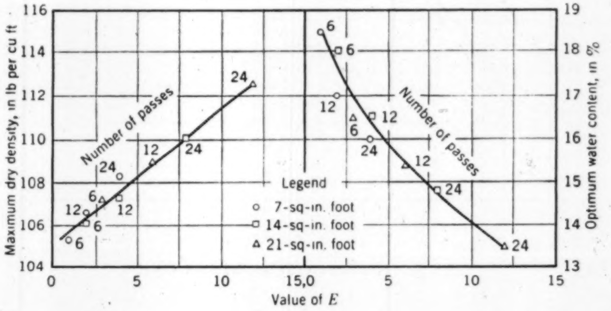


FIG. 7.—COMBINED EFFECT OF FOOT SIZE AND NUMBER OF PASSES ON MAXIMUM DENSITY AND OPTIMUM WATER CONTENT

The results of the CBR-tests, water-content tests, and density tests were plotted individually, as for the sheepsfoot-roller tests. Composite curves of the CBR versus water content and density versus water content were prepared. In the tests the effect of tire pressure was greater than that of repetitions, and the results were prepared accordingly. Fig. 9 shows the composite curves for 4 coverages. Separate curves are shown for the different tire pressures. An increase in tire pressure produces the same effect as an increase in compactive effort (number of blows) in the laboratory test. As the compactive effort is increased, the maximum density increases and the optimum water content decreases. The combined effect of tire pressure and coverages can be shown best by curves such as those shown in Fig. 10. Plots are shown for a

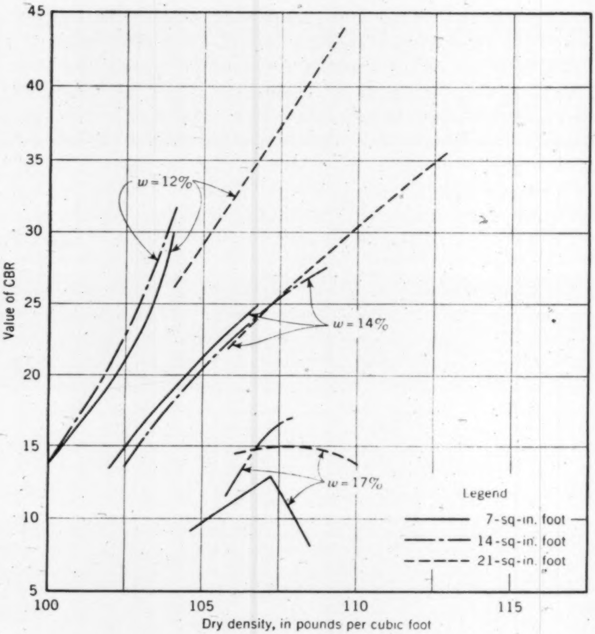


FIG. 8.—EFFECT OF FOOT SIZE ON CBR

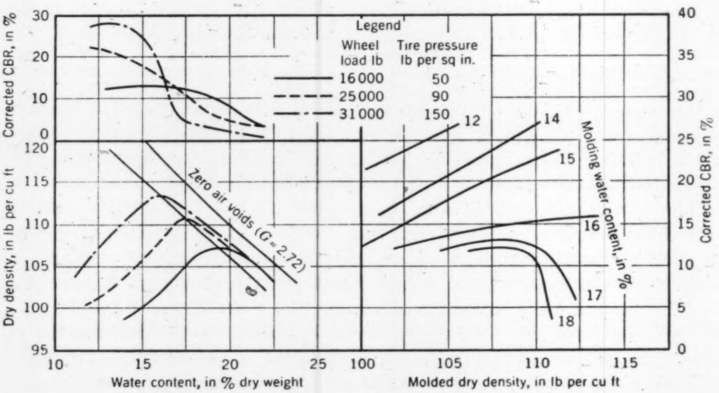


FIG. 9.—CBR, DENSITY, AND WATER-CONTENT DATA FOR LEAN CLAY TEST FILLS (4 COVERAGES—RUBBER-TIRED ROLLER)

relatively dry water content, one at approximately optimum, and one on the wet side of optimum. For water contents on the dry side of optimum and at optimum, there is a definite increase in density with an increase in tire pressure and coverages. On the wet side of optimum there is little increase in density with an increase in either tire pressure or coverages.

The strength measured by the CBR could not be related to the specific tire pressure but did show some variation, depending on whether a given condition of water content and density was obtained with a few repetitions of the higher tire pressure or with more repetitions of the lower tire pressure. Fig. 11 shows CBR-curves versus density for three selected water contents—one dry, one at approximately optimum, and one wet of optimum. It can be seen that at the higher water content (18%) the number of coverages had little effect. For the other two water contents the lesser coverages produced the highest CBR at low densities; at high densities the reverse was true. This feature was studied to a limited degree in the laboratory, but no conclusions were reached.

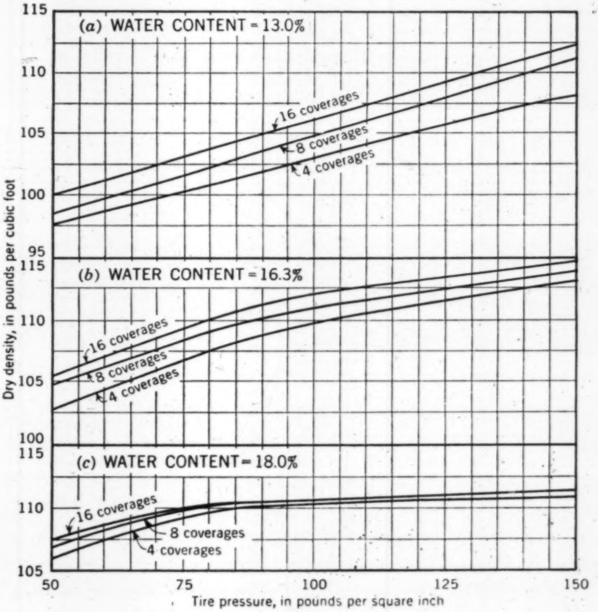


FIG. 10.—EFFECT OF TIRE PRESSURE AND COVERAGES ON DENSITY

Lift Thickness.—The effects of variations in lift thickness were studied by constructing test sections of the lean clay and by compacting with the rubber-tired roller with the tires inflated to 90 lb per sq in. and 150 lb per sq in. Lift thicknesses varied from approximately 6 in. to 24 in. after compaction.

The results of the CBR-tests, water-content tests, and density tests were plotted in individual charts, as illustrated for the sheepsfoot-roller tests. A large volume of data was collected, but only one outstanding feature is illustrated herein—the lower density found in the lower levels of the thicker lifts. Fig. 12 shows the density gradients that were measured in the sections compacted with the roller at a tire pressure of 90 lb per sq in. Data are shown for 6-in. lifts, 12-in. lifts, and 24-in. lifts at three water contents and were interpolated from plots of intermediate type similar to those described. Near the compaction surface the density was approximately the same regardless of the lift thickness. It decreased with depth below the compaction surface; the gradient was nearly the same in the 12-in. lift as in the 24-in. lift. It is apparent from these tests that lifts that are thicker than normal will result in a layered fill. Whether or not such conditions could be tolerated would depend on the specific purpose for which the fill was constructed. For fills that would be subjected to heavy wheel loads, as in airfield construction, the layered effect would not be tolerable because the repetitions of heavy wheel-load traffic of airplanes would produce compaction in the relatively uncompacted bottom zone of the thick lifts, thus resulting in objectionable settlement. The layered fill might be suitable in levee construction if the

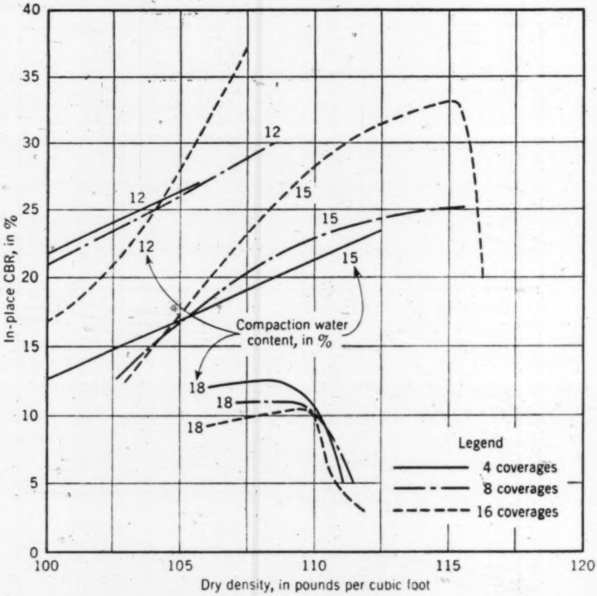


FIG. 11.—EFFECT OF COVERAGES ON CBR FOR LEAN CLAY TEST FILLS (RUBBER-TIRED ROLLERS—TIRE PRESSURES OF 50 LB PER SQ IN., 90 LB PER SQ IN., AND 150 LB PER SQ IN.)

strength and permeability of the bottom zones of the thick lifts met the design requirements.

The decrease in density with depth below the compaction surface is a result of a decreased compactive effort at the lower depths. Fig. 13 is a plot of density versus water content for conditions produced by the rubber-tired roller in a 24-in. lift. Curves are shown for different water content-density conditions at different zones in the lift and were interpolated from intermediate-type plots. The pattern is similar to that developed in the laboratory test by changing the compactive effort.

A study of the CBR-values measured in the different lift thicknesses indicated that the primary variables were water content and density, and no effects of lift thickness were discernible.

Operational Characteristics.—No attempt is made herein to consider the economics of compaction by the various procedures. However, during the tests numerous observations were made of the operation of the equipment, the most important of which are:

1. The 34,000-lb crawler-type tractor could easily pull the sheepsfoot roller at the light and intermediate loads but labored considerably at the heavy one. The same was essentially true for the rubber-tired roller at the three loads.

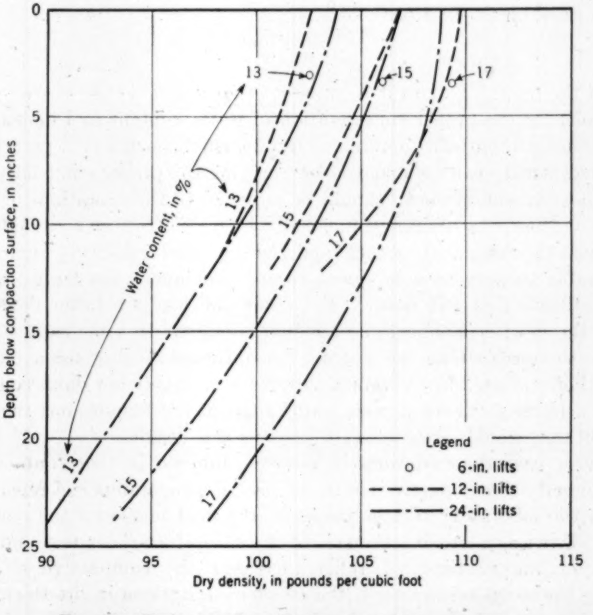


FIG. 12.—EFFECT OF LIFT THICKNESS ON DENSITY (TIRE PRESSURE = 90 LB PER SQ IN.)

2. The bond between lifts was better in the lanes compacted with the sheepsfoot roller than in those compacted with the rubber-tired roller. Both rollers produced laminations at water contents that were wet of optimum. Laminations produced by the rubber-tired roller were more pronounced than those produced by the sheepsfoot roller.

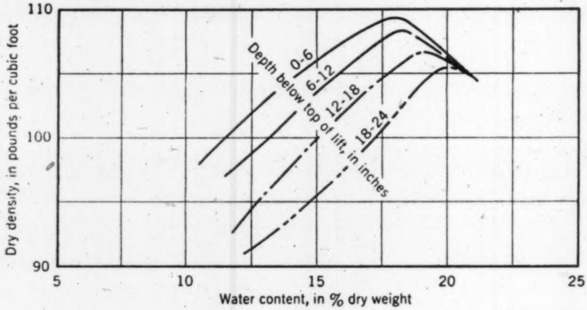


FIG. 13.—VARIATION OF DENSITY IN DIFFERENT ZONES OF A LIFT (TIRE PRESSURE = 90 LB PER SQ IN.)

3. Rolling was very difficult for lift thicknesses of more than 12 in. because of ruts formed on the first pass.

SUMMARY

From the preceding data it is apparent that a wide range of strengths can be obtained by the proper consideration of water content and density. The desired water content can be obtained by the usual construction practices, and the desired density can be achieved by applying the proper compactive effort.

The density and strength should be adjusted to the conditions of the job. There is no merit in obtaining unusually high densities and strength unless they benefit the design. In airfield and highway work relatively high densities are desirable to prevent subsequent compaction under the large number of load repetitions that will occur. For dams and levees a lower density may produce the most economical and adequate design. In some cases it may be desirable to construct on the wet side of optimum to produce a plastic fill. This method was used by the senior writer[5] for relatively low dams constructed on loess. Large settlements were anticipated in the foundation; therefore, a plastic fill that would settle without cracking was desired.

In every case the variations in density observed in these data could be related directly to compactive effort. Increasing the number of repetitions of blows in the laboratory test or passes in the field increased the compactive effort. Likewise, increasing the size of the sheepsfoot-roller feet and the tire pressure of the rubber-tired roller increased the compactive effort. By reversing the usual terminology, the effect of variations in lift thickness can be related to compactive effort; decreasing the lift thickness increased it. In all cases the increase in compactive effort produced an increase in maximum density and a decrease in optimum moisture. Generally, the strength measured by the CBR followed the variations in water content and density. The relationship between the CBR, water content, and density is rather complicated and can best be shown by curves, such as those in Fig. 2(*a*) (for conditions immediately after compaction) and in Fig. 2(*b*) (for conditions after soaking).

---

[5] "Utility of Loess as a Construction Material," by W. J. Turnbull, *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Eng., Rotterdam, Vol. V, 1948, pp. 97-103.

Conclusions

Based on the data presented in this paper, the following conclusions can be made:

1. Cohesive soils can be stabilized to yield a given strength over a fairly wide range of values by a proper consideration of the water content and density.
2. The relationship of strength, water content, and density is complex but follows the general patterns illustrated in Fig. 2.
3. Variations in load repetitions, foot size, tire pressure, and lift thickness produce variations in compactive effort.
4. An increase in compactive effort produces an increase in maximum density and a decrease in optimum water content.

Acknowledgments

The studies described herein were conducted under the direction of the Office of the Chief of Engineers as a part of the over-all development of design criteria for airfield pavements for the United States Air Force (Department of Defense). Gayle McFadden, M. ASCE, Thomas B. Pringle, M. ASCE, and F. B. Hennion of the Airfields Branch of the Office of the Chief of Engineers monitored the study, for which guidance was furnished by Arthur Casagrande, Donald W. Taylor, Kenneth B. Woods, Omer J. Porter, and Philip C. Rutledge, Members, ASCE, acting as consultants. The laboratory and field work was accomplished by the Flexible Pavement Laboratory of the Waterways Experiment Station.

DISCUSSION

Gerald A. Leonards,[6] A. M. ASCE.—Among the important factors that influence the strength and compressibility of compacted clay soils are (*a*) molding water content, (*b*) type and amount of compactive effort, (*c*) presence or absence of a source of water, (*d*) soil type, (*e*) confining pressure, and (*f*) nature of the applied stresses.

An excellent summary of the effects of factors (*a*), (*b*), and (*c*) on the stability of a compacted clay soil has been presented in the paper. However, conclusion 1 might be amplified by adding "and with due consideration for the effects of soil type, confining pressure, and nature of applied stresses." Failure to consider these factors may lead to erroneous conclusions in establishing compaction requirements to achieve a specified stability.

Fig. 14 shows compaction and soaked CBR-data for a limestone residual soil sampled near Bedford (Ind.). The data were obtained by J. Richard Bell,[7] J. M. ASCE, as part of a more comprehensive investigation under the direction of E. J. Yoder, A. M. ASCE. The clay has a liquid limit of 80 and a plasticity index of 45, and 55% (by weight) of the particles are smaller than 2 microns. All samples were compacted in a CBR-mold in five layers by a 10-lb hammer falling 18 in. and were subsequently soaked for four days with surcharges as indicated. Of particular significance is the important effect of confining pressure on the soaked CBR-values obtained. For a compactive effort equivalent to the test developed by the American Association of State Highway Officials (AASHO) as modified[8] by the Corps of Engineers, the peak value of soaked CBR falls from approximately 6.5 to approximately 1.5 when the confining pressure is reduced from 1.4 lb per sq in. to 0.7 lb per sq in. Comparable effects on the compressibility were previously measured by H. C. Woodsum,[9] and the differences in behavior, depending on whether water comes in contact with the compacted soil before or after the confining pressure is applied, have also been shown.[10]

Fig. 15 shows curves of soaked CBR versus dry density at equal molding water contents. These curves were drawn from the data presented in Fig. 14 and are similar to those shown in Fig. 2(*b*). The data for a silty clay presented by the authors show a continuing increase in the peak value of soaked CBR as the dry density increases; data cited by the writer for a highly plastic clan show an initial increase followed by a decrease in the peak value of soaked CBR. This difference in behavior is only one manifestation of the effect of soil type on the properties of compacted clay.

---

[6] Associate Prof. of Soil Mechanics, Purdue Univ., Lafayette, Ind.

[7] "Plastic Moisture Barriers for Highway Subgrade Protection," by J. Richard Bell, thesis presented to Purdue University, Lafayette, Ind., in 1956, in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering.

[8] "Airfield Pavement Design, Flexible Pavement," *Engineering Manual for Military Construction*, Pt. VII, Corps of Engrs., U. S. Dept. of the Army, July, 1951, Chapter 2, paragraph 3(*b*), appendix (*b*).

[9] "The Compressibility of Two Compacted Clays," by H. C. Woodsum, thesis presented to Purdue University, Lafayette, Ind., in 1951, in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering.

[10] Discussion by G. A. Leonards of "Effect of Compaction on Soil Properties," by S. D. Wilson, *Proceedings*, Conference on Soil Stabilization, Massachusetts Inst. of Technology, Cambridge, 1952, p. 159.

For a given confining pressure (Figs. 14 and 15) the largest value of the soaked CBR is obtained when the clay is compacted at optimum moisture content but at an intermediate degree of compaction. The writer has used the word "overcompaction" in referring to this phenomenon,[11] which is of considerable importance when compacting highly plastic or expansive clay subgrades for highway and airport pavements.

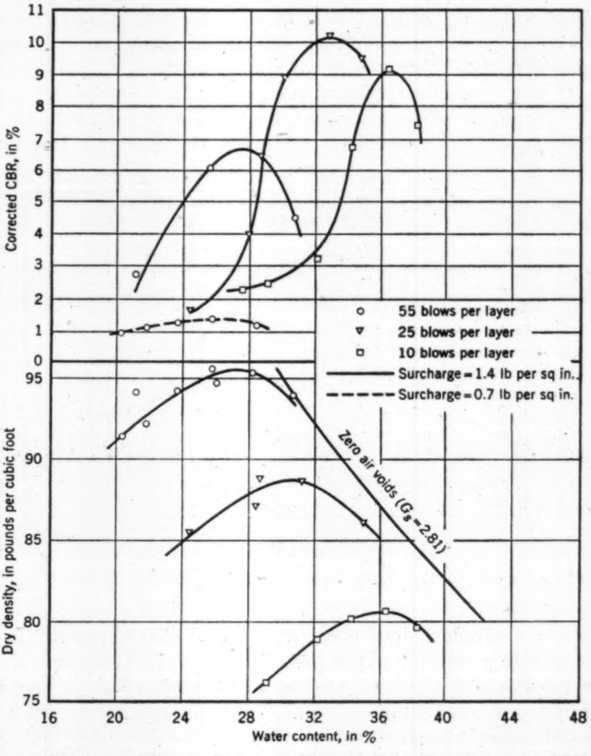


FIG. 14.—MOLDING WATER CONTENT VERSUS DRY DENSITY AND CBR

The strength-and-stress-deformation characteristics of compacted soils differ markedly, depending on whether the soil is subjected to a continuously increasing stress or to repetitive stress applications of varying magnitude.[12, 13]

---

[11] "Strength Characteristics of Compacted Clays," by Gerald A. Leonards, *Transactions*, ASCE, Vol. 120, 1955, p. 1451.

[12] "Effect on Vibratory and Slow Repetitional Forces on the Bearing Properties on Soils," by G. P. Tschebotarioff and G. N. McAlpin, *Technical Development Report No. 57*, Civil Aeronautics Administration, U. S. Dept. of Commerce, Washington, D. C., October, 1947.

[13] "Effects of Repeated Loading on the Strength and Deformation of Compacted Clay," by H. B. Seed, C. K. Chan, and C. L. Monismith, *Proceedings*, Highway Research Board, National Research Council, Washington, D. C., Vol. 34, 1955, pp. 541-558.

Under the action of repeated loads the stability of compacted clay is also dependent on the nature of the materials in contact with the clay.[14] These effects are not fully understood, and the CBR-test is not an adequate measure of stability under this type of loading condition.

For a given type and amount of compactive effort, a clay soil compacted on the wet side of the optimum moisture content will have a higher degree of saturation; be more compressible, more flexible, and more impervious; have a lower shear strength; and lose less strength on coming in contact with water than if it were to be compacted on the dry side of optimum. It is important to recognize that the differences in magnitude of these effects are primarily functions of soil type and confining pressure. For example, an earth dam subject to differential settlements may be compacted somewhat on the dry side of optimum if it is constructed of highly plastic clay. Imperviousness would not be a problem, and the higher initial shear strength (and lower construction pore pressures) would permit the use of steeper side slopes than would be possible if compaction on the wet side of optimum were specified. The confining pressures are generally sufficient to prevent a large loss in strength when the soil comes in contact with water, and the dam will be flexible enough to adjust to the differential settlements without cracking. Compaction on the dry side of optimum, using soils of low plasticity, may result in excessive leakage and dangerous cracks in the dam. Conversely, the low confining pressures afforded by highway pavements may cause the loss in strength, where moisture content is increased by swelling, to dominate the performance of the subgrade.

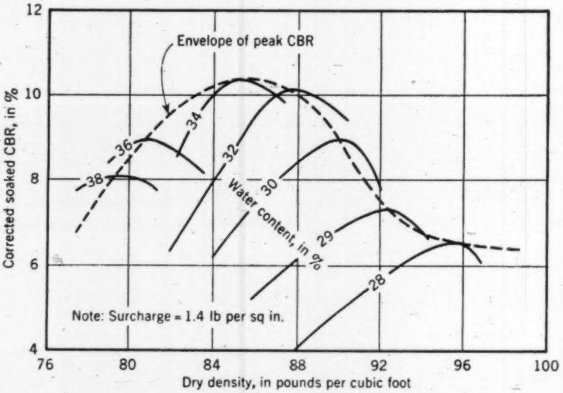


FIG. 15.—CBR VERSUS DRY DENSITY

In summary, the behavior of compacted clay soils can be wholly evaluated in terms of moisture content and density if—and only if—the following factors have been given proper consideration: (1) Soil type and interaction with adjacent soil types; (2) type and amount of compactive effort; (3) confining pressure; (4) presence or absence of a source of water; (5) nature of applied loads, sustained or repetitive; and (6) strength, stiffness, and permeability requirements of the structure.

---

[14] "Interactions of Selected Combinations of Subgrade and Base Course Subjected to Repetitive Loading," by J. A. Havers, thesis presented to Purdue University, Lafayette, Ind., in 1956, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Unfortunately, the quantitative effects of some of these factors are imperfectly known, and a completely rational basis for design has not been formulated. Nevertheless, failure to consider all the factors involved may lead either to inadequate compaction requirements or to uneconomical design.

John A. Focht, Jr.,[15] A. M. ASCE.—Of particular interest are the curves in Fig. 2, showing that the shear strength of compacted cohesive soil (measured by CBR-tests), both in the unsoaked and soaked condition, may decrease with increasing density. Fig. 2 indicates conclusively the necessity for proper moisture control during compaction so that attainment of the specified density will produce the desired stability and strength. At a low compaction moisture content, the specified density may be attained easily by increasing the effort, thus producing a strong fill but one which may become saturated and lose its high initial strength.

The curves for a molding water content of 12% in Fig. 3 establish the fact that very strong, dense fill may lose most of its strength when saturated. Unless moisture control is required, density specifications for many types of fill are incomplete and will not necessarily result in the desired characteristics of the fill.

For too many engineers and contractors the terms, "100% Proctor" and "100% Modified," describe a fill possessing "ideal" qualities, particularly structural fill. The following accounts of two fills that were compacted to the so-called "100% density," but were not stable, illustrate the difficulties that may result from reliance on only a density specification.

In one case an 18-in.-thick fill beneath a concrete slab floor for an office building was constructed of high liquid limit clay. The contractor was requested to compact the clay heavily, even though it was dry, in an effort to achieve high density. After the building was in service, the fill increased in moisture content and heaved the floor slab up approximately 3 in. The slab was severely cracked along utility conduits, thus disrupting the use of the building.

In another instance the backfill of a deep footing excavation was controlled to produce 100% of the laboratory maximum density, but the moisture content was greater than optimum. The ring wall foundation for a water tank, 40 ft in diameter and 80 ft high, overlapped the backfilled excavation approximately 12 ft. When the tank was first filled with water, plastic failure of the fill occurred, permitting a downward movement of 5 in. on one side of the ring wall and resulting in significant tilt of the tank. The backfill, even though at 100% laboratory maximum density, did not have adequate strength to support the tank.

---

[15] Chf. Design Engr., McClelland Engrs., Inc., Houston, Tex.

In these two instances, erroneous concepts of stabilization by compaction produced unsatisfactory fills. Consideration of the general relationship of strength, water content, and density, as presented by the authors, would have helped to avoid the difficulties that developed. These situations are cited to emphasize further the authors' statement, under the heading, "Summary," that "the density and strength should be adjusted to the conditions of the job."

It is hoped that the paper will cause engineers to consider why they are compacting their fill, what they want to achieve, and what should be done to obtain the desired stability, rather than merely requiring the contractor to produce 95% or 100% laboratory density without any consideration of moisture content.

EDMUND J. ZEGARRA,[16] A. M. ASCE.—Stabilization of soils by compaction has been, and still is, an empirical subject and, in many cases, an amateurish one. For a long time there has been a pressing need for a firm criterion for the degree and method of compaction required to produce a predetermined strength. Few serious, detailed studies are available in the professional literature. The subject has been considerably illuminated by the authors. For their research they selected the lean clay native to Vicksburg; this clay is one of the soils whose properties are best known. From the findings of the research they have chosen highlights and significant facts—they refer to the comparison of results achieved by conventional laboratory compaction techniques with the results obtained by field methods and equipment.

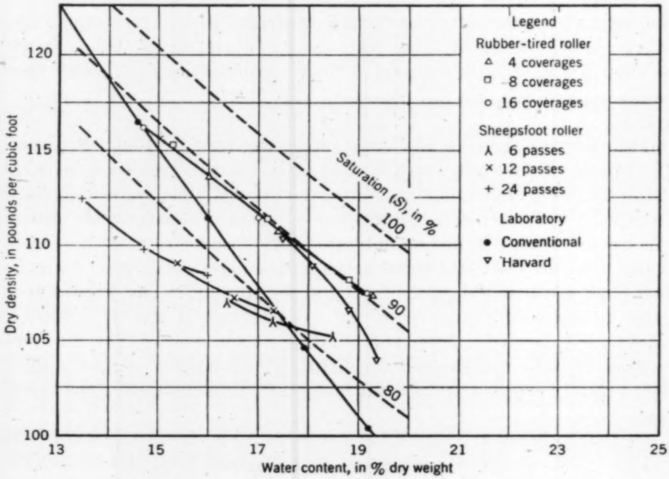


FIG. 16.—WATER CONTENT VERSUS DRY DENSITY

---

[16] Soils and Foundation Engr., M. W. Kellog Co., New York, N. Y.

Some practical conclusions will be drawn, deduced by analyzing certain results given in the paper and additional data furnished by the authors. These data consist of the test results for field compaction with 8 coverages and 16 coverages of rubber-tired rollers, and the effect of soaking on unit weight and molding moisture content for the laboratory specimens.

In order to visualize the laboratory tests and the field results in a different relationship, the data have been rearranged and summarized in Figs. 16 and 17. Fig. 16 shows the envelope of the peak values of the individual curves of molding water content versus dry density for field compaction, using sheepsfoot rollers and rubber-tired rollers; these values are also shown for laboratory methods designated as conventional (blows delivered by a hammer) and as "Harvard" (apparatus developed[17] by Stanley D. Wilson, A. M. ASCE). All these curves may be directly compared with the theoretical values for 100%, 90%, and 80% saturation. Fig. 17 shows similar envelopes of CBR-values for peak dry density and optimum water content, for laboratory results, and for field compaction with rubber-tired rollers. The term, compaction, as used herein will refer to peak values of dry density and optimum water content. The following trends appear from Fig. 16:

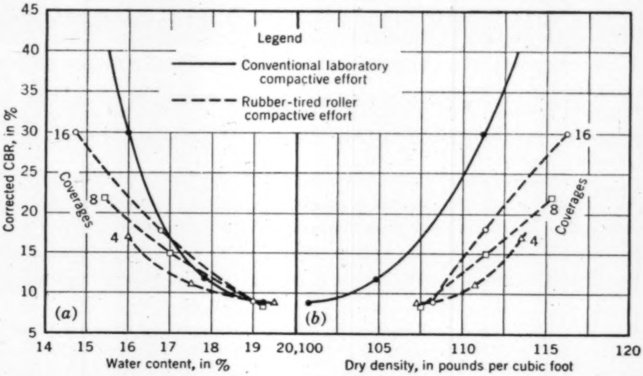


FIG. 17.—WATER CONTENT AND DRY DENSITY VERSUS CORRECTED CBR

1. Field compaction by rubber-tired rollers appears capable of achieving compaction values that correspond at least to those at 90% saturation.

2. Light and medium rubber-tired rollers are ineffective to increase compaction regardless of the number of coverages. Heavy rollers do increase compaction moderately with increasing coverages.

3. Field compaction with sheepsfoot rollers of uniform pressure have a consistent trend toward increased compaction with respect to the size of foot and number of passes. However, the envelope curve has no relationship to

---

[17] "Comparative Investigation of a Miniature Compaction Test With Field Compaction," by S. D. Wilson, paper presented at the annual meeting of the ASCE, New York, N. Y., January, 1950.

the saturation curve, the conventional laboratory curve, or the rubber-tired compaction curve. The low dry unit weights obtained by this equipment indicate that the resulting mass of soil would be compressible and, therefore, would cause larger settlements.

4. Conventional laboratory compaction methods indicate no relationship to field compaction curves or to saturation curves. By coincidence, the highest compaction achieved by conventional laboratory methods (Modified AASHO) has the same numerical value as field compaction by the heaviest rubber-tired roller at maximum coverages.

5. From Fig. 17(*a*) at the high water contents (19%), a constant CBR is obtained regardless of the compactive effort. At low water contents (approximately 15%) the CBR increases with pressure and number of coverages. There is no relationship with the CBR of laboratory compaction.

6. Rubber-tired roller compaction yields a family of curves with respect to tire pressure and number of coverages for medium and heavy rollers.

7. From Fig. 17(*b*) there is no relationship between laboratory and field compaction with respect to the CBR.

8. Field compaction with light, rubber-tired rollers produces a constant CBR regardless of coverages.

9. There is a reasonable correlation, resulting in families of curves, showing correspondence between the CBR, pressure, and coverages for medium and heavy rubber-tired rollers.

The diversity of results shown by these comparisons has a common explanation, which has been suggested as the underlying fact—the soil structure resulting from the different compactive efforts affects directly, in an unknown ratio, the density, water content, and shearing resistance as measured by the CBR.

The CBR-values for sheepsfoot-roller compaction compared with maximum dry density and with optimum water content have not been plotted. In general, the CBR versus dry density points show a greater scattering for sheepsfoot rollers than for rubber-tired rollers, but there is a good agreement in the CBR versus optimum water content with the dynamic laboratory compaction method. This agreement is somewhat amazing in view of the low dry densities obtained with sheepsfoot rollers, but it probably has a relationship to the type of structure obtained.

Conventional laboratory effort is dynamic; Harvard laboratory effort is nearly static; sheepsfoot-roller effort is static and kneading; and rubber-tired-roller effort is static.

Each type of effort evidently achieves compaction by a unique arrangement of particles in the soil mass, which has an undetermined effect on the shearing resistance. Furthermore, it is believed that under the influence of saturation the original particle arrangement is disturbed and eventually becomes stable. For example, it may be shown that the laboratory test specimens after soaking reach a state of compaction nearly parallel to the 95%-saturation line, but only at considerably high water contents, ranging from 16% to 21% for the 55-blow effort and from 24% to 28% for the 6-blow effort. Whether compaction in the field shows a similar trend toward the 100%-saturation line is not known, but it probably would.

Unfortunately, the data on the CBR are not available for the Harvard method of compaction. The compaction curve shown in Fig. 16 was compiled by Wilson in a paper presented at a Society meeting in January, 1950. Because the Harvard method produces compaction values that are in close agreement with field compaction by rubber-tired rollers and because the effort is static in both, the resulting structure and the CBR-values would probably be in agreement.

Since the early days (1940–1941), when the Corps of Engineers adopted design by the CBR method, it has been recognized that no correlation existed between field and laboratory compaction results. As a matter of historical record, the AASHO method replaced the original (1929) California compaction method (static load of 2,000 lb per sq in.) for expediency in field operations. Perhaps this fact has been obscured in the intervening years, but it has been vividly illustrated by Turnbull and Foster.

With this background, the following conclusions may be made:

a. Rubber-tired rollers having high tire pressures are capable of producing a range of high compaction values with varying numbers of coverages.

b. Laboratory compaction procedures (Harvard and Modified AASHO) yielding compaction values near to or equal to 90% saturation are most likely to be duplicated by field compaction using rubber-tired rollers.

c. Sheepsfoot rollers used for compacting clays are unable to effect proper compaction, and soils stabilized in this fashion are likely to be more compressible.

In addition to these conclusions, two facts appear to show a more realistic method for the design of fills: First, laboratory compaction methods (Modified AASHO and Harvard) that yield a maximum dry density equal to that at 90% saturation or better should be specified for field compaction with rubber-tired rollers (when a percentage of maximum compaction is specified, the value should be selected from the 90%-saturation curve); and second, rubber-tired rollers of heavy and medium weights should be specified for field compaction in preference to light-weight rollers or sheepsfoot rollers.

The authors have reported clearly and concisely the results of a well-planned and carefully conducted research. Their efforts shouldbe a stimulant to those who are concerned with this subject both in the field and laboratory. Data, especially those from the field, are notoriously scarce, and yet this is precisely where significant observations can be made on different soils. When sufficient evidence is available to generalize the foregoing conclusions with respect to one type of soil, the subject of compaction will have become clarified.

Kazuo B. Hirashima,[18] M. ASCE.—Interesting information on soil compaction has been presented. Among other things, it has been emphasized that a high degree of compaction does not necessarily lead to high strength. The moisture content at which the soil was compacted was important. In fact, the data presented showed that, on the wet side of optimum, a high compactive effort resulted in a lowering of strength. This was interesting because for many years the writer worked on projects calling for compaction of soils with very high, natural moisture contents.

---

[18] Cons. Engr., Honolulu, Hawaii.

The optimum value of the water content for the curves in Fig. 1 (*a*) ranges from approximately 15% to 20%, depending on the applied compactive effort. The optimum moisture of the soils encountered in highway work in Hawaii is usually much higher—almost twice the foregoing values. This high optimum moisture makes the use of the Modified-AASHO method of doubtful value.

Several modern highway projects have been completed, involving soils having natural moisture contents of from two to three times the optimum value. For instance, on one recently completed project, the liquid limit of the soils was in the range of from 60 to 70, whereas the natural moisture content ranged at times from between 70% and 100%. Such soils were very stable in cut but presented a difficult problem in fill construction. Drying out the soil was not economically feasible. The authors have stated, under the heading, "Summary," that "in some cases it may be desirable to construct on the wet side of optimum to produce a plastic fill." The economical solution to the problem was, indeed, the construction of such a fill. Experiments showed that a TD-24 tractor was best suited for the work. The fills were all constructed by end-dumping in one lift of from 15 ft to 20 ft, and the applied compactive effort consisted simply of running the tractor (with a bulldozer blade) over the top of the lift to level it off. Many times it was not possible to do even as much as this. Therefore, the compactive effort was very slight, in accordance with the principle illustrated in Fig. 1, which shows that, as the optimum moisture increases, the compactive effort must be lessened. If the compactive effort is not lessened, it could lead to instability.

Laboratory consolidation tests can be performed on remolded samples. It is possible to show in this manner that an end-dumped fill, especially one that is dumped in one 15-ft lift, will settle many inches. Actually the highway project mentioned previously, which has been completed and opened to traffic for approximately 1½ yr, showed very little settlement subsequent to the final paving of the surface. Levels were taken at numerous points, both longitudinally and transversely, shortly after paving and approximately one year thereafter. The maximum difference in elevation noted at any place was approximately 0.06 ft, or approximately ¾ in. If any large settlement took place, it did so during the construction period, and the prolonged settlement predicted by laboratory tests on remolded samples did not materialize.

The compactive effort must be made to suit the nature or characteristics of the soil, and one should not arbitrarily insist on applying the Modified-AASHO compaction procedure or Standard-AASHO compaction procedure to all soils (even for highway work). Again, as the authors have stated under the heading, "Summary," "there is no merit in obtaining unusually high densities and strength unless they benefit the design." To this one might add, "unless the same can be attained economically."

WILLARD J. TURNBULL,[19] M. ASCE, AND CHARLES R. FOSTER,[20] A. M. ASCE.—It was not intended to present the subject as solved; instead the writers meant to show that the effect of compaction on a soil is complicated, to present one method of study, and to stimulate additional thought and study.

Mr. Leonards correctly calls attention to the fact that the patterns of the CBR, water content, and density shown in the paper are typical of soils that show little or no swell. Soils that swell appreciably in the presence of water have different patterns because of that swelling. Mr. Leonards mentions that the peak CBR occurs at an intermediate degree of compaction. It is suggested that in working with swelling soils first consideration be given to the swelling characteristics. The procedures of compacting at a range of moisture contents and compactive efforts can be followed and each sample tested for swell during the soaking procedure. A study of expansion, molding water content, and molding density should be made to determine the molding water content and density that will result in a tolerable amount of expansion. The CBR-value for these conditions must be accepted even if it does not represent the peak CBR that can be achieved with the specific soil. The design density in these cases may be lower than that usually encountered for normal subgrades; therefore, in airfield work or in highways involving heavy traffic, it may be necessary to bury the layer deeper than indicated by the CBR-design in order to prevent compaction by traffic.

Mr. Leonards mentions that the CBR-test is not an adequate measure of the stability of a clay under repeated loads or when the adjacent materials affect the clay. The CBR-penetration test can measure only the stability that exists at the time the test is made. By itself, the test cannot predict the strength that the soil will have when its characteristics are changed by repeated loads or by the effect of an adjacent layer (by desiccation, ion exchange, or other processes). To predict future strength, the test must be coupled with other procedures that adjust the sample to the future condition. In this respect, it is no different from any other because no test can do more than measure the strength that exists at the time it is made.

As a side issue, the writers do not believe that it is good economy to construct a highly plastic clay for an earth dam on the dry side of optimum in order to achieve a steeper side slope. The capacity to accept differential movements without cracking outweighs many times the advantage of steeper slopes.

The writers can only agree with and endorse the comments made by Mr. Focht and are certain that the two cases he cites could be expanded from his files and from other files. Moisture control is certainly as important as density control. As indicated by Mr. Focht, the engineer should determine why compaction is desired, what he wants to achieve by it, and how he intends to accomplish it.

The comparison made by Mr. Zegarra of the percentage of saturation obtained by the various compaction methods is interesting. The writers generally agree with his examination of the trends but have the following comments:

---

[19] Chief, Soils Div., Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss.

[20] Chief, Flexible Pavement Branch, Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss.

a. With reference to trend 2—the rubber-tired roller tests at Vicksburg were made with the same roller loaded to three different weights. These gross weights were 64,000 lb, 100,000 lb, and 124,000 lb, and the designations, "light, medium, and heavy," should refer to these loadings rather than to conventional equipment.

b. With reference to trend 3—the maximum unit weights of soil achieved with the sheepsfoot rollers, at the various numbers of coverages, ranged from 90% of Modified AASHO maximum (6 passes with a 7-sq-in. foot) to 97% of Modified AASHO maximum (24 passes with a 21-sq-in. foot). These are relatively high densities for cohesive soils and are low only by comparison with the values obtained with the rubber-tired rollers. The writers cannot agree that the soil would be compressible and cause large settlements.

In general, the writers agree with Mr. Zegarra's first and second conclusions, but they do not agree with his third conclusion because it is believed that sheepsfoot rollers may be entirely satisfactory for compacting lean clay soils for dams, levees, and fills.

Mr. Hirashima draws attention to problems encountered with an unusual soil in Hawaii. The writers have had no experience in such cases but wish to compliment him for using common sense rather than blindly following a practice established for entirely different types of soils. However, it is felt that the soil in the fill, end-dumped in one 15-ft lift, probably went through a hardening or cementing process after being placed in the fill, thus accounting for such a small settlement. The clause added by Mr. Hirashima, "unless the same can be attained economically," is especially apt. It was felt that the phrase, "benefit the design," was synonymous with economics because the production of the most economical design that will get the job done (within acceptable maintenance limits) is the engineers' task. However, it is better to be specific in this case, and the writers welcome Mr. Hirashima's additional phrase.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2908

TRENDS IN HYDRAULIC GATE DESIGN

By Dow A. Buzzell,[1] M. ASCE

With Discussion by Messrs. Abraham Streiff; George R. Latham; William G. H. Holt; Kandaswami S. Chetty; William Gustave Weber; and Dow A. Buzzell

Synopsis

During the past 8 yr the Corps of Engineers (United States Department of the Army), anticipating its requirements for hydraulic gates of exceptional size and exacting serviceability, has developed and tested several types of sluice, crest, and intake gates, some of which are novel in design or application. The most successful gates from a practical standpoint are an improved, hydraulically operated slide gate that has operated under 285 ft of head and an eccentric-trunnion tainter sluice gate designed for 200 ft of head.

All the gates described herein—with the exception of the skimming tainter gate—have proved in service to be satisfactory from an engineering standpoint. They represent important contributions to engineering knowledge and practice.

Introduction

Striking advances have been made in hydraulic gate design by the Corps of Engineers (United States Department of the Army). These advances resulted from real demands arising from project conditions and were made only after much experimentation and many model tests. At times, however, it was requisite that a bold decision regarding the use of some of these devices be made. It is of interest to note that as late as 1946 it was firmly believed that no greater depth of water than 25 ft could be discharged over an ogee spillway without shaking the structure to pieces, and that sluice gates could not be operated under heads exceeding 100 ft for an extended period of time without being destroyed by cavitation and erosion. Because both these values have been more than doubled (as of 1957) without any harmful consequences, quite a

---

Note.—Published, essentially as printed here, in October, 1954, as *Proceedings-Separate No. 517*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Chf. Design Engr., Tippetts, Abbett, McCarthy & Stratton, Cali, Colombia, South America.

few other so-called "criteria" regarding the size and use of crest, intake, and sluice gates have also been discarded. At the present time (1957) there are no arbitrary limitations on the size or operating head of any type of gate except possibly on the hydraulic slide gate.

TRENDS IN GATE DESIGN

In 1946–1947 the Corps conducted a survey of all known and accessible hydraulic gates including those built and operated by other agencies and by private corporations. The purpose of this survey was to determine in so far as possible the operating experience of various types of gates and, also, the types best suited to specific conditions. At the time many diverse opinions and partly substantiated criteria were current among engineers in this field. The survey was launched with the expectancy that all bothersome questions would be answered and that henceforth the Corps would make no errors either in choice of gate type or in the detailed design of the type chosen. The Corps was particularly interested in high-head gates that when partly open could operate in closed conduits.

Unfortunately, the results of this survey were, at the time, rather inconclusive. Almost every type of gate has been used at some time to control water under almost every possible condition. Butterfly gates, usually used as shutoff valves for penstocks, had proved successful as free-discharge sluice gates under high head at Ross Dam (Skagit River), and many roller-train gates of the Broome or similar types were serving with varying success in conduits as high-head sluice gates. The reports on slide gates showed the frequent occurrence of erosion, cavitation, and vibration of the gates. The tainter gate had been suggested because of its desirable hydraulic properties for high-head discharge, but at the time no such use of the gate had been attempted in the United States. The Stoney gate, the cylinder gate, and many varieties of the needle valve had already resulted in excessive maintenance costs, which were verified during the survey, and none of these gates have been used in a Corps project since 1935.

In 1946 the Corps planned several projects in which high-head gates of unusual size or application were required. Spurred by necessity and dissatisfied with information obtained from the survey, the Corps undertook a series of full-scale tests on slide gates and tainter gates. The first of these tests was performed on a 4-ft-by-6-ft sluice gate at Norfork Dam (North Fork River). Eventually the Corps developed a sluice slide gate that could be used in small sizes at any gate opening under heads as high as 200 ft and a sluice tainter gate suitable for the same usage, but practically unlimited in size. Essentially the improvements accomplished by tests of the Norfork slide gate were (1) the reshaping of the contour of the gate bottom and (2) the beveling of the downstream faces of the gate guides. The tainter-gate tests at Norfork were a demonstration rather than a new development. However, the tests did prove that a satisfactory closure could be made either with expansible rubber seals or by using solid rubber seals and traversing the gate with an eccentric trunnion.

The Norfork Slide Gate.—The slide gate, with a cast-steel body operated by a hydraulic cylinder, is the most versatile of all high-head gates up to a size of approximately 6 ft by 10 ft. Several hundred of them are in use (1957) and, since the development of the Norfork type, have proved uniformly satisfactory for continuous operation at any opening under heads up to at least 200 ft. One gate has actually been successfully operated at a 285-ft head. Because the standard slide gate when installed costs between $400 per sq ft and $500 per sq ft, it is too expensive to be used in every case.

Practical limitations on the size of cylinder and on the intensity of oil pressure make 6 ft by 10 ft the maximum size for this gate. Thus, this type of gate is unsuitable for larger conduits. Slide-gate bodies are made of cast steel; attempts to make them of welded steel plates have not proved economically practicable (as of 1957).

The Garrison Tainter Gate.—The Garrison tainter gate (Garrison Dam, Missouri River) is 18 ft wide and 24.5 ft high with a head of 169 ft; the total water thrust is 4,700,000 lb, but only one 7.5-hp motor is required for operation. This design was tested as a model at Vicksburg, Miss., and further tested as a 4-ft-by-6-ft model under a 182-ft head at Norfork Dam. The three Garrison gates have been installed and have been tested under 180 ft of head; operation is entirely satisfactory.

At Garrison Dam it was desired to control the discharge of a 22-ft-diameter conduit with a single gate and to regulate the flow to any degree of fineness without risking cavitation or erosion. The tainter gates have accomplished this at a cost comparable to that of an equivalent area of small standard slide gates.

Although there is every reason to believe that the tainter gate will be entirely satisfactory for its intended task of discharging up to 25,000 cu ft per sec of water without vibration of the gate or damage to the gate or conduit, it is felt that the hydraulically actuated rubber seals mounted on the gate frame are unnecessarily elaborate and costly. The Corps has developed, tested, and built an improved and much cheaper version of the high-head tainter valve for Lookout Point Dam (Middle Fork, Willamette River).

The Lookout Point Tainter Sluice.—One of the series of tests at Norfork had shown that it was quite practicable to move a tainter gate transversely against a solid rubber sealing frame by the use of an eccentric-trunnion mounting. There were strong indications that the water passages below the gate frame should be abruptly enlarged not only to provide a mounting for the seal but also to provide ample aeration for the water jet. In this fashion almost free-discharge conditions would be obtained. This type of gate mounting is much simpler than the Garrison design, both structurally and hydraulically, and its initial cost proved to be much lower. At Lookout Point it was desired to use a sluice gate for a head of 200 ft that was somewhat larger than the largest practicable slide gate. The gate finally developed is 6 ft 9 in. by 12 ft and is mounted on a power-operated eccentric trunnion, which enables the operator to withdraw the gate to clear the frame-mounted solid rubber seals before raising or lowering. This clearance eliminates rubbing friction and makes it possible to operate the gate with a small, although exceedingly long, hydraulic cylinder. The hydraulic cylinder is more suitable than a mechanical device because it tends to "dampen out" small oscillations of the gate itself. The gap at the seal causes a tremendous shower of spray to form in the chamber below the gate during movements, but this has proved to be more spectacular than harmful.

The seals of this gate consist of a uniform section of hard rubber securely clamped to all four sides of the gate mounting. Rotation of the eccentric trunnion through the beveled gear train moves the gate backward or forward through a total movement of ½ in. The eccentric member of the trunnion is mounted between inner roller bearings and outer roller bearings that take the entire thrust of approximately 1,000,000 lb. The traversing mechanism is driven by a 15-hp motor. The gate is opened and closed by a 7⅝-in.-diameter oil cylinder, which acts on a crosshead moving in fixed guides. The oil pump motor is also 15 hp.

In operation, the eccentric member withdraws the gate to clear the seals, the required vertical movement is made, and, then, the gate is moved forward to clamp firmly against the seals. With the gate securely clamped, vibration cannot occur and, in fact, there is no perceptible vibration under any operating condition. In the unsealed position, water jets freely through the seal gap and fills the gate chamber with spray, but this condition lasts only a few minutes and the operator is protected by a concrete and steel enclosure.

One of the attractive features of this gate is its low cost—approximately $90 per sq ft of gate opening. Another distinct advantage lies in the possibility of making this gate much larger than the largest practicable slide gate. Because no difficulty has been encountered in designing tainter gates with total thrusts of 4,000,000 lb using plain trunnions instead of roller trunnions, by using a somewhat heavier eccentric-operating mechanism one might well quadruple the area of this gate before encountering mechanical difficulties. There is no reason why gates of this type should not prove to be feasible up to 300 sq ft in area and for 300 ft or more of head.

Detroit Dam Fixed-Roller Vertical-Lift Gate.—At about the same time that the Lookout Point tainter gate was undergoing development, the 400-ft-high Detroit Dam (North Santiam River) was also being planned. Ordinarily the Corps would use three tiers of standard slide gates in a dam of this height so that operation of the gates would not have to be undertaken at heads exceeding 100 ft. Flood routing and stream regulation at Detroit Dam demanded possible sluice operation at any reservoir level. The results of certain tests, including those from Norfork Dam, indicated that, if mechanical difficulties could be overcome, the slide gate could be designed to operate at a 200-ft head and, thus, eliminate one entire tier of gates. (The gate dimensions are the Corps' standard 5 ft 8 in. by 10 ft.)

Because sliding friction is the limiting factor in slide-gate design, fixed rollers were used and this, in turn, dictated the use of a welded-plate gate body. The seals are a standard J solid rubber section for top and sides. This gate has been entirely successful in operating at any gate opening up to 285 ft. The necessity of using a welded-plate gate body has, however, made this a rather costly gate, and it is certain that the design will not be repeated. (The discharge capacity is 7,000 cu ft per sec.) The eccentric-trunnion tainter gate is far more suitable and economical for this type of location.

McNary Intake Gates.—At McNary Dam (Columbia River) a novel solution was attempted for the problem of a large penstock intake gate. In order to supply 15,000 cu ft per sec of water to the 110,000-hp Kaplan wheels, a gate opening 20 ft wide and 51 ft high was required. This presented no structural problem because gates have been built to greater widths for the 107-ft head involved. The solution offered an opportunity to facilitate operation by providing, at low cost, means for quickly closing any gate without using the traveling gantry crane.

The gates at McNary Dam are standard roller-train gates with rubber seals. Each gate body consists of five units joined together so that the gate body as a whole is semiflexible and, thus, compensates for inequalities of track and permits even bearing under load. Each gate is suspended from a fixed lifting beam through two hydraulic cylinders with safety dogs to prevent involuntary gate drop. The gates hang just high enough to clear the water passage, and any gate can be quickly lowered by the operation of two oil valves. When it is necessary to work on a gate, the gantry crane is used to withdraw the entire gate and beam assembly. Thus, individual gate operation is secured at a fraction of the cost of individual hoists. Ordinarily two large cranes would have been used for a twenty-unit powerhouse of this type. As designed, one crane can be eliminated, and this alone more than pays for the extra cost of the beams and cylinders.

Modern Tainter Gate.—The tainter gate has undergone a minor revolution in design and usage. Although the well-designed tainter crest gate used (1930's) for Caddoa Dam (Arkansas River) was of low-alloy steel having a limiting stress of 27,000 lb per sq in., the gate was overly heavy and, because of the lavish use of curved plates and surfaces, it was rather costly. The anchorages were eye-bars securing a 20-in.-diameter trunnion pin 14 ft long, which was subjected to heavy bending stresses. The tainter gate that has been evolved during the past ten years has a gate body made of carbon steel, and with inclined end frames it is lighter and no more complex than an average bridge structure. The anchorage is a continuous, doubly overhung box girder to which the anchor girders are welded. A typical gate, 40 ft wide by 38.5 ft high, costs approximately $37 per sq ft including hoists. It is operated by a 7.5-hp motor, which lifts both ends of the gate simultaneously by means of a high-speed countershaft. The Corps has built larger tainter gates, up to 40 ft by 50 ft, and several hundred more are planned (as of 1957). The tainter crest gates are well standardized in a simple design with a minimum number of members. Full advantage is taken of continuity. The weight is low and bid prices are usually little higher than those for average bridge steel. The Corps is now investigating the use of post-tensioning for gate anchorages. By simply anchoring the usual trunnion girder to the concrete pier by long, high-strength steel bars or wire cables, it should be possible to eliminate 75% of the weight and 60% of the cost of the heavy anchorages.

Large Vertical-Lift Roller-Train Gates.—It has been found from extensive inquiry that a single gate of the vertical-lift type is more economical than two or more, at least for sizes up to 18 ft wide by 25 ft high under high heads. The Corps invariably uses one gate for each power intake. However, for outlet sluices it is desirable to have at least three gates so that some discharge capacity will be available even though one gate is inoperable. Where there are three or more outlet tunnels or sluices, advantage can be taken of the low cost of a single-gate leaf. For example, the 11-ft-by-23-ft roller-train sluice gate at Fort Randall Dam (Missouri River) is a welded body gate with the improved bottom shape of the type of the Norfork Dam. Hitherto these large high-head gates have been built with nearly square bottoms because of structural considerations. The Fort Randall gate was designed with a 45°-sloped bottom member of cast steel in order to minimize downpull resulting from low-pressure areas that occur at this point when the gate is partly open. This innovation has proved to be practicable from a structural point of view, and the gate operates satisfactorily at all normal heads. During diversion, when the gate was operated at abnormally low heads, some vibration occurred because the upstream pressure was insufficient to secure the gate against the guides. (The seals are of solid rubber.)

New Skimming Tainter Gate.—In connection with studies of the proposed navigation locks for the Ohio River, it was found that several spillway gates of a span of approximately 100 ft by a height of 36 ft would be required. Because considerable time was available for planning, some extensive cost comparisons were made of single vertical-lift and double vertical-lift gates and of rolling gates of the type used on the Mississippi River. The double vertical-lift gate had been favored, but studies were undertaken of a double-leaf tainter gate.

The problem was to provide a skimmer gate that could be lifted to a great height to clear floods. Skimming was requisite in order to pass ice and drift and also to conserve water for a future power plant. By placing a fixed cable or chain hoist high on the upstream end of the gate pier and by running the hoist cable over a set of fixed rollers mounted on a circular segment, it was possible to maintain a fixed-position hoist line. After passing over this segmental roller bed, the cable extended to an idler mounted on a trolley, which also moved on a segmental track. The hoist line passed over the idler and then dropped vertically to the tainter gate. The final result of this indirect rigging was that the hoist line was always vertical to the gate, and the required pull on the line was almost constant, varying only with the water-pressure friction. Minimized hoist loads for these large gates are highly desirable. The skimming action of this gate is secured by overrunning the hoist in the gate-closed position, the weight of the separate skimming leaf lowering it the desired amount.

CONCLUSIONS

On the basis of the information presented herein, it is believed that the utility of both the hydraulic slide gate and the eccentric tainter gate has been extended by the research and development work of the Corps. Slide gates of moderate size can be designed to serve in a partly open position for discharge under high heads without adverse hydraulic or mechanical effects. Tainter gates up to the larger dimensions can be used economically as high-head sluice gates. The operation of large power intake gates has been simplified by the use of auxiliary hydraulic cylinders. Several useful innovations in gate design and operation have been developed and have proved practical.

DISCUSSION

Abraham Streiff,[2] M. ASCE.—There is probably no field of design that has attracted the inventive engineer more than that of the hydraulic gate. This field is as old as civilization itself; Herodotus (450 B.C.) wrote: "The great King of Persia blocked up all the passages between the hills with dykes and flood gates." In modern times there are hydraulic gates of every conceivable system, shape, and size. Vibration and cavitation are the nemesis of hydraulic gate design and are not necessarily confined to high heads. They were encountered in 1890 at the then comparatively new Stoney gates of the Chèvres power plant (on the Rhône River below Geneva, Switzerland).

The account given by Sir William Willcocks, who, at the beginning of the twentieth century, tried to find an acceptable design for the deep sluice gates of Assuan Dam (Nile River), is interesting.[3] He chose the Stoney gate, now (1957) discarded in its original form.

More recently, the Bureau of Reclamation, United States Department of the Interior (USBR), launched a program of high-head structures and acquired invaluable experience in the design of high-pressure outlets. The adverse experiences encountered by the USBR are carefully discussed in the work of John L. Savage,[4] Hon. M. ASCE, on high-pressure outlets. The lessons contained therein have sometimes been overlooked, with dismal results, as in the case of the outlets of Madden Dam (the Canal Zone), Denison Dam (Red River), and Keystone Dam (North Platte River).

The first comment to the author's statement that the Corps has undertaken a survey of existing gate systems is that engineers would welcome publication of that report.

Mr. Buzzell's belief, stated in the "Introduction," that "as late as 1946," no greater depth than 25 ft could safely be discharged over an ogee spillway is not generally shared. The USBR had 50-ft-by-50-ft gates long before 1946. The gates at Laufenburg (on the Rhine River), although on a low ogee, were 56 ft 9 in. by 52 ft 6 in. and were built in 1914. The writer designed one spillway containing 50-ft-by-30-ft tainter gates, which have passed a flood of 500,000 cu ft per sec with an overflow depth of 41.5 ft (1952), and another spillway, built in 1945, containing four tainter gates, 78 ft 9 in. by 36 ft. Both of these stand on gravity ogee sections more than 80 ft high; the first spillway spans the Colorado River in Texas and the second, the Duero River in Spain. The latter are the largest tainter gates in existence.

The main conclusion derived from the paper is the reversal of the findings in the USBR manual on high-pressure outlets.[4] This manual concluded that best results will be obtained with the gate placed at the downstream outlet of the conduit. This principle was also used by the Corps in many installations, such as at Mud Mountain Dam (White River). Larger conduits virtually compelled a step backward in fact and in effect; the gate moved back into the conduit. The USBR also reversed itself; slide gates had always been retained under moderate heads with good ventilation, and the USBR has a standard set of these in use for many years. However, in recent years Paradox gates and Tube valves have been placed "inside" the spilling tubes, as at Marshall Ford Dam (Colorado River) and Grand Coulee Dam (Columbia River). The results have not been altogether satisfactory because it is practically impossible to design a perfect hydraulic gate under conditions of inaccessibility, confinement, and incipient cavitation. Underwater corrosion, mud, calcareous deposits, and barnacles soon cover everything, and the roller tracks deteriorate with the gates. The decision to use tainter gates in locations so afflicted is a bold step on a hitherto unprecedented scale and one that deserves commendation. A cautious attitude should be maintained until years of operation have confirmed the expectations. The partly open Garrison gate, deprived of its fixed support, may be subject to heavy vibration. At a discharge of 25,000 cu ft per sec under 169 ft of head, the gate would shoot a rectangular jet of approximately 15 ft by 18 ft at 90 ft per sec into a 22-ft-diameter round conduit flowing at 66 ft per sec when full and laid through the embankment. This will represent an enormous hydraulic compressor of no less than 480,000 hp in a 22-ft concrete conduit under highly turbulent conditions. The writer fears that such use for any length of time would soon cause adverse results similar to those at the spill tunnels of Hoover Dam (Colorado River). It would seem more advisable to disperse the flow into smaller units. The value of the deep intake is debatable for many reasons not connected with the gate design. Such deep intakes are found in many foreign installations that have up to 300 ft of head and turbine intakes; they are then provided with filler gates, as at Hohenwarte Dam (Saale River, Germany). Inside free-discharge gates in such cases are avoided.

---

[2] Cons. Engr., New York, N. Y.

[3] "60 Years in the East," by Sir William Willcocks, E. & F. N. Spon Ltd., London.

[4] "High-Pressure Reservoir Outlets," by J. M. Gaylord and J. L. Savage, Bureau of Reclamation, U. S. Dept. of the Interior, Washington, D. C., 1923.

The McNary Dam intake gates for large Kaplan units are a big improvement over the system used by the Tennessee Valley Authority (TVA). In Germany, hydraulic operation of head gates for large Kaplan units is widespread. Oil pressure is usually 800 lb per sq in.; for starting under high head a second pump for 1,680 lb per sq in. is added temporarily. The closing time varies from 10 sec to 30 sec; at the TVA dams it is 30 min or more. The McNary intake seems, therefore, better equipped; it is a considerable advance over other systems. For a smaller unit having a capacity of 4,000 cu ft per sec, the writer has used one tainter gate, 45 ft by 29 ft.

The tainter gate has a long history.[5] Inclined struts have been used exclusively in hundreds of gates designed by William G. Fargo, M. ASCE. The Corps' designs are apparently still considerably heavier than existing designs and they cost a great deal more. The moving part of a 40-ft-by-25-ft gate at Buchanan Dam (Colorado River) weighs 50,000 lb; at Granite Shoals Dam (downstream of Buchanan Dam) the 50-ft-by-30-ft gates weigh 110,000 lb. In 1951 the cost of the latter (including anchorages) was $21,650. Two traveling hoists were used for nine gates, the cost per gate being $3,910; the total cost of the gate (including anchorages and hoist) was only $17 per sq ft. The use of long steel bars to anchor the gate by bond into the pier has been the usual practice for half a century. The thrust of 1,753,000 lb of each strut of the 78-ft-9-in-by-36-ft gates is supported by twenty-three imbedded and upset, prestressed 3-in. bars held only by bond. Half a century of experience in the design of this type of gate has produced a surface gate that is lighter and more economical than any other type of hydraulic gate for surface spillways and that requires a minimum of maintenance.

---

[5] "Tainter Gates of Record Size Installed in Spanish Dam," by Abraham Streiff, *Civil Engineering*, March, 1950, pp. 29–30.

It is not known whether the Fort Randall roller-train sluice gates are to be operated at "part gate"; if they are, their ultimate failure will only be a matter of time in the light of all previous experience covering the period from the introduction of roller trains (circa 1890). It would be of interest to know what innovations were introduced to warrant a new trial.

GEORGE R. LATHAM,[6] M. ASCE.—Design of hydraulic gates is a specialized field that engages relatively few engineers. Fewer still have had the experience of actually operating the equipment. Because interchange of this information is meager, Mr. Buzzell's contribution is particularly valuable and welcome.

The Corps' survey of hydraulic gates was a very thorough undertaking; it was only practical to be undertaken by a government agency. Undoubtedly much valuable information was obtained, and it is regretted that it could not be made available to interested engineers. Many designers are not fully aware of the objections to various types of gates and of why some gates have fallen into disuse. The manual[7] of the USBR has proved very helpful and is referred to by many gate designers. The writer appreciates this opportunity to learn of the large number of special gates described by Mr. Buzzell.

On several projects with which the writer was associated, sliding, flat, steel gates without wheels were used to close off diversion tunnels that bypassed the stream flow during the construction of the dam. These gates are designed for the full reservoir head but are closed during the low flow period; a permanent concrete plug is then constructed in the tunnel. Burning through the supporting hanger and allowing the gate to drop freely is the usual method for closing these gates. It has been used successfully for gates up to 16 ft wide by 23 ft high.

Another large project required three diversion gates, each approximately 18 ft wide by 36 ft high. They were to be designed for a full reservoir head of 180 ft and were intended to be closed under a probable head of 50 ft. Each gate weighed approximately 50 tons; the impact from dropping a gate of this size would be excessive. Therefore, it was decided to lower the gate by utilizing the intake gate hoists. The gates were equipped with rubber J seals and bushed wheels on cantilever axles. The gate body was designed to withstand the reservoir head of 180 ft. The wheel assemblies were designed to withstand only the closing head of 50 ft under normal working stresses. After closure, the reservoir filling would increase the pressure on the gate and deflect the cantilever axles by $\frac{1}{8}$ in. until solid blocks on the gate came in contact with the steel guides, giving solid bearing from gate to guides to withstand the pressure from the full reservoir. This method permitted a considerable saving in the design of the wheel assemblies.

---

[6] Chf. Architectural and Structural Engr., Ebasco Services, Inc., New York, N. Y.

[7] "Valves, Gates & Steel Conduits," *Design & Construction Manual, Design Supplement No. 7*, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo., Vol. 10.

William G. H. Holt[8].—It is stated (in the "Introduction") that " * * * as late as 1946 it was firmly believed that no greater depth of water than 25 ft could be discharged over an ogee spillway without shaking the structure to pieces * * *." In Canada in 1923, vertical-lift spillway gates of the Stoney type, as large as 50 ft wide by 30 ft high, were in regular use. The Stoney design was superseded soon by the fixed-roller construction. Since then many gates of the latter type even larger in size have been installed. Three 50-ft gates, 36 ft high, were installed in 1932 and are still in regular use. These gates are operated with little or no freeboard. The great majority are exposed to severe winter conditions, some being located where temperatures drop to −50°F. The tainter gate has been used comparatively little in Canada. Climatic conditions undoubtedly are largely responsible for this.

The statement (under the heading, "Trends in Gate Design: Large Vertical-Lift Roller-Train Gates") that, until the " * * * Fort Randall gate was designed with a 45°-sloped bottom member * * * high-head gates had been built with nearly square bottoms because of structural considerations * * *," is rather surprising. For more than 35 yr, the 45°-sloped bottom has been used extensively in Canada for intake gates and free-discharge gates. More than six hundred of these gates have been built during this period. Normally the skin plate is placed on the downstream side, and large vent holes are provided in the sloping bottom and in the bottom cross girder. This construction reduces vacuum effects on the bottom to the point where they have little effect on the hoisting effort with gates of normal size. Gates as wide as 45 ft and as high as 34 ft with heads up to 75 ft have presented no insuperable difficulties in design or construction. Head gates of this type are self-lowering against full-turbine flow and can be raised against full head without the use of bypass valves.

Kandaswami S. Chetty[9].—This is an excellent summary of operating experiences with high-head gates, on which, formerly, little data had been available; it forms a valuable contribution to the design of control structures. The developments in hydraulic gate design have been necessary in view of the rising costs of materials, which have made conventional and satisfactory gates, such as hollow-jet valves, extremely expensive. The developments may be summarized as:

1. The reshaping of the bottom of the gate leaf to have a well-defined control point from which the high-velocity jet could shoot into the conduit (in past designs, the continuous shifting of the control point from the upstream end to the downstream end of the gate leaf resulted in the fluctuations of pressures at the bottom of the gate leaf and, hence, vibrations);
2. The reshaping of the downstream end of the gate slot to minimize disturbance due to sudden expansion of the area at the gate slot; and
3. The enablement of the conduit to flow only partly full so as to allow sufficient air on the downstream side of the gate and obviate the negative pressures.

---

[8] Mech. Engr., Eastern Div., Dominion Bridge Co., Ltd., Lachine, Quebec, Canada.

[9] Deputy Director, Central Water & Power Comm., New Delhi, India.

These improvements were made to the existing slide gates and fixed-wheel gates, which were used as regulating sluice gates, but the gates still developed trouble at higher heads. The use of tainter gates has proved satisfactory and economical and is a definite improvement. It is of interest to note that the USBR has developed radial gates for high heads, and in Davis Dam (Colorado River) a 22-ft-by-19-ft gate is installed to operate under a 113-ft head. The design for this gate has been conventional without any provision for hydraulically actuated or pneumatic seals or for an eccentric pin. These gates have proved entirely satisfactory. It will be useful to compare the extra cost of the arrangement of pneumatic rubber seals or of an eccentric pin with that of the hoist if the seal used at Norfork Dam is used.

It is generally found that slide gates are arranged in tandem so that one gate is used as a service gate and the other in front as an emergency gate. The utility of this method is appreciated when there are only a small number of sluices. However, if there are a greater number of sluices, it may be more economical to provide one regulating gate for each of the sluices and only one emergency gate operated by a gantry crane and capable of closing any of the sluices. In practically all the projects there is always a gantry crane on the top of the dam for anumber of other purposes. This crane will be used for operating the emergency gate.

It will be useful to know the design criteria for the conduit liners so as to determine their thicknesses and their lengths in front of, and downstream of, the gate. It is not known whether Mr. Buzzell's cost estimate of from $400 per sq ft to $500 per sq ft is the total cost of service and emergency gates or the individual cost for each gate.

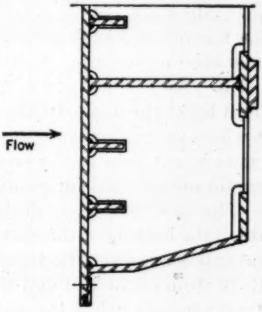


Fig. 1.—Bottom Shape of the Fixed-Wheel Gate

It is stated that the fixed-wheel gate arrangement has proved to be too costly because of the welded-plate gate body. In Europe and in India, however, the welded-gate body is more economical than cast-steel slide gates. Hence, fixed-wheel gates have found greater application than slide gates. This has been reflected in the installation of fixed-wheel gates for regulating purposes rather than slide gates. Similarly for conduit liners, structural steel plates are preferred to cast-iron or cast-steel liners.

Because the conduit downstream of the gate at Detroit Dam is designed to flow only partly full, the gate shaft may not be subjected to any water pressure except for spray; hence, the necessity for the liners in the gate shaft is not clear (Fig. 1).

If the gate closes by its own weight, will a mechanical hoist with chain or rope be more economical than a hydraulic hoist?

William Gustave Weber[10].—Some of the more notable developments in hydraulic gate design made by the Corps have been presented. The paper is of great value to engineers engaged in similar work particularly because of the broad field covered by the Corps' projects and the unexcelled opportunities its designers had for preliminary model testing and for subsequent observation of actual operating characteristics of the installed equipment. The author's comments also are of special value to organizations such as the USBR when similar problems are encountered.

Slide Gates.—Mr. Buzzell has arrived at some conclusions with which USBR engineers agree. The latter have long since abandoned the idea that sluice-gate operations should be restricted to installations where the total effective head does not exceed 100 ft.

The USBR has numerous slide gates in many ways similar to the earlier type that are in service at heads usually below 100 ft, but in some instances at heads up to 125 ft. In general, their performance has been satisfactory. However, in some installations, not necessarily the ones at higher heads, there have been indications of cavitation on the bottoms of the gate leaves and along the lower edges of the downstream gate frames. In a few of these instances, cavitation has reached such an advanced stage that extensive repairs and alterations have been necessary. The normal service gate has a bottom slope of from 1.0 to 1.5. An earlier type of gate of the USBR has a horizontal bottom with a slight lip at the downstream edge. Each gate has a critical percentage opening at which cativation is most apt to occur. If operating conditions are such that a gate is not held in the critical range for appreciable periods of time, it may remain undamaged for many years of service.

The 45°-test type of design is a definite improvement over the older type in which the bottom of the gate is parallel or nearly parallel with the bottom of the water passageway; the trend is toward this newer design, with the contour of the bottom of the leaf and the angularity of the faces on the sides of the downstream frames below the slots not greatly different from the ones used on the test gate. This trend applies not only to slide gates but also to the larger roller-mounted gates and wheel-mounted gates, such as the penstock intake gates at Grand Coulee Dam, Hungry Horse Dam (South Fork, Flathead River), and Palisades Dam (South Fork, Snake River). On the Palisades gate the bottom plate is omitted entirely, but the effective slope is the same as though it were there.

The estimate that 6 ft by 10 ft is a maximum size for slide gates at a 200-ft head is questionable. The Palisades outlet gates have a greater area (67.5 sq ft) and will operate under 20% more head. In this gate there is a 30-in. cylinder and a 2,000-lb-per-sq-in. design hoist; the actual operating pressure is 1,500 lb per sq in. Because 30 in. appears to be about as large a diameter as is desirable for practical use if more operating force were required, it is probable that the USBR would consider use of higher operating pressures, perhaps to a maximum of 5,000 lb per sq in.

The USBR studies indicate that seat-bearing pressures rather than hoist capacities would be the limiting considerations. Bearing pressures as high as

---

[10] Mech. Engr., Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo.

4,000 lb per sq in. would not be unreasonable if the conventional bronze-on-bronze seals and seats were changed to bronze-on-monel or to bronze-on-stainless steel with lubrication grooves. The latter combination would give an added benefit because the coefficient of friction would be somewhat lower. It was proposed that this alloy combination be used on the Palisades gates, but the restrictions on the use of high-nickel alloys prevented consideration when final designs were prepared. In determining bearing pressures, the USBR prefers to limit effective bearing surfaces to 4 in. (a 6 in. maximum) transversely to the seat length, and to make the gates rigid enough to avoid extreme concentration of stresses at the inner edges of the seats.

The statement that gate bodies cannot be constructed economically if they are made of weldments is not in agreement with USBR experience with the Shasta Dam (Sacramento River) and Palisades Dam outlet gates in which combinations of steel weldments and castings were used. Furthermore, because there is no reason for limiting bodies to steel, gray iron frequently has been used, as in the Carter Lake (Big Thompson Project, Colorado) 3-ft-by-3-ft outlet gates where heads of 170 ft will occur.

Tainter Gates.—The use of top-seal tainter or radial gates in conduits appears to have several desirable aspects and bears further investigation and consideration by the USBR. Consequently, the USBR is greatly interested in following the service records of the gates installed by the Corps. It is particularly interested in the performance of the gate at Lookout Point Dam with its eccentric trunnion and with the water passageway enlarged to provide seal mounting. The USBR prefers gate water passageways with continuously uniform cross sections but believes the Lookout Point contour can be used without detriment in many installations. The arrangement appears definitely to provide firm seating with a minimum of vibration and leakage. However, unless the tendency to vibrate is pronounced, a suitable top seal and rigid hoist connection without eccentric trunnions may give equally acceptable results.

The USBR questions the use of roller bearings on the trunnions of tainter gates and their serviceability over an extended period of time. Sleeve bearings have been preferred and have given satisfactory service and, when properly lubricated, have shown little need for replacement.

The typical tainter crest gate, as described by Mr. Buzzell, has inclined arms and recessed trunnions. Gate fabrication appears more difficult than for rectangular connections. Also, the arrangement requires a relatively wide pier. Where space is at a premium and piers as narrow as practicable are desired, the writer questions the advisability of such a design, although it has desirable features, among them weight and cost reduction. The latter may be more apparent than real, however, if increased pier width is considered.

Roller-Mounted and Wheel-Mounted Gates.—The McNary intake gate is 20 ft wide by 51 ft high with a single, continuous roller train on either side of the gate. Such a train is much longer than is usually used, and because the failure of even one link in the chain would put the entire train out of service, more than normal inspection and maintenance work would seem needed. The USBR has found that, where feasible, wheels mounted on self-lubricating bronze bushings are preferable to roller trains from the standpoint of first cost and maintenance charges. Wheel-rail contact stresses are a limiting factor, however, and, although the USBR has installed many wheel-mounted gates, it occasionally is faced with a load problem to which rollers appear to be a preferable solution. The usual USBR practice with either roller-mounted or wheel-mounted gates is to suspend each gate from a single hydraulic hoist. Gate operation is timed for a closing period of 3 min or less, with the gate closing by gravity after release from its hanger. In some more recent USBR installations, the hanger has been omitted, and the gate is supported on the hydraulic fluid—usually oil—in the hoist cylinder. Cylinder pistons are packed so that downward drift is slight, and an automatic recovery system is used to restore each gate to its fully raised position immediately above the penstock intake after a predetermined amount of drift. One advantage of this type of suspension is the fact that the gate may be readily lowered even in case of total power failure, whereas the latched gates require power for a brief period during which the latches are being released. Where there are a number of gates installed, a gantry crane is usually installed for maintenance work. This crane has a smaller capacity and is much lighter than necessary if it were required to place a gate in an emergency over the entrance to a runaway penstock.

Sleeve-Type Gates and Valves.—Mr. Buzzell is justifiably critical of the high maintenance costs of many of the older types of cylinder gates and needle valves. However, the USBR has found, that in some instances the causes of high maintenance costs could be removed by relatively minor design changes. There are occasional combinations of conditions that make cylinder gates appear attractive in spite of a somewhat unfavorable history. The USBR has not provided needle valves for outlet works for several years largely because, where a valve of this type is desired, a hollow-jet valve will answer the purpose at less initial expense and annual maintenance costs. This is true even though modifications in needle-valve design, as in the valves at the Madera Canal outlets of Friant Dam (San Joaquin River), have greatly improved performance records.

It is to be hoped that experiments of the type conducted by the Corps will be continued and that observations as to the performance of the future gates and valves will be extended for sufficiently long periods of time to establish their worth or to point to further desirable modifications.

The type of gate or valve selected for any given installation will depend not only on the initial cost of the equipment, the maintenance costs, and out-of-service time, but also on its functional suitability and the costs of the related structures.

Dow A. Buzzell,[11] M. ASCE.—The comments of Mr. Streiff reveal an exhaustive knowledge of the history of hydraulic gate development in the United States and abroad. A study of the history of hydraulic gates, such as that attempted in the Corps' survey, will show that it is difficult to draw rigid and fixed conclusions regarding these structures. Several attempts have been made to correlate the results of this survey so that such conclusions could be reached, but when all the variants were taken into consideration, this was found to be impracticable. The principal factors were the hydraulic design of the water passages, the design of the gate leaf including the metals used, and the actual usage of the facility. Many of the gates examined had almost never been used or had functioned only briefly during flood routing or for emergency closure. Naturally such gates had a favorable history because only periodic painting and lubrication preserved them indefinitely. Severe corrosion in polluted waters, as on the Pennsylvania rivers, had long been known, and the liberal use of high-alloy metals had supplied the answer. The trouble at Denison Dam resulted from faulty fabrication, and after suitable repairs the original gates are serving satisfactorily. In a few cases faulty operative procedures had injured gates.

---

[11] Chf. Design Engr., Tippetts, Abbett, McCarthy & Stratton, Cali, Colombia, South America.

One firm conclusion of the survey was that high-head gates can be mounted within the water passages. Most of the Corps' earth dams are so equipped and have had up to 17 yr of successful operation of roller-train gated outlets, normally closed to hold a conservation pool but frequently operated during flood routings. These gates are usually exposed to the erosive action of bed load and have an excellent history. The Fort Randall gates will be operated at all heads but in such a manner as not to be held at the one low-head point where some vibration occurs. The objections to end-outlet gates are (1) the greater initial cost of the pressurized conduit and (2) the more elaborate stilling basin, even though there is some saving in the control tower. There is also some apprehension regarding the safety of a long pressurized conduit through a high earth dam.

The Corps has come to rely more and more on model tests in formulating the design of hydraulic-control works. The technique of model building and testing has been much improved in recent years, and instances of the prototype failing to operate as satisfactorily as the model are exceedingly rare.

The deflection-bearing device described by Mr. Latham is an ingenious method of economizing on a gate intended for a special use. However, it could not be used for an operating gate.

Mr. Holt's remarks, together with those of Mr. Streiff, regarding heavy discharges over ogee crests only show that truth can be stranger than theory. Some engineers have acted as observers in the galleries of concrete dams during heavy spillway discharges and have been alarmed by the vibration observable during these discharges. Some of the Corps' early model tests also predicted this vibration. Because better models have been developed, or perhaps because of more experience with them, the fears regarding deep spillway discharges have largely been dissipated.

Mr. Chetty has summarized the principal improvements that have been made in gate leaf and mounting design. The writer is familiar with the Davis Dam gate design and is pleased to hear of its favorable action. The Corps adopted the eccentric pin to assure longer seal life and to make certain that there would be no vibration at partly open gates. Successful performance of the Davis gates should encourage the designer to use this simpler and more economical design. The writer would prefer the eccentric gate for very high heads because small seal leaks can cause "wire-cutting" when under high pressure.

The Corps usually provides two gates in tandem and an emergency upstream bulkhead handled by a hoist or mobile crane. If this upstream closure arrangement is entirely dependable for closure against full flow through the outlet, there would seem to be justification for omission of the second internal gate. In many of the Corps' projects, rapid closure in case the operating gate failed when open is so important that the Corps has been unwilling to accept the risk of the much longer time required to make the bulkhead closure. However, there are undoubtedly projects where no such hazard would be involved. The cost figures given are for one gate only. Conduit liners are placed only over areas in which model tests indicate danger from erosion or cavitation and are usually of cast steel, but there are no known criteria for thickness.

The Detroit fixed-wheel, welded-body gate proved to be expensive but this may be an isolated instance. The Corps has used fixed-wheel gates for power intakes and elsewhere, and the writer has always felt that this type has been neglected. When wheel loads become excessive, the designer is forced to use roller trains, but the roller-train gate has the disadvantage of exposing a great area of rollers, links, and bearing plates to the effects of the stream. The fixed-wheel gate with sealed bearings will suffer far less from the contamination of bed load, polluted water, and marine life.

The writer would consider a fixed, double-flanged wheel gate with sealed roller bearings as the first choice for power-intake closure. The Detroit sluice gate was not economical and proved to have the hydraulic disadvantage of the spreading jet causing some erosion of the conduit walls. A steel liner to cover the area of impingement would have prevented this. The Meridian gate is believed to be both economical and hydraulically sound. A mechanical hoist is usually less expensive than a hydraulic hoist, but the hydraulic hoist can be closed after power failure and it tends to dampen out vibrations.

Mr. Weber's remarks are appreciated because the Corps has closely studied the USBR's work and frequently profited from it. The hydraulic slide gate undoubtedly can be used in larger sizes than the one the Crops has arbitrarily set as the limit. The critical factor in the opinion of designers is the bearing pressure. This side bearing can never be uniform over the area, and the allowable intensity, considering the deviations in gate fabrication, is a question for the designer's judgment. The Grand Coulee experiments showed that very high intensities of bearing were permissible for certain metallic combinations, providing that the bearing was uniform.

The writer's previous remarks cover possible simplification of the eccentric tainter gate. He cannot agree with Mr. Weber concerning crest tainter gates. The Corps does not use wider piers for these gates (the standard width being 8 ft), and unit bid prices are no higher for the inclined-arm type. Total cost of the entire spillway is less for the inclined-arm gate, which will be found in some old wooden tainter gates.

The writer agrees with Mr. Weber regarding the wheeled, rather than the roller, type of gate. When the load is too heavy for the flanged wheel on a crane rail, the Corps has used flat rim wheels bearing on a massive embedded bearing plate, which permits much higher loads. The bushed bearing should be used whenever possible.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2909

TRANSITION PROFILES IN NONUNIFORM CHANNELS

By Francis F. Escoffier,[1] A. M. ASCE

With Discussion by Messrs. Achille Lazard; Léon J. Tison; and Francis F. Escoffier

Synopsis

If a water-surface profile passes without abrupt change from tranquil to rapid flow, or from rapid to tranquil flow, the point at which it does so lies on a profile that can conveniently be called a "transition profile," and the depth at that point can be termed a "transition depth." Such a profile is fixed in that it does not vary in position with changes in discharge as do normal-depth profiles and critical-depth profiles. Mathematically, the point of transition is a singular point, and the solution to the differential equation for steady flow does not yield the traditional vertical water-surface profile at that point. A graphical practical method for constructing transition profiles is outlined herein.

The Transition Profile in Uniform Channels

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically for convenience of reference in the Appendix.

Considering the differential equation for water-surface profiles in uniform channels,

$$\frac{dy}{dx} = S_o \frac{1 - \left(\frac{Q}{Q_n}\right)^2}{1 - \left(\frac{Q}{Q_c}\right)^2} \qquad (1)$$

Note.—Published, essentially as printed here, in June, 1956, in the Journal of the Hydraulics Division, as *Proceedings Paper 1006*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Hydr. Engr., Corps of Engrs., U. S. Dept. of the Army, Mobile Dist., Mobile, Ala.

in which y denotes the depth of water; x, the distance along the channel; S_o, the slope of the bottom of the channel; Q, the discharge; Q_n, the normal discharge; and Q_c, the critical discharge. If it is assumed that in the differential equation for water-surface profiles in uniform channels (Eq. 1) there exists a depth for which the condition,

$$Q_n = Q_c \neq Q \qquad (2)$$

is satisfied, then for that depth it can be seen that the expression on the right side of Eq. 1 is equal to S_o. This means that the depth, y, increases in relation to the distance, x, at a rate equal to the bottom slope, S_o, and that, therefore, the water-surface profile at the point in question is horizontal. The profile defined by $Q_n = Q_c$ is conveniently termed a transition profile, and it is a general characteristic of such a profile that the water-surface profiles intersecting it are horizontal at the point of intersection as long as $Q \neq Q_c$, or as long as the water surface does not pass through critical depth as it intersects the transition profile.

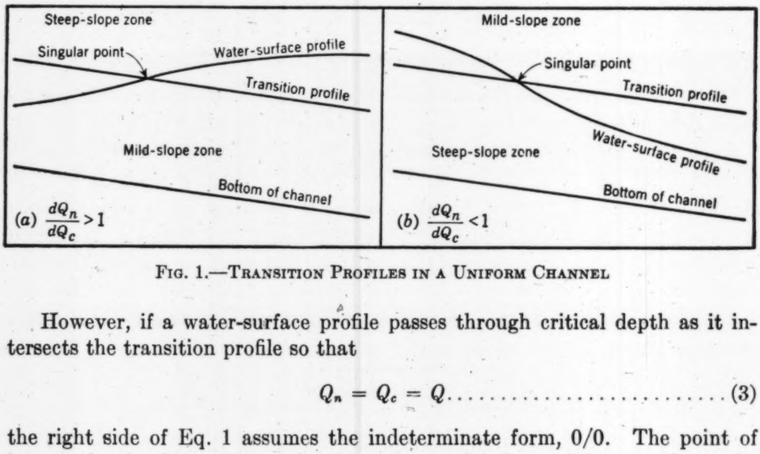

Steep-slope zone
Singular point
Water-surface profile
Transition profile
Mild-slope zone
Bottom of channel
(a) $\frac{dQ_n}{dQ_c} > 1$
Mild-slope zone
Singular point
Transition profile
Water-surface profile
Steep-slope zone
Bottom of channel
(b) $\frac{dQ_n}{dQ_c} < 1$

FIG. 1.—TRANSITION PROFILES IN A UNIFORM CHANNEL

However, if a water-surface profile passes through critical depth as it intersects the transition profile so that

$$Q_n = Q_c = Q \qquad (3)$$

the right side of Eq. 1 assumes the indeterminate form, 0/0. The point of intersection in this case is a singular point, and it is possible to evaluate the expression by replacing the numerator and the denominator by their derivatives. The result is

$$\frac{dy}{dx} = S_o \frac{dQ_n}{dQ_c} \qquad (4)$$

a formula that can be used to determine the slope of a water-surface profile (in a uniform channel) as it intersects the transition profile and simultaneously passes through critical depth. Two such intersections are shown in Fig. 1.

In Fig. 1(a), $dQ_n/dQ_c > 1$, and the profile passes from rapid to tranquil flow, whereas in Fig. 1(b), $dQ_n/dQ_c < 1$, and the opposite change occurs. (Ed. note: See closing discussion.)

An important property of the transition profile is that it forms a line of separation between two zones. In one zone, $Q_n < Q_c$, and the water-surface profiles have the properties normally associated with a mild-slope channel; in the other zone, $Q_n > Q_c$, and they have the properties normally associated with a steep-slope channel. These two zones will be referred to subsequently as mild and steep zones, respectively.

Transition depths in uniform channels have been examined by Achille Lazard[2] who referred to them as "characteristic" depths. He credits G. Mouret[3] with the development of the idea. The writer prefers the term "transition" to "characteristic" because the profiles represent the normal location of transitions from tranquil to rapid flow, a factor of considerable importance in nonuniform channels.

Lazard notes that in rectangular channels and in closed conduits, the number of transition profiles will be two, one, or zero depending on whether the channel slope is greater than, equal to, or less than a value that he designates as the limiting slope. If two transition profiles exist, the zone below the lower profile and that above the upper profile are both mild, whereas that between the two profiles is steep. As the slope of the channel is reduced, the two transition profiles approach each other until they coincide, at which time the slope becomes equal to the limiting slope and the steep zone disappears. For that slope and for lesser slopes, all water-surface profiles have the properties normally associated with mild-slope channels. Lazard also introduces the concept of a characteristic discharge, or in the terminology of this paper, "a transition discharge," which is equal to the common value assumed by both the normal and the critical discharges on the transition profile.

The writer, in a discussion of a paper by Marvin E. Von Seggern,[4] A.M. ASCE, noted that the functions developed in that paper could be used to show the existence of transition profiles. At that time the writer was unaware of the existence of Lazard's paper.

TRANSITION PROFILES IN NONUNIFORM CHANNELS

To extend the concept of a transition depth to nonuniform channels, it is necessary to derive an appropriate generalization of Eq. 1. Beginning with the dynamic equation in the form,

$$d\left(z + \frac{v^2}{2\,g}\right) = -\left(\frac{Q}{K}\right)^2 dx \qquad (5)$$

in which z denotes the elevation of the water surface; v, the velocity; g, the acceleration of gravity; and K, the conveyance; and choosing x and z as independent variables, it is possible to write

$$d\left(\frac{v^2}{2\,g}\right) = -\frac{Q^2}{g\,A^3}\left(\frac{\partial A}{\partial x}\,dx + \frac{\partial A}{\partial z}\,dz\right) = -\left(\frac{Q}{Q_c}\right)^2 (S_A\,dx + dz) \qquad (6)$$

---

[2] "Contribution à l'étude théorique du mouvement graduellement varié en hydraulique," by Achille Lazard, *Annales des Ponts et Chaussées*, March–April, 1947.

[3] "L'Hydraulique générale et appliquée," by G. Mouret, Paris, 1927.

[4] "Integrating the Equation of Non-Uniform Flow," by M. E. Von Seggern, *Transactions*, ASCE, Vol. 115, 1950, p. 71.

in which

$$S_A = -\left(\frac{dz}{dx}\right)_{A\text{—constant}} = \frac{\dfrac{\partial A}{\partial x}}{\dfrac{\partial A}{\partial z}} \qquad (7)$$

A denotes the cross-sectional area of the channel, and S_A is the slope of the profile for A = constant. The slope term, S_A, which is analogous to the bottom slope, S_o, is a function of both x and z.

Substituting Eq. 6 into Eq. 5 results in

$$dz - \left(\frac{Q}{Q_c}\right)^2 (S_A\, dx + dz) = -\left(\frac{Q}{K}\right)^2 dx \qquad (8a)$$

which can be arranged in the form,

$$\frac{dz}{dx} + S_A = \frac{S_A - \left(\dfrac{Q}{K}\right)^2}{1 - \left(\dfrac{Q}{Q_c}\right)^2} \qquad (8b)$$

The concept of a paranormal discharge, Q_n', is now utilized. This quantity, which is defined by

$$Q_n' = K\sqrt{S_A} \qquad (9)$$

was introduced by Pierre Massé[5] in his study of channels having uniform, rectangular cross sections but variable bottom slopes. When this equation is substituted into Eq. 8b,

$$\frac{1}{S_A}\frac{dz}{dx} + 1 = \frac{1 - \left(\dfrac{Q}{Q_n'}\right)^2}{1 - \left(\dfrac{Q}{Q_c}\right)^2} \qquad (10)$$

is obtained, which is the desired generalized form of Eq. 1.

The left side of Eq. 10 cannot be replaced by the term, dy/dx, as in Eq. 1 because the appropriate differential expression, $dy = dz + S_A\, dx$, is not an exact differential and, consequently, the supposed function, y, does not exist.

The transition profile can now be defined as the profile along which $Q_n' = Q_c$; this profile again forms the boundary between a zone with mild-slope characteristics and one with steep-slope characteristics. The transition discharge for any cross section is equal to the common value assumed by the paranormal and the critical discharges on the transition profile in that cross section. If the water surface for a given discharge passes from subcritical to supercritical flow, it will do so on the transition profile and in the cross section where that discharge is equal to the transition discharge. The cross section in question is the control for that particular discharge, and if the discharge changes, the control will move upstream or downstream (except in a uniform channel). A ready means for locating the controlling cross sections in a channel is obtained by plotting the transition discharge as a function of the variable, x. As in the foregoing, if

---

[5] "Ressaut et ligne d'eau dans les cours d'eau à pente variable," by Pierre Massé, *Revue Générale de l'Hydraulique*, Nos. 19 and 20, January–April, 1938, pp. 7–11 and pp. 61–64.

$$Q_n' = Q_c \neq Q \quad \text{(11)}$$

the numerator and the denominator on the right side of the equation are equal and, therefore, $dz/dx = 0$, which implies that the water-surface profile is horizontal at the point of intersection to the transition profile. Also, if the profile passes through critical depth as it crosses the transition profile,

$$Q_n' = Q_c = Q \quad \text{(12)}$$

and the right side of Eq. 10 assumes the indeterminate form, 0/0. The point of intersection is singular, and the expression is evaluated by replacing the numerator and the denominator by their derivatives. The result is

$$\frac{1}{S_A}\frac{dz}{dx} + 1 = \frac{dQ_n'}{dQ_c} \quad \text{(13)}$$

However, this expression is not as simple to evaluate as that in Eq. 4 because Q_n' and Q_c are now functions of the two variables, x and z. In fact,

$$dQ_n' = \frac{\partial Q_n'}{\partial x}\,dx + \frac{\partial Q_n'}{\partial z}\,dz \quad \text{(14)}$$

and

$$dQ_c = \frac{\partial Q_c}{\partial x}\,dx + \frac{\partial Q_c}{\partial z}\,dz \quad \text{(15)}$$

which can be substituted into Eq. 13 to yield

$$\frac{1}{S_A}\frac{dz}{dx} + 1 = \frac{\dfrac{\partial Q_n'}{\partial x} + \dfrac{\partial Q_n'}{\partial z}\dfrac{dz}{dx}}{\dfrac{\partial Q_c}{\partial x} + \dfrac{\partial Q_c}{\partial z}\dfrac{dz}{dx}} \quad \text{(16)}$$

If the water-surface slope is designated by S, so that

$$S = -\frac{dz}{dx} \quad \text{(17)}$$

and if

$$r = \left(\frac{dQ_n}{dQ_c}\right)_{x=\text{constant}} = \frac{\dfrac{\partial Q_n'}{\partial z}}{\dfrac{\partial Q_c}{\partial z}} \quad \text{(18)}$$

$$S_n = -\left(\frac{dz}{dx}\right)_{Q_n'=\text{constant}} = \frac{\dfrac{\partial Q_n'}{\partial x}}{\dfrac{\partial Q_n'}{\partial z}} \quad \text{(19)}$$

and

$$S_c = -\left(\frac{dz}{dx}\right)_{Q_c\text{—constant}} = \frac{\dfrac{\partial Q}{\partial x}}{\dfrac{\partial Q_c}{\partial z}} \qquad (20)$$

then Eq. 16 can be rewritten as

$$1 - \frac{S}{S_A} = r\,\frac{S_n - S}{S_c - S} \qquad (21a)$$

Eq. 21*a* can be arranged in a more useful form as

$$\frac{S}{S_A} = \frac{(S_c - r\,S_n) + S\,(r - 1)}{S_c - S} \qquad (21b)$$

It is convenient at this point to introduce the slope of the transition profile, S_t. Because along this profile, $Q_n' = Q_c$, it is also necessary that

$$\frac{d\,(Q_n' - Q_c)}{dx} = \frac{\partial Q_n'}{\partial x} + \frac{\partial Q_n'}{\partial z}\frac{dz}{dx} - \frac{\partial Q_c}{\partial x} - \frac{\partial Q_c}{\partial z}\frac{dz}{dx} = 0$$

and, therefore,

$$S_t = -\left(\frac{dz}{dx}\right)_{Q_n'-Q_c} = \frac{\dfrac{\partial Q_n'}{\partial x} - \dfrac{\partial Q_c}{\partial x}}{\dfrac{\partial Q_n'}{\partial z} - \dfrac{\partial Q_c}{\partial z}} \qquad (22)$$

On substituting from Eqs. 18, 19, and 20, Eq. 22 is further reduced to

$$S_t = \frac{r\,S_n - S_c}{r - 1} \qquad (23)$$

Simultaneously substituting Eq. 23 into Eq. 21*b* and introducing the slope,

$$S_m = (1 - r)\,S_A \qquad (24)$$

produces

$$S = S_m\,\frac{S_t - S}{S_c - S} \qquad (25)$$

Eq. 25 reduces to the quadratic equation,

$$S^2 - (S_c + S_m)\,S + S_m\,S_t = 0 \qquad (26)$$

the solution to which is

$$S = \frac{S_c + S_m \pm \sqrt{(S_c + S_m)^2 - 4\,S_m\,S_t}}{2} \qquad (27)$$

Eq. 27 gives the slope of a water-surface profile passing through a singular point on a transition profile.

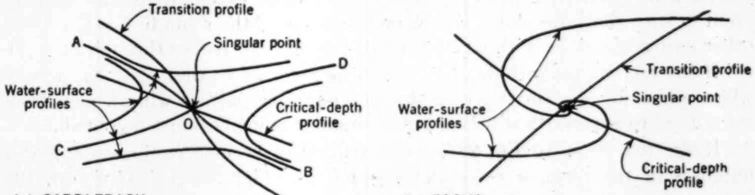
In the mathematical theory of singular points developed by H. Poincaré,[6] it can be shown that four types can occur, depending on the nature of the roots to the equation,

$$\lambda^2 - (S_c - S_m)\,\lambda + S_m\,(S_t - S_c) = 0 \ldots\ldots\ldots\ldots\ldots (28)$$

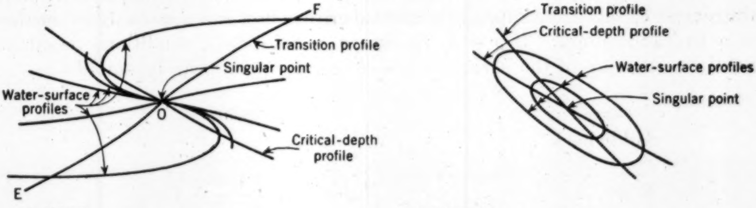
known as the characteristic equation. This equation can be obtained from Eq. 26 by means of the substitution,

$$S = \lambda + S_m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

which also defines the dimensionless variable, λ. (According to Theodore von Kármán, Hon. M. ASCE, and Maurice A. Biot,[7] the four types of singular points are saddle point, nodal point, spiral point, and vortex point; according to the Poincaré theory, the types are the saddleback, node, focus, and center.)


Transition profile
A
Singular point
D
Water-surface profiles
Critical-depth profile
O
C
B
Transition profile
Singular point
Water-surface profiles
Critical-depth profile

(*a*) SADDLEBACK (*c*) FOCUS


F
Transition profile
Singular point
Water-surface profiles
O
Critical-depth profile
E
Transition profile
Critical-depth profile
Water-surface profiles
Singular point

(*b*) NODE (*d*) CENTER

FIG. 2.—VARIOUS SINGULAR POINTS

If the two roots to Eq. 28 are real but unlike in sign, the singular point is a "saddleback" (Fig. 2(*a*)). The profile, AOB, in Fig. 2(*a*) represents a transition from subcritical to supercritical flow, and the point is therefore a control. The water-surface profile, COD, however, represents a transition from supercritical to subcritical flow, and point O is not a control. Segment CO represents a type of flow that can occur only if a suitable control exists farther upstream.

---

[6] "Goursat's Mathematical Analysis—Differential Equations," by Edouard Goursat, Pt. II, Ginn and Co., New York, N. Y., Vol. II, 1917, pp. 179–180.

[7] "Mathematical Methods in Engineering," by Theodore von Kármán and Maurice A. Biot, McGraw-Hill Book Co., Inc., New York, N. Y., 1940, pp. 150–158.

Similarly, segment OD represents a type of flow that can exist only if a suitable control exists farther downstream. If these two suitable controls do not exist, the continuous transition is not possible and a hydraulic jump must form either upstream or downstream from point O.

Several water-surface profiles that do not pass through the singular point O are also shown in Fig. 2(*a*). These profiles are always vertical where they intersect the critical-depth profile and are always horizontal where they intersect the transition profile.

If the two roots to Eq. 28 are real but alike in sign, the singular point is a node. Such a singular point, for which $S_c > S_m$, is shown in Fig. 2(*b*). All the water-surface profiles shown in this diagram are transitions from supercritical to subcritical flow, and, therefore, point O is never a control. A continuous profile is possible only if suitable controls exist upstream and downstream from point O. All the water-surface profiles except EOF pass through point O with the same slope, this slope being one of the roots to Eq. 27. The other root is the slope with which the one profile EOF passes through point O.

If the relative magnitude of S_c and S_m is reversed so that $S_c < S_m$, a node will occur again, but in this case the water-surface profiles will all pass from subcritical to supercritical flow and the singular point becomes a control.

If the two roots to Eq. 28 are not real, they become conjugate complex numbers of the form, $a + b\,i$ and $a - b\,i$, in which $b \neq 0$. If $a \neq 0$, the singular point is a focus, but if $a = 0$, it is a center (Figs. 2(*c*) and 2(*d*)). In either case a transition from subcritical to supercritical flow is impossible, and the reverse change from supercritical to subcritical flow can occur only by means of a hydraulic jump. If $\rho = S_c/S_m$ and $\sigma = S_t/S_m$, the conditions required for the occurrence of the various types of singular points can be stated as

Type	Condition
Saddleback................	$\sigma < \rho$
Node......................	$\rho < \sigma < \left(\frac{1+\rho}{2}\right)^2$
Focus.....................	$\sigma > \left(\frac{1+\rho}{2}\right)^2$ and $\rho \neq 1$
Center....................	$\sigma > 1$ and $\rho = 1$

These conditions are represented diagrammatically in Fig. 3. Each type of singular point except the center is represented by a zone. The center is represented by a line because it is simply a special case among the foci.

The use of the theory of singular points in the study of water-surface profiles was first introduced by Massé who also introduced the term "paranormal depth." In his studies he assumed a channel of uniform rectangular cross section and varying bottom slope. He also assumed that Q_n' and Q_c varied as the same power of the depth, which is equivalent to assuming that $r = 1$. As a result of the last assumption, the transition profile becomes a line normal to the bottom of the channel and is, therefore, confined to the one cross section where the bottom slope is critical. The formulas derived by Massé can be obtained by substituting $r = 1$, $S_c = S_A = S_o$, and $m = S_o - S_n$ into Eq. 21*a* to give a characteristic equation from which the following conditions are derived:

Type	Condition
Saddleback	$M < 0$
Node	$0 < m < \frac{S_o}{4}$
Focus	$m > \frac{S_o}{4}$

The singular point known as a center does not appear because it is impossible to satisfy the equation $\rho = 1$ under the assumptions made by Massé.

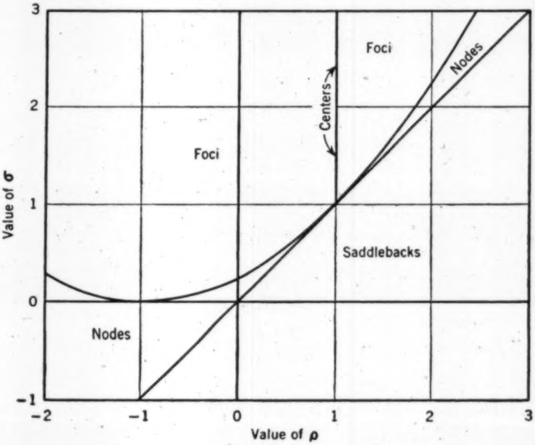


FIG. 3.—SINGULAR POINTS FOR FRICTIONLESS CHANNELS

TRANSITION PROFILES IN NONUNIFORM CHANNELS WITHOUT FRICTION

Unfortunately some of the formulas that have been derived heretofore fail in the case of a frictionless channel. Because $K = \infty$ in such a channel, Eq. 8*b* becomes

$$\frac{dz}{dx} + S_A = \frac{S_A}{1 - \left(\frac{Q}{Q_c}\right)^2} \qquad (30)$$

If the profile along which $S_A = 0$ is considered, it can be seen that any water-surface profile intersecting this profile will have a horizontal slope at the point of intersection unless $Q = Q_c$, in which case the right side of Eq. 30 becomes indeterminate and the point of intersection becomes a singular point.

The profile along which $S_A = 0$ is, therefore, a transition profile and

$$S_t = -\left(\frac{dz}{dx}\right)_{S_A=0} = \frac{\dfrac{\partial S_A}{\partial x}}{\dfrac{\partial S_A}{\partial z}} \tag{31}$$

As in the foregoing, the numerator and the denominator on the right side of Eq. 30 are replaced by their derivatives with the result that

$$\frac{dz}{dx} = \frac{1}{2}\,\frac{Q_c^3}{Q^2}\,\frac{dS_A}{dQ_c} \tag{32a}$$

$$\frac{dz}{dx} = \frac{1}{2}\,\frac{Q_c^3}{Q^2}\,\frac{\dfrac{\partial S_A}{\partial x} + \dfrac{\partial S_A}{\partial z}\dfrac{dz}{dx}}{\dfrac{\partial Q_c}{\partial x} + \dfrac{\partial Q_c}{\partial z}\dfrac{dz}{dx}} \tag{32b}$$

or

$$\frac{dz}{dx} = \frac{1}{2}\,\frac{Q_c^3}{Q^2}\,\frac{\dfrac{\partial S_A}{\partial z}}{\dfrac{\partial Q_c}{\partial z}}\,\frac{S_t + \dfrac{dz}{dx}}{S_c + \dfrac{dz}{dx}} \tag{32c}$$

By substituting $-S$ for dz/dx and noting that for a singular point $Q = Q_c$,

$$S = -\frac{Q_c}{2}\,\frac{\dfrac{\partial S_A}{\partial z}}{\dfrac{\partial Q_c}{\partial z}}\,\frac{S_t - S}{S_c - S} \tag{32d}$$

To transform Eq. 32*d* further it will be shown that

$$\frac{\partial Q_c}{\partial x} = -\frac{Q_c}{2}\,\frac{\partial S_A}{\partial z} \tag{33}$$

Applying the operator, $\partial/\partial z$, to Eq. 7, the equation obtained is

$$\frac{\partial S_A}{\partial z} = \frac{\dfrac{\partial A}{\partial z}\dfrac{\partial^2 A}{\partial x\,\partial z} - \dfrac{\partial A}{\partial x}\dfrac{\partial^2 A}{\partial z^2}}{\left(\dfrac{\partial A}{\partial z}\right)^2} \tag{34}$$

However, $\partial A/\partial x = 0$ on a transition profile, and, therefore,

$$\frac{\partial S_A}{\partial z} = \frac{\dfrac{\partial^2 A}{\partial x\,\partial z}}{\dfrac{\partial A}{\partial z}} \tag{35}$$

The critical discharge, Q_c, is given by

$$Q_c = A\sqrt{\frac{g\,A}{W}} \tag{36}$$

in which W is the width of the channel at the water surface. In view of the relationship, $W = \partial A/\partial z$, Eq. 36 becomes

$$Q_c = A\sqrt{\frac{g\,A}{\frac{\partial A}{\partial z}}} \tag{37}$$

Applying the operator, $\partial/\partial x$, to Eq. 37 renders

$$\frac{\partial Q_c}{\partial x} = \frac{1}{2}\left(\frac{g\,A^3}{\frac{\partial A}{\partial z}}\right)^{-\frac{1}{2}} \frac{3\,g\,A^2\,\frac{\partial A}{\partial x}\,\frac{\partial A}{\partial z} - g\,A^3\,\frac{\partial^2 A}{\partial x\,\partial z}}{\left(\frac{\partial A}{\partial z}\right)^2} \tag{38}$$

Because $\partial A/\partial x = 0$ on the transition profile, Eq. 38 reduces to Eq. 33. Making the appropriate substitution from Eq. 33 into Eq. 32*b* yields

$$S = S_c\,\frac{S_t - S}{S_c - S} \tag{39}$$

which is the desired equation.

In comparing Eq. 39 with Eq. 25, a singular point in a frictionless channel is a special case among those for which $S_m = S_c$. This implies that $\rho = 1$, and from Fig. 3 it is seen that the singular points for a frictionless channel are all either saddlebacks or centers.

GRAPHICAL CONSTRUCTION OF TRANSITION PROFILES

The mathematical methods described previously have shown some of the more important properties of transition profiles. However, they have not provided a practical way of plotting these profiles. A graphical method is presented in which use is made of the three functions,

$$F = \frac{1}{2\,g\,A^2} \tag{40a}$$

$$F' = \frac{1}{2\,g\,A^2} + \frac{L}{2\,K^2} \tag{40b}$$

and

$$F'' = \frac{1}{2\,g\,A^2} - \frac{L}{2\,K^2} \tag{40c}$$

These functions are plotted horizontally in rectangular coordinates against the water-surface elevation, z, which is plotted vertically. The use of these functions in the graphical determination of water-surface profiles has been described elsewhere[8,9,10] and will be mentioned only as it relates to frictionless channels. It suffices to note that, if the function F' for the cross section at the downstream end of a reach and the function F'' for the cross section at the upstream end are plotted, the intersection of the two curves will provide an approximate elevation for the transition profile at the midpoint of the reach. The accuracy of the estimate can be improved to any degree desired by suitably shortening the reach. The construction of a transition profile in this way is illustrated in Fig. 4(*a*).

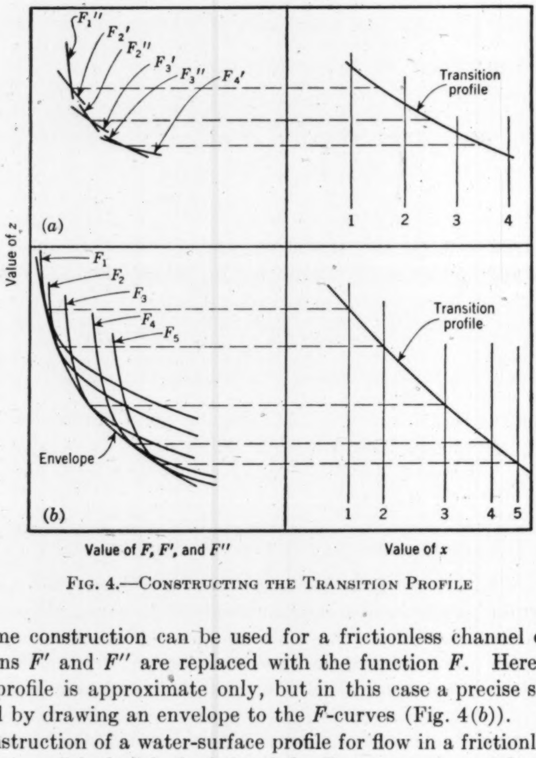


FIG. 4.—CONSTRUCTING THE TRANSITION PROFILE

The same construction can be used for a frictionless channel except that the functions F' and F'' are replaced with the function F. Here again, the transition profile is approximate only, but in this case a precise solution can be obtained by drawing an envelope to the F-curves (Fig. 4(*b*)).

The construction of a water-surface profile for flow in a frictionless channel is readily accomplished with the help of the F-curves. A straight line having a slope equal to $-Q^2$ and drawn so as to intersect the F-curves will determine, by these intersections, a water-surface profile, and the elevation of that profile in any cross section will be the same as the elevation of the intersection on the

---

[8] "Determination graphique de la ligne d'eau et calcul des remous," by N. Raytchine and P. Chatelain, *La Houille Blanche*, No. 3, May–June, 1950.

[9] *Engineering Manual*, Chapter 9, Pt. CXVI, Civ. Works Constr., Corps of Engrs., U. S. Dept. of the Army, Washington, D. C., May, 1952.

[10] "Backwater Profiles Solved by Escoffier-Raytchine-Chatelain Method," by John R. Stipp, *Civil Engineering*, August, 1953, p. 63.

corresponding F-curve. The intersection of the same straight line with the z-axis will also determine the energy head for the water-surface profile. An important characteristic of the foregoing construction is that if flow at critical depth occurs in any of the cross sections represented in the diagram, the straight line in question will be tangent to the corresponding F-curve. Figs. 5(a) and 5(b) illustrate a saddleback and a center, respectively, and show how the construction is effected. In Fig. 5(a), line AB determines the two hyperbola-like profiles; line CD, the two profiles that pass through the singular point; and line EF, the two profiles that pass above and below the singular point. Because flow at critical depth occurs at points O, M, and N, line CD is tangent to curve F_3, and line AB is tangent to curves F_2 and F_4. In Fig. 5(b), line IJ determines the oval-shaped profile. Flow at critical depth occurs in cross sections 1 and 5, and line IJ is, accordingly, tangent to curves

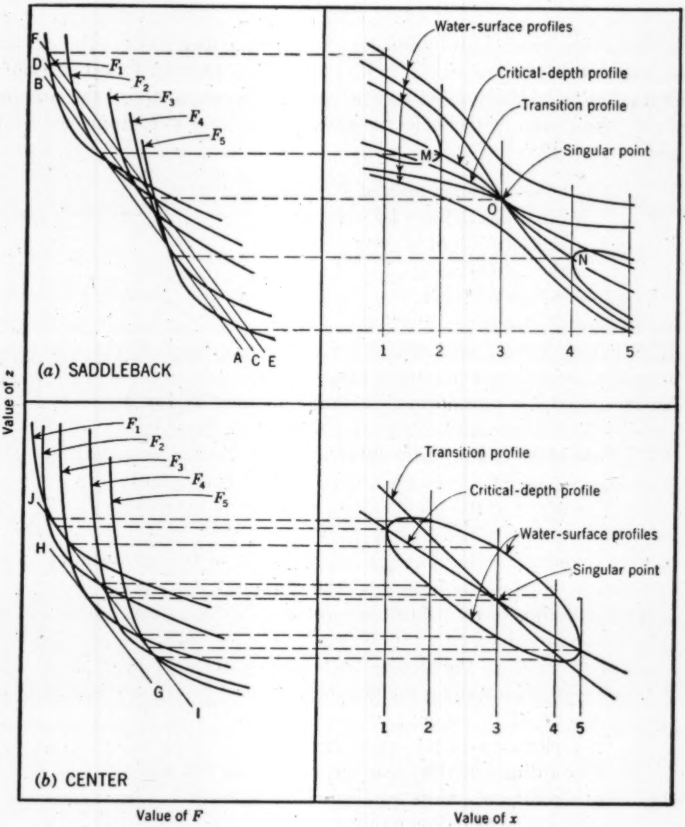

F
D
B
F_1
F_2
F_3
F_4
F_5
A C E
(a) SADDLEBACK
Water-surface profiles
Critical-depth profile
Transition profile
Singular point
M
O
N
1
2
3
4
5
Value of z
F_1
F_2
F_3
F_4
F_5
J
H
G
I
(b) CENTER
Transition profile
Critical-depth profile
Water-surface profiles
Singular point
1
2
3
4
5
Value of F
Value of x

FIG. 5.—SADDLEBACK AND CENTER IN A FRICTIONLESS CHANNEL

F_1 and F_5. The tangency of line GH to curve F_3 implies flow at critical depth at the singular point. However, because this line does not intersect any other F-curve, no water-surface profile passes through the singular point.

APPENDIX. NOTATION

The following symbols, adopted for use in this paper and for the guidance of discussers, conform essentially with "American Standard Letter Symbols for Hydraulics" (ASA Z10.2.—1942) prepared by a committee of the American Standards Association with Society representation, and were approved by the Association in 1942:

A = cross-sectional area of the channel;
F, F', F'' = functions defined by Eqs. 40;
g = acceleration of gravity;
K = the conveyance;
L = length of reach;
Q = discharge;
  Q_c = critical discharge;
  Q_n = normal discharge;
  Q_n' = paranormal discharge, defined by Eq. 9;
r = a dimensionless number, defined by Eq. 18;
S = slope of the water surface;
S_A = slope of the profile along which A is constant;
S_c = slope of the profile along which Q_c is constant;
S_m = slope defined by Eq. 24;
S_n = slope of the profile along which Q_n or Q_n' is constant;
S_o = slope of the bottom of the channel;
S_t = slope of the transition profile;
v = velocity;
W = width of the channel at the water surface;
x = distance along the channel;
y = depth of water;
z = elevation of the water surface;
λ = a dimensionless number, defined by Eq. 29;
ρ = a dimensionless number = S_c/S_m; and
σ = a dimensionless number = S_t/S_m.

DISCUSSION

Achille Lazard[11].—The paper permits the systematic extension of concepts such as "transition profiles" and "transition discharges."

The writer has no objection to the substitution of the word "transition" for that of "characteristic."[2] In creating the idea of characteristic depth, Mouret[3] merely specified that this depth is a characteristic of the channel bed and is independent of the discharge, in contrast to normal and critical depths in which the depths are dependent on the discharge.

In considering a characteristic discharge (the discharge for which the critical slope is equal to the slope of the channel), the writer wished to emphasize the fact that this discharge separates two types of flow in the channel. The expression "transition discharge" is certainly preferable to the expression "characteristic discharge."

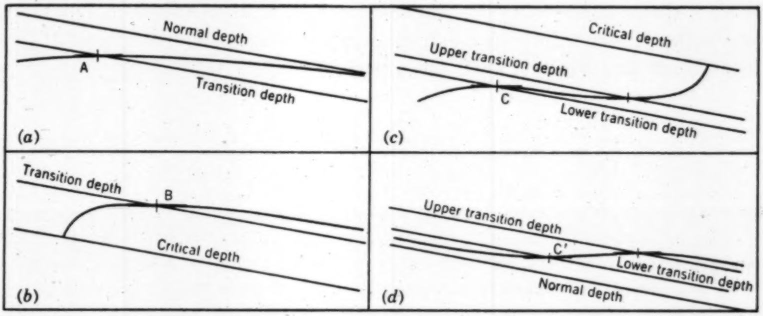


Fig. 6.—Various Types of Water-Surface Profiles

Because the water-surface profile is normally horizontal at its point of intersection with the transition profile, a name or an index should be provided for each section of the water-surface profile that can be defined. Mouret numbered the sections of water-surface profiles that he had studied with the arabic numerals 1 to 6.[3] (These correspond to the types, M_3, M_2, M_1, S_3, S_2, and S_1, in the order named, that are used by engineers in the United States.) The writer has designated new sections by A, B, C, and C′ (Fig. 6). Sections C and C′ are practically horizontal with an inflection between the two transition profiles.

In combining some of Mouret's sections with those of the writer, water-surface profiles very similar to those that are described in the classical manuals are obtained. However, Mouret's sections do not seem to represent real water surfaces.

---

[11] Chf. Engr. of Bridges and Roads, National Soc. of French Railways, Paris, France.

The author has in turn introduced new sections of water-surface profiles, some of which probably do not represent real water surfaces. It would be useful to designate the others in some convenient way.

It is important in numerical computations not to omit from the formulas the classical coefficient, α, which accounts for the unequal distribution of velocity about the mean velocity, V. This coefficient, about which very little is known, should in principle vary when the cross sections and the channel slopes vary. The evaluation of this term can present difficulties. In the case treated by the writer, in which the channel is uniform and the channel slope is constant, a constant value could be ascribed to α.

The introduction of this coefficient into the formulas modify profoundly the numerical values obtained for the transition depths and discharges because these values are determined by the intersections of lines that are nearly parallel. This is shown in the following tabulation, which contains values computed by the writer for the trapezoidal, tailrace canal for the Soulom powerhouse (Pyrenees Mountains, France) which has a slope of 0.004:

α	Transition depth, in centimeters	Transition discharge, in cubic meters per second
1.0	90	7
1.2	45	2

Laboratories should seek, by means of model experiments with canals of appropriate cross sections, to verify or disprove the theory of water-surface profiles for gradually varied flow.

LÉON J. TISON[12].—The paper is particularly interesting because several of the writer's predecessors at the University of Ghent in Belgium and the writer himself have previously made studies in the same field. The great merit of the paper lies in its extension of the theory in question to the field of nonuniform channels.

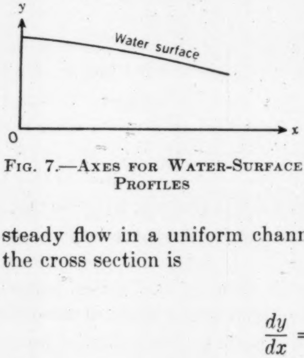


FIG. 7.—AXES FOR WATER-SURFACE PROFILES

In these studies of water-surface profiles, the axis of abscissa was taken parallel to the bed of the channel and the axis of ordinate, perpendicular thereto (Fig. 7). Under these conditions, the equation for steady flow in a uniform channel with a uniform distribution of velocities in the cross section is

$$\frac{dy}{dx} = \frac{S_o - \dfrac{P\,Q^2}{C^2\,A^3}}{\sqrt{1 - S_o^2} - \dfrac{W\,Q^2}{g\,A^3}} \quad \text{(41)}$$

---

[12] Prof. of Hydraulics, University of Ghent, Ghent, Belgium.

in which P is the wetted perimeter and C is the Chezy coefficient. The numerator on the right side of Eq. 41 vanishes for one or more values of y; these values will be denoted by Y_n. Similarly the denominator vanishes for a given value of y, which will be denoted by Y_c.

For a given discharge the condition, $Y_n = Y_c = Y_p$, will be satisfied for one value of the bottom slope and this value is designated as S_p. The slope, S_p, will be termed the passage slope. The quantities, S_p and Y_p, are given by

$$S_p = \frac{P_p\, Q^2}{C^2\, A_p{}^3} \qquad (42)$$

and

$$\sqrt{1 - S_p} = \frac{W_p\, Q^2}{g\, A_p{}^3} \qquad (43)$$

from which

$$\frac{S_p}{\sqrt{1 - S_p{}^2}} = \frac{g\, P_p}{W_p\, C^2} \qquad (44)$$

is obtained, which is the equation for the passage slope. In fact, either Eq. 42 or Eq. 43 must be used with Eq. 44 to determine S_p and Y_p for a given discharge and a given cross section (in a uniform channel). For a uniform channel, concave upward, and for a constant value of C, S_p and Y_p have but a single value each. Theoretically, C might vary in such a way that several values of S_p are possible. However, too little is known concerning the variation of C for small values of y and for circular conduits flowing nearly full to justify the assertion that several values are really possible.

The three different terms—horizontal elements, transition depths, and characteristic depths—relate to the same concept. The term "horizontal element" (élément d'horizontalité) was introduced in 1863 by M. Boudin.[13] For each form of water-surface profile (termed "hydraulic axes" by Boudin) obtained, Boudin sought to determine whether it presented a depth for which the tangent to the water-surface profile is horizontal. The condition to be satisfied is

$$\frac{S_o}{\sqrt{1 - S_o{}^2}} = \frac{dy}{dx} = \frac{S_o - \dfrac{P\, Q^2}{C^2\, A^3}}{\sqrt{1 - S_o{}^2} - \dfrac{W\, Q^2}{g\, A^3}} \qquad (45a)$$

which lead to

$$\frac{S_o}{\sqrt{1 - S_o{}^2}} = \frac{g}{C^2}\left(\frac{P}{W}\right)_k \qquad (45b)$$

That is, a horizontal element exists for the depth, y_k, for which the quantity, P/W, assumes the value, $(P/W)_k$, required by Eq. 45b.

---

[13] "De l'axe hydraulique des cours d'eau contenus dans un lit prismatique," by M. Boudin, *Annales des Travaux Publics de Belgique*, Vol. XX, 1863.

This is always impossible for depths that lie between the critical depth, Y_c, and the normal depth, Y_n, because dy/dx is negative in that region, and a horizontal element corresponds to the positive value,

$$\frac{dy}{dx} = \frac{S_o}{\sqrt{1 - S_o^2}}$$

Boudin dealt separately with mild and steep bottom slopes. For mild slopes,

$$\frac{S_o}{\sqrt{1 - S_o^2}} < \frac{g}{C^2}\left(\frac{P}{W}\right)_{y=Y_n} \qquad (46)$$

It follows from Eq. 45*b* that

$$\frac{g}{C^2}\left(\frac{P}{W}\right)_k < \frac{g}{C^2}\left(\frac{P}{W}\right)_{y=Y_n} \qquad (47)$$

and, as P/W increases in value with Y (for cross sections that are concave upward or circular), Boudin, assuming C to be constant, concluded that $Y_k < Y_n$. That is, a horizontal element can exist only for depths that are less than Y_n and, consequently, also less than Y_c.

For steep bottom slopes Boudin showed by similar reasoning that a horizontal element can exist only for depths that are greater than Y_n and, consequently, greater than Y_c.

He did not seek to find the conditions under which horizontal elements occur in the foregoing cases. However, the writer has shown in a paper[14] that in mild-slope channels (in which C is constant) a transition slope or transition depth will not occur if

$$\frac{S_o}{\sqrt{1 - S_o^2}} < \left(\frac{P}{W}\right)_0 \qquad (48)$$

in which $(P/W)_o$ is the value of P/W for $y = 0$. Thus, because P/W is an increasing function of y, it is impossible to satisfy Eq. 45*b* if the inequality of Eq. 48 holds true.

Similarly a horizontal element is impossible in a steep-slope channel if

$$\frac{S_o}{\sqrt{1 - S_o^2}} > \frac{g}{C^2}\left(\frac{P}{W}\right)_\infty \qquad (49)$$

Boudin's development was completed by A. Merten,[15] who clearly established several points in regard to passage slopes and stopped with the consideration of inflexion points, a matter closely related to that of transition depths. Boudin had recognized the existence of inflexion points for steep-slope channels, and Merten established the possibility of such points in mild-slope channels.

---

[14] "Cours d'Hydraulique," by L. J. Tison, University of Ghent, Louvain, 1953, pp. 19–20.

[15] "Recherches sur la forme des axes hydrauliques dans un lit prismatique," by A. Merten, Series 3, *Annales*, l'Association des Ingénieurs sortis des Écoles Spéciales de Gand, Belgium, Vol. V, No. 3, 1906.

The writer investigated[14] horizontal elements with the significant change of introducing a varying value of C. This coefficient, in reality, is not a constant but a function that increases with y. As a consequence for small values of y, the function, $g\,P/C^2\,W$, is not necessarily a diminishing function of y, but presents an appearance similar to that shown in Fig. 8. It follows that 0, 1, or 2 horizontal elements (transition depths) are possible, corresponding to $S_o/\sqrt{1 - S_o^2}$ less than, equal to, or greater than $(g\,P/C^2\,W)_{min}$. This result is equally applicable to cross sections that are concave upward and to circular cross sections.

It should always be remembered that the study of horizontal elements is, in practice, greatly complicated by the fact that the variation of C with y is not well understood for small values of y and for values of y approaching the diameter of a circular section. Tests performed by O. Wilcox[16] show, in the latter case, that the theoretical variation of the normal discharge with the depth of water does not correspond to reality.

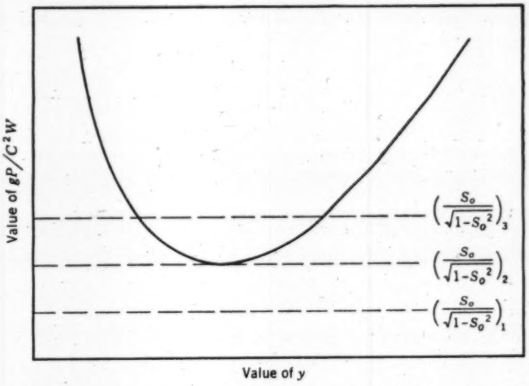


FIG. 8.—RELATIONSHIP OF C AND y

As the writer has already stated, Boudin was of the opinion that the condition for the occurrence of a horizontal element is represented by Eq. 45*b*, and, furthermore, that in the case of $S_o = S_p$ the tangent to the water-surface profile should be horizontal for a depth equal to the depth of passage, $Q_n = Q_c = Q$.

Nevertheless, this result is not general. In effect, Eq. 41, which may be written as

$$\frac{dy}{dx} = \frac{N}{D} \qquad (50)$$

---

[16] "A Comparative Test of the Flow of Water in 8-Inch Concrete and Vitrified Clay Sewer Pipe," by O. Wilcox, *Bulletin 27*, Eng. Experiment Station, Univ. of Washington, Seattle, Wash., 1924.

in which N and D represent the numerator and denominator, respectively, of the right side of that equation, yields for $y = Y_p$,

$$\frac{dy}{dx} = \frac{0}{0} \qquad (51)$$

The application of the rule of the Marquis de l'Hôpital yields

$$\frac{dy}{dx} = \frac{N'}{D'} \qquad (52)$$

The tangent to the water-surface profile for $y = Y_p$ will be horizontal if

$$\frac{S_o}{\sqrt{1 - S_o^2}} = \frac{N'}{D'} \qquad (53)$$

Eq. 53 is equivalent to $\frac{d\,Q_n}{d\,Q_c} = 1$ in the paper.

The foregoing equation is not always satisfied. Thus, in a rectangular channel having a width, W, it can be shown that

$$\frac{S_p}{\sqrt{1 - S_p^2}} > \left(\frac{dy}{dx}\right)_{Y_p} \qquad (54)$$

Therefore, it is clear that a horizontal element does not occur in that case. However, if W approaches infinity, the tangent approaches a horizontal position.

The normal-depth profiles and critical-depth profiles divide the space above the bottom of the channel into three regions. In the highest and the lowest of these regions the sign of dy/dx is positive, but in the intermediate region it is negative. A bottom slope equal to the passage slope causes the intermediate region to vanish, and, as a consequence, dy/dx must always be positive. All this relates to a uniform channel concave upward. As a result, it follows that in this case a water-surface profile cannot pass through the depth, Y_p, with diminishing depths. This conclusion can be considered as a logical consequence of another property of uniform channels that are concave upward —that in such channels a chute (abrupt passage from a depth greater than Y_c to one less than Y_c) is impossible, whereas a jump, the inverse phenomenon, is possible.

By contrast, in the case of a circular section, the existence of a passage slope is accompanied by a region in which dy/dx takes on negative values so that a water-surface profile with diminishing depths is possible.

Without wishing to detract from the author's valuable work in this field, the writer would like to indicate another method for the study of transition profiles in nonuniform channels, a method that follows logically from the foregoing.

For a nonuniform channel, Eq. 41 becomes

$$\frac{dy}{dx} = \frac{S_o - \frac{P\,Q^2}{A^3\,C^2} + \frac{Q^3}{g\,A}\frac{\partial A}{\partial x}}{\sqrt{1 - S_o^2} - \frac{Q^2\,W}{g\,A^3}} \qquad (55)$$

A horizontal element corresponds to the condition,

$$\frac{S_o}{\sqrt{1 - S_o^2}} = \frac{\frac{P}{A^3}\frac{Q^2}{C^2} - \frac{Q^2}{g}\frac{\partial A}{A^3\,\partial x}}{\frac{Q^2}{g}\frac{W}{A^3}} \qquad (56a)$$

or

$$\frac{S_o}{\sqrt{1 - S_o^2}} = \frac{g\,P}{C^2\,W} - \frac{1}{W}\frac{\partial A}{\partial x} \qquad (56b)$$

This result can be used to investigate channels with simple cross sections (rectangular, for example).

Francis F. Escoffier,[17] A. M. ASCE.—It appears that varied terminology exists in regard to the depth which the writer has termed "transition depth." In the studies by Mouret and Mr. Lazard this depth is called the "characteristic depth." In the studies by Boudin, Merten, and Mr. Tison a distinction is made between the case in which the tangent to the water-surface profile is horizontal and the case in which it is not. In the first case, $Q_n = Q_c \neq Q$, or $Q_n = Q_c$ and $dQ_n/dQ_c = 1$, and the segment of the water-surface profile passing through the depth in question is called a horizontal element. Although no specific name is assigned to this depth, it is identified by the subscript, n, as $y = Y_n$. In the second case, $Q_n = Q_c = Q$, and the depth is termed a "passage depth." It appears, therefore, that for a given channel (with a given bottom slope) the depth in either case is the same and, thus, the term "passage depth" could be used in both cases.

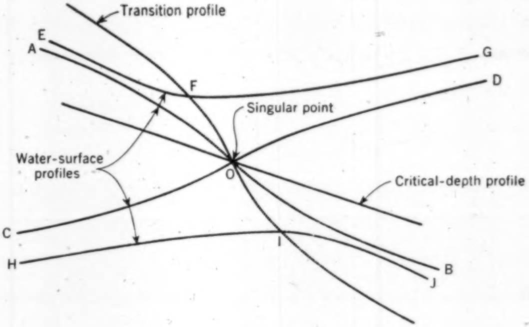


Fig. 9.—Types of Transitions for a Saddleback

Mr. Lazard has expressed doubt as to the reality of some of the water-surface profiles obtained by Mouret and by the writer. The writer emphasizes the fact that his profiles, and no doubt those of Mouret, were deduced logically from the dynamic equation for flow in open channels (Eq. 5) and are, therefore, as valid as that equation. Possibly the most significant source of error arises from the neglect of the curvature of filaments. For that reason the profiles become completely unreliable in the vicinity of a passage through critical depth if the equation yields a vertical water surface.

---

[17] Hydr. Engr., Corps of Engrs., U. S. Dept. of the Army, Mobile Dist., Mobile, Ala.

The importance of the velocity-distribution coefficient stressed by Mr. Lazard is conceded. However, it should be noted that the neglect of this coefficient is not ordinarily as great a source of error in nonuniform channels as it is in uniform channels. This coefficient can be introduced into the computations by rewriting Eqs. 40*b* and 40*c* as

$$F' = \frac{\alpha}{2\,g\,A^2} + \frac{L}{2\,K^2} \qquad (57a)$$

and

$$F'' = \frac{\alpha}{2\,g\,A^2} - \frac{L}{2\,K^2} \qquad (57b)$$

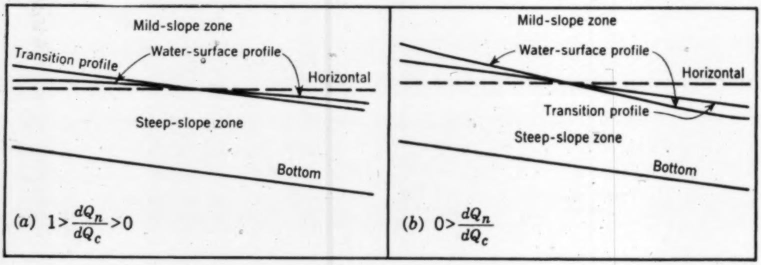

Mild-slope zone
Transition profile
Water-surface profile
Horizontal
Steep-slope zone
Bottom
(a) $1 > \frac{dQ_n}{dQ_c} > 0$
Mild-slope zone
Water-surface profile
Horizontal
Transition profile
Steep-slope zone
Bottom
(b) $0 > \frac{dQ_n}{dQ_c}$

FIG. 10.—TRANSITION PROFILE IN A UNIFORM CHANNEL

The writer shares Mr. Lazard's wish that laboratories become interested in the various water-surface profiles arising from the theory of transition profiles and undertake to study them in models.

The writer was interested in learning of the work done by Boudin, Merten, and Mr. Tison on the subject of transition depths and regrets that research preliminary to the writing of his own paper failed to disclose this literature. The early date of Boudin's work (1863) is significant.

Mr. Lazard has suggested that the different types of water-surface profiles that arise in the theory of transition profiles be named or designated in some way. Unfortunately, the number of such types is so great that the list of names would be too long to be useful. However, it is felt that it would be helpful to distinguish between four types of transitions. These are illustrated by means of the saddleback shown in Fig. 9. The four terms and their counterparts in Fig. 9 are as follows:

Transitional minimum	EFG
Transitional maximum	HIJ
Transitional drop	AOB
Transitional rise	COD

The first two terms may be called "nonsingular" transitions because F and I, the points of intersection with the transition profile, are not singular points. On the other hand, the last two terms may be called "singular" transitions because they pass through the singular point, O.

Mr. Tison's comments in regard to the possibility of a water-surface profile passing with diminishing depths through a transition profile (in a uniform channel) were made with Fig. 1(*b*) in mind. However, this figure is quite misleading because it really corresponds to the condition, $0 > dQ_n/dQ_c$, rather than simply $1 > dQ_n/dQ_c$, and represents a type of water-surface profile than can occur only in a closed conduit that is flowing nearly full. Fig. 10, which supersedes Fig. 1(*b*), was prepared to clarify this matter. The water-surface profiles in Fig. 1(*a*) and Fig. 10(*a*) pass from rapid to tranquil flow, whereas the water-surface profile in Fig. 10(*b*) passes from tranquil to rapid flow.

Mr. Tison's Eqs. 55 and 56 provide another method for investigating the properties of transition profiles. They include the factor, $\sqrt{1 - S_o^2}$, to correct for the fact that the channel cross sections are not vertical. The importance of this correction increases with the slope of the channel.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2910

MINIMUM-WEIGHT DESIGN OF A PORTAL FRAME

BY WILLIAM PRAGER,[1] M. ASCE

WITH DISCUSSION BY MESSRS. ROBERT L. KETTER, AND WILLIAM PRAGER

SYNOPSIS

Plastic design of structural frames for minimum weight has been the subject of several papers. In these the weight per unit length of a structural member has been assumed to be proportional to its fully plastic moment, although the effect of deviations from this simple relationship has occasionally been examined qualitatively. This paper is concerned with the minimum-weight design of frames under the assumption that the unit weight of a structural member is proportional to the α-th power of its fully plastic moment, the positive exponent, α, being smaller than unity. For $\alpha = 2/3$, a chart is developed that gives, at a glance, the minimum-weight design for various geometries and loading conditions of a portal frame.

INTRODUCTION

Using concepts of limit analysis, several authors[2,3,4] have explored the design of structural frames for minimum weight. J. Heyman[5,6] showed that the method of inequalities, which Bernard George Neal and Paul Southworth

---

NOTE.—Published, essentially as printed here, in October, 1956, in the Journal of the Engineering Mechanics Division, as *Proceedings Paper 1073*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Prof. of Applied Mechanics, Brown Univ., Providence, R. I.

[2] "The Design of Steel Frames," by J. F. Baker, *The Structural Engineer*, London, Vol. 27, 1949, pp. 397–431.

[3] "Limit Design of Beams and Frames," by H. J. Greenberg and W. Prager, *Transactions*, ASCE, Vol. 117, 1952, pp. 447–484.

[4] "Extended Limit Design Theorems for Continuous Media," by D. C. Drucker, W. Prager, and H. J. Greenberg, *Quarterly of Applied Mathematics*, Vol. 9, 1952, pp. 381–389.

[5] "Plastic Design of Beams and Plane Frames for Minimum Material Consumption," by J. Heyman, *Quarterly of Applied Mathematics*, Vol. 8, 1951, pp. 373–381.

[6] "Plastic Design of Plane Frames for Minimum Weight," by J. Heyman, *The Structural Engineer*, London, Vol. 31, 1953, pp. 125–129.

Symonds,[7] M. ASCE, developed for the determination of the load-carrying capacity of a frame of given dimensions, could be easily adapted to the problem of designing a frame for minimum weight if the weight per unit length of a structural member could be taken as proportional to the fully plastic bending moment of this member. J. Foulkes[8] noted that under this assumption the minimum-weight design of a structural frame constituted a problem in linear programming, as described[9] by A. Charnes, W. W. Cooper, and A. Henderson. In a later paper,[10] Foulkes established the prerequisites for a minimum-weight design, assuming again that the unit weight of a member was proportional to its fully plastic moment. Making the same assumption, R. K. Livesley[11] used a different approach and studied the programming of the problem for an automatic computer.

For most structural sections the unit weight is nearly proportional to a power of the fully plastic moment, with a positive exponent, α, that is smaller than unity. For example, $\alpha = 2/3$ if all available sections are geometrically similar. Although Heyman and Foulkes have taken $\alpha < 1$ in a few simple examples, their over-all results are based on the assumption that $\alpha = 1$. Therefore, it seems worth while to reconsider the problem of minimum-weight design in terms of such general assumptions regarding the relationship between the unit weight and the fully plastic moment. The subsequent statements (under the heading, "Theory") are based on the premise that the unit weights of the available structural sections are proportional to a certain power of their fully plastic moments, the positive exponent being smaller than unity.

THEORY

The plastic design of beams and frames is based on an idealized relationship between bending moment and curvature. Accordingly, the curvature is negligible at all cross sections in which the absolute value of the bending moment remains below a critical value, M_o; however, a yield hinge may develop at any cross section in which the absolute value of the bending moment reaches this critical value. Thus, a beam or frame will remain practically rigid until yield hinges have developed at a sufficient number of cross sections to transform the structure, or some part of it, into a mechanism. The appearance of such a system of yield hinges is considered, therefore, as an indication that the practical load-carrying capacity of the frame has been reached.

A system of infinitesimal displacements, made possible by the insertion of an adequate number of yield hinges into the otherwise rigid members of the structure, specifies a flow mechanism. Two systems of infinitesimal displacements, obtained from each other by multiplication with a constant factor, are regarded as determining the same flow mechanism.

---

[7] "The Calculation of Collapse Loads for Framed Structures," by Bernard George Neal and Paul Southworth Symonds, *Journal*, Inst. C. E., London, Vol. 35, 1950–1951, pp. 21–40.

[8] "Minimum-Weight Design and the Theory of Plastic Collapse," by J. Foulkes, *Quarterly of Applied Mathematics*, Vol. 10, 1953, pp. 347–358.

[9] "An Introduction to Linear Programming," by A. Charnes, W. W. Cooper, and A. Henderson, John Wiley & Sons, Inc., New York, N. Y., 1953.

[10] "The Minimum-Weight Design of Structural Frames," by J. Foulkes, *Proceedings*, Royal Soc. of London, Series A, Vol. 223, 1954, pp. 482–494.

[11] "The Automatic Design of Steel Frames for Minimum Weight," by R. K. Livesley, *Quarterly Journal of Mechanics and Applied Mathematics*, London, Vol. 9, 1956, pp. 257–278.

Given loads are beyond the load-carrying capacity of a beam or frame if a flow mechanism exists for which the work of these loads exceeds the energy dissipated in the plastic bending at the yield hinges. Conversely, the absence of such a flow mechanism indicates that the given loads are within the load-carrying capacity of the structure.[3] A flow mechanism for which the energy dissipated in the yield hinges equals the work of the given loads will be termed a failure mechanism for these loads.

The following comments are restricted to frames that consist of straight prismatic members subjected to concentrated loads only. In a frame of this kind, yield hinges can occur only at loaded sections or at end sections of members, and the number of possible flow mechanisms remains finite. Because, for practical purposes, any curved member of varying cross section may be approximated by a polygon of straight prismatic members, and any distributed load may be approximated by an equipollent group of concentrated loads, the subsequent remarks need not be understood nor applied in a limited sense.

In the design problem considered herein the geometrical arrangement of the members is given. The fully plastic moments of the members are to be determined so that: (1) The given loads just exhaust the load-carrying capacity of the frame; and (2) the total weight of the frame is as small as possible, the unit weight, w_i, of a generic member being $w_i = c\,M_i^{\alpha}$ (in which c has the same value for all members, M_i is the fully plastic moment of the considered member, and the exponent, α, has the same value for all members and satisfies $0 < \alpha < 1$). It is assumed that n fully plastic moments are at the choice of the designer although the frame may have more than n-members. For instance, considerations of structural symmetry may cause a reduction in the number of independent fully plastic moments.

For the geometrical analysis of the design problem, the fully plastic moments, M_i $(i = 1, 2, \ldots, n)$, are taken as the rectangular Cartesian coordinates in an n-dimensional space ("design space"). Any point (design point) in the positive orthant, $M_i \geq 0$, of this space represents a specific design although not necessarily one that satisfies the conditions stipulated previously.

For any flow mechanism the condition that the work of the given loads must not exceed the energy dissipated in the yield hinges is expressed by a linear inequality in the fully plastic moments. Interpreted geometrically, such an inequality specifies a half space. The given loads exceed the load-carrying capacity of any design represented by a design point situated outside this half space. The positive orthant of design space and the half spaces corresponding to the various flow mechanisms have a convex region in common. Only design points on the boundary of this region are admissible if the given loads are just to exhaust the load-carrying capacity of the frame. Because the number of possible flow mechanisms is finite, this boundary is a polyhedron with a finite number of $(n - 1)$-dimensional faces. A design will be called admissible if it is represented by a design point on this polyhedral surface.

If l_i denotes the combined length of all members with the fully plastic moment, M_i, the structural weight of the frame is

$$W = c\,(l_1 M_1^{\alpha} + l_2 M_2^{\alpha} + \ldots + l_n M_n^{\alpha}) \quad \ldots\ldots\ldots\ldots (1)$$

For a fixed positive value of W, Eq. 1 represents a surface in design space that is convex with respect to the origin because $0 < \alpha < 1$. From this surface, the surface corresponding to any other value of W is obtained by a similarity transformation with the origin as the center.

Fig. 1 shows a typical situation for $n = 2$. The curves correspond to various values of the structural weight, W, and any point on the polygon ABCDE indicates a design whose load-carrying capacity is just exhausted by the given loads. Because the polygon and the curves are convex with respect to the origin, O, the design of minimum weight must be represented by a vertex, namely, point C in Fig. 1. Because several vertices of the polygon may fall on the same curve of constant weight, the problem of minimum-weight design may have several solutions.

Each side of the polygon corresponds to a failure mechanism in the sense that all points of a side, except the end points, represent designs that develop the same failure mechanism under the given load. A vertex, however, represents a design that may fail in either of the mechanisms corresponding to the adjacent sides, or in any positive, linear combination of these mechanisms. Because the minimum-weight design is represented by a vertex, it admits of such a family of failure mechanisms.

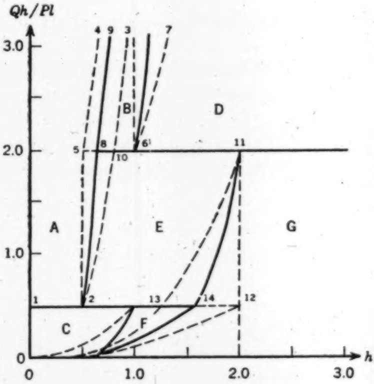


FIG. 1

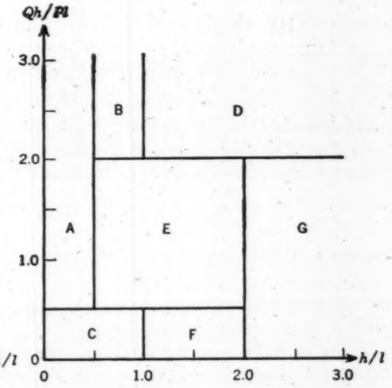


FIG. 2

In Fig. 1 any of these failure mechanisms is represented by a straight line through the considered vertex, which has only this point in common with the polygon. If the vertex represents a minimum-weight design, these lines include the tangent to the curve of constant weight through the vertex. The equation of this tangent is

$$M_1 \frac{\partial W}{\partial M_1} + M_2 \frac{\partial W}{\partial M_2} = \text{constant} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which the derivatives must be evaluated at the given vertex.

For any failure mechanism the balance between the work, L, of the given loads and the energy dissipated in the yield hinges is expressed by

$$M_1 \theta_1 + M_2 \theta_2 = L \quad (3)$$

in which θ_i denotes the sum of the absolute values of the angle changes at all yield hinges with the fully plastic moment, M_i. For the failure mechanism represented by the considered tangent, the coefficients of the fully plastic moments in Eq. 3 must be proportional to the coefficients of these moments in Eq. 2. Because of Eq. 1 this means that a minimum-weight design admits of a failure mechanism for which

$$\theta_1 : \theta_2 = l_1 M_1^{\alpha-1} : l_2 M_2^{\alpha-1} \quad (4)$$

Although necessary, this condition is not sufficient for the considered design to be admissible as a minimum-weight design. For example, the design represented by point B in Fig. 1 admits of a failure mechanism satisfying Eq. 4 because the tangent of the curve of constant weight through point B has only this point in common with the polygon. Nevertheless, the structural weight of design B exceeds that of design C. The latter is characterized by the facts that it admits of a failure mechanism satisfying Eq. 4 and that no admissible design of lesser weight admits of such a failure mechanism.

The situation is much the same for $n > 2$. The prerequisites for a minimum-weight design are: (1) It must admit of a failure mechanism satisfying

$$\theta_1 : \theta_2 : \cdots : \theta_n = l_1 M_1^{\alpha-1} : l_2 M_2^{\alpha-1} : \cdots : l_n M_n^{\alpha-1} \quad (5)$$

and (2) there must not be any admissible design of lesser weight admitting of such a failure mechanism.

EXAMPLE

As in the paper by Foulkes,[9] the portal frame shown in Fig. 2 is considered as the example, and it is proposed herein to derive a design chart that exhibits the dependence of the minimum-weight design on the ratios, h/l and $(Q\,h)/(P\,l)$. The coefficient α in Eq. 1 will be considered as 2/3.

For positive values of loads P and Q, only the flow mechanisms of Fig. 3 and their positive linear combinations need to be considered. The following inequalities correspond to mechanisms a through e (Fig. 3) and must be fulfilled if the given loads are not to exceed the load-carrying capacity of the frame:

$$4\,M_1 \geq P\,l \quad (6)$$

$$2\,M_1 + 2\,M_2 \geq P\,l \quad (7)$$

$$2\,M_1 + 2\,M_2 \geq Q\,h \quad (8)$$

$$4\,M_2 \geq Q\,h \quad (9)$$

$$4\,M_1 + 2\,M_2 \geq P\,l + Q\,h \quad (10)$$

$$2\,M_1 + 4\,M_2 \geq P\,l + Q\,h \quad (11)$$

To examine the import of these inequalities, assume first that $M_1 < M_2$. Any yield hinge that forms at a corner of the frame will then be in the beam rather than in the column. This means that only mechanisms a, c, and e (Fig. 3) need to be considered when $M_1 < M_2$. Inspection of the inequalities corresponding to these mechanisms reveals that the inequality expressed by Eq. 10 is more critical than the sum of Eqs. 6 and 8. Therefore, it is unnecessary to investigate the combination of mechanisms a and c.

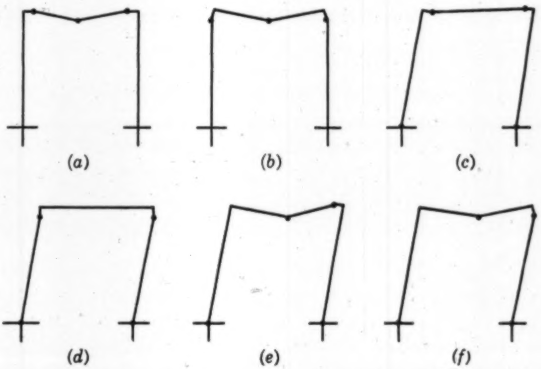


FIG. 3

Considering next the combination of mechanisms a and e, if both are to be failure mechanisms,

$$M_1 = \frac{P\,l}{4} \qquad (12)$$

and

$$M_2 = \frac{Q\,h}{2} \qquad (13)$$

Because it was assumed that $M_1 < M_2$, it follows from Eq. 12 that

$$\frac{Q\,h}{P\,l} > \frac{1}{2} \qquad (14)$$

With $\alpha = 2/3$, the condition expressed by Eq. 4 becomes

$$\frac{\theta_2}{\theta_1} = \frac{h}{l}\left(\frac{M_1}{M_2}\right)^{1/3} \qquad (15)$$

In Eq. 6 the ratio of the coefficients of M_2 and M_1, θ_2/θ_1, has the value of 0; in Eq. 10 it has the value of $\frac{1}{2}$. Because mechanisms a and e are combined with arbitrary positive coefficients, θ_2/θ_1 varies from 0 to $\frac{1}{2}$. According to Eq. 15,

$$0 \leq \frac{h}{l}\left(\frac{M_1}{M_2}\right)^{1/3} \leq \frac{1}{2} \qquad (16)$$

The substitution of M_1 and M_2 of Eq. 12 into Eq. 16 furnishes

$$0 \leq \left(\frac{h}{l}\right)^3 \leq \frac{1}{4}\frac{Q\,h}{P\,l} \qquad (17)$$

In the design chart of Fig. 4, h/l and the ratio, $(Q\,h)/(P\,l)$, are used as rectangular coordinates. Because these are restricted to positive values, a specific geometry of the frame, h/l, and a type of loading, Q/P, are represented by a point in the first quadrant. The heavy lines divide this quadrant into regions labeled from A through G. For each region the minimum-weight design is specified by one of the following:

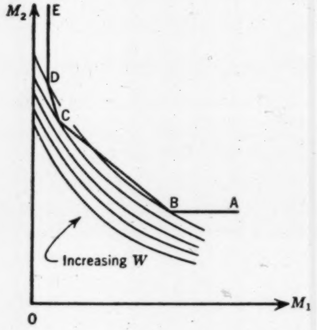


FIG. 4

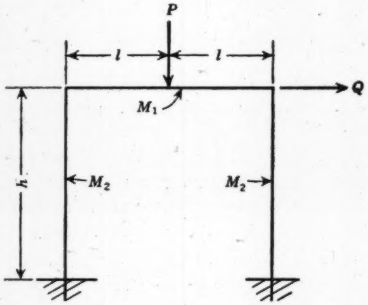


FIG. 5

Region A: $M_1 = P\,l/4$, $M_2 = Q\,h/2$
Region B: $M_1 = P\,l/2$, $M_2 = (Q\,h - P\,l)/2$
Region C: $M_1 = M_2 = P\,l/4$
Region D: $M_1 = M_2 = Q\,h/4$
Region E: $M_1 = M_2 = (P\,l + Q\,h)/6$
Region F: $M_1 = (P\,l - Q\,h)/2$, $M_2 = Q\,h/2$
Region G: $M_1 = P\,l/2$, $M_2 = Q\,h/4$

Each of these regions corresponds to the combination of two of the flow mechanisms shown in Fig. 3. Region A, for instance, corresponds to the combination of mechanisms a and e, which has been investigated in the foregoing. According to Eqs. 14 and 17, region A cannot extend below line $Q\,h/P\,l = \frac{1}{2}$, to the left of line $h/l = 0$, or to the right of line $(h/l)^3 = Q\,h/4\,P\,l$. In Fig. 4 these are line 1–2, the vertical axis, and the curve 2–3.

Region B corresponds to the combination of mechanisms c and e. If c and e are to be failure mechanisms,

$$M_1 = \frac{P\,l}{2} \qquad (18)$$

and

$$M_2 = \frac{Q\,h - P\,l}{2} \tag{19}$$

An examination similar to that presented for region A shows that region B cannot extend beyond broken line 4–5–6–7 in Fig. 4.

At this point a frame geometry and a type of loading that are represented by a point between line 4–5 and line 3–10 are considered. The foregoing would indicate that the design specified by Eqs. 12 and 13, as well as the design specified by Eqs. 18 and 19, admits of a failure mechanism satisfying Eq. 4. In general, however, these two designs will have different structural weights; thus, only one of them is a minimum-weight design. Only when the considered point lies on curve 8–9 do the two designs have the same structural weight. To the left of this curve the first design is lighter, and to the right of this curve the second design is lighter. Therefore, curve 8–9 is the common boundary of regions A and B. Its equation is found by setting the weight of the first design,

$$W_A = c\left[2\,l\left(\frac{P\,l}{4}\right)^{2/3} + 2\,h\left(\frac{Q\,h}{2}\right)^{2/3}\right] \tag{20}$$

equal to the weight of the second design,

$$W_B = c\left[2\,l\left(\frac{P\,l}{2}\right)^{2/3} + 2\,h\left(\frac{Q\,h - P\,l}{2}\right)^{2/3}\right] \tag{21}$$

The boundaries of all regions in Fig. 4 can be found in this manner. Regions E and G, for instance, correspond to the mechanism combinations ef and df, respectively. From Eq. 4 it is found that region E cannot extend to the right of broken line 11–12, and region G cannot extend to the left of curve 11–13. By equating the structural weights of designs E and G, the common boundary of these regions, curve 11–14, is obtained.

Foulkes[10] has derived a similar design chart for $\alpha = 1$. This is shown in Fig. 5; the formulas specifying the minimum-weight design for each region are the same as those given in Fig. 4. Comparison of Fig. 4 and Fig. 5 reveals that the change from $\alpha = 1$ to the more realistic value, $\alpha = \frac{2}{3}$, does not change the general arrangement of the regions although it introduces considerable distortion.

CONCLUSIONS

Plastic design for minimum weight is usually based on the simplifying assumption that the weight per unit length of a structural member is proportional to its fully plastic moment. For the portal frame analyzed by Foulkes,[10] the more realistic assumption that the unit weight is proportional to the $\frac{2}{3}$-th power of the fully plastic moment leads to a design chart having the same general arrangement of design region as Foulkes' chart but involving considerable distortion of the latter. Accordingly, both assumptions regarding the unit weight lead to the same minimum-weight design only if the frame specifications are represented by a point well in the interior of one of the design regions. When the specifications do not fulfil this condition, the usual simplifying assumption regarding unit weight may not yield the true minimum-weight design. However, because the points on the border between adjacent design regions represent specifications for which the corresponding designs have equal weights, the difference in weight between the true minimum-weight design and the design obtained from the usual simplifying assumption is likely to be small. This remark justifies the use of the simplifying assumption for practical purposes.

ACKNOWLEDGMENTS

The results presented herein were obtained in the course of research sponsored by the Office of Naval Research, United States Department of the Navy, under Contract Nonr-562 (10).

The writer is indebted to Livesley for supplying an advance copy of his paper.[11]

DISCUSSION

Robert L. Ketter,[12] J. M. ASCE.—In discussing the problem of the minimum-weight design of portal frames, it should be noted that seldom, if ever, will a structure as found in practice be subjected to only one condition of loading. Therefore, it is necessary to determine or to check the required size of the component parts of the frame for each of the possible loading configurations. The problem becomes even more involved for the case of multiple-span structures.

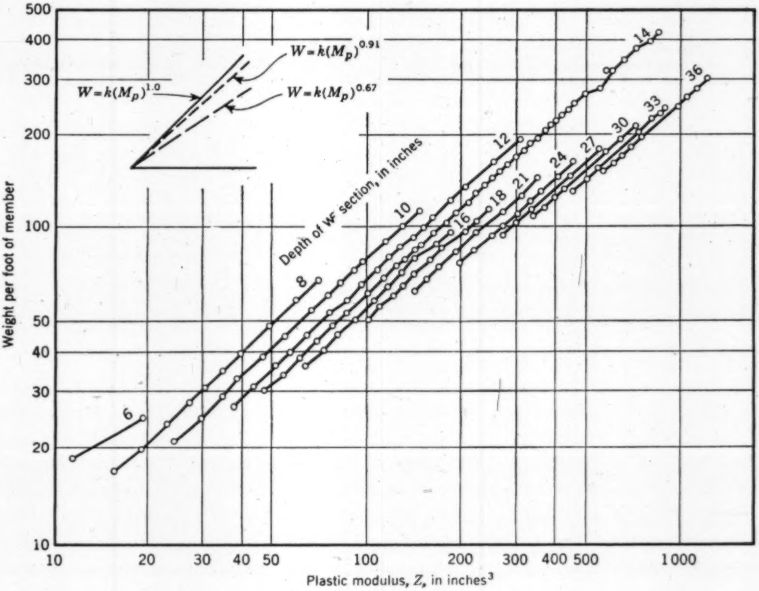


Fig. 6

As a second and final point of this discussion, it is desirable to show how well the value of $\alpha = 2/3$ used by Mr. Prager checks currently (1957) available structural sections. In Fig. 6 the weight per foot of members is plotted versus the plastic modulus (which is proportional to the full plastic moment, M_p) for each of the rolled WF shapes. As indicated, within a given range of nominal depth of section, weight increases approximately as the plastic modulus raised to the 0.91-power. Considering the lower extremities of these sections available—that is, the most economical sections—if the weight is less than approximately 40 lb, the weight increases approximately as the 0.55-power of the plastic modulus. For cases in which the weight is greater than 40 lb, $W = k(M_p)^{0.63}$ seems to be a better approximation. The value of $\alpha = 2/3$ used by the author is a reasonable value for the over-all range of available wide-flange shapes.

---

[12] Research Asst. Prof., Fritz Eng. Lab., Lehigh Univ., Bethlehem, Pa.

William Prager,[13] M. ASCE.—Thanks are extended to Mr. Ketter for his comments.

The purpose of the paper was to investigate the influence of the usual linearization of the expression for structural weight. For this purpose, it seemed sufficient to consider a single condition of loading. It should be noted that minimum-weight design for multiple conditions of loading has been treated by Foulkes.[14]

---

[13] Prof. of Applied Mechanics, Brown Univ., Providence, R. I.

[14] "Linear Programming and Structural Design," by J. Foulkes, *Proceedings*, 2d Symposium on Linear Programming, U. S. Bureau of Standards, Washington, D. C., Vol. I, 1955, pp. 177–184.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2911

DEVELOPMENTS IN SEPTIC TANK SYSTEMS

By John E. Kiker, Jr.,[1] M. ASCE

Synopsis

Some important changes in the design criteria for septic tanks and subsurface sewage-disposal systems have occurred. These changes are set forth and evaluated.

Introduction

Important changes in design criteria for septic tanks and subsurface sewage-disposal systems are being made (as of 1957). These changes are the result of field research, including studies and evaluations of available field data, and actual observations of field installations during construction and normal operation and after failures.

Public health workers, in general, and sanitary engineers, in particular, have become fairly familiar with the extensive studies that were begun in 1946 by the United States Public Health Service (USPHS) on household sewage-disposal systems. These studies resulted in three comprehensive progress reports.[2,3,4] Few people in the field are probably aware of the precautions being taken by the USPHS in attempting to correlate the results found at the Robert A. Taft Sanitary Engineering Center in Cincinnati (Ohio) with practical observations in the field, and in attempting to limit recommendations based on the studies to practices which may reasonably be expected under field conditions. To this end and to assist in the preparation of a new manual,[5] the USPHS has engaged consultants who have had extensive experience in the design, construction, operation, and repair of various types of septic tank systems. The addition of their knowledge to the considerable talents already within the USPHS promises that the manual will be of

---

Note.—Published, essentially as printed here, in October, 1956, in the Journal of the Sanitary Engineering Division, as *Proceedings Paper 1088*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Prof. of Civ. Eng., Univ. of Florida, Gainesville, Fla.

[2] "Studies on Household Sewage Disposal Systems," Pt. I, by S. R. Weibel, C. P. Straub, and J. R. Thoman, U. S. Govt. Printing Office, Washington, D. C., 1949.

[3] *Ibid.*, Pt. II, by T. W. Bendixen, M. Berk, P. J. Sheehy, and S. R. Weibel, U. S. Govt. Printing Office, Washington, D. C., 1950.

[4] *Ibid.*, Pt. III, by S. R. Weibel, T. W. Bendixen, and J. B. Coulter, U. S. Govt. Printing Office, Washington, D. C., 1954.

[5] "Manual of Septic Tank Practice," *U.S.P.H.S. Publication No. 526*, U. S. Govt. Printing Office, Washington, D. C., 1957.

particular value in reducing the number of failures of individual disposal systems.

This paper summarizes the more important changes which that will be examined in the new manual. There is no assurance that all the changes mentioned herein will be included in the manual because the final manuscript has not as yet (1957) been approved. Therefore, this report represents only the views of the writer.

SEPTIC TANK CAPACITY

Capacity is one of the most important considerations in the design of septic tanks. Contrary to popular belief, sewage solids that have been subjected to anaerobic treatment in a septic tank cause much less clogging than the same quantity of solids in the effluent from a primary settling tank where the detention time is short. Solids that are of a gelatinous nature in fresh sewage become more crystalline in sewage that has been subjected to anaerobic treatment during prolonged storage in a septic tank.

Large tanks providing for liberal sludge and scum storage capacities obviously have the additional advantage of permitting longer intervals between cleanings. In some cases, a 50% increase in capacity will double the time interval between cleanings; hence, liberal tank capacity is not only important from a functional standpoint but from the standpoint of economy. When it is further considered that septic tanks were designed for the treatment of domestic sewage before the development of household washing machines, synthetic detergents, and garbage grinders, and that many failures in existing systems have been attributed to increased loads resulting from the expanding use of such appliances, it becomes apparent that an upward revision of tank capacities is needed; the capacities recommended are given in Table 1.

TABLE 1.—LIQUID CAPACITY OF TANK, IN GALLONS

Number of bedrooms	Recommended tank capacity	Equivalent capacity per bedroom
2 or less	750	375
3	900	300
4[a]	1,000	250

[a] For each additional bedroom add 250 gal.

Because less clogging is caused by an effluent from a tank in which prolonged storage has been provided, it is no longer considered justified to drastically reduce the detention time in large tanks, such as those serving institutions, recreational areas, and business establishments.

COMPARTMENTATION

It is a well-established fact that the performance of a two-compartment septic tank is superior to that of a single-compartment tank in the removal of suspended solids and organic colloids. One of the reasons for this is the trapping action of the second compartment, serving as it does to allow settling of particles scoured out of the first compartment. The advantage of the extra removal in the second compartment is more significant than may be first apparent. For example, if 70% of the suspended solids is removed in a single-compartment tank, and 80% is removed in a two-compartment tank of the same total volume, the suspended-solids load on the leaching system is one-half greater from the single-compartment tank. Considering that failures of leaching systems occur most frequently in tight soils, the superior performance of a two-compartment tank may mean the difference between success and failure in many cases.

Based on information presently available (1957), the best design for home installations is believed to be a properly fitted, two-compartment tank, with the volume of the first compartment equalling about two-thirds the total volume of the tank. It is not considered practical, however, to recommend definite requirements for two-compartment tanks in areas in which single-compartment tanks have been satisfactory.

TANK PROPORTIONS

Available data indicate that relatively shallow tanks function as well as deep ones provided there is no sacrifice involved as to capacity or surface area. Also, for tanks of a given capacity and depth, elongated tanks are as effective as those having length-to-width ratios of 3 to 1 or less. Thus, for large tanks especially, it should not be required that the length of a tank be no more than three times its width. Depths may range, generally, between 30 in. and 60 in.

PERCOLATION TESTS

The purpose of percolation tests is to obtain a quantitative measure of the hydraulic capacity of a soil to absorb clean water. The results are translated empirically into a measure of the ability of soil to absorb the effluent from a septic tank. The length of time required for percolation tests will vary in different types of soil. The safest method is to make the tests in holes that have been allowed to soak and to swell overnight. This is particularly desirable when the tests are being made by inexperienced personnel, but it is not usually necessary if they are made by an experienced engineer. Tests should be made in soil that is saturated, however, and several tests should be made in separate holes until the water seeps away at a constant rate,[6] or until equilibrium conditions are reached.[7,8] The need for continuing percolation tests for sewage-absorption systems until such a degree of consistency is obtained in the results was noted first in 1948,[6] and again in 1949[7] and 1950.[8,9] It was later re-emphasized in the Cincinnati studies,[4] which advocated overnight swelling for certain types of soils. Although soaking and overnight swelling have some advantages, it has been found that, in most cases, the shorter or "single-visit" tests will give comparable results if they are performed properly—that is, if the instructions for them are followed and if the tests are continued until the results are consistent. Failure to do this has been the primary reason for such discrepancies as have been reported. Pending the development of further basic data, both types of tests are being recognized and are considered acceptable under conditions that will be explained in greater detail in the new manual.

---

[6] "Subsurface Sewage Disposal," by John E. Kiker, Jr., *Bulletin No. 23*, Florida Eng. and Industrial Experiment Station, Gainesville, Fla., December, 1948, p. 24.

[7] "Improved Soil Percolation Test," by Harvey F. Ludwig and Gordon W. Ludwig, *Water and Sewage Works*, Vol. 96, May, 1949, pp. 192–194.

[8] "Equilibrium Percolation Test," by H. F. Ludwig, W. D. Ward, W. T. O'Leary, and E. Pearl, *ibid.*, Vol. 97, December, 1950, pp. 513–516.

[9] "Rational Design Criteria for Sewage Absorption Fields, " by John E. Kiker, Jr., *Sewage and Industrial Wastes*, Vol. 22, September, 1950, pp. 1147–1153.

LEACHING-SYSTEM DESIGN

For the most part, the design of different types of leaching systems has been based on original work of Henry Ryon, who recommended, in effect, that the leaching surface area of seepage pits equal approximately 75% of the area required in subsurface irrigation fields or in absorption trenches. In some sections of the United States, however, it has been found that seepage pits which are so designed will fail much earlier than absorption trenches. As a result, some localities have required larger areas for seepage pits than for absorption trenches—which is the opposite of Ryon's recommendation.

These experiences indicated a need for further investigation and for a reappraisal of the factors that were formerly thought to influence the functioning of subsurface-disposal systems of all types. It was commonly believed that the permissible loading on seepage pits could be greater than that on seepage trenches because of the greater head of water in pits. In most places, it was also common practice to allow for only the sidewall area in seepage pits and for only the bottom area of seepage trenches. In 1952, Harvey F. Ludwig, M. ASCE, and John M. Stewart, J. M. ASCE,[10] noted that this was inconsistent and expressed the belief that all the available contact area should be equally considered. It may be added that the inconsistency Ludwig and Stewart noted becomes glaring when combinations of the two types of systems are used in a single installation.

The reports of failures of seepage-pit systems in places in which seepage trenches functioned satisfactorily under reasonably comparable conditions (except for reductions in leaching area as described previously) discredited the importance of head and other hydraulic factors in the operation of leaching systems. Although such factors are significant in percolation tests with clean water because the rate of percolation depends on the true hydraulic head, which "* * * includes the pull of the suspended water"[9] below the test hole, different phenomena predominate when sewage effluent is applied to a leaching area.

First, the degree of sewage applied is much smaller than the quantity of water used in a percolation test, the ratio being approximately 1 to 40. A controlling factor when sewage is applied to a leaching system is the surface area available for the decomposition of the suspended material and organic colloids filtered out in the leaching process; such decomposition is essential to prevent the surface from clogging prematurely. Thus, the hydraulic factors are relatively unimportant as compared with the available surface filtering area, and a given area should be considered about as equally effective whether it is horizontal or vertical. This leads to the conclusion that the same design criteria should be used in computing the required areas for seepage pits and seepage trenches, and for combinations of the two. Considering the wide variations in the dimensions of seepage pits—which may range from 4 ft in diameter and 90 ft deep in one part of the country to upwards of 10 ft in diameter and 4 ft deep in another part of the country—there seems to be little justification in adhering to concepts based on limited work in an area where pit dimensions varied over a relatively narrow range. Indeed, the extremely deep pits of small diameter may appear to be merely vertical trenches, and the similarity is worth noting.

---

[10] "Equilibrium Percolation Test for Estimating Soil Leaching Capacity," by Harvey F. Ludwig and John Stewart, *Modern Sanitation*, Vol. 4, October, 1952, pp. 61–66.

For percolation rates not less than 1 in. in 30 min, the writer recommends that the same absorption areas be required for seepage pits and trenches. Soils having rates slower than 1 in. in 30 min are considered unsuitable for seepage pits, but shallow trenches are considered acceptable if the rate is not slower than 1 in. in 60 min. This allows for some evapotranspiration where the disposal area is shallow. The recommended absorption-area requirements are summarized in Table 2.

TABLE 2.—ABSORPTION-AREA REQUIREMENTS FOR PRIVATE RESIDENCES

Percolation rate,[a] in minutes	Required absorption area,[b] in square feet per bedroom[c]	Percolation rate,[a] in minutes	Required absorption area,[b] in square feet per bedroom[c]
1 or less		15	
2		30	
3		45	300[d]
4		60	330[d]
5		more than 60	...[e]
10			

[a] Time required for water level to fall 1 in. [b] The absorption area in standard, narrow, and shallow trenches is considered to be the bottom trench area. For extra deep, wide trenches, allowance should also be made for the effective sidewall area below the first foot underneath the distributing pipe. For leaching pits the absorption area is equal to the outside area of walls in the pervious strata beneath the inlet. In computing the outside area of leaching pits, the "dug" diameter is used and full credit is allowed for rock backfill outside the walls. [c] In every case, sufficient area should be provided for at least two bedrooms. [d] Unsatisfactory for seepage pits. [e] Unsuitable for any leaching system.

Although not recommended for household installations, trenches that are deeper and wider than those generally permitted prior to 1957 are now recognized as having practical application in special cases or in large systems. This is based on the premise that a deep trench is essentially a combination of a horizontal trench and a vertical pit, and it acknowledges the observed success of wide trenches in some places. Due allowance should be made for extra depths and widths.

FURTHER RESEARCH NEEDS

Approximately twelve hundred products, including some containing enzymes, have been placed on the market for use in septic tanks. Although extravagant claims have been made for some of them, none of them, so far as is known, has proved advantageous in properly controlled tests conducted by educational institutions or by reputable testing laboratories. Several universities have excellent facilities for fundamental studies that could be used on such properly sponsored research projects. Laboratories operated on government funds derived from taxes are seldom permitted to make tests on proprietary products.

There is conflicting information concerning the effects of synthetic detergents on septic tank systems. Studies at the Robert A. Taft Sanitary Engineering Center showed that none of seven brands of household detergents interfered with normal sludge digestion when used in quantities representing average household use for all purposes; however, several of the brands noticeably interfered with this digestion when used in quantities representing more than average use. Other research undertaken at the center has indicated that detergents do not cause more short-time clogging than any soap, but it was also found that one detergent did slightly more damage to soil structure.

Testimony by experienced engineers, sanitarians, septic tank installers, and others has indicated that the new detergents have a marked effect in shortening the life of soil-absorption systems. There is a real need to compare the effects of proprietary products in parallel installations under properly controlled conditions in which the use and nonuse of synthetic detergents are the only variables. Here again, however, no manufacturer has as yet been willing to sponsor a research program that would make comprehensive comparisons in installations where actual field conditions could be simulated for prolonged periods. Although it may be agreed that average amounts of detergents will have no adverse effects on septic tank systems, the fact remains that there are approximately 6,000,000 septic tank installations in the United States. If detergents caused annual failures of only 1% of this number, that would still account for 60,000 failures a year. Therein may lie a reason for the discrepancy between published research results and the field observations of those who insist that the new detergents have been responsible for many failures.

There is an encouraging note to this problem, however, in the fact that septic tank capacities now recommended are reasonably liberal—they provide for a 50% increase to allow for household appliances, including garbage grinders. It is apparent that this increased capacity should result in better conditioning of the effluent that is discharged to subsurface disposal systems and that this, in turn, should result in fewer failures.

The need for further research was noted recently by the Building Research Advisory Board (BRAB), which, under a contract between the Federal Housing Administration (FHA), and the National Research Council, evaluated various opinions, observations, and research. A special advisory committee was appointed for this study, the members of which "* * * were selected on the basis of their knowledge and experience as individuals in the subject of the study." [11]

In addition to the observation that further research is needed, the major conclusion of the BRAB was that:

> "A properly designed individual septic tank and subsurface drainage system can be used effectively in disposing of all liquid household wastes

---

[11] "The Effect of Automatic Sequence Clothes Washing Machines on Individual Sewage Disposal Systems," *Report to the Federal Housing Administration*, Div. of Eng. and Industrial Research, Building Research Advisory Board, Washington, D. C., March 22, 1956, p. 24.

including both sewage and the discharge of automatic sequence clothes washing machines in the same system." [12]

The committee also agreed that it is unnecessary to have separate systems for the disposal of sanitary wastes and laundry wastes from an automatic washing machine; also, laundry wastes may be advantageously treated by the buffering action, the lint removal, and the biochemical action that occur in the septic tank used for the sanitary wastes. According to the report, "except in rare cases, there will be an economic advantage in having a combined system for disposal of sanitary wastes and laundry wastes." [13]

However, the report does not disparage the use of separate systems for laundry wastes, and it is believed that separate systems should be allowed when home owners wish to go to the extra expense of installing them. Of 159 replies from health departments to a letter of inquiry from the BRAB, 89 indicated that individual sewage-disposal systems will function satisfactorily with combined sanitary wastes and wastes from automatic sequence washing machines if properly sized for the increased liquid volume. However, 36 indicated that "separate sewage disposal facilities should be provided to dispose of waste from automatic sequence washing machines." (Thirty reported no information on the subject of the inquiry.) [14] Although septic tank installations should not be allowed in new developments where a single disposal system would not be satisfactory, separate systems for laundry wastes are sometimes desirable in marginal areas or in areas that were developed before home water consumption was increased as a result of the use of automatic washing machines. In such a separate system, a replaceable rock filter may be found to be better than a septic tank for the removal of lints and fines from the wastes.

Conclusions

At best, septic tanks and subsurface sewage-disposal systems are poor substitutes for central collection and treatment systems. Septic tanks were designed for the treatment of domestic sewage before the advent of household washing machines, synthetic detergents, and garbage grinders. Because they are likely to create public health problems which may be difficult to correct, septic tank disposal systems should be avoided wherever possible. In general, they should be used only in rural areas of large acreage where suitable soil is available for disposal of the effluent by subsurface means. They should not be used in places where the soil is impervious, or where the maximum ground-water table comes within 4 ft of the surface during the wettest season, or where the subsoil conditions are otherwise unsatisfactory. They also should not be used in locations where there is any likelihood of contaminating a water supply, lake, or other water course.

When septic tanks must be used, they should be of adequate capacity to allow for increased loads from modern home appliances. The disposal systems following the tanks should also be liberally designed in accordance with sound engineering principles, which include determinations of soil characteristics by subsurface explorations in addition to properly conducted percolation tests. There should be no compromise by allowing installations of inadequate septic tank systems as a means of competing with central sewerage and sewage-treatment facilities. At the same time, antiquated traditions should be abolished and reasonable departures from standard practices should be allowed where they have proved successful.

---

[12] "The Effect of Automatic Sequence Clothes Washing Machines on Individual Sewage Disposal Systems," *Report to the Federal Housing Administration*, Div. of Eng. and Industrial Research, Building Research Advisory Board, Washington, D. C., March 22, 1956, p. 2.

[13] *Ibid.*, p. 3.

[14] *Ibid.*, pp. 22-23.

These objectives may be accomplished by following the procedures recommended by the USPHS.

ACKNOWLEDGMENTS

The Joint Committee on Rural Sanitation, sponsored by the USPHS under the chairmanship of Malcolm C. Hope, is performing an important role in reviewing and revising the manuscript for the manual on which this paper is based. Most of the writer's initial work on the manual was done at the Robert A. Taft Sanitary Engineering Center in collaboration with James B. Coulter, Samuel R. Weibel, and others associated with the Cincinnati studies.

Reports prepared in 1952 and 1954 by the Rural Sanitation Committee of the American Public Health Association were used as major sources of information. Contributions of exceptional practical value were made by Ludwig and James J. Spear.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2912

COST ALLOCATION FOR MULTIPLE-PURPOSE WATER PROJECTS

By Newcomb B. Bennett, Jr.,[1] M. ASCE

With Discussion by Messrs. Frederick L. Hotes; Eugene W. Weber;
Wendell E. Johnson and Charles A. Cocks; and
Newcomb B. Bennett, Jr.

Synopsis

The separable costs-remaining benefits method of cost allocation has not been used long enough to understand precisely its procedural details and basic concepts. The dependence of this method on benefits and alternate costs is emphasized herein, together with the meanings attached to single-purpose alternate costs, specific and separable costs, joint costs, and remaining project costs. These concepts are examined in relation to each other and to the broad principles of cost allocation, use of project works, project formulation, and legislative history.

Allocation of Costs

Need.—Government agencies in the water-development field find it necessary to assign or allocate costs of multiple-purpose structures, such as dams, among the functions to be served by that structure. This operation becomes necessary because of: (1) The utilization of one structure to provide services of different natures; and (2) the requirements of federal laws that make some services reimbursable, others nonreimbursable, some interest-bearing, and others noninterest-bearing.

Allocation is the process of assigning an equitable share of the cost of a multiple-purpose project to each function in the project. Within each such project there will generally be specific costs—that is, those costs that are readily identified with a single function—and joint costs—that is, those costs that are related to two or more functions.

Note.—Published, essentially as printed here, in May, 1956, in the Journal of the Investigation and Drainage Division, as *Proceedings Paper 961*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Chief, Div. of Project Development, Bureau of Reclamation, U. S. Dept. of the Interior, Washington, D. C.

Methods.—There are many methods of allocating costs, and any single method, or combination of methods, might be applicable to a particular situation. All will not be covered herein, but some of the basic concepts that should govern the preferable ones will be set forth. In March, 1954, the three agencies of the United States dealing principally with multiple-purpose projects—the Federal Power Commission, the Corps of Engineers (United States Department of the Army), and the United States Department of the Interior—agreed to use three methods. These were the methods of separable costs-remaining benefits, alternative justifiable expenditure, and use of facilities, in order of preference.

TABLE 1.—BRIEF DESCRIPTION OF STEPS INVOLVED IN METHODS OF COST ALLOCATION

SEPARABLE COSTS-REMAINING BENEFITS METHOD	ALTERNATIVE JUSTIFIABLE EXPENDITURE METHOD
(1) The benefits of each purpose are estimated.	(1) The benefits of each purpose are estimated.
(2) The alternative costs of single-purpose projects to obtain the same benefits are estimated.	(2) The alternative costs of single-purpose projects to obtain the same benefits are estimated.
(3) The separable cost of each purpose is estimated.	(3) The specific cost of each purpose is estimated.
(4) The separable cost of each purpose in the multiple-purpose project is deducted from the lesser of each purpose's benefits or alternative cost. The lesser figure is used because alternative cost is used in this method only if it represents a justifiable expenditure—that is, if it does not exceed the benefits.	(4) The specific cost of each purpose in the multiple-purpose project is deducted from the lesser of that purpose's benefits or alternative cost. The lesser figure is used because alternative cost is used in this method only if it represents a justifiable expenditure—that is, if it does not exceed the benefits.
(5) From the total cost of the project all separable costs are deducted to determine residual costs.	(5) From the total cost of the project all specific costs are deducted to determine joint costs.
(6) Residual costs, designated as joint costs in this method, are distributed in direct proportion to the remainders found in step (4).	(6) Joint costs of the multiple-purpose project are distributed among purposes in direct proportion to the remainders found in step (4).
(7) To determine the cost allocated to each purpose, the separable and distributed costs for each purpose are added, and, in the case of power, from that sum is subtracted the amount of taxes foregone which was used in computing power costs under steps (2) and (3).	(7) Allocation of the project cost is determined in the same manner as under the separable costs-remaining benefits method.

TABLE 2.—SIMPLIFIED ILLUSTRATION OF APPLICATION OF SEPARABLE COSTS-REMAINING BENEFITS METHOD (COSTS IN THOUSANDS OF DOLLARS)

Item	PROJECT PURPOSES			
	Flood control	Irrigation	Power	Total
1. Benefits	8,000	23,000	35,000	
2. Alternative single-purpose cost	10,000	27,000	30,000	
a. Lesser of item 1 or item 2	8,000	23,000	30,000	
3. Separable costs	0	8,000	29,000	37,000
4. Remaining benefits (item 2*a* − item 3)	8,000	15,000	1,000	24,000
Percentage of total	33	62	5	100
5. Unallocated joint cost (project cost − item 3)	...	...	...	19,000
6. Allocated joint cost	6,300	11,800	900	19,000
7. Total allocation (item 6 + item 3)	6,300	19,800	29,900	56,000

The separable costs-remaining benefits method is one in which each purpose is assigned its separable cost, in addition to a share of the joint costs proportionate to the remainders found by deducting the separable cost of each purpose from the lesser benefits or the alternate cost for that purpose (Tables 1 and 2).

The alternative justifiable expenditure method is one in which each purpose is assigned its specific cost, in addition to a share of the joint costs proportionate to the remainders found by deducting the specific cost of each purpose from the lesser benefits or the alternate cost for that purpose (Table 1).

The use of facilities method is one in which each purpose is assigned its specific cost or its separable cost, in addition to a share of the joint costs proportionate to the use of facilities in terms of comparable measures, such as water releases, reservoir capacity, power capacity, or consumption of energy. However, this method has only limited application, such as in dividing the cost of a canal between municipal water and irrigation water on a capacity basis. It is useful, then, in making suballocations on the basis of a proportionate use of a given facility as measured in physical terms.

The alternative justifiable expenditure method is the better known of the other two methods and has only one significant difference from the separable costs-remaining benefits method in that with the latter method separable costs are used rather than specific costs as in the alternative justifiable expenditure method; this difference will be cited subsequently.

Each of these two methods has seven major steps, which are shown in Table 1. Many of the problems dealing with the selection of alternatives that will be examined herein are common to both methods.

Principles.—The use of one structure to provide more than one service allows the services to be provided at less cost than the total cost of separate structures for each service. The incremental cost of adding each function to the other functions of the combined structure should be less than the cost of the most economical single-purpose alternative method of producing similar benefits for that function. This being the case, a basic principle of cost allocation is that the saving derived through the use of the combined structure for numerous functions should be shared equitably by all functions. There are certain benefit-cost relationships that must be recognized—such as, no function is to be assigned costs in excess of its benefits or to be supported by benefits attributable to another purpose, and no function is to be assigned costs greater than the cost of an alternative single-purpose project. The allocation must be consistent also with existing laws, treaties, and compacts affecting the project plan.

The following broad principles are applicable:

1. Each purpose should share equitably in the savings resulting from multiple-purpose construction within the limits of maximum and minimum allocations.
2. The minimum allocation is its specific cost or its separable cost, depending on the method of allocation.
3. The maximum allocation is its benefits or its alternative single-purpose cost, whichever is less.
4. Joint costs should be apportioned without regard to the capability of any particular purpose to repay its costs.
5. Legal priorities for the use of water and existing laws, treaties, and compacts must be recognized.

Inherent Weaknesses.—Any allocation method that uses benefits at any stage is subject to the question of their validity. The term "benefits" may be defined briefly as the estimated value of the services or products provided by the various functions of the project. As will be noted from Table 1, benefits are used in both methods as a rein on alternate costs, a measurement of the maximum allocation, and a basis of proportioning joint costs. Therefore, they are of critical importance in the determinations under certain conditions. Theoretically an equitable apportionment will result only if the benefits are equally valid among themselves—that is, power benefits, or municipal and industrial water benefits, should be computed with the same validity as irrigation and flood-control benefits. It can be questioned whether this is currently (1957) being done. Power and municipal water-supply benefits are usually computed as functions of alternative costs. Irrigation benefits are measured as increased net income, whereas flood control is measured as savings in damages.

In addition there may be intangible benefits on which no monetary value can be placed; yet they are real benefits not entering into the allocation process.

Another major weakness is the inadequacy of the basic data used in estimating alternatives for the single purpose. Many times an alternative will be a different dam site for which little, if any, basic data are available. The cost estimate is not likely to have the same degree of validity as that used for the project.

These are matters of concern for aspects of project development in addition to cost allocations. They can affect project formulation and justification. However, they do not affect the application of the cost-allocation method.

Basic Concepts

Uniform application of the separable costs-remaining benefits method requires acceptance and adoption of basic concepts covering alternative single-purpose costs, specific costs, separable costs, and, perhaps, remaining project costs. The adoption of a particular method does not guarantee uniform procedures and results. To use separable costs is not the answer because they are subject to acceptable definition and the application thereto of guiding concepts. Methodology is still being explored to clarify the relationships between generally accepted concepts and the desirable degree of refinement.

Example of an Allocation

In order to understand the material presented subsequently, Table 2 has been prepared, showing in a simplified manner the application of the major steps involved in the separable costs-remaining benefits method. (Table 1 shows what those steps are.) Such items as interest during construction, addition of taxes foregone, and operation and maintenance costs have been excluded to facilitate understanding and because the points raised hereinafter are not affected by them.

Selection of the Single-Purpose Alternative.—To provide service to each project at least cost, all practical alternative means of single-purpose development in the project area should be considered. The most economical alternative method of providing benefits equivalent to those provided by the selected project should be used in formulation and in cost allocation. This alternative does not have to be located at the multiple-purpose project site and it may differ physically—for example, a steam-electric development may be the most economical alternative to a hydroelectric plant. Because of the scarcity of available sites, construction of a single-purpose dam may preclude using the same site for other purposes for which that site is necessary or most suitable. This practical difficulty does not prevent the use of the site in considering an alternative to computing single-purpose costs for allocation purposes. The alternative should be realistic in that it can be built and, if built, would produce equivalent benefits. The single-purpose alternative need not have benefits in excess of its costs because it may be unjustifiable as a separate project. In such cases the benefits, rather than the alternative single-purpose costs, are used as upper limits in cost allocation. The alternative single-purpose costs properly represent one limit on costs that may be included in project plans and allocated to a purpose; the other upper limit is the evaluation of benefits.

As stated, it is not required that the single-purpose alternative be justified. In many cases there is no justifiable alternative of any magnitude. If the alternative must produce benefits equivalent to those of the selected project, the chances of justification are even less. Such a hypothetical requirement would then become meaningless because there would be no alternative. It would probably be necessary to rely solely on benefits as a measure of the maximum allocation. All that could be gained by requiring justification of the alternative would be the refinement of the costs of that alternative, which would result from the efforts made to find justifiable alternatives.

Specific Costs.—The readily determinable costs of facilities that are clearly for one purpose only should be allocated specifically to that purpose; examples of such facilities are power plants, transmission lines, and irrigation canals. They are termed specific costs and represent the minimum allocation in the alternative justifiable expenditure method. At times, there will be space in a jointly used reservoir that is assigned exclusively to a single purpose, as for example in the case of the inviolate flood-control space in many reservoirs. The specific cost of providing such space is not readily determinable because the space occurs at the top of the reservoir. The cost of adding the increment can be determined by subtracting the cost of the structure without the space from that with the space, but this encroaches on the determination of separable costs. By definition, specific costs are reconcilable with engineering cost estimates, which list the costs of separate physical features, such as dams, canals, tunnels, penstocks, power plants, and electric-transmission systems. The remainder of multiple purpose-project costs would be those items in engineering cost estimates that represent the total cost of jointly used physical features. An alternate approach would so segregate specific costs that it would encroach on the apportionment of joint costs, and the cost of a physical feature would not be used as the unit of computation. For example, the specific cost of a power plant or penstock embedded in a dam may be listed as the cost of the power plant or penstock adjusted to reflect the alternate cost of constructing a dam without the embedded features, which may greatly modify the specific cost and in some cases might eliminate it. Such adjustments may be worth the effort when major costs are involved, but they apply to separable costs rather than to specific costs, as defined in the March, 1954, interagency agreement.

SEPARABLE COSTS

Separable costs include specific costs plus that part of the joint costs traceable solely and clearly to the inclusion of a single purpose in the multiple-purpose project. The concept is an extension of the basic idea of the direct assignment of specific costs. The separable cost is the minimum allocation to a function when the separable costs-remaining benefits method is used. It is quite possible that no part of the joint cost can be traced to a particular purpose. Separable costs for any purpose may be zero, specific costs only, or specific costs together with some part of the joint costs. When these costs for all purposes consist of specific costs or are zero, the separable costs-remaining benefits method is identical to the alternative justifiable expenditure method in computation and results, although more time, effort, and money may have been spent in exploratory studies of possible separable costs.

Identification of Separable Costs.—Although separable costs might conceivably be obtained by addition of component elements, in practice and by definition they are obtained by subtraction. This cost for each purpose is defined as the difference between the cost of the multiple-purpose project and the cost of the same project without the purpose. To avoid favoring one purpose over another, each purpose should be treated as if it were the last increment to a diminished project serving all other multiple purposes. This computation is intended to identify the added costs of increased size of structures, changes in design, or other factors necessary to add the purpose to the project. A series of cost estimates must be prepared, representing the multiple-purpose project without each purpose. A clear distinction should be maintained between the concepts underlying these estimates, which may be termed remaining project costs, and the concepts of alternative single-purpose costs previously cited.

REMAINING PROJECT COSTS

If the project has been properly formulated, the remaining project cost will be justifiable by an excess of benefits over costs. For cost-allocation purposes, however, the remaining costs do not need to be justified and do not need to be the most economical source of providing multiple-purpose benefits to the remaining project functions. To find and to use the most economical alternative in this instance might disperse the functions to other sites and substitute other features, such as a steam-electric plant rather than a hydroelectric plant. Such a dispersal or substitution would not reveal increments of separable cost by identifying the remaining part of the multiple-purpose project. Instead, it would present a comparison of the multiple-purpose project with a competitive plan of different nature and location. Such a comparison is brought into the cost allocation by the alternative single-purpose costs. The cost-allocation process should not be complicated by introducing the additional category of alternative multiple-purpose costs.

Other approaches to the remaining project costs would be to seek the most economical plan regardless of location or type of physical facilities, or to use the most economical plan for each combination of purposes at the multiple-purpose site. However, these approaches appear to lead to inconsistencies, as, for example, in the case of the omission of irrigation from a project including hydroelectric power. If the water required for irrigation depletions were used for power, the most economical source of power benefits equal to those obtained from the multiple-purpose project might be a less expensive power plant. In this case the reduction in specific power costs (reflected in the cost of certain generating units) would become part of the separable cost of irrigation, which is obviously illogical. Therefore, it appears necessary to leave specific costs unchanged in the remaining project and to reject the definition based on the most economical plan at the multiple-purpose site.

In the example cited, remaining project costs might also be based on an unchanged power plant but one with reservoir storage reduced to keep power benefits constant. They might be based on an unchanged power plant with reservoir capacity reduced only by the irrigation storage, which would possibly result in power benefits greater than those obtained from the multiple-purpose project. The choice will depend in part on the storage capacity required to avoid interference with other project purposes. The use for power of the water required for irrigation depletions, in addition to omission of irrigation storage capacity, might drastically reduce the size of the reservoir in the remaining project. The reduction might well reach the point where the separable cost of irrigation would exceed the alternative single-purpose cost of irrigation. This would establish the minimum allocation at a greater cost than the maximum allocation. Cases will arise in which greater benefits to some purpose are unavoidable if basic concepts of allocation are to be preserved. However, the remaining project should provide benefits to each remaining purpose that are at least equal to those obtained from the multiple-purpose project.

Joint Use of Reservoir Capacity

As previously noted, the cost of exclusive space assigned to a single project purpose should be omitted from the remaining project in determining the separable cost of that purpose. The treatment of seasonally exclusive space is less clearly defined, as in the case of seasonal flood-control space, which at other times of the year serves other purposes. In order to avoid interfering with these purposes in determining separable flood-control costs, the seasonal-flood-control space should be treated as an inseparable joint cost included wholly in the remaining project. As another example, if all water releases for navigation were also used for power generation, the omission of navigation would not allow a reduction in reservoir capacity, and the separable cost of navigation would be limited to specific costs. In the case in which one purpose, such as irrigation, is served by pumping from the reservoir, the change of reservoir capacity by omitting another function would increase pump lifts and interfere with irrigation operations and benefits. Truly separable costs include those parts of jointly used works that can be omitted without interfering with other functions; if interference is caused, that segment is an inseparable joint cost.

EMPHASIS ON SEPARABLE COSTS

Some aspects of the separable costs-remaining benefits method are relatively new, and procedures for deriving these costs are still being explored (as of 1957). Confusion and overlapping of concepts are major problems as well as the degree of refinement that is worth while. Extreme refinement is involved when attempts are made to assign the maximum amount of joint costs to those that are separable. By intensive effort, if none of the project purposes have separable costs limited to specific costs or to zero, it might be possible to obtain a total for all purposes of more than the multiple-purpose project costs. Emphasis should be placed on separable costs rather than on remaining project costs; the latter are only a statistical residue in deriving the former.

PROJECT FORMULATION AND LEGISLATIVE HISTORY

The separable costs-remaining benefits method may not yield equitable results if applied arbitrarily to a project in which the scale of development for each purpose was determined on other concepts, such as the maximum use of a favorable site. The method may also have a limited use if applied retroactively to a project governed by a legislative background and by justifying reports which base project authorization on other cost allocations, such as fixed proportionate uses of project works. The criteria used in formulating, authorizing, and constructing the project will need to be recognized in the cost allocation.

CONCLUSIONS

The basic principle in the separable costs-remaining benefits method of cost allocation is that the savings achieved through use of the multipurpose structure should be shared by all purposes. The maximum allocation to a purpose is limited by the lesser of benefits or alternative costs, whereas separable costs represent the minimum allocation. In applying the method, the alternative projects need not be economically justified nor the remaining project the most economical source of providing the remaining benefits. Determining separable costs should be emphasized rather than remaining costs because they are assignable directly to the functions.

Using present methods of computing benefits, the weakest part of the allocation process lies with the validity and comparability of the benefits. For equitable results the allocation process must be consistent with the project-formulation process.

ACKNOWLEDGMENTS

The writer wishes to thank Ira A. Watson, John D. Abrahamson, James C. Bradley, and George E. Tomlinson, M. ASCE, for their material contributions to the paper and their aid in its final preparation.

DISCUSSION

Frederick L. Hotes,[2] **M. ASCE.**—Cost-allocation methods for water-resources-development projects have been required in the past primarily for use in the analysis of proposed federal works. When state governments begin actively financing, constructing, and operating multiple-purpose water projects, it can be expected that cost allocation will also be necessary in this area. Furthermore, the growing use of federal funds in projects built by local districts makes it imperative that all engineers in the water-development field know the principles and details of federal cost-allocating methods. It is almost axiomatic that any flood-control benefits that might accrue from a local project will be presented before the United States Congress with a request for federal financing on a nonreimbursable basis; other nonreimbursable benefits may, on occasion, also be claimed. Although Congress, in principle, might favor such requests, details of the benefits will be required, together with the proper allocation of costs, in accordance with federal practices. Federal financing of small projects will also require the use of some accepted method of cost allocation. Mr. Bennett has performed a distinct service in reviewing the most current practices.

Although there are many methods of cost allocation, they all share a common purpose—attempting to distribute the total costs of the project equitably among the various functions or beneficial effects of the project. This simple aim is amazingly difficult to achieve, as Mr. Bennett has so ably demonstrated by presenting the results of some of his experiences with the separable costs-remaining benefits method of cost allocation, which is presently (1957) one of the better techniques available in this field.

The author has indicated (under the heading, "Project Formulation and Legislative History") that the method does not always yield equitable results. In this connection the writer would like to discuss a limitation in item 3 (under the heading, "Allocation of Costs: Principles") of the broad principles set forth.

There is a danger in using hypothetical examples, but the subsequent illustration of the use of the method demonstrates a definite limitation in certain instances.

Example.—A certain project has only irrigation and power benefits. The total project benefits are $58,000,000 and exceed the total project costs of $56,000,000. At this stage of the analysis the project analyst does not know whether or not each purpose will yield benefits greater than cost because he does not know what proportion of the cost should be allocated to each function. Following Mr. Bennett's outline, the costs are allocated as in Table 3.

The analysis now reveals that the irrigation costs exceed the benefits. The proper conclusion is that the inclusion of irrigation under these circumstances is uneconomical and unjustified. It would not be right to apply item 3 and limit the cost to benefits. In addition, including irrigation in this instance yields a power-assessment cost that is greater than the cost of an alternative project, which is also uneconomical.

---

[2] Water Conservation Engr., Development and Resources Corp., New York, N. Y.; presently located in Ahvaz, Iran; formerly Associate Prof. of Civ. and Irrig. Eng., Univ. of California, Berkeley, Calif.

It would appear, then, that a proper and necessary extension of item 3 of Mr. Bennett's cost-allocation principles would be that, if the allocation costs for any particular function exceed the corresponding benefits, that function should be removed from the project, or the project be redesigned, rather than reducing the allocated costs to the level of the benefits. The foregoing is in full accord with the criteria set forth in a 1950 report,[3] wherein it is stated that "* * * each separable segment or purpose provides benefits at least equal to its costs."

TABLE 3.—HYPOTHETICAL APPLICATION OF SEPARABLE COSTS–REMAINING BENEFITS METHOD REVEALING UNECONOMIC PURPOSE (COSTS IN THOUSANDS OF DOLLARS)

Item	PROJECT PURPOSE		
	Irrigation	Power	Total
1. Benefits	$23,000	$35,000	$58,000
2. Alternative single-purpose cost	27,000	30,000	
a. Lesser of step (1) or step (2)	23,000	30,000	
3. Separable costs	8,000	29,000	37,000
4. Remaining benefits (item 2a − item 3)	15,000	1,000	16,000
Percentage of total	94	6	
5. Unallocated joint cost (project cost − item 3)	...	...	19,000
6. Allocated joint cost	17,800	1,200	19,000
7. Total allocation (item 6 + item 3)	25,800	30,200	56,000

Another limitation of the separable costs-remaining benefits method is shown in Table 4 for the case of a reservoir designed entirely for irrigation yield, but which, inherently, provides some recreation benefits. Both the separable costs and the specific costs for recreation are assumed to be zero—that is, the recreation benefits accrue even though no specific part of the structure was designed solely to provide these benefits.

TABLE 4.—HYPOTHETICAL APPLICATION OF SEPARABLE COSTS–REMAINING BENEFITS METHOD ILLUSTRATING LIMITATION FOR A CASE WHEN SEPARABLE COST IS ZERO (COSTS IN THOUSANDS OF DOLLARS)

Item	PROJECT PURPOSE		
	Irrigation	Recreation	Total
1. Benefits	$23,000	$2,000	$25,000
2. Alternative single-purpose cost	23,000	5,000	
a. Lesser of step (1) or step (2)	23,000	2,000	
3. Separable costs	20,000	0	20,000
4. Remaining benefits (item 2a − item 3)	3,000	2,000	5,000
Percentage of total	60	40	
5. Unallocated joint cost (project cost − item 3)	...	...	0
6. Allocated joint cost	0	0	0
7. Total allocation (item 6 + item 3)	20,000	0	20,000

---

[3] "Proposed Practices for Economic Analysis of River Basin Projects," Report by the Subcommittee on Benefits and Costs, U. S. Federal Inter-Agency River Basin Committee, U. S. Dept. of the Interior, Washington, D. C., May, 1950, p. 37.

It is logical and fair that the recreation function be allocated some share of the cost as some benefit is gained. Perhaps the best method of allocation in this case would be by proportional benefits—that is, to have the total cost allocated in proportion to the benefits derived.

Whereas the proportional-benefits method is equitable in the case of Table 4, it is not correct to use it in the first case shown in Table 3. One of the most vital weaknesses of the proportional-benefits method is that it permits the inclusion of an uneconomical component as long as the total benefits exceed the costs. It is this very deficiency in this simple approach to cost allocation that compels the use of more complicated methods.

It is probably safe to say that, as yet, there is no single method of cost allocation that gives equitable results in all cases. Engineers and economists must use the best of several methods, or a combination of several methods, to attain an equitable distribution of costs.

Although Mr. Bennett has eliminated details that do not pertain to his basic thesis, one point was mentioned that perhaps needs amplification. He has indicated (under the heading, "Example of an Allocation") that for the sake of simplicity discussion of "taxes foregone" was omitted. However, in item (7) of Table 1 (under the heading, "Separable Costs-Remaining Benefits Method") the following appears:

> "* * * and, in the case of power, from that sum is subtracted the amount of taxes foregone which was used in computing power costs under steps (2) and (3)."

The full significance of this phrase is somewhat confusing. Apparently these taxes foregone would be included in items (2) and (3) only if the benefits in item (1) included comparable taxes. The cost-allocation process would yield approximately the same results if taxes were omitted from the first three steps. The inclusion or exclusion of taxes would probably depend directly on the most convenient method of determining the benefits and indirectly on the policies of the agency conducting the study.

EUGENE W. WEBER,[4] M. ASCE.—The concepts and principles for the application of the separable costs-remaining benefits method of cost allocation have been thoughtfully analyzed and effectively presented.

Experience in applying this method of cost allocation has indicated that it may be desirable to depart from the rigid interpretation of separable costs defined by the author on the basis of the original concept presented in the May, 1950, report of the Subcommittee on Benefits and Costs of the United States Federal Inter-Agency River Basin Committee (United States Department of the Interior). Usually it will be possible to obtain satisfactory estimates of separable costs by computing the difference between the cost of a multiple-purpose project and the cost of the same project with a purpose omitted. Occasionally, however, it may be necessary to take into account the fact that the most logical project for all purposes, except the one for which separable costs are being computed, is a project that is not a variation of the multiple-purpose project in question. When this is the case, it is believed that a broader concept should be followed and that the separable cost for any purpose should be computed as the difference between the multiple-purpose project cost and the cost of the best available and most economical project that will afford equivalent results for all the other purposes.

---

[4] Special Asst. to Asst. Chief of Engrs. for Civ. Works, Corps of Engrs., U. S. Dept. of the Army, Washington, D. C.

If this broader concept is not followed when there are, in fact, more economical ways of achieving the desired results for all but one of the purposes, an equitable sharing in the advantages of the multiple-purpose project will not be fully realized in the cost allocation.

Wendell E. Johnson,[5] **M. ASCE, and Charles A. Cocks**[6].—The problems of distributing costs equitably among functions served by multiple-purpose water projects are of considerable importance. As the development of water resources progresses, the multiple-purpose project becomes more important. Mr. Bennett has performed a necessary service in attempting to rationalize the concepts appropriate for use in cost-allocation studies. The exchange of ideas and theories and the improvement in basic concepts and their use will lead to more uniform and more appropriate solutions to the problems.

As stated in the paper, the separable costs-remaining benefits method of cost allocation is relatively new, and, as such, a precise understanding of its procedural details and basic concepts has not been fully achieved (as of 1957). Mr. Bennett has noted that the method depends on benefits and alternate costs, and he has cited the meanings attached to single-purpose alternate costs, specific and separable costs, joint costs, and remaining project costs. In many respects the separable costs-remaining benefits method is closely allied to the alternate justifiable expenditure method of cost allocation,which has been in use for some time and whose procedures and policies relative to its use are quite well established. Thus, the factors common to both methods have been understood and well stated.

The only significant difference in the two methods is the use of separable costs in lieu of specific costs. With regard to specific costs, Mr. Bennett states (under the heading, "Example of an Allocation: Specific Costs"): "The readily determinable costs of facilities that are clearly for one purpose only should be allocated specifically to that purpose * * *". The author further states (under the heading, "Separable Costs"): "Separable costs include specific costs plus that part of the joint costs traceable solely and clearly to the inclusion of a single purpose in the multiple-purpose project." The 1950 report,[3] in which the separable costs-remaining benefits method was proposed, states: "The separable cost for each project purpose is the difference between the cost of the multiple-purpose project and the cost of the project with the purpose omitted."

In explaining his concept of separable costs, Mr. Bennett states that (1) it is an extension of the basic idea of the direct assignment of specific costs; (2)

---

[5] Chief, Eng. Div., Missouri River Div., Corps of Engrs., U. S. Dept. of the Army, Omaha, Nebr.
[6] Chief, Planning Section, Missouri River Div., Corps of Engrs., U. S. Dept. of the Army, Omaha, Nebr.

separable costs are the difference between the cost of the multiple-purpose project and the cost of the same project with the purpose omitted; (3) the remaining project need not be justified for cost-allocation purposes; (4) the remaining project need not be the most economical source of providing multiple-purpose benefits to the remaining functions; (5) although the alternative single-purpose costs bring a comparison of plans of different nature or location into the cost-allocation study, the remaining project should not do so; (6) the remaining project should provide benefits to each remaining purpose at least equal to or in excess of those obtained from the multiple-purpose project; and (7) separable costs include only costs of parts of the multiple-purpose project; which may be omitted without interfering in any way with service to the remaining functions and without requiring any adjustments in facilities to provide service to the remaining functions.

Some of these restrictions on the process of determining separable costs appear to negate, in part at least, the theory of separable costs indicated in the 1950 report. This report, after defining separable costs as quoted previously, continues as follows:

> "Separable costs include more than the direct or specific costs of physically identifiable facilities serving only one purpose, such as an irrigation distribution system. They also include all added costs of increased size of structures and changes in design for a particular purpose over that required for all other purposes, such as the cost of increased reservoir storage capacity. In effect, separable costs are computed from a series of project cost estimates, each representing the multiple-purpose project with one purpose omitted. Such information will be readily available when the recommended practices of project formulation have been followed. Where project formulation has not been of the detail suggested in the recommended procedure, it may be necessary to use specific costs in lieu of separable costs in those cases where re-estimating would be unduly burdensome."

In describing the application of procedures recommended in project formulation, the report states:

> "The next step in river basin study should be to examine and analyze the physical possibilities for improvement or development of the basin's resources to meet the needs or objectives. At all stages of such analysis, preliminary, intermediate, and final, the advantages and disadvantages of the various physical possibilities can and should be evaluated and compared in terms of benefits and costs, measured with successively increasing degrees of refinement, as required, to eliminate the obviously unjustified and least favorable possibilities, until the optimum plan of development is formulated."

It further states:

> "At various stages of project formulation, the program, project, or segment of a project under consideration must satisfy the criterion that it would be more economical than any other actual or potential available means, public or private, of accomplishing the specific purpose involved. A program, project or segment of a project should not be undertaken if it would preclude development of any other means of accomplishing the same results at less cost. This limitation applies to alternative possibilities which would be displaced or eonomically precluded from development if the project is undertaken. Other means of obtaining similar benefits which would not be

> precluded from development are not limitations on project justification but are, in effect, additional projects which may be compared in an array to determine which should be given prior consideration from the standpoint of economic desirability . . . The alternative possibilities to be considered in applying this limitation should include all practicable means of accomplishing the desired results which are within the purview of the agency making the economic analysis. In theory, the broadest possible range of alternatives for any given objective should be considered but it is recognized that in practice, the range of alternatives that can be considered at regional levels may be limited by the information available at such levels. Also, there may be alternative possibilities which are not known to an agency responsible for project analysis. Nevertheless, consideration of alternatives on the broadest possible basis should be given at all levels of responsibility and necessary information for that purpose should be exchanged among the Federal agencies involved and utilized at appropriate levels of project analysis and review."

It appears evident that the separable costs-remaining benefits method, as outlined in the aforementioned report, depended on the full process of project formulation, in which the engineer had the full responsibility of determining the most economical means of accomplishing the purposes involved, including determining the costs of including a function. It is believed that the remaining project, or projects, should be formulated to serve the remaining functions by the most economical source, or sources, of providing the same benefits that would be provided by the multiple-purpose project and that such remaining project, or projects, should be economically justified. The separable costs of a function would, therefore, be the difference in costs of the multiple-purpose project and the remaining project with the function omitted, with the remaining project adjusted as required to serve the remaining functions comparable to the multiple-purpose project. In this way the true net increase in the cost of including a function is obtained. If a remaining project omitting one function were allowed to provide beneficial effects significantly in excess of the multiple-purpose project, it would be left to individual discretion to set the amount of such excess, which would preclude the derivation of comparable estimates. In order to assure comparable estimates for remaining functions, the service rendered by the remaining project should, therefore, be comparable to the service rendered by the multiple-purpose project. This might involve providing, in the remaining project, facilities of lesser scope and cost specifically for a single purpose. This is not inconsistent because it is a measure of the extra costs involved for that function to meet a specific need when the separable function is included. As stated in the 1950 report, during project formulation all possible alternatives should be investigated. This would involve, in some instances, the comparison of alternate locations for remaining projects to determine the most economical means of providing service.

Mr. Bennett has described a modification of the alternate justifiable method, which would slightly extend the concept of specific costs, rather than clarify what separable costs should be under the separable costs-remaining benefits method proposed in 1950. Future clarification of the apparent difference in concept is necessary.

NEWCOMB B. BENNETT, JR.,[7] M. ASCE.—The discussions of cost allocations emphasize the dependence of cost allocation on project formulation. The hypothetical example presented by Mr. Hotes in Table 3 is an improperly formulated project because the total maximum allocations of $53,000,000 in item 2*a* of the table are insufficient to cover the total project costs of $56,000,000 in item 7. This would have been more evident if item 2*a* had been designated "justifiable expenditure," which is the maximum allocation assignable to a function. Mr. Hotes correctly concludes that in such a case one function should be removed from the project or the project redesigned, rather than that the costs allocated to each purpose should be reduced to an amount less than the benefits for each purpose. The necessity for such an alteration of the project plan does not limit or extend the basic principle that "the maximum allocation to each purpose is its benefits or alternative single-purpose cost, whichever is less." This principle does not require an adjustment of the final step in the allocation process.

The economic justification of segments and purposes of projects or units should be determined and tested in the process of project formulation. Cost allocation is not a device for testing the justifiability of segments or purposes; it is a method of distributing costs of a properly formulated and justifiable project and should not be initiated until the formulation and justification have been completed.

During project formulation, segments of a project are tested in terms of the relationship between separable costs and incremental benefits for the segment under consideration. This process is not dependent on the relationship of the total allocation to a particular purpose in comparison to the total benefits of that purpose. When the formulation process is properly executed, Mr. Hotes' illustration could not happen.

Separable costs used in the cost-allocation process are derived under the assumption that each purpose, in turn, is the last increment of the project, whereas project formulation frequently depends on increments which serve parts of several purposes. For example, the last increment in project formulation may be an increase in the height of a dam, affecting power head, flood-control capacity, and the quantity of water for irrigation and municipal use. Thus, in project formulation a whole purpose of the final project is seldom a single increment. Because of this difference some of the necessary data for estimating separable costs may be available in studies of plan formulation. However, additional estimates are needed, and the treatment of an entire purpose as a last increment may not be fully compatible with the actual size and order of increments used in project formulation.

Mr. Hotes observes that the mention of taxes foregone in Table 1 is confusing. Inadvertently, the modifying phrase regarding taxes foregone was included in the brief description of the allocation steps. The inclusion of taxes foregone, however, as Mr. Hotes correctly concludes, would yield approximately the same allocation results. The more significant effect of the use of taxes foregone occurs in benefit-cost analysis.

---

[7] Chief, Div. of Project Development, Bureau of Reclamation, U. S. Dept. of the Interior, Washington, D. C.

Messrs. Johnson and Cocks propose that the remaining project, or projects, be economically justified. Mr. Weber goes somewhat further and proposes that, in certain cases at least, the remaining project should be the best available and the most economical. In most cases, when a project has been properly formulated, the remaining project will be economically justified. It is entirely conceivable, however, that instances will arise when this is not the case, particularly in a project with one prime function and one or more incidental functions—for example, recreation and fish and wildlife. As the writer indicated (under the heading, "Remaining Project Costs"), to find and use the most economical remaining project might disperse the project benefits to a point where separable costs would not be revealed and might well lead to illogical conclusions; nevertheless, the matter should be explored.

Basically the questions raised concern the concept of remaining costs, whereas the writer believes emphasis should be on the concept of separable costs. For the present, at least, it is believed that determination of remaining costs should be considered essentially as a statistical or a mathematical device designed to provide a tool for distributing joint costs. To go further than this may be a refinement not warranted by the accuracy of other basic data.

The 1950 report is the "first word" rather than the "last word" on project formulation and separable costs. This report is now (1957) being revised to take advantage of some of the actual experience obtained during the last five years. It is hoped that the revision will result in sharper definitions and will recognize the many subtle differences encountered in the actual analysis of multiple-purpose water projects. Some of the unresolved problems are (*a*) consideration of the relationship between the most economical and the most likely alternative; (*b*) the difference between separable costs of an entire purpose and those of parts of several purposes used in project formulation; (*c*) the need for economic justification of the remaining project; (*d*) whether separable costs for cost allocation should be traceable solely and clearly to a single purpose; (*e*) whether such costs should be confined to multiple-purpose sites; (*f*) the treatment of intangible and incidentalpurposes; and (*g*) the treatment of interest during construction and taxes foregone.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

TRANSACTIONS

Paper No. 2913

DESIGN OF STABLE CANALS AND CHANNELS IN ERODIBLE MATERIAL

By Pete W. Terrell,[1] M. ASCE, and Whitney M. Borland,[2] A. M. ASCE

Synopsis

The design of stable channels in erodible material is complex and depends on the incomplete technical development of open-channel hydraulics, sediment transport, and fluvial morphology. Considerable engineering experience and judgment are required in an approach to this problem. Major factors that must be integrated into the design of a canal are listed and examined herein. The development and present method of the design of canals are outlined.

Factors causing a change in stream regimen are enumerated. Several methods by which the proper size and shape of a channel can be computed are suggested, and one example is presented. Practical considerations and field experience in channel-stability problems are given. An outline of the basic data needed for the adequate design of a channel is presented.

Introduction

Irrigation projects require (1) the conveyance of water to land by canals and laterals and (2) the removal of the water not consumed by the irrigable areas by a constructed drainage system that frequently includes improvement of natural drainage ways. Usually both the irrigation and drainage water must be conveyed by channels in erodible material. Basinwide water-resource development and transbasin diversion have radically changed the regimen of rivers and streams so that channel rectification, bank protection, and grade stabilization must be used to protect adjacent property. In all natural drainage ways and in some drains, it is essential to consider the sediment load of

---

Note.—Published, essentially as printed here, in February, 1956, in the Journal of the Hydraulics Division, as *Proceedings Paper 880.* Positions and titles given are those in effect when the paper was approved for publication in *Transactions.*

[1] Asst. Chief, Canals Branch, Div. of Design, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo.

[2] Head, Sedimentation Section, Hydrology Branch, Div. of Project Investigations, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo.

the water in the design of the channel cross section and in the improvement of the channel.

For canal design the presence of clay and fine silt in the water requires no special consideration other than that taken for sediment-free water. An attempt is made to maintain as high a velocity as possible to prevent deposition and without causing scour. The presence of coarser sediments has not been considered because the canals receive water from storage reservoirs or from diversion works where most of the coarser sediments are kept from entering the canal.

Two important factors must be considered in the design of canals and drains—water salvage and maintenance cost. In canals various types of linings are considered because excessive seepage not only wastes water but waterlogs the land, making it nonproductive. The rectification of river channels is necessary to decrease the water lost in transit from high water tables and nonbeneficial consumptive use of phreatophytes. Canal and channel designs should result in the smallest annual total cost and therefore must prevent conditions leading to progressive increase in maintenance cost.

Canals.—A brief outline will be presented of the background and development of the method used by the Bureau of Reclamation, United States Department of the Interior (USBR), in arriving at a usable section for a canal prism. The limitations of the present USBR practice as well as an exposition of this practice are also given. The background data are essential because irrigation systems in this country differ materially from those in other parts of the world. Until these basic differences are evaluated, it is not apparent why rules, formulas, and equations that are acceptable elsewhere have proved unusable in the work of the USBR.

It is possible to develop a stable channel in erodible material where the capacity and silt load are constant. It is also possible to develop a channel that will erode and refill during certain seasons so that at any time during a year the channel will be in the same state that it was at a corresponding time the previous year. However, neither of these conditions has a wide practical application, and it is this failure of theoretically perfect channels to function properly in practice that causes the difficulties of canal design.

It is necessary to recognize the field conditions making a highly theoretical approach impracticable, so that the engineer designing a canal section can properly evaluate the usefulness of a theoretically computed section. Some of the major considerations in channel choice that must be accounted for in design are as follows:

1. The terrain traversed by a canal may extend for more than a hundred miles. In such distances, soils vary widely. The major types, such as silts, sands, clays, broken rock, or solid rock, can be considered separately. However, the soils within any class also vary radically; thus, assumed channel factors do not apply from section to section, as is displayed clearly in loessal areas.

2. The soil in place may be stable against scour, but when worked in excavation it becomes easily erodible. This would indicate that the water prism should be in undisturbed earth. However, the increased cost of such a location is obviously not justifiable, and the alinement chosen is dependent

on an evaluation of costs of various possibilities and types of equipment. The development of earth-moving equipment itself has changed design. A bank built with excavating and hauling equipment is considerably different from that placed by draglines. (These factors influence the selection of the channel sections and must be considered in determining the side slopes and alinement.)

3. After construction the canal must not only be intrinsically stable but must also hold water. Leakage results in considerable water loss that may ruin land by waterlogging. Moreover, excessive losses require larger sections to provide for deliveries in addition to loss; hence, lining is a major consideration. There are several types of membrane linings which require earth cover to protect the membrane, and the cover must resist scour. The selection of the type of lining requires a study of costs among the various possibilities for a given section. An adequate theoretical approach is valuable for this comparison.

4. Some scour can be tolerated in cut, but, where extensive reaches must be in fill, scour cannot be allowed. Hence, judgment dictates the "safety factor" that is to be used against scour.

5. A knowledge of land use and development is also necessary. A new project will result in years of relatively low demand and may never fully utilize the capacity provided. On the other hand, a developed project may require the extensive use of sections almost as soon as the new supply canal is constructed. A new system can, therefore, safely carry a higher scour potential if serving an undeveloped area than it can in serving a developed area because complete seasoning is possible before full use will be required.

6. Irrigation operations also influence designs. Is the canal to be in service consistently, or will it be empty during winter months? Are weed problems the primary maintenance item, or will blow sand tend to plug the section? Will the canal be checked up for side deliveries at low flows, or will the flow seek its own level for various demands? All these problems influence the section and must be evaluated properly to realize the minimum over-all project cost of construction, operation, and maintenance.

The foregoing factors illustrate design problems which, if known—as is rarely the case—serve to alter the theoretically perfect section. The unknown factors will be examined in the analysis of a theoretical design approach.

The design studies[3] made for a stable channel in the All-American Canal (California) embrace the initial approach to the problem of the design of stable channels. Earlier data and formulas were studied, but the application seemed beyond their possible extension. The canal would be forced to carry clear water while the reservoir filled in and then would probably carry a considerable silt load. Furthermore, the canal traversed many miles of desert terrain of widely varying soil conditions. After a study of the available material and data, it was decided to install clarifying devices to maintain a clear-water canal and to design accordingly.

The shearing forces on the sides and banks were evaluated, and the main canal was designed with a value of B/D of approximately 8 to 1 (B being base width and D, depth) with 1.75-to-1 (horizontal-to-vertical) side slopes and a maximum velocity of 3.75 ft per sec. This section is still stable after more than fifteen years of almost constant use. Naturally such a channel gives rise to the question of whether it could have been smaller and still be stable.

---

[3] "Boulder Canyon Project Final Reports," Pt. IV of "Design and Construction," *Bulletin 1, General Features*, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo., 1941, p. 278 and p. 282.

The results of past experience have been tabulated[4] showing the maximum and minimum hydraulic properties, depending on soil types. These data serve as a guide to the designer in establishing proper canal sections. To guard against water loss or land waterlogging, linings of all types have been used. With a few exceptions, these linings are resistant to scour and, hence, do not require study of erosion; however, these linings involve other problems. The question of sediment transport in lined and in erodible channels will be presented subsequently in this paper. Hard-surface linings are very often used in noncohesive soils and, thus, limit the number of available canals for the study of erosion criteria in these types of soils.

Between 1948 and 1950, a program of investigation was formulated and, under the direction of Emory W. Lane, M. ASCE, was put into effect. The basic approach was the extension of the tractive force principle. The results of a great many independent developments and equations were compared with each other and with the results of USBR canal sections; little correlation between formulas, or between formulas and field conditions, was found. When the tabulated sections[4] were analyzed (using tractive force for evaluation), a better agreement seemed obtainable, although the variations indicated that further study was warranted.

Tractive force is simply the pull of water on the wet perimeter. For an infinitely wide channel, the tractive force, F_t, on the bottom is

$$F_t = w\, d\, s \qquad (1)$$

in which F_t is in pounds per square foot, w is the weight of water in pounds per cubic foot, d is the depth in feet, and s is the slope of hydraulic gradient.

A report[5] prepared by the USBR gives a full explanation of this analytical work and the influences of sides of different slope and base widths. Reports[6, 7] were also prepared developing the theoretically perfect shape and evaluating the critical tractive force.

One of the reports[5] contains charts from which various (B/D)-ratios and corresponding side slopes can be evaluated for design use. It is necessary to know the mechanical analysis of the soil being investigated. Studies are in progress to determine the effect of cohesion, which as yet cannot be accurately evaluated. This report computes the minimum section for a given soil and is usable as a basis for design. Physical limitations of survey work and excavating machinery prohibit the use of the section as computed. Also, operation roads and structure design alter the shape. However, a realization of the theoretical minimum section is of value to the designer.

---

[4] *Reclamation Manual*, Design Supplement No. 3, Bureau of Reclamation, U. S. Dept. of the Interior, Washington, D. C., Vol. 10, 1952, Chapter 1, Fig. 5.

[5] "Progress Report on Results of Studies on Design of Stable Channels," *Hydr. Lab. Report No. Hyd-352*, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo., June, 1952.

[6] "Stable Channel Profiles," *Hydr. Lab. Report No. Hyd-325*, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo., September, 1951.

[7] "Critical Tractive Forces on Channel Side Slopes," *Hydr. Lab. Report No. Hyd-366*, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo., February, 1953.

Another report[7] is a theoretical evaluation of the maximum limit of tractive force as applied to any soil particle and provides a better understanding of the mechanics of this approach.

As the reports explain, the equations do not answer a great many questions. The basic principle seems correct, but boundary layers and the turbulent losses certainly need further study and analysis to improve the basic equations. The reports contain laboratory and field research data obtained prior to 1952. This investigation is still continuing on a reduced scale, but several canals have been designed to check certain phases.

The current investigations cover three principal phases: (1) Application of the tractive force analysis to channels in noncohesive soils, (2) development of procedures and equipment to determine cohesive properties of the in-line soils, and (3) investigations of soil properties not defined by either the plasticity index or mechanical analysis.

The application of the design procedure is being checked by using three reaches of a canal in almost identical soils consisting of fine wind-blown sands and silts. One reach utilizes a tractive force of 0.033 lb per sq ft. Other reaches have progressively higher forces of as much as 0.075 lb per sq ft. The higher values are expected to scour; therefore, sections were selected where scour could be controlled by checking. These sections have not been operated at full capacity as yet.

The effective value of cohesion is a troublesome problem. A natural tendency to reduce a section for economy induces an overdependence on cohesion, which, in turn, probably results in high maintenance costs. A reasonable value is obviously needed; but to the writers' knowledge a dependable method of evaluation has yet (1957) to be devised. The USBR's approach is to develop an apparatus similar to a vane borer to measure shear resistance, but this has many uncertainties. Another approach is to use a submerged controlled jet to give comparative values; the third is to obtain saturated shear values from a loading machine or correlation with plasticity index. To date, progress has been too limited to judge the efficiency of any of these steps.

It has been noticed that certain canals scour, whereas others under very similar conditions do not. These conditions are similar as to width, depth, side slopes, velocities, reasonable freedom from sediment load, mechanical analysis, and plasticity index of soils. Because these factors do not account for the dissimilarity of results, soil samples were taken for chemical examination. The examination may show that an ion exchange between water and soil hydration of soil material provides a binder in some localities. Should this prove a dependable condition, future evaluation of a proposed section should include whatever advantage can be taken of such benefits.

If all these steps prove fruitful, many factors will still remain to judgment, and considerable investigation will still be necessary before theoretical design can accurately predict a stable channel. However, even in its present stage of development, the use of tractive force seems to provide a useful tool, and it is as accurate an approach as has been devised.

The USBR design consists of selecting from experience and judgment charts an approximate section, which is used in preliminary surveys. For this alinement, geological investigations are made. These investigations are essentially a series of test holes and samples. The soil is classified and analyzed for sizes of particle and for plasticity. Shear tests are made for use in determining bank stability.

With these data, the designer would investigate the section proposed by applying the tractive force analysis to determine probable stability by reaches and to determine the minimum section that appears usable. Up to this point, the analysis is theoretical. This proposed section must then be evaluated for the effect of cohesion, which is still beyond an exact analysis. It is further considered in relation to (1) the cost of original construction, (2) the rights of way, (3) the risks involved should some scour occur, (4) the desirability of a scouring velocity to move wind-blown sand in small sections, (5) the section changes, (6) the structure costs, and (7) the advisability of lining. All these factors will, to a varying degree in different projects and areas, influence the section selected.

In short, even with an improved theoretical analysis for the design of a stable section, practical considerations, economic conditions, bank stability, and operation and maintenance practices all combine so that, at best, the analysis is a guide. It will not, in the foreseeable future, supplant experience and sound engineering judgment in the design of canals.

CHANNEL TRANSPORTING A SEDIMENT LOAD

The design and rectification by the USBR of channels carrying sediment-laden water is a major problem. Natural streams can be divided into two main classes, "perennial" and "ephemeral."

Perennial Streams.—A perennial stream is one in which continuous flow is maintained for long periods of time even though it is dry for some periods. In contrast, an ephemeral stream is one in which flow is directly related to storm runoff and does not flow for periods greater than twenty-four hours.

The stability of sediment-carrying perennial streams may be upset by: (1) Decreasing the flow, (2) increasing the flow, (3) increasing the sediment load, (4) decreasing the sediment load, or (5) regulating the flow. These changes have been found to occur under the following conditions:

Decrease in Flow.—The major problem encountered by a decrease in flow occurs by diversion of the clear flows as they emerge from the mountainous areas. Because these flows are historically mixed with highly sediment-laden water from high sediment-producing parts of the drainage, a serious problem results when they are diverted. What actually occurs is an annual channel flushing as a part of the channel regimen. Many of the streams draining the Rocky Mountains present this problem—for example, the South Platte River, the Arkansas River, and the Rio Grande River.

Increase in Flow.—Diversion of water to a stream from another drainage basin may often increase the flow by volume greater than the original flow. Some of the transmountain diversion schemes present this problem. Also, off-channel storage reservoir installations may use a small channel for conveyance of releases back to the main channel. Return flow from an irrigable area will often accrue to a small channel draining the area.

Increase in Sediment Load.—Changes in agricultural practices and land use have been one of the largest factors encountered in increasing the sediment load of a stream. The sediment load of a stream may be increased by adding highly sediment-laden waste water from an irrigable area.

Decrease in Sediment Load.—The sediment load of a stream is decreased by storage reservoirs in a stream system. Clear-water releases from a reservoir that stores highly sediment-laden water very often develop a degradation problem downstream.

Regulation of Flow.—A storage reservoir, particularly for flood control, may materially change the flow pattern or the flow-duration relationships. This may change the discharge that has been most influential in establishing the stream regimen. For example, floods that originally were conveyed by large overbank areas may, after storage, be released at bankfull capacity for a considerable period of time. The stream regimen may not have been established by long periods of bankfull flow.

Ephemeral Streams.—The stability of an ephemeral-type stream presents one of the most acute channel problems facing the USBR. The problem appears to be different from that of perennial stream channels and is also more sensitive. This may be due, in part, to the more divergent changes that upset the stability. The factors that upset the stability of ephemeral channels are much the same as for perennial streams, but the channel reactions appear to be different and, for that reason, are mentioned separately. They are: (1) Increasing the flow, (2) regulating the flow, (3) increasing the sediment load, or (4) decreasing the sediment load.

Increasing the Flow.—Project development is often responsible for an increase in flow of an ephemeral channel and may change the flow from ephemeral to perennial. Ephemeral channels may be used as carriers of transbasin diversions for the conveyance of water to and from off-channel storage reservoirs, and as canals in project distribution systems. The flow in small, ephemeral channels is often increased when storm flows are rerouted from their natural courses to protect irrigable areas, canals, or structures. Irrigation wastes are often dumped into ephemeral channels to be conveyed back to a main stream channel.

Regulation of Flow.—The flow pattern of an ephemeral channel can be changed so that it will affect the stability of the channel. Storm detention in either main-stem dams or in several tributary dams may change the flow pattern to the extent that flows may appear in the channel for several days, whereas under natural conditions the flow was a rush of water. Also, water that under natural conditions appeared as overbank flow may be confined to the channel and, in many cases, at near channel capacity. Conservation measures on the drainage area, such as contour farming, contour terracing, and changes in land use and cropping pattern, may be effective in changing the flow pattern.

Increase in Sediment Load.—Such an increase can occur both from upstream channel failures and from production in the drainage area. Increases in load from the drainage area usually occur because of modifications in this area, such as clearing forest areas, developing forest burn areas, cultivating pasture land, and developing irrigation areas. Surface wastes from irrigable land often contain sediments.

Decrease in Sediment Load.—When the sediment load of a stream is decreased, there is less transport capacity required of the stream to move the sediment. The difference in the load may be made up in part from the bed and banks if the proper material is available from this source. A decrease can occur due to storage in detention dams and due to drainage-area conservation measures.

DESIGN OF ARTIFICIAL CHANNELS

The USBR has the problem of designing artificial channels to convey water with sediment loads. These channels are generally not associated with irrigation distribution systems, but are channels that convey diverted stream flow to off-channel storage reservoirs and river conveyance channels, such as the salvage of water in the delta area above Elephant Butte Reservoir (New Mexico). Heavy loads of sediment usually cannot be prevented from entering project drains when storm flows are carried. These drains may be either perennial or ephemeral. The design of a channel to carry sediment loads requires that certain basic data be available. The data desired are:

1. A flow-duration curve of the flows the channel must carry;
2. A sediment-rating curve showing the average sediment load in tons per day for a given discharge that the channel must carry;
3. Size-distribution curves showing the average texture of the sediments at various instantaneous discharges throughout the range of the sediment-rating curve;
4. Representative bed-material, size-distribution curves above the potential diversion point;
5. The slope of the channel above the potential diversion point;
6. Representative material, size-distribution curves for the material through which the channel will be constructed (variations in texture with depth and plasticity index if applicable);
7. The slope of the proposed channel and the limitations on variation;
8. The type and vigor of the existing native vegetation in the area the channel may traverse; and
9. An outline of any limiting conditions on the channel design and future operation.

A method that has been used by the USBR in developing a channel design is the transport method.[8] Essentially, it is a "cut-and-dried approach," with transport computations being made until the point is determined at which the transport capability of the channel matches that required by the sediment load the channel must carry. Only the sediments coarser than 0.0625 mm are considered in this type of analysis. Such a computation was performed for the conveyance channel above Elephant Butte Reservoir using a function developed by Hans A. Einstein,[9] M. ASCE. In this case, the slope was that of the valley, and very little variation from the natural slope was practical. The bed material of the channel from which the water was diverted was assumed to be the same as that which could be expected in the proposed channel because the native material and the slopes were comparable. With a given slope and bed material, the transport capability in pounds per second per foot of width was determined by size fractions (0.0675 mm, 0.120 mm, 0.125 mm, 0.250 mm, and 0.50 mm) for various hydraulic radii that might be expected. The results were plotted on semilog paper, and transport curves were drawn for each size fraction (Fig. 1). Using the bed-load function developed by Einstein (Fig. 2), discharge-hydraulic radii relationships were

---

[8] "Conveyance Channel Widening Study—Middle Rio Grande Project," by J. M. Lara and C. R. Miller, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo., September, 1951.

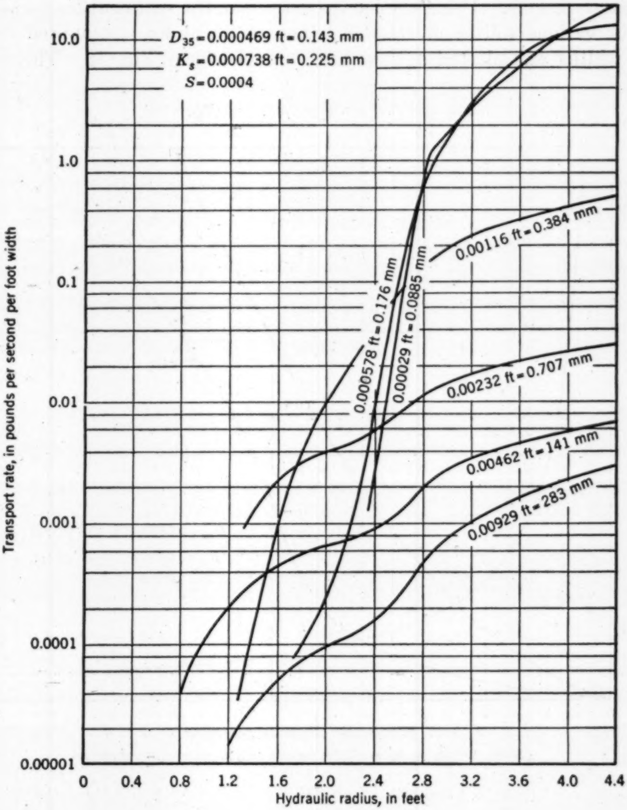


FIG. 1.—UNIT SEDIMENT TRANSPORT FOR VARIOUS PARTICLE SIZES FOR CONVEYANCE CHANNEL

determined for channels of various widths. These relationships were expressed by curves (Fig. 3). Table 1 was then arranged to represent, essentially, a mechanical integration of the flow-duration curve for the flows the channel will be expected to take. It may be noted that some of the coarser fractions do not show adequate transport. The total sand transport was used rather than the individual fractions because of the necessary assumptions and the limitations of the formula to compute accurately the transport of the individual size fractions. Using the curves described, the sediment load was computed for various widths of channel. The sediment load computed for the various widths was plotted against the channel width (Fig. 4). The plot of the points reasonably well defined a straight line. From this relationship, the width at which the expected average sand-sediment load would be transported was determined.

---

[9] "Bed-Load Function for Sediment Transportation in Open Channel Flows," by Hans A. Einstein, *Technical Bulletin No. 1026*, U. S. Dept. of Agriculture, Washington, D. C., September, 1950.

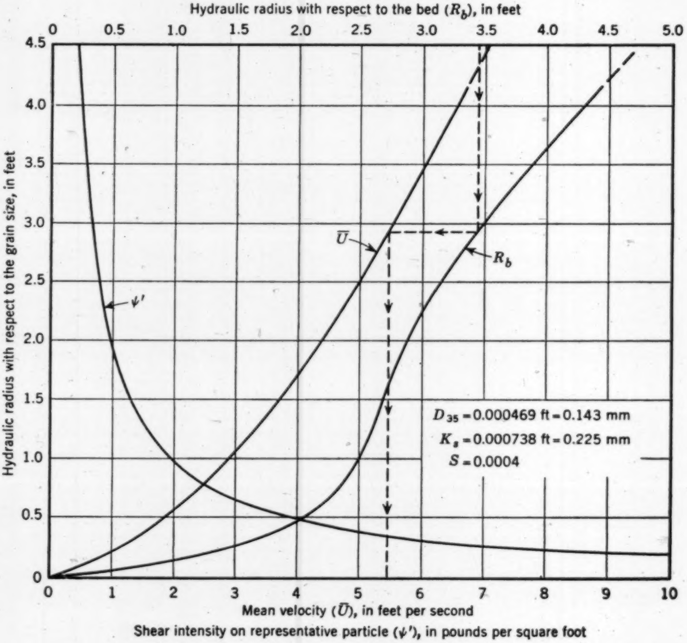


FIG. 2.—EINSTEIN CHANNEL CHARACTERISTICS FOR CONVEYANCE CHANNEL

However, the width computed was not the width constructed because of the limitations of the earth-moving equipment. A narrower channel was constructed, which is being allowed to erode. A representative curve of widening rate was determined using the transport curves. The curve of widening rate appears to approach asymptotically the optimum width computed. The width to which the channel is expected to erode is where the widening rate curve

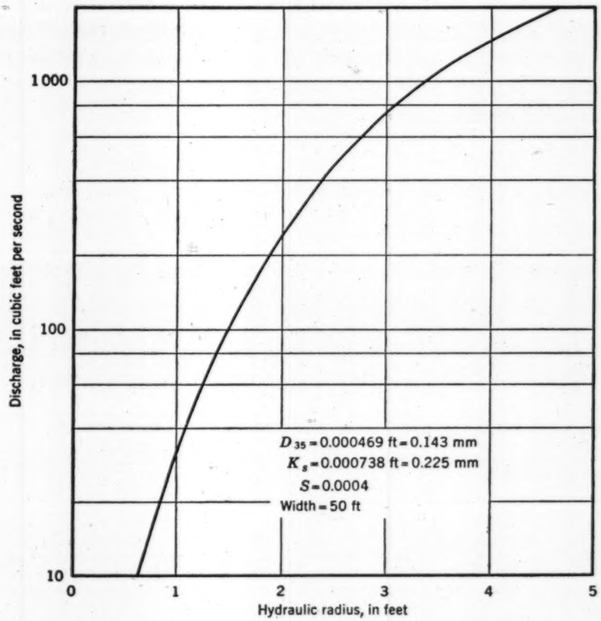


FIG. 3.—DISCHARGE AND HYDRAULIC RADIUS FOR CONVEYANCE CHANNEL

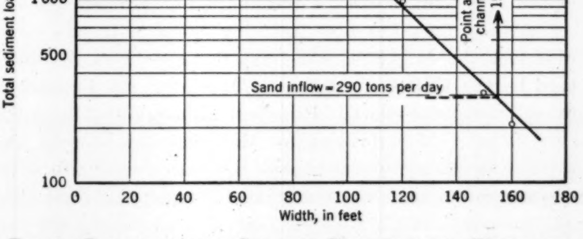


FIG. 4.—RELATIONSHIP OF SEDIMENT TRANSPORT AND WIDTH FOR CONVEYANCE CHANNEL

TABLE 1.—Transport Load for Conveyance
(Rectangular Channel, 50-Ft

Q_w, in cubic feet per second	Flow duration, in %	Hydraulic radius, R_b, in feet	Channel width, P_b, in feet	Transport					
				Particle sizes,					
				290		578		1,160	
				lb/sec/ft	tons/day/ft	lb/sec/ft	tons/day/ft	lb/sec/ft	tons/day/ft
1.4	1	0.13	50						
11	4	0.67	50						
78	10	1.38	50					0.00019	0.000821
244	10	2.03	50			0.000285	0.001231	0.01400	0.068480
415	10	2.43	50	0.004	0.01728	0.013500	0.058320	0.04500	0.194400
580	10	2.74	50	0.280	1.20960	0.395000	1.706400	0.11600	0.501100
720	10	2.97	50	1.630	7.04160	1.190000	5.140800	0.17900	0.773300
840	10	3.13	50	2.510	10.84320	2.100000	9.072000	0.21900	0.946100
1,080	10	3.46	50	5.300	22.90000	5.300000	22.900000	0.29100	1.260000
1,500	7	4.02	50	13.500	40.82400	13.500000	40.824000	0.42000	1.270000
Totals	82[a]			23.224	82.83568	22.498785	79.702751	1.28419	5.014201
Tons per day for Q_w <2,000 cu ft per sec				4,141.68		3,985.14		250.71	
Tons per day for sand in flow <2,000 cu ft per sec[b]				0.02		6.38		131.47	
Difference[c]				4,141.66		3,978.76		119.24	

[a] Zero flow occurred 18% of the time; therefore, total is 82%. [b] Sand inflow determined by another study
ft, 0.000578 ft, and 0.001160 ft scouring may theoretically occur; for the three remaining size ranges, deposition

breaks and starts to approach a straight line. It was thought that the narrower width chosen would become stable because of bank-restraint barriers offered by the vegetation in the area.

This approach can be used together with several of the other acceptable sediment-transport functions, but it has three weaknesses that preclude the results from being exact computations. The bed-material size is a significant component of the computation, and this variable is assumed throughout. The width of compatible transport may still have an erosive force that will produce bank failure, and therefore, stability is not assured. The experience of the USBR shows that existing transport functions and formulas have not been developed to a point of exact determination of sediment movement.

Another method that has been used by the USBR was developed by Thomas Maddock, Jr., M. ASCE, and Luna B. Leopold, A. M. ASCE.[10] The formula used by the USBR is not that of Maddock and Leopold but rather is a result of subsequent work. However, although the formula may not represent the latest (1957) thinking by Maddock and Leopold, the formula when adapted showed a significant enough correlation coefficient with observed field conditions to warrant consideration for practical application. Their

---

[10] "The Hydraulic Geometry of Stream Channels and Some Physiographic Implications," by Thomas Maddock, Jr. and Luna B. Leopold, *Professional Paper 252*, Geological Survey, U. S. Dept. of the Interior, Washington, D. C., 1953, p. 57.

Channel, Middle Rio Grande Project
Width, Slope = 0.004 Ft Per Ft).

Load

in 10^{-6} feet						Totals		
2,320		4,620		9,290		in pounds per second per foot	in tons per day per foot	in tons per day
lb/sec/ft	tons/day/ft	lb/sec/ft	tons/day/ft	lb/sec/ft	tons/day/ft			
0.00120	0.005184	0.000310	0.001339	0.000032	0.000138	0.001732	0.007482	0.374
0.00405	0.017496	0.000680	0.002938	0.000103	0.000445	0.019118	0.090590	4.530
0.00610	0.026350	0.000960	0.004150	0.000180	0.000780	0.069740	0.301280	15.064
0.01000	0.043200	0.001700	0.007300	0.000400	0.001700	0.803100	3.469300	173.465
0.01410	0.060910	0.002680	0.011600	0.000710	0.003100	3.016490	13.031310	651.565
0.01600	0.069120	0.003230	0.014000	0.000930	0.004000	4.849160	20.948420	1,047.220
0.02010	0.086800	0.004300	0.020000	0.001430	0.010000	10.916830	47.176800	2,359.000
0.02600	0.079000	0.005700	0.017000	0.002320	0.007000	27.454020	83.021000	4,151.050
0.09755	0.388060	0.019560	0.078327	0.006105	0.027163	47.130190	168.046182	8,402.268
19.40		3.92		1.36		...	...	8,402.21
108.47		32.62		10.58		...	...	...
−89.07		−28.70		−9.22		...	...	8,112.67

using hydraulic characteristics under natural conditions. [e] Difference indicates that for particle sizes of 0.00290 may theoretically occur.

formula is

$$\frac{w}{d} = \left(\frac{\dfrac{S^{1/2}}{n}}{0.225\,(C_s\,V_s)^{0.395}} \right)^3 Q^{0.555} \qquad \text{(2)}$$

in which w denotes the width of the water surface in feet; d, the mean water depth in feet; S, the slope of the hydraulic gradient; n, the rugosity coefficient; C_s, the sediment concentration in parts per million by weight; V_s, the mean fall velocity of the sediments in feet per second at the appropriate temperature; and Q, the dominant discharge in cubic feet per second. The (w/d)-ratio can be resolved into separate components by applying the Manning equation. For wide, shallow channels the Manning equation can be resolved to

$$Q = 1.486 \left(\frac{S^{1/2}}{n} \right) w\, d^{5/3} \qquad \text{(3)}$$

by letting the mean depth equal the hydraulic radius and the area equal the product of width and depth.

The term "dominant discharge" has been used by many authors and has been defined several ways. It has been recognized by students of stream morphology that a natural stream channel does not maintain a constant unwavering condition of stability. At one discharge the channel may be degrading; at another discharge the channel may be aggrading; at a third it might be widening; and at a fourth it may be narrowing. In fact all these conditions may act in a stream simultaneously and within separate reaches. However, the over-all effect is not one of continued aggradation, degradation, widening, or narrowing. The dominant discharge is herein defined as that discharge most influential in dictating the general condition of stability. In USBR usage it has been defined quantitatively as the discharge that will carry the greatest sediment load coarser than 0.0625 mm, with respect to time. Generally the dominant discharge is slightly greater than the median discharge. The procedure of Maddock and Leopold, as described in the foregoing, has three weaknesses that prevent the results from being conclusive. The empirical correlation was developed from data obtained mostly from natural streams whose shapes were developed through long periods of time. Therefore, the channel shape computed may be that which the channel tends to assume and which it will attain in time. Because of natural perimeter barriers, such as heavy cobbles, rock, vegetation, and highly plastic soils, the time needed to attain the width may be beyond economic consideration. Another weakness may be seen by examination of the formula. If the $(C_s V_s)$-value approaches the condition of clear water, the (w/d)-ratio becomes larger than would be reasonably expected. Also, the rugosity coefficient, n, of the design channel, as affected by variable sediment movement, cannot be accurately predicted.

Another empirical approach[11] used was developed by Thomas Blench, M. ASCE, and, as with other approaches, requires considerable judgment and experience in its application.

The tractive force principle, presented in the section on "Canals," does not consider the influence of sediment loads, particularly heavy loads of coarse material. However, it does give a factor for bank resistance in noncohesive materials. Also, the report[5] does not give a good definition of allowable tractive forces for cohesive materials, the influence of vegetation, or the ephemeral flow condition. Field investigation has been made of approximately seventy artificial and natural channels with varying conditions of cohesiveness, vegetative growth, and for both perennial and ephemeral flow. Most of the channels have twenty years of service or more and were selected because they were effectively stable, although some erosion may occur at times. Rough working curves have been developed and are now being checked with field experience.

Observations of Phenomena.—In working with channel-stability problems, several observations of channel phenomena have influenced the approach to the problems. For instance:

1. There are conditions in which the sediment load of a stream exceeds the transport capability, yet serious bank erosion will occur.

2. Continuity of tractive force and smooth hydraulic transition are very often a requirement for stability. High tractive forces in one reach may erode bed and bank material that will deposit in reaches of low tractive forces, and aggradation will result. Constrictions, such as natural, narrow rock

---

[11] "Hydraulics of Sediment-Bearing Canals and Rivers," by Thomas Blench, Evans Industries Ltd., Vancouver, British Columbia, Canada, 1951, p. 111.

controls and inadequate bridge openings, may cause backwater and eddy conditions that produce aggradation areas above and scour areas below. The upstream area can often be more serious when coarse materials deposited during flood stage force the lower flows to vulnerable bank areas.

3. The allowable tractive forces for ephemeral streams in cohesive soils are higher than perennial streams in the same soil. Erosion can usually be expected, particularly at the historical flood magnitudes, even when only small quantities of continuous flow are added.

4. When large volumes of low flow water are added to meandering streams sufficient to create heavy bend erosion, the pattern of meander movement is upset. When floods are no greater than in the past, the deposition on the inside of bends is not as fast as the accelerated bend erosion. The end result is a loss of the meander belt, increased slopes, increased scour ability, and general channel failure.

5. Vegetation is effective in increasing the allowable tractive force on both the bed and the banks. However, vegetation will not grow in the perennially inundated parts of the channel unless velocities are exceedingly low.

6. Sediments will not follow the water into the underlying gravels of an influent stream. Influent streams that carry large amounts of sediment are special stability problems.

7. Sediments finer than 0.031 mm generally do not seriously affect channel stability problems except for side berming action in areas of heavy grass vegetation.

Conclusions

The approach to channel-stability problems requires the application of all that is known about river hydraulics and sediment transport. The USBR's approach has been a combination of the approaches herein described. For example, the procedures of Maddock and Leopold and of Blench will give a width at the dominant discharge. This width is then checked to see if the sediment will be transported. If so, the bank tractive forces are checked. If the bank tractive forces are too high for the bank material to resist, additional barriers such as riprap, vegetation, and revetments are needed. If the bank and bed material show considerably more allowable tractive force, the channel will remain stable at a narrower width, as dictated by the allowable tractive force of the bank and bed. Transport will almost always be greater than required with the narrower widths. At the same time, one must always check the seven special conditions mentioned. It is expected that, with experience, more of these special conditions will be added to the list.

The problem of stable channels is complex. The type of engineering concerned with this problem does not lend itself to precise design and depends on the judgment, experience, and skill of the engineer. USBR engineers believe that those faced with these problems should continue to strive cooperatively to develop procedures that assure simpler and more conclusive solutions.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2914

SOIL LYSIMETERS IN WASTE WATER RECLAMATION STUDIES

BY GERALD T. ORLOB[1] AND ROBERT G. BUTLER,[2]
JUNIOR MEMBERS, ASCE

WITH DISCUSSION BY MESSRS. RALPH STONE; AND GERALD T. ORLOB AND ROBERT G. BUTLER

SYNOPSIS

Interest in waste water reclamation by surface spreading has dictated a need for study of the mechanism of infiltration, the phenomena that determine the rates of water percolation, and the interaction of porous media and degraded waters. Soil lysimeters are shown to be well suited for the investigation of many important aspects of the reclamation problem and to offer a convenient, economical means of predicting full-scale field spreading performance. The behavior of five typical pervious agricultural soils under water and sewage spreading conditions are evaluated, and a comparison is made between field and lysimeter performance for two of these soils.

INTRODUCTION

Concern over water shortages in many western areas, notably the Central Valley and the southern coastal plains in California, has directed considerable attention toward techniques for the reclamation of all types of waste waters. Recharge of underground supplies by surface spreading and infiltration has often been considered feasible where the water might otherwise be lost by surface runoff to salt water. For example, proposals for the "conjunctive use" of surface-water and ground-water reservoirs have embodied the use of in-

---

NOTE.—Published, essentially as printed here, in June, 1956, in the Journal of the Sanitary Engineering Division, as *Proceedings Paper 1002*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Asst. Prof. of Civ. Eng., Univ. of California, Berkeley, Calif.

[2] Asst. Prof. of Civ. Eng., North Dakota State College, Fargo, N. Dak.

filtration basins as an integral part of the reclamation scheme.[3,4] Waste irrigation water has been introduced into ground-water storage by surface spreading,[5,6] and several experiments,[7,8] some on full scale, have been concerned with the reclamation of domestic sewage by this procedure.

One of the most complete field studies[9] of sewage spreading was conducted over a period of twenty-eight months (1950 to 1952) near Lodi, Calif. In contrast to many prior experiments with sewage reclamation by spreading, this investigation incorporated extensive physical, chemical, and bacteriological analyses in an attempt to determine some of the fundamental phenomena relating to the problem. Although only a single soil, Hanford fine sandy loam, was studied, the experiments revealed that the rate of infiltration was dependent on (1) the nature of surface treatment, (2) the duration of application, (3) the period of resting between applications, and (4) the quality of the effluent applied. It was also concluded that a bacteriologically safe water could be produced from primary settled sewage after percolation through only 4 ft of Hanford soil. Infiltration rates were generally less than 0.3 cu ft per sq ft per day (or feet per day), except during a short period immediately following the beginning of spreading. When the basin was drained, dried, spaded, and rested for several weeks, infiltration rates substantially improved when the spreading was resumed.

Information has been obtained by L. Schiff,[6,10] A. M. ASCE, and E. S. Bliss, C. E. Johnson, and Schiff[5] for water spreading on two agricultural soils, Hesperia and Exeter loams near Bakersfield, Calif. Using waste irrigation waters they reported sustained infiltration rates ranging from 1 ft per day to 3 ft per day. Special soil preparation, notably the use of soil additives such as cotton gin trash and the growth of forage crops, was found to be beneficial to infiltration. Pond arrangement and construction were also noted to influence performance.

OBJECTIVES OF INVESTIGATION

Detailed studies such as those described can provide much in the way of a fundamental understanding of soil-waste water relationships. However, owing to the extreme diversity in soil and waste water characteristics, it may be difficult to extend information obtained in such experiments to new situations. Also, such investigations are costly and not easily justifiable as a component of modest engineering studies. Consequently, a less costly method is needed

---

[3] "A Comprehensive Plan for the Utilization of a Surface Reservoir with Underground Storage for Basin Wide Water Supply Development: Solano Project California," by F. B. Clendenen, thesis presented to the University of California, at Berkeley, in 1954, in partial fulfilment of the requirements for the degree of Doctor of Engineering.

[4] "Planned Utilization," by Robert O. Thomas, in "Ground-Water Development: A Symposium," *Transactions*, ASCE, Vol. 121, 1956, p. 422.

[5] "Report on Cooperative Water Study with Emphasis on Laboratory Phases," by L. Schiff, E. S. Bliss, and C. E. Johnson, *Provisional Report*, S.C.S., U. S. Dept. of Agriculture, Washington, D. C., 1950.

[6] "Some Development in Water Spreading," by L. Schiff, *ibid.*, 1953.

[7] "Reclamation of Treated Sewage," by R. F. Goudey, *Journal*, A.W.W.A., Vol. 23, 1931, pp. 230–240.

[8] "Report Upon the Reclamation of Water from Sewage and Industrial Wastes in Los Angeles County, California," by C. E. Arnold, H. E. Hedger, and A M Rawn, County Sanitation Dist. of Los Angeles County, Calif., April, 1949.

[9] "Reclamation of Sewage Waters by Spreading," *Publication No. 6*, State Water Pollution Control Board, Sacramento, Calif., 1953.

[10] "An Exploratory Study on the Effect of Surface Head on Infiltration Rates Under Water Spreading Conditions," by L. Schiff, *Provisional Report*, S.C.S., U. S. Dept. of Agriculture, Washington, D. C., 1953.

for generalizing the findings of such studies and to provide data on which to base engineering decisions for the design of spreading installations.

In planning the studies that are reported (in part) herein, it was postulated that the behavior of Hanford loam under field conditions at Lodi and Hesperia loam under water spreading conditions at Bakersfield might be correlated with the performance of the same materials in pilot-scale lysimeters. If a dependable correlation could be obtained, it might then be possible to use these relationships to predict the behavior of other soils under field spreading conditions with a reasonable degree of accuracy. The cost of lysimeter construction and operation, nominal in contrast to field test installations, would not be so burdensome that the widest choice of possible solutions for a reclamation problem could not be considered.

Although the entire investigation included detailed consideration of physical, chemical, and bacteriological factors and how they might influence the feasibility of spreading practices, this paper is concerned only with the effect of physical phenomena on the rates of infiltration. Other aspects of the reclamation problem, particularly the fate of bacteria during movement through porous media in underground waters, are still being studied.

INVESTIGATION

Installation.—Twenty lysimeters, each 5 ft deep and 3 ft in diameter, constructed from galvanized, corrugated iron pipe sections welded to a steel base plate, were utilized for holding the soils under investigation. Graded-gravel underdrains and collection manifolds were provided for each unit, the percolant being collected in a reservoir where accumulated discharge could be determined by hook gage observations. The discharge pipe from the collection manifold was constructed in such a way that the point of discharge was at the same elevation as the gravel-soil interface in the lysimeter, thus producing a "perched aquifer" and a plane of atmospheric pressure at the bottom of the soil column.

Specially constructed devices designed to measure tension (or pressure) in the liquid phase of partly saturated soil columns were installed in five of the lysimeters. These tensiometers consisted of a glass manometer leg (3-mm inside diameter), attached by a flexible tube to a porous filter cylinder tip. Various types of tips were investigated, including glass tubing packed with glass wool or plaster of Paris, but a 10-mm-by-55-mm, porosity No. 2 filter cylinder was found to be the most satisfactory for the range of tensions encountered.

In order to make the soil as nearly homogeneous as possible; large extraneous particles and debris were removed by screening through a mesh wire screen with 1/8-in openings prior to placing. The soil was placed in the lysimeter unit in a random manner without artificial compaction. Each soil was allowed to consolidate under its own weight. During the course of placing the soils in the lysimeters, samples were taken and composited for use in obtaining physical and chemical analyses. The construction details of a typical lysimeter are shown in Fig. 1.

Fresh Water and Sewage Supply.—Fresh water used in the studies was obtained from the distribution system of the East Bay Municipal Utility

District, California. The chemical characteristics of the water are shown in Table 1.

Sewage used in the investigations was pumped directly from a trunk sewer serving a typical residential-commercial area in Richmond, Calif. No significant amount of industrial waste was present. Primary treatment was provided by settling and skimming in a 7,000-gal circular clarifier with a theoretical detention time of 2 hr and a surface loading of 900 gal per sq ft per day. Supplementary treatment was provided as needed by additional sedimentation or mechanical filtration. Typical composition of the settled sewage utilized is given in Table 2.

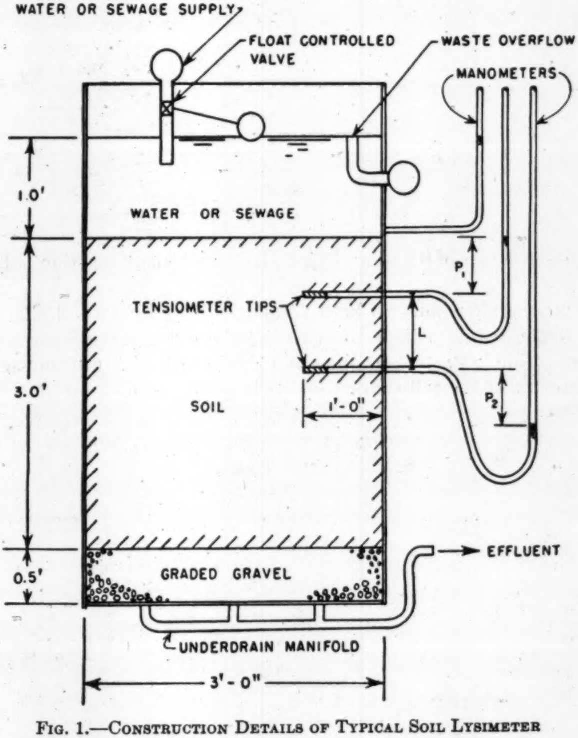

WATER OR SEWAGE SUPPLY
FLOAT CONTROLLED VALVE
WASTE OVERFLOW
MANOMETERS
1.0'
WATER OR SEWAGE
P_1
TENSIOMETER TIPS
L
3.0'
SOIL
1'-0"
P_2
0.5'
GRADED GRAVEL
EFFLUENT
UNDERDRAIN MANIFOLD
3'-0"

FIG. 1.—CONSTRUCTION DETAILS OF TYPICAL SOIL LYSIMETER

Both sewage and water were applied to the lysimeters through a manifold system connected directly to the source of supply, and the rate of application was regulated by adjustable float valves and by overflow pipes.

Soil Characteristics.—The soils used in the investigation were selected as being typically representative of California agricultural soils from areas in which the spreading of water or sewage may offer a potential solution to water shortages. Of the five types chosen, three—Columbia, Hesperia, and Yolo—

are classified as sandy loams. Hanford soil, the same as that studied at Lodi, is defined as a fine sandy loam, whereas Oakley soil is considered a sand. Each of the soils studied is frequently found in California and is usually associated with permeable subsoils and deep permeable substrata. None contained soluble salts in excess of 0.1%, and all exhibited a neutral reaction (pH between 6 and 8).

TABLE 1.—CHEMICAL CHARACTERISTICS OF FRESH WATER USED IN SPREADING STUDIES

Constituent	AVERAGE	RANGE
	Concentration, in parts per million	
Hardness ($CaCO_3$)	57.6	23.6 to 93.2
Turbidity	1	...
	Value	
pH	8.9	8.3 to 9.7
Monovalent—total cation ratio	0.164	0.08 to 0.29

The extreme variability in both the physical and chemical characteristics of naturally occurring soils is illustrated by the five materials considered in this investigation. Particle grading and size distribution are shown in Fig. 2, and a summary of the soil characteristics is presented in Table 3.

TABLE 2.—COMPOSITION OF SETTLED SEWAGE APPLIED TO THE SOILS IN THE LYSIMETERS

Constituent	AVERAGE	RANGE
	Concentration, in parts per million	
B.O.D. (5-day at 20°C)	187	90 to 322
Suspended solids	135	44 to 324
Total nitrogen	40.8	32.7 to 55.9
Ammonia nitrogen	31.3	17.2 to 50.4
Organic nitrogen	9.0	4.3 to 18.4
Nitrate nitrogen	0.5	0.0 to 1.8
Total alkalinity ($CaCO_3$)	209	120 to 300
	Value	
pH	7.4	7.1 to 7.9
Monovalent—total cation ratio	0.641	0.523 to 0.760

Water Spreading.—In order to have a basis for determining the effect of the constituents of sewage on the infiltration of sewage effluents into soils, it was necessary to establish the behavior of each of the five soils under spreading with fresh water and to consider the factors that control infiltration rates.

In studies of infiltration of water into undisturbed soil plots and into core samples, J. E. Christiansen and O. C. Magistrad observed the frequent recurrence of a characteristic time-rate curve, which was comprised of three distinct phases. Infiltration rates on samples investigated by Christiansen and Magistrad declined rapidly in the period following the outset of spreading until a minimum was reached. They rose again to a maximum, which in some cases was greater than the initial infiltration rate, and then subsequently declined until such time as an equilibrium rate was established or the soil became completely sealed. The initial, rapid decrease in rate was attributed to structural changes in the soil due in part to swelling and dispersion of dry soil upon wetting, and in part to dispersion of clay particles by ion exchange between the soil and the percolating liquid. The second phase, an increase in rate, was apparently a function of the solution of gases entrapped within the soil structure into the percolating liquid. Finally, percolation rates diminished under the continued action of bacteria in producing "polysaccharides," which clogged the soil voids. L. E. Allison[11] subsequently demonstrated that the gradual decline of the time-rate curve during the third phase, as observed by Christiansen and Magistrad, was nonexistent in sterile soil columns, thus supporting their theory of microbial clogging. Actually, it appears reasonable that each of the effects noted is operative to some extent throughout the entire period of spreading, and the characteristic curve results from a combination of all effects.

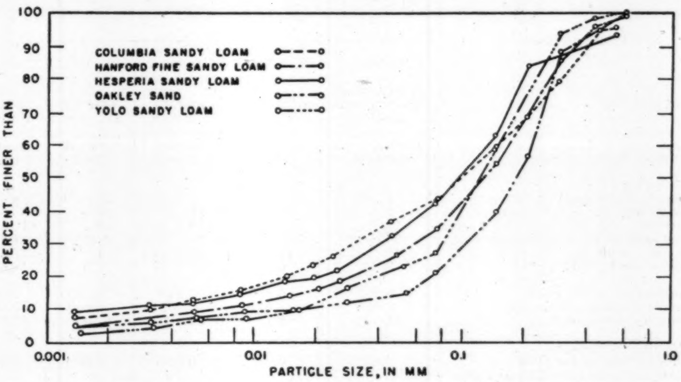


FIG. 2.—SOIL PARTICLE SIZE DISTRIBUTION

The soils used in the studies reported herein also exhibited the characteristic time-rate curve, as may be seen from Fig. 3 through Fig. 7, although in a few instances the first phase was almost nonexistent. Oakley sand, the most permeable of the soils, showed practically no initial decrease in infiltration, and in Yolo soil the initial drop in rate was confined to the first few days of spreading water. However, Columbia loam showed a continual decrease in infiltration for the first 30 days, the rate dropping from approximately 1.75 ft per day to 0.30 ft per day.

---

[11] "Effect of Microorganisms on Permeability of Soil Under Prolonged Submergence," by L. E. Allison, *Soil Science*, Vol. 63, 1947, pp. 439–450.

Before sewage spreading was begun, water application was continued until each soil was well established on the third phase of the time-rate curve. The time required to reach the equilibrium condition, however, varied considerably among the different soils. For example, Yolo sandy loam required only from 12 days to 15 days of water spreading preliminary to sewage application, whereas Columbia sandy loam received water for more than 70 days before infiltration rates were clearly established in the final phase of the typical curve.

TABLE 3.—PHYSICAL AND CHEMICAL CHARACTERISTICS OF FIVE PERVIOUS CALIFORNIA SOILS

Characteristics	Soil				
	Oakley	Yolo	Hanford	Hesperia	Columbia
Parent material	sandy-mixed alluvium, wind modified	sedimentary alluvium	granitic alluvium	granitic alluvium	mixed alluvium
Soil reaction	slightly acid	neutral	neutral	neutral	neutral
Subsurface drainage	excessive	good	good	good	good
Particle size, in %:					
Clay (<0.002 mm)	5	3	6	10	8
Silt (0.002 mm to 0.05 mm)	9	19	21	24	30
Very fine sand (0.05 mm to 0.1 mm)	15	20	16	18	12.5
Fine sand (0.1 mm to 0.2 mm)	26	33	25	29.5	17.5
Medium sand (0.2 mm to 0.5 mm)	42	24.5	29	11	27.5
Coarse sand (>0.5 mm)	3	0.5	3	7.5	4.5
Modal size, in millimeters	0.205	0.170	0.210	0.180	0.180
Effective size,[a] in millimeters	0.020	0.021	0.0074	0.002	0.0033
Uniformity coefficient[b]	11.2	8.1	24.9	67.3	47.3
Monovalent cations, in milliequivalents per 100g	0.24	0.43	0.72	0.89	0.51
Divalent cations, in milliequivalents per 100g	2.79	13.98	5.66	8.02	5.67
Exchange capacity, in milliequivalents per 100g	3.03	14.41	6.38	8.91	6.18

[a] Effective size = 10% less than size, in mm. [b] Uniformity coefficient $= \frac{60\% \text{ less than size, in mm}}{10\% \text{ less than size, in mm}}$.

A comparison of water spreading performance for the five soils during the first 30 days reveals Oakley sand to be the most permeable, at 40.3 ft per day, followed by Yolo, Hanford, and Hesperia with rates of 12.9, 2.53, and 2.37, respectively. Columbia soil, which had the widest distribution of particle sizes, passed only an average of 0.57 ft per day. A summary of water spreading for the five soils is given in Table 4.

Sewage Spreading.—After the third phase of the typical time-rate curve had been established in each of the soils, fifteen of the units (three of each soil type) were inundated with primary sewage effluent for periods of up to 150 days. One unit of each soil continued to receive fresh water and served as a control for the three columns receiving sewage. All units were subject to the same surface head, temperature, and climatic conditions during the period of study.

Infiltration rates were determined daily, and samples of influent and effluent were regularly analyzed by the standard[12] chemical, biological, and physical tests considered important to the interpretation of lysimeter performance.

Immediately following the application of sewage to the lysimeters, several significant differences in the performance of the various soil types were noted (Fig. 3 through Fig. 7). As shown in Fig. 8, the infiltration rate on Unit 20 of Oakley sand dropped from approximately 26 ft per day at the outset of sewage spreading to less than 0.6 ft per day at the end of 48 hr; within 15 days the rate had declined to an insignificant 0.12 ft per day. Fig. 9 shows that such a change in rate was typical also of Yolo sandy loam, but the change was not so pronounced. Unit 11 of this soil showed a decrease in infiltration rate from 9 ft per day to 0.72 ft per day in 48 hr and to approximately 0.3 ft per day at the end of 14 days.

However, the three finer-grained soils appeared to be little affected by the application of sewage, the trend of the time-rate curves for these soils being nearly uniform whether the soil was spread with water or with sewage. For example, infiltration rates on Columbia Unit No. 18 ranged from 0.7 ft per day at the outset of spreading to 0.52 ft per day at the end of 14 days. After one month of sewage spreading, however, the infiltration rates for each of the fine soils began to decrease at a rate slightly greater than that of the corresponding water controls. All three fine soils were filtering sewage at the rate of approximately 0.2 ft per day at the end of 10 weeks. Corresponding rates for Oakley and Yolo soils at the end of the 10-week period were approximately 0.08 ft per day and 0.25 ft per day, respectively.

TABLE 4.—Observed Infiltration of Water into Soils

Infiltration rate, in feet per day	Soil				
	Oakley	Yolo	Hanford	Hesperia	Columbia
Initial	29	10	6.4	5.2	1.75
After 15 days	47	13	2.3	1.85	0.40
After 30 days	43	10.5	1.4	1.15	0.30
After 45 days	34	9	0.85	...	0.50
After 60 days	26	7.5	0.60	...	0.74
After removal of entrapped air from soil	47 at $t = 15$ hr	20 at $t = 4$ hr	4.25 at $t = 6$ hr	4.5 at $t = 5$ hr	0.70 at $t = 50$ hr
Average for first month	40.3	12.9	2.53	2.37	0.57
Total water introduced into soil during first month, in feet	1,213	386	75.9	71.0	17.0

The abrupt clogging of Oakley and Yolo soils, particularly the former, was somewhat surprising. Even though some intrusion of sewage solids into the surface of the media was anticipated, it was believed that the ultimate rates for each of these soils (both very permeable to water) would remain at a fairly high level even after sewage application. Such was not the case, and it has been subsequently noted that this behavior of Oakley soil under certain

---

[12] "Standard Methods for the Analysis of Water, Sewage and Industrial Wastes," Am. Public Health Assn., A.W.W.A., and Federation of Sewage and Industrial Wastes Assns., 10th Ed., 1955.

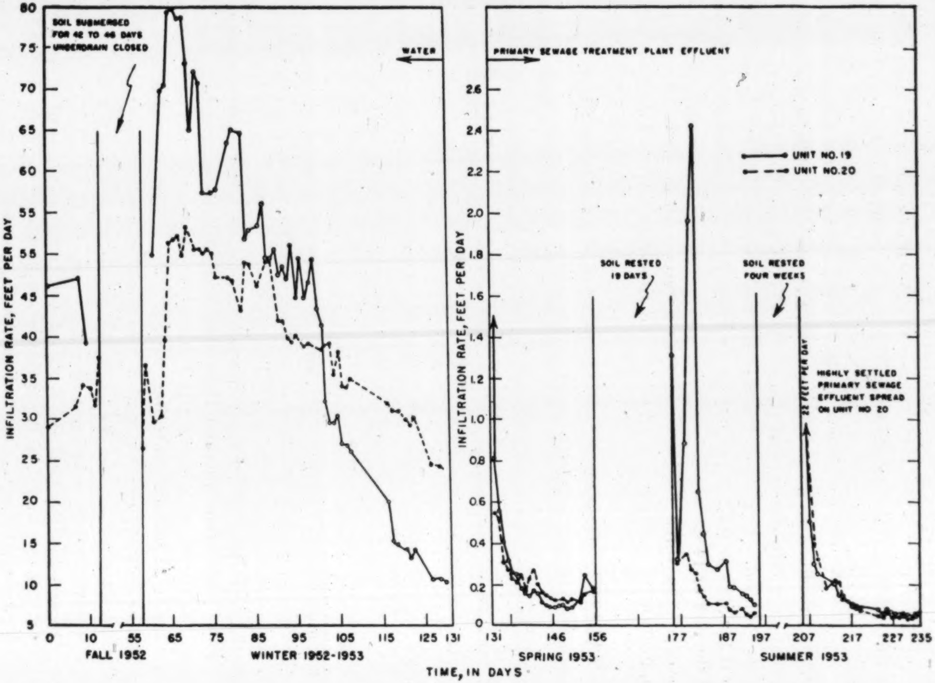


FIG. 3.—RATES OF INFILTRATION OF WATER AND SEWAGE INTO OAKLEY SAND

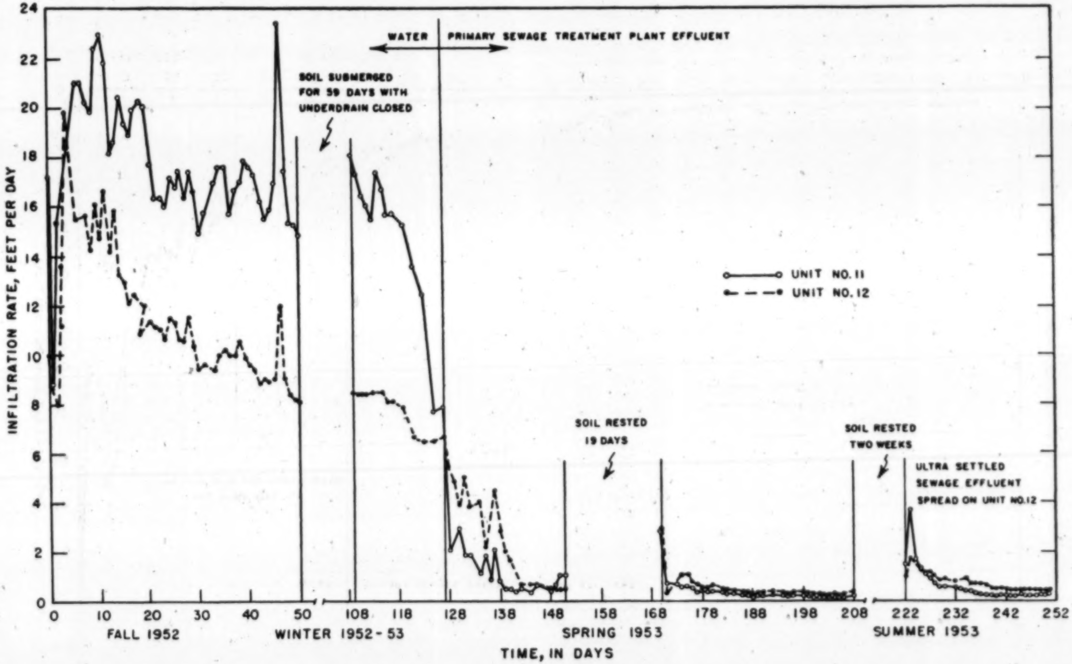


FIG. 4.—RATES OF INFILTRATION OF WATER AND SEWAGE INTO YOLO SANDY LOAM

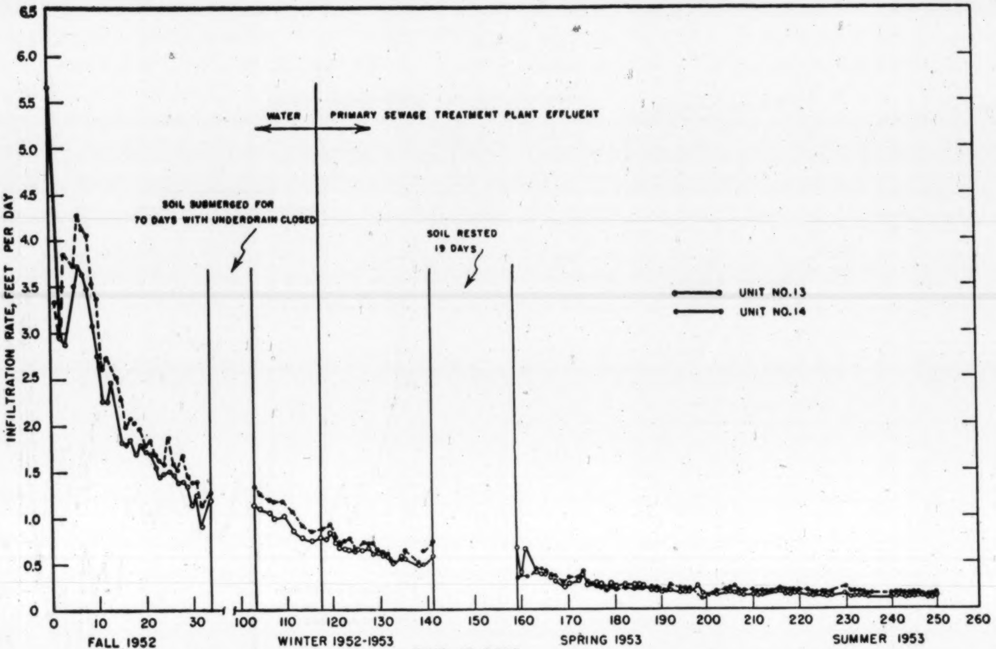


FIG. 5.—RATES OF INFILTRATION OF WATER AND SEWAGE INTO HANFORD FINE SANDY LOAM

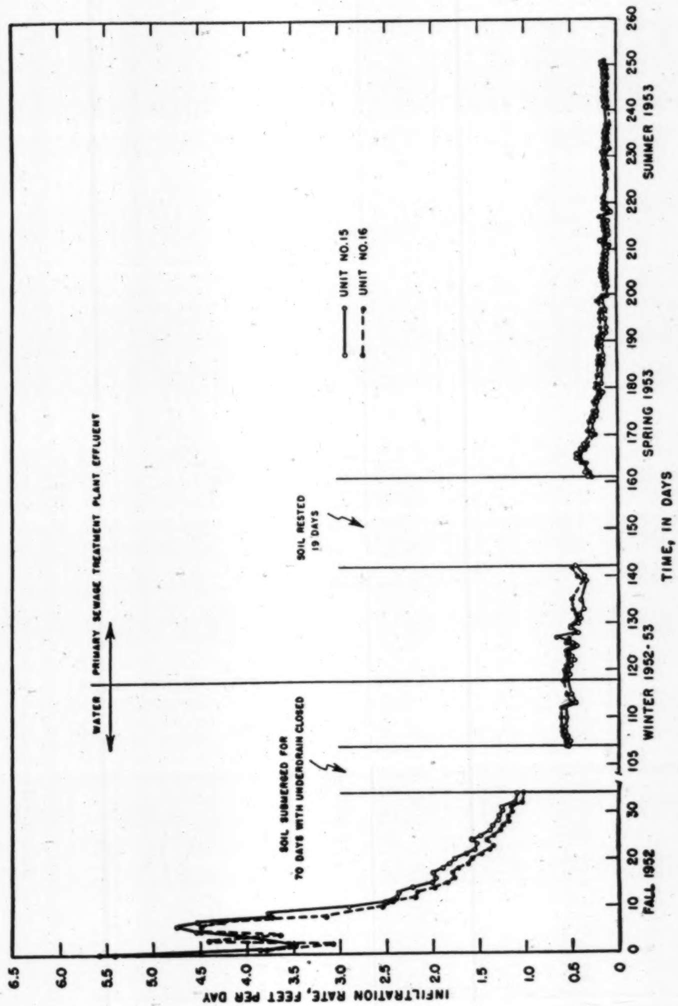


FIG. 6.—RATES OF INFILTRATION OF WATER AND SEWAGE INTO HESPERIA SANDY LOAM

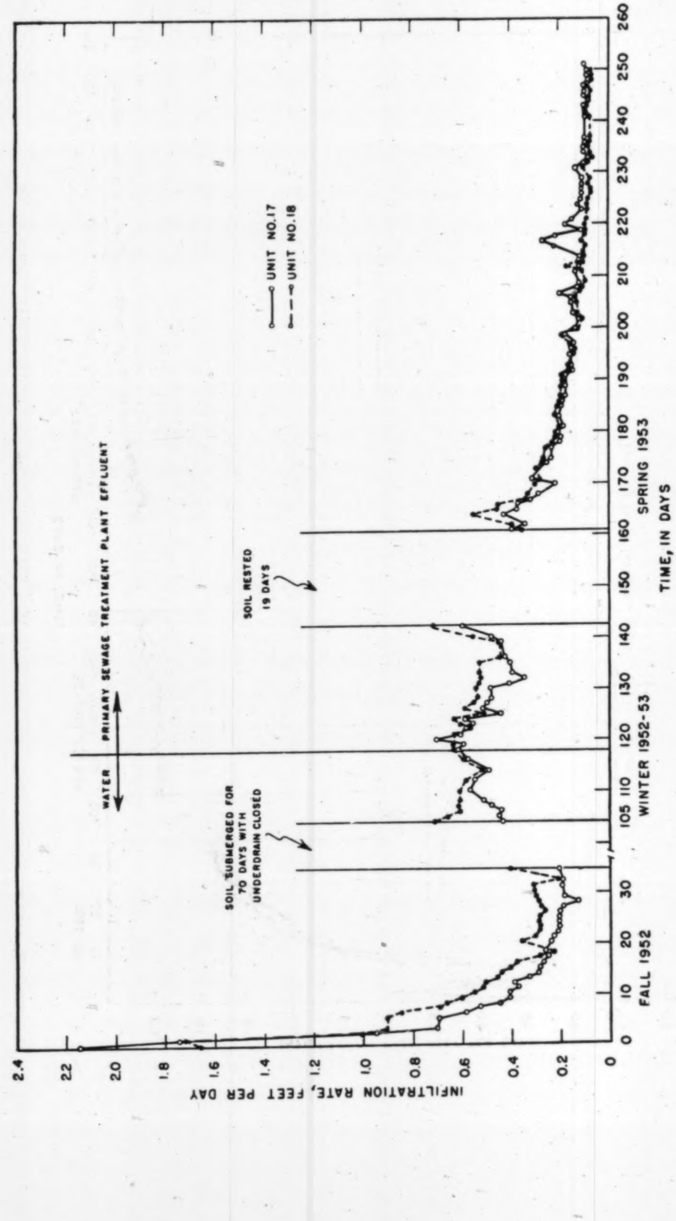


FIG. 7.—RATES OF INFILTRATION OF WATER AND SEWAGE INTO COLUMBIA SANDY LOAM

field conditions is not uncommon although seldom anticipated by engineers. Storm drainage facilities for the town of Oakley, Calif., have been considerably dependent on the infiltration capacity of this soil with some unfortunate results. Catch basins originally designed to act as sumps or "leaching pits" returning surface water to the underground soon become completely ineffective due to clogging with fine particles in the soil by surface runoff. Recent experimentation at the town's water filtration plant with infiltration basins for disposal of filter backwash have also shown that the infiltration capacity of the soils is greatly reduced by the turbidity carried by the wash water.

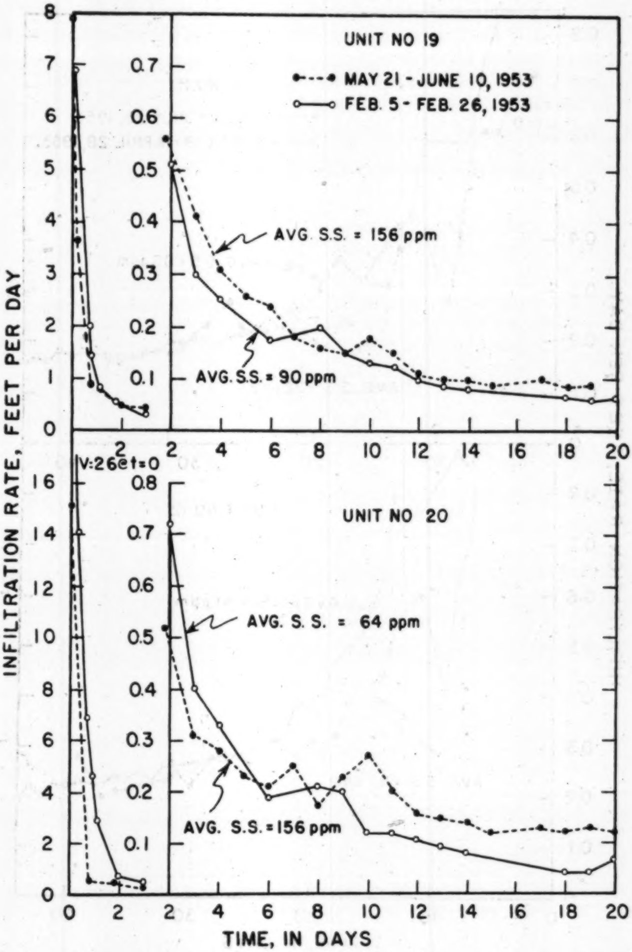


FIG. 8.—EFFECT OF SOIL CLOGGING ON INFILTRATION RATE (OAKLEY SAND)

Inspection of the surface of soil lysimeters and core samples, and observations of effluent characteristics indicated that during the progress of sewage spreading all the suspended material in the applied sewage was removed and accumulated at or near the soil surface. Penetration of particulate material into the soil structure was confined to a surface stratum a few centimeters deep. Analyses of core samples indicated that concentrations of organic material, as evidenced by the presence of carbon, were significantly increased only in the surface 2 in. of Oakley sand and Yolo sandy loam. Further

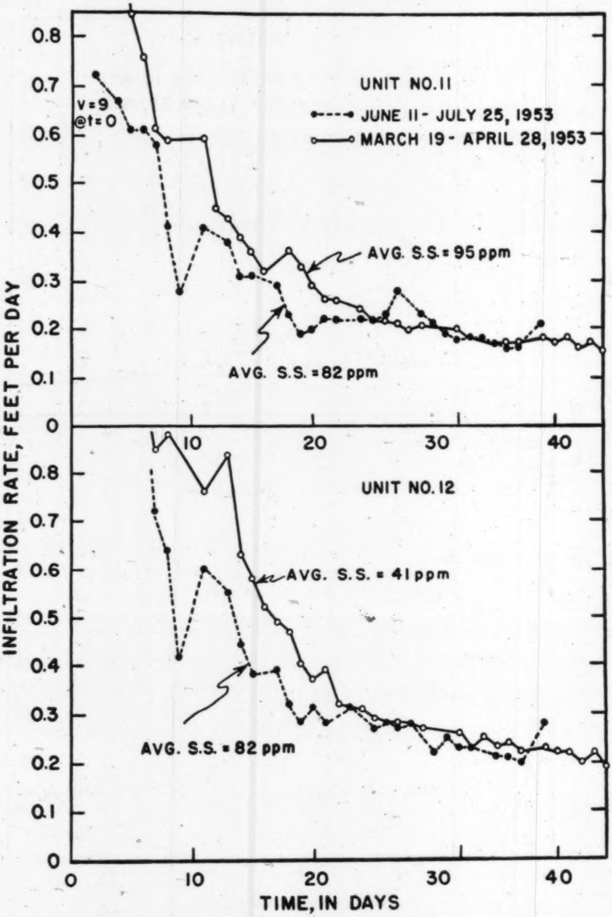


FIG. 9.—EFFECT OF SOIL CLOGGING ON INFILTRATION RATE (YOLO SANDY LOAM)

analyses of core samplings from the top 1 cm of all soils revealed substantial accumulations of organic matter on the surface of the soil.

At first observation, subsurface variations in organic components did not appear significant. However, when the change in carbon content of surface strata with respect to the normal carbon content of the soil before sewage spreading was noted (Fig. 10), it became apparent that increases in organic matter in this region were significantly different for each of the soils. For example, Oakley sand showed increases in carbon content, ranging from six to fifteen times normal content within the first centimeter of soil, and the distribution of carbon with depth indicated even deeper penetration. Yolo soil also showed substantial organic increases at the surface but little indication

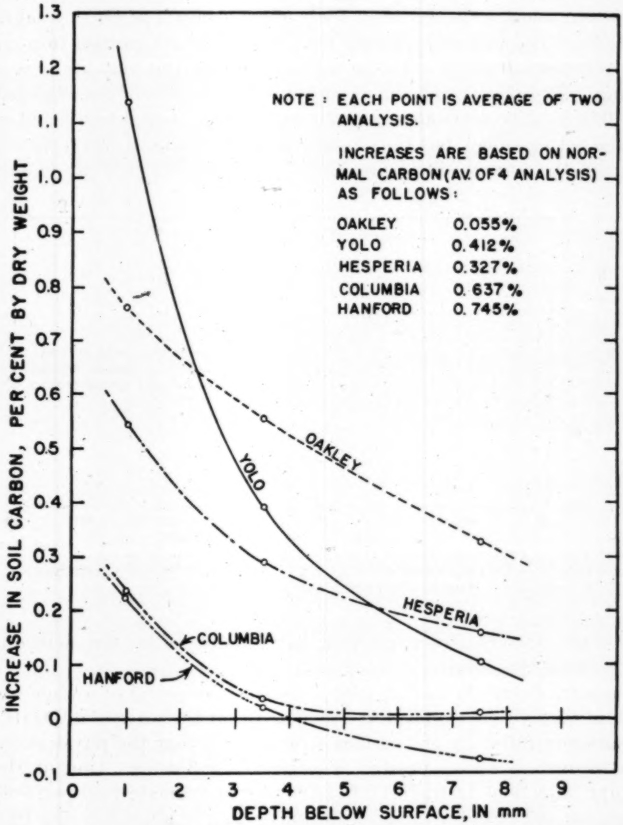


Fig. 10.—Changes in Soil Carbon with Depth Following Sewage Spreading

of penetration beyond 1 cm. Hesperia sandy loam exhibited a tendency toward increased surface organics and an appreciable increase at depths greater than 1 cm. On the other hand, Columbia and Hanford soils appeared to have limited the accumulation of sewage organics to the first 4 mm.

Observation of pressures or tensions in the liquid phase during spreading also demonstrated the effect of solids intrusion on soil permeability. Although all the soils exhibited some changes in permeability during spreading, the changes were most pronounced in the surface stratum of Oakley sand (Figs. 11 and 12). The loss in hydraulic head from the soil surface to a depth of 4.6 cm increased from 31.2 cm to 68.0 cm during the first 24 hr of sewage spreading. The coefficient of permeability correspondingly changed from 4.35 to 0.072. The Yolo and Hesperia loams also showed significant changes in surface permeability, but the changes noted in Hanford and Columbia soils were slight, the permeability being little affected by the application of sewage.

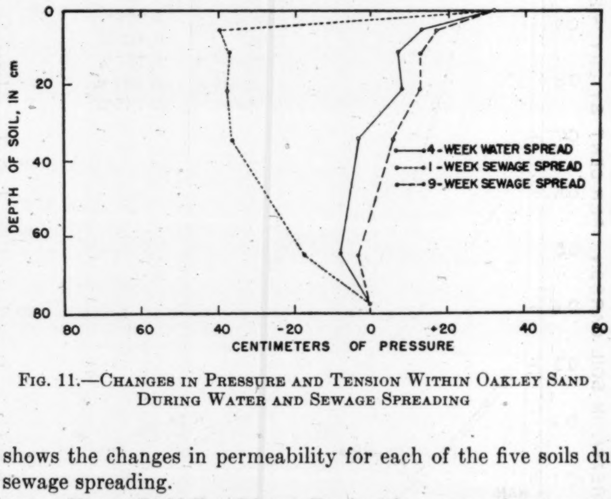


FIG. 11.—CHANGES IN PRESSURE AND TENSION WITHIN OAKLEY SAND DURING WATER AND SEWAGE SPREADING

Fig. 12 shows the changes in permeability for each of the five soils during 40 days of sewage spreading.

Lysimeters Versus Field Test Plots.—During the course of sewage spreading on each of the five soils studied, it was noted that the rates of infiltration were essentially controlled by the surface stratum and that the physical condition of the subsurface strata exerted a negligible influence. One of the more important objections to the use of lysimeters to estimate field performance—the fact that soil structure and permeability may be altered during transfer of the soil to the experimental column—thereby becomes comparatively unimportant. The field practice of spading, harrowing, or otherwise conditioning the surface of spreading basins to improve infiltration capacities necessarily modifies soil structure and permeability. Therefore, lysimeter soils, especially those near the surface in the zone that actually controls infiltration, may be expected to be reasonably comparable to their field counterparts. It would seem probable that in situations where suitably deep, pervious strata exist in the field, the practicality of water reclamation by surface spreading should be subject to determination by lysimeter studies. That such is indeed the case can be shown by a comparison of field and lysimeter infiltration rates observed for Hanford fine sandy loam and Hesperia sandy loam.

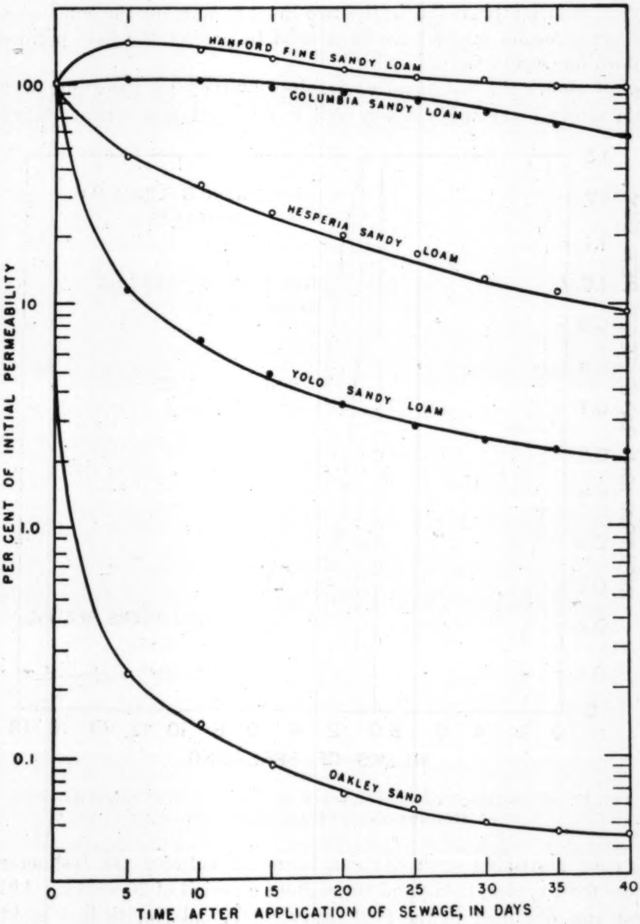


FIG. 12.—CHANGES IN PERMEABILITY OF SURFACE STRATA DURING SEWAGE SPREADING

A comparison of rates of infiltration of sewage into the Hanford soil during similar periods for both lysimeters and a plot designated as Basin D′ in the Lodi studies[9] are shown in Fig. 13. Basin D′, which produced higher infiltraion and percolation rates than those obtained by any other method studied at Lodi, was subjected to periods of inundation followed by drying and spading of the surface stratum. Thus, it was disturbed in a manner comparable to that used in lysimeter operation. The sewage applied to both Basin D′ and the lysimeter was treated by primary sedimentation only, and the depths of surface inundation were substantially the same. Fig. 13 shows that changes in rate are sufficiently similar in the two cases to warrant the conclusion that the lysimeter results might have been used to predict the field performance of Hanford fine sandy loam.

Fig. 14 shows the comparative results obtained by spreading water on Hesperia sandy loam in lysimeters and in field test ponds near Bakersfield.

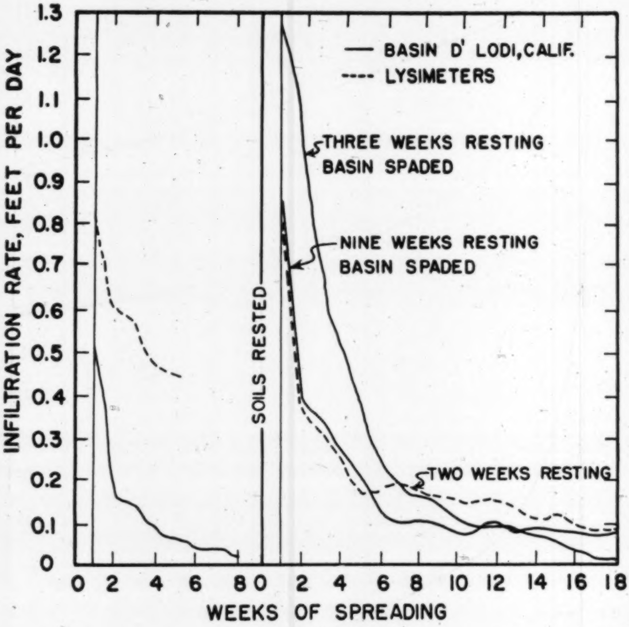


Fig. 13.—Comparison of Performance of Hanford Fine Sandy Loam in Lysimeters and Field Test Plots

In this case, a striking similarity can be noted between the lysimeter data and those for a typical field pond throughout a period of 20 weeks. Although the best run obtained in the field study[6,10,5] (Pond No. 19 in Fig. 14) was appreciably better than either the lysimeter or Pond No. 25 in terms of infiltration rate, the curves seem to approach the same equilibrium value after 14 or 15 weeks. On the basis of Fig. 14 it is reasonable to conclude that a lysimeter study could have been used to predict the field performance of Hesperia sandy loam under water spreading conditions.

Engineering and Economic Factors.—Field scale experiments, such as those conducted in the Lodi and Bakersfield studies, require relatively long periods of time and the investment of considerable sums of money. Therefore, they are not adaptable to engineering studies in many localities in which water reclamation by spreading might be proposed and where it might be seriously undertaken if the suitability of the soil for spreading purposes could be evaluated by less costly means. From the results presented, it seems possible to secure reliable data on the probable field performance of any particular soil by using relatively inexpensive laboratory lysimeters and comparatively short time studies. This means that the costs and time involved in the preliminary engineering investigation of a water reclamation project should compare favorably with preliminary work on other engineering undertakings. Experimental results on soils should be regarded in much the same manner as are the properties of other engineering materials; when used conservatively they should provide an adequate basis for satisfactory engineering decisions.

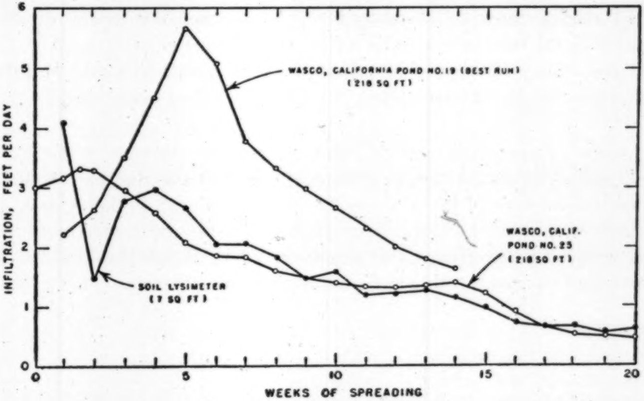


Fig. 14.—Comparison of Performance of Hesperia Sandy Loam in Lysimeters and Field Test Plots

Conclusions

The infiltration capacities of five pervious, California agricultural soils during prolonged spreading with water and primary sewage effluent were determined using soil lysimeters. A summary of the results and conclusions follows.

1. Each of the soils exhibited time-infiltration rate curves of a characteristic shape. The typical curve included a phase of initial rate decrease occasioned by soil dispersion or swelling; a second phase in which the rates increased due to solution of entrapped gases; and a third phase in which the rate of infiltration gradually decreased due to microbial clogging. Infiltration rates for continuous water spreading ranged from 0.6 ft per day to 40 ft per day.

2. Spreading of primary sewage produced an abrupt decrease in the infiltration rates and surface permeabilities of two of the coarsest soils within

the first few hours. Little or no change occurred in the rates of infiltration into the finer soils for a period of one month after the onset of spreading. Sustained infiltration rates during prolonged sewage spreading ranged from approximately 0.1 ft per day to 0.3 ft per day.

3. The abrupt decrease in infiltration capacity of the soils studied was attributed to the intrusion of sewage solids into the soil voids of a comparatively shallow surface stratum. As a rule surface strata controlled the rates of infiltration during either water of sewage spreading although permeability changes were most noticeable in the latter case. Disturbance of the soil samples during the assembly of lysimeters produced no significant changes in performance from the changes experienced in field test plots of much larger scale.

4. Lysimeters provide a convenient and inexpensive means of studying the fundamental behavior of soils under the various conditions of water and sewage spreading. They may be valuable as aids in estimating the performance of large-scale spreading operations.

ACKNOWLEDGMENTS

The studies reported herein were performed at the Sanitary Engineering Research Laboratory of the University of California at Berkeley as a part of a research program on the fundamentals of waste water reclamation. The writers wish to acknowledge the assistance of P. H. McGauhey, M. ASCE, who reviewed the manuscript and offered valuable criticism.

DISCUSSION

Ralph Stone,[13] **A. M. ASCE.**—Useful information, indicating that simple soil lysimeters may have value in certain waste water reclamation studies, has been developed by the authors. However, it may be desirable to mention some of the limitations of these devices for waste water reclamation studies.

Perimeter effects can cause variation in percolation between a lysimeter and a field prototype. The lysimeter will normally provide poorer bacteriological treatment than that which is encountered in actual field operation because of possible cross-connection and water movement at the perimeter; this also results in differences in the percolation rates. The surface of the lysimeter may be disturbed when effluent is introduced into its limited area and may, thereby, provide inaccurate data. Special inlet devices are needed for spreading water in lysimeters to avoid such disturbances.

In treating waste water containing organic matter the efficiency of the biological treatment is often associated with algae and sunlight as well as other factors. Test-lysimeter construction may be such that there is limited direct sunlight to dry out the soil surface or to provide energy for algae in the spread effluent. It is true that when constructing lysimeters it is possible to provide surface conditions similar to a prototype. However, when spreading in an actual basin with a depth to ground water of as much as several hundred feet below the ground surface, the soil and gas conditions would be expected to be different than those encountered in a restricted test lysimeter.

Because of the intricacies of waste water spreading, it is recommended that data for a large-scale spreading program be developed in test plots with a minimum size of one half acre. The lysimeters remain a worthwhile tool for qualitative tests and for basic laboratory research work in which it is desirable to limit and control test conditions.

Gerald T. Orlob[14] **and Robert G. Butler,**[15] **Junior Members, ASCE.**—As Mr. Stone observes, the use of soil lysimeters in waste water reclamation studies must be tempered with an appreciation of certain limitations. The results of such investigations are certainly not a complete substitute for a thorough engineering investigation of conditions at the proposed recharge site. However, they are valuable in guiding the engineer in preliminary investigations and can be used at comparatively much lower costs than those for development of the half-acre plots advocated by Mr. Stone.

The investigations presented in the paper, as well as subsequent studies and observations of the field performance of spreading basins and storm water disposal sumps, have demonstrated that infiltration of waste waters into soil is controlled by the surface stratum. For this reason it is of small consequence that the lysimeter medium is a disturbed specimen. The surface layer of soil on any field basin also has its basic structure altered in the construction and operation of the basin.

---

[13] Cons. Engr., Ralph Stone & Co., Beverly Hills, Calif.

[14] Asst. Prof. of Civ. Eng., Univ. of California, Berkeley, Calif.

[15] Asst. Prof. of Civ. Eng., North Dakota State College, Fargo, N. Dak.

Perimeter effects or the contribution to surface infiltration by unrestricted lateral flow are most pronounced on small test plots and tend to diminish, percentagewise, as the basin size is increased. These effects may be particularly significant when an impermeable subsurface horizon exists. Under such circumstances surface spreading may not be a suitable water reclamation technique. In general, when the prototype basin is large, flow is predominantly vertical as in the soil lysimeter.

If lysimeter walls are constructed of corrugated steel, as were those used in the studies reported in the paper, short-circuiting along the perimeter surface will be minimal. The danger of "piping" contaminated water through the medium is greatest near the onset of spreading on a previously dried soil, but is rapidly minimized by the intrusion of suspended material. In the natural soil profile also, root channels and media discontinuities exist which may aid bacterial penetration.

Soil gases at great depths are of comparatively small consequence in restricting percolation when compared with the intrusion of particulates into surface soil voids and the production of gases from biological stabilization of organic matter. These phenomena are confined to a comparatively shallow depth of medium and may be virtually the same in lysimeter as in prototype basins.

It is important to analyze the results of lysimeter studies critically before indulging in the expense of large-scale test plots. A potentially promising spreading site may prove to be less than feasible by an inexpensive preliminary investigation.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2915

WAVE RUN-UP ON SHORE STRUCTURES

By Thorndike Saville, Jr.,[1] A. M. ASCE

Synopsis

Laboratory tests determining run-up on shore structures as a result of wave action are described, and curves relating the run-up to wave steepness, structure type, and depth at structure toe are presented.

Introduction

A need for more adequate design data on the height of wave run-up on shore structures has long been evident. Many protective structures along the shores of rivers, lakes, reservoirs, and oceans have been designed to meet run-up requirements—that is, freeboard—essentially by "rule-of-thumb" methods rather than on a sound factual basis. The vertical height to which water from a breaking wave will run up on a given structure determines the height to which the structure must be raised to prevent overtopping and resultant flooding on the landward side, and, if the structure is a dam or levee, to prevent possible destruction by rearface erosion. These crest elevations have usually been determined by applying a rather arbitrarily selected ratio of run-up to wave height, R / H. A value of R / H equal to 1.5 is most frequently used, although values as low as 0.9 and as high as 2 have been suggested for particular designs. It has been generally recognized that the correct value would depend on structure characteristics (that is, shape and roughness), water depth at its toe, and wave characteristics, but sufficient data are not available (as of 1957) to determine these relationships accurately. The value of 1.5 is the one that is given for general use by the Beach Erosion Board, Corps of Engineers (United States Department of the Army),[2] for structures in, or landward of, the breaker zone. The Waterloopkundig Laboratorium at Delft, Netherlands, has determined an approximate value from relatively few observations of

---

Note.—Published, essentially as printed here, in April, 1956, in the Journal of the Waterways and Harbors Division, as *Proceedings Paper 925*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Asst. Chief, Research Div., Beach Erosion Board, Corps of Engrs., U. S. Dept. of the Army, Washington, D. C.

[2] "Shore Protection Planning and Design," *Technical Report No. 4*, Beach Erosion Board, Corps of Engrs., U. S. Dept. of the Army, Washington, D. C., 1954, p. 89.

$R / H = 8 \tan \theta$, in which θ is the structure slope[3]; if a berm of width B is placed at the mean water level, the ratio can be reduced by a factor of $1 - B / L$, in which L is the wave length. This ratio applies to "an open regularly set stone revetment and width of berm of about $\frac{1}{4}$ of the wave length L." It is further stated that a smooth impervious facing increases the run-up by approximately 15%, whereas a rough stepped facing can decrease it by approximately 10%. Some model-study data for smooth slopes of 15°, 30°, and 45° have been obtained at the University of California, at Berkeley, by Kenneth N. Granthem.[4] In addition to the need for data for the more conventional types of shore structures, the increasingly frequent use of artificially placed fill as a beach-protective structure has led to a need for data on relatively gentle slopes—that is, from 1 on 10 to 1 on 30—to enable an adequate design of beach berm elevations.

Recognizing the lack of basic data, the Beach Erosion Board initiated in 1952 a general comprehensive test program on wave run-up and its accompanying factor, wave overtopping; this was part of the Board's general research program on factors basic to shore protection and on the design of shore structures. Some of these tests, including all those on wave overtopping, have been performed at the Waterways Experiment Station of the Corps at Vicksburg, Miss., in accordance with a test schedule prepared by the Board; others have been conducted in the Board's laboratory. A preliminary analysis of the overtopping data on the vertical wall by Joseph M. Caldwell, M. ASCE, and the writer,[5] and a nonanalytical, tabular summary of the Vicksburg data [6] have been published. The present report contains an analysis of the run-up data taken at both laboratories.

TEST FACILITIES AND PROCEDURE

The test program at the Waterways Experiment Station was conducted in a concrete flume 120 ft long, 5 ft wide, and 5 ft deep. Waves were generated by a plunger-type wave machine, in which the wave period could be regulated by varying the speed of the motor and the wave height could be regulated by varying the eccentricity of the plunger arm, hence the plunger's depth of penetration. The tests at the Beach Erosion Board were conducted in a steel wave flume 96 ft long, 1.5 ft wide, and 2 ft deep. The pusher-type wave generator that was used consisted of a vertical bulkhead attached eccentrically through a pusher arm to a rotating driving disk 2 ft in diameter. The disk was driven by a 2½-hp motor through a vari-drive unit. The wave period could be adjusted by varying the gear ratio in this unit, and the wave height could be adjusted by varying the eccentricity of the connecting rod and, hence, the travel distance of the bulkhead. In both cases, waves of known characteristics were propagated against the test structure at the opposite end of the tank (Fig. 1). The Vicksburg test structures were tested in one, two, or three different depths of water at each structure toe, these depths being obtained by varying the depth in the deep part of the tank. The structures tested were a vertical wall; a curved wall, which was based on the Galveston (Tex.) seawall section; a similar curved wall but with recurvature at the top; smooth slopes of 1 on 3 and 1 on 1½; a step-faced wall of 1-on-1½ slope; and a riprap-faced wall of 1-on-1½ slope consisting of one layer of riprap on an impermeable base. All structures were fronted by a 1-on-10 beach slope. All the Board test structures were tested in four depths at each structure toe, the depths being obtained by keeping the water level in the deep part of the tank constant at 1.25 ft and varying the location of the structure (Fig. 1). The structures tested were smooth slopes of 1 on 1½, 1 on 2¼, 1 on 3, 1 on 4, and 1 on 6. As with the Vicksburg tests, all structures were fronted by a 1-on-10 beach slope. In addition, beach slopes of 1 on 10 and 1 on 30 were tested for the full depth of 1.25 ft at their toe. The 1-on-3 slope and the 1-on-10 slope were also tested with varying depths in the deep part of the tank to determine the effect of this depth, if any, on the wave run-up. The wave heights involved ranged from 0.17 ft to 0.70 ft at Vicksburg and from 0.03 ft to 0.58 ft at the Board; wave periods ranged from 0.63 sec to 3.64 sec at Vicksburg and from 0.61 sec to 4.70 sec at the Board. Roughly ten different combinations of wave height and period were tested for each structure and depth at Vicksburg, where the emphasis was on testing quickly as many structures as possible and where overtopping measurements were also made. At the Board, about fifty-one combinations of wave characteristics were tested for each structure and depth. Run-up alone was measured, and a much more thorough program could be accomplished.

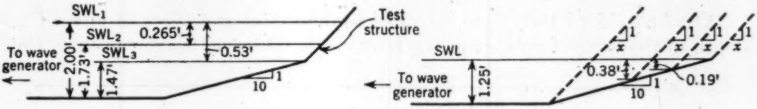
---

[3] "New Designs of Breakwaters and Seawalls with Special Reference to Slope Protection," by W. F. Van Asbeck, H. A. Ferguson, and H. J. Schoemaker, Question I, Section II, 18th International Navigation Cong., Rome, September, 1953, p. 174.

[4] "Wave Run-up on Sloping Structures," by Kenneth N. Granthem, *Transactions*, Am. Geophysical Union, Vol 34, 1953, pp. 720–724.

[5] "Experimental Study of Wave Overtopping on Shore Structures," by Thorndike Saville, Jr. and J. M. Caldwell, *Proceedings of the Minnesota International Hydraulics Convention*, International Assn. for Hydr. Research, Minneapolis, Minn., 1953, pp. 261–269.

[6] "Laboratory Data on Wave Run-up and Overtopping on Shore Structures," by Thorndike Saville, Jr., *Technical Memorandum No. 64*, Beach Erosion Board, Corps of Engrs., U. S. Dept. of the Army, Washington, D. C., 1955.



(*a*) WATERWAYS EXPERIMENT STATION (*b*) BEACH EROSION BOARD

FIG. 1.—EXPERIMENTAL ARRANGEMENTS

To put the experimental values in perhaps a somewhat better perspective for field engineers, the Vicksburg tests could be visualized as a 1-to-17 linear model and the Board tests, as a 1-to-23.7 linear model. For the Vicksburg tests, depths of 25 ft, 29.5 ft, and 34 ft would exist at the toe of the 1-on-10 slope, and the structures would be tested with depths of 0 ft, 4.5 ft, and 9 ft at their respective toes; deep-water wave characteristics would vary from 3-ft-to-12-ft heights and from 3-sec-to-15-sec periods. For the Board tests the depth at the toe of the beach would be 29.5 ft, and, again, depths of 0 ft, 4.5 ft, and 9 ft would exist at the toes of the structures; deep-water wave heights would range from approximately 0.67 ft to 14 ft, and periods would range from 3 sec to 23 sec. Essentially the full range of prototype conditions was covered.

At both test locations, in order to permit a stable condition to become established before measurements were taken, the first two to four waves were ignored; measurements were then taken on the next six to fifteen waves or so, but were, in any case, stopped before reflected waves from the structure could reach the generator and return to the structure. In the run-up tests at the Board, actual measurements were taken on each of the six to fifteen waves, and these were averaged to obtain a mean value. At Vicksburg, only the maximum of the six to fifteen waves was recorded; the water was then stilled and the test repeated several times. The maximum values from these tests were then averaged to obtain the value reported. The Board tests thus indicate an average value of run-up, whereas the Vicksburg tests indicate an average maximum value. In general, however, these values should not differ markedly. For the two structures tested at both locations for the same wave conditions—that is, the 1-on-1½ smooth slope and the 1-on-3 smooth slope—the Vicksburg values averaged 28% higher than the Board data for the 1-on-1½ slope and 6% lower for the 1-on-3 slope. The degree of scatter of the points appeared to be of the same order of magnitude, and the Vicksburg points were practically all included within the envelope band of the far greater number of points obtained at the Board.

EXPERIMENTAL RESULTS

The first Board tests were intended to determine the effect, if any, of the depth in the generating part of the tank on the observed wave run-up. Consequently, tests were initially run for the 1-on-10 slope with two different depths, and for the 1-on-3 slope with four different depths in the deep part of the tank. For each of these cases the slope extended uniformly through the entire depth to the bottom of the tank. The points obtained are plotted in Fig. 2.

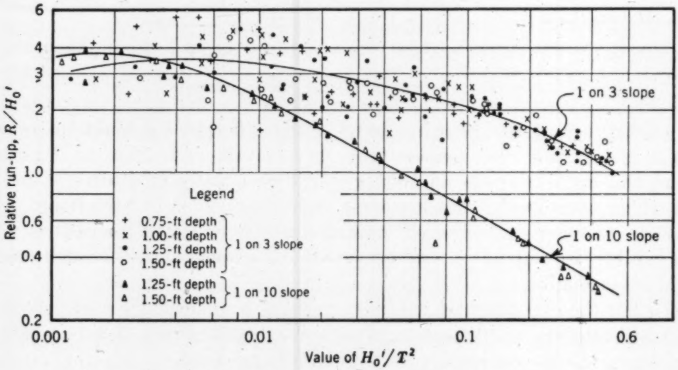


FIG. 2.—EFFECT OF TANK DEPTH

The values plotted are relative run-up—that is, the ratio of vertical rise of water on the structure face above still-water level to the deep-water wave height and the ratio of deep-water wave height to the square of the wave period. The latter value, H_o' / T^2, is directly proportional to the dimensionless value of deep-water wave steepness, H_o' / L_o. Actually $H_o' / L_o = H_o' / (g\, T^2/2\pi)$; thus, $H_o' / T^2 = g\, H_o' / 2\, \pi\, L_o = 5.12\, H_o' / L_o$. However, it is felt that the term, H_o' / T^2, is easier to use because the wave period is usually directly available either from forecasts or from measurements, whereas the wave length generally must be computed. An additional complexity in using the wave length as a parameter is that it changes quite appreciably with depth, and a depth position must therefore be specified with the value given. The wave period, on the other hand, remains constant regardless of depth variation. The height parameter used is the deep-water height corrected for refraction—that is, the height which would exist in deep water offshore from the structure if there were no refraction effect and the refraction coefficient were 1.0. The breaker height would perhaps be a more desirable parameter, but it is extremely difficult to obtain both in the laboratory and in the field. Any other height, such as that which would obtain in the deep part of the tank or at the toe of the structure, again requires definition of the depth involved and complicates the usage of the laboratory results. However, this depth effect is not as important for wave height as for wave length because the height change is usually relatively small as long as the breaking condition is not approached.

As may be seen from Fig. 2, no depth effect was apparent, and curves have been drawn through the general mass of points. From a practical standpoint, this result might be important, for example, in the consideration of design criteria for an offshore breakwater, and in determining whether design run-up values would differ were the breakwater to be located in depths of 50 ft, 70 ft, or perhaps 100 ft. It might also be important in determining whether variation of the relatively large depths existing at the base of a reservoir or river levee would make any difference in freeboard requirements. Actually the independence of run-up on depth for relatively large depths (as, for example, several times the wave height) might well be expected, but considerable question has been raised about it in the past. This result is also confirmed by other tests performed at Vicksburg for the design of the Lake Okeechobee levees [7] in Florida.

The scatter of the points about the derived curves is quite typical; the degree of scatter throughout the tests was small for the gentle slopes but became relatively large for the steeper slopes. This, also, is to be expected because a relatively large horizontal difference in wave uprush means only a small difference in the vertical value for the gentle slopes but a larger difference for the steeper slopes. It should be noted that the scatter is much smaller for the steeper waves—that is, those having relatively large values of H / T^2—than for those having relatively low (H / T^2)-values. In almost every case the scatter of points was not excessive—that is, greater than approximately ±20%—for (H / T^2)-values greater than 0.08 or 0.1. However, the scatter of points was frequently as great as from ±100% to 200% for values of H / T^2 less than approximately 0.01 or 0.02, the worst cases being for the steeper

---

[7] "Summary Report, Waves and Wind Tides in Shallow Water," Corps of Engrs., Jacksonville Dist., U. S. Dept. of the Army, Jacksonville Fla., 1955, pp. 42–43.

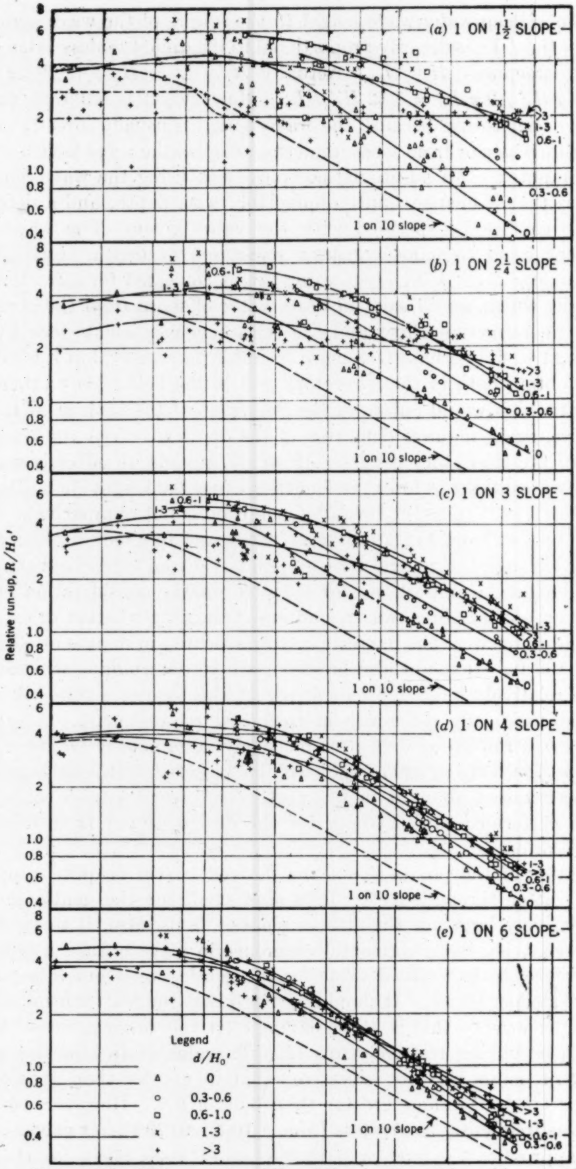
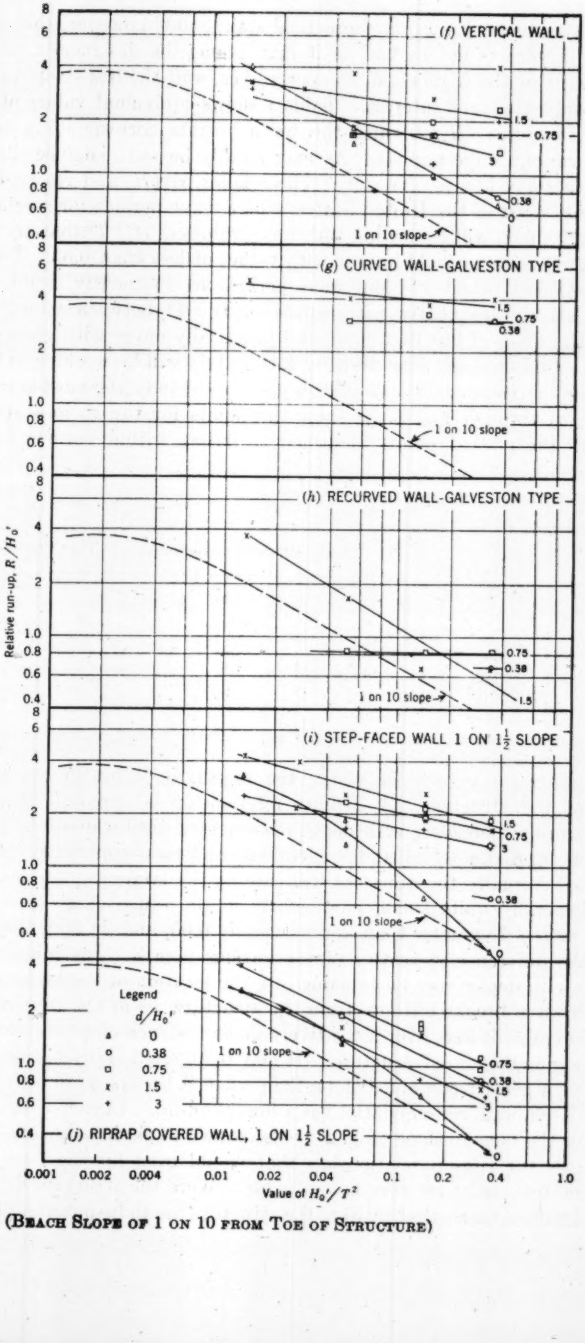


FIG. 3.—RUN-UP AS A FUNCTION OF STRUCTURE DEPTH AND WAVE



STEEPNESS (BEACH SLOPE OF 1 ON 10 FROM TOE OF STRUCTURE) slopes, such as 1 on $1\frac{1}{2}$. From a practical standpoint, however, this particular scatter pattern is not as bad as it may seem; the designer is interested almost always in the higher and steeper waves, and the less steep waves are of almost only academic interest. Table 1 shows equivalent values of H / T^2 and wave steepness (H / L) and the wave heights corresponding to these values for various wave periods. As may readily be seen, engineer designers dealing with small bodies of water, such as lakes, rivers, and reservoirs, and with the east coast of the United States, where wave periods for storm waves are relatively short, will be dealing with wave values of H / T^2 that are usually greater than 0.1 and probably never with values of less than 0.05. Designers on the west coast, where incident wave periods are frequently larger, will be dealing relatively frequently with waves of H / T^2 between 0.05 and 0.1, seldom with values of less than 0.05, and practically never with values of less than 0.02. Consequently, the designer fortunately will be working in the area in which the curves have the least scatter and appear to be the most accurate.

Fig. 3 (*a*) through Fig. 3 (*j*) show the points for the various structures tested with the plotted points identified as falling within various ranges of depth-height ratio values, in which the depth, *d*, indicates the depth at the toe of the structure. A definite relationship is apparent throughout, with the run-up increasing with depth of structure until a depth-height ratio of between approximately 1 and 3 is reached and then apparently decreasing somewhat. Actually the apparent decrease as the larger depths are reached appears generally quite small, particularly in the range of greatest interest of values of H / T^2 greater than approximately 0.05, and, in fact, may be due only to the difference in scatter of the various points. The initial increase in run-up with depth may be explained by the location of the breaking wave. When the wave breaks offshore from the structure, as in the case of shallow structure depths, it acts over a relatively great distance of gentle beach slope before reaching the actual structure. It may be thought of, then, as essentially acting over a gentler combined slope than when it breaks more nearly on the structure itself, and, consequently, has a lower run-up. In each case the curve derived for the 1-on-10 slope (Fig. 2) is also shown to aid in comparison. This curve shows the run-up relationship that would exist for the 1-on-10 beach slope if the test structure were not in place. Were the structure depth to be decreased still further—that is, were the structure toe to be put at a plus eleva-

TABLE 1

$\frac{H_o'}{T^2}$	$\frac{H_o'}{L_o}$	Wave Height Corresponding to Time Periods			
		5 sec	10 sec	15 sec	20 sec
0.5	0.0975	12.5	50	...	...
0.2	0.039	5.0	20	45.0	...
0.1	0.0195	2.5	10	22.5	40
0.05	0.00975	1.25	5	11.25	20
0.02	0.0039	0.5	2	4.5	8
0.01	0.00195	...	1	2.25	4

tion—the resulting run-up curves would approach the curve for the 1-on-10 slope still more closely. The depth-height ratio ranges chosen were purely arbitrary. The particular ranges used were selected as most conveniently fitting the groups of data taken. The Vicksburg data were taken for five values of this ratio only, and these are shown as specific values rather than as ranges. However, the symbols used correspond to those used for the applicable ranges for the Beach Erosion Board data.

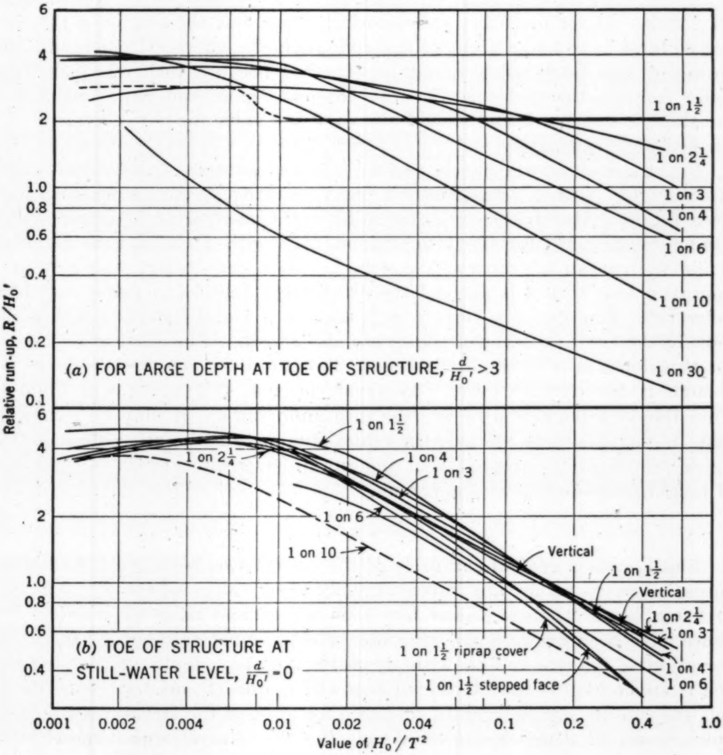


FIG. 4.—RELATIVE RUN-UP FOR ZERO DEPTH AT TOE OF STRUCTURE

Fig. 4 shows the curves derived for the various slopes tested for conditions of deep water at the structure toe—that is, where, in the laboratory, the structure extended to the full depth of the tank. The points delineating these curves were essentially all for depths equal to, or greater than, three times the wave height. For the 1-on-$1\frac{1}{2}$ slopes through 1-on-6 slopes, they are identical with the curves shown in Fig. 3 for a value of d / H greater than three. The data for the 1-on-10-slope curve are shown in Fig. 2. The points establishing the curve for the 1-on-30 slope are not shown, but approximately thirty-three points were used to determine the curve, which does not deviate more than ±10% from any point. As noted earlier, the scatter was quite small for the gentle slopes (1 on 30, 1 on 10, and 1 on 6), but became appreciable as the 1-on-1½ slope was approached. This may also be seen from the plotted points in Fig. 3. As is to be expected, the run-up increases as the slope becomes steeper—at least in the case of the steeper waves. A slight decrease in run-up with slope increase is indicated for slopes steeper than 1 on 6 for (H / T^2)-values of less than approximately 0.01, but the scatter in this area was so large as to make accurate determination of a curve extremely difficult. However, this relationship might be expected because with relatively deep water fronting the structure the wave becomes more and more reflected as the slope is steepened, and should approach the value for pure clapotis as the vertical slope is neared.

Fig. 4 also shows the curves derived for the condition of zero depth at the structure toe, with a 1-on-10 beach slope seaward; all curves were obtained from Fig. 3. Also drawn is the curve for the 1-on-10 slope, which shows the run-up relationship for the beach slope only with no structure in place. Again the run-up increases as the slope becomes steeper, but after a slope of approximately 1 on 3 is reached the exact slope appears to make very little difference. Curves for a vertical wall and for a 1-on-1½ stepped wall and a 1-on-1½ slope with a layer of riprap are also shown. The vertical wall curve for this depth is only slightly different from those for the steeper slopes, although it lies slightly lower. However, because this curve was obtained from the Vicksburg data of average maximum run-up, it may be somewhat high in comparison to the average values obtained for the other slopes. The lower curves for the stepped wall and riprap cover indicate the effectiveness of roughness in decreasing the run-up.

SUMMARY

Fig. 5 gives a general summary of the information obtained for all structures. Run-up values are shown for only three values of H / T^2—0.05, 0.1, and 0.3; however, these values represent the general range of practical use. Lines have been drawn connecting points of an equal value of d / H_o'. The variation of relative run-up with structure type and depth at the structure toe is apparent. It may be noted that the frequently used value of 1.5 for relative run-up, although quite adequate for most of the structures tested for steep waves of many design cases (an (H / T^2)-value of approximately 0.3), is not adequate for many types of structures in which waves of lower steepness are considered. The vertical wall resulted in lower run-ups than the slopes steeper than approximately 1 on 4 for all conditions except that of zero depth at the structure toe where almost the same run-up resulted regardless of slope. This decrease in run-up for the vertical wall may be explained possibly by greater ease of reflection of the uprushing water (if the wave has broken seaward of the structure toe). It may also be explained by the fact that the horizontal momentum of the uprushing water may be applied directly to carry the water forward on the inclined slopes and that it is only gradually changed

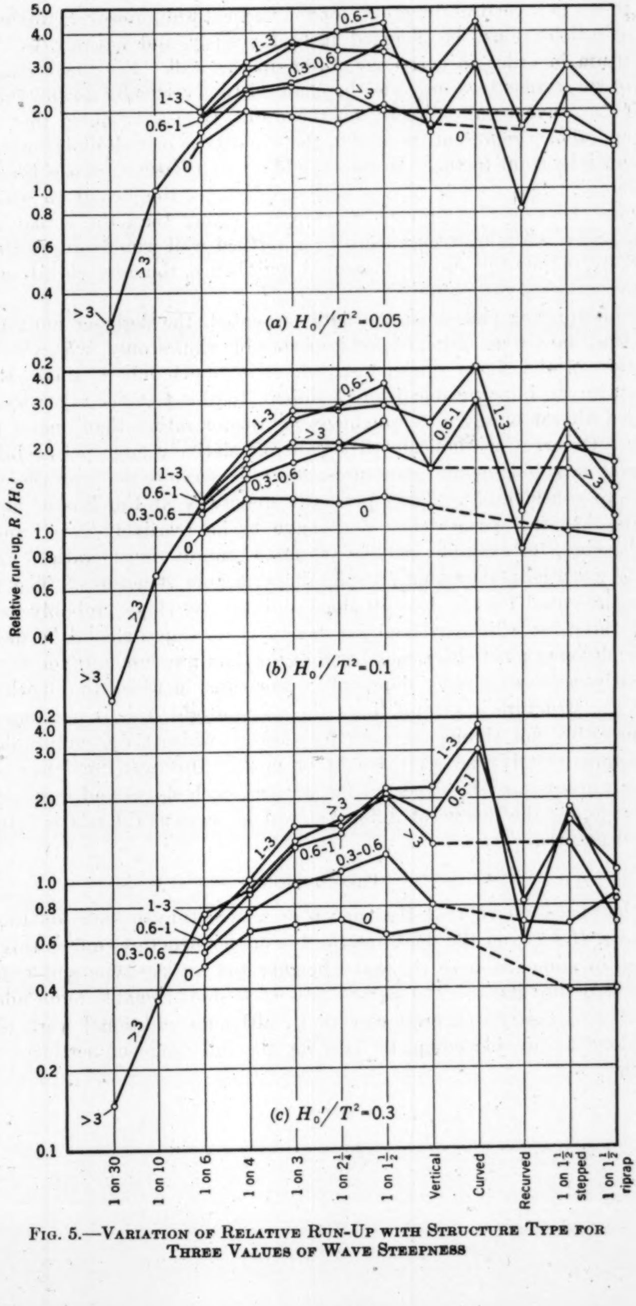


Fig. 5.—Variation of Relative Run-Up with Structure Type for Three Values of Wave Steepness

there to act in a vertical direction; for the vertical wall, however, this momentum essentially must be changed instantaneously and entirely to vertical momentum in order to carry the water up the wall. For the vertical wall some of the momentum may also be changed to a vertically downward direction if the wave breaks directly on the wall. It may be noted that curved walls apparently represent, generally, the worst type of wall design when run-up is an important factor. However, adding recurvature reduces the run-up considerably; here, wave run-up is determined as the height to which the wall must be raised to prevent overtopping rather than the actual vertical rise of water. Adding recurvature to a vertical wall would presumably also materially reduce the run-up measured and, hence, the crest elevation necessary to prevent overtopping.

In interpreting and using the data presented, the designer must bear in mind that data were obtained for impermeable slopes only, with the partial exception of the single riprap test, which involved only a single layer of riprap over an impermeable 1-on-1½ slope. In this test the layer should be regarded almost purely as a roughness parameter rather than one of permeability. Observations indicate that walls of relatively large permeability, as for riprap or rubble-mound structures, generally serve to decrease the amount of run-up considerably. Tests presently underway at the Board, however, indicate that the permeability does have to be relatively large, and that sand beaches, for example, behave (as far as run-up considerations are concerned) essentially the same as solid impermeable structures. The run-up curves presented for the 1-on-10 slope and 1-on-30 slope probably are very nearly those for which artificial beaches of these slopes should be designed.

The designer must also remember that the data presented are for structures fronted by a 1-on-10 beach slope only. For cases in which the depth at the toe of the structure is several times the wave height, this is not important; it is probably not significant even for cases in which the structure depth is only approximately one wave height or more. However, for lesser depths, greater run-ups may be expected for steeper beach slopes and lesser ones for gentler slopes; the degree of difference will increase as the relative structure depth decreases.

CONCLUSIONS

It has been shown that the run-up on a structure depends on the wave steepness, H / T^2; on the depth at the toe of the structure, unless this depth is large in comparison to the wave height; and on the type and roughness of the structure itself. The curves presented should enable more adequate designs of necessary wall crest elevations, although additional work remains to be done to provide complete data for the full range of pertinent design problems.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2916

EXPERIENCES WITH LOESS AS FOUNDATION MATERIAL

BY WILLIAM A. CLEVENGER,[1] A. M. ASCE

WITH DISCUSSION BY MESSRS. HARRY R. CEDERGREN; RALPH B. PECK AND HERBERT O. IRELAND; AND WILLIAM A. CLEVENGER

SYNOPSIS

Certain characteristic properties governing the behavior of loess as a foundation material have been defined through extensive laboratory and field studies. These studies were primarily limited to loess or loess-like materials. In this paper broad generalizations of many pertinent properties of loess are presented, and their significance is examined on the basis of specific typical experiences with loess as foundation material. Some interesting data gathered by the writer on foundation failures of residences are described, as well as results of laboratory and field studies of the properties of the loess connected with these failures.

INTRODUCTION

True loess is a loose, wind-deposited soil covering vast areas of several continents, including North America. It is generally composed of uniform, silt-sized particles, which are loosely deposited and are bonded together with relatively small fractions of clay to form the typical loess "structure." Normally loess has a high shearing resistance and will withstand high loadings without significant settlement when natural moisture contents are low. However, upon becoming wet, the clay bond tends to soften and the loess structure collapses, thus inducing large settlements under small loads and a loss of shearing strength.

A thorough basic knowledge of the engineering properties of loess is necessary to utilize this material safely as a foundation. Much information is available on the origin and deposition of loess, its mineralogical composition, and its

---

NOTE.—Published, essentially as printed here, in July, 1956, in the Journal of the Soil Mechanics and Foundations Division, as *Proceedings Paper 1025*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Vice-Pres., Woodward-Clyde-Sherard & Associates, Denver, Colo.

engineering properties as defined by laboratory and field tests; however, there is little information on the results of actual experiences with loess as a foundation material. A few typical examples of such experiences will be presented herein, and an attempt will be made to correlate them with the properties of the materials involved, as determined by laboratory tests. Although the experiences are quite varied, such information may be of benefit to those confronted with the foundation problems associated with construction in loess areas.

Much of the laboratory test data contained herein were obtained by the Bureau of Reclamation, United States Department of the Interior (USBR), in connection with the construction of hydraulic works in the Missouri River Basin areas of Nebraska and Kansas; these data have been reported[2] by Wesley G. Holtz, M. ASCE, and Harold J. Gibbs, A. M. ASCE. Data applying to loess deposits in the Denver (Colo.) area were furnished by resident soils consultants. Properties of the particular loess being examined are compared with those of the average Missouri River Basin loess in order to indicate the peculiarities of the individual deposits.

Index Properties of Loess—Missouri River Basin Area

Appearance and Structure.—The typical, undisturbed Missouri Basin loess is from tan to light brown in color, crumbly, lightweight, and contains many visible voids and root holes. At a low natural moisture content, it is fairly brittle but can usually be powdered easily between the fingers. Few individual particles can be seen with the unaided eye, and practically no stratification is visible. Most of the root-like channels are in a vertical direction, thereby giving vertical cleavage to massive deposits of loess. This can be observed in the field in the splitting off of columnar sections of dried loess in steep-sided gullies.

Gradation and Plasticity.—Loess contains primarily silt-sized particles of quartz and feldspars bonded together by a relatively small proportion of montmorillonite-type clay. Most of the grains are between 0.019 mm and 0.074 mm except in the more clayey loesses, as indicated in Fig. 1(*a*). Approximately three-fourths of the samples from the Missouri River Basin area fell within the boundaries indicated by "silty loess," one-fifth were classified as "clayey loess," and approximately one-twentieth were termed "sandy loess." Almost all the loess has some plasticity when remolded, as indicated by the Atterberg limits chart in Fig. 1(*b*). The relatively small amount of clay that is present has a significant influence on the engineering properties of the loess.

Undisturbed Density.—In its natural state, loess from the Missouri River Basin area is normally very loose because of its method of deposition, in addition to its not having been subjected to sufficient saturation in nature to produce a "breakdown" of the typical structure. Undisturbed densities of true, wind-deposited loess normally range from approximately 75 lb per cu ft to 85 lb per cu ft in this area. If the material has been wetted and consolidated or has been reworked, the natural density is higher—sometimes as much as 100 lb per cu ft or more. In unusual cases the wind-deposited material has been found to have a density of as low as 65 lb per cu ft. Studies made by the USBR have shown that the natural density is perhaps the most important index property of loess because the ultimate settlement that can be expected and the shearing resistance of the material after wetting are dependent to a great extent on the natural density. Large settlements and low shear resistance can be anticipated for Missouri River Basin loess at a density of 80 lb per cu ft or less, whereas soil with a density greater than 90 lb per cu ft will settle a relatively small degree and will have a fairly high shearing strength. The properties are transitional between 80 lb per cu ft and 90 lb per cu ft. Standard Proctor maximum densities vary from approximately 105 lb per cu ft to 110 lb per cu ft for the loess to which the foregoing criteria apply.

---

[2] "Consolidation and Related Properties of Loessial Soils," by W. G. Holtz and H. J. Gibbs, *Special Publication No. 126*, A.S.T.M., 1952.

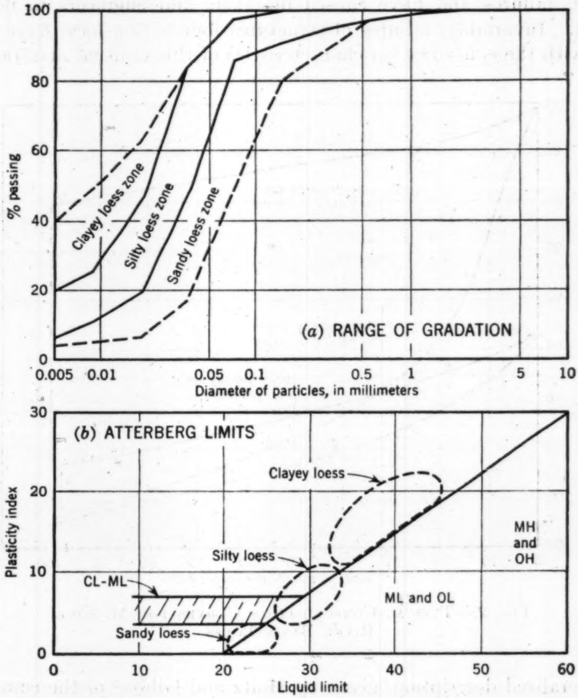


FIG. 1.—IDENTIFICATION OF MISSOURI RIVER BASIN LOESS

Natural Moisture.—Moisture contents of the undisturbed loess are usually on the order of 10%, and, regardless of the density, the supporting capacity of loess at this moisture is high. With increasing natural moisture to about 15%, the supporting capacity is reduced only slightly, but additional increase in moisture is associated with a pronounced decrease in the supporting capacity.

Because of the open, porous structure, Missouri River Basin loess will retain only approximately from 25% to 28% moisture, even if thoroughly wetted by artificial means. At moisture contents greater than 15%, the structural properties of loess appear to depend primarily on the density.

STRUCTURAL PROPERTIES OF LOESS—MISSOURI RIVER BASIN AREA

Consolidation.—The outstanding structural property of loess is probably consolidation. Large consolidations under footings have resulted in many foundation failures and have caused designers and engineers to be greatly concerned. Invariably, failures of structures founded on loess have been associated with the consolidation characteristics of this unusual material.

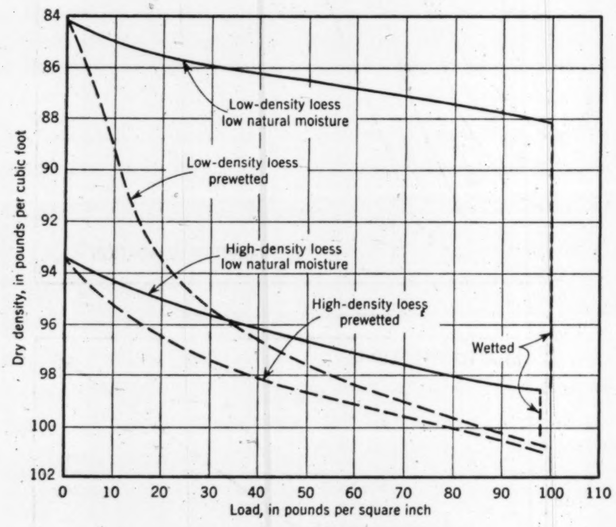


FIG. 2.—TYPICAL CONSOLIDATION CURVES FOR MISSOURI RIVER BASIN LOESS

A generalized description, given by Holtz and Gibbs,[2] of the consolidation properties of the Missouri River Basin loess shows that the potential settlement of a loess foundation is governed primarily by the in-place density and by the highest moisture content attained by the soil. Other factors, such as gradation and past and present natural loadings, also have an effect, but, by comparison, they factors are of relatively minor importance.

Several typical laboratory consolidation curves for test specimens of loess have been plotted in Fig. 2 to demonstrate the effect of in-place density and of wetting on consolidation properties. Test specimens at low natural moisture consolidate comparatively little at high density or at low density. Prewetted, low-density specimens consolidate between 15% to 20%, and for high-density specimens, either at natural moisture or prewetted conditions, there is little consolidation. The effect of saturation can be observed from the additional consolidation that results from wetting the specimens while under a load of 100 lb per sq in. The approximate effect of saturation at other loadings can be estimated from the foregoing curves by observing the difference in density between the natural and prewetted specimens at any particular load.

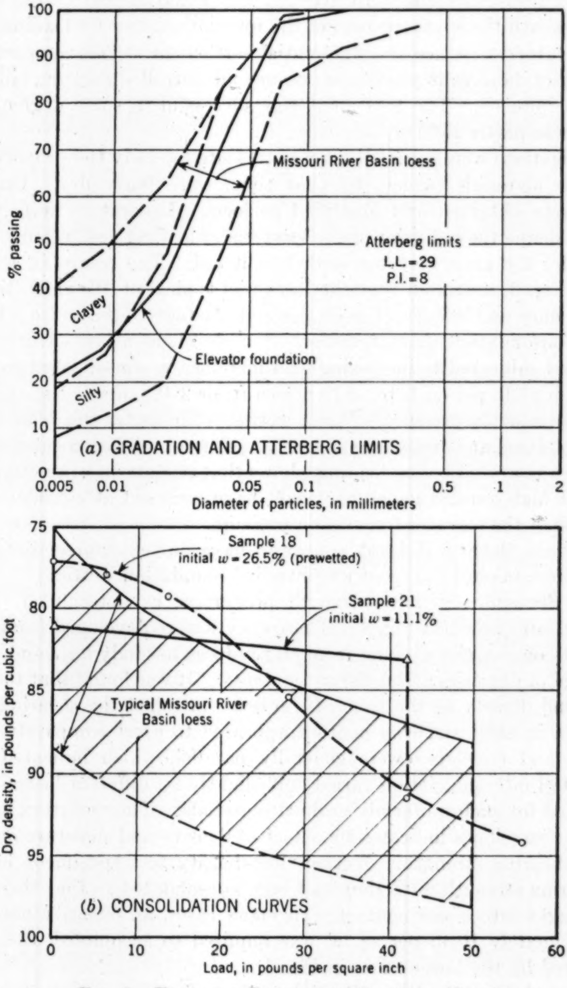


FIG. 3.—ELEVATOR FOUNDATION LOESS IN KANSAS

There have been numerous large engineering structures of various types founded on loess in the Missouri River Basin area, a few of which have failed. One example of unsatisfactory performance is a grain elevator in Kansas, data for which were furnished by a soils consultant who analyzed the foundation conditions. This elevator was completed during July, 1952. After heavy rains had occurred in August of that year, the structure settled quite badly on the north side, causing a noticeable tilting in that direction. It was believed that water had ponded on the north side because of poor surface drainage and had caused the settlement. An attempt was made to correct the tilting by wetting beneath the southern part of the foundation, and by loading this part of the elevator to reduce the differential settlement. This appears to have been a rather dangerous procedure because uncontrolled shearing failure could have been induced. The treatment was successful in some degree, and the structure was partly righted.

Investigations were initiated later at this site to study the properties of the foundation materials, which, by that time, were fairly dry. Undisturbed samples were obtained and identified as loess. Laboratory tests were performed to define the behavior of the loess under loading and wetting and thus explain why the grain elevator settled as it did. The results of these tests, shown in Fig. 3, indicated that the loess was typical of Missouri River Basin loess, not only on the basis of grain size and plasticity but also from the standpoint of consolidation characteristics. Sample 21, at a low natural moisture content and subjected to increasing load increments, consolidated from a density of from 81 lb per cu ft to 83 lb per cu ft, or 2.4%, under the approximate elevator foundation pressure. When wetted while under this load, the specimen consolidated an additional 8% or 9%. The consolidation curve for sample 18, which was wetted before loading, shows that settlement of the wetted loess, or loess at high natural moisture, would have occurred at considerably lower loadings than the elevator foundation pressure.

This grain elevator did not appear to have moved appreciably after precautions were taken to prevent uncontrolled foundation wetting.

Shear Strength.—Shearing-strength properties, as determined by laboratory tests, are shown in Fig. 4. These curves were developed from results of many tests on samples ranging from clayey loess to sandy loess and covering a wide range of densities and moisture contents. It was found that the moisture content and density at the time of the test controlled the shearing strength. Differences in sand and clay content appeared to exert comparatively minor influence, and envelopes were generally parallel, which indicated that the coefficient of internal friction varied only slightly for different loesses, as might be expected for such uniformly grained materials. The variation in the location of the envelopes indicates the effect of density and moisture on the total average shearing strength. Wetted, low-density test specimens had almost zero shearing strength until they had been consolidated so that the soil grains were brought into closer contact. In many cases, an effective normal stress of approximately 10 lb per sq in. was required to accomplish the foregoing, as indicated by the lower curve in Fig. 4.

The usual Missouri River Basin loess has effective coefficients of internal friction varying from approximately 0.60 to 0.65. The cohesion of the loess ranges from zero for low-density wetted loess to from 10 lb per sq in. to 20 lb per sq in. for loess at natural moisture content.

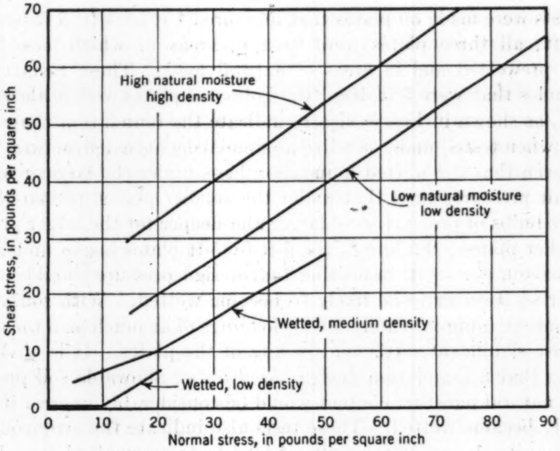


FIG. 4.—TYPICAL SHEAR ENVELOPES FOR LOESS

Field Load Tests.—Plate-bearing tests conducted at various sites within the Missouri River Basin Project of the USBR consistently confirm the properties of loess as defined by laboratory tests and field experiences. Data obtained from a comprehensive study performed on loess near Ashton Dam, Nebraska, have been reported[2] by Holtz and Gibbs and are summarized briefly herein. This method of investigating foundations is particularly important because of its popularity in the United States.

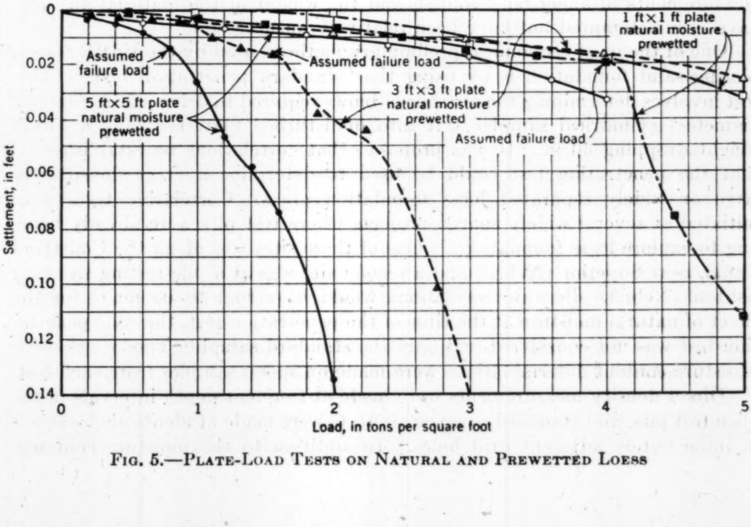


FIG. 5.—PLATE-LOAD TESTS ON NATURAL AND PREWETTED LOESS

Load tests were made on plates that measured 1 ft by 1 ft, 3 ft by 3 ft, and 5 ft by 5 ft; all three plates were used in areas in which loess had been thoroughly prewetted and in areas of natural loess. These tests were conducted in holes that were 5 ft deep; the holes and plates were of the same size. The results, as shown in Fig. 5, clearly indicate the foundation characteristics of the loess when tested under conditions simulating actual structural loadings. It may be seen that the wetted foundation loess under the larger plates failed at lower unit pressures than that under the smaller plates, primarily because the effective bulbs of pressure were larger and deeper for the larger plates than for the smaller plates. Failure of the 5-ft-by-5-ft plates began under loads of as low as 0.8 ton per sq ft, indicating that design pressure should not exceed 0.25 ton per sq ft on material likely to become wetted. With soil at natural moisture content (approximately 10%), loadings of as much as 5 tons per sq ft did not cause significant settlement of any of the plates. It is obvious from these curves that a foundation design based on an allowable soil pressure for material at natural moisture content would be considerably in error if the loess subsequently became wetted. These tests also indicate the erroneous conclusions that may be drawn from results of load tests on small plates when these results are used to design larger footings.

Use of Penetration Tests to Investigate Loess Foundations

As previously noted, the shearing and consolidation characteristics of natural loess that will ultimately be subjected to wetting depend on the natural density. Therefore, in investigating loess foundations, it is very important to utilize methods of exploration that will yield numerical values of in-place density. Field-density measurements in open test pits, or direct density measurements on undisturbed samples obtained by any means, are the most desirable and reliable methods. Others, including computation of density from direct measurements of auger-hole volume and the weight of the material augered, have been attempted but have been relatively unsuccessful.

One of the most economical preliminary methods used by the USBR for exploring sand foundation is by using the "standard penetration test." This test involves determining the number of blows required to drive a 2-in. (outside diameter) cylindrical sampler 1 ft into undisturbed material using a 140-lb weight dropping 30 in. It was proposed that correlations be established so that the penetration test could be used to determine average densities of large or widely separated loess foundation areas. Correlative tests were initiated at several widely spaced projects where test pits were already being dug to explore loess foundations. One of these sites was along the Courtland Canal, near Superior (Nebr.), and a second site was at a pile-testing site near Ashton (Nebr.). Because corrections would have had to be made for the effect of natural moisture at the time of the penetration test, the cone penetration test was not considered and only the standard sampling spoon was used. Moisture-content determinations were made on spoon samples from each test.

Direct density measurements were made at frequent depth intervals in the open test pits, and standard penetration tests were made at identical elevations in immediately adjacent drill holes. In addition to the moisture contents, gradations and Atterberg-limits tests were also made to establish correlation between materials. The results of the penetration resistances and densities for identical materials at natural moistures ranging from 10% to 15% are shown in Fig. 6. Without detailed mathematical analysis, it is apparent that the scattering of points is too great to allow the use of penetration tests to determine in-place densities. Consequently, when foundations might possibly become wetted and the soil properties depend on density, penetration testing is not recommended.

However, such tests have been occasionally used by the USBR in investigations of loess foundations in which the natural water content is not expected to change appreciably from that which exists at the time of testing. The bearing capacity of the materials depends to a considerable extent on the natural shearing strength of the loess, which can be roughly measured by the penetration tests.

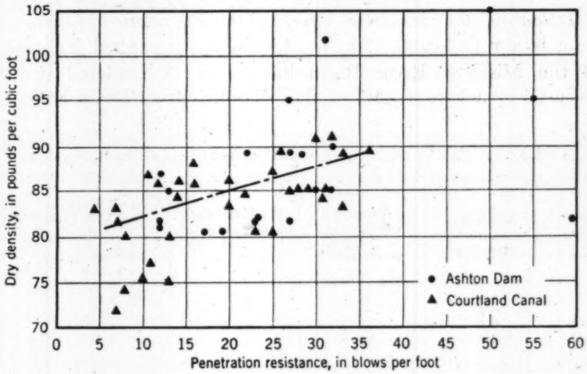


FIG. 6.—RELATIONSHIP BETWEEN DRY DENSITY AND PENETRATION RESISTANCE

PRECONSOLIDATING LOESS FOUNDATIONS BY PONDING

The results of laboratory testing of undisturbed samples have shown that prolonged wetting of loess produces softening of the particle-to-particle clay bond, causing collapse of the soil structure. This results in large consolidations or possibly shear failures, depending on load conditions. Furthermore, according to the laboratory test curves, such soil-structure breakdown upon wetting frequently occurs at pressures lower than the previous overburden pressure, indicating that the natural material has never been wetted sufficiently in nature to consolidate. From this it would be expected that thorough wetting of a natural loess deposit would cause settlement because of its own weight only. In fact, evidences of this have been reported,[3] and the settlement of large areas has been noted on newly irrigated lands and in the reservoir areas of

---

[3] Discussion by Karl Terzaghi of "Consolidation and Related Properties of Loessial Soils," by W. G. Holtz and H. J. Gibbs, *Special Publication No. 126*, A.S.T.M., 1952.

USBR dams. The addition of loads in excess of the overburden pressure, as in the case of most construction, produces considerable consolidation.

The possibility of preconsolidating loess foundations has appealed to many engineers and designers, particularly for hydraulic structures such as dams and canals in which the materials will eventually become wetted. For earth dams, which exert high pressures but are rather flexible, the information obtained indicated that by ponding settlement could be attained during construction rather than after completion and filling of the reservoir. For concrete canal structures, many of which are fairly rigid and can be damaged by large settlement, ponding presented a method of securing settlement before construction. It was believed that surcharge loads equivalent to structure loads placed on the loess foundation during wetting would cause almost complete settlement of the loess.

Preconsolidating the Medicine Creek Dam Foundation Loess.—Medicine Creek Dam is a rolled-earth dam located in south central Nebraska in the center of the Missouri River Basin loess area. Completed in 1949, this 2,750,000-cu-yd structure is part of the Missouri River Basin Project, storing approximately 196,000 acre-ft of water for irrigation, flood control, and recreation purposes. The dam is approximately 5,665 ft long, 115 ft high above the stream bed, and has a 65-ft cutoff trench resting on shale foundation.

TABLE 1.—TEST RESULTS ON UNDISTURBED SOIL SAMPLES

Depth below ground surface, in feet	Average moisture, in percentage	Average density, in pounds per cubic foot	Fill plus overburden pressure, in pounds per square inch	CONSOLIDATION AT OVERBURDEN PLUS FILL PRESSURE	
				Natural, in percentage	Wetted, in percentage
5	8.8	79	25	8.4	10.9
17	9.7	77	33	1.3	9.1
19	9.6	81	34.5	1.0	3.9
50	6.6	92	55	1.5	6.5

The problem of constructing this dam without there being ultimate failure through foundation settlement became apparent in the initial investigations. On the right abutment, dry, undisturbed loess occurred to depths averaging about 40 ft, with some loess being as deep as from 60 ft to 70 ft. Laboratory tests on undisturbed samples obtained in a test pit at a representative site indicated that the loess density was generally low and that large settlement could be expected.

Table 1 summarizes partial results of tests on undisturbed samples representing the loess stratum obtained at four depths in a test pit. From Table 1 the average settlement of the wetted loess from the overburden pressure plus the maximum fill pressure would be approximately 7.6%. Assuming that the foundation loess would be subjected to an average of the overburden pressure plus the fill pressure and that the loess ranges from 40 ft to 80 ft deep, the approximate, anticipated total foundation settlement would be on the order of from 3 ft to 6 ft.

Because of this indicated danger of postconstruction settlement upon saturation by reservoir water, the foundation in this area was thoroughly wetted before fill construction by ponding and sprinkling. Moisture content of the loess in the critical area was raised from an average of 12% to an average of 28% in from twenty-five days to two months using 33,000,000 gal of water. Settlement-measuring points were established throughout these ponded areas. It is significant that no settlement occurred from saturation alone. Base plates for measuring the foundation settlement caused by fill construction were installed

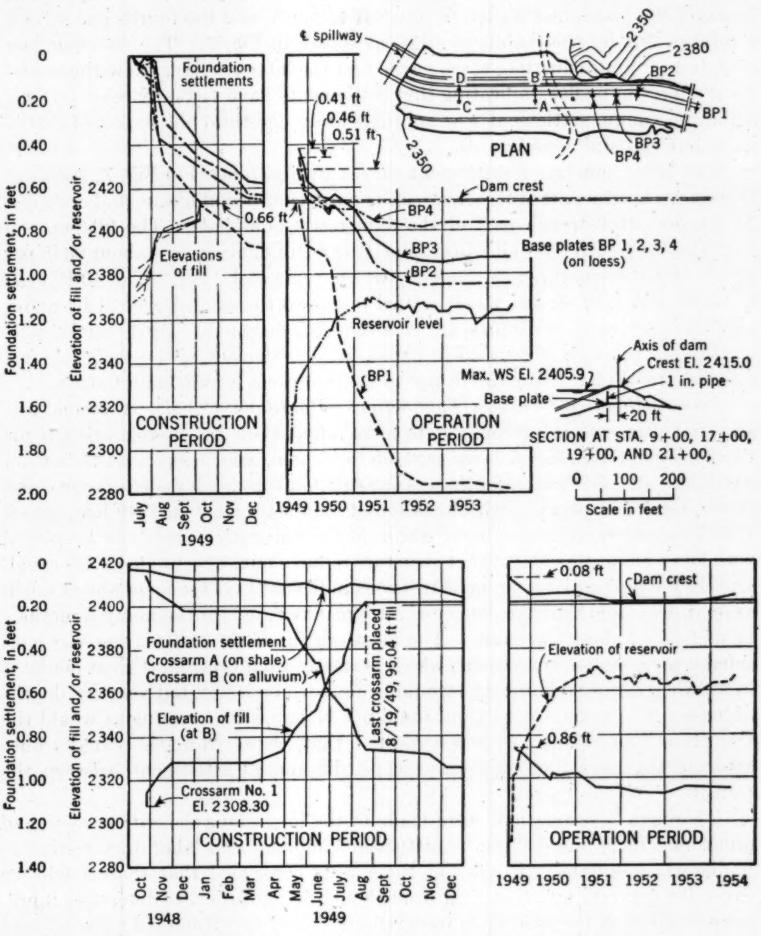


FIG. 7.—FOUNDATION SETTLEMENT AT MEDICINE CREEK DAM

on top of the loess stratum in four locations designated BP-1 through BP-4 (Fig. 7).

Fill construction on the loess began in about the middle of 1949 and was completed later that year, with measurements of base-plate settlement being made periodically from the beginning of fill construction until July, 1954. Three of the plates had settled from 0.8 ft to 1.0 ft, and the fourth had settled a total of 2 ft by the middle of 1954, as shown in Fig. 7. The reservoir had been full for approximately three years, and the later readings show the settlement curves to be flat, indicating that settlements have virtually ceased. The actual maximum settlement was approximately one-third the expected settlement but was still considerable.

Two other foundation-settlement curves are also plotted in Fig. 7; crossarm B is shown for the part of the dam resting on stream-bed alluvium and crossarm A, for the cutoff-trench part of the dam resting on shale. The fill pressure over the 50-ft-deep river alluvium (mostly sand and gravel) was about 90 lb per sq in., and the measured settlement was approximately 1 ft. The 40-ft loess stratum (BP-2, BP-3, and BP-4) settled to almost the same degree (1 ft) under only 45 lb per sq in. of fill pressure, and the 80-ft-deep loess (BP-1) settled 2 ft, under a fill pressure of only 35 lb per sq in. Settlement of the shale under a fill pressure of more than 150 lb per sq in. (crossarm A) was only 0.02 ft.

Preconsolidating Structure Foundations—Courtland Canal.—A second example of attempts to preconsolidate loess foundations by presaturating is for the Courtland Canal. A large part of this canal, which has a 26-ft bottom width and an 8.5-ft water depth, was constructed through low-density silty and clayey loess. Experience with loess foundation settlement in 1949 had caused USBR engineers to become more aware of the foundation problems associated with loess, and it was feared that structure failures might occur along this canal. During the initial planning, investigations indicated that the loess was as much as 50 ft deep, and that the danger of settlement existed for the many structures to be located along the canal. The most critical of these structures was considered to be siphon transitions, which normally would rest on a great depth of loess, whereas the siphon barrel would be founded on lesser, but varying, depths of loess. The barrel foundation would not be subject to wetting as would the transition foundation. It was expected that water from the canal would saturate the loess thoroughly and cause differential settlement between the transition and the barrel.

Thorough investigations were made of the loess along the critical section of alinement, including drilling, undisturbed sampling, and laboratory testing of undisturbed samples. Results of these tests confirmed that there might be cause for concern, and that field tests should be performed to determine if preconsolidation of the transition foundations should be attempted prior to construction. One site was selected for field trials. At this site a transition would eventually be located, and unfavorable conditions were found by exploration and testing. The results of the laboratory testing showed that the loess at this site was medium plastic, about 95% of its particles were finer than the No. 200 sieve size, and fell well within the gradation range for Missouri River Basin loess, as shown in Fig. 8. Consolidation-test specimens at natural moisture content and subjected to loads varying from 0 lb per sq in. to 40 lb per sq in. consolidated very little, but upon saturation under a load of 40 lb per sq in. the specimens consolidated a considerable additional amount. Other specimens, which had been wetted before testing, consolidated to approximately the same end density as the specimens that had been saturated while under load. From these data it was determined that under its own weight approximately 1.5 ft or 2 ft of settlement could occur upon field saturation of the loess.

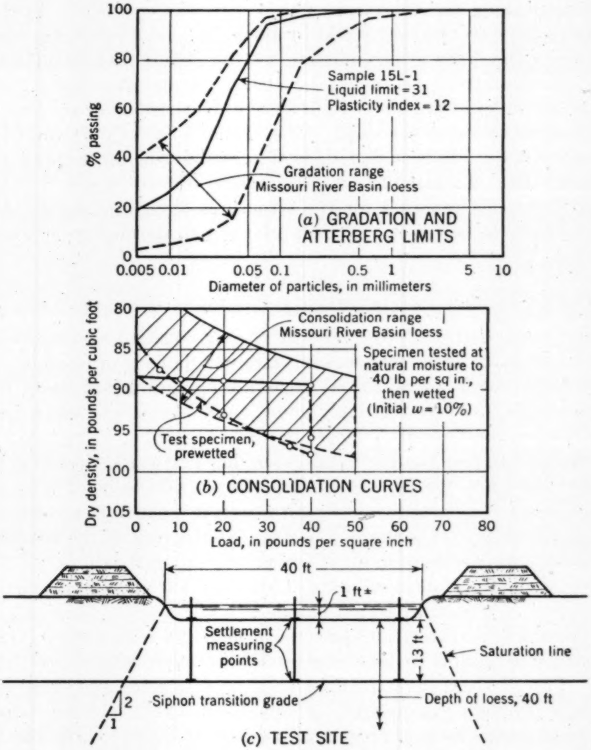


FIG. 8.—PRECONSOLIDATION OF LOESS FOUNDATION BY SATURATION AT COURTLAND CANAL

A 40-ft-sq area at the transition site was selected for the field tests; the transition grade was approximately 13 ft below the ground surface. The area was diked and kept filled with water for three weeks, during which time 330,000 gal of water permeated the foundation. Test holes were drilled outside the area to determine the progress of saturation. It was found that there was good saturation of the loess within an area bounded by a truncated cone with side slopes of $\frac{1}{2}$ horizontal to 1 vertical from the edges of the ponded area, as shown in Fig. 8. Tests showed that the moisture content of the loess after the ponding was completed ranged from about 20% to 28% with an average of approximately 24%, indicating that the loess was sufficiently wetted to cause breakdown of the loess structure. However, elevation measurements made on the various settlement-measuring points showed that the maximum settlement was only 0.02 ft. This ponding experiment proved that saturation would not necessarily cause consolidation of the loess; therefore, this plan was not utilized in construction along the Courtland Canal. It was concluded, in this case, that the loess may or may not settle by its own weight. If a surcharge had been placed on this loess after complete wetting, some settlement would undoubtedly have occurred. However, because the foundation loading of such a canal structure is less than the weight of the overburden to be removed, no surcharge was necessary in this case. This section of the Courtland Canal was completed, and water has been flowing in the canal during most of the irrigation seasons since 1951. To date (1957) no significant difficulties due to foundation settlement have been reported for the structures along the canal. If settlements had begun to occur, it had been planned to densify specific structure foundations.

EXAMPLES OF RESIDENCE-FOUNDATION FAILURES—DENVER AREA LOESS

With the expansion of Denver and increasing construction of subdivisions away from downtown areas, large subsidences and damage of residence foundations have been noted. Residential areas are spreading out away from the main stream valleys, where relatively good foundation conditions exist, to the hilltops where major deposits of loess occur. According to reports,[4] loess is quite widespread in the Denver area, extending eastward across the uplands from each of the main valleys. These deposits are said to be of Wisconsin-age loess, which is sandy near the valleys; in the east it is sandy silt to silt, which is finer in texture than that near the valleys. Weathering of the silt has developed some clay, which has been identified petrographically (in only a few samples) as being chiefly montmorillonite. Much of the loess is marked by the presence of lime in small, separated nodules.

Fairly complete investigations have been made by soils consultants of residence-foundation failures in loess and of the detailed soils properties of the material. Two of these investigations have been selected as typical examples and will be described subsequently. Comparisons of the soil properties of the specific loesses in the Denver area have been made with those of the Missouri River Basin loess.

Denver Residence No. 1.—This residence was built just outside the southeastern city limits on the heights east of, and overlooking, the South Platte River. It is constructed of flagstone (sandstone blocks) and is founded on spread footings. The initial cost of the house and appurtenant structures was more than $100,000.

In 1953, soils consultants were asked to study foundation conditions because extreme damage had occurred due to footing movements. A chimney

---

[4] "Pleistocene and Recent Deposits in the Denver Area, Colorado," *Bulletin 996-C*, Geological Survey, U. S. Dept. of the Interior, Denver, Colo., 1954.

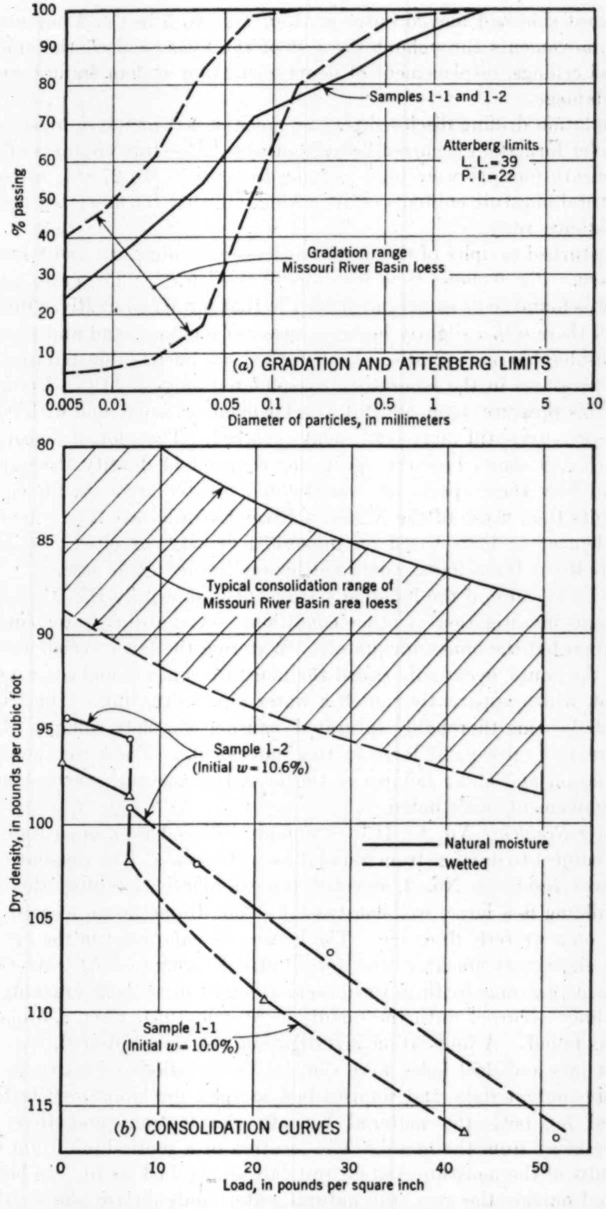


FIG. 9.—DENVER AREA LOESS, RESIDENCE NO. 1

on the west side had moved outward from 4 in. to 5 in.; and horizontal and vertical movements throughout the rest of the house caused the cracking of walls and ceilings, displacement of floors, distortion of door frames, and other serious damage.

Foundation drilling disclosed that as much as 7 ft or 8 ft of loess overlying the Denver formation occurred below footings. Moisture contents of samples from beneath footings were high, ranging from 19.3% to 27.8%, as compared with natural moisture contents of approximately 10% for unwetted materials a short distance away.

Undisturbed samples of the unwetted loess were obtained and tested in the laboratory. Fig. 9 shows typical results of these tests. The grain-size distribution was found to be somewhat similar to that for Missouri River Basin loess, although there was a slightly higher content of medium sand and the samples were considerably more plastic. Undisturbed specimens subjected to estimated footing pressures in the laboratory consolidated only slightly. Upon saturation at this pressure, large additional settlement occurred, and with increasing applied pressures still more settlement occurred. The plot of density versus load in Fig. 9 shows how the specimens changed in density throughout the test, and how these specimens consolidated greater amounts for equal load increments than those of the Missouri River Basin loess. The broad criteria for settlement of loess based on numerical density, as cited previously for Missouri River Basin loess, would not be applicable in this case.

The basic cause of the building settlement is shown by the laboratory test curves and investigation of other conditions. At natural water content the loess supported the house adequately. Watering the lawn, which was planted around the house, eventually raised the moisture content, and some settlement occurred, which apparently caused a water pipe to rupture. The foundation materials became thoroughly saturated, causing complete collapse of the soil structure and subsequent large footing movements. There were evidences of consolidation and shear failures of the loess because both vertical and horizontal movements were noted.

Denver Residence No. 2.—It is commonly believed that wood-frame houses are not subject to damage from foundation settlement. The foundation failure of Denver residence No. 2, selected for examination, refutes this premise. This building is a large, well-constructed, wood-frame house located south of Denver on a westerly drainage. The house was completed in the fall of 1953, and no significant cracking was noted until February, 1954. At this time, movement of garage footings was observed, and considerable cracking in walls and ceilings occurred until the middle of March, 1954, when a broken water pipe was found. A foundation investigation was then initiated.

Test pits and drill holes were sunk at the locations indicated in Fig. 10. Moisture-content data and undisturbed samples for laboratory testing were obtained for both the material beneath the footings and that at some distance away from the house. The location of a septic-tank drain field and the results of the moisture-content tests are shown in Fig. 10. In holes 1 and 2 located outside the area, the natural water contents are almost all on the order of 10%. Below the footings the water contents average more than 20%, indicating that the supporting soil became wet. The materials were found to be typical, Denver clayey loess to 5 ft or 6 ft below footings. Natural densities were low, ranging from 76 lb per cu ft to 95 lb per cu ft.

Laboratory test results on representative samples, shown in Fig. 11, identified the loess at this site as being the same as that at Denver residence

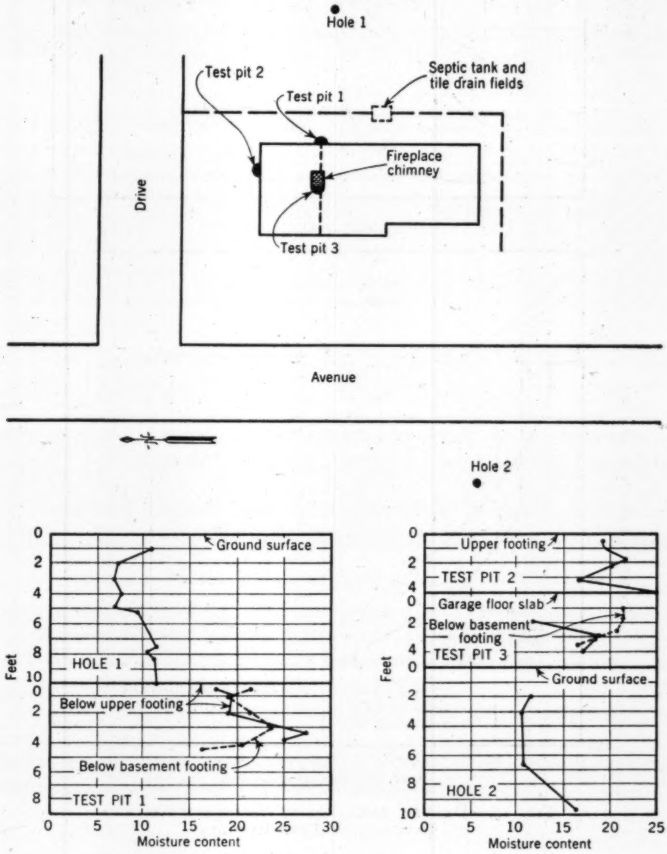


FIG. 10.—DENVER AREA LOESS, RESIDENCE NO. 2, MOISTURE-CONTENT DETERMINATIONS

No. 1. The materials were also similar to Missouri River Basin loess except that both the sand and clay contents were slightly higher. The consolidation curves, plotted as dry density versus load, show the comparison of these properties with those of the Missouri River Basin loess. The Denver loess exhibited similar soil-structure collapse with wetting at low pressure, but the

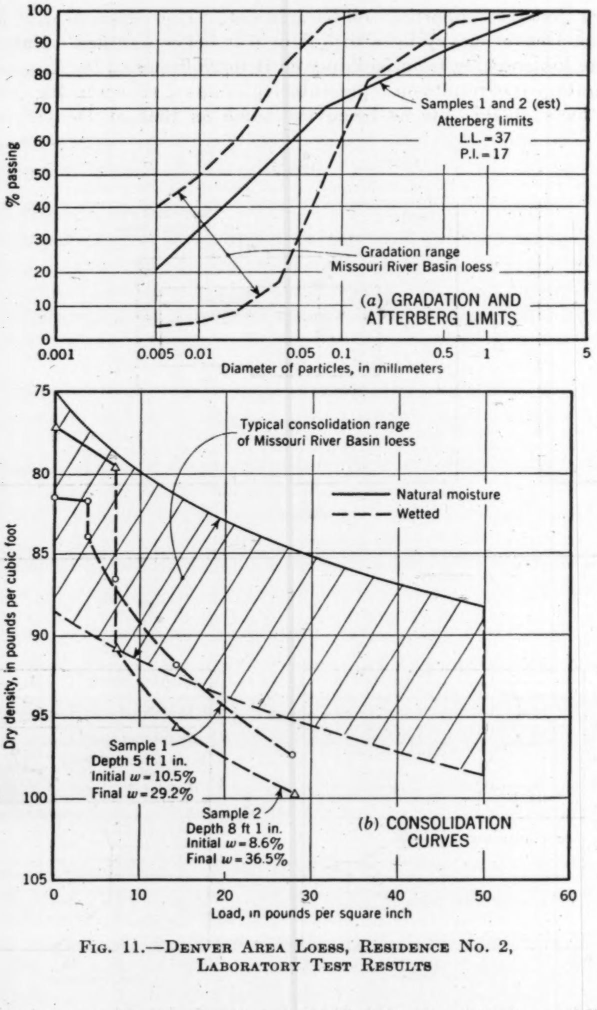


FIG. 11.—DENVER AREA LOESS, RESIDENCE NO. 2, LABORATORY TEST RESULTS

settlements were even more pronounced. Again, the criteria established to indicate the structural properties of loess in one area on the basis of density alone are not safe to use in other areas without full investigations.

In the case of Denver residence No. 2, the cause of failure was attributed to saturation of the loess by the sewage drain field. This saturation caused movement with the resultant rupture of a water pipe, which then wetted the foundation thoroughly, causing still further movement.

In the case of the foregoing two residences, a foundation investigation by competent engineers equipped to perform consolidation and shear tests in the laboratory would have disclosed the serious nature of the foundation problems. Both of these buildings could have been supported economically on cast-in-place concrete piers founded on bedrock that was at shallow depths. This type of foundation has often proved to be as economical in the Denver area as the usual spread-footing construction, particularly in instances in which the depth to bedrock is small.

CONCLUSIONS

Missouri River Basin Area Loess.—Missouri River Basin loess is composed primarily of silt-sized particles bonded together by small amounts of montmorillonite-type clay to form a typical, porous, open structure.

Natural loess normally occurs at densities ranging from approximately 75 lb per cu ft to 95 lb per cu ft. Upon wetting, low-density loess (less than 80 lb per cu ft) settles excessively and has low shear strength. At densities of from 80 lb per cu ft to 90 lb per cu ft, these properties improve, and at densities greater than 90 lb per cu ft, the loess is capable of supporting loads normally assigned to soil.

At low moisture content (15% or less) natural loess will support the normally assigned loadings for silty soil regardless of density. At high natural moisture (more than 20%) the supporting capacity depends on the density.

Plate-load tests indicate that the bearing power of low-density loess may be more than 5 tons per sq ft in the dry state and as low as 0.25 ton per sq ft when wetted.

Reliable density-in-place measurements cannot be made by standard penetration tests. Estimates of the bearing capacity of loess can be made by these tests if there is assurance the loess will never become wetter than at the time of the tests.

Presaturating a loess deposit will not necessarily cause consolidation by the weight of the loess stratum except, perhaps, when the density is very low. An external load, applied by embankment construction or by other methods while the loess is wet, will cause consolidation.

Denver Area Loess.—This loess is better graded and contains more clay than that of the Missouri River Basin.

Settlement and shear-strength properties of the wetted loess are even more unfavorable than for the Missouri River Basin loess.

The density criteria developed for Missouri River Basin loess are not valid for that in the Denver area. The criteria for the moisture content required to cause loess-structure collapse appear valid.

In residential areas, wetting of foundation loess is accomplished by lawn sprinkling sufficiently to cause structure collapse with resultantly significant settlements. Therefore, residences should not be founded on loess unless complete foundation investigations by competent soils engineers show that the material will support the building even after wetting.

DISCUSSION

Harry R. Cedergren,[5] **A. M. ASCE.**—In presenting the experiences with loess in two areas of the United States, the author has described a type of soil that has rather remarkable and spectacular properties. Examples of foundation failures resulting from great losses in strength caused by the saturation of the loose soil have been given.

Another outstanding characteristic of loess is its extremely low resistance to erosion, even in the compacted state. Such a feature is important in designing and constructing earth dams and levees on loess foundations. Special treatment of slopes exposed to the elements is also necessary.

Loess deposits in some areas in the Pacific northwest have been a cause of concern to engineers and builders. Chimneys have toppled over when wetting of their foundations was not controlled. Airfield and highway pavements have settled and cracked badly when water was permitted to penetrate through joints or cracks and to wet the subgrade. Other structural problems and failures have been common wherever loess has been encountered; however, difficulties with this type of soil can be minimized if its characteristics are considered. The loess in the area near Walla Walla (Wash.) is predominately silt, with the clay content ranging usually from 8% to 20% and the sand content ranging from 2% to 10%. Its plasticity index normally is less than 8, and its liquid limit varies from 16 to 30. Its loss in strength, measured by the California Bearing Ratio (CBR) tests before and after soaking, increases rapidly with decrease in density, as illustrated by the curve in Fig. 12. This curve represents the results of 140 CBR-tests on Walla Walla loess in the unsoaked and soaked condition. The tests were made on undisturbed samples taken from the subgrade of a newly constructed military airfield.

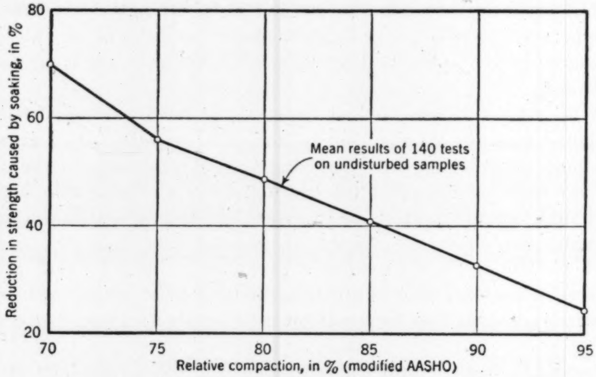


Fig. 12.—Loss in Strength Caused by Soaking, Walla Walla (Wash.) Loess

---

[5] Senior Materials and Research Engr., State Div. of Highways, Sacramento, Calif.

Although the unstable structure of loess is of great interest to engineers desiring to eliminate structural failures, its low resistance to erosion may lead to failures that can endanger life. For example, an earth dam located in the Walla Walla area will never be capable of functioning as the designers intended without major reconstruction because serious erosion has occured within the structure. The soils engineers who designed the dam were fully aware of the treacherous nature of loess and incorporated features that should have guaranteed a stable dam. Major excavation and recompaction of the loess foundation reduced settlements to tolerable amounts. An internal drainage blanket was furnished to control underseepage. Unfortunately, the inspector and the builder of the dam were not familiar with the characteristics of loess and permitted major segregation to occur within the blanket. Consequently, several hundred cubic yards of compacted loess were washed through the blanket to a pipe drain and out of the dam within two years after its completion. This inherent instability greatly impared the usefulness of the structure, which would have been of great value if the intentions and concepts of the designers had been realized.

Whenever loess must be used as the foundation for a major segment of an earth dam or levee, designers and builders should take unusual precautions to avoid designing or building an unsafe structure. Furthermore, those familiar with the properties of loess should be available for consultation throughout all stages of the project.

RALPH B. PECK,[6] M. ASCE, AND HERBERT O. IRELAND,[7] A. M. ASCE.—A distinct service has been performed by the author in summarizing his experiences and those of the USBR with loess, a material about which too little is known despite its widespread distribution. The writers hope that the paper will stimulate the publication of many other records of experience with loess.

Since 1949, engineers at the University of Illinois, at Urbana, have collected test data and field observations concerning loess, not only in the Missouri River Basin but also in the contiguous loessial areas of the Mississippi and Illinois Rivers; participants in this work have been Thomas S. Fry, J. M. ASCE, Tien Hsieng Wu, J. M. ASCE, Leonardo Zeevaert, A. M. ASCE, and James J. Pearson, Jr., J. M. ASCE. Sites from which data have been obtained are shown in Fig. 13. These sites extend from the semiarid regions, with which Mr. Clevenger's experience has been associated, into the semihumid areas of the central lowlands. Against this broader background of climatic conditions, the experiences and correlations of the author have a regional aspect and, if not interpreted in this light, might give a restricted or narrow conception of the physical properties of the material. The writers regard loess not as a soil of remarkably constant and uniform properties, but as one possessing local and regional variations almost as striking as those of some glacial materials.

The strength and rigidity of loess appear to depend to a considerable extent on the moisture content of the clay binder, which is reflected in the moisture content of the soil as a whole. This is admittedly a broad generalization, to which there are many exceptions, but it serves as a starting point for a discussion. The writers' data indicate significant differences in the structural properties of loess from different parts of the United States that could not be correlated strictly with such index properties as grain size, density, or penetration resistance. However, an instructive correlation was found to exist between the average natural water content of loess deposits and the average annual rainfall of the locality, as shown in Fig. 14. This correlation indicates that the behavior of loess in humid regions should be expected to differ from that in semiarid regions.

---

[6] Prof. of Foundation Eng., Univ. of Illinois, Urbana, Ill.

[7] Associate Prof. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

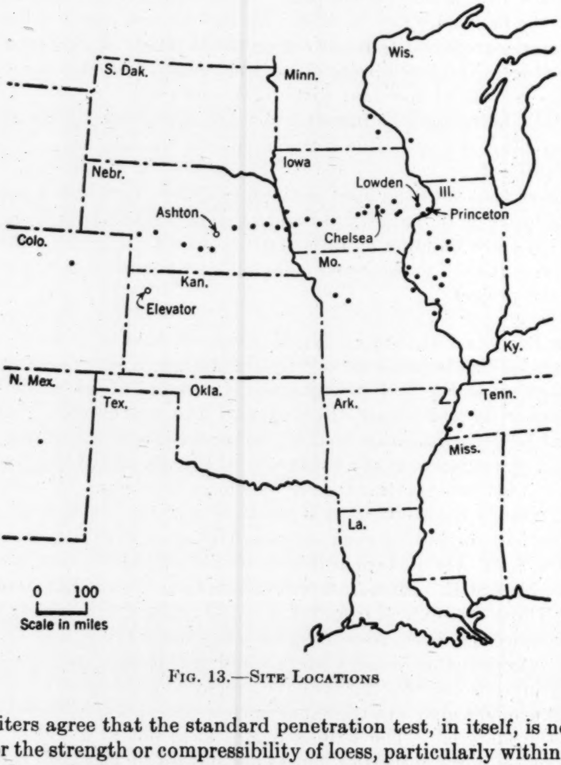


FIG. 13.—SITE LOCATIONS

The writers agree that the standard penetration test, in itself, is not a good criterion for the strength or compressibility of loess, particularly within a limited geographical area. Two more dependable measures of compressibility are believed to be the load at ½-in. settlement of a 1-ft-sq plate, and the pressure corresponding to the break in the curve of e versus log p from a standard consolidation test on a hand-carved sample; e is the void ratio and p is the load per unit area. If an attempt is made to correlate either of these quantities with the N-values for one part of the United States, no trend is apparent; N is the number of blows per foot from the standard penetration test. In Fig. 15(a), for example, the relationship is shown for several sites in eastern Iowa and western Illinois. The sites are represented by the solid dots in the lower left corner of the plot. Similarly, results reported by the USBR,[8] indicated by the open circles, show no correlation among each other. However, if both sets of data are considered together, there emerges a fairly satisfactory relationship between the *N*-values and the results of the consolidation tests. In similar fashion there appears to be a fair correlation between the *N*-values and the results of field load tests, as illustrated in Fig. 15(*b*).

If the subsoil of a structure can be protected from the effects of prolonged wetting, as can usually be done (except possibly in the case of residences), the designer seems to be faced with two alternatives if he uses soil-supported footings or a raft. Because the load-settlement curves for loess often exhibit a fairly well-defined break (Fig. 16) marking the pressure at which the open structure of the soil begins to crush or collapse, the designer may choose to restrict the footing pressure to a value well below the break in the curve so that the structure will experience minor settlements. However, he may choose a higher value and accept larger settlements. The latter alternative is feasible because there is little likelihood of a complete bearing-capacity failure by rupture in the usual sense. The second alternative has often been used by constructors of grain elevators, whereas the first alternative is more suitable in connection with conventional structures such as school buildings.

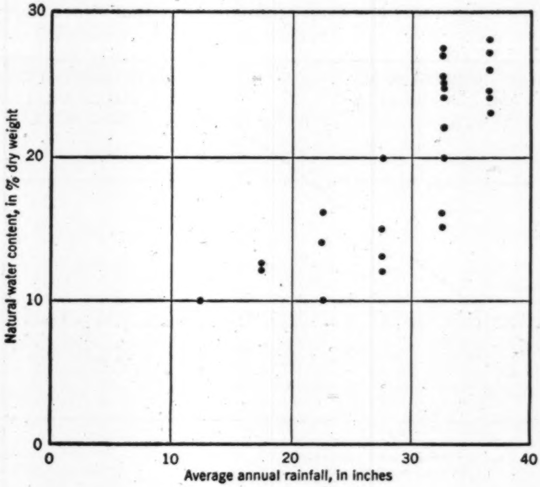


FIG. 14.—RELATIONSHIP BETWEEN ANNUAL RAINFALL AND NATURAL WATER CONTENT

---

[8] "Report of Loess Research Studies for the Ashton Pile Testing Program—Lower Platte River Area—Missouri River Basin Project, Nebraska," *Earth Lab. Report No. EM-278*, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo., December 17, 1951.

In connection with the design of two concrete towers for which the tolerable settlement was very small, the writers chose to restrict the soil pressure under dead load to approximately 1/2.5 times the pressure at which the break occurred in standard 1-ft-by-1-ft load tests. The tower at Princeton (Iowa) rests on a 36-ft-sq raft and is underlain by 85 ft of loess. The tower at Lowden (Iowa) rests on a 46-ft-sq raft and is underlain by 22 ft of loess. Typical load-settlement curves for the load tests and the observed maximum settlement for each structure are shown in Fig. 17; DL refers to the dead load of the structure, and DL + LL + WL to the dead, live, and wind loads. Observations utilizing specially constructed, deep bench marks were conducted for four years and showed no tendency toward progressive settlement.

The settlement records demonstrate that, at equal unit pressures, the settlement of the structures was less than approximately four times that of the test plates, although the width of the structures was roughly forty times that of the plates. The natural moisture content of the loess at these two sites was about 23%, considerably higher than the value of 15% indicated by Mr. Clevenger as being the upper limit for undisturbed loess of good supporting value.

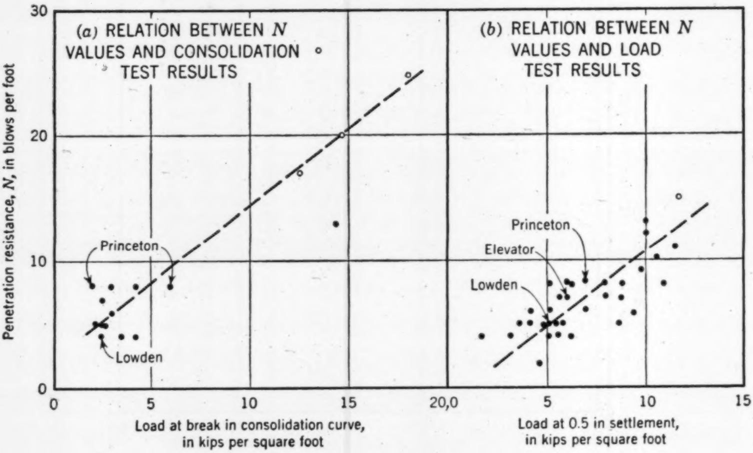


Fig. 15.—Relationship Between N-Values and Test Results

The writers are familiar with the grain elevator in Kansas cited by the author as an example of settlement caused by wetting of the loess after construction. They believe that there is no evidence that ponding of the water caused the settlement, but believe rather that the soil pressure was excessive and distributed too nonuniformly for the loess in its original condition. According to Mr. Clevenger, the elevator was completed in July, 1950, and heavy rains in August, 1950, caused ponding and settlement on the north side. Fig. 18 shows the record of movements of the elevator, the daily rainfall at the weather station in the same town, and the total amount of grain in storage. It

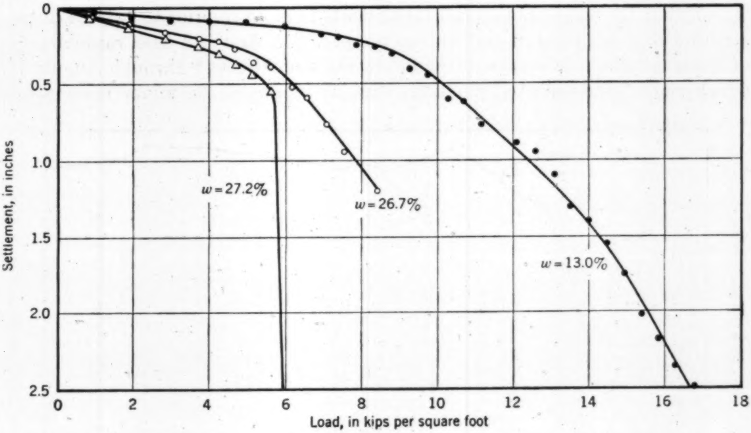


FIG. 16.—TYPICAL LOAD RESULTS

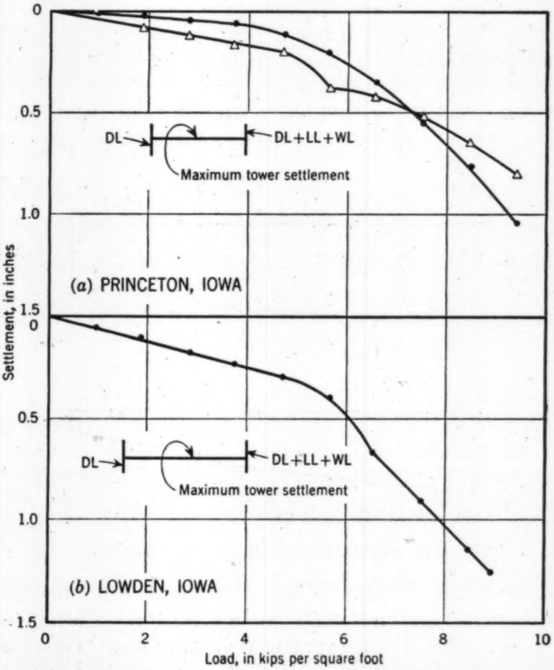


FIG. 17.—RESULTS OF LOAD TEST AND TOWER SETTLEMENT

is seen that the most pronounced movements took place prior to July 24 during a period of small rainfall, but during the period of first and most rapid loading of the structure. In contrast, the heavy rains of August 9 through August 15, which caused the reported ponding, were accompanied by minor movements,

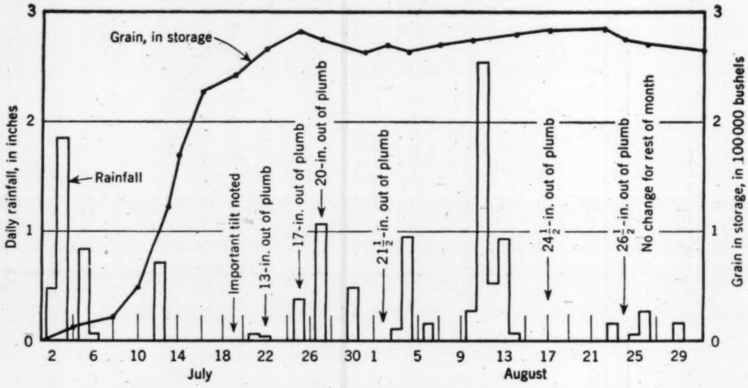
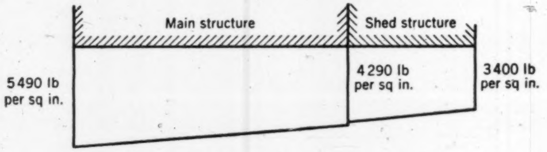


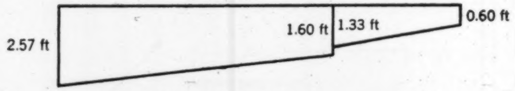
FIG. 18.—RAINFALL RECORD AT SITE AND LOADING HISTORY OF GRAIN ELEVATOR

and the load on the foundation was nearly constant. There is evidently an excellent correlation between movement and load, and none between movement and rainfall.

The structure consisted of eight bins resting on a raft 103 ft by 59 ft in plan, with a loading shed on the south side rigidly attached to the bins. Because of



(a) SOIL PRESSURE



(b) SETTLEMENT OCT 24

FIG. 19.—SETTLEMENT AND CONTACT PRESSURES

this arrangement the soil pressure was nonuniform when the structure was loaded; the computed pressures are shown in Fig. 19(*a*). Corresponding settlements are shown in Fig. 19(*b*), approximately three months after filling. The settlement evidently depended on soil pressure instead of any other factor, such as wetting on the north side.

Several load tests were made prior to placing the concrete for the raft. One indicated a settlement of 3 in. under a unit load of 5,000 lb per sq ft; three other tests showed settlements of approximately 1.3 in. at 10,000 lb per sq ft, but the settlements at intermediate pressures were not recorded. In any event the actual soil pressure of approximately 5,500 lb per sq ft seems excessive in view of the results of the load tests. Had the loading been concentric and the settlement uniform, probably no difficulties would have arisen because uniform settlements on the order of 1 ft are not considered unusual or undesirable for such structures in the locality.

The Kansas elevator demonstrates the crushing type of failure that occurs in loess. The footings for the shed punched downward without any accompanying heave of the adjacent soil or floor (Fig. 20). The settlement is the result of a decrease in volume of the loess, probably within a fairly shallow depth. For this reason the settlement of a structure that causes such crushing should not greatly exceed that of a small test plate at the same unit load.

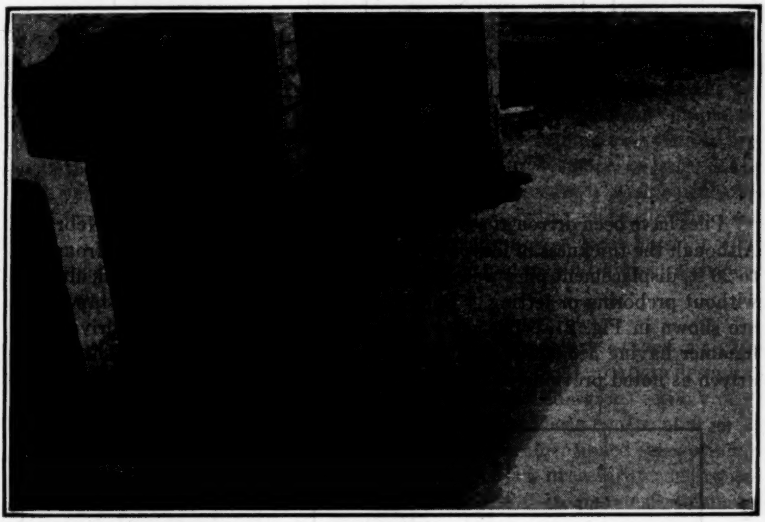


FIG. 20.—PUNCHING FAILURE OF ELEVATOR FOOTINGS

Pile foundations in undisturbed loess were not cited in the paper probably because of the high strength of such materials in the semiarid part of the Missouri River Basin. The Ashton tests of the USBR indicated the necessity for predrilling in order that displacement piles could be driven through the strong loess in that area. Only three displacement piles were driven without preboring or jetting; these piles reached practical refusal at depths of from 23 ft to 35 ft and did not penetrate through the loess stratum. Two of the piles were timber, and one was a step-taper pile; all were driven by a hammer having a 5,000-lb ram falling 3 ft.

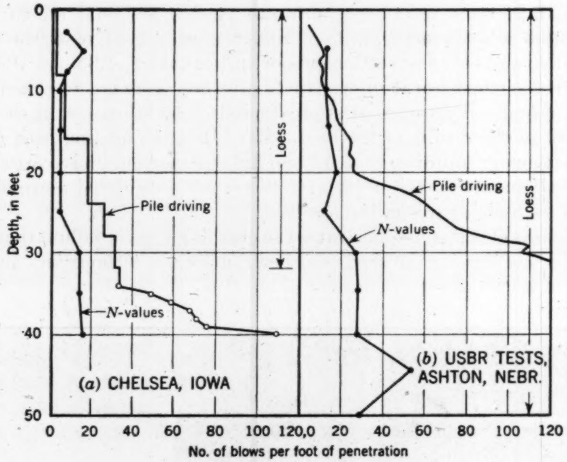


FIG. 21.—PILE-DRIVING RECORDS

Piles have been driven at several sites in western Iowa and eastern Nebraska. Although the thickness of loess at many of these sites did not exceed from 15 ft to 20 ft, displacement piles were driven through the loess without difficulty and without preboring or jetting. Typical pile-driving records for step-taper piles are shown in Fig. 21. The piles driven at Chelsea (Iowa) were driven by a hammer having a 3,000-lb ram falling 4 ft, and those driven at Ashton were driven as noted previously. The Chelsea record (Fig. 21 (*a*)) is representative

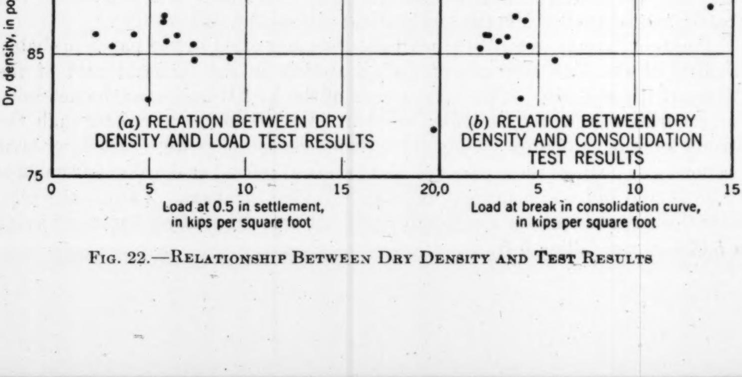


FIG. 22.—RELATIONSHIP BETWEEN DRY DENSITY AND TEST RESULTS

of the semihumid areas, whereas the Ashton test results (Fig. 21(*b*)) are representative of the semiarid regions.

Mr. Clevenger has concluded that the supporting capacity of natural loess is closely related to the density. It is doubtful if this conclusion is valid for the entire range of loess deposits in the central United States.

Fig. 22(*a*) shows the dry density, as determined from hand-carved samples, plotted against the load at 0.5-in. settlement, as determined from standard load tests. A similar relationship is shown in Fig. 22(*b*), in which dry density is plotted against the load at the break in the curve of e versus log p for the same material. It appears from these data that no satisfactory correlation exists between density and the supporting capacity of loess.

The author indicates that field-density measurements in open test pits or direct-density measurements on undisturbed samples obtained by any means are the most desirable and reliable measurements. It is doubtful if direct-density measurements on undisturbed samples, as usually obtained from a boring, have any relationship to the in-place density. The writers have observed many thin-walled tube samples of loess that have been compacted by the sampling operation until the typical structure of the loess has been completely destroyed. In fact, this collapse of structure during sampling may tend to prevent the samples from being identified as loess. Furthermore, because of the high vertical permeability of loess, extreme care is necessary if wash borings are used in order to avoid an increase in the natural moisture content of the samples. Therefore, it appears that undisturbed sampling of loess must be done by special techniques in order to preserve the structure and moisture content of loessial soils.

William A. Clevenger,[9] A. M. ASCE.—Low erosion resistance is an important characteristic of loess that must be considered, particularly in designing hydraulic structures, as has been emphasized by Mr. Cedergren. Many miles of lined and unlined canals and drainage channels have been constructed in the Missouri River Basin loess area. Because of the aforementioned susceptibility of loess to erosion, the slopes exposed to the weather are necessarily constructed as steep as possible in order to minimize such erosion. In many cases slopes of $\frac{1}{4}$ on 1 have been used. The longitudinal slopes of unlined canals and channels are also controlled to a large extent by the susceptibility of loess to erosion, whether in the undisturbed or disturbed condition.

Messrs. Peck and Ireland have questioned the sequence of rainfall, loading, and settlement of the Kansas elevator. The words, "July" and "August," were used in general terms in the paper. Part of the settlement occurred in July and with the bin loading. However, there was considerable rainfall in July. The drainage conditions, which led to ponding between the north side of the building and the adjacent spur railroad track, existed throughout the period of settlement. Therefore, any heavy rainfall in the drainage area would cause an accumulation of water at the site. The fact that the tilting was later corrected to a considerable extent by artificially wetting the opposite side of the elevator foundation further supports the opinion that the soil structure collapsed from wetting as well as from loading.

---

[9] Vice-Pres., Woodward-Clyde-Sherard & Associates, Denver, Colo.

Messrs. Peck and Ireland have stated that there is no correlation between the density and supporting capacity of loess and have presented data to support their opinion. These data show the relationship between (1) density and load-test results and (2) density and consolidation-test results for loess tested at the natural water content. For this moisture condition the writer agrees with the data presented, as indicated in Fig. 2. However, experience with structures that have undergone foundation failures on loess shows that these failures are almost always associated with foundation wetting. This conclusion leads to the belief that the ultimate supporting capacity is related to the ultimate settlement to be expected upon wetting, which is particularly true for loess foundations of hydraulic structures that are certain to become wetted. There is excellent correlation between supporting capacity and density if it is assumed that the loess will become thoroughly wetted.

The discussers' comments on the disturbance of loess samples caused during normal, undisturbed sampling operations are appreciated, and any possible misconception in this respect should be corrected. However, the word "undisturbed" was intended to apply to truly undisturbed samples taken by any method that might be devised.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2917

A FLOODWAY FOR HOUSTON, TEX.

By Kenneth Heagy,[1] M. ASCE

Synopsis

The most suitable plan for the flood protection to the city of Houston, Tex., is one that provides for the continued operation of Barker Reservoir and Addicks Reservoir; for the rectified channel immediately downstream therefrom; and for the clearing, straightening, enlarging, and lining, where necessary, of the channels of Buffalo Bayou, Brays Bayou, and White Oak Bayou. Extensive studies were made and retention and diversion of flood waters were considered, but the rectification of natural channels proved to be the most desirable and economical plan.

Introduction

The city of Houston, Tex., is located in the center of the Buffalo Bayou watershed. The principal streams passing through the city are Buffalo Bayou and its major tributaries, Brays Bayou and White Oak Bayou. The flood plains of the maximum floods of record on Buffalo Bayou, Brays Bayou, and White Oak Bayou encompass both urban and rural properties. The municipal development of the city of Houston and adjacent towns comprising the Houston metropolitan area extend within these flood plains and include extensive industrial, commercial, and residential properties as well as utilities and transportation facilities. The center of Houston, situated at the confluence of White Oak Bayou and Buffalo Bayou, has experienced heavy damages to commercial and industrial properties. Since 1945 extensive residential expansion within the flood plain of Brays Bayou has greatly increased the damage potential from the floods on this stream. The location and extent of the Buffalo Bayou watershed and the locations of Buffalo Bayou, Brays Bayou, and White Oak Bayou are shown in Fig. 1.

Channel-Flow Capacities

The present (1957) flow capacity of the channel of Buffalo Bayou at bankfull stage decreases rapidly from approximately 30,000 cu ft per sec between

---

Note—Published, essentially as printed here, in April, 1956, in the Journal of the Waterways and Harbors Division, as *Proceedings Paper 926*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Chief, Eng. Div., Galveston Dist., Corps of Engrs., U. S. Dept. of the Army, Galveston, Tex.

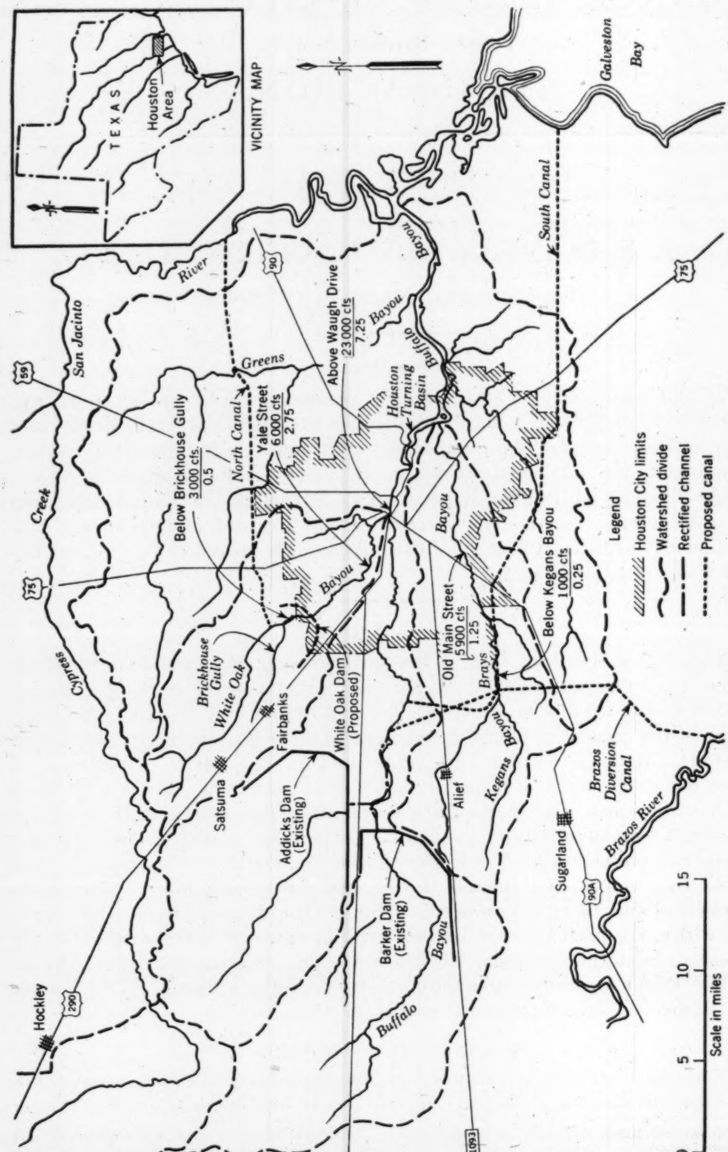

VICINITY MAP
TEXAS
Houston Area
N
Galveston Bay
South Canal
San Jacinto River
Greens Bayou
North Canal
Above Waugh Drive
23 000 cfs
7.25
Yale Street
6 000 cfs
2.75
Below Brickhouse Gully
3 000 cfs.
0.5
Houston Turning Basin
Buffalo Bayou
Bayou
Old Main Street
5 900 cfs
1.25
Below Kegans Bayou
1 000 cfs
0.25
Brays Bayou
Kegans Bayou
Alief
Sugarland
Brazos Diversion Canal
Brazos River
Cypress Creek
Brickhouse Gully
White Oak Bayou
Fairbanks
Satsuma
Addicks Dam (Existing)
White Oak Dam (Proposed)
Barker Dam (Existing)
Buffalo Bayou
Hockley
290
90
1093
90A
75
59
Legend
Houston City limits
Watershed divide
Rectified channel
Proposed canal
0
5
10
15
Scale in miles

FIG. 1.—HOUSTON, TEX., AND VICINITY

the Houston turning basin and the mouth of White Oak Bayou, mile 21.7, to approximately 25,000 cu ft per sec upstream from the mouth of White Oak Bayou, and, thence, the channel capacity decreases gradually to approximately 6,000 cu ft per sec at mile 43.6.

The flow capacity of the channel of Brays Bayou at bankfull stage is about 12,000 cu ft per sec at the mouth and remains relatively constant to mile 8.7, MacGregor Drive; thence, the channel capacity decreases gradually to approximately 5,900 cu ft per sec at mile 10.8, Old Main Street; 2,500 cu ft per sec at mile 14.6, Willow Waterhole Bayou; and approximately 1,000 cu ft per sec at mile 19.4, Kegans Bayou.

The present flow capacity of the channel of White Oak Bayou at bankfull stage is approximately 10,000 cu ft per sec at the mouth and remains relatively constant to mile 1.5, Little White Oak Bayou. Above Little White Oak Bayou the channel capacity decreases gradually to approximately 6,000 cu ft per sec at mile 4.3, Yale street; approximately 3,000 cu ft per sec at mile 14.3, Brickhouse Gully; and approximately 2,000 cu ft per sec at mile 17.3, Cole Creek.

HYDROLOGY

Storms.—The major storms that have occurred on the Buffalo Bayou watershed for which rainfall data are available, together with information regarding the location of the storm centers, the total precipitation, the duration at the centers, and the average depth over the Buffalo Bayou watershed, are listed in Table 1.[2]

TABLE 1.—MAJOR STORMS

Period of storm	STORM CENTER			Average depth over watershed, in inches
	Location	Duration, in hours	Rainfall, in inches	
May 24 to 31, 1929	Fairbanks	42	12.95	7.9
December 6 to 8, 1935	Near Satsuma	34	20.80	11.7
September 23 to 25, 1941	Sugarland	48	9.55	...[a]
November 1 to 3, 1943	Near Alief	25	12.20	7.9
August 27 to 30, 1945	Near Hockley	30	20.06	11.6

[a] Not determined.

Floods.—Records indicate that floods occurred on Buffalo Bayou and its tributaries in 1854, 1875, 1879, 1907, and 1929. The city of Houston was founded in 1835, and, based on testimony and records of local interests, the maximum known flood on Buffalo Bayou, White Oak Bayou, and Little Oak Bayou occurred in December, 1935. During this flood some overflow occurred from White Oak Bayou into Buffalo Bayou, and considerable overflow occurred from Buffalo Bayou into Brays Bayou. The peak discharge of the December, 1935, flood on Buffalo Bayou, White Oak Bayou, and Little White Oak Bayou are shown in Table 2. The maximum known stages along Brays Bayou between mile 14.2 and mile 2.8 were produced by the flood of

---

[2] "Interim Report on Buffalo Bayou and Tributaries, Texas," *House Document No. 250* (published in part), 83rd Cong., 2d Session, 1953.

TABLE 2.—FLOOD OF DECEMBER, 1935

Stream	Location	River mile	Peak discharge, in cubic feet per second
Buffalo Bayou	Eureka cutoff	29.4	40,030
Buffalo Bayou	Lockwood Drive	18.4	52,750
White Oak Bayou	Railroad bridge west of Stude Park	3.4	14,750
Little White Oak	Sylvester Street	3.8	4,950

August, 1945; the maximum known stages over the remainder of the bayou were produced by the flood of May, 1929. Flood flows on Buffalo Bayou, White Oak Bayou, and Brays Bayou have been measured by the Geological Survey, United States Department of Interior (USGS), since 1936. The floods of record in 1941, 1943, and 1949 were of a damaging nature on Brays Bayou. Table 3 shows the maximum flood discharges during the period of record of the USGS stream-gaging stations.

TABLE 3.—MAXIMUM FLOOD DISCHARGE OF UNITED STATES GEOLOGICAL SURVEY RECORD, 1936–1949

Stream	Locality	River mile	Elevation, mean sea level, in feet	Peak discharge, in cubic feet per second	Date
Buffalo Bayou	Addicks	46.6	80.93	11,200	August 29, 1945
Buffalo Bayou	Waugh Drive	24.7	33.83	10,900	August 30, 1945
White Oak Bayou	Yale Street	4.2	38.37	8.600	November 2, 1943
Brays Bayou	Old Main Street	10.8	46.70	8,120	November 2, 1943
South Mayde Creek	At Addicks Dam	1.3	89.98	2,250	April 23, 1949
Buffalo Bayou	At Barker Dam	49.8	90.10	2,840	August 30, 1945

Frequency of Flooding.—Table 4 shows the approximate flood-stage discharges at selected points on Buffalo Bayou, White Oak Bayou, and Brays Bayou, and their frequency of occurrence. Frequencies are based on the assumption that Barker Reservoir and Addicks Reservoir do not exist and that runoff occurs under 1950 watershed and channel conditions.

TABLE 4.—FREQUENCY OF FLOOD-STAGE DISCHARGES

Bayou	Location	Discharge, in cubic feet per second	Frequency, in years
Buffalo	Above Waugh Drive	23,000	7.25
White Oak	Below Brickhouse Gully	3,000	0.5
White Oak	Yale Street	6,000	2.75
Brays	Old Main Street	5,900	1.25
Brays	Below Kegans Bayou	1,000	0.25

On Brays Bayou at Old Main Street, the stage at which flooding begins under 1950 conditions represents an estimated discharge of 5,900 cu ft per sec. However, during the period of record, flooding has occurred at less discharge because of varying channel conditions. As shown in Table 4, the frequency of occurrence of flood stage at Old Main Street is approximately once each year. The maximum discharge of record at Old Main Street (Table 3) is 8,120 cu ft per sec, which is only 1.4 times the estimated flood-stage discharge.

PLANS CONSIDERED

Following the destructive flood of December, 1935, which caused the loss of eight lives, damaged property in Houston to the extent of approximately $2,500,000, and severely hindered navigation in the Houston ship channel, a federal flood-control project for Buffalo Bayou was authorized by the River and Harbor Act of June 20, 1938, and the Flood Control Act of August 11, 1939. This project authorized improvement of Buffalo Bayou and its tributaries above the Houston ship channel turning basin to provide for the control of floods, the protection of Houston from flood damages, and the prevention of the deposit of silt in the turning basin by use of detention reservoirs, enlargement and rectification of channels, the construction of control works, and any diversions that may be found advisable.

In 1940 the Corps of Engineers, United States Department of the Army, and the Harris County Flood Control District (the state agency representing local interests) formulated a joint general plan of development for the detail design and construction of flood-control works on Buffalo Bayou. The basic features of this joint plan, with subsequent minor revisions, were:

a. Construction of the White Oak detention reservoir, the north diversion canal, and appurtenant works to prevent discharging flood flows from upper White Oak Bayou through Houston;

b. Construction of Barker Reservoir and Addicks Reservoir in the vicinity of Addicks to control the runoff from upper Buffalo Bayou, and of the south diversion canal, including 7.4 miles of channel rectification from the twin reservoirs to Galveston Bay to prevent this runoff from passing through the city of Houston;

c. Removal of obstructions in the channel of Buffalo Bayou within the city of Houston; and

d. Protection of the floodway areas against further encroachment by the establishment and enforcement of building limit lines on existing streams, and the prevention of dumping of waste materials on the banks of existing streams within building limit lines.

The diversion plan[3] (Fig. 1) was considered desirable because during 1940, when it was being considered, the enlargement and rectification of the channels of Buffalo Bayou and White Oak Bayou, which involved excavation of large quantities of material; the alteration of many of the present bridges or their replacement by larger and more expensive structures; and the construction of special works to protect the Houston ship channel from the deposition of silt during floods would have been more expensive. Under this diversion plan, Barker Reservoir and Addicks Reservoir were completed in 1945 and 1948, respectively, and channel rectification from the reservoirs to the point of diversion into the proposed south canal was completed in 1948.

---

[3] "Report on Houston Ship Channel and Buffalo Bayou, Texas," *House Document No. 456* (published in part), 75th Cong., 2d Session, 1937.

However, since 1940 the rapid residential and industrial development of Houston and other communities on, or in the vicinity of, the sites of the proposed White Oak reservoir and on or in the north and south canals so increased the cost of the proposed White Oak reservoir and of the north and south canals that it was impracticable to construct these features at that time (1957).

The area along Brays Bayou has been highly improved by recent extensive urban construction and is subject at this time to potentially severe flood damages. The project formulated in 1940 would not provide adequate flood protection to this area. The south canal feature of the project would provide only such incidental protection as would result from the diversion of the runoff from the upper parts of the Brays Bayou watershed. Therefore, it has become necessary to afford adequate flood protection to the Brays Bayou area.

In view of these changed conditions, the local interests obtained authority for a review of reports on Buffalo Bayou to determine the best comprehensive plan for the betterment of navigation in the Houston ship channel and for the control of floods throughout the entire Buffalo Bayou watershed, including modifications of the existing plan of improvement and of the requirements of local cooperation.

In order to expedite completion of the flood-control project on Buffalo Bayou, it was decided to formulate a partial plan of improvement for the Buffalo Bayou watershed limited to providing flood protection to Houston, and to prepare an interim report thereon. Because Buffalo Bayou above the Houston turning basin, Brays Bayou, and White Oak Bayou causes most of the flood damages in Houston, the investigations were limited to the determination of plans for control of floods on those streams.

Investigation of plans of improvement indicated two basic plans as being most feasible for providing flood protection to Houston. The main feature of one plan is the diversion of parts of Buffalo Bayou and Brays Bayou to the Brazos River. The other plan is for the enlargement and rectification of the natural stream channels to carry the flood discharges through Houston without damages.

Each plan is designed to provide the same degree of flood protection to Houston—that is, to control the standard project flood to nondamaging stages through the city. The benefits, consisting principally of prevention of flood damages, would be approximately the same and are estimated at $3,345,000. They are estimated on the basis of present conditions with allowance for future growth and would result from the complete plan of improvement, including the two existing reservoirs and channel rectification, and the proposed channel rectification or diversion.

The estimated first cost, the estimated annual charges, and the ratio of annual benefits to annual charges for each plan of improvement, including the cost of existing Barker Reservoir and Addicks Reservoir and the existing rectified channel, are presented in Table 5. Construction costs are based on December, 1950, prices. Land and damage costs are based on appraisals furnished by the local interests.

The diversion plan offers a solution to the flood problem that would cause less disruption in Houston than would the rectification plan. However, there are several features of the former plan which lessen its desirability. These features have not been evaluated and are not reflected in the economic ratios given in Table 5. Two considerations are the greater length of improved channel that would require maintenance, and the possibility that all phases of channel maintenance have not been given comparable treatment. Also the Brazos River flood plain would be subject to increased flood damages in the event of coincident floods on the two watersheds. Available data indicate that certain coincident flows from Buffalo Bayou and the Brazos River may cause increased damages of $500,000 in the Brazos River flood plain from a single flood, and that the increase in average annual damages in the plain, resulting from diversion of the Brazos River, may exceed a range of from $50,000 to $100,000. Furthermore, Brays Bayou and its tributaries would suffer damages from backwater from high stages on the Brazos River which, when concurrent with floods on Buffalo Bayou, would interfere with the functioning of the diversion channel. These factors are serious disadvantages and would reduce the favorable economic ratio of the diversion plan.

In addition to the unfavorable engineering features of the diversion plan, there are certain legal aspects that arise from the diversion of water from one watershed to another. Local interests would have to obtain any necessary authority for the diversion; furthermore, they would have to assume liability for flood damages caused along the Brazos River by the floodwaters diverted from Buffalo Bayou. In view of the probability of such damages, despite the lack of estimates of the damages, the Harris County Flood Control District advised that it would not be able to assume the liability for such damages. Examining all aspects of the diversion plan, evaluated features, unevaluated and intangible features, and engineering and legal difficulties, it is considered that the diversion plan would not be feasible.

TABLE 5.—ESTIMATED FIRST COST, ANNUAL CHARGES, AND RATIO OF BENEFITS TO CHARGES

Plan of improvement	First cost	Total annual charges	Ratio of benefits to charges
Rectification plan	$61,761,000	$2,775,300	1.21
Diversion plan	62,861,000	2,877,700	1.16

The rectification plan provides for a partly lined channel for most of Brays Bayou and White Oak Bayou. Paved channels result in better flow conditions. They require less area of rights of way, and, being more stable, they are easier to keep free of objectionable stagnant pools.

There are several parts of Brays Bayou and Buffalo Bayou that are in highly developed areas with structures built on the banks of the streams. On the basis of investigations to date, the best treatment of the channel cannot be determined. It may be more economical to provide vertical sheet-pile walls at these points than to provide for sloping sides. (The cost estimates are sufficient to include several sections of wall should they prove necessary.)

From the standpoint of navigation, any channel rectification work by itself would be detrimental. However, Barker Reservoir and Addicks Reservoir control the runoff from 279 sq miles or 57% of the area draining into the Houston turning basin, thus affording considerable reduction to the currents in the Houston ship channel during floods. A proposed desilting basin immediately upstream from the Houston turning basin, provided with the rectification plan, would reduce the quantity of silt that may be carried from White Oak Bayou and upper Buffalo Bayou into the deep-draft channel of the Houston ship channel.

A desilting basin is considered to be a necessary feature of the rectification plans to protect the Houston turning basin from excessive sediment deposition during floods. The basin would reduce the quantity of maintenance dredging required in the turning basin and ship channel. However, the sediment deposits would have to be removed from the desilting basin after each flood. The total maintenance dredging would be approximately the same with or without the desilting basin and, therefore, probably would not differ materially from the amount that is required under present conditions. Because maintenance of the desilting basin would reduce the necessary maintenance of the turning basin, this work should be part of the cost of maintaining the existing navigation project, and should be done by the United States under the Houston ship channel navigation project at no increase in the estimated cost of maintenance. The authorization of the existing navigation project, which provides for construction and maintenance of off-channel silt basins, would permit the incorporation of the proposed desilting basin in the navigation project. The initial funds for the desilting basin will be provided under the flood-control project.

The Reservoir Design Storm

The Buffalo Bayou watershed is in an area subject to all the conditions that make large storms possible. The Westfield, Tex., storm of December, 1935, was the most intense storm during the period of record. However, had this storm been centered over the basin, it would have produced a more severe flood than the one that actually occurred.

The storm of record in the United States showing the greatest depth of rainfall over a large area occurred in 1899 at Hearne, Tex., only 90 miles from Houston, under meterological conditions that could be approximated closely over the Buffalo Bayou watershed. After due consideration by Corps of Engineers and United States Weather Bureau personnel, it was concluded that the maximum probable storm that might occur over the Buffalo Bayou basin was a transposition of the Hearne storm of from June 28 to July 1, 1899. Should such a storm visit the area, the average rainfall over the basin would be in excess of 27 in., which is almost twice the average of 15 in. that produced the record flood of 1935.

Addicks Reservoir and Barker Reservoir are of such magnitude as to limit the runoff produced by the maximum probable storm (design storm of 31.4 in. in a period of three days) to a maximum total regulated discharge of approximately 15,000 cu ft per sec. The ultraconservative design capacities of these reservoirs were considered justified because failure of either dam might result in loss of life and damage far in excess of their cost.

DETERMINATION OF STANDARD PROJECT FLOOD

In order to achieve more uniform results in the hydrological investigations of its field agencies, the Corps of Engineers has developed the concept of a standard project storm and a standard project flood for its flood-control investigations. A standard project storm for a particular drainage area should represent the most severe flood-producing rainfall quantity-intensity sequence relationship and areal distribution of any storm that is considered reasonably characteristic of the region in which the drainage basin is located, giving due consideration to the runoff characteristics of the basin.

The standard project flood is the runoff from the standard project storm. This flood represents a practical measure of the flood potentiality of a particular drainage basin, corresponding to storms and runoff conditions observed in the region on a sufficient number of occasions to demonstrate that a distinct danger exists of such a flood occurrence. The standard project flood reflects a generalized analysis of flood potentialities in a region as contrasted to an analysis of limited flood records at the specific locality, which may be misleading because of the inadequacies of records or abnormal sequences of hydrologic events. The statistical probability of occurrence of this standard project flood is not of primary importance. The principal purposes of the standard project flood are:

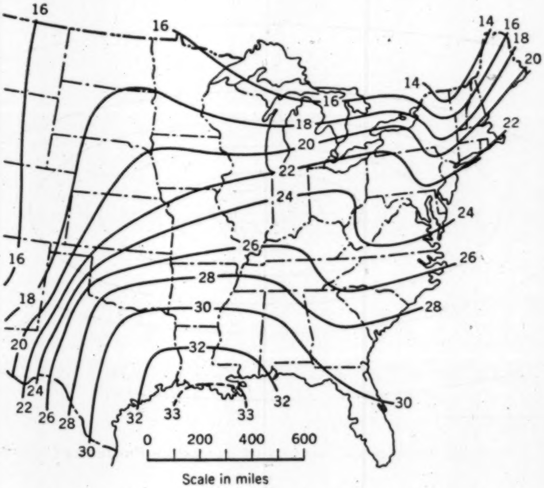


FIG. 2.—GENERALIZED ESTIMATES OF THE MAXIMUM POSSIBLE PRECIPITATION, IN INCHES, OVER AN AREA OF 200 SQ MILES IN 24 HR

a. To serve as a standard against which the degree of protection finally selected for the project may be judged and compared with protection provided at similar projects; and *b.* To represent the flood discharge that should be selected as the design flood for the project, or approached as nearly as practicable in consideration of economic or other governing limitations, if an unusually high degree of protection is justified by hazards to life and high property values within the area to be protected.

The standard project flood used in the investigation of Buffalo Bayou is derived from an over-all study made by the Corps of Engineers of all recorded major storms in the United States east of 105° longitude for small drainage basins of 1,000 sq miles or less.[4] The 105th meridian crosses the extreme western tip of Texas, and Buffalo Bayou, therefore, is located in the area encompassed by this study. The rainfall criteria used in the over-all study are based primarily on an analysis of major convective-type storms and relate to a total storm duration period of 24 hr. The average depth over an area of 200 sq miles for a duration period of 24 hr was used as the unit for comparing and correlating the storms.

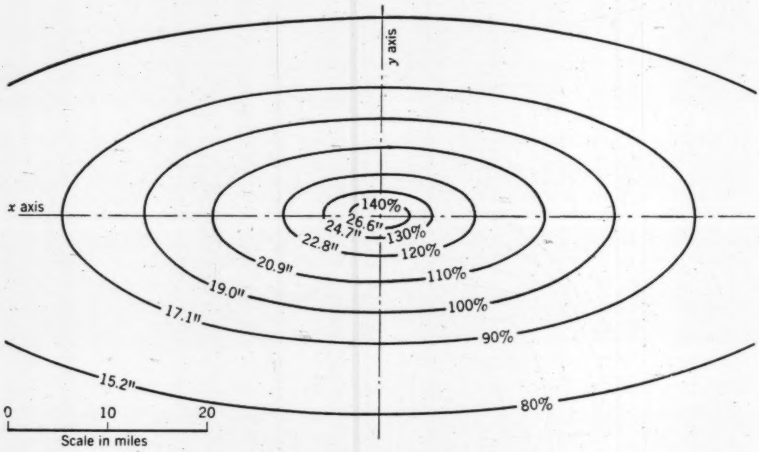


FIG. 3.—ISOHYETAL PATTERN OF STANDARD PROJECT STORM

The average depths of 24-hr rainfall over an area of 200 sq miles for all storms that have been investigated to date were plotted in their geographical positions. Generalized isohyets representing maximum possible 24-hr rainfall over 200 sq miles were obtained from the United States Weather Bureau (Fig. 2) and superimposed on this map. Studies of the relationship between maximum possible rainfall and the major storms of record indicated that, in general, 50% of maximum possible rainfall includes all but approximately 15% of the major storms of record. Accordingly, 50% of the maximum possible rainfall in the vicinity under consideration was adopted for the standard project storm.

---

[4] *Civil Works Engineer Bulletin No. 52-8*, Corps of Engrs., U. S. Dept. of the Army, Washington, D. C., March 26, 1952.

The standard project storm isohyetal pattern that is being used in the design of the Houston project is shown in Fig. 3. The rainfall values shown on this pattern were derived by multiplying the index rainfall for the locality shown on Fig. 4 by the percentages shown on the pattern.

After fitting the standard project storm over the section of the watershed being studied to give the maximum peak discharges, the total 24-hr storm volume is determined in the usual way by planimetering the isohyetal patterns and determining the average depth of storm rainfall. This depth is then divided into four 6-hr periods in accordance with percentages determined in the over-all study. The Buffalo Bayou standard project storm has an average depth of 19 in. over 200 sq miles in 24 hr. For this depth the time distribution of rainfall for the four 6-hr periods is as follows: First period, 9.5%; second period, 20.1%; third period, 56.7%; and fourth period, 13.7%. Infiltration losses of 1.0 in. for the first period and 0.6 in. for each of the three remaining periods were subtracted to obtain the rainfall excess. This excess was then developed into the standard project flood by use of the Snyder Unit Hydrograph Method. The coefficients used were determined from the reproduction of actual floods on Buffalo Bayou and its tributaries.

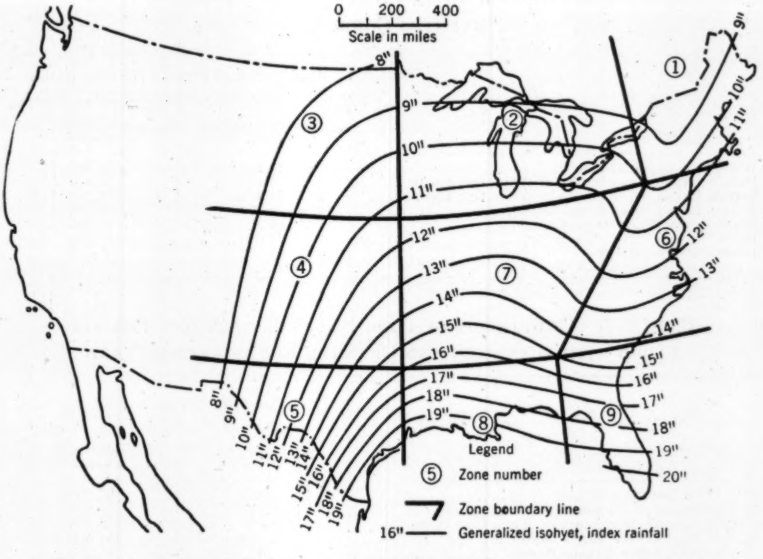


FIG. 4.—GENERALIZED ESTIMATES OF THE AVERAGE PRECIPITATION, IN INCHES, OVER AN AREA OF 200 SQ MILES IN 24 HR

Under existing and improved conditions with Addicks Reservoir and Barker Reservoir in operation, the peak flows that would be produced by the standard project storm fitted over the principal parts of the Buffalo Bayou watershed, are shown in Tables 6 and 7. For comparison the peak flows that would be produced under present conditions by the 1935 storm increased 50% are also shown.

CONSTRUCTION FEATURES OF THE RECTIFICATION PLAN

As previously stated, the rectification plan provides for the control of the standard project flood to nondamaging stages by detention storage in Barker Reservoir and Addicks Reservoir and by enlarging and rectifying the stream channels.

Barker Dam and Addicks Dam and Existing Rectified Channel.—These features are considered to be a part of the rectification plan. It is proposed to operate each reservoir with the three gated conduits remaining closed and with all discharge through the two ungated conduits during passage of floods through the reservoirs. The combined flood-peak discharge from the two reservoirs would be approximately 6,700 cu ft per sec for the standard project flood. The existing rectified channel has sufficient capacity to carry the standard project flood and would not be modified.

TABLE 6.—PEAK FLOWS PRODUCED BY STANDARD PROJECT STORM AND BY 1935 STORM INCREASED 50%, UNDER EXISTING CONDITIONS

Location	Peak Flows, in Cubic Feet Per Second	
	Standard project storm	1935 storm increased 50%
White Oak Bayou at mouth	25,200	24,700
Buffalo Bayou above mouth of White Oak Bayou	26,400	22,500
Buffalo Bayou below mouth of White Oak Bayou	49,200	46,200
Buffalo Bayou below Brays Bayou	74,000	70,700
Brays Bayou below Kegans Bayou	16,200	17,800
Brays Bayou at Old Main Street	25,000	24,600
Brays Bayou at mouth	31,200	30,700

TABLE 7.—COMPARISON OF PEAK FLOWS OF STANDARD PROJECT FLOOD WITH DIVERSION AND CHANNEL RECTIFICATION PLANS

Location	Peak Flows, in Cubic Feet Per Second	
	Diversion plan	Channel rectification plan[a]
White Oak Bayou at mouth	12,400	27,700
Buffalo Bayou above mouth of White Oak Bayou	27,600	31,300
Buffalo Bayou below mouth of White Oak Bayou	34,300	55,400
Buffalo Bayou below Brays Bayou	52,700	87,400
Brays Bayou below Kegans Bayou	0	16,200
Brays Bayou at Old Main Street	15,800	28,000
Brays Bayou at mouth	23,000	35,700

[a] Peak flows are for improved channel conditions.

Enlargement and Rectification of Buffalo Bayou.—It is proposed to enlarge and rectify Buffalo Bayou from the turning basin of the Houston ship channel, mile 16.0, to the lower end of the existing channel rectification, mile 43.6.

The proposed rectified channel would be approximately 20 miles long, trapezoidal in cross section, and with sodded banks with slopes of 2½ to 1, except in reaches where adjacent development is such that channel paving or the use of flood walls is more feasible to obtain the necessary capacity. This plan includes a desilting basin just above the turning basin to reduce the amount of silt entering the turning basin and the Houston ship channel. The proposed basin would have a bottom width of 300 ft at El. −25 ft, mean sea level, and a length of 686 ft. In addition it is proposed to construct erosion-control works in the channel above the turning basin.

Rectification of Brays Bayou.—It is proposed to enlarge and rectify Brays Bayou from its mouth to Westheimer Road, mile 25.5. The proposed channel would be 25.5 miles long and trapezoidal in cross section. Concrete channel lining will be used on the lower parts of the slopes and on the bottom of the channel from tide water, mile 4.4, to Kegans Bayou, mile 19.4 The side slopes in the tidal section and from Kegans Bayou to the upper end of the improvement, mile 25.5, will be sodded with slopes of 2½ to 1. Fig. 5 shows a typical section of the partly lined channel.

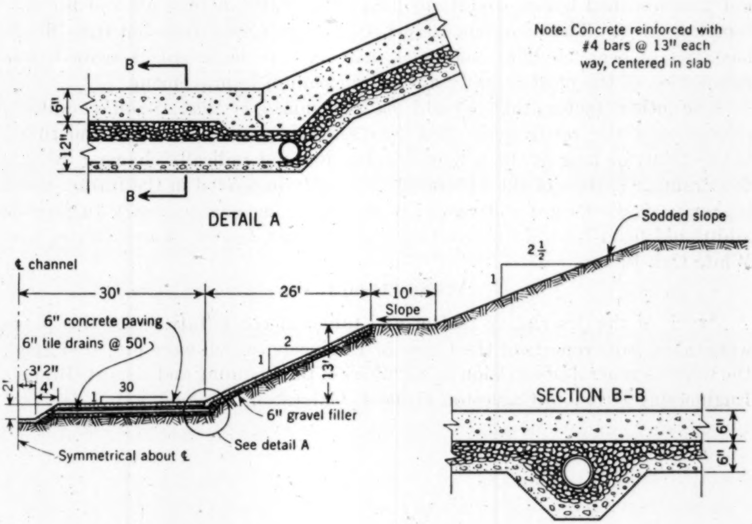

Note: Concrete reinforced with #4 bars @ 13" each way, centered in slab
B
6"
12"
B
DETAIL A
Sodded slope
2½
1
℄ channel
30'
26'
10'
Slope
6" concrete paving
6" tile drains @ 50'
3'2"
4'
2'
1
30
2
1
13'
6" gravel filler
See detail A
Symmetrical about ℄
SECTION B-B
6"
6"

FIG. 5.—CHANNEL IMPROVEMENT AT BRAYS BAYOU

Rectification of White Oak Bayou.—It is proposed to rectify White Oak Bayou from its mouth to the Burlington-Rock Island Railroad bridge, mile 13.5. The proposed channel would be about 8.9 miles long and trapezoidal in cross section. The plan provides for concrete channel lining except that the upper parts of the slopes would be sodded. Lining is considered necessary to prevent excessive scouring of the channel bed and slopes.

Changes in Utilities.—The proposed channel enlargement and rectification plan will necessitate changes in five railroad bridges, nine highway bridges, and in some utilities adjacent to or across Buffalo Bayou; in eight railroad bridges, nineteen highway bridges, and in some utilities adjacent to or across Brays Bayou; and in a highway bridge across White Oak Bayou.

Miscellaneous.—The estimates of cost for the proposed plans of improvement include allowances to provide for any erosion-prevention works that may prove necessary in those channels in which no protection was provided.

CONCLUSIONS

The development of a comprehensive flood-control plan for a rapidly growing city such as Houston becomes complex if the plan of improvement is not accomplished before conditions change radically in the city and the surrounding area. The comprehensive plan for protecting Houston from floods has already been modified once and may have to be modified again before completion of the most recently authorized plan of improvement.

One other factor that should be recognized is that runoff conditions change, and the relationship that was assumed between rainfall and runoff is valid only as long as the watershed area does not radically change. Should the drainage system of the watershed be greatly improved in the future, much higher peak discharges will result; it may then become necessary to provide additional flood-control improvements for Buffalo Bayou, Brays Bayou, and White Oak Bayou.

ACKNOWLEDGMENTS

Much of the descriptive matter and tables of computations for this paper were taken from reports of the Corps of Engineers, which were prepared under the writer's general supervision by members of the Planning and Reports Branch, Engineering Division, Galveston District, Galveston, Tex.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2918

SEDIMENT-TRANSPORT MECHANICS IN STABLE-CHANNEL DESIGN

By Emmett M. Laursen,[1] A.M. ASCE

With Discussion By Messrs. Sam Shulits, and Emmett M. Laursen

Synopsis

The three requisites for a stable alluvial channel are explicitly stated, and the role of sediment transport in each is assessed. The paper demonstrates the similarity of the many sediment-transport formulas, and the general method of their application to design is illustrated. The use of these formulas as scaling relationships between different channels is advocated.

There is not enough information on the mechanics of sediment transportation for this theory alone to be used with confidence in the design of an alluvial channel. However, there is sufficient information available on transport theory to aid the designer in using engineering judgment. In part, this aid consists of dispelling confusion by demonstrating that design methods and criteria that are seemingly different are instead basically equivalent.

By definition, a stable, sediment-bearing canal in erodible material must not only convey the required water discharge, but must also transport the sediment load supplied and must not scour or silt either the bed or the banks. These three factors delineate the designer's problem—the solution of which is usually single valued. However, there are exceptions because there may be a narrow but finite range of flow conditions above which the banks will not be affected by silting and below which erosion will not take place. The designer's problem is the selection of the "correct" combination of slope, size, and shape of channel, which will result in a stable canal for a particular combination of water discharge and sediment load (or range thereof) and the particular materials

---

Note.—Published, essentially as printed here, in August, 1956, in the Journal of the Hydraulics Division, as *Proceedings Paper 1034*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Research Engr., Iowa Inst. of Hydr. Research, Iowa City, Iowa.

through which the canal must pass. The selection must be based on personal experience guided by such relationships as have been deduced from general experience with existing channels and with laboratory flumes. The difficulty in selecting the correct combination stems from an always limited personal experience and from a lack of confidence in the guiding relationships.

In order to design a stable channel, three relationships are needed, one for each of the factors in the previous definition—a flow equation, a sediment-transport equation, and a bank-erosion criterion. Although the flow conditions in the stable canal must satisfy all three relationships simultaneously, for clarity they will be considered separately in the following.

THE FLOW EQUATION

Of the three relationships needed for the design of a stable alluvial channel, the flow equation is the best understood. Basically this relationship is the Chezy equation,

$$Q = C\,A\sqrt{R\,S} \qquad (1)$$

In the use of this equation the designer must assign, arbitrarily, a numerical value to the discharge coefficient, C. In order to aid his judgment, he will usually prefer to use some empirical formula such as that of Manning:

$$Q = \frac{1.49}{n}\,A\,R^{2/3}\,S^{1/2} \qquad (2)$$

This form of the flow equation was widely accepted because it was generally believed that the coefficient, n, was primarily, if not entirely, a function of the boundary material. Although it is realized that n can be affected by several factors, a wealth of information has been accumulated over the years on the n-values for existing channels covering a wide range of size, shape, alinement, bank and bed material, and condition. Because of this store of information, n-values for channels similar to that being designed can usually be found. The importance of this fact is self evident, but it is noteworthy because the same sort of helpful information is needed for the other two relationships.

Sediment transportation will affect the flow, and, hence, it will be reflected in the flow equation in so far as it affects the discharge coefficient. Dunes are almost always associated with the transport of sediment; the resistance coefficient, which is the inverse of the discharge coefficient, is affected by the resulting increase in boundary roughness. That sediment transport may influence the resistance coefficient in other ways has been claimed[2] but has not been proved unequivocally. However, no matter what the causal reaction the rate of sediment transport is one of the several criteria that should be considered in appraising the similarity of channels.

Two attempts to relate the resistance coefficient to sediment transport should be mentioned. Hans A. Einstein, M. ASCE, and Nicholas L. Bar-

---

[2] "Some Effects of Suspended Sediment on Flow Characteristics," by V. A. Vanoni, *Bulletin No. 34*, Proceedings of the 5th Hydraulics Conference, Univ. of Iowa Studies in Eng., Iowa City, Iowa, 1953, pp. 137–158.

barossa,[3] A.M. ASCE, divided the bed resistance into two parts, one due to the sand roughness and the other to bar or dune roughness. To evaluate the former they used Strickler's formula,

$$n = \frac{\sqrt[6]{K_s}}{29.3} \qquad (3)$$

in which K_s is the diameter (measured in feet) of the particle size coarser than 65% of the sediment. After eliminating bank effects in narrow channels, the effect of the sand roughness was subtracted from the total resistance, and this remainder was related to a modification of Einstein's ψ-function for the transportation of bed load.

James R. Barton and Pin Nam Lin[4] have proposed a relationship between the Chezy coefficient, the Reynolds number of the flow, and the ratio between the sediment size and the hydraulic radius. Although sediment transport does not enter the relationship explicitly, the data were obtained from the flows transporting sediment.

Both of these proposals are so new that there has not been sufficient time to evaluate their usefulness or their weaknesses. However, they both bypass the problem of predicting dune size, which is probably the major effect of sediment transport on resistance to flow. Alvin G. Anderson,[5] A. M. ASCE, obtained a solution for the length of dune as a function of the Froude number and presented a limited quantity of data corroborating his mathematical solution. His solution had a limit of a plane bed in which the dune length became infinite. In addition, he presented one example of a plane bed at a Froude number of 1.6. However, a plane bed (that is, one without dunes) has been obtained at a Froude number as low as 0.85 from measurements taken at the Iowa Institute of Hydraulic Research at Iowa City. The sand in this experiment was 0.1 mm in diameter—finer than the sand of Anderson's example. It is possible for dunes to disappear because the dune height reduces to zero as well as because the dune length increases to infinity. The height of the dune, which was not, and cannot be, a part of Anderson's analysis, is fully as important as the length. Very likely it is more a function of the particle size and the Reynolds number of the flow than of the Froude number.

The empirical equations describing stable channels that have been developed in India[6] include relationships that are, in essence, evaluations of the Chezy *C*, just as the Manning equation is. These flow relationships naturally are subject to the same criticism that is often made of other Indian criteria—that is, because they are based on data from Indian canals, they are tailored to fit conditions of that area. This criticism, although valid, does not mean that due consideration should not be given to modifying the relationships to fit the conditions in other areas of the world.

---

[3] "River Channel Roughness," by Hans A. Einstein and Nicholas L. Barbarossa, *Transactions*, ASCE, Vol. 117, 1952, pp. 1121–1146.

[4] "A Study of Sediment Transport in Alluvial Channels," by J. R. Barton and P. N. Lin, *Research Report*, Civ. Eng. Dept., Colorado Agri. and Mech. College, Fort Collins, Colo., 1955.

[5] "The Characteristics of Sediment Waves Formed by Flow in Open Channel," by A. G. Anderson, *Proceedings*, 3d Midwestern Conference on Fluid Mechanics, Univ. of Minnesota, Minneapolis, Minn., 1953, pp. 379–395.

[6] "Historical Note on Empirical Equations, Developed by Engineers in India for Flow of Water and Sand in Alluvial Channels," by C. Inglis, *Proceedings*, 2d Meeting of the International Assn. for Hydr. Research, Stockholm, 1948.

Regardless of the flow equations and the numerical value of a resistance coefficient that the designer chooses, he will have placed a restriction on the physical character of the channel eventually to be selected. Before the final decision, two more restrictions involving the same type of evaluation must be stated explicitly.

THE SEDIMENT–TRANSPORT EQUATION

If the flow entering the canal is not clear but carries with it a sediment load, the capacity of the channel must equal the rate at which sediment is supplied to the channel for regime conditions to prevail. This requirement places a second restriction on the physical characteristics of the channel to be selected. Some of the many sediment-transport equations for bed load that have been proposed are listed in Table 1. This table is not comprehensive, nor is it intended to rate or judge the many expressions that can be found in the literature. In fact, the problem of picking a sediment-transport equation must be faced by each individual designer. All the equations are slightly different in form; all are based on limited data, most of which is obtained from the laboratory; each fits some data better than the others; none fits all the data (which is why there are so many); and none has gained the wide acceptance shared by the relatively few flow equations. If the designer is unable to decide with con-

TABLE 1.—SIMILARITY OF BED–LOAD FORMULAS

P. DuBoys (Lorenz G. Straub)[a]	$q_s = A_1 \tau (\tau - \tau_c)$	$= B_1 n^4 \left(\frac{V^4}{y^{2/3}}\right)$
Armin Schoklitsch[b]	$q_s = \frac{A_2}{D^{1/2}} S^{3/2} (q - q_c)$	$= B_2 \frac{n^3}{D^{1/2}} \left(\frac{V^4}{y}\right)$
Eugene Meyer-Peter[c]	$q_s = (A_3 q^{2/3} S - A_4 D)^{3/2}$	$= B_3 n^3 \left(\frac{V^4}{y}\right)$
Waterways Experiment Station[d]	$q_s = \frac{A_5}{n} (\tau - \tau_c)^m$	$= B_5 n^{2m-1} \left(\frac{V^{2m}}{y^{m/3}}\right)$
Arthur C. Shields[e]	$q_s = \frac{A_6}{D} q S (\tau - \tau_c)$	$= B_6 \frac{n^4}{D} \left(\frac{V^5}{y^{2/3}}\right)$
Brown-Einstein[f]	$q_s = \frac{A_7}{D^{3/2}} \tau^3$	$= B_7 \frac{n^6}{D^{3/2}} \left(\frac{V^6}{y}\right)$
Brown-Kalinske[f]	$q_s = \frac{A_8}{D} \tau^{5/2}$	$= B_8 \frac{n^5}{D} \left(\frac{V^5}{y^{5/6}}\right)$

[a] *House Document 238*, 73rd Cong., 2d Session, 1935, p. 1135.

[b] "The Schoklitsch Bed Load Formula," by S. Shulits, *Engineering*, London, Vol. 139, June, 1935, pp. 644–646 and p. 687.

[c] "Neure Versuchresultate über den Geschiebetrieb," by E. Meyer-Peter, H. Favre, and H. A. Einstein, *Schweizerische Bauzeitung*, Vol. 103, 1934.

[d] "Studies of River Bed Materials and Their Movement with Special Reference to the Lower Mississippi River," *Bulletin No. 17*, Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss., January, 1935.

[e] "Anwendung der Aechlichkeitsmechanik und der Turbulenzforschung auf die Geschiebebewegung," by A. Shields, *Mitteilungen*, Preussische Versuchsanst für Wasserbau und Schiffbau, Berlin, 1936.

[f] "Sediment Transportation," by C. B. Brown, Chapter XII of *Engineering Hydraulics*, John Wiley & Sons, Inc., New York, N. Y., 1950, pp. 796–799.

fidence on one equation, he may use more than one, or even all, as the hydrologist does with the many available flood formulas.

In Table 1, it should be noted that the equations suggested by Einstein[7] and Anton A. Kalinske,[8] M. ASCE, have not been used. Instead use was made of modifications.[9] In these the original parameters are utilized, but the forms of the equations are changed to fit the data for the high rates of sediment transport. That these modified equations are not strikingly different from the other empirical sediment-transport equations should not be surprising, because in their development the assumption is made that the velocity at the edge of the laminar sublayer, v_δ (whether or not actually existent), can be substituted for the velocity at the level of the moving grain of sediment, v_g. Because v_δ is, as usual, equated to $11.6\sqrt{\tau/\rho}$ and bears little relation to v_g, except that both are small and both are at levels near the bed, this is equivalent to assuming that q_s is a function of τ.

In order to compare the different transport equations more easily, they have been put into a form indicating the dependence of q_s on the mean velocity, V, and the depth of flow, y, by neglecting the critical shear, τ_c, and using the Manning equation to eliminate the slope. The similarity between not only these but all the commonly used sediment-transport equations can thus be made apparent. One might also suspect, along with G. K. Gilbert,[10] that the exponent as well as the coefficients in the transport equation should be variable.

If a sizable bed load is to be expected, the simplified forms can be used to estimate the transport characteristics of the canal. Because it is difficult to measure the bed load or to predict the bed-material load entering the headworks of the proposed canal, a further modification of the forms may be helpful. For example, if the first equation in Table 1 is written for the sediment-discharge ratio, and then the ratios of the various parameters are taken for the proposed channel and an existing "similar" channel, relative, rather than absolute, values can be estimated:

$$\frac{\dfrac{q_s}{q}\ \text{(proposed)}}{\dfrac{q_s}{q}\ \text{(existing)}} = \left[\frac{B_1\ (p)}{B_1\ (e)}\right]\left[\frac{n^4\ (p)}{n^4\ (e)}\right]\left[\frac{V^3\ (p)}{V^3\ (e)}\right]\left[\frac{y^{-5/3}\ (p)}{y^{-5/3}\ (e)}\right] \ldots\ldots (4)$$

It is interesting to note that Chezy proposed his flow equation in this ratio form,[11] and that the accumulation of n-values, together with descriptions of the channels, is essentially only a somewhat more sophisticated manner of handling the problem.

A further rearrangement of the simplified transport equations will result in the form of the Kennedy equation and the variations thereof, as shown in

---

[7] "Formulas for the Transportation of Bed Load," by H. A. Einstein, *Transactions*, ASCE, Vol. 107, 1942, pp. 561–577.

[8] "Movement of Sediment as Bed Load in Rivers," by A. A. Kalinske, *Transactions*, Am. Geophysical Union, Vol. 28, No. 4, 1947, pp. 615–620.

[9] "Sediment Transportation," by C. B. Brown, Chapter XII of *Engineering Hydraulics*, John Wiley & Sons, Inc., New York, N. Y., 1950, pp. 796–799.

[10] "The Transportation of Debris by Running Water," by G. K. Gilbert, *Professional Paper No. 86*, Geological Survey, U. S. Dept. of the Interior, Washington, D. C., 1914.

[11] "A History of Hydraulics to the End of the 18th Century," by S. Ince, thesis presented to the State University of Iowa, at Iowa City, in 1952, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Table 2. The similarity between the modified transport equations and the Kennedy type of equations indicates that the physical basis for the latter is the requirement for sediment-transport capacity. That only Sir Claude C. Inglis, M. ASCE, has considered this factor explicitly is probably due to the fact that the equations were derived from data of canal systems in which the sediment-discharge ratio did not vary greatly.

On the basis of personal judgment, aided to some extent by the sediment-transport equations of one form or another, the requirement for sediment capacity must be satisfied just as the requirement for flow capacity was satisfied, and a second restriction on the combination of size, shape, and slope of channel is imposed.

TABLE 2.—SIMILARITY OF BED-LOAD AND KENNEDY FORMULAS

P. DuBoys (Lorenz G. Straub)	$\frac{V}{y^{5/9}} = C_1 \frac{1}{n^{4/3}} \left(\frac{q_s}{q}\right)^{1/3}$
Armin Schoklitsch	$\frac{V}{y^{2/3}} = C_2 \frac{D^{1/6}}{n} \left(\frac{q_s}{q}\right)^{1/3}$
Eugene Meyer-Peter	$\frac{V}{y^{2/3}} = C_3 \frac{1}{n} \left(\frac{q_s}{q}\right)^{1/3}$
Waterways Experiment Station[a]	$\frac{V}{y^{m'}} = C_5 \frac{1}{n} \left(\frac{q_s}{q}\right)^{1/(2m-1)}$
Arthur C. Shields	$\frac{V}{y^{5/12}} = C_6 \frac{D^{1/4}}{n} \left(\frac{q_s}{q}\right)^{1/4}$
Brown-Einstein	$\frac{V}{y^{2/5}} = C_7 \frac{D^{3/10}}{n^{6/5}} \left(\frac{q_s}{q}\right)^{1/5}$
Brown-Kalinske	$\frac{V}{y^{11/24}} = C_8 \frac{D^{1/4}}{n^{5/4}} \left(\frac{q_s}{q}\right)^{1/4}$
R. G. Kennedy[b]	$\frac{V}{y^{0.64}} = 0.84$
Gerald Lacey[c]	$\frac{V}{y^{1/2}} = 1.15 \sqrt{f}$
Thomas Blench[d]	$\frac{V}{y^{1/2}} = \sqrt{b}$
Claude C. Inglis[e]	$\frac{V}{y^{1/2}} = C_9 \frac{g^{1/3} w^{1/4}}{\nu^{1/6}} \left(\frac{q_s}{q}\right)^{1/4}$

[a] $m' = \frac{(m + 3)}{(6\,m - 3)}$.

[b] "The Prevention of Silting in Irrigation Canals," by R. G. Kennedy, *Minutes of Proceedings*, Inst. of Ceramic Engrs., Vol. 119, 1895, pp. 281–290.

[c] "A General Theory of Flow in Alluvium," by G. Lacey, *Journal*, Inst. of Ceramic Engrs., Vol. 27, November, 1946, pp. 16–47.

[d] "Regime Theory for Self-Formed Sediment-Bearing Channels," by Thomas Blench, *Transactions*, ASCE, Vol. 117, 1952, pp. 383–408.

[e] *Annual Reports (Technical)*, Central Irrig. and Hydrodynamic Research Station, Poona, India, 1940–1941, p. 50; 1941–1942, p. 33.

THE BANK-EROSION CRITERION

If the banks of the canal were vertical and nonerodible, the final selection of channel characteristics satisfying only the first two requirements could be made either arbitrarily or on the basis of economics. However, an alluvial channel will have sloping banks of erodible material. Flow conditions along the banks, therefore, must be such that sediment will not deposit on the banks and that the bank material will not be scoured. The first of these requirements will govern for canals designed for a minimum slope; the second, for canals designed for a minimum area. Satisfactory solution of the nonsilting problem requires an understanding of sediment-transport mechanics in much greater detail than is presently available, particularly with respect to the suspended-sediment load. Knowledge of the nonscouring problem is somewhat more advanced, but the bank-erosion criterion is still the least satisfactory of the three relationships needed for design.

In the case of banks composed of granular material such as sands and gravels, erosion will occur if the forces tending to move the particles are sufficiently large to overcome the resisting force of gravity. In the case of banks composed of clays or other cohesive materials, the force of cohesion will also resist movement and in fact may be much greater than the resisting force of gravity.

In associating the flow conditions in the canal to the bank erodibility, the canal shape enters the problem on both sides of the relationship. The slope of the banks will obviously affect the susceptibility of the banks to erosion, and the cross-sectional shape will affect the distribution of the boundary shear.

Emory W. Lane and Enos J. Carlson,[12] Members, ASCE, have presented the relationship for the ratio of the critical tractive force on a bank slope, τ_{cs}, to the critical tractive force, τ_{cb}, as a function of the angle of the side slope and the angle of repose of the material. That is,

$$K = \frac{\tau_{cs}}{\tau_{cb}} = \cos\phi \sqrt{1 - \frac{\tan^2\phi}{\tan^2\theta}} \qquad (5)$$

in which ϕ is the angle of the side slope and θ is the angle of repose. Other studies under the direction of Lane[13] have attempted to obtain the distribution of shear in a trapezoidal section. This knowledge of the effect of cross-sectional shape allows data or equations on the critical tractive force for bed material to be transferred to the banks.

The two formulas for critical tractive force most widely quoted are those of C. M. White[14]:

$$\tau_{cb} = 12\,D \qquad (6)$$

and of the late Arthur C. Shields,[15] M. ASCE:

$$\tau_{cb} = (S_s - 1)\,D\,f\left(11.6\,\frac{D}{\delta'}\right) \qquad (7)$$

Any of the relationships for τ_c of the equations in Table 1 can also be used at the discretion of the designer. However, these relationships may result in a low rate of movement rather than zero movement. A criterion based on zero movement is perhaps unduly restrictive because a small degree of scour can usually be tolerated and because suspended sediment will tend to add cohesive material to the banks. Application of the relationship from the first equation in Table 1 will result in values of permissible canal velocities that are approximately the same as those given by Samuel C. Fortier, A.M. ASCE, and Fred C. Scobey,[16] M. ASCE; application of the relationships of White and Shields will result in smaller permissible velocities.

---

[12] "Some Factors Affecting the Stability of Canals Constructed in Coarse Granular Materials," by E. W. Lane and E. J. Carlson, *Proceedings*, Minnesota International Hydraulics Convention, Minneapolis, Minn., 1953, pp. 37–48.

[13] "Design of Stable Channels," by Emory W. Lane, *Transactions*, ASCE, Vol. 120, 1955.

[14] "Equilibrium of Grains on Bed of Stream," by C. M. White, *Proceedings*, Royal Soc. of London, Series A, Vol. 174, 1940, pp. 322–334.

[15] "Anwendung der Aechlichkeitsmechanik und der Turbulenzforschung auf die Geschiebebewegung," by A. Shields, *Mitteilungen*, Preussische Versuchsanstalt für Wasserbau und Schiffbau, Berlin, 1936.

Little is known concerning the resistance to movement of clays except that some clays can withstand very high boundary shear. One might speculate that the resistance to boundary shear should be related to the internal shearing characteristics of the clay mass in an undisturbed saturated condition. For naturally deposited material, which is not likely to be completely homogeneous, the weakest part of the mass would govern. Erosion of seams or pockets of weak material would expose the adjacent stronger material to greater forces. The only published work concerning the resistance of cohesive materials is contained in the paper by Fortier and Scobey, and even in this paper there is insufficient information about the character of the material to be considered as more than a rough guide for the designer.

The third relationship, together with the first and second, fixes the width, depth, and slope of the channel. No method is available to determine the best geometrical shape of the cross section except examination of other channels. In fact, canals are usually dug to a shape easily handled by construction machinery, and then the flow is allowed to modify the shape by a moderate amount of scouring and filling.

CONCLUSIONS

Three relationships are needed to specify a stable alluvial channel. They may be those directly stating the three requirements for the stable canal as presented herein, or they may be any three independent forms derived therefrom. The three independent regime-theory criteria implicitly contain these same factors.

The difficulties of designing a stable canal lie not in using the general relationships, but in assigning numerical values to coefficients and in choosing a formal equation. The essence of design procedure is shown in Fig. 1. Choice of the equation form and numerical values to describe the sediment-transport capacity will fix a relationship between the mean velocity of flow and the depth of flow, as indicated by the heavy curve in the schematized diagram. The choice of the equation form and numerical values to describe the flow capacity will result in a family of curves relating velocity and depth for various canal slopes, as indicated by the dashed curves. The dot-dash curve of constant tractive force describing the bank competence where it intersects the transport-capacity curve will specify one of the family of flow-capacity curves. The velocity, depth, and slope are thereby fixed, and the width can be obtained from the continuity principle.

---

[16] "Permissible Canal Velocities," by S. Fortier and F. C. Scobey, *Transactions*, ASCE, Vol. 89, 1926, pp. 940–984

The interrelationship previously stated constitutes the general application of sediment-transport mechanics to stable-canal design. Only a complete, comprehensive, rigorous theory could substitute for a full-scale model canal. It is by observation of similar canals that the designer can assign the needed numerical values; sediment-transport mechanics can aid in assessing the degree of similarity of different canals. The parameters of the various equations can be used as comparative criteria and the equations themselves as approximate scaling relationships. If the proposed and existing conditions are treated as ratios, the discrepancies between absolute values as computed by the equations should be greatly reduced.

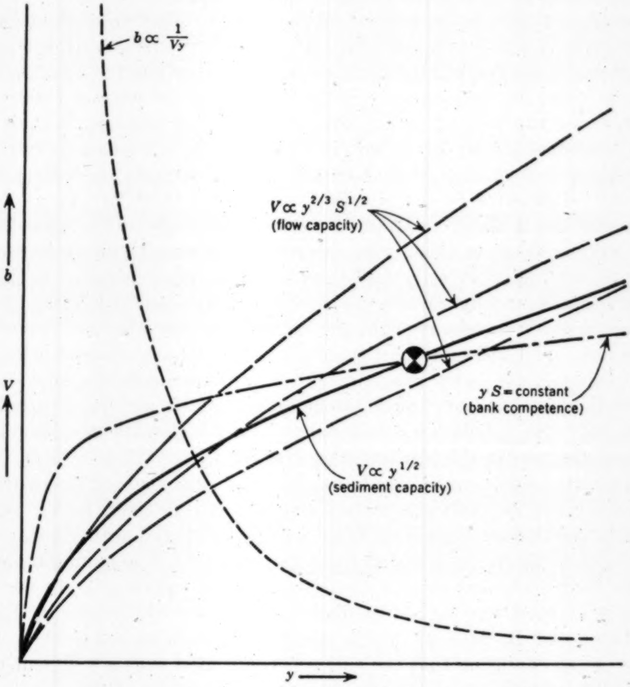


FIG. 1

Few, if any, canals are completely stable. In planning remedial and maintenance work, the principles of sediment-transport mechanics can be used in the same manner as in stable-canal design, the principal difference being that a canal almost completely similar is then readily available for observation.

DISCUSSION

Sam Shulits,[17] M. ASCE.—In a complete and an illuminating manner, Mr. Laursen has summarized just how a hydraulic engineer should proceed to design a stable channel with the available information. At the same time his cogent comparison of available methods and formulas shows clearly the difficulties and uncertainties in dealing with stable-channel design.

From around 1930 to the present, research in sediment transportation has been increasingly active and inspired. This has been particularly true in the United States in the last ten to fifteen years. It is startling to be reminded by the author, first, that substitution of the velocity at the edge of the laminar sublayer, v_δ, for the velocity at the level of the moving grain, v_g, is equivalent to the deduction made by P. DuBoys in 1879—that q_s is a function of τ; and second, that transformation of the DuBoys bed-load formula yields the permissible canal velocities of Fortier and Scobey obtained in 1926. To many hydraulic engineers, it may be a strained procedure to apply at the open-channel bed the laminar sublayer velocity equation based on smooth pipe experiments, whether or not the sublayer actually exists. The only conclusion is that research in open-channel hydraulics and in sediment transport still has not found, or perhaps has not sought enough, the appropriate fundamental concepts of fluid mechanics. Until more particularized facts for open channels are available, one can only use, in the analytical process of fluid mechanics, the information that is available from the fields of aerodynamics and pipe turbulence. Unfortunately, the results are occasionally the old tools with new handles; at least this is the case in the two instances just cited.

Comparative studies of bed-load formulas have been made at the hydraulics laboratory of The Pennsylvania State University at University Park. One result of this work[18] is presented in Table 3 to illustrate the author's statement that the difficulties in stable-channel design lie in "* * * assigning numerical values to coefficients and even in choosing a formal equation." In addition, there exist at least two more formulas—the formula[19] developed by Eugene Meyer-Peter and R. Mueller, and a modified Einstein procedure.[20,21] These formulas are so confusing that the engineer confronted with a sediment-transport problem usually resorts to the choice of one or two of the most widely quoted.

In Table 3, X and Y are coefficients varying from formula to formula and sometimes within one formula; T is the tractive force and T_0, the inceptive or critical tractive force; n, the Manning or Ganguillet and Kutter roughness coefficient; V, the mean velocity and V_0, the critical mean velocity; m, an exponent varying from formula to formula; q, the discharge and q_0, the critical discharge, both per unit width; S, the slope; d, the grain diameter; R, the hydraulic radius; γ_B and γ_F, the densities of bed material and fluid (water), respectively; and f, the mathematical symbol for "a function of."

---

[17] Prof., Dept. of Civ. Eng., The Pennsylvania State Univ., University Park, Pa.

[18] "The Dilemma of Bed-Load Formulas," by Sam Shulits, C. D. Sims, Jr., and D. J. Stull, paper presented at 36th Annual Meeting, Am. Geophysical Union, Washington, D. C., May, 1956.

[19] "Formulas for Bed-Load Transport," by E. Meyer-Peter and R. Mueller, Report on the 2d Meeting of the International Assn. for Hydr. Structures Research, Stockholm, 1948, pp. 39–64.

[20] "Computations of Total Sediment Discharge, Niobrara River near Cody, Nebraska," by B. R. Colby and C. H. Hembree, *Water Supply Paper No. 1357*, Geological Survey, U. S. Dept. of the Interior, Washington, D. C., 1955.

[21] "Application of the Modified Einstein Procedure for Computation of Total Sediment Load," by K. B. Schroeder and C. H. Hembree, *Transactions*, Am. Geophysical Union, Vol. 37, No. 2, April, 1956, pp. 197–212.

Although Table 3 incorporates an attempt to demonstrate similarities, these resemblances in no way imply even a similar range of quantitative results. Some of the formulas contain unsuspected sensitivities that render ineffective any simplifying assumptions. For example, a glance at the Schoklitsch expression for the inceptive tractive force for sand,[22]

TABLE 3.—BED LOAD PER UNIT WIDTH (G_1)

P. DuBoys (Lorenz G. Straub Fritz Schaffernak)	$X\,T\,(T - T_0)$
Y. L. Chang	$\frac{X\,n}{10^2}\,T\,(T - T_0)$
Waterways Experiment Station	$\frac{X}{n}\,(T - T_0)^m$
Shiou-Dean Chyn	$X\,V^m\,(T - T_0)$
Arthur C. Shields	$X\,\frac{q\,S}{d}\,(T - T_0)$
Armin Schoklitsch (old)	$\frac{X}{d^{0.5}}\,S^{1.5}\,(q - q_0)$
Armin Schoklitsch (new)	$X\,S^{1.5}\,(q - q_0)$
C. H. MacDougall	$X\,S^m\,(q - q_0)$
Hugh J. Casey	$X\,S^m\,(q - q_0)$
Gilbert K. Gilbert	$X\,(q - q_0)$
Eduard Fabre	$X\,S^{1.5}\,q - Y\,d$
Eugene Meyer-Peter	$\begin{cases} X\,(S\,q^{0.67} - Y\,d)^{1.5} \\ X\,S^{1.5}\,(q^{0.67} - q_0^{0.67})^{1.5} \end{cases}$
Oliver G. Haywood	$X\,d^{1.5m}\left(\frac{S\,q^{0.67}}{d^{0.33}} - Y\right)^{1.5}$
H. Nakayama	$(X + Y\,S)\,q\,(V^2 - V_0^2)$
Morrough P. O'Brien	$X\left(\frac{V}{R^{0.33}}\right)^m$
Hans A. Einstein	$\frac{G_1}{\gamma_B\,g}\left(\frac{\gamma_F}{\gamma_B - \gamma_F}\right)^{0.5}\left(\frac{1}{g\,d^3}\right)^{0.5} = f\left(\frac{\gamma_B - \gamma_F}{\gamma_F}\right)\frac{d}{R'\,S}$
Anton A. Kalinske	$\frac{G_1}{d\,T^{0.5}} = X\,f\left(\frac{T_0}{T}\right)$

$$q_0 = 0.6\,\frac{d^{3/2}}{S^{7/6}} \qquad (8)$$

might lead to the conjecture that the 7/6th power of S is an exaggerated refinement—that is, the first power might be an adequate approximation. However, there is an appreciable difference between $S^{7/6}$ and S because at $S = 0.00001$, $S^{7/6} = 0.146\ S$, and at $S = 0.01$, $S^{7/6} = 0.464\ S$.

---

[22] "Handbuch des Wasserbaues," by A. Schoklitsch, Springer-Verlag, Vienna, 2d Ed., Vol. 1, 1950, p. 175.

Even for those bed-load formulas in which the basic data, usually from flume tests, cover the same range, the computed results may be from 1% to 200% apart, or more. The foregoing pertains particularly to the DuBoys-Straub formula, the Waterways Experiment Station formula, the Schoklitsch formula, the Casey formula, the Haywood formula, and the Meyer-Peter formula in the comparisons made at The Pennsylvania State University. Although the occurrence of such disagreements is well known, the array of formulas in Tables 1 and 2 and in Table 3 underlines the difficulties of the formula dilemma.

Emmett M. Laursen,[23] A. M. ASCE.—The original contentions set forth in the paper have been amplified and reinforced by Mr. Shulits. The "formula dilemma," as he has stated, is such that at the present time the design of stable channels cannot proceed with any substantial degree of confidence only on the basis of sediment-transport mechanics; experience is absolutely essential. However, sediment-transport mechanics can serve as a guide, and the many formulas can serve as scaling relationships and in that way supplement experience. When the formulas are used to compare different channels, the gross quantitative disagreements largely disappear and the similarities are emphasized.

---

[23] Research Engr., Iowa Inst. of Hydr. Research, Iowa City, Iowa.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2919

MORRO BAY STEAM ELECTRIC PLANT

BY J. GEORGE THON,[1] M. ASCE, AND
GORDON L. COLTRIN,[2] A.M. ASCE

WITH DISCUSSION BY MESSRS. GLENN B. WOODRUFF; JACOB FELD; WILLIAM W. MOORE; GEORGE L. JORDY; AND J. GEORGE THON AND GORDON L. COLTRIN

SYNOPSIS

The principal features of the Morro Bay Steam Electric Plant of California that are of particular interest to the civil engineer are (1) a condenser cooling-water system that utilizes a surge chamber; (2) a 24-in.-diameter, fuel-oil supply line extending 4,400 ft into the ocean; (3) sea-water evaporators that were the first to be used for the industrial production of fresh water in the United States; (4) a continuous-mat foundation under the main power building; (5) a 450-ft-high, reinforced concrete stack incorporating the latest concepts in seismic design; and (6) turbine generator foundations that were dynamically designed. Some of the problems encountered in designing these features are described.

INTRODUCTION

During the period from 1946 to 1954 the demand for electric power increased markedly in the San Joaquin Valley of California. The peak load produced primarily by agricultural pumping increased 150% as compared with a power system growth of 105% during the same period.

It soon became apparent that additional capacity would be required by 1955 to meet this growth in load. The economics of fuel costs and transmission losses coupled with the all-important requirements for cooling water were the main factors in choosing Morro Bay, California, as the location of this source of capacity.

LOCATION

Morro Bay is situated on the Pacific Ocean half way between San Francisco and Los Angeles, and 13 miles northwest of San Luis Obispo, all in California.

NOTE.—Published, essentially as printed here, in June, 1956, as *Proceedings Paper 737*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Chf. Civ. Engr., Power Div., Bechtel Corp., San Francisco, Calif.

[2] Superv. Civ. Engr., Pacific Gas and Electric Co., San Francisco, Calif.

The plant site is a 140-acre tract at the north end of the bay between the beach line and state highway 1 near famous Morro Rock. Railroad facilities were available, with a railhead at Camp San Luis Obispo on state highway 1 approximately 10 miles from the steam plant site.

General Design

The plant arrangement is based on an ultimate installation of eight complete units with two control centers each controlling four units. The initial installation consisted of two 156,250-kw reheat turbine generators and two 1,135,000-lb-per-hr boilers. The turbine-inlet conditions are a 1,800-lb-per-sq-in. pressure, 1,000° F temperature, and a reheat temperature of 1,000° F.

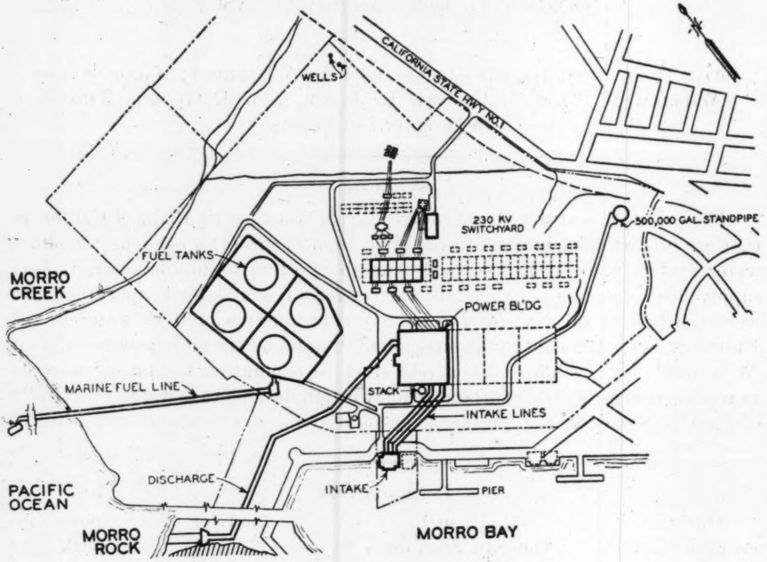

CALIFORNIA STATE HWY NO.1
WELLS
230 KV SWITCHYARD
500,000 GAL. STANDPIPE
FUEL TANKS
MORRO CREEK
POWER BL'DG
MARINE FUEL LINE
STACK
INTAKE LINES
DISCHARGE
INTAKE
PIER
PACIFIC OCEAN
MORRO ROCK
MORRO BAY

Fig. 1.—Site Plan for Morro Bay Steam Plant, California

The fuel is oil, with provision for future conversion to pulverized coal and petroleum coke if necessary. The first unit was placed into commercial operation in October, 1955, and the second unit in July, 1956. The total cost of the initial two units was $44,300,000.

The principal engineering features of the plant are outlined in Fig. 1. Power is generated at 18,000 volts and is increased to 230,000 volts with three single-phase transformers for each unit. Two 230,000-volt circuits will transmit most of the power to the Gates substation in the San Joaquin Valley near Coalinga, approximately 65 miles northeast of Morro Bay. Power will also be transmitted at 110,000 volts over one circuit to the San Luis Obispo substation for local distribution.

MAIN CONDENSER COOLING-WATER SYSTEM

A major problem in the design of a plant of this size is that of pumping large quantities of water to the main condensers for cooling purposes and returning this heated water to its source without recirculation.

The cooling-water system of this plant is ideal in this respect in that recirculation is virtually impossible. A flow of 200,000 gal per min, or 450 cu ft per sec, enters the intake structure and is pumped by four mixed-flow-type pumps through four 700-ft lengths of 54-in. (inside diameter) reinforced concrete pipe to the condensers. The discharge from each condenser is collected in a common 7-ft-by-10-ft concrete tunnel, conveyed 2,850 ft to the base of Morro Rock, and discharged into the Pacific Ocean.

In addition to the long discharge tunnel to Morro Rock, the following alternatives were investigated: *(a)* Discharge into the ocean at a point where the marine fuel line enters the ocean, or *(b)* discharge back into Morro Bay.

The fact that the beach to the north of Morro Rock is a part of the clam beds along the central California coast precludes discharge directly into the surf by either an open channel or a buried conduit. Therefore, the only solution for alternative *(a)* would be a buried conduit across the beach and a marine pipeline of sufficient length so as not to create a hazard to the public from the standpoints of fishing and bathing. An economic comparison of this system with that of discharging back into the Pacific Ocean at Morro Rock showed a savings in favor of the latter method, due largely to the high construction costs of the proposed marine pipeline.

Alternative *(b)* involved a study of water temperatures in the bay and their effect on the vacuum in the condensers, and, hence, the efficiency of the plant.

Morro Bay has a tidal prism between mean lower low water and mean higher high water of 8,800 acre-ft. The temperature of the water in the summer varies from 54° F at flood tide to 62° F at ebb tide. During the winter these same temperatures are practically constant at 50° F. Inasmuch as the major demands for power will occur during the summer months, tidal conditions at that time were the main concern.

While passing through the main condensers, the cooling-water temperature is raised 15° F at maximum load. The problem, then, was to determine the temperature rise that would occur in the bay as a result of adding 450 cu ft per sec of water that was 15° warmer. Several assumptions were made as to the degree of mixing of the discharge water with the tidal water. These assumptions were as follows:

1. There will be a complete mixing of the water during the tidal cycles under consideration, thus resulting in a uniform heating of the tidal water.
2. The discharge water will remain stratified and channeled and, under this same tidal condition, will flow directly into the intake structure, thereby producing a maximum rise in temperature.
3. The discharge water will remain stratified and channeled and will flow past the intake structure, thereby producing no temperature increase.

A study of the tidal currents and the topography of the bay indicated that the actual conditions lie between assumptions 2 and 3 and were quite closely represented by assumption 1. Based on the assumption of complete mixing of the discharge water with the water of the bay, the conclusions in Table 1 were reached as to variations in temperature.

Any temperature increase in the cooling water will impair the condenser vacuum by a corresponding increase in fuel consumption. The maximum capacity of the turbine generator will also decrease with an inferior vacuum. Based on a 2° to 5° rise for two units (14% of the time) the additional capitalized fuel costs and capacity losses for summer conditions and peak-load demand would be \$270,000. The additional cost of discharging at Morro Rock due to the increased conduit and related structures was \$200,000, or \$70,000 less.

TABLE 1.—TEMPERATURE VARIATIONS IN DEGREES FAHRENHEIT

Number of condensers operating	Temperature rise from recirculation	Percentage of time
2	0° to 1°	66
	1° to 2°	20
	2° to 5°	14
8	0° to 5°	75
	5° to 10°	12
	10° to 15°	8
	15° to 20°	5

Conditions other than the purely economic ones also had to be considered. A fairly large oyster industry operates at the lower or southerly end of Morro Bay. Consultation with biologists indicated that any increase in water temperature over the normal variation might affect the oyster culture. Should any damage occur to this culture, it might be difficult to prove that the warm discharge was not a contributing factor. The possible danger to small boats within the harbor as a result of the exit velocities from the discharge also had to be considered.

Surge Tank.—Although the long discharge tunnel to Morro Rock was the most desirable from a standpoint of operation and economy, it presented a serious hydraulic problem. As can be seen from Fig. 2, the invert of the discharge tunnel is at El.-7.0, and the crown is at El.0.0 referred to mean lower low water datum. With this arrangement the tide affects the entire length of the tunnel and provides (*a*) an effective seal for the siphon or discharge leg of the condenser and (*b*) a maximum regain of head on the siphon leg of the condenser under all tidal conditions.

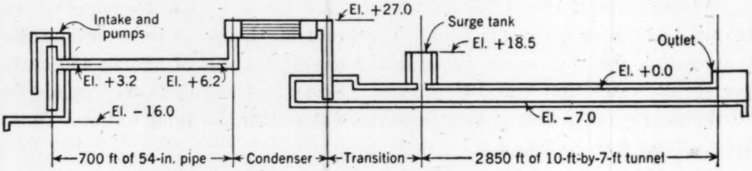


FIG. 2.—PROFILE OF COOLING-WATER SYSTEM

The disadvantage of the foregoing arrangement occurs under conditions of low tide and after a shutdown when the condensers are empty. A start-up of one circulating water pump may cause a serious water hammer in the 54-in. reinforced concrete intake line during the acceleration of the large mass of water in the discharge tunnel.

Under normal conditions the mixed-flow-type pump will start with an impulse head of 53 ft with Q (the flow) equal to zero. Approximately 36 sec later the pump will operate at a uniform flow of 130 cu ft per sec with a total developed head of 19 ft. An investigation of what occurs in this time interval revealed that when the condenser started to spill, the flow in the intake lines would be 105 cu ft per sec and the flow in the discharge tunnel would be zero. Approximately 6.5 sec later the flow in the intake lines would drop to 82 cu ft per sec, and the flow in the tunnel would be 22 cu ft per sec, which would fill the system for the first time; at this instant a water hammer would occur in the intake lines. The velocity would then drop from 5.16 ft per sec to 1.41 ft per sec, causing a 282,000-ft-lb shock to the system and developing a static head of 367 ft.

It would not have been economical to design for a pressure rise of that magnitude; however, there were two other solutions:

1. Butterfly valves could be installed at the circulating water pump in the intake line in an effort to throttle the flow and allow the pump flow to increase gradually until the system was in equilibrium.
2. A surge tank could be placed in the system to absorb the energy produced by the water hammer.

Of the two solutions, the surge tank was the more economical and desirable from the standpoint of reliable operation because there were no mechanical parts to fail when their operation was most needed.

The surge tank is a restricted-orifice type with a 48-in.-diameter orifice in the crown of the tunnel. The tank is 20 ft by 8 ft, extends 18.5 ft above the tunnel, and allows a maximum surge of 9.0 ft.

Thermal Mussel Control.—Mussels are indigenous to the Pacific Ocean coastal waters, and their growth in the circulating water system could be quite serious if allowed to flourish without control. Experience at other plants indicates that mussels will cling to the walls of the circulating water conduits up to depths of 6 in., thereby materially reducing the capacity of the conduits.

It is proposed to control the growth of mussels at this plant by recirculating the warmed water between the condensers and the intake structure and allowing the temperature to rise excessively for short periods of time. Experience at other plants has shown that by raising the temperature to approximately 110° F for a period of 20 min every three weeks the mussels can be killed, loosened from the walls of the system, and then flushed through the condensers into the discharge water. The frequent application of this operation is necessary in order to prevent their growth beyond a point where they might plug the condenser tubes. The exact temperature, duration, and frequency will have to be determined by operation experience.

This method of combating the growth of mussels is not new to the power industry; however, it is believed that at the Morro Bay plant improvements have been made over other methods of handling the problem. Normally, a cross connection is made just behind the pumps, thus confining the mussel control to the pipelines. With the addition of the motor-operated sluice gates in front of the traveling screens, the mussel control has been extended to include the entire intake structure. It is believed that this system will eliminate lengthy shutdowns to clean out the screen and pump wells.

FUEL-OIL SYSTEM

The fuel-oil delivery and storage system consists of a marine anchorage, 4,483 ft of marine pipe, 1,258 ft of land pipe, and four 168,000-bbl storage tanks. The system is designed to received 8,000 bbl per hr of 600 SSF viscosity oil. Initially 200 SSF viscosity oil will be delivered at approximately 135° F.

The marine anchorage, 4,400 ft offshore, provides permanent anchors of from 5 tons to 10 tons; there are two anchors amidship and three anchors aft. The forward anchorage is provided by using the ship anchors. Each of these permanent anchors is located approximately 800 ft from the ship and is attached to a mooring buoy with 400 ft of chain. Each buoy consists of a tank that is 11 ft long, has an outside diameter of 6.5 ft, and is provided with a quick release hook to fasten the mooring lines of the ship. Buoys are also provided to mark the ends of the pipe and the hose. A radar-reflector buoy is provided for navigational assistance during foul weather.

The anchorage is located in the prevailing currents in such a manner that the ship can set its forward anchors, and a crew in a small launch can fasten the lines to the three mooring buoys aft. With the ship approximately in position with the fore and aft anchors, springing lines are then made fast to the mooring buoys amidship.

Delivery of oil is accomplished by picking up the end of the 200-ft-long by 12-in.-diameter sea hose from the ocean bottom by the chain attached to its marker buoy and connecting it to the ship pumps.

The marine part of the line is a 24-in.-outside-diameter, ½-in.-thick wall pipe with ¾ in. of somastic coating. In order to provide a negative buoyancy of approximately 15 lb, a 1¼-in. gunite coating was added. The land part of the line is a 24-in.-outside-diameter, ⅜-in. wall pipe with ¾-in. somastic coating but with the gunite coating omitted. The marine part of the line is cathodically protected against corrosion.

The fuel-oil tanks are located 8 ft above plant grade, thus allowing the oil to flow to the plant by gravity. All future tanks will be located at plant grade. Each tank is 195 ft in diameter and 32 ft high; the temperature in the tanks is maintained between 100° F and 120° F by steam-recirculation heaters.

The installation of the marine fuel-oil line was quite spectacular from the standpoint of coordination between land and sea construction forces. The pipe was initially delivered in lengths of 39 ft. These lengths were welded on the beach into five strings of approximately 820 ft each and a shorter string of 380 ft. The joints were somastic and gunite covered. The 24-in.-by-12-in. reducer was welded to the 380-ft section and then mounted on a structural steel sled. A barge with a 60-ton winch anchored in the mooring area provided a 1¼-in. pulling cable, which was attached to the lugs provided behind the reducer. Side-boom tractors then picked up the first string, and with the barge pulling and the tractors pushing, the section was launched into the surf. After the first string was launched, an additional string was welded to the first string, and it, in turn, was launched. A total of twelve side-boom tractors were required for each string. Communication between the barge and land forces was maintained by ship-to-shore radio, and coordination of land forces was accomplished by visual hand signals. After launching started, the joints between sections were only welded and somastic coated. Wood battens were substituted for the gunite coating at these joints to protect the somastic coating and to provide a smooth surface for the pulling operation. The total operation was performed in 15 hr and 30 min. The average pulling time for each section was 30 min, whereas the welding and somastic coating of the joints averaged 3 hr and 15 min each.

Erosion Control.—When the property was first purchased, the area to be occupied by the fuel-oil tanks consisted of high sand dunes rising above the plant elevation by as much as 30 ft. This condition presented not only a problem from the standpoint of wind erosion but from the standpoint of utilizing the material to construct dikes to enclose the tanks for protection against spillage or breaks in the tanks. Several solutions to the problem were studied, ranging from removing the dune sand entirely and constructing fire walls of precast and prestressed concrete to constructing the dikes of dune sand and guniting the slopes to prevent erosion.

The method finally adopted was ideal in that it was as effective and more economical than any other method heretofore adopted at other steam plant sites.

The dikes were constructed of the dune sand by grading the area down to approximately 3 ft below the final grade of the tank foundation and then rebuilding it to final grade by controlled compaction. Compaction was accomplished with a 50-ton, pneumatic rubber-tired roller to a field density of 95% at optimum moisture content of the material. A strict control over the compaction was made by making tests at 20-ft intervals over the tank-base area and 50-ft intervals in the dike areas. Relative compaction was determined by using a modified standard method[3] of the American Association of State Highway Officials.

During the interim between completion of the dike construction and wind-erosion control treatment, the field forces protected the finished slopes from wind or water erosion by maintaining the moisture content of the surface particles to such an extent that they remained in a cohesive state. This was accomplished with "rainmaker" pipe equipped with fixed-mist nozzles and installed along the top of the dikes. Care was taken to avoid concentrating the water in any given place, thus causing channeling and sloughing. The construction forces were also permitted to maintain these slopes during this period by applying thin layers of straw to the surfaces at the rate of from 2 tons per acre to 4 tons per acre and by tamping the straw into the sand by means of a modified sheepsfoot roller. If these methods did not maintain the slopes, the construction crew was also allowed to add more straw and cover the straw with chicken-wire mats laid flat on the ground and held in place by stakes. In practice, weather conditions were such that the latter two methods were not necessary.

---

[3] "Soil Testing for Engineers," by T. William Lambe, John Wiley & Sons, Inc., New York, N. Y., p. 46.

Final erosion control was accomplished by planting "ice plant" (mesembryanthemum edule) on the dike slopes. The plant is native to the area and was actually obtained by thinning existing growths on the site.

The area was prepared by broadcasting commercial pelleted fertilizer over the area at the rate of 10 lb per 1,000 sq ft several days before planting. Rice straw was then spread over the area at the rate of 6 tons per acre on slopes of 5 to 1, or on steeper slopes, and 4 tons per acre on flatter slopes. The straw was incorporated into the sand by use of a modified tamping roller consisting of thin lugs protruding 6 in. from the roller, 6 in. wide, and ½ in. thick instead of the usual sheepsfoot. This procedure imbedded the straw and did not pull it out as the lug was withdrawn.

Cuttings of ice plant approximately 10 in. long were planted through the straw and into the ground at intervals of 12 in. on the slopes and 18 in. on the flat areas. The second application of fertilizer was made at the same rate approximately two months after planting was completed. The nurseryman contracting the job was required to maintain the growth for a period of ninety days after the initial planting. During this period he watered the plants and replaced any that did not appear to be taking hold. Approximately 13 acres were planted in this manner, and the results have been extremely satisfactory.

Fresh-Water Supply

The fresh-water requirements for the plant were determined to be approximately 80 gal per min per unit. The largest single items are 25 gal per min for boiler make-up and 25 gal per min for vertical pump-bearing lubrication; however, salt water can be used for the latter item with a fresh-water stand-by. The remaining 30 gal per min consist of drinking and sanitary water, water for cracking the scale from the tubes in the evaporators and for flushing, and general utility water, such as that for flushing the boiler tubes during maintenance periods. The initial requirements for fresh water were set at 160 gal per min for the initial two-unit installation and 640 gal per min for the ultimate installation of 8 units.

Complete dependability of the source was of paramount importance both in quantity and quality, especially from the standpoint of the water furnished for boiler make-up.

The possible sources of this water were (*a*) the Upper Salinas River Dam, 10 miles east of San Luis Obispo; (*b*) local wells; (*c*) the effluent from the sewage treatment plant of the Morro Bay Sanitary District; (*d*) local runoff; and (*e*) sea-water evaporators.

Upper Salinas River Dam.—The Upper Salinas River Dam stores water for five public agencies; the city of San Luis Obispo and Camp San Luis Obispo have a joint allocation and are the largest users. Their needs are served by a joint pipeline that terminates at Chorro Creek Reservoir at Camp San Luis Obispo. To supply the ultimate plant needs fully would require a 12-in.-diameter pipeline approximately 11 miles long. Further investigation into priorities and allocations for storage indicated that a firm supply of water could not be assured.

Local Wells.—Historic data on the wells in the area indicate that the wells are subject to deterioration both as to yield and quality of water. Shortages of ground water occurred during dry years, and some sea water has encroached into the ground-water basins along the central coast. There were two wells on the property with an estimated combined yield of 250 gal per min when the property was purchased. In view of the historical background, it did not seem advisable to rely on these wells as principal sources of water, but it was felt that they could be used as stand-bys. A later experience during the early periods of construction justified this decision. At that time, both wells were continuously pumped to provide water for a concrete batch plant and for other construction needs. One well went dry, and the yield per foot of drawdown greatly decreased in the other. Since then a third well has been drilled, and the total combined yield of this well and the remaining one is 200 gal per min. A series of observation wells has been placed around these wells consisting of 1½-in. perforated pipe. With intermittent pumping, the yield per foot of drawdown has remained constant, and the observation wells indicate that the seasonal ground-water table is remaining consistently high. Monthly salinity tests of the water in the observation wells has not yet indicated salt-water intrusion under the present conditions. During a dry year and low ground-water table, such intrusion could occur if the wells were pumped excessively.

Effluent from Sewage-Treatment Plant.—The Morro Bay Sanitary District, which includes the towns of Cayucos to the north, Morro Bay to the south, and the intervening areas, has completed a sewage-treatment plant to the north of and adjacent to the steam-plant site. This installation is a clarifier-and-filter-type plant. An investigation indicated that the quantity of effluent could supply four steam units, but a supply for the ultimate installation of eight units would be doubtful. The quality of the effluent from a plant of this type is such that the detergents in the water would cause excessive foaming in the evaporators and restrict the chemical action of the water softener in the steam plant's own water-treatment installation. The presence of ammonia because of the high organic content in the effluent would cause excessive corrosion of the brass tubing in the evaporators. In the event of a breakdown in the sewage-treatment plant, no guarantee could be made of a continuous source of treated effluent. In view of these objections the clarifier-and-filter-type plant was not considered reliable.

Impounding Local Runoff.—One of the most promising sources of fresh water was the impounding of local runoff. The local annual rainfall varies from 30 in. in the coastal range east of Morro Bay to 16 in. in Morro Bay. Although not abundant in runoff, there were several small streams that could furnish the ultimate requirements of 1,300 acre-ft per yr.

What seemed to be the most economical site for a dam was located on the Toro Creek watershed about 3 miles north and 1 mile east of the steam plant. The watershed above the dam was 14 sq miles. No long-term runoff records were available for Toro Creek; however, a gaging station was established before the winter rains of 1952. The nearest comparable stream with gaging records was Arroyo Grande Creek, which was approximately 25 miles south of Morro Bay and had a drainage area of approximately 106 sq miles.

The straight ratio of drainage areas shows that Toro Creek drains only 13.5% of the area of Arroyo Grande; however, by comparing areas of average normal rainfall and relating runoff to rainfall, it appeared that Toro Creek should flow approximately 20% of the rate of Arroyo Grande. Comparison of actual measurements during the winter of 1952–1953 showed that 20% was a conservative assumption.

A mass curve plotted for the entire period of record was based on this assumption, which indicated that a storage of 3,700 acre-ft would be required to yield the ultimate demands of 1,300 acre-ft per yr.

Material was available within ½ mile of the site to construct an earth-fill-type dam. The dam would be 90 ft high, require approximately 200,000 cu yd of fill, and create a storage reservoir covering 175 acres. Consideration of the possibility of flash floods in the area indicated that a 7,000-cu-ft-per-sec, side-channel spillway would be required. Locating a dam at this site would also require a 4-mile pipeline to the steam plant, relocating 2 miles of county road, relocating existing oil-transmission pipelines in the storage-reservoir area, and in addition to the purchase of land.

Sea-Water Evaporators.—Although the process of sea-water evaporation has been common aboard ships, it is believed that the use of sea-water evaporators for producing fresh water for a stationary steam plant is the first industrial application of this system in the United States.

Each steam-generating unit will require a set of triple-effect evaporators designed to produce 50 gal per min. This capacity was required to provide for both boiler make-up and utility water. Because approximately half of the water produced from sea-water evaporators need not be of the purity required for boiler make-up, a purity of 50-ppm total solids was specified. A part of this water would then be redistilled in the station evaporators to produce the required purity of less than 1 ppm for boiler make-up.

Fig. 3 shows a simplified outline of the three-effect evaporation cycle. Each of these three evaporators consists of a ⅜-in.-thick, welded copper bearing steel shell that has a 6-ft outside diameter and is 20 ft long. The lower half of the shell contains 290 aluminum-brass tubes 1 in. in diameter.

Salt water with a concentration of 36,000 ppm is introduced to the cycle into the shell of the first-effect evaporator. The resulting brine is transmitted to the shells of each succeeding effect and finally wasted to the main condenser water-discharge tunnel with a concentration of 54,000 ppm.

Steam from the station evaporator is introduced into the tube bundle of first effect to heat the salt water and create steam to heat the second and third effects; steam so produced is finally condensed at the evaporator condenser. Vapor created by pure condensate in the tube bundles of the second and third effects is also collected at the evaporator condenser. The result is about 50 gal per min of fresh water with a purity of 50 ppm. A part of this is drawn off and transmitted to the raw water storage tank for utility purposes, and the remainder is piped to the station evaporator for redistilling. Pure condensate so created is drawn from the first-effect tube bundle and transmitted to the main condenser at a purity of less than 1 ppm. At the main condenser it is introduced into the boiler-feed water system.

Summary.—A comparison of the sources of water indicated that the Upper Salinas River Dam and the sewage-treatment plant should be eliminated because a firm supply could not be assured.

An economic comparison of Toro Creek Dam and the sea-water evaporators indicated that the capital cost of the evaporators was approximately one-third that of the dam, but that the annual cost was greater beyond an installation of two units. However, if the operation of the evaporators were limited solely to the production of boiler make-up water and if the utility water were supplied by wells, installation of the evaporators would be economically favorable up to a six-unit installation.

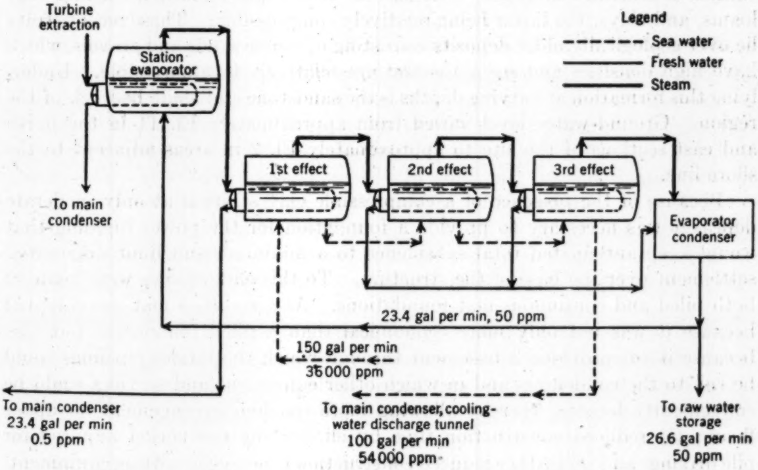

Turbine extraction
Station evaporator
Legend
Sea water
Fresh water
Steam
To main condenser
1st effect
2nd effect
3rd effect
Evaporator condenser
23.4 gal per min, 50 ppm
150 gal per min
36000 ppm
To main condenser
23.4 gal per min
0.5 ppm
To main condenser, cooling-water discharge tunnel
100 gal per min
54000 ppm
To raw water storage
26.6 gal per min
50 ppm

FIG. 3.—THREE-EFFECT EVAPORATION

This latter scheme was finally adopted with a slight modification of the cycle (Fig. 3). The pump-bearing lubrication-water requirements are fulfilled by using salt water. The additional capacity of the evaporators serves as a stand-by in the event of a failure in the wells or in the bearing-lubrication system.

FOUNDATIONS

The Morro Bay Steam Plant is located immediately west of the town of Morro Bay on a low section of land, which terminates in the promontory leading to the dacite sea stack of Morro Rock. To the west, between the site and the Pacific Ocean, there are shifting sand dunes having an average height of more than 30 ft above the adjacent ground. Just to the south, and on the opposite side of Morro Rock from the ocean, is the navigation channel leading to Morro Bay; to the east there is a 60-ft-high sandstone bluff consisting predominantly of poorly cemented, reddish-brown, fine-to-coarse sand. Originally, the low and flat area now occupied by the plant had an average elevation of 6 ft, according to the Bureau of Coast and Geodetic Survey (United States Department of Commerce) datum. However, during 1942, when the site was developed as a United States Navy training base, the area was filled hydraulically with sand and gravel to an average elevation of 14 ft.

Subsurface exploration of the site was performed during the early months of 1953 at which time forty-three holes were drilled to depths varying between 60 ft and 101 ft below ground surface. The various soil strata were classified by field examination, which was supplemented by laboratory tests on representative undisturbed core samples.

In general, the top 54 ft to 82 ft of natural deposits underlying the hydraulic fill are of recent geological origin. The deposits appear to be lenticular and consist of alluvial soil types—that is, fine sands, sandy and silty loams, clay loams, and clays, the latter being relatively compressible. These recent strata lie over geologically older deposits consisting of coarse sands and gravels, which have high densities and strengths and are relatively incompressible. Underlying this formation at varying depths is the sandstone and shale bedrock of the region. Ground-water level varied from approximately El. 11 in the north and east sections of the site to approximately El. 2 in areas adjacent to the shore line.

Because of the presence of a compressible clay stratum at only moderate depth, it was necessary to provide a foundation for the power building that would keep anticipated total subsidence to a minimum and limit differential settlement over the base of the structure. To this end, studies were made of both piled and continuous-mat foundations. As a result, a mat was adopted because it was not only more economical than a piled foundation but also because it (*a*) provided a basement through which the intake pipelines could be run to the condensers and in which other equipment and services could be conveniently located, thereby allowing more spacious arrangement on upper floors; (*b*) reduced construction time by eliminating the period required for pile driving; and (*c*) further reduced construction time by permitting equipment, piping, and cable installation in the basement to proceed concurrently with erection of the steelwork above.

The 5-ft-thick mat of reinforced concrete, founded at El. −2.0, is 267 ft long and 186.5 ft wide. The mat was designed to act monolithically with the ground floor slab and beams at El. 15.0, with the perimeter walls and piers transferring the shears. The rigidity of this 17-ft-deep substructure not only bridges localized areas of weakness in the supporting soils, but also assures minimum differential settlement due to either differences in soil pressures or variations in thickness of compressible layers. All superstructure steel columns are supported on the foundation piers at ground-floor level. The turbine-generator pedestals, because of the magnitude of the machine loads and because of the necessity of avoiding transfer of vibrations, are separated from the rest of the structure and are supported directly on the mat. Together, the dead and live loads of both superstructure and substructure produce soil pressures varying from 2,500 lb per sq ft to 2,800 lb per sq ft. However, as a result of the removal of 16 ft of soil and the buoyant effect of the ground water, the effective pressure on the subsoil is reduced by approximately 2,000 lb per sq ft, varying from approximately 500 lb per sq ft to 800 lb per sq ft.

After the magnitude of the foundation loads had been determined, studies were made to determine probable settlements. In order to arrive at these, it was necessary to consider the construction sequence. As the earth load is reduced during excavation for the substructure, the underlying clayey soils swell and are recompressed later by the addition of the building loads. Settlement of the structure is affected by the length of time the excavation remains open, the length of time that the dewatering system remains in operation, the rate at which loading is applied, and the relative rigidity of the mat. For the investigation of settlements, it was assumed that the excavation would remain open less than one month, that the dewatering period would be six months, and that the loads would be applied gradually at a uniform rate of increase over a period of eighteen months.

Because the structural behavior of the foundation would lie somewhere between that of a perfectly flexible mat without restraint, in so far as differential settlements are concerned, and that of a perfectly rigid mat, allowing no warping of the mat surface, ultimate settlements were computed for these two limiting conditions. Results for the flexible mat are shown in Fig. 4(*a*), and in Fig. 4(*b*) the results are compared. In addition, the anticipated rate of vertical movement of any point on the mat as a function of time is given by the curve in Fig 4(*c*). In this connection it should be noted that almost 70% of the total settlement was expected to occur during the construction period, although settlement is expected to continue during approximately ten years at a progressively decreasing rate.

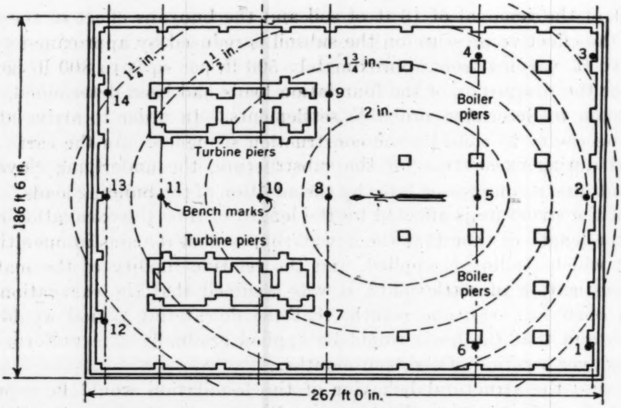
The accuracy of the computed settlements depends on the validity of these assumptions. Actually, substructure excavation was started on July 3, 1953, and proceeded from the north end of the building under the turbine room to the south end under the boiler area; this excavation was completed on August 28, 1953.

Construction of the mat, also from north to south, began on August 17, 1953, and was completed on October 30, 1953. Both excavation and concreting proceeded in stages rather than in a continuous operation.

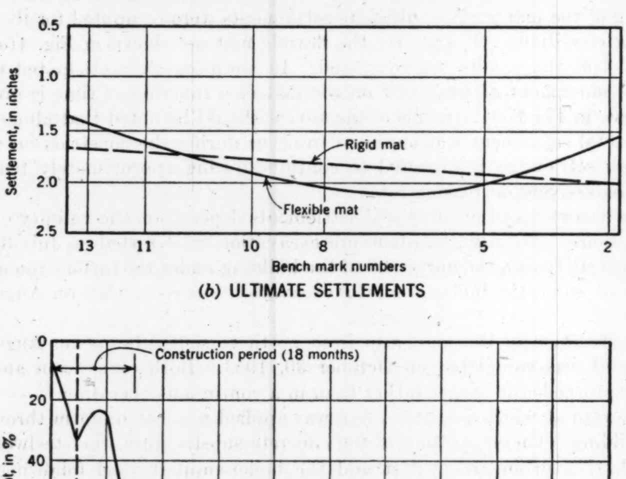
The rate at which actual loading was applied was not uniform throughout the building. On completion of the concrete substructure, the steelwork was erected, the turbine room first and the boiler-support steel following later.

Erection of the first boiler located in the southeast quadrangle began in June, 1954, and was completed in November, 1954; work on the second boiler unit began immediately thereafter. The installation of the first turbine-generator unit began in January, 1955.

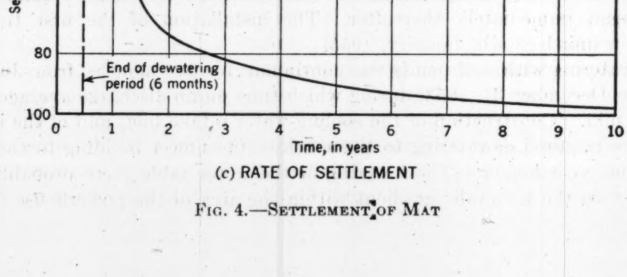
Dewatering with well points was continuous for five months, from July 27, 1953, to December 31, 1953, during which time pump discharge averaged 450 gal per min. Construction of the cooling-water intake lines and of the intake structure required dewatering to the south of the power building to continue about one year longer. The lowering of the water table there probably had an effect on the hydraulic gradient within the area of the powerhouse itself.


1½ in.
1¼ in.
1¾ in.
2 in.
4
3
14
9
Boiler piers
Turbine piers
13
11
10
8
5
2
Bench marks
Turbine piers
Boiler piers
12
7
6
1
186 ft 6 in.
267 ft 0 in.

(a) ULTIMATE-SETTLEMENT CONTOURS UNDER FLEXIBLE MAT



(b) ULTIMATE SETTLEMENTS



(c) RATE OF SETTLEMENT

FIG. 4.—SETTLEMENT OF MAT

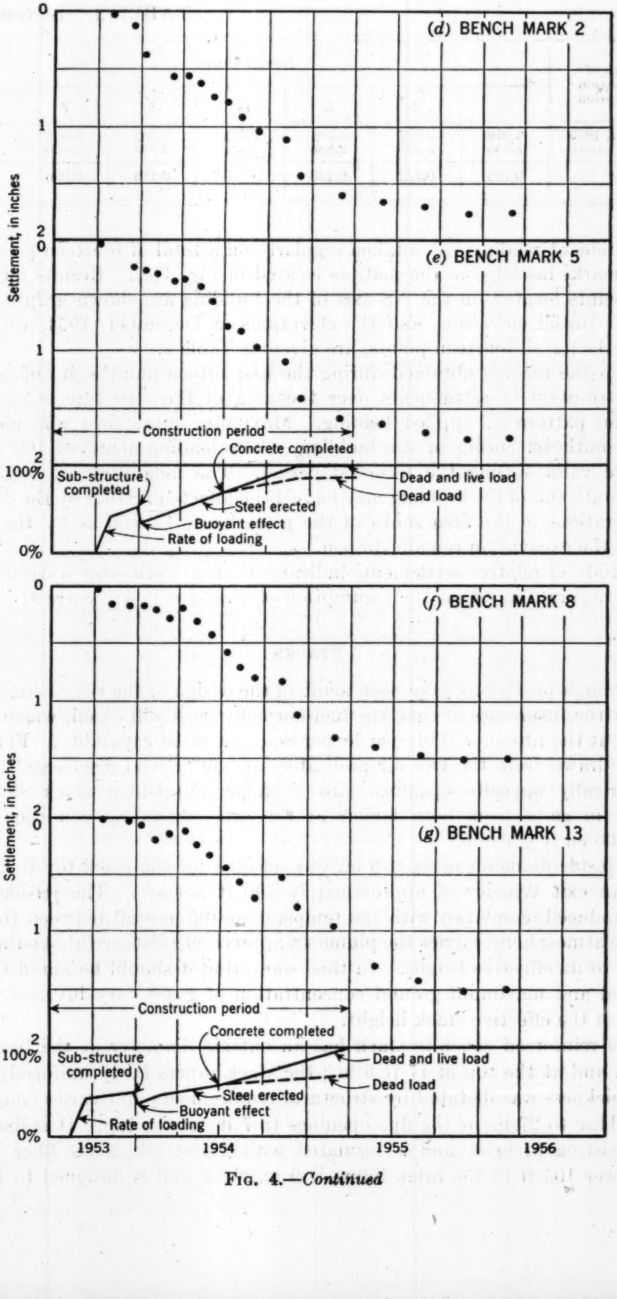


FIG. 4.—*Continued*

TABLE 2.—BENCH-MARK

Month of observation of elevation	BENCH MARK						
	1	2	3	4	5	6	7
September, 1953	3.019	3.017	3.026	2.975	3.002	3.004	2.983
December, 1954	2.915	2.895	2.883	lost	2.883	2.908	2.906
Difference	0.104	0.122	0.143	...	0.119	0.096	0.077

Settlement readings were taken regularly on a total of fourteen permanent bench marks installed in the mat, as located in Fig. 4(*a*). Results for four of these points located on the NS-axis of the building are shown graphically in Fig. 4. Initial elevations and the elevations in December, 1954, with their differences for all fourteen points, are given in Table 2.

From the records obtained during the first fifteen months, it appears that the distribution of settlements over the area of the structure is consistent with the pattern of applied loading. Maximum subsidence was measured in the southeast corner of the building, where loading achieved 100% of its ultimate value during the recorded period. The magnitude of settlements at the south end of the building may have been affected further by the dewatering operations in the area south of the powerhouse as well as by the length of time the excavation remained open.

A study of relative settlements indicates that no noticeable warping of the surface occurred and that the assumption of a rigid mat was correct.

STACKS

Although provisions have been made in the design of the two steam generators for the future use of coal, the fuel normally used will be oil, which will be burned at the rate of 250 bbl per hr per boiler at rated capability. Flue gases will be drawn from the two air preheaters at 330° F and discharged through diametrically opposite openings into a single 450-ft-high stack, the total volume of gases from both boilers at the same operating conditions being 1,100,000 cu ft per min.

An inside diameter of 14 ft 9 in. was selected for the stack top to give the gases an exit velocity of approximately 100 ft per sec. The jet-like effect thus produced, combined with the temperature differential between the gases and the atmosphere, carries the plume an appreciable distance above the stack, adding to its effective height. In this connection it should be noted that the dispersal and maximum ground concentration of gases vary inversely as the square of the effective stack height.

The reinforced concrete stack has an outside diameter at the base of 42 ft 8 in. and at the top of 17 ft 6 in.; the stack tapers fairly uniformly. The shell thickness was dictated by structural requirements and varies from 24 in. at the base to 27 in. at the flue openings to 7 in. at the top. It is lined with acid-resisting firebrick and is insulated with 2-in.-thick, glass fiber boards. The lower 105 ft of the brick lining is 9 in. thick and is designed to be self-

ELEVATIONS, IN FEET

BENCH MARK							Month of observation of elevation
8	9	10	11	12	13	14	
2.966	3.013	2.998	2.954	2.970	2.999	2.999	September, 1953
2.871	2.900	2.911	2.886	2.903	2.935	2.929	December, 1954
0.095	0.113	0.087	0.068	0.067	0.064	0.070	Difference

supporting. Above this section, it is 4½ in. thick and is supported at every 30 ft on concrete corbels formed monolithically with the shell.

The weight of the stack and lining and the overturning moments from both wind and seismic loadings are distributed to the foundation subsoils through a heavily reinforced concrete mat supported on 289 step-tapered concrete piles. This mat is octagonal, is 90 ft from face to face, and is 10 ft thick under the stack, the thickness decreasing to 6.5 ft at the mat periphery. The piles, carrying load by both end bearing and friction, have an average length of 68 ft and are driven through the recent soils to a penetration of at least 3 ft into the dense, underlying sands and gravels of the older stratum. For dead load only the maximum allowable load per pile was limited to 45 tons. For the condition of combined loads, vertical and seismic, this was increased to 90 tons. The 2,030 cu yd of concrete required for construction of the pile cap were placed in a continuous operation for 28 hr. Fig. 5 shows the reinforcing steel for the foundation and dowels for the stack shaft.

Except for seismic loadings, structural design of the stack followed the 1953 recommendations[4] of the American Concrete Institute (ACI).

Recent investigations have raised strong doubts regarding the validity of the heretofore widely used assumption that all parts of a structure are subjected to a uniform seismic acceleration during an earthquake. For this to be true it is necessary that the structure be infinitely rigid. Because of the inherent flexibility of the average structure, designs based on the application of seismic forces, which are the product of mass and constant acceleration, will produce a structure that is weaker in its upper sections. This is especially true in the case of a relatively limber structure such as a tall stack, a fact that was confirmed by a survey of damage to chimneys caused by the 1923 Kanto earthquake and the 1948 Fukui earthquake in Japan.[5] It was found that only in five out of twenty-two fractured stacks was the point of fracture located below the third point of the height. Since the 1923 earthquake, Japanese building codes have required that stacks be designed using a constant seismic coefficient of 0.15.

In the United States two codes for seismic design acknowledge that there is a functional relationship between height and the acceleration factor. One of these codes published[6] in 1952 recommends that the total base shear should be distributed over the height of the structure in accordance with

---

[4] "Specification for the Design and Construction of Reinforced Concrete Chimneys," *Journal*, A.C.I., Vol. 26, 1954, pp. 1–48.

[5] "The Fukui Earthquake," Office of the Engr., General Headquarters, Far East Command, U. S. Dept. of the Army, Tokyo, Japan, Vol. II, February, 1949.

[6] "Lateral Forces of Earthquake and Wind," by the Joint Committee of the San Francisco (Calif.) Section, ASCE, and the Structural Engrs. Assn. of Northern California, *Transactions*, Vol. 117, 1952, p. 716.

$$F_x = \frac{V\ W_x\ h_x}{\Sigma(W\ h)} \qquad (1)$$

and

$$V = C\ W \qquad (2)$$

in which F_x is the lateral force at the section under consideration, V denotes the base shear, W_x is the weight of the section under consideration, h_x represents the height of section above the base of the stack, $\Sigma(W\ h)$ is the summation of the products of weight of each section and its height above the base, and C is a lateral-force coefficient or a seismic coefficient.

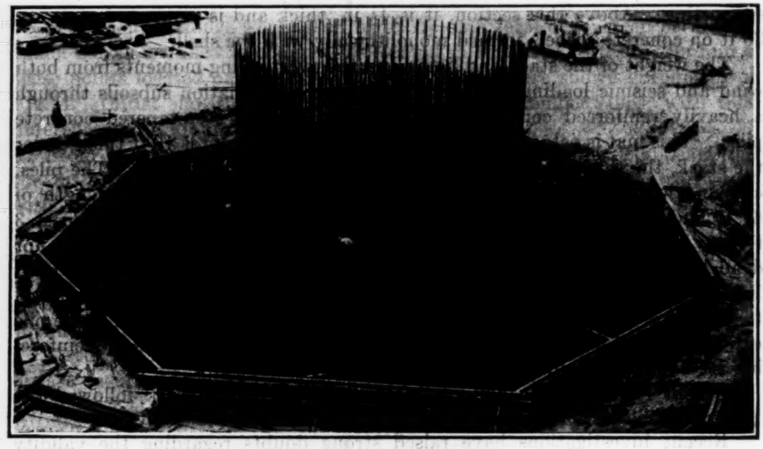


Fig. 5.—Reinforcing for Stack Foundation

The other code[4] recommends that a constant acceleration factor be used in the design of the lower one-fifth of the stack, but in the upper four-fifths a magnification factor should be applied to the seismic coefficient determined by the empirical formula,

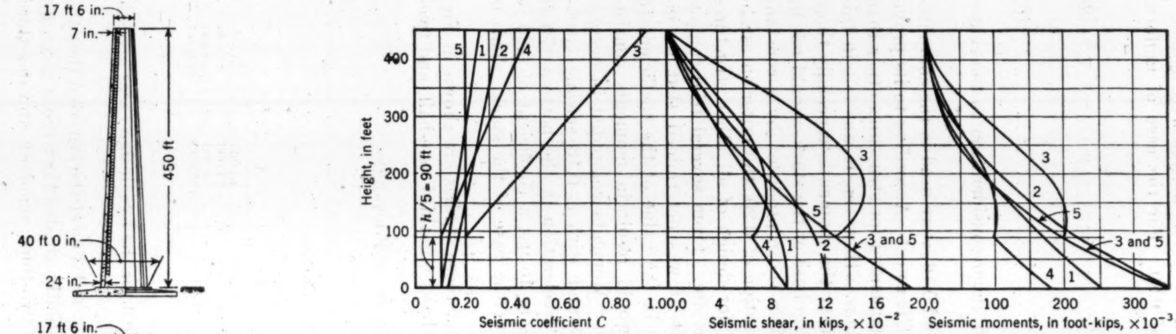
$$K_x = K_e\left(1 + \frac{h'}{100}\right) \qquad (3)$$

which results in

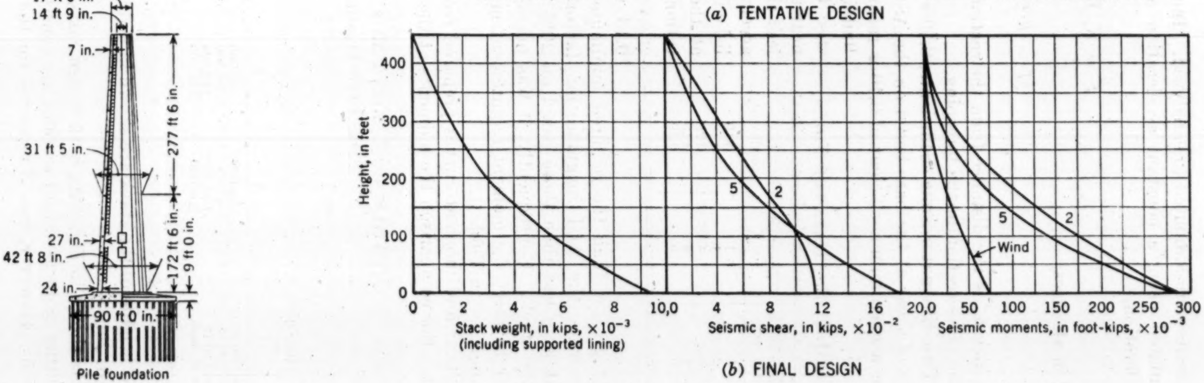
$$F = K_e\ W\left(1 + \frac{h'}{100}\right) \qquad (4)$$

in which K_e is the seismic coefficient and h' is the distance from the section under consideration to the section that is one-fifth of the total height of the chimney above the base.

In order to determine how these recommendations would affect the stack design, a trial section was selected (Fig. 6(a)) for which seismic shears and



(*a*) TENTATIVE DESIGN



(*b*) FINAL DESIGN

FIG. 6.—DESIGN OF CONCRETE STACK

moments were computed and analyzed. Included in this analysis were moments computed using a constant acceleration of 0.20. For the case of uniform acceleration,

$$F = K_e W \qquad (5)$$

and in all cases,

$$M = F h'' \qquad (6)$$

in which h'' is the distance from the section under consideration to the center of the mass section in the foregoing.

Two different seismic coefficients, 0.10 and 0.13, were used in the computations of shear at the base based on the ASCE code.[6] According to its provisions a maximum coefficient of 0.10 is recommended for structures other than buildings. However, it was felt in this case that the moments, especially those near the base of the stack, should not be less than those produced by a constant acceleration of 0.20, the criterion established for design of all other structures at the site. From several trial computations, it was determined that an increase in the value of the coefficient to 0.13 would result in moments consistently greater than the required minimum. Moments computed in accordance with the ACI magnification factor[4] and with a 0.10 seismic coefficient were less than those based on 20% acceleration in the bottom 175 ft of the stack. Consequently, it was necessary to use a coefficient of 0.20 in order to meet the moment requirement near the base of the stack. This is summarized in Table 3.

TABLE 3.—DATA FOR FIG. 6

Curve and design case	Equation for F	Seismic coefficient
1	1	0.10
2	1	0.13
3	4	0.20
4	4	0.10
5	5	0.20

Shear and moment curves prepared from these studies are shown in Fig. 6(*a*). For comparison the moments at the base, one-third, one-half, and two-third heights of the stack are shown in Table 4 for the five design cases investigated.

TABLE 4.—BENDING MOMENTS, IN FOOT-KIPS, FOR FIVE DESIGN CASES

Height of section	Design Case				
	1	2	3	4	5
$\frac{2}{3} h = 300$ ft	28,000	36,000	86,000	43,000	26,000
$\frac{1}{2} h = 225$ ft	67,000	83,000	153,000	74,000	61,000
$\frac{1}{3} h = 150$ ft	120,000	152,000	196,000	98,000	128,000
$h = 0$, base	250,000	354,000	350,000	175,000	350,000

As shown in Fig. 6(*a*), both design cases 2 and 3 meet the criterion that the resulting moments at all sections are consistently greater than those given by case 5. However, case 3 moments for the two-third, one-half, and one-third heights are 230%, 151%, and 53% greater than those for case 5 at the same sections. Those for case 2 are only 38%, 36%, and 19% greater than the corresponding moments for case 5. To accommodate these high moments for case 3, a second trial design was prepared, the resulting gravity load further increasing final moments. For this condition the stack shaft alone would require 2,945 cu yd of concrete, an increase of 30% over the 2,260 cu yd required for one designed in accordance with case 2.

Consistent with what was felt to be a sound engineering approach, the final stack design was prepared following the ASCE seismic recommendations[6] using a coefficient of 0.13. Stack outline, weight, and seismic shears and moments are shown in Fig. 6(*b*). It will be seen from the curves and from Table 5 that the resulting final moments (case 2) are greater over the entire stack height, and that they are considerably larger for the upper, critical sections than those produced by a 20% constant acceleration (case 5).

The fact that the stack weight and stress functions are smaller than those of the tentative design is attributable to certain changes introduced in the final design. These changes consisted of increasing the thickness of the bottom 105 ft of the lining to 9 in., of making the bottom structurally self-supporting, and of adjusting the thickness of the concrete shell to actual stress requirements.

TABLE 5.—COMPARISON OF FINAL MOMENTS, CASE 2 WITH CASE 5

Case	Height of Section, in Feet									
	0	50	100	150	200	250	300	350	400	450
2	287	229	176	130	89	57	32	14	4	0
5	283	205	144	97	62	37	20	8	1	0
Difference, in %	1.4	11.7	22.2	34.0	43.6	54.1	60.0	75.0	300.0	0

The construction of the stack shaft began on July 12, 1954, and was completed on December 1, 1954. It is believed that this stack of the Morro Bay Steam Plant is the first to be designed for seismic loading in accordance with recommendations of the 1952 ASCE code.[6]

STRUCTURAL FEATURES

Functionally, the plant consists of the boilerhouse, the turbine room, and a firing aisle between. Structurally, the three elements are designed to act as a single unit (Fig. 7).

Supported from a common reinforced concrete substructure, the steel superstructure was arranged to provide adequate support and convenient access to equipment as well as a weather enclosure for the plant. Apart from vertical loading, the structure was designed to withstand safely lateral forces caused by an earthquake of an intensity equal to 20% of gravity.

The boilers each weigh approximately 7,000,000 lb and are suspended from the top of the boilerhouse steelwork by hangers, an arrangement necessitated by the 6-in. thermal expansion of the boiler casing while in operation. Lateral movements during an earthquake of this huge pendulum are restrained by a system of buckstays. Reactions from these buckstays are carried at six elevations by horizontal trusses to adjacent boiler-support columns. Although oil is the only fuel presently available, provisions were made for future coal firing. In line with this the end row of full-height columns, as well as the stub columns, were designed to support coal bunkers of 1,200 tons capacity per unit.

A Warren-type system of bracing in both vertical planes provides stability of the structure for lateral forces. The roof framework and the concrete floors provide bracing in the horizontal plane. Column shafts were embedded to a depth of 3 ft in the concrete piers of the substructure. Vertical loading is transferred through the piers directly to the mat. Horizontal shears at column bases are distributed by the ground-floor slab among the piers and from there are carried to the foundation.

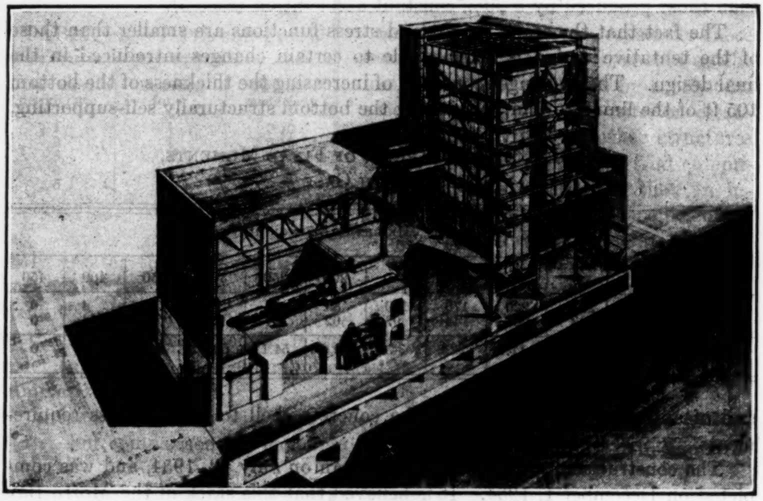


FIG. 7.—SECTION THROUGH POWERHOUSE

The turbine generators, supported on isolated pedestals, are set on the same axis as the steam generators. This arrangement of boilers and turbines allows for short runs of primary and reheat piping, and also results in a minimum length of bus ducts between generator terminals and the station and main transformers. Such a plant layout, which afforded considerable over-all economies, required an almost 120-ft-wide turbine room unobstructed by intermediate columns. To bridge this span, 12.5-ft-deep roof trusses were adopted. Although no portal frame action was considered when sizing the columns, a certain rigidity of connection between column and truss was assumed when designing the trusses. The stepped columns of the turbine room carry, apart from the roof framework, crane girders for a 60-ton-capacity bridge crane.

Bracing, which is provided between the columns as well as between the trusses, enables the turbine room structure to withstand seismic forces acting in a longitudinal direction. Transversely to the building, lateral loading is transferred to the boilerhouse structure by the combined action of the roof framework and concrete floors.

The operating controls of the plant are concentrated in the firing aisle. There, on the operating floor, is the "nerve center" of the entire plant—the control room. Through its glassed walls, the operator must have an unobstructed view of the turbine ends, the boiler feed pumps, and the various control panels. To achieve this a 40-ft-wide bay was required. This was obtained by moving the turbine room and boilerhouse structures apart and providing horizontal framing supported by adjacent columns.

Although the two-story office building, the machine shop, and warehouse adjoin the powerhouse, they are structurally independent of it.

A total of 3,950 tons of riveted structural steelwork was used in the construction of the plant.

CONCRETE

The problem of chemically reactive aggregates is common throughout central California,[7] where sedimentary rocks of the opaline shale type form one of the sources of aggregate. The outward manifestation of the presence of reactive aggregate is widespread pattern cracking of the concrete. The mechanism of this alkali-aggregate reaction is a chemical process in which soluble alkalies released by the hydration of cement react with siliceous constituents of the aggregates to produce an alkali silica gel. Expansion of this gel, which accompanies the process, causes cracking and deterioration of the concrete.

Extensive tests were performed on aggregates from quarries in the vicinity of the plant. Coarse aggregate was required to show less than 12% loss of weight after five cycles of sodium-sulfate tests and the fine aggregates, a loss of less than 8%. As a further precautionary measure, 18% of the cement was replaced by pozzolanic material,[8] a calcined diatomaceous earth, and low-alkali, type II cement was used exclusively.

Although the design of concrete mixes was adjusted periodically to meet particular needs, two of the mixes were used predominantly. In heavy sections, such as the powerhouse mat and the stack foundation where high strength was of secondary importance, a crushing strength at twenty-eight days of 2,500 lb per sq in. was attempted. This mix contained 4.75 sacks of cement per cu yd of concrete. In all other structures a mix with a compressive strength of 3,500 lb per sq in. at twenty-eight days was specified. The average cement content of this mix was six sacks per cu yd.

All concrete was furnished from a batching plant, which was installed at the job site and had a capacity of 75 cu yd per hr. A careful scrutiny of the quality of concrete and aggregates was maintained by qualified personnel, who had a fully equipped testing laboratory at their disposal. A total of 41,500 cu yd of concrete was placed, and a total of 3,650 tons of reinforcing steel was used in the construction of the plant.

---

[7] "Alkali-Aggregate Reaction in California Concrete Aggregates," by Richard Merriam, *Report 27*, Div. of Mines, State of California, 1953.

[8] "Symposium on Use of Pozzolanic Materials in Concretes," *Technical Publication No. 99*, A.S.T.M. 1949.

TURBINE-GENERATOR FOUNDATIONS

The two turbine-generator units of the Morro Bay Steam Plant have a rated capability of 156,250 kw; this places them among the largest machines installed and in operation (as of 1954). These turbines are 3,600 rpm, tandem-compound triple flow, and the generators are 3,600 rpm, 18,000 volt, three phased, and hydrogen cooled.

The condensers are of the horizontal, single-pass type. Part of the weight of the condenser is carried on the turbine exhaust, and the balance, directly by the foundation through an arrangement of spring supports.

In view of the importance and the value of the equipment they carry, great care was exercised in the design and physical arrangement of the turbine-generator foundations. This required a cooperative effort by mechanical and structural engineers. Based on information obtained from the turbine and condenser manufacturers, an outline of the foundation was prepared by the mechanical engineers, who also established its principal dimensions.[9] It was the function of the structural engineers to analyze the behavior of the foundation under static and dynamic loading and to prepare a detailed structural design.

Static loading, impact factors, and maximum permissible deflections of members directly supporting the machine were specified by the turbine manufacturer. These recommendations also included allowable stresses to be used in the design and, as a guide, gave the desired ratio between the weight of the machine and that of the concrete foundation. With these criteria established, the structural design of the turbine pedestal followed the pattern of analysis established for rigid reinforced concrete frames.[10]

The capability of turbine generators has increased sharply in recent years, and there has been a corresponding increase in the weight and speed of machines. The need for a more rational approach to the problem of dynamic design of turbine foundation has therefore become apparent.

Such investigators as G. Ehlers,[11] H. Kayser,[12] and E. Rausch[13,14] have contributed much to the development of suitable methods of analysis. The German Bureau of Standards in recognizing this fact has included in a specification[15] for the design and construction of turbine foundations a requirement for their dynamic investigations.

---

[9] "Turbine Generator Foundations," *Publication No. GET-1749*, General Electric Co.

[10] "Foundation for a Large Turbogenerator," by Paul Rogers, *Journal*, A.C.I., Vol. 23, November, 1951, pp. 213–222.

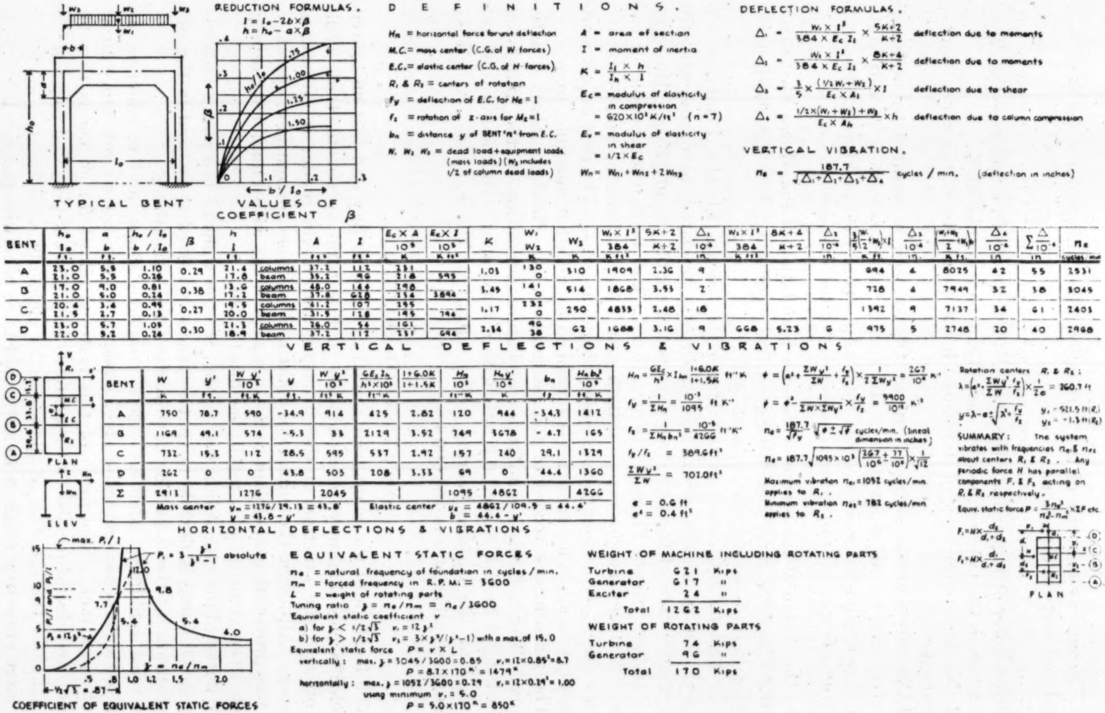
[11] "Berechnungen der Schwingungen von Turbinenfundamente," by G. Ehlers, *Beton und Eisen*, Vol. 22, June 22, 1929, pp. 409–415.

[12] "Theoretische Betrachtungen und ausgefuehrte Versuche über Fundamentschwingungen," by H. Kayser and A. Troche, *ibid.*, Vol. 24, January 5, 1930, pp. 15–18.

[13] "Zur Berechnung von Dampfturbinenfundamenten," by E. Rausch, *ibid.*, Vol. 30, August 20, 1931, pp. 294–298.

[14] "Maschinenfundamente und andere dynamische Bauaufgaben," by E. Rausch, Verein Deutscher Ingenieure, Berlin, Vol. 3, 1942.

[15] "Stuetzkonstruktionen für Rotierende Maschinen (Tischfundamente für Dampfturbinen)," *DIN 4024*, Deutsch Normen, Berlin, January, 1955, pp. 1–4.



COEFFICIENT OF EQUIVALENT STATIC FORCES

REDUCTION FORMULAS.

$l = l_0 - 2b \times \beta$

$h = h_0 - a \times \beta$

D E F I N I T I O N S.

H_n = horizontal force for unit deflection
M.C. = mass center (C.G. of W forces)
E.C. = elastic center (C.G. of H forces)
R_1 & R_2 = centers of rotation
f_y = deflection of E.C. for $H_E = 1$
f_z = rotation of z-axis for $M_z = 1$
b_n = distance y of BENT "n" from E.C.
W_1 W_2 W_3 = dead load + equipment loads (mass loads) (W_3 includes 1/2 of column dead loads)

A = area of section
I = moment of inertia
$K = \frac{I_l \times h}{I_h \times l}$
E_c = modulus of elasticity in compression = 620×10^3 K/ft² (n = 7)
E_v = modulus of elasticity in shear = $1/2 \times E_c$
$W_n = W_{n1} + W_{n2} + 2W_{n3}$

DEFLECTION FORMULAS.

$\Delta_1 = \frac{W_1 \times l^3}{384 \times E_c I_l} \times \frac{5K+2}{K+2}$ deflection due to moments

$\Delta_2 = \frac{W_2 \times l^3}{384 \times E_c I_l} \times \frac{8K+4}{K+2}$ deflection due to moments

$\Delta_3 = \frac{3}{5} \times \frac{(1/2 W_1 + W_2)}{E_v \times A_l} \times l$ deflection due to shear

$\Delta_4 = \frac{1/2 \times (W_1 + W_2) + W_3}{E_c \times A_h} \times h$ deflection due to column compression

VERTICAL VIBRATION.

$n_e = \frac{187.7}{\sqrt{\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4}}$ cycles / min. (deflection in inches)

BENT	h_0 l_0 ft.	a b ft.	h_0/l_0 b/l_0	β	h l ft		A ft²	I ft⁴	$\frac{E_c \times A}{10^6}$ K	$\frac{E_c \times I}{10^6}$ K ft²	K	W_1 W_2 K	W_3 K	$\frac{W_1 \times l^3}{384}$ K ft³	$\frac{5K+2}{K+2}$	$\frac{\Delta_1}{10^{-4}}$ in.	$\frac{W_2 \times l^3}{384}$ K ft³	$\frac{8K+4}{K+2}$	$\frac{\Delta_2}{10^{-4}}$ in.	$\frac{3}{5}\left(\frac{W_1}{2}+W_2\right) \times l$ K ft.	$\frac{\Delta_3}{10^{-4}}$ in.	$\left(\frac{W_1+W_2}{2}+W_3\right)h$ K ft.	$\frac{\Delta_4}{10^{-4}}$ in.	$\sum\frac{\Delta}{10^{-4}}$ in	n_e cycles/min
A	23.0 21.0	5.8 5.5	1.10 0.26	0.29	21.4 17.8	columns beam	37.2 35.2	112 96	231 218	595	1.03	130 0	310	1909	2.36	9				994	4	8025	42	55	2531
B	17.0 21.0	9.0 5.0	0.81 0.24	0.38	13.6 17.2	columns beam	48.0 37.8	144 628	298 234	3894	3.45	141 0	514	1868	3.53	2				728	4	7949	32	38	3045
C	20.4 21.5	3.4 2.7	0.95 0.13	0.27	19.5 20.0	columns beam	41.2 31.5	107 128	255 195	794	1.17	232 0	250	4833	2.48	18				1392	9	7137	34	61	2403
D	23.0 22.0	5.7 5.2	1.05 0.24	0.30	21.3 18.9	columns beam	26.0 37.2	54 112	161 231	694	2.34	96 38	62	1688	3.16	9	668	5.23	6	975	5	2748	20	40	2968

V E R T I C A L D E F L E C T I O N S & V I B R A T I O N S

BENT	W K	y' ft.	$\frac{W y'}{10^3}$ ft.K	y ft.	$\frac{W y^2}{10^3}$ ft² K	$\frac{6E_c I_h}{h^3 \times 10^3}$ ft⁻¹K	$\frac{1+6.0K}{1+1.5K}$	$\frac{H_n}{10^3}$ ft⁻¹K	$\frac{H_n y'}{10^6}$ K	b_n ft	$\frac{H_n b_n^2}{10^9}$ ft. K
A	750	78.7	590	-34.9	914	425	2.82	120	944	-34.3	1412
B	1169	49.1	574	-5.3	33	2129	3.52	749	3678	-4.7	165
C	732	15.3	112	28.5	595	537	2.92	157	240	29.1	1329
D	262	0	0	43.8	503	208	3.33	69	0	44.4	1360
Σ	2913		1276		2045			1095	4862		4266
	Mass center $y_m = 1276/29.13 = 43.8'$ $y = 43.8 - y'$					Elastic center $y_e = 4862/109.5 = 44.4'$ $b = 44.4 - y'$					

HORIZONTAL DEFLECTIONS & VIBRATIONS

$H_n = \frac{6E_c}{h^3} \times I_{hn} \frac{1+6.0K}{1+1.5K}$ ft⁻¹K

$f_y = \frac{1}{\Sigma H_n} = \frac{10^{-3}}{1095}$ ft K⁻¹

$f_z = \frac{1}{\Sigma H_n b_n^2} = \frac{10^{-9}}{4266}$ ft⁻¹K⁻¹

$f_y/f_z = 389.6$ ft²

$\frac{\Sigma W y^2}{\Sigma W} = 702.0$ ft²

$e = 0.6$ ft

$e^2 = 0.4$ ft²

$\phi = \left(e^2 + \frac{\Sigma W y^2}{\Sigma W} + \frac{f_y}{f_z}\right) \times \frac{1}{2 \Sigma W y^2} = \frac{267}{10^6}$ K⁻¹

$\psi = \phi^2 - \frac{1}{\Sigma W \times \Sigma W y^2} \times \frac{f_y}{f_z} = \frac{5900}{10^{12}}$ K⁻²

$n_e = \frac{187.7}{\sqrt{f_y}} \sqrt{\phi \pm \sqrt{\psi}}$ cycles/min. (lineal dimension in inches)

$n_e = 187.7 \sqrt{1095 \times 10^3 \left(\frac{267}{10^6} \pm \frac{77}{10^6}\right)} \times \frac{1}{\sqrt{12}}$

Maximum vibration $n_{e1} = 1052$ cycles/min applies to R_1.
Minimum vibration $n_{e2} = 782$ cycles/min applies to R_2.

Rotation centers R_1 & R_2:

$\lambda = \left(e^2 + \frac{\Sigma W y^2}{\Sigma W} - \frac{f_y}{f_z}\right) \times \frac{1}{2e} = 260.7$ ft

$y = \lambda - e \pm \sqrt{\lambda^2 + \frac{f_y}{f_z}}$ $y_1 = 521.5$ ft (R_1) $y_2 = -1.3$ ft (R_2)

SUMMARY: The system vibrates with frequencies n_{e1} & n_{e2} about centers R_1 & R_2. Any periodic force H has parallel components F_1 & F_2 acting on R_1 & R_2 respectively.

Equiv. static force $P = \frac{3 n_e^2}{n_e^2 - n_m^2} \times \Sigma F$ etc.

$F_1 = H \times \frac{d_2}{d_1 + d_2}$

$F_2 = H \times \frac{d_1}{d_1 + d_2}$

E Q U I V A L E N T STATIC FORCES

n_e = natural frequency of foundation in cycles / min.
n_m = forced frequency in R.P.M. = 3600
L = weight of rotating parts
Tuning ratio $j = n_e/n_m = n_e/3600$
Equivalent static coefficient v
a) for $j < 1/2\sqrt{3}$ $v_1 = 12j^2$
b) for $j > 1/2\sqrt{3}$ $v_2 = 3 \times j^2/(j^2 - 1)$ with a max. of 15.0
Equivalent static force $P = v \times L$
vertically: max. $j = 3045/3600 = 0.85$ $v_1 = 12 \times 0.85^2 = 8.7$
$P = 8.7 \times 170^K = 1479^K$
horizontally: max. $j = 1052/3600 = 0.29$ $v_1 = 12 \times 0.29^2 = 1.00$
using minimum $v_1 = 5.0$
$P = 5.0 \times 170^K = 850^K$

WEIGHT OF MACHINE INCLUDING ROTATING PARTS

Turbine	621	Kips
Generator	617	"
Exciter	24	"
Total	1262	Kips

WEIGHT OF ROTATING PARTS

Turbine	74	Kips
Generator	96	"
Total	170	Kips

FIG. 8.—DYNAMIC ANALYSIS OF TURBINE AND GENERATOR FOUNDATION

Provisions of this specification consider that:

1. Natural frequencies of the foundation must be computed for both horizontal and vertical directions.
2. In order to avoid resonance, natural frequencies of the structure should be either lower or higher than the speed of the unit. In either case the difference between the two frequencies should not be less than 20%.
3. Natural frequencies of individual bents should be of the same order of magnitude.
4. The dynamic stability of the foundation can be assured by designing the structure for an equivalent static force. The magnitude of this static force depends on the weight of the rotating parts of the machine and the ratio of natural frequencies to forced frequencies of the foundation.

In the design of the turbine-generator foundation of the Morro Bay Steam Electric Plant, the foregoing criteria were incorporated. In Fig. 8 the dynamic investigation of the foundation is shown.

The turbine generators of the Morro Bay Steam Electric Plant had not yet been set in service when this paper was prepared, and, consequently, performance data of the pedestals were not available (as of 1954). However, the Pittsburg Steam Plant, Pittsburg (Calif.) had been in operation for two years. The turbine generators of both plants are of the same rated capacity, and the design of the foundations followed similar lines. The only adjustments made were to allow for condensers furnished by different manufacturers.

The performance record of the pedestals in the Pittsburg plant is satisfactory. The measured amplitudes of induced vibration of the foundation at deck level do not exceed 0.2 mils with a maximum amplitude of 0.53 mils recorded at the turbine bearing. The output of the machine when these readings were taken was 160,000 kw.

Architectural Features

The location of the plant near the city of Morro Bay and just off a well-traveled scenic highway prompted an architectural treatment that would not obscure the function of the structure and would render it esthetically pleasing. Although the climate is mild and suitable for outdoor operation, the close proximity of the plant to the ocean combined with a humid atmosphere dictated the necessity of a totally enclosed plant.

A fluted aluminum siding was chosen for the walls of the power building. Such sheet siding is flexible in that it can be removed or re-used in the case of expansion; at the same time it provides a tight sheathing.

Service buildings, such as offices, machine shops, and warehouses, are faced with porcelain-enameled steel paneling chosen for its resistance to atmospheric attack. The finish of this material allows for choice as to color, texture, and size of panels. In the Morro Bay plant it provided contrast and distinguished the function of these smaller elements, avoiding the monotony that large areas of a single material would have otherwise produced.

DISCUSSION

Glenn B. Woodruff,[16] M. ASCE.—An excellent description has been presented of the civil engineering features of a modern steam electric power plant. However, to the designer of a comparable facility this is less important than the description of the careful analysis of the various possible alternates which preceded each step in the design. In the latter respect Messrs. Thon and Coltrin have described methods of approach permitting considered judgment as contrasted with following precedent or making hasty decisions.

The upper strata are recent alluvial deposits subject to consolidation under load. Snap judgment would have dictated piles extending to the lower and firmer strata. The careful investigation indicated that differential settlements could be kept within tolerable limits by the rigid frame foundation structure, which, in addition to economy, contained the other advantages mentioned by the authors.

Again it would have been natural for the designers to have followed, without question, the specifications of the ACI code[6] in the design of the heavy concrete stack.

Although it is believed that the assumption of $C = 0.13$, using the Society recommendations,[7] is somewhat conservative, the resulting sections give the best possible distribution of the material for resisting seismic forces.

Similar analyses were used in securing the most economical solutions of the other problems. In this respect the paper is an outstanding example of the approach that should be given to all engineering designs.

Jacob Feld,[17] M. ASCE.—In addition to the many engineering ideas advanced by the authors, a general description has been made of how aluminum materials were used for architectural surfacing of an industrial building. The usual blank wall surfaces, which make such buildings resemble odd-shaped barns, were given pleasing detail by using commercially available fluted sheets in both horizontal and vertical positions. The contacts between the areas so obtained were covered by flat aluminum strips to form frames or borders, which create differences in color reflection and an attractive pattern.

As with many other structures, the control of materials during the construction period was quite difficult and led to possible failure. One problem encountered was the preparation and maintenance of the fine dune sand into the desired slopes. The continuous fine-spray control of the moisture content to the proper amount so as to prevent wind erosion and at the same time protect against liquification and sloughing of 13 acres of surface is no small project. In a somewhat similar problem, the writer used a spray of emulsified asphalt into which fertilizer and selected grass seed had been mixed. The asphalt skin aided considerably in holding the surface, kept the seed where desired, and at the same time discouraged feeding by birds. The asphalt in no way retarded germination and actually seemed to act as a delayed fertilizer once the grass sprouted.

---

[16] Cons. Engr., San Francisco, Calif.

[17] Cons. Engr., New York, N. Y.

With population drifting to arid and semiarid areas, the supply of water becomes more of a controlling problem. The large-scale, sea-water evaporation equipment will be a model for studies in many foreign countries. The problem consists of the preparation of (*a*) potable water, which is known to be expensive; (*b*) water for industrial use, in which appreciable chemical contents can often be tolerated or compensated; and (*c*) water for agriculture where not enough is yet known (1957) as to how far plants can be modified to reduce the necessity for salt removal. With the heavy plant growth in salt-water marshes, it seems possible that food plants can be developed to permit irrigation with sea water from which only a part of the salinity is removed. This is parallel to the development of metal alloys and bearings, which permit the use of sea water for lubrication and cooling, instead of normal equipment, which requires potable water to prevent excessive corrosion and maintenance.

The description of foundation designs, estimates of settlement, and the record of actual settlements is another valuable contribution. The reasons for the choice of a spread-footing mat on the soil as against a piled foundation for the main plant and its heavy equipment are given completely, and the result certainly justifies the decision. However, the mat under the stack is supported on piles. It would be helpful to know why the opposite decision was made in the latter structure, even though the soil conditions seem to have been the same and the loads are comparable. Also, with the expected settlements so closely checked by the measured dishing of the main mat, could not at least half have been compensated for by cambering the mat, thus reducing the necessity for equipment and piping adjustments? However, the scheme of hanging the boilers free of the foundation must have eliminated many adjustment problems.

William W. Moore,[18] M. ASCE.—A most useful description of the many unusual design features of the Morro Bay Plant has been presented. These data will no doubt suggest valuable design concepts and greatly aid designers in other projects. In reference to the foundation features of the plant, there are two factors that warrant particular notice. The first is the results of frequent collaboration between the design engineers and foundation engineers throughout the design period. The second is the attention given to the effects on the structural and design details of the settlement patterns expected as a result of the sequence of foundation construction and loading.

Different foundation design criteria were used for the different plant structures—that is, a rigid mat structure for the main power building, driven piling for the stacks, and spread foundations for tankage and several other smaller structures. These selections were the result of having considered the cost and behavior of each unit individually, and the resulting economy was substantial in comparison with the use of a single foundation design for the entire site. Such results could not be obtained if the foundation exploration and subsequent reports had been made and completed prior to and separate from the development of the detailed structural designs. For this project an initial phase of the foundation exploration was undertaken as an aid to the site selection; this provided some data for preliminary design studies. Then, as the structural requirements and possible foundation types received detailed consideration, supplementary explorations were made of the soil structure at the particular locations of important units. The data obtained from laboratory tests of these soils were used in studies of the capacities, settlements, and costs of alternate foundation schemes being considered to fulfil the structural requirements. This process frequently involved several preliminary consultations between the structural engineers and the foundation engineers, and approximate design studies were often made before the final design was selected for complete analysis.

---

[18] Senior Res. Partner, Dames & Moore ,San Francisco, Calif.

It has often been stated that differential settlements usually cause trouble in engineered structures. However, the construction of an entire project or even of a complete single unit of a project cannot be performed so that the foundation load is applied at a uniform rate over the entire area of the unit. Therefore, the time rates of settlements as well as the total settlements often have an important bearing on differential movements between parts of the plant and on conditions of strain or stress that may be created by these movements. For the subject project the time-settlement studies presented by the authors (Fig. 4(*c*)) were prepared to enable consideration of the effects on the structure that might be caused by settlement during the dewatering period and during the periods of applying structural loads. For these studies the stress conditions in the soils were evaluated by use of tables[19] of stress distribution prepared by Nathan M. Newmark, M. ASCE. The studies were based on consolidation test data obtained in the laboratory, using 2½-in.-diameter-by-1-in.-thick core samples of the soils retained in thin brass sleeves. The sleeves surrounding the samples enabled some of the friable sandy loam and weak clayey soils to be handled and tested with a minimum of sample disturbance.

The settlement data presented by the authors is of particular importance because it gives detailed information from the start of the foundation loading period. In the writer's experience, it has often proved difficult or impossible to maintain reliable settlement records during the construction period. Such data are of major value if valid comparisons are to be made between settlement analyses based on laboratory tests of the soils and those based on the actual field behavior. Too often the only settlement data available are taken after the construction period, and the results may indicate slight settlements, which are sometimes assumed to show that the computed settlements were far too large. The profession certainly has much to learn about analysis and prediction of soil behavior, and data of the nature presented in the paper are invaluable to progress in this direction.

George L. Jordy.[20]—The thoroughness of each step made prior to the selection of the various equipment is quite impressive. Of particular note is the method of determining the proper type of foundation, especially considering the varying strata and subsoil encountered and the complex loading of the mat.

---

[19] "Influence Charts for Computation of Stresses in Elastic Foundations," by Nathan M. Newmark, *Bulletin No. 338*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., November 10, 1942.

[20] Mgr., Chimney Div., The Rust Eng. Co., Pittsburgh, Pa.

However, the writer's primary interest is in the 450-ft-high reinforced concrete chimney which is one of the newer types that handle high-outflow-velocity gases. These chimneys are designed on the basis of various empirical equations concerning the additional stack height that is obtained by emitting the gases at a high velocity so as to nullify, in so far as possible, the combined effect of humidity, temperature, and wind velocity. Naturally, the effect of these three factors can usually be achieved before the plume is bent by one or all of these agents, and this estimated height of plume is termed "added stack height." By utilizing this plume effect, the actual structural height of a chimney is somewhat reduced, and this, in turn, reflects on the initial economy of the structure both above grade and in the foundation and subfoundation requirements.

The Pacific Coast area is one that must be scrutinized carefully as a result of the earthquake potential. The thoroughness with which the chimney was investigated, both for the tentative design—which materially established certain clearances—and for the final design, does credit to the authors.

For more than three decades the writer has witnessed the development and growth of the chimney industry. One must marvel at the ability of some chimneys to resist conditions beyond the scope of their design; on the Pacific Coast there are chimneys that have been designed to resist a wind load of 20 lb per sq ft of projected area, and many of these chimneys have successfully undergone fairly heavy earthquake disturbances. The writer has designed chimneys for the Pacific Coast area as well as in other earthquake areas of the world. It is surprising how engineers have been attempting for many years to establish a correct design procedure in order to provide a safe structure under severe earthquake conditions. Most chimneys involve a difficult design problem, and it is because of this that the professional societies have attempted to achieve a uniform solution. These activities have been emphasized in the method of investigation of the Morro Bay chimney structure.

The writer does not believe that it is proper to design a chimney by applying a uniform earthquake coefficient to the entire structure. Chimney designers must use the theoretical formulas that have been established. At the same time, means must be found whereby the over-all economy of the structure, from a design standpoint, will guarantee the safety desired through use of the theoretical formulas. The graphs shown in Fig. 6 indicate the comparison of certain codes. In principle, they embody the Society code using the seismic coefficient of 0.13 and the ACI code with a uniform seismic coefficient of 0.20, which was generally used in areas of greatest seismic probability. Comparing the curves of the final design with those of the tentative design, and particularly with curve 3, which is predicated on the ACI 1953 code using a seismic coefficient of 0.20 for the lower one-fifth of the height and an increasing coefficient for the remainder of the height, it will be noted that this latter code will provide an excessive amount of rigidity, particularly in the central three-fifths of the unit.

It is the writer's opinion that the chimney in question could only have been improved from a structural-design point of view by either of the following:

1. A seismic coefficient of 0.20 could have been used for the bottom one-fifth of the chimney, and a seismic coefficient of 0.13 or 0.15, for the upper four-fifths of the chimney; increasing either of these latter coefficients in the manner prescribed by the ACI 1953 code would have provided a somewhat more rigid structure in the middle three-fifths without materially increasing the requirements of the final design.

2. For a chimney slightly less rigid, a seismic coefficient of 0.13 could have been used for the bottom 90 ft and increased for the remainder of the height according to the ACI 1953 code.

These recommendations for an improvement of the design are not intended as a reflection on the designers' judgment, but rather as an expression of the writer's opinion that additional rigidity is needed in the central three-fifths of any chimney structure subject to earthquake forces. In the Pacific Coast area the most pertinent illustration of this requirement might be the earthquake effect on the 375-ft chimney at the Seal Beach Power Station, Seal Beach (Calif.). This chimney was superimposed on a structure, and the upper part of the chimney was substantially affected by an earthquake in 1933.

The writer again wishes to compliment Messrs. Thon and Coltrin for their excellent analysis in connection with the Morro Bay chimney. The writer believes that a chimney structure has been provided that will give many years of excellent service. The best available materials have been used in all parts of the unit to resist any possible defects due to erosion, corrosion, and velocity pressure that might develop in these high-velocity units.

J. George Thon,[21] M. ASCE, and Gordon L. Coltrin,[22] A. M. ASCE.—The opinion was expressed by Mr. Woodruff that the selection of the seismic factor of 0.13 for the design of the 450-ft-high concrete stack was conservative. As explained by the writers, this factor was selected because of the desire to have the bending moments at all sections at least as large as those produced by a constant acceleration equal to 20% of gravity. The latter criterion has been commonly used in California when designing structures as important as a central steam electric station, where interruption of services might not only cause substantial financial loss but also widespread public inconvenience.

Mr. Feld requested the reasons for selecting a piled foundation under the stack, whereas a floating-mat foundation was used under the main building. The reasons were:

1. The unit soil pressure under the buildings was reduced by 1.6 kips per sq ft by the removal of 16 ft of soil and the construction of a basement. This was not readily adaptable to the foundation of the stack. Also, in order to keep the unit pressures within allowable limits, an excessively large mat would have been required.

2. The layer of weak to moderately firm clayey soils at the south end of the plant proved to be of variable thickness. The consolidation of this stratum might have caused uneven settlements, a most undesirable feature in a 450-ft-high slender structure.

---

[21] Chf. Civ. Engr., Power Div., Bechtel Corp., San Francisco, Calif.

[22] Superv. Civ. Engr., Pacific Gas and Electric Co., San Francisco, Calif.

Mr. Moore noted the value of the settlement records. It should be added that settlement readings were continued and that subsidence of bench marks 2 and 5 in Fig. 4 ceased at approximately $1\frac{3}{4}$ in. Points 8 and 13 came down to $1\frac{1}{2}$ in., and no further movement has taken place. These observations were taken at monthly intervals, the last being in February, 1956.

Mr. Jordy suggested that it might have been more structurally sound if a constant seismic coefficient of 0.20 had been used for the bottom one-fifth of the stack, and if 0.13 or 0.15 had been adopted for the upper four-fifths and distributed according to the 1953 ACI code. The writers considered a similar solution; however, after considerable thought, they decided against such an expediency, which would have failed to comply with either the ASCE or the ACI codes.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2920

DEVELOPING PORT FACILITIES ON HOUSTON'S SHIP CHANNEL

BY FRANK H. NEWNAM, JR.,[1] M. ASCE

WITH DISCUSSION BY MESSRS. ERSEL G. LANTZ; AUSTIN E. BRANT, JR.; AND FRANK H. NEWNAM, JR.

SYNOPSIS

In the past virtually all public and private wharves at the Port of Houston (Tex.) have been of the marginal type because of the unusual nature of the Houston Ship Channel. Unlike most deepwater ports, the turning basin of the Port of Houston is 50 miles from the open sea (the Gulf of Mexico) and was developed by dredging a narrow, shallow, winding stream known as Buffalo Bayou. As an inducement to industry, a strip of land, 2,500 ft deep on each side of the center of the channel, was taken into the city of Houston but was declared exempt from city taxes. As a result, industrial facilities and their private wharves were forced to be developed as close as possible to the channel, and the utilization of marginal wharves was thereby dictated.

The property owned by the Port of Houston, on which the present public wharves are located, is relatively shallow in depth, and its wharves have also been of the marginal type. However, the port has recently acquired a tract of land of sufficient depth to consider constructing pier-type wharves and slips. Because of this fact and the desirability of properly planning in advance for the most economical ultimate layout of the new area, detailed studies were undertaken for an approved plan of development for this area and for the estimated cost of the incremental parts. These studies included visits to other major deepwater ports in the United States and conferences with representatives of the various agencies that will utilize the new facilities.

As a result, a table of basic criteria and dimensions was adopted for marginal-type facilities and pier-type facilities. After a careful examination was made of several possible layouts, the "marginal-wharf" type of development was recommended as being the most satisfactory for the site conditions.

NOTE.—Published, essentially as printed here, in May, 1956, in the Journal of the Waterways and Harbors Division, as *Proceedings Paper 965*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Partner, Lockwood, Andrews & Newnam, Cons. Engrs., Houston, Tex.

Research at the beginning of the study revealed a lack of information and basic criteria. Therefore, it is believed that the criteria developed, the factors considered, and the conclusions reached will be of value to others considering an expansion of port facilities.

INTRODUCTION

The Houston Ship Channel of Houston (Tex.) is probably the most unusual deepwater port in the United States. As shown in Fig. 1, the harbor frontage, which is referred to as the Port of Houston, extends along both sides of the Houston Ship Channel for 25 miles. The turning basin, at the head of deepwater navigation, is 50 miles from the Gulf of Mexico and 58½ miles from the end of the jetties. The upper part of the channel has a bottom width of 250 ft and a depth of 36 ft, but it has a proposed ultimate project depth of 40 ft and a channel width of 300 ft. The history of the ship channel is interesting because local interests paid one-half of the initial cost of construction in a field in which the federal government has traditionally assumed all the burden. In the amount of tonnage handled, the Houston Ship Channel has ranked second of the deepwater ports in the United States.

In order to attract industry a strip of land, 2,500 ft deep on each side of the center of the channel, was taken into the city of Houston but declared exempt from city taxes. This forced the development of industrial facilities to be as close as possible to the channel, thereby helping to dictate the utilization of marginal-type wharves.

Although the Harris County-Houston Ship Channel Navigation District (the Port of Houston) owns and operates considerable port facilities, these facilities are a small percentage of the total along the ship channel. The tracts of land owned by the district are relatively shallow in depth, and therefore all wharves constructed by it to date (1956) have also been of the marginal type.

However, the Port of Houston purchased a tract of land southeast of the public grain elevator and less than 1 mile downstream from the turning basin, which, together with two small adjacent tracts planned for purchase, contains 328 acres and sufficient depth to permit the consideration of the construction of slips for virtually the first time. This property has a total channel frontage of 9,000 ft and an average depth of approximately 1,600 ft. The tract is easily served by adjacent roads, utilities, and railroads.

An important factor in considering the development is the topography of the land, which is at El. +25 to El. +30 along the channel and rises to approximately El. +40 at the rear. The disposal of spoil from the area will be expensive inasmuch as the closest dumping grounds for hydraulic spoil disposal are more than 1 mile distant, and a considerable distance farther for spoil hauled away in vehicles. The problem of providing spoil-disposal areas for maintenance dredging and for widening and deepening operations is rapidly becoming a serious problem for the upper section of the ship channel.

The 328-acre tract of land is the last tract of any size that is available to the Port of Houston near the turning basin for the expansion of the port facilities.

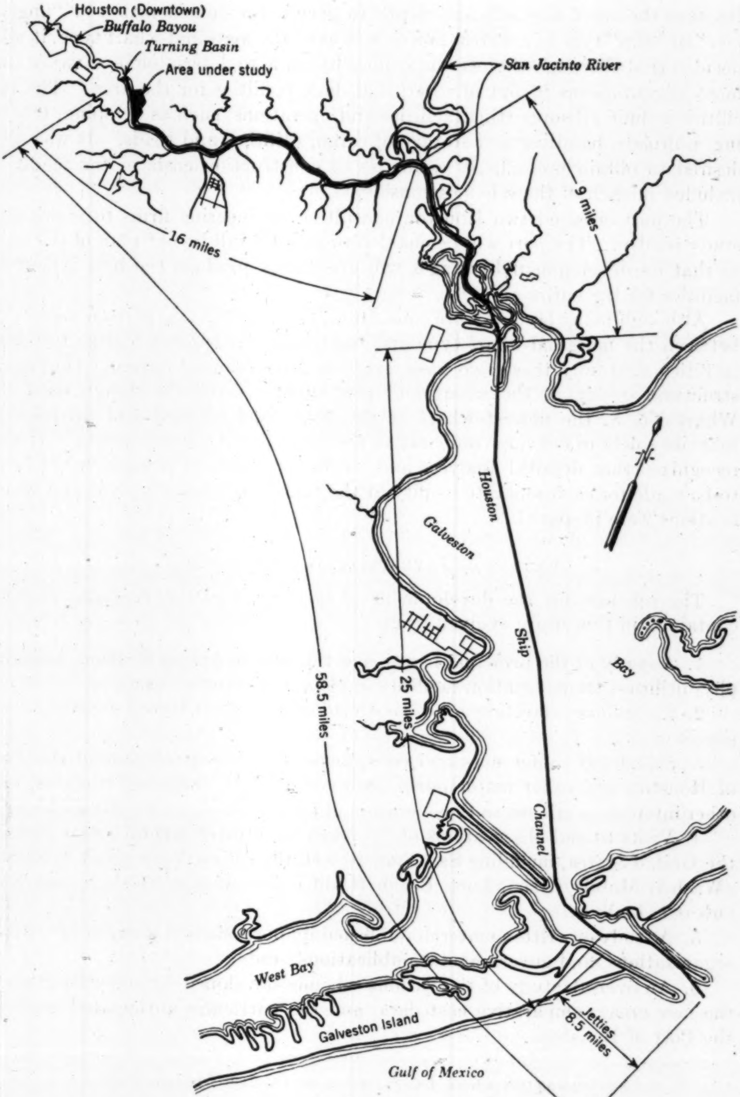

Houston (Downtown)
Buffalo Bayou
Turning Basin
Area under study
San Jacinto River
9 miles
16 miles
Houston
Ship
Channel
Galveston
Bay
25 miles
58.5 miles
West Bay
Galveston Island
Jetties
8.5 miles
Gulf of Mexico

FIG. 1.—HOUSTON (TEX.) SHIP CHANNEL

Because the tract has sufficient depth to permit the consideration of "finger-pier" or "slip" type of construction as well as of the marginal-wharf type, it was decided that thorough engineering studies be made to determine in advance the most advantageous layout of wharf and dock facilities for the area. The facilities include all supporting facilities and operations, such as shipping, trucking, railroads, handling and storage of cargo, utilities, and roads. It was also desired to obtain preliminary estimates of construction costs of the facilities included in each of the schemes considered.

The port engaged two Houston consulting engineering firms to make this study jointly. The port will be enabled to adopt a full layout plan of the area so that its development by stages will eventually produce the best layout of facilities for the entire area.

Although many layouts were considered, the essential comparison made was between the marginal-wharf type and the finger-pier type of marine terminal facilities, and only these two basic schemes are examined herein. One basic structural design of the wharf and pier substructure—the design used for Wharf No. 8, the newest wharf at the port—was selected and used in all schemes solely to develop comparative cost data based on 1955 prices. It was recognized that detailed analyses and, probably, different designs would have to be made for each separate facility at the time that contract plans and specifications were prepared.

EXTENT OF INVESTIGATIONS AND STUDIES

The schemes for the development of the area and the recommendations contained in this study evolved from:

1. A study of the principal features of the site, including location, topography, utilities, transportation facilities, and probable future usage;
2. Preliminary development of several basic types of layout considered for the site;
3. Scheduled conferences and consultations with port officials of the Port of Houston and other major ports, railroad officials, shippers, truckers, and other interests pertinent to the operation of the facilities and handling of cargo;
4. Visits to and observations of the latest facilities of several major ports of the United States, including San Francisco (Calif.), New York (N. Y.), Seattle (Wash.), Mobile (Ala.), Long Beach (Calif.), Los Angeles (Calif.), and New Orleans (La.);
5. A study of written material published by port officials, surveys by various organizations, and governmental publications; and
6. An over-all study of the various schemes developed for the utilization of the new area, comparative cost data, and the particular anticipated needs of the Port of Houston.

INFORMATION FROM INSPECTION OF OTHER MAJOR PORTS

General.—The major ports of San Francisco, New York, Long Beach, Los Angeles, New Orleans, Seattle, and Mobile were visited in order to observe the most recent and modern layouts of marine terminal facilities of both marginal-wharf type and finger-pier type, and to discuss such layouts with and to receive the recommendations of key port officials (both operators and engineers) of each port authority. Basic dimensions, arrangement of facilities, amount of storage and handling space desirable, and other principal features of both the finger-type and marginal-type piers were studied. From the standpoint of operation the advantages and disadvantages of the use of finger-type piers and marginal-type wharves, or combinations of both, were studied, and recommendations were sought concerning the best utilization and development of the property.

The information gained in observing the new wharf facilities at each major port has served as an invaluable guide and check. Observing older and outmoded installations has also been beneficial because the faults and deficiencies stressed by the port officials serve as adequate warning in the planning of similar installations. In examining the information and recommendations received, the basic differences between the ports observed and the Port of Houston—such as types of cargo handled, arrangement and size of waterways, configuration of waterfront, land space, access to waterfront, and types of land transportation principally used in handling cargo, among others—were considered in preparing the layouts of wharves and other facilities.

San Francisco.—The marine terminal facilities serving the Port of San Francisco are virtually all of the pier type and are located along the west shore line of San Francisco Bay. This bay, with its vast area of deep water, provides ample space for anchorages and the proper handling and turning of vessels and, therefore, is ideally suited for finger-type piers. Because this port is one of the older ones the majority of the piers were designed at a time when land shipment was by rail almost exclusively. At the present time, such shipments are approximately 25% by rail and 75% by truck. New facilities at this port, such as the Mission Rock Terminal, make full provision for a large paved area inside the pier for the maneuvering, parking, and docking of a large number of trucks.

It was recommended that piers be only one ship's length (from 800 ft long to 850 ft long) because of the difficulty of berthing the inside ships for piers that are two ships in length. However, when the latter piers are used they should be from 1,200 ft long to 1,400 ft long. In either case the piers should be wide enough to permit a ship to be berthed across the end of the pier. Other recommendations were that (*a*) the width of the slips should be one ship's length, with 225 ft as a minimum (300 ft desirable); (*b*) the desirable width of the transit shed should be 200 ft; (*c*) the width of the paved area between back aprons should be a minimum of 125 ft; (*d*) there should be no railroad trackage in transit sheds; (*e*) the desirable dock height for trucks should be 48 in.; and (*f*) the width of the rear apron should be 15 ft.

New York.—New York is the largest port in the United States and also one of the oldest. It has a variety of wharf types, but the finger-pier type of construction predominates. For all the port facilities there is an ample area of water in front of the wharves to permit the maneuvering of vessels during docking operations without interfering with the other water traffic.

One of the newest facilities, and one apparently considered to be one of the most satisfactory in the area, is the Port of Newark (N. J.) located on a large artificial channel connecting into the west shore of Newark Bay. This facility is of the marginal-wharf type and is considered to be more efficient and satisfactory from the standpoint of operation than any of the finger-pier type of wharves. Two interesting features of this facility are (1) the offices located in each shed opposite each berth at a second story level, which permits full view of all operations and provides storage space under the offices; and (2) special heated cargo rooms for valuable cargo storage in the sheds.

From 80% to 90% of the land shipment is by truck, and a 100-ft-wide area is paved behind the rear apron of the transit sheds. There is a definite advantage in having large paved areas behind and separate from the shed area that can be used for open storage and transhandling of items that are not fast moving.

The officials of the Port of New York Authority, as in the case of the Port of San Francisco, considered that the ideal width for a transit shed was 200 ft. It was also felt that a minimum distance of 900 ft beyond the end of the piers was needed for the maneuvering of vessels in docking for this type of facility. This distance would be exclusive of any space required for the passage of other vessels.

Other recommendations included:

1. The transit shed area per berth—a minimum of 90,000 sq ft for Class A berths and from 45,000 sq ft to 90,000 sq ft for Class B berths;
2. The width of slips—a 300-ft minimum if the slips are at right angles to the channel, otherwise they should be wider;
3. The width of the front apron—50 ft;
4. The width of the rear apron—17 ft with an 18-ft canopy;
5. The dock height for trucks—48 in.; and
6. The lighting in transit sheds—industrial-type, fluorescent lighting at 20-ft centers, providing 8 ft-c in the warehouse area plus transparent sheets in the roof for daylight help.

Long Beach.—The Port of Long Beach is essentially a man-made harbor consisting of an inner and an outer harbor area and a large number of modern facilities constructed by the Port of Long Beach and the United States Department of the Navy. Although both pier and marginal wharves are operated, port officials believed that marginal wharves were the most desirable and economical to operate. Here again, the new transit sheds are 200 ft wide, and a clear height of 20 ft was desirable. The wharf apron is 50 ft wide with two railroad tracks 9 ft and 22 ft from the fender line. There are two spotting tracks and one running track behind the sheds.

There is a 300-ft open wharf and storage area between sheds, which ramps from wharf-apron level at the front to track level behind the shed, and the wharf lead tracks run up this ramp grade. This scheme presents an easy truck and railroad car approach to the wharf apron and apparently does not present any handicap in handling cargo on this open storage area.

Los Angeles.—This port also has both the finger-pier and marginal-type wharves, located at San Pedro Bay, which provide ample space for the maneuvering of vessels. Front-apron widths are generally from 35 ft to 37 ft, rear aprons are 16 ft wide with 16-ft canopies, and transit-shed widths are 200 ft. Paved areas, from 100 ft wide to 150 ft wide, were provided behind the transit sheds for the maneuvering and docking of trucks. The open wharf between sheds is ramped up from truck and track level behind the shed to the wharf-apron level.

The piers and slips on the terminal island fronting the main channel are built diagonally to the channel, with the slips being 300 ft wide and 100 ft long and with piers being approximately 450 ft wide. These piers and slips are considered ideal for the operational needs of the railroads, which handle a majority of the tonnage, the railroad classification and storage yard being located immediately behind the end of the slip.

New Orleans.—The marine terminal facilities of the Port of New Orleans are located along the Mississippi River in Louisiana and consist generally of marginal-type wharves. The Mississippi River is sufficiently wide to permit the safe handling and berthing of ships at piers, but the developed area adjacent to the river in New Orleans restricts the depth of land available for such terminal facilities.

Many of the marine facilities at New Orleans are quite old and outdated, and, because of the extensive use of manual labor in handling cargo in this area, many of the transit sheds are 100 ft wide or less. There is a minimum of railroad tracks and very little yard space because a major part of the cargo is handled by trucks.

This port is correcting many of its unsatisfactory conditions by constructing the new Napoleon Avenue wharf, which will have a wharf apron 47 ft wide, a transit shed 200 ft wide with a 19-ft clear height, and a rear apron 30 ft wide with a 15-ft canopy.

Mobile.—The most modern facility at this port is a marginal-wharf-type marine terminal with a 30-ft-wide apron and a 200-ft-wide transit shed with a clear height of 20 ft. As in the case of the Port of Newark, there is an office at second story level for each berth. The loading apron at the back of the shed is 25 ft wide and differs from most of the other new facilities in that no permanent cover is provided.

Seattle.—Seattle has two harbors: One is a fresh-water harbor serving the lakes, and the other is a salt-water harbor with the principal facilities located in Elliott Bay. This bay is spacious and deeply sheltered. It is free of hazards to navigation and is ideally suited to the pier-type layout, and the majority of facilities are of this type. The observations made in this visit and the comments and drawings furnished by port officials have added useful data for studies of pier-type layouts.

CONFERENCES WITH USING AGENCIES

General.—Conferences were scheduled with railroad officials, operators of steamship and stevedoring companies, and representatives of trucking interests. A separate conference was scheduled with each group because each interest had its own operational requirements. It was the main purpose of these conferences to obtain a frank examination of the basic needs and operation requirements of each group and to incorporate their recommendations and proposals, in so far as possible, in the layout of the new terminal facilities. These interests are necessarily conflicting in many respects, and therefore the final layout and arrangement must be a compromise.

Steamship and Stevedoring Companies.—All steamship and stevedoring company representatives favored the marginal-type layout. This type facilitates the berthing of ships and is considered safe in general for the handling and berthing of ships in the comparatively small waterfront area provided by the Houston Ship Channel. It was felt that the pier-type construction would result in slow, costly, and hazardous berthing operations on the narrow channel. It was also felt that any type of finger-pier layout should provide a minimum distance of 900 ft from the end of the pier to the far edge of the ship channel in order that ships may safely enter and leave slips. This 900-ft distance does not include any room for the passing of other vessels, which would delay traffic on the ship channel.

TABLE 1.—DIMENSIONS AND CRITERIA AGREED ON BY STEAMSHIP AND STEVEDORING COMPANIES

Item	Dimension or Criteria
(*a*) FINGER PIER	
Width of slip	300 ft
Slip length (one ship length)	800 ft
Width of warehouse (transit shed)	200 ft
Overhead clearance in warehouse	18 ft
Width of paved loading area between warehouses (transit sheds)	150 ft
Width of front apron	50 ft
Width of rear loading platform[a]	20 ft
Number of railroad tracks on front apron[b]	2
Number of tracks behind each warehouse	2
Double railroad crossovers, front apron	1 at center of each berth
Single railroad crossovers, front apron	1 at each end of berth
(*b*) MARGINAL WHARF[c]	
Width of paved loading area behind warehouse (transit shed)	100 ft
Number of railroad tracks on front apron[d]	3
Number of railroad tracks at rear of warehouse[e]	2 or 3

[a] Rear loading platform should be covered. [b] Front apron tracks should be far enough apart to permit operation of crane on second track. [c] Dimensions or criteria are the same as in (*a*) with noted exceptions. [d] Running track needed along with two spotting tracks because tracks serve a greater number of berths than in finger-pier layout. [e] Minimum number of tracks is two, but preference for three tracks was expressed.

The minimum basic dimensions and arrangements of facilities shown in Table 1 for the finger-pier type of layout were generally agreed on.

Railroad Companies.—In the conferences with representatives of the various operating railroads, these representatives all agreed that the finger-type layout is most desirable from the standpoint of the operation requirements of the railroads, because this layout effects the most economical and most simple switching and handling of cars within the wharf area and makes it possible to reduce the time required in moving cars from the spotting tracks at the loading aprons to the car storage yards.

It was agreed that the following factors were desirable and applicable to both the marginal-type construction and finger-pier-type construction:

1. A maximum grade of 1% for connecting tracks;
2. A minimum curve of 12° 30′, except in a few special critical areas in which 15° curves will be satisfactory;
3. A minimum spacing of 13 ft between adjacent tracks, although in order to provide double crossovers the limited space available at the front apron may require an increased spacing of approximately 19 ft; and
4. A storage yard capable of containing 500 cars for the entire area with storage track lengths of approximately 50 car lengths.

TABLE 2.—SUMMARY OF ANSWERS RECEIVED REGARDING LONG BEACH (CALIF.) HARBOR FACILITIES

HARBOR FACILITY	Minimum	Maximum	Average
Depth, at low water, of main channel to outer and inner harbor, in feet	32	40	40
Bottom width of main channel to outer and inner harbor, in feet	300	2,000	600
Depth of slips alongside marine terminals, in feet	32	40	35
Width of slips between piers of approximately 2,000-ft length, in feet	300	500	400
Length of berth for largest ocean-going vessel, in feet	500	1,200	850
Length of berth for average-size, ocean-going vessel, in feet	450	600	500
Width of transit shed, in feet	140	250	200
Vertical clearance between floor and lower chord of truss in shed, in feet	16	25	20
Longitudinal width between columns, in feet	30	75	40
Water-side door openings, continuous (cont.) or every other panel (e.o.p.)	cont.	e.o.p.	e.o.p.
Land-side door openings, continuous (cont.) or every other panel (e.o.p.)	cont.	e.o.p.	e.o.p.
Vertical clearance of water-side door openings, in feet	12	20	16
Vertical clearance of land-side door openings, in feet	12	18	14
Widths of water-side door openings, in feet	12	22	18
Widths of land-side door openings, in feet	12	20	16
Vertical clearance of door openings in end of shed, in feet	14	25	20
Width of door openings in end of shed (two doors), in feet	14	20	18
One-story or two-story transit sheds	1	1	1
Width of roadway space between sheds (including track facilities), in feet	150	500	200
Width of rear loading platform of transit shed, in feet	10	15	12
Number of railroad tracks on apron wharf of shed	2	2	2
Number of spot tracks in rear of transit shed	2	3	2

For a marginal-type wharf, three tracks, two spotting tracks, and one running track would be desirable for the front apron, with a single crossover at the end of each berth extended to the running track adjacent to the transit shed. An outlet from the running track to the storage yard should be located at the end of each warehouse.

At the rear of the rear apron there should be two spotting tracks, with the first track 8½ ft from the rear apron edge, the second track 13 ft on centers behind, and the rearmost (running) track approximately 100 ft from the rear apron edge to allow for truck loading and unloading at the rear apron. Crossovers for these tracks should be provided at the end of each warehouse. The independent switching of cars for each berth could also be obtained by having independent "dead-end" spotting tracks at each berth, with lead-ins from the running track at one end of each berth.

Trucking Companies.—Representatives of the trucking companies had no particular preference for either the marginal-wharf layout or finger-pier-type layout provided that there should be a paved area 100 ft deep behind the rear apron for marginal-type wharves and a 150-ft paved loading area between back aprons of warehouses for finger piers.

It was recommended that the rear aprons of warehouses be 20 ft wide, be covered for the full width, and have a platform height of 48 in.

Driving lanes inside the transit sheds were also recommended to permit the handling of large volumes of packaged merchandise, such as sugar, flour, and salt. The clear span inside should be a minimum of 18 ft with a few 18-ft-high doors, although a 10-ft door height or a 12-ft door height was considered generally satisfactory for truck handling.

One large (150-ton) loading device and one small loading device should be provided for loading and unloading cargo in the open storage area. Ample space for parking and turning of trucks in the parking and holding areas was also requested. Attention was directed to the increasingly higher percentage of land shipments by trucks instead of by rail from major ports of the country.

Other References.—Studies were made of other useful data concerning desirable basic dimensions and layout of marine terminal facilities. These references included United States Government publications, such as the port series published by the Corps of Engineers (United States Department of the Army) and the United States Maritime Commission, and the 1953 report of the Port Development and Construction Committee of the American Association of Port Authorities. This committee sent questionnaires to various ports concerning the basic dimensions of terminal facilities and a second set of questionnaires to steamship and stevedore companies on the Pacific Coast for use in future development at the Port of Long Beach. The information in these questionnaires is summarized in Table 2.

Basic Dimensions and Criteria Adopted

After all the data previously described had been reviewed and analyzed, it was considered necessary to adopt certain standards for basic dimensions, arrangement of facilities, and other criteria common to the various wharf facilities in order to compare alternate schemes on an equivalent basis. Although an attempt was made to select the dimensions and criteria that at this time seemed the most satisfactory for these facilities, it was emphasized that each separate facility should receive a more thorough and careful analysis during the preparation of contract plans and specifications. It was also felt that many of these basic dimensions could be varied to suit the particular facility as long as the variations did not interfere with the ultimate coordinated development of the layout that was finally selected and adopted for this area.

For the marginal-wharf type of development a pattern was adopted that provided two berths with sheds followed by one open wharf. Because each berth was 600 ft long and because the shed allocated to each covered berth was only 500 ft long, considerable study of the spacing of sheds along the wharf was required, and first consideration was given to centering the sheds on each berth. This left an open space between sheds that apparently would have little use; therefore, it seemed more desirable to place the two 500-ft sheds together and make one shed 1,000 ft long with firewalls. This arrangement leaves an 800-ft-long open wharf between sheds on the marginal-type facility. It should be mentioned that the arrangement of two berths with sheds followed by one berth with an open wharf not only provides an economical balance between the relative needs for open storage and sheds, but it also permits railroad entrance to the front tracks at each third berth.

In adopting criteria and dimensions for transit sheds, because of the problem of pilferage no provisions were made for truck and railroad-car traffic inside the shed as part of the normal cargo-handling operation. However, using the plans presented herein, it would be possible for trucks to enter the shed from the rear side or the ends.

Visits to other major ports of the United States have indicated the widespread use of, and the urgent need for, paved areas behind the wharf facilities for personnel parking, holding yards for trucks, and open-cargo storage. When new marine terminal facilities are built, thorough studies should be made to determine the quantity and location of such paved areas. Although the exact usage cannot be defined at this time, the need for such paved areas has been recognized, and an amount has been included in the cost estimate for these areas, located along the access road paralleling the waterfront, and for a truck holding yard.

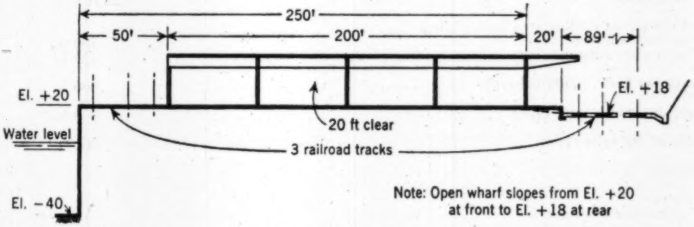

250'
50'
200'
20'
89'-1
El. +20
El. +18
Water level
20 ft clear
3 railroad tracks
El. -40
Note: Open wharf slopes from El. +20 at front to El. +18 at rear

FIG. 2.—SECTION OF WHARF AND SHED, MARGINAL-WHARF PLAN

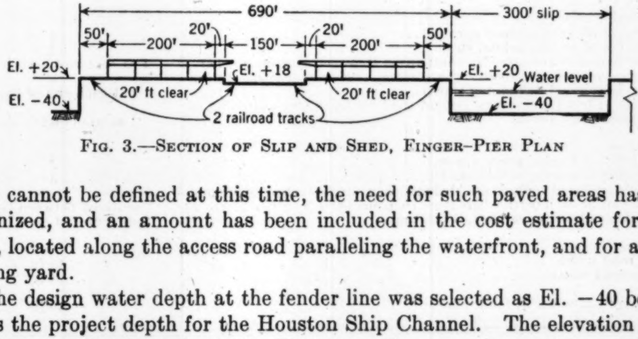

690'
300' slip
50'
200'
20'
150'
20'
200'
50'
El. +20
El. +18
El. +20
Water level
El. -40
20' ft clear
20' ft clear
El. -40
2 railroad tracks

FIG. 3.—SECTION OF SLIP AND SHED, FINGER-PIER PLAN

The design water depth at the fender line was selected as El. −40 because this is the project depth for the Houston Ship Channel. The elevation of the wharf apron adjoining Wharf 16 is El. +18. However, the two newest wharves in the turning-basin area are at El. +21. El. +20 was selected as the apron height in the new area because a 2-ft differential can be tied into Wharf 16 without too long a transition section. The reasons for selecting El. +20 or El. +21 for aprons at this port are:

1. The high ground on which the wharves must be built makes a high elevation desirable from the standpoint of economy in the removal of earth and the connection with adjacent railroads.

2. The steamship and stevedoring companies prefer the high elevation.
3. The combination of high tides from tropical hurricanes, although infrequent, plus heavy rainfall in Buffalo Bayou and San Jacinto River (Texas), which usually accompanies tropical hurricanes, can produce extremely high water levels in the ship channel.

TABLE 3.—Basic Dimensions and Criteria Adopted

Item	Dimension or criterion
Wharf apron	
Height above mean low water	20 ft
Width	50 ft
Transit shed	
Width	200 ft
Length per berth	500 ft
Clear height (bottom of roof truss to floor)	20 ft
Width of rear loading platform	20 ft
Width of canopy, rear loading platform (extended to center line of first track)	28 ft 6 in.
Width of paved loading area back of shed	
Marginal wharves	100 ft
Finger piers (distance between rear loading platforms)	150 ft
Length per berth (marginal type)	600 ft
Single slip length (pier type)	800 ft
Width of slip (pier type)	300 ft
Railroads	
General	
Minimum track spacing	13 ft
Maximum degree of curvature (where economically possible)	12°30′
Maximum grade (where economically possible)	1.0%
Storage yard capacity	500 cars
Length of storage yard	50 car lengths
Marginal type	
Number of tracks, wharf apron	3
Number of tracks, behind shed	3
Double railroad crossovers, wharf apron (between spotting tracks)	1 at center of each berth
Single railroad crossovers, wharf apron (between spotting tracks and extended to running track)	1 at each end of berth
Single railroad crossovers, behind shed (independent switching of cars at each berth)	1 at each end of shed
Pier type	
Number of tracks, wharf apron	2
Number of tracks, behind shed	2
Single railroad crossovers, behind shed (independent switching of cars at each berth where more than one berth length of pier)	1 centered between berths
Double railroad crossovers, wharf apron	1 at center of each berth
Roadways	
Main road width	44 ft
Access road width	24 ft
Design depth of water at fender line	− 40 ft

The basic dimensions and arrangements adopted are listed in Table 3. Fig. 2 shows the principal dimensions and elevations for a typical section of wharf and shed under the marginal-wharf plan of development. Fig. 3 shows the principal dimensions and elevations for the slip, the shed, and the area between sheds on a typical cross section of the finger-pier plan of development.

At the beginning of the studies, every effort was made to find previous reports, articles, or other publications that furnished criteria and basic dimensions for the development of marginal-type wharf facilities, or pier-type wharf

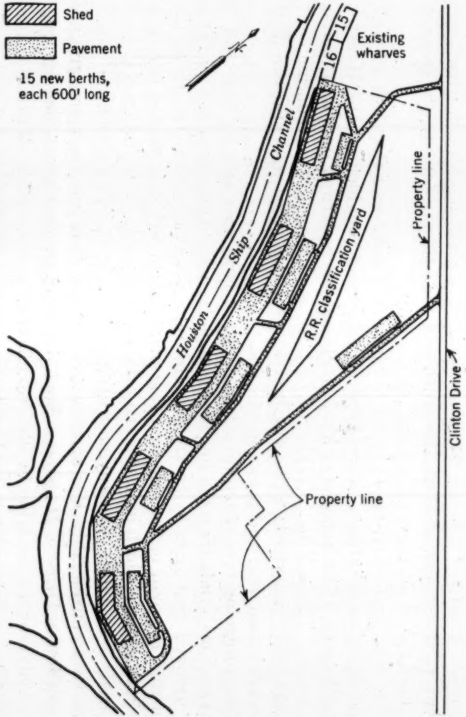

Shed
Pavement
15 new berths, each 600' long
Existing wharves
16
15
Houston Ship Channel
R.R. classification yard
Property line
Clinton Drive
Property line

FIG. 4.—MARGINAL-WHARF PLAN

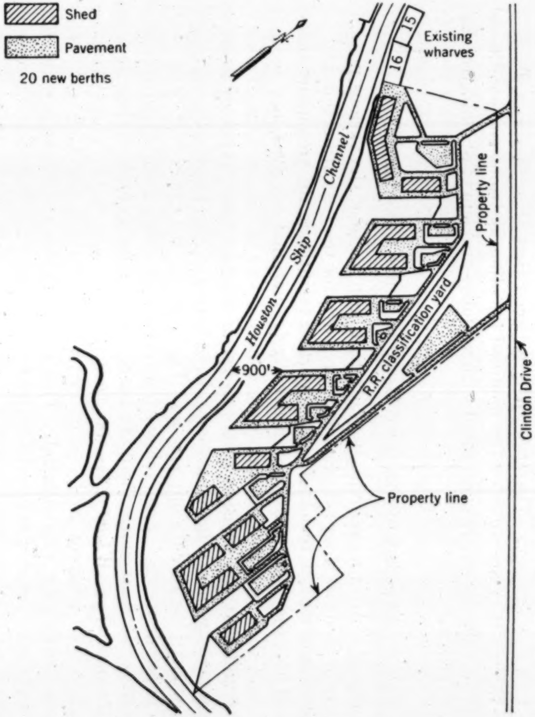

Shed
Pavement
20 new berths
Existing wharves
16
15
Houston Ship Channel
900'
R.R. classification yard
Property line
Clinton Drive
Property line

FIG. 5.—FINGER-PIER PLAN

facilities, or both. Remarkably little information was found to have been published on this subject. Therefore, it is believed that the criteria in Table 3, even though they are considerably modified for other locations and conditions, will be of value to others confronted with a similar problem of planning the layout of terminal facilities.

ANALYSIS OF THE TWO BASIC LAYOUTS CONSIDERED

General.—After a preliminary study had been made of the various possible layouts of facilities, four schemes were prepared for the best utilization and development of marine terminal and supporting facilities in the area under study. One of the four schemes concerned entirely marginal wharves, one entirely piers and slips, and two schemes were combinations of the preceding two. It was found that the decision rested between the two basic types of layouts; therefore, only these schemes are considered herein. Fig. 4 shows the marginal-type-wharf plan of development, and Fig. 5 shows the finger-pier-type-wharf plan of development. Each scheme has certain advantages and disadvantages. Therefore, the final choice should be based on a careful analysis of these factors.

Marginal-Type-Wharf Plan of Development.—The best layout utilizing marginal-type wharves is shown in Fig. 4, in which transit sheds and wharf aprons parallel the ship channel. This plan provides for fifteen new berths; ten first-class, covered steamship berths; and five open wharves. Because of the small scale the railroads are not shown in Fig. 4; however, the location of railroads in relation to the front apron and rear apron are shown in cross section in Fig. 2. From the running track in the rear and from the classification yard, railroad tracks connect with the tracks on the front of the apron by crossing each open wharf. There are two tracks entering across each open wharf, one passing to the transit shed on the south and the other passing to the front of the transit shed on the north of the open wharf.

The marginal-wharf-type layout is considered safer and the most suitable for navigation operations on the type of waterfront provided by the Houston Ship Channel, which consists of a relatively narrow waterway on a meandering alinement. This scheme is preferred by both steamship and stevedoring companies and is considered to be the best one for the safest and also the most economical handling and berthing of ships.

The foregoing layout required less depth of land area and far less spoil of excavated materials than the finger-pier-type layout. Quantities of excavated and dredged materials to be spoiled are approximately one-sixteenth of the quantities required in the scheme using piers, which will be reflected herein in the cost estimate which indicates the total costs of excavation and spoil of such materials. In addition to any cost savings the selection of this scheme would mean that considerably less spoil would be placed in the dumping grounds across Clinton Drive in Houston. Such dumping grounds are needed for long-term maintenance dredging in the ship channel, and, because all areas around the turning basin are becoming highly developed and industrialized, such spoil areas are becoming increasingly difficult to obtain. An additional advantage is that no land is lost to dredging behind the harbor line, and the depth of land not used for waterfront facilities may be used for open storage, personnel parking, truck holding yards, storage warehouses, and for other future needs.

There are no slips or waterways behind the harbor line in this scheme as in the finger-pier-type layout. Therefore, the federal government will maintain the ship channel at project depth and up to within 50 ft of the fender line of the wharves.

The unbroken waterfront apron facilitates the moving of cargo in emergency operations from one shed or berth to another shed, or to a waiting ship up or down the channel. This type of waterfront also provides ready access and ample space for fire boats during fires. In addition, fires at one berth would be more remote from, and therefore less hazardous to, other berths than in schemes utilizing a pier-type construction.

The 100-ft wide paved area behind the transit sheds provides ample room for the maneuvering and parking of trucks, and trucks have access to the full length and the ends of every transit shed. Because the running track behind sheds serves the entire waterfront, the rail traffic through the paved area is heavier in this scheme than in the other. However, there are fewer tracks crossing the access road that parallels the waterfront than in the pier-type layouts.

The railroad companies do not consider the marginal-wharf-type plan of development as desirable as the finger-pier-type facility inasmuch as the latter plan is considered to provide the better scheme for the most economical handling and switching of cars between the wharf apron and the storage and classification yard.

Fewer lineal feet of bulkhead per berth are required for wharves of the marginal type than for the pier-and-slip type because end treatment of slips and extra length of slips over berthing distance are required in the pier-and-slip type of wharf facility.

The foregoing layout has one-fourth less berths because the marginal-wharf layout parallels the channel and does not utilize any land depth for berthing space.

Finger-Pier-Type-Wharf Plan of Development.—Fig. 5 shows what is apparently the best layout utilizing this type of pier, which has the major advantage of providing the greatest number of berths (twenty) for the area as compared with fifteen berths for the marginal-type layout. It would also be physically possible to add additional berths in the future by increasing the length of several slips and by rearranging tracks and paving. Here again, for the sake of clarity on the small scale map, the railroad lines are not shown in Fig. 5. A connection is made between the classification yard and the running track extending the full length of the area to the tracks on the front apron and behind the rear platform; these tracks are shown in the typical section in Fig. 3. There are no railroads inside the transit sheds or at the ends of the bulkheads.

Nineteen covered berths provide a gross shed area of 1,900,000 sq ft as compared with almost half that amount in the marginal-wharf-type layout; however, this scheme has only one open wharf, and possibly more open storage space would be desirable during actual development. The cost per berth is considerably higher because a greater length of bulkhead is required per berth than in the marginal-wharf construction.

Concerning navigation and the berthing of ships, the advantages enumerated in the foregoing for the marginal wharves are reversed in this type of layout in that the relatively narrow and meandering ship channel does not provide the open waterfront space that usually accompanies finger-pier layouts. At least 900 ft are needed from the end of the piers to the far edge of the channel for the safe berthing of ships, with no allowance being made for the passing of other ships. This arrangement would hinder traffic.

The layout of berths at an angle to the channel and conformance with the clear distance requirements outlined previously result in 106 acres of land being lost to dredging behind the harbor line and in a considerable loss of land depth needed for open storage and parking areas in the rear. The slips in this scheme are waterways behind the harbor line and outside the scope of dredging operations now being performed by the federal government in maintaining the ship channel at project depth. Therefore, the dredging of the slips, as well as the area between the channel and piers, would have to be a maintenance expense of the Port of Houston. The land required by the piers at the downstream end of the property results in the use of the total depth of land at that end of the area under study. In addition to the loss of land depth, this plan causes the greatest amount of dredged material to be hauled over the relatively long haul distances and spoiled in the dumping grounds now used for long-term, maintenance-dredging spoils.

In the finger-pier-type layout, there are no tracks on the wharf apron and behind the sheds which serve the entire waterfront area, and therefore there would be less rail traffic in these paved areas. However, this scheme has the greatest number of tracks crossing the access road that parallels and serves the waterfront area.

Railroad officials definitely prefer this plan, and it is generally considered that the pier layout is best suited for the most economical switching of cars in the wharf area.

Cost Comparison.—Preliminary cost estimates have been developed for each of the two layouts, and these cost data and other information used in evaluating and comparing the cost totals are shown in Table 4. As described previously, one basic structural design of the wharf and pier structures was selected and used in all schemes in order to develop the comparative cost data, based on 1955 prices, and was not intended to indicate proposed structural design. Preliminary structural design studies were not within the scope of this study and should be based on thorough field investigations of soil and site conditions for each wharf site whenever contract plans are prepared.

The estimated cost of each scheme does not include amounts required for land acquisition, legal fees, bonds, and other financing costs. The total cost of each is based on the consideration that any plan adopted would undoubtedly be developed in increments over a period of years. This total cost is not intended to represent the estimated cost of constructing all the facilities at one time.

As shown in Table 4, the marginal-wharf layout has the lower total cost. Items that vary with the arrangement of facilities in each scheme, such as number of berths, number of open wharves, and total gross shed area, must necessarily be included as factors in evaluating the total estimated cost for each plan. These evaluation factors are included in Table 4 immediately below the total cost figures.

Using the foregoing factors, the total cost per scheme was reduced to cost per berth, with all marine terminal facilities included, and the cost per berth of the marginal-wharf layout is lower than that of the finger-pier layout.

Because the gross shed area per plan varies from 1,000,000 sq ft to 1,900,000 sq ft, the most exact cost comparison is reflected in the last item, which shows "cost per berth less sheds," and the cost per berth is again lower for the marginal-wharf-type layout.

TABLE 4.—COST ESTIMATE COMPARISON

DESCRIPTION	MARGINAL-WHARF-TYPE LAYOUT	FINGER-PIER-TYPE LAYOUT
Wharf structure from front fender through second rear railroad track, including transit-shed floor and apron floor	$21,175,000	$39,600,000
Transit-shed superstructure, including electrical, plumbing, and other facilities; inside shed (no sprinkling)	3,500,000	6,650,000
All dredging to front of fender line, including spoil disposal	125,000	1,656,000
Land excavation and spoil disposal not included in first and third items	720,500	953,000
Access road, concrete paving, and drainage necessary thereto	242,324	231,075
Overpass	250,000	250,000
Underpass	200,000	200,000
Additional paved areas not included in first item	482,000	432,000
Railroads, except those included with wharf structures and appurtenant drainage	675,000	639,000
Utilities to wharf area	102,000	82,000
Fencing, site grading, drainage, and miscellaneous	500,000	500,000
Total	$27,971,825	$51,193,075
+15% engineering and contingencies	4,195,775	7,678,960
Total estimated cost	$32,167,600	$58,872,035
Number of berths	15	20
Cost per berth	$ 2,144,505	$ 2,943,600
Number of covered berths	10	19
Number of open wharves	5	1
Gross shed area, in square feet	1,000,000	1,900,000
Area behind harbor line lost to dredging, in acres	0	106
Cost per berth less sheds (including engineering and contingencies)	$ 1,876,175	$ 2,561,225

The fact that the marginal-wharf layout is lower in cost per berth, even with transit sheds excluded from the cost figures, is attributed largely to the excess amount of bulkhead required per berth in the pier-and-slip-type layout as compared with the marginal-type layout. The excavating, dredging, and spoil items also increase the unit cost per berth of the pier-type construction above the unit cost per berth of the marginal-wharf-type construction.

CONCLUSIONS

The conclusions presented herein evolved from (*a*) studies and comparisons of all schemes considered for development of the area and of the two basic types prepared for the particular area under study, using basic criteria and dimensions adopted after numerous conferences with Houston operators and port authorities; (*b*) visits to other major ports in the United States; (*c*) studies of bulletins and reports published by various port authorities; and (*d*) from an over-all study of the information obtained from the foregoing in order to apply it to the area being considered. These conclusions are as follows:

1. Cost comparison favors marginal wharves because this scheme has the least cost per berth (less sheds)—$1,876,175 as compared to $2,561,225—and the least total cost—$32,167,600 as compared to $58,872,035 for the finger-pier type.

2. The pier-type layout can be utilized for the site but is less suitable for berthing operations because the limited waterfront area provided by the relatively narrow ship channel is not as satisfactory as the large open bays and wide rivers usually prevailing in the areas in which finger piers are used.

3. The pier-type layout provides more berths (twenty) than the other scheme by utilizing land depth as well as channel frontage for berthing spaces.

4. The use of the pier-type layout would result in the greatest quantity of land (106 acres) lost to dredging behind the harbor line as compared with virtually no land lost for the marginal-wharf type.

5. The cost for maintenance dredging would be the least for the marginal-wharf-type layout because the maintenance dredging of the ship channel (to within 50 ft of the fender line) is financed by the federal government, and the maintenance dredging of slips behind the harbor line would be financed by the navigation district.

6. The marginal-wharf type requires a relatively small quantity (834,000 cu yd) of dredged material from the front of the fender line to be spoiled (as compared with 13,800,000 cu yd for the pier type). Therefore, selection of this layout would conserve the port-owned dumping grounds, which are needed for long-term, maintenance-dredging operations in the turning-basin area.

7. The pier-and-slip layout provides the best arrangement of facilities for railroad operations. The marginal-wharf layout provides the best arrangement of facilities for trucking, steamship, and stevedoring operations. Trucking concerns, which now handle a major part of the cargo, would have access to the full length and both ends of sheds in the marginal-wharf layout.

8. The unbroken waterfront apron of the marginal wharves provides a ready means for moving cargo along the waterfront in emergency operations from one shed to another shed, or to a waiting ship up or down the channel.

9. In case of fire, a ship moored at a marginal wharf is more accessible to a fire boat than one moored in a slip. The U-shaped transit sheds of the pier-type layout, with three sheds under one roof, present a greater hazard from fires and explosions than the marginal type.

On the basis of the foregoing, the marginal-wharf layout shown in Fig. 4 would be preferred. Each plan has advantages and disadvantages; however, the advantages of the marginal-wharf scheme, which outweigh the advantages of the finger-pier type and which effected this recommendation, can be briefly summarized as follows:

a. It costs less to construct.

b. It provides the easiest berthing for ships, interfering the least with other traffic in the channel.

c. It satisfies best the operational needs of steamship, stevedoring, and trucking companies.

d. It utilizes the land area to the greatest possible extent, with less land being lost to dredging operations.

e. The cost of long-term maintenance dredging to maintain project depth is the least.

f. It conserves best the port-owned dumping grounds for long-term, maintenance-dredging operations.

g. The hazard from waterfront fires and explosions is the least.

h. The unbroken wharf apron provides a ready means for emergency handling of cargo between adjacent sheds or ships along the waterfront.

DISCUSSION

Ersel G. Lantz,[2] **A. M. ASCE.**—Several valuable criteria to be used in port development have been set forth by Mr. Newnam. The tables containing the answers to the questionnaires sent to railroads, trucking lines, steamship lines, and stevedoring companies illustrate the wide difference of opinion among the users of port facilities. The rapid growth of the use of mechanical handling equipment, especially fork trucks, has raised problems in port design that could not have been foreseen by the builders of the older docks, sheds, and wharves.

TABLE 5.—Dimensions and Criteria Used for Port of Brownsville (Tex.) Docks

Item	Dimension or Criterion
Wharf apron	
Height above mean low water	12 ft
Width	50 ft
Transit shed	
Width	200 ft
Length per berth	500 ft
Clear height (bottom of roof truss to floor)	19 ft
Width of rear loading platform	30 ft
Width of canopy, rear loading platform (extended to center line of first track)	none
Width of paved loading area back of shed (marginal wharves)	100 ft
Length per berth (marginal type)	600 ft
Single slip length (pier type)	...
Width of slip (pier type)	...
Railroads	
General	
Minimum track spacing	14 ft
Maximum degree of curvature (where economically possible)	18°
Maximum grade (where economically possible)	1.5%
Storage yard capacity	200 cars
Length of storage yard	25 car lengths
Marginal type	
Number of tracks, wharf apron	2
Number of tracks, behind shed	2
Double railroad crossovers, wharf apron (between spotting tracks)	1 at center of dock area
Single railroad crossovers, behind shed (independent switching of cars at each berth)	1 for three sheds
Roadways	
Main road width	26 ft paved
Access road width	30 ft paved
Design depth of water at fender line	−32 ft

The comparatively small Port of Brownsville (Tex.) has just completed (as of October, 1956) a $3,500,000 marginal-type dock and shed facility. Table 5 compares the Port of Brownsville facility with Table 3 of the paper.

The writer does not believe that the 20 ft of apron height included in the first item of Table 3 can always be justified. One advantage would be in the use of shipside cranes operating from a railroad track inasmuch as the high dock would give the crane operator a better view when handling cargo in or out of the hold. The wide variety of sizes and loaded conditions prevailing among the ships operating out of any port that does not handle a specialized cargo makes the distance traveled by the load so variable that any savings as a result of the shorter distance would be negligible. Many of the small freighters and banana boats operating into the Port of Brownsville would have difficulty operating over a dock with a 20-ft elevation. The expense added to the bulkhead cost by the extra 8 ft of height far outweighs any known advantages.

---

[2] Engr., Port of Brownsville, Brownsville, Tex.

The Port of Brownsville used an apron width of 50 ft for the new dock because past experience with 35-ft and 40-ft widths of apron had shown the need for wider aprons in giving more maneuverability to equipment and providing space for the temporary storage of dunnage, ship supplies, and cargo. The extra width of the apron causes the cargo to travel farther on its way to the hold of the ship, but the advantages gained by the extra working space provided by the wider apron in stacking and classifying cargo and for storing dunnage far outweigh the extra expense caused by the longer travel.

This port's new shed width is 200 ft, which is greater than the distance longshoremen will move cotton with hand trucks without an extra handling charge. However, it was felt that the extra space could be used for cotton storage during the peak season. In handling general cargo or cotton by mechanical equipment, the extra longshoreman charge does not apply. Five hundred feet of shed space for each ship's berth are used; however, the 1,000 ft of shed on the new dock is divided into three sections by firewalls. This was done to satisfy an insurance requirement that will not permit more than 7,500 bales of cotton in one fire division without penalty.

The clear height of the shed at the Port of Brownsville is approximately the same as that recommended in the criteria—that is, a minimum shed clearance with the present-day use of fork machines for the stacking of cotton and palletized cargo. The port built the rear platform 30 ft wide to allow the unloading of the cotton trucks on the apron outside the shed because cotton insurance requirements will not permit trucks to enter into the sheds. Because of the climate prevailing in the port area, a canopy over the rear apron and track was not felt to be economically justified, and cotton warehouse regulations do not permit the canopy without penalty.

With reference to the third item of Table 5, the Port of Brownsville is providing 100 ft of paved unloading space in the back of the shed to allow trucks to back up and unload onto the rear apron.

The 600 ft of berthing space provided, mentioned in the fourth item of the table, is ample. There is extra space at each end of the shedded area to give approximately 600 lineal ft per berth.

The port set a minimum track spacing of 14 ft (seventh item in the table). The maximum degree of curvature was set at 18°; however, local conditions caused some curves to be 22° 30 min. Special gage rods were installed on all curves, and, so far, even the extreme curves have given no trouble. The maximum grade is set at 1.5% in the tracks installed for the new docks. Present track storage is for only 50 cars; however, space has been left for future tracks if needed.

It is felt that a third track on the apron would not be worth the extra cost involved.

The road sizes (eighth item) need not be examined except to add that extra space is provided on the access road for parking trucks waiting to go onto the dock. Parking for dock personnel is in back of the loading area.

The design depth of 32 ft for the dock (ninth item) is considered sufficient for many years to come; however, special provision is made for increasing this depth should it ever become necessary.

The only item in Table 4 that can be compared herein is the cost per berth less sheds, the last item in the table.

The port's new dock unit provides three deep-sea berths and one 20-ft-deep berth, which can be converted to a deep-sea, 32-ft dock for $2,900,000, and a future dredging cost of $60,000 would give four deep-sea berths at $2,960,000, or $740,000 per berth.

The writer believes that much of the extra $1,360,000 per berth shown in Table 4 is due to the greater height of the apron as well as to the greater design depth of the channel.

Austin E. Brant, Jr.,[3] J. M. ASCE.—A concise presentation of the difficult subject of design criteria for general-cargo facilities and of the detailed studies has been made. It is interesting to compare the criteria and estimated costs developed by Mr. Newnam with those determined during an engineering and economic survey[4] made in 1953 for the Harris County-Houston Ship Channel Navigation District. The primary purpose of the survey was the establishment of a master plan for the future development of the physical facilities of the port, and it included the determination of basic criteria for general-cargo marine terminals at Houston.

Basis for Determination of Criteria.—Any modern general-cargo wharf must necessarily be designed as a compromise of the primary features proposed by steamship operators, stevedores, terminal operators, and railroad and trucking interests. The criteria recommended[4] for general-cargo facilities at Houston combine as many of the desired characteristics as possible to provide an efficient marine terminal at reasonable cost. In the development of these criteria the opinions of those who ultimately must be responsible for the operating efficiency of Houston's public marine terminals were particularly important.

Type of Wharf.—Early in the survey, it was concluded that marginal wharves would be the most suitable type for development along the Houston Ship Channel because this type of wharf permits vessels to dock and undock without blocking the channel. Moreover, a marginal wharf with an adjacent transit shed and supporting area usually is more efficient in operation than any other type of wharf. Such a wharf also has a high degree of flexibility under varying usages and permits effective utilization of the upland areas for warehouses, open storage, railroad yards, and water-oriented industries.

Layout of Berths.—It was also believed that general-cargo ship terminals should be built with units of two covered berths separated by a single-berth open wharf. This arrangement obviates the necessity of providing wide clearances between individual berths, thus permitting the most economical utilization of bulkhead frontage, sheddage, and road and track construction while still preserving independent rail and truck access to each berth. Under the usual operating procedures, operations at the two berths would be separate. In the case of large cargoes or in an emergency, it would be relatively simple to expand or contract the storage areas at adjacent berths.

---

[3] Project Engr., Tippetts-Abbett-McCarthy-Stratton, Cons. Engrs., New York, N. Y.

[4] "Master Plan Survey for Port of Houston," Knappen-Tippetts-Abbett-McCarthy, New York, N. Y., June, 1953.

TABLE 6.—Principal Criteria for Covered General-Cargo Ship Berths

Item	1953 Report
Berth	
Length	600 ft
Depth (below mean low water)	40 ft
Wharf apron	
Width	40 ft
Elevation (above mean low water)	25 ft
Transit and warehouse storage	
Net transit storage area, per berth[a]	85,000 sq ft
Net warehouse storage area, per berth	40,000 sq ft
Gross transit storage area, per berth	117,500 sq ft
Gross warehouse storage area, per berth	57,500 sq ft
Length of shed, per berth	500 ft
Width of shed, per berth	350 ft
Width used for transit storage	235 ft
Width used for warehouse storage	115 ft
Clear stacking height	20 ft
Width of rear loading platform	25 ft
Truck facilities	
Width of main terminal roadway	48 ft
Width of feeder roadways	24 ft
Maximum grade[a]	5%
Width of truck maneuvering area at rear of transit sheds	125 ft
Railroad facilities	
Maximum permissible degree of curvature[a]	19.1°
Maximum desirable degree of curvature	18°
Maximum grade	1%
Holding yard capacity, per berth	from 50 to 60 cars
Tracks on wharf apron	2

[a] Not given by Mr. Newnam.

As a result of an investigation of warehousing costs and requirements in the port area, it was concluded that facilities for warehouse storage of waterborne general cargo should be provided at shipside. Warehouse space constructed integrally with the transit storage space would aid in reducing the congestion caused by cargoes delayed excessively in transit storage.

Desirable Characteristics of Covered General-Cargo Ship Berths.—The principal criteria developed during the survey are summarized in Table 6. The major differences between the criteria given by the author and those presented in the report[4] are in the elevation of the wharf apron, the number of tracks on the apron, and the size of the transit storage space.

The elevation of 25 ft above mean low water given for the wharf apron in the report would be higher than any recorded high water in the ship channel. Moreover, it would permit the establishment of reasonably convenient, safe grades on the rail connections between the proposed development area and both the Clinton Division of the Port Terminal Railroad and the existing wharves on the north side of the Turning Basin. Because the elevation of Wharf 16 is approximately 18 ft and the elevation of the mainline of the railroad near Clinton Drive is approximately 42 ft, any significant change in the basic 25-ft elevation proposed for these wharves would render more difficult one or the other of the proposed rail connections. This would be particularly important in relation to the rail connection to the Port Terminal Railroad, which would serve as the primary means of rail access to the area. An appreciable reduction in the proposed apron elevation would result in a steepening of the grade on this connection and would add measurably to the hazard of rail operation. Lower elevations would increase excavation costs, whereas higher elevations would increase the cost of the bulkhead structure. It is considered that, with the elevations that have been chosen, an economic balance can be achieved.

TABLE 7.—ESTIMATED COSTS FOR CONSTRUCTION OF GENERAL-CARGO WHARVES IN AREA SOUTH OF PUBLIC GRAIN ELEVATOR

Item	Cost
(*a*) 600-FT SHIP BERTH (COVERED)	
Wharf platform 60 ft wide and shed platform on fill 355 ft wide, including dredging, excavation, pavement on compacted fill, track wells, trackage, mooring facilities, and other appurtenances	$1,130,000
Shed superstructure (350 ft by 500 ft) and footings; semifireproof construction and sprinklers	820,000
Excavation and grading at rear of wharf proper for holding yard, roadways, truck pavements, parking, and open storage areas	90,000
Railroad holding yard, including track leads extending to wharf proper	60,000
Roadways and truck pavements	45,000
Parking and open storage areas	30,000
Subtotal	2,175,000
Engineering, legal, administration, and contingencies @ 20%	435,000
Total	$2,610,000
(*b*) 575-FT SHIP BERTH (OPEN)	
Wharf platform 60 ft wide, 355-ft pavement on compacted fill, including dredging, excavation, trackage, mooring facilities, and other appurtenances	$1,080,000
Third, fourth, fifth, and sixth items for covered ship berth	230,000
Subtotal	1,310,000
Engineering, legal, administration, and contingencies @ 20%	260,000
Total	$1,570,000

Two tracks are proposed for the wharf apron because it is believed that one loading track and one running track are usually adequate for operations at a general-cargo terminal.

Modern cargo vessels, such as the C4-S-A4 cargo class, have a bale cubic capacity of as much as 640,000 cu ft. A study of the general-cargo commodities stored in the covered transit sheds at Houston indicates that the average weight of cargo handled is approximately 40 lb per cu ft. In the transit storage areas at the port, it has been found economical to stack most general cargo three pallets high for an average floor loading of approximately 300 lb per sq ft of net storage area. The net transit storage area required to accommodate the contents of a general-cargo vessel with a capacity of 640,000 cu ft can be determined as follows:

$$640{,}000 \text{ cu ft } \frac{40 \text{ lb per cu ft}}{300 \text{ lb per sq ft}} = 85{,}000 \text{ sq ft.}$$

This area should be increased about 40% to allow for aisles and sorting space, giving a gross transit-shed space required of approximately 117,500 sq ft.

Estimated Costs of Construction.—Preliminary cost estimates for the construction of covered and open general-cargo wharves in the area covered by the paper were made during the survey. These estimates, which include the costs of all rail, roadway, and supporting facilities, are given in Table 7. Using those costs, the total estimated cost for the fifteen-berth development presently (1953) proposed would be approximately $34,000,000. In comparing this cost estimate with that of Mr. Newnam, allowances should be made for the increase in the cost of construction since 1953 and for the different size and type of sheds.

The estimates given in the report are considerably higher than the actual costs for wharves previously built in the turning basin area. These increased costs result from provisions to improve the functional layout and to increase the operating capacity per berth. The principal provisions are:

1. From 75% to 100% more covered storage than at existing berths to provide sufficient area to handle not only the largest foreseeable requirements for transit storage, but also considerable shipside warehousing;
2. Semifireproof construction of sheds and the provision of sprinklers required for transit and warehouse operations under the same roof;
3. A railroad holding yard for from 50 to 60 cars per berth, which is required because of the already saturated condition of the North Yard of the Port Terminal Railroad and because of the distance (average, in excess of 2 miles) between North Yard and the proposed wharves;
4. An independent roadway network because the existing roads in the turning basin area would be inadequate for additional traffic to or from the endowment area; and
5. Considerable earth moving and grading required for the development of adequate back-up areas and for the new rail and road facilities.

Conclusions.—The criteria and cost estimates presented by Mr. Newnam for the construction of marine terminal facilities at Houston agree in general with those developed in the 1953 survey. The substantial agreement between these two studies should encourage the Port of Houston in the development of additional facilities to insure its future growth and the maintenance of its standing as one of the major ports of the United States.

Frank H. Newnam, Jr.,[5] M. ASCE.—Messrs. Brant and Lantz have not only contributed much that is of value to the development of this subject but have provided additional information concerning design criteria for general-cargo wharf facilities. The outstanding feature of the discussions is the high percentage of concurrence in the criteria developed because it is believed to be generally accepted that no two sets of conditions are ever exactly the same at two different ports. This point is emphasized because it will always be necessary to consider and study carefully the prevailing conditions in preparing the design for specific facilities.

---

[5] Partner, Lockwood, Andrews & Newnam, Cons. Engrs., Houston, Tex.

Because the discussion by Mr. Brant concerns the same area at the Port of Houston, explanations are offered concerning certain points he has raised. He has indicated (under the heading, "Desirable Characteristics of Covered General-Cargo Ship Berths") some of the advantages of an apron elevation of 25 ft above mean low water. Studies outlined in the paper have recommended an elevation of + 20 ft for the following reasons:

1. Transition could be made within a reasonable distance to increase the height from + 18 ft (existing at Wharf 16) to + 20 ft and thus permit the flow of rail and truck traffic between the aprons of adjacent wharves.
2. Subsequent to 1953 there has been considerable excavation (earth borrow) from this area, which greatly reduces the expense of utilizing an apron elevation of +20 ft.
3. The rapid increase in the cost of the substructure that accompanies a higher apron elevation is believed to offset the additional cost of earth removal in this area.
4. Sufficiently flat railroad grades could be worked out for the area, utilizing the elevation of +20 ft for the apron. It will be noted that the studies recommended a maximum railroad grade of 1% and a maximum desirable degree of curvature of 12.5° (Table 1).

As a matter of interest, the Port of Houston subsequently decided to adopt an apron elevation of 18 ft above mean low water for the four new wharves in this area.

Mr. Brant has indicated (under the heading, "Desirable Characteristics of Covered General-Cargo Ship Berths") that only two tracks are recommended for this area. In visits to and studies of other ports, the writer and other investigators found that there was no agreement among port operators on this subject. The 50-ft apron width does permit the use of three tracks, which greatly facilitates the handling of cars and is preferred by the Port of Houston; this fact definitely influenced the recommendation on this item.

It is gratifying to note that the cost estimates of the two reports are remarkably close when allowance is made for the difference in size of the transit sheds. The difference in area of the transit storage sheds is due to a different concept in the arrangement and use of the sheds and particularly in the design loading. The study cited by the writer recommended a design loading of 600 lb per sq ft (as compared with the 300 lb per sq ft used by Mr. Brant), which materially reduces the required shed area. This and other differences indicate that normally there is more than one engineering approach to solving a specific problem.

Mr. Lantz has indicated that the recommended apron elevation of 20 ft is 8 ft higher than that selected for the new facility at the Port of Brownsville. This again emphasizes the fact that each specific facility must be designed for its own governing conditions, and Mr. Lantz states some of the conpitions at this port that differ from the Port of Houston. If the artillery method of "bracketing the target" were used, however, an average between the elevation of 25 ft recommended by Mr. Brant and the elevation of 12 ft recommended by Mr. Lantz would be 18½ ft.

The cost per berth for the facilities described by Mr. Lantz is considerably less than the estimates listed in the report outlined by the writer and those mentioned by Mr. Brant. This is undoubtedly due largely to the difference in channel depth and apron height. In designing different waterfront facilities, the rapid increase in cost accompanying any relatively small increase in apron height or channel depth, or both, is a source of constant amazement.

For the items not affected by the apron height, such as the transit sheds and lengths of berths, there appears to be close agreement between the dimensions selected at the two ports. For the rear loading platform it is agreed that the width of 30 ft used at the Port of Brownsville is preferable to the width of 20 ft recommended for the Port of Houston whenever the owner is willing to accept the slight increase in cost.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2921

DYNAMIC STRESSES IN CONTINUOUS PLATE-GIRDER BRIDGES

By Roy C. Edgerton[1] and Gordon W. Beecroft,[2]
Associate Members, ASCE

With Discussion By Messrs. Robert K. L. Wen; and Roy C. Edgerton and Gordon W. Beecroft

Synopsis

This paper describes and presents results of the tests conducted on two three-span, continuous, plate-girder bridges in Oregon. Included herein are measurements of the deflection of the girders and strains in the girders, stringers, and floor beams under test vehicles. The test records provide values for the frequency of vibration, the frequency of strain oscillation, the amplitude of vibration, and the amplitude of strain oscillation. Test data are presented as curves, which show the variation of stress and deflection with the speed of the test vehicles. Comparisons are made with stresses and deflections computed according to existing specifications. Measured stresses, in general, are found to be lower than the computed values. Analytical work pertaining to theoretical frequencies and amplitudes of vibration is summarized. Good correlation was obtained between measured and computed frequencies of vibration. Computations relative to the amplitude of vibration do not provide a correlation with the measured values; however, they suggest that a substantial part of the vibration can be attributed to deck roughness. Measured strains are used to determine the degree of fixity of the ends of the floor beams, and measured deflections are used to evaluate the torsional rigidity of the structures.

Introduction

Bridge specifications have been based on computed static-load stresses, empirical impact allowances, and satisfactory past performance. The need for

---

Note.—Published, essentially as printed here, in May, 1956, in the Journal of the Structural Division, as *Proceedings Paper 973.* Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions.*

[1] Research Engr., Oregon State Highway Dept., Salem, Ore.

[2] Asst. Research Engr., Oregon State Highway Dept., Salem, Ore.

strain measurements in structural members under moving loads has long been recognized. Since the development and application of electronic strain-recording equipment made these measurements possible, a number of tests of bridges under moving loads have been made. The present report adds two three-span, continuous, plate-girder bridges to the growing list of dynamic bridge studies.

During 1952 the Physical Research Branch of the Bureau of Public Roads, United States Department of Commerce (BPR), completed the assembly of a portable bridge-testing unit. The unit, contained in a house trailer, consisted of the equipment necessary to record varying strains and deflections. Agreement was reached between the BPR and the Oregon State Highway Department for the test unit to be used on an Oregon highway bridge during the summer of 1953 as a cooperative project of the two agencies. The North Dillard Bridge over the South Umpqua River on U.S. 99 in Douglas County was selected for the test. At the conclusion of the tests on the North Dillard Bridge, it was decided to repeat a part of the test program on the Troutdale Bridge over the Sandy River on U.S. 30 in Multnomah County.

Test Bridge

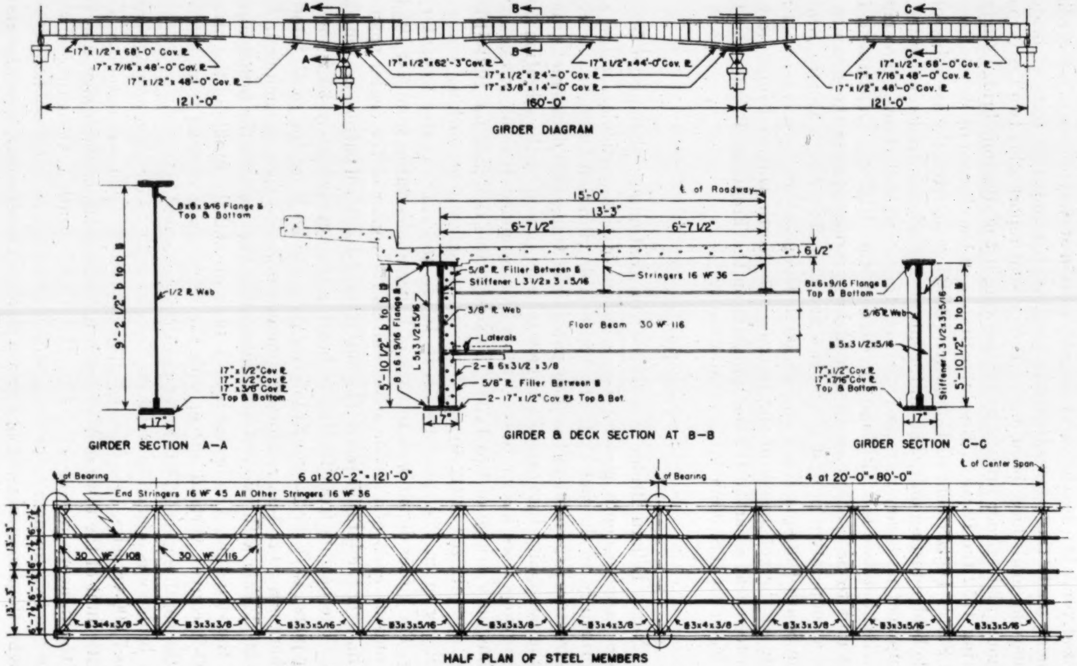
The North Dillard Bridge is a three-span, continuous, plate-girder bridge with two variable depth girders. The selection of this particular bridge was based primarily on the fact that: (1) Appreciable vibration had been observed in the structure; (2) most of the floor panels had transverse deck cracks, indicating the possibility that some damage was being caused by the vibration; and (3) no previous work known to the writers had examined dynamic stresses and vibrations on a bridge of this type. The bridge was designed for H20-S16-44 loading[3] and utilized A242-46 steel[4] having an allowable working stress of 27,000 lb per sq in. The spans are 121 ft, 160 ft, and 121 ft center to center of bearing. The girder depth back to back of flange angles varies from 5 ft 10½ in. at the outside ends of the side spans and at the center of the center span to 9 ft 2½ in. over the interior piers. The four piers of dumbbell pattern are founded on rock. The heights from footing to bearing are approximately 35 ft for the exterior piers, 43 ft for the north interior pier, and 38 ft for the south interior pier. There is a 50-ft 3-in. reinforced concrete approach span at each end. The deck is 30 ft wide with two 3-ft 6-in. sidewalks; Fig. 1 shows the principal details of the structure. The bridge is on a tangent, and the profile of the bridge is a 600-ft convex vertical curve, with the deck grade varying from level to 0.8%; the south end is 2.77 ft lower than the north end.

The bridge was opened to traffic on November 29, 1950. The average daily traffic is 7,000 vehicles, 13% of which are heavy trucks and combinations. The south city limit of the town of Winston is 0.2 mile north of the bridge, and the edge of the unincorporated town of Dillard is 0.5 mile south. Along the highway between the two towns are three roadside fruit stands, several residences, and road connections to a sawmill and a gravel plant. These features were important considerations in the operation of test vehicles because they necessitated additional precautions to prevent traffic interference.

---

[3] "Standard Specifications for Highway Bridges," A.A.S.H.O., 4th Ed., 1944, pp. 125–142.

[4] "1946 Book of A.S.T.M. Standards," Pt. 1*a*, A.S.T.M., 1947, p. 27.


GIRDER DIAGRAM

GIRDER SECTION A—A

GIRDER & DECK SECTION AT B—B

GIRDER SECTION C—C

HALF PLAN OF STEEL MEMBERS

A
B
C
17"x 1/2"x 68'-0" Cov. ⅊
17"x 7/16"x 48'-0" Cov. ⅊
17"x 1/2"x 48'-0" Cov. ⅊
17"x 1/2"x 62'-3" Cov. ⅊
17"x 1/2"x 24'-0" Cov. ⅊
17"x 3/8"x 14'-0" Cov. ⅊
17"x 1/2"x 44'-0" Cov. ⅊
121'-0"
160'-0"
8x6x9/16 Flange ∠ Top & Bottom
1/2 ⅊ Web
9'-2 1/2" b to b ∠
17"x 1/2" Cov. ⅊
17"x 1/2" Cov. ⅊
17"x 3/8" Cov. ⅊
Top & Bottom
17"
℄ of Roadway
15'-0"
13'-3"
6'-7 1/2"
6 1/2"
5/8" ⅊ Filler Between ∠
Stiffener L 3 1/2 x 3 x 5/16
Stringers 16 WF 36
3/8" ⅊ Web
Floor Beam 30 WF 116
Laterals
2-∠ 6x3 1/2 x 3/8
5/8" ⅊ Filler Between ∠
2-17"x 1/2" Cov ⅊s Top & Bot.
5'-10 1/2" b to b ∠
8 x 6 x 9/16 Flange ∠
L 5x3 1/2x5/16
8x6x9/16 Flange ∠ Top & Bottom
5/16" ⅊ Web
∠ 5x3 1/2x5/16
17"x 1/2" Cov ⅊
17"x 7/16" Cov ⅊
Top & Bottom
Stiffener L 3 1/2x3x5/16
℄ of Bearing
6 at 20'-2" = 121'-0"
℄ of Bearing
4 at 20'-0" = 80'-0"
℄ of Center Span
End Stringers 16 WF 45 All Other Stringers 16 WF 36
30 WF 108
30 WF 116
13'-3"
6'-7 1/2"
∠3x4x3/8
∠3x3x3/8
∠3x3x5/16
∠3x3x5/16
∠3x3x3/8
∠3x4x3/8
∠3x4x3/8
∠3x3x3/8
∠3x3x5/16
∠3x3x5/16

FIG. 1.—BRIDGE DETAILS

TEST PROGRAM

The program consisted of four series of tests, with vehicles approximating the H20-S16-44 truck traveling at speeds of from 5 miles per hr to 45 miles per hr. In the interest of public safety, speeds greater than 45 miles per hr were not attempted due to the restrictions imposed by the towns and the roadside development. In series I, girder deflections and positive-moment strains were measured; negative-moment girder strains were measured in series II; stringer strains were measured in series III; and floor-beam strains in series IV.

Magnetic strain gages were used in series I and series II, and SR-4 resistance strain gages were used in series III and series IV. The deflectometers operated on the same principle as a magnetic strain gage. A coil housed in a pipe coupling was attached to the lower grider flange. An iron core in the coil was attached by a brass rod to a pipe mast from the ground or stream bed. Deflection of the girder produced a relative motion between the coil and the core, thereby producing a change in the characteristics of the circuit and, ultimately, a galvanometric deflection.

The strain-recording equipment consisted of a power supply, bridge and attenuator panels, and two oscillographs. Light-beam galvanometers in the oscillographs exposed the strain traces on sensitized paper. Each oscillograph was connected to fifteen active traces. A separate power supply and separate bridge and attenuator panels were provided for the deflectometers. The output was wired to the same two oscillographs used for strain recording. Dark-room facilities for developing the oscillograms were provided in the trailer.

The magnetic strain gages and the deflectometers were calibrated with dial indicators before installation and after removal. The strain-recording equipment contained calibrating facilities for SR-4 resistance strain gages.

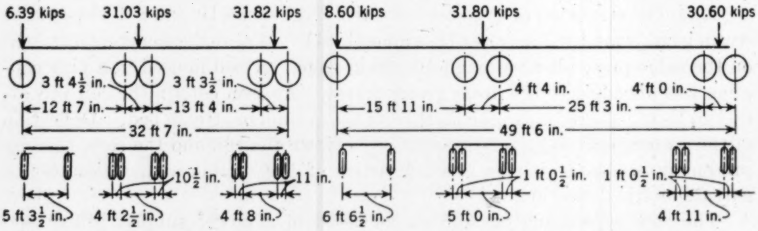
Road tubes were installed across the bridge deck at each pier and at the center of the center span. The road tubes operated air switches, which recorded the progress of the test vehicle on one trace on each oscillogram. This record and $\frac{1}{10}$-sec lines that were printed automatically on every oscillogram permitted accurate determination of the location of the test vehicle at all times. It also made possible an accurate determination of vehicle speed.

The strain gages and the strain-recording equipment, including the oscillographs, were components of the BPR test unit. The deflectometers, deflectometer power supply, bridge panels, and attenuator panels were loaned to the project by the Institute of Traffic and Transportation Engineering of the University of California at Berkeley (Calif.).

Two test vehicles were used. Vehicle A was a logging truck and trailer unit based on a 2½-ton, military-type truck chassis. The vehicle was loaded with steel sheet piling. Vehicle B was a diesel truck-tractor with an equipment semitrailer. The load was a track-type tractor. Fig. 2 shows the essential dimensions and axle loads of the two vehicles.

Series I.—For series I, six deflectometers and twenty-four magnetic strain gages were installed. A deflectometer was installed on the outside lower flange of each girder at the center of each span. Strain gages were installed on the inside and outside top and bottom flanges of each girder at the center of each span. The gages were installed on the lower surface of the top flanges and on the upper surface of the bottom flanges. The axis of each gage was located $\frac{7}{8}$ in. from the outside edge of the outstanding leg of the flange angle.

Series I was divided into four subseries. Series I*a* was a set of runs, with the test vehicle centering the roadway center line. Runs were started with vehicle A. Four or six runs, two or three in each direction, were made at each multiple of 5 miles per hr, from 5 miles per hr to 30 miles per hr as indicated on the speedometer. The highest speed attainable by vehicle A under the test conditions was 30 miles per hr. Runs with vehicle B were made at 5 miles per hr, and, at each multiple of 5 miles per hr, from 20 miles per hr to 45 miles per hr according to the speedometer. There was some variation between speedometer speed and actual speed as computed from the air switches and the oscillogram time lines. The highest actual speed was approximately 43 miles per hr.



(*a*) TRUCK A, 69240 LB (*b*) TRUCK B, 71000 LB

FIG. 2.—TEST VEHICLES

For an understanding of the terms used on the curves and in the examination of the test results, Fig. 3 should be studied. This figure is a typical deflection trace from an oscillogram. For clarity the ratio of ordinate to abscissa of the trace has been doubled. The trace takes the form of a vibration curve with a frequency of approximately 2 cycles per sec superimposed on a deflection curve representing the passage of the vehicle across the center span. To take the results from a trace, envelopes are drawn to the vibration curve. The maximum ordinate of the upper envelope and the maximum ordinate of a line midway between the envelopes are converted to deflection by the calibration curve for the individual gage and are termed the total deflection, Δ_t, and the mean deflection, Δ_m. The difference between the total deflection and the mean deflection is the amplitude of vibration. Strain traces are analyzed in an identical manner. Strains are converted to stress by an assumed modulus of elasticity of

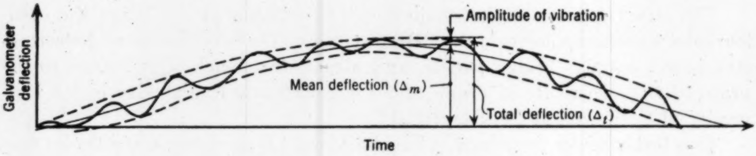


FIG. 3.—TYPICAL DEFLECTION TRACE

30,000,000 lb per sq in. The terms used are total stress, σ_t; mean stress, σ_m; and amplitude of stress oscillation. In the examination of the test results, the measured mean stress will be compared with the computed static stress, and the measured total stress will be compared with the computed stress, with the American Association of State Highway Officials (AASHO) impact allowance[5] included. The latter comparison is valid, but the former requires explanation because the computed static stress should be compared with a measured static stress. With one exception, static stresses were not recorded because it was expected that the mean stress at 5 miles per hr would furnish an adequate measure of static stress. In the instance in which static stresses were recorded, in which vehicle A was parked near the west curb at the center of the center span, the stresses were approximately 2% lower in the heavily loaded girder and approximately 6% lower in the less loaded girder than corresponding mean stresses at a vehicle speed of 5 miles per hr.

Because of the low maximum speed of 30 miles per hr attainable with vehicle A, this vehicle was used only in series I. Except for brief comments, only the results from runs with vehicle B will be presented. The stresses cited and shown by the curves in the subsequent illustrations will be the lower-flange stresses because these are the critical stresses in each section. The total stress and the mean stress in the lower flanges of each girder in the center span for all series I*a* southbound runs and northbound runs were averaged separately to see whether or not the direction of the vehicle had an effect on the stress and deflection. Because no directional effect was evident, the results of the runs in the two directions were averaged to give a single stress curve and a single deflection curve for the center span of each girder for series I*a*. Figs. 4(*a*) and 4(*b*) show the stress curves for the two girders. The curves are the result of averaging the stress values for the test runs within each of the various 5-mile-per-hr speed brackets. The plotted points show the maximum stress measured for an individual run at each speed. The stress values presented herein will be those indicated by the curves unless it is specified that the value is for an individual run. In Figs. 4(*a*) and 4(*b*), the highest mean stress is approximately 90% of the computed stress without impact based on AASHO specifications.[6]

Because shear-developing angles were used at 21-in.-to-24-in. spacing in the region of positive moment, the computed girder stresses and deflections presented herein are those obtained by considering composite action of the girder with that part of the deck allowed under AASHO specifications. A value of $n = 10$ was used in computing both stresses and deflections. The highest total stress is approximately 7% greater than the computed stress without impact, but approximately 10% less than the computed stress with impact. The increase in total stress resulting from an increase in the amplitude of vibration from between 20 miles per hr and 30 miles per hr is typical of the span. For series I*a*, the maximum stress measured for vehicle A in the center span was approximately 3% higher than the computed stress without impact, but approximately 13% lower than the computed stress with impact. The maximum stress for an individual run is about 6% less than the computed stress with impact for vehicle B. Figs. 4(*c*) and 4(*d*) show the deflection curves for series I*a* with vehicle B in the center span. The deflection follows the same general pattern as the stress, except that a sharp increase in amplitude in the east girder occurs from approximately 29 miles per hr to 35 miles per hr. The greatest observed deflection is 0.55 in. and the greatest amplitude of vibration is 0.13 in., both occurring at about 42.6 miles per hr in the east girder. The maximum amplitude of vibration for vehicle A in the center span was 0.04 in.

---

[5] "Standard Specifications for Highway Bridges," A.A.S.H.O., 6th Ed., 1953, pp. 164–165.

[6] *Ibid.*, pp. 210–212 and pp. 250–251.

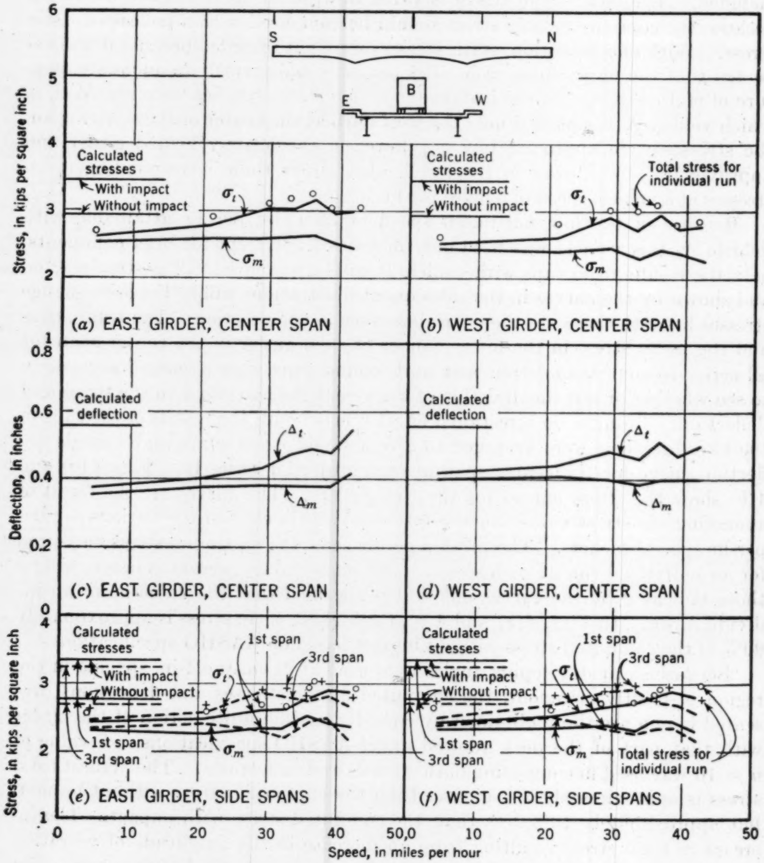


FIG. 4.—STRESSES AND DEFLECTIONS, NORTH DILLARD BRIDGE, SERIES I*a*

In the case of the side spans, a difference in stress and deflection characteristics due to the direction of the vehicle might be expected for two reasons. The influence line for moment at the midpoint of the end span of a continuous series is not symmetrical, nor is the axle and load arrangement of the vehicle longitudinally symmetrical. Computations indicate that the vehicle should produce higher stresses when proceeding across a side span from the approach than when proceeding across a side span from the center span. On the other hand, when the vehicle proceeds from the center span to a side span, the latter is already in a state of vibration at the time the vehicle enters the span. Figs. 4(*e*) and 4(*f*) show the stress curves for the side spans. First-span effects and third-span effects are shown by separate curves, and different symbols are used to distinguish first-span maximum stresses for individual runs from corresponding third-span stresses. Although the highest total and mean stresses are similar, the reaction to speed is different. In the first-span effect, where the structure is motionless until the vehicle enters the span, the highest stresses occur at approximately 40 miles per hr; in the third-span effect, where the structure vibrates from the passage of the vehicle across the other two spans, the highest stresses occur at approximately 30 miles per hr. For the latter effect the highest stress for an individual run occurs at 38 miles per hr in the east girder and at 33 miles per hr in the west girder. On the side spans a greater amplitude of stress oscillation was caused by the passage of vehicle A than by that of vehicle B. The total stress for the third-span effect of vehicle A traveling at 31 miles per hr is approximately 3% less than the computed stress with impact. Unfortunately, this was the highest speed attainable with vehicle A, so it is not known what effect higher speeds would have. However, the maximum amplitude of stress oscillation for the third-span effect of vehicle B occurred at approximately 30 miles per hr; therefore, it is likely that this value is near the maximum.

Series I*b* was a repetition of series I*a* with the exception that all runs were made with the east tires of the vehicle 1 ft from the east curb. Hereafter the heavily loaded east girder will be termed the "loaded" girder, and the less loaded west girder will be termed the "unloaded" girder. Fig. 5 shows the stresses in the center span. The computed stresses are based on a simple-beam distribution to girders. The curves for measured stress show that a greater proportion of the load than that assumed was transferred to the unloaded girder primarily because of the torsional rigidity of the structure. The laterals are in the plane of the bottom flange of the floor beam and, with the floor slab and main girders, form a box section over 4 ft deep having a significant torsional stiffness. The deep floor beams and their connections are stiff enough to prevent distortion of the cross section. An evaluation of the torsional rigidity of the structure will be presented subsequently. Again the abrupt increase in amplitude from between 20 miles per hr and 30 miles per hr in the loaded girder will be noted. The unloaded girder generally shows a greater amplitude of stress oscillation than the loaded girder. The amplitudes of stress oscillations recorded for vehicle A in this subseries were smaller than those shown in Fig. 5 for vehicle B. The measured stresses in the unloaded girder for individual runs show a high degree of scatter. The maximum value for an individual run is approximately twice as high as the mean stress at 5 miles per hr (approximately equal to the static stress), indicating an impact of 100%. This extreme value is attributed to the fact that the girder is lightly loaded.

To determine the damping effect of stationary mass or dead load on the vibration of the structure, vehicle A was parked at the center of the center span with tires touching the west curb. Series I*b* was then repeated as series I*c* with vehicle B making all the runs. Fig. 6 shows the stress curves under these conditions. New strain and deflection "zeros" were established after vehicle A was parked on the span. The stresses in Fig. 6 are those caused by the moving vehicle only; they do not include the stress caused by the parked vehicle. The highest total stresses in the loaded girder occur at a different speed than in series I*b*, but the magnitude of these stresses is virtually unchanged by the parked vehicle. The maximum stress in the unloaded girder for an individual run is about 12% lower than the corresponding value for series I*b*.

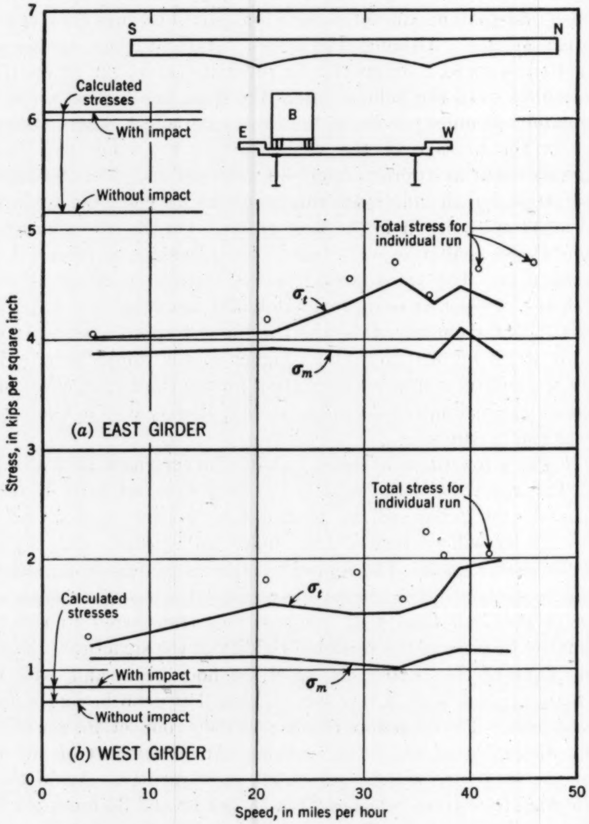


FIG. 5.—STRESSES IN CENTER SPAN, NORTH DILLARD BRIDGE, SERIES I*b*

The fourth subseries under series I was I*t*, which comprised the readings taken under conditions of normal heavy truck traffic on the bridge. The traffic was allowed to flow undisturbed during these tests, resulting in numerous records in which several passenger vehicles were on the bridge in addition to a truck. A few records were obtained with two trucks on the structure. The tabulated data from these records were carefully examined to locate any unusually high stresses or vibrations. No stresses or deflections greater than those measured in the loaded girder in series I*b* were found.

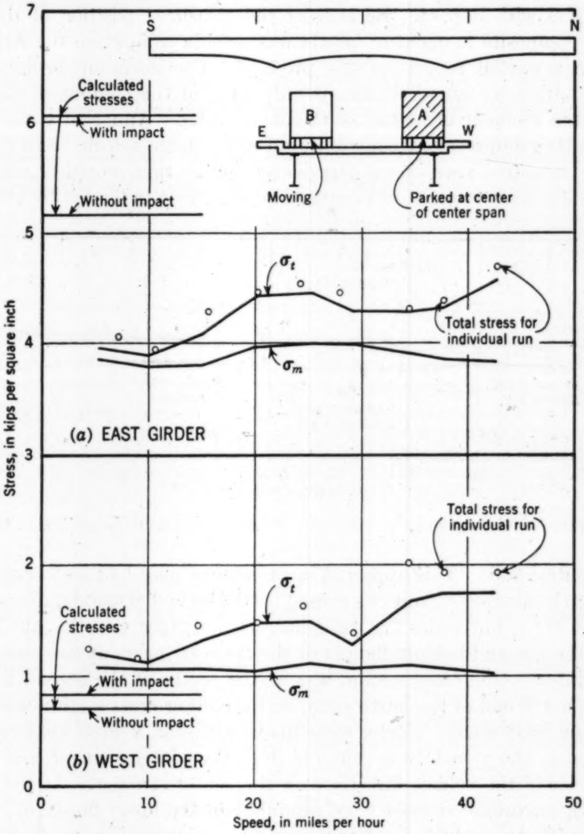


FIG. 6.—STRESSES IN CENTER SPAN, NORTH DILLARD BRIDGE, SERIES I*c*

Series II.—For series II, the twelve strain gages were removed from the inside girder flanges and were installed on the girder flanges near the north interior pier. Eight were installed in the center span on the inside and outside of top and bottom flanges on both girders, and four in the north span on the outside of top and bottom flanges of both girders. All gages were 2 ft 6 in. from the pier center line. This distance was necessary to avoid the special girder details at the pier. Fig. 7 shows the stress curves for the bottom-flange gages on the center-span side of the pier. The negative-moment stresses show a higher degree of uniformity and less oscillation at higher speeds than the positive-moment stresses. Disregarding composite action, because this measurement is for negative-moment stresses, the computed stresses without impact are almost identical to the measured mean stress. The amplitude of stress oscillation is greater than the specified impact allowance, resulting in a maximum measured stress 16% higher than the computed stress with impact. Because of the measured position of the neutral axis, the composite moment of inertia was used in computing the stresses presented in a partial report on this project.[7] The mean upper-flange stress, computed from measured strains, is only 34% of the computed stress based only on the moment of inertia of the steel girder. Apparently the concrete slab and its reinforcement contribute an appreciable tensile resistance; however, the measured lower-flange stresses are higher than would be expected for a section having a moment of inertia as large as that indicated by the position of the neutral axis. This apparent inconsistency may be due to experimental error or to local stress conditions caused by the special girder details at the pier.

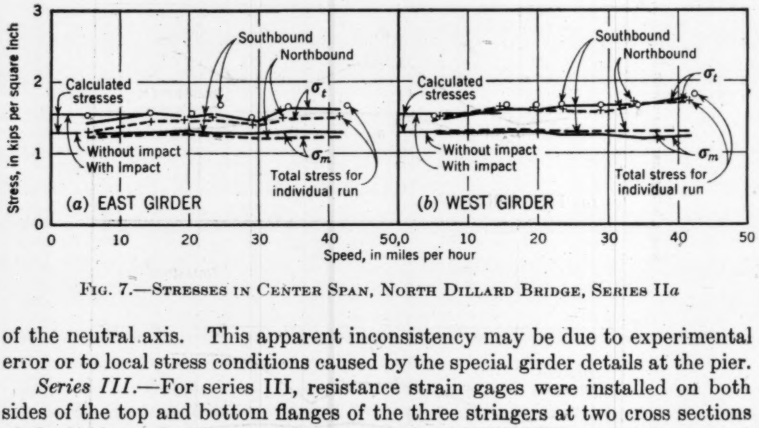


FIG. 7.—STRESSES IN CENTER SPAN, NORTH DILLARD BRIDGE, SERIES II*a*

Series III.—For series III, resistance strain gages were installed on both sides of the top and bottom flanges of the three stringers at two cross sections of the bridge. One cross section was at the center of the second floor panel from the north end of the north span, and the other cross section was over the floor beam next south. Runs were made with the vehicle centered on the deck; hence, the vehicle was centered over the center stringer and with the west wheels of the vehicle directly over the center stringer. At the center of the panel, maximum stresses were produced in the lower flange of the center stringer with the vehicle centered over the stringer; Fig. 8(*a*) shows these stresses. The unusual feature of these curves is the variation of both mean stress and total stress with speed and the comparative uniformity of the amplitude of stress oscillation. The plotted points showing the maximum stress for individual runs indicate a fairly uniform increase in total stress with an increase in speed. The highest total stress for an individual run is about 94% of the computed stress without impact. The difference between the measured stress and the computed stress is explained, in part, by the distribution of the wheel loads to the stringers. Strain measurements in the three stringers indicate that the part of the total load carried by the center stringer is about 8% less than computations based on AASHO specifications indicate. Also, composite action of the concrete deck is not considered in the computed stress because no shear devices were installed on the stringers; however, composite action is evident in the relationship between upper- and lower-flange stresses. The magnitude of the mean upper-flange stress is about 75% of the mean lower-flange stress. The computed lower-flange stress, based on a moment of inertia indicated by the measured position of the neutral axis, is approximately 11% lower than that computed when composite action was disregarded. Stresses computed from the measured load distribution and indicated composite action are similar to the measured stresses. With the vehicle centered over the center stringer, the outer stringers carry a greater part of the total load than indicated by the specifications and exemplify less composite action than the center stringer indicates. These factors result in a computed stress for the outer stringers that is very similar to the measured mean stress.

---

[7] "Vibration and Stresses in Girder Bridges," *Bulletin 124*, Highway Research Board, National Research Council, Washington, D. C., 1956, p. 41.

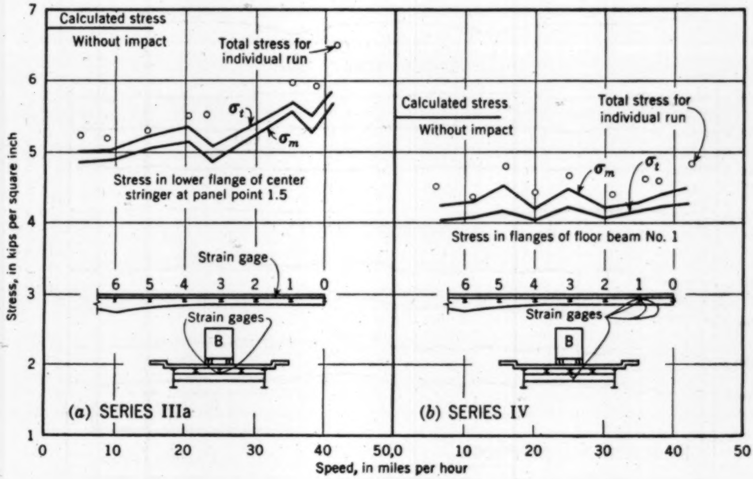


FIG. 8.—FLOOR-SYSTEM STRESSES, NORTH SPAN, NORTH DILLARD BRIDGE

Series IV.—Fig. 8(*b*) shows for series IV the flange stresses at the center of the first floor beam from the north end of the north span. The floor beam showed a different response to speed than any other member tested. Here, also, the measured stresses are much lower than the computed stresses. Analyses of the records of gages near the ends of the floor beams show that the low stresses at the center of the floor beam are due to a partial fixity at the connection of the floor beam to the girder. The analysis of the degree of fixity provided by this connection is presented subsequently.

DECK ROUGHNESS

During the testing of the North Dillard Bridge, it was concluded that the deck roughness might be a contributing factor to the vibration. To verify this conclusion it was decided that measurements of strain and deflection should be made on a similar bridge having a smooth deck. The Troutdale Bridge was chosen because it was erected from the same superstructure plans as the North Dillard Bridge.

Profiles in each of the four wheel tracks were obtained on both bridges by taking elevations with a level at 10-ft intervals and making an accurately traced graphical profile between each pair of level points. These profiles included 60 ft of roadway and approach structure at each end of the steel structure. Fig. 9 shows the deviations of the 10-ft points from true grade, the true grade for each wheel track being represented by a horizontal line. To arrive at a numerical comparison of the roughness of the two structures, total deviations from true grade at the 10-ft points were computed. The total deviations in 520 ft for the four wheel tracks at North Dillard were 2.02 ft, 2.09 ft, 2.05 ft, and 2.00 ft—for an average of 2.04 ft. The deviations at Troutdale were 1.37 ft, 1.08 ft, 0.84 ft, and 0.84 ft—for an average of 1.03 ft. On the basis of this comparison, the Troutdale Bridge was selected for making comparable measurements of strain and deflection.

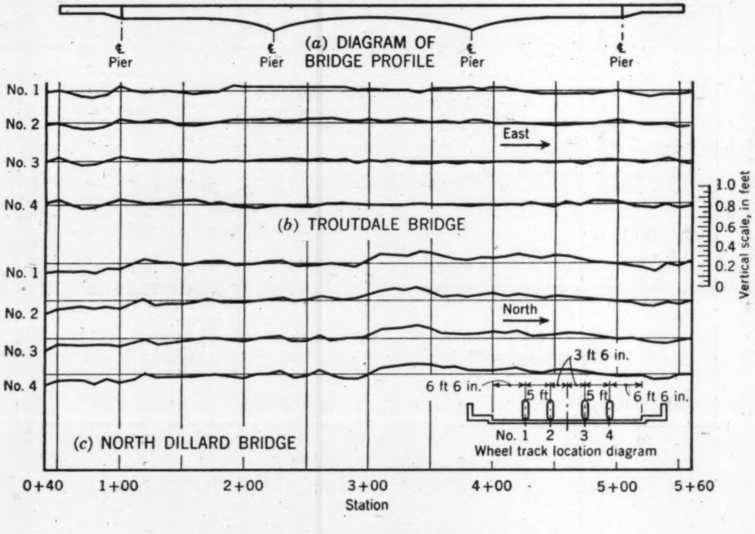


FIG. 9.—DEVIATIONS FROM TRUE GRADE

Differences other than deck roughness that might be significant in the vibratory characteristics of the structures include the fact that the Troutdale Bridge is on a level grade. In addition, its deck has few transverse cracks, and its piers are shorter (about 28 ft from footing to bearing) and are of slightly different detail than those of the North Dillard Bridge. These piers are founded on piling as compared with the slight grade and vertical curve, the extensively cracked deck, and the higher piers founded on rock of the North Dillard Bridge.

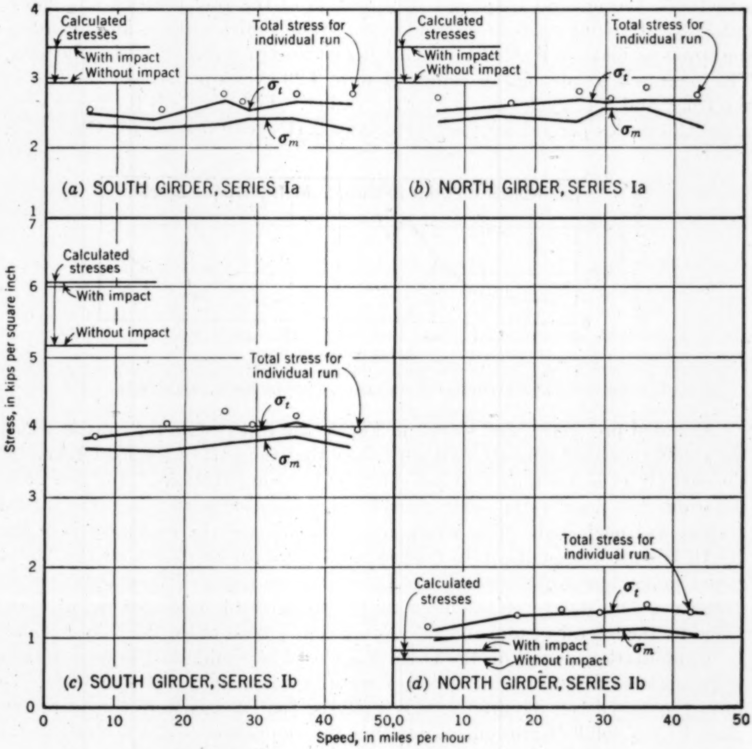


FIG. 10.—STRESSES IN CENTER SPAN, TROUTDALE BRIDGE

Series I*a* and I*b* of the North Dillard Bridge tests were duplicated on the Troutdale Bridge, except that resistance strain gages were used and strain gages were placed only on the top and bottom outside flanges of the girders at the centers of the west and center spans. Vehicle B was used for all runs in these tests. Fig. 10 shows stress curves for the center span of the Troutdale Bridge. The mean stresses in the Troutdale Bridge girders average about 5% less than those in the North Dillard Bridge, and the mean deflections average approximately 4% less. The amplitude of vibration and stress oscillation on the Troutdale Bridge is much less, as can be seen by comparing Figs. 10(*a*) and 10(*b*) with Figs. 4(*a*) and 4(*b*), and Figs. 10(*c*) and 10(*d*) with Fig. 5.

Fig. 11 shows the average variation in amplitude of vibration with speed for both girders in the center span of the two structures as taken from the deflection records for series I*a*.

VIBRATION

In addition to the measurements of strain and deflection made on the two structures, the natural frequency of vibration of the test vehicles was determined by attaching resistance strain gages to the vehicle springs and recording the strain oscillations resulting from having let the truck roll off a 4-in. platform. The measured frequencies of vehicle A were 3.29 cycles per sec for the rear of the truck and 3.42 cycles per sec for the trailer. For vehicle B the frequencies were 2.36 cycles per sec for the tractor and 2.41 cycles per sec for the trailer.

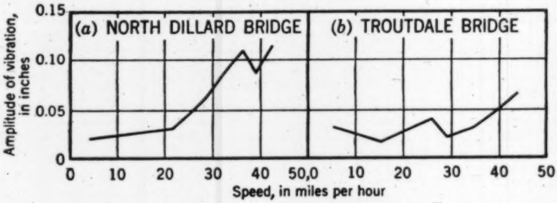


FIG. 11.—AMPLITUDE OF VIBRATION, CENTER SPANS, SERIES I*a*

As noted previously and shown in Fig. 5, the test results for runs in which the vehicle traveled near the east curb show that the part of the load taken by the east girder is less than that indicated by the computations. In these computations the floor beams were considered simply supported and the lateral system was neglected. The stress measurements for the center span of the North Dillard Bridge at a vehicle speed of 5 miles per hr show that the loaded girder takes approximately 9% less than simply supported floor-beam values indicate. This difference is caused by the considerable torsional rigidity provided by the lateral system and the riveted connections of the deep floor beams.

Computations based on the twist of a closed cell subjected to pure torsion were made to determine the effective resisting moment of the structure. To form the closed cell required in this analysis, the laterals were considered replaced by a solid diaphragm by relating the elongation of the laterals to the shear deformation of a thin plate. The cell thus formed consists of the concrete deck, the girder webs, and the solid diaphragm described above. For the center span, the resisting torsional moment results in an indicated transfer of 7% of the vehicle weight from the loaded girder to the unloaded girder for a 10-ft vehicle eccentricity. A simple-beam stress distribution for this vehicle position shows that the east girder would take 87.7% of the total load. The load transferred from this girder due to torsional rigidity reduces the computed value to 80.7%. The measured value, based on relative strains, shows that 78.5% of the total load is carried by the east girder. The computation of the load transfer due to torsional rigidity was based on measurements of the differential deflection of the girders. Therefore, the computations explain the deviation from a simple-beam distribution in the test structure, but the method cannot be applied to predict the behavior of another bridge except for estimates that might be justified due to similarity of design. An analysis that considered deflection and torsion simultaneously would be of greater value; however, no such analysis was attempted due to the extreme complexity of the problem.

The natural frequency of vibration of the test structures was computed by replacing the variable moment of inertia, based on AASHO specifications, with an equivalent uniform moment of inertia. Because vibration is a function of the internal energy alternately stored and released by the beam, a uniform moment of inertia required to do the same internal work as that done by the actual variable depth girders was determined for each span. Although this computed moment of inertia may not provide precisely the same vibratory characteristics as the actual girder, it seemed preferable to base the computations on internal energy rather than on deflections. The computed values were 132,720 in.^4 for the center span and 124,310 in.^4 for the end spans. Another approximation used in the solution of frequency of vibration was that of assuming a uniform dead load on the structure. The total dead load for each span was divided by the span length to give a mean value that was considered a uniform load. The resultant values were 2,320 lb per ft for the center span and 2,297 lb per ft for the end spans.

The solution for the frequencies of vibration for the first symmetric mode and the first asymmetric mode followed the method outlined by S. Timoshenko,[8] who assumed each span had the same unit weight and moment of inertia. To apply the method to the present study, it was necessary to include the ratio of the k-values for the center and end spans as well as the span lengths in obtaining the trial-and-error solution. The computed frequency of the first symmetric mode is 1.57 cycles per sec and that of the first asymmetric mode is 2.50 cycles per sec. The measured values are 1.7 cycles per sec and 2.7 cycles per sec, respectively, for the North Dillard Bridge. For each mode the measured frequency is 8% higher than the computed frequency. This difference might be due to the use of an equivalent uniform moment of inertia; however, the computed value is reasonably close to the measured value, and the difference is in line with the computed-stress and measured-stress values. The measured stresses were lower than those determined on the basis of AASHO specifications for composite girders, indicating that the moment of inertia is actually larger than the computed value. This larger moment of inertia would cause higher frequencies of vibration, as measured.

Although the Troutdale Bridge was erected from the same superstructure plans as the North Dillard Bridge, the measured frequency of the first symmetric mode on this structure was 1.87 cycles per sec, 10% higher than the natural frequency measured on the North Dillard Bridge. The reason for this higher frequency cannot be determined, but the most probable cause of the difference lies in the relative number of transverse deck cracks; the North Dillard Bridge has numerous cracks, whereas the Troutdale Bridge has very few. The shorter piers and the pile foundation of the Troutdale Bridge might also contribute to this difference. Because the measured mean stresses averaged approximately 5% lower in the Troutdale Bridge, a slightly higher frequency could be expected for this bridge. No records for the Troutdale Bridge were observed in which a residual vibration in the first asymmetric mode was evident. Many of the records for the North Dillard Bridge tests indicate this mode clearly. Because the same test vehicle was used in testing both bridges, the factor that provides the most plausible reason for this difference in vibratory characteristics is the excessive roughness of the North Dillard Bridge deck. This roughness, in conjunction with the oscillation of the vehicle load on its springs, may cause the asymmetric mode of vibration to be impressed on the structure before the truck leaves the span. This vibration would persist at its natural frequency after the passage of the truck.

---

[8] "Vibration Problems in Engineering," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 2d Ed., 1937, pp. 345–348.

The existing theory pertaining to amplitudes of vibration of highway bridges is limited. No one approach can begin to cover the innumerable variables that might be encountered in multiple-span highway structures. There are some basic theoretical concepts that might be applied with reasonable success to simple-span structures, provided enough information about the bridge characteristics and the vehicle characteristics was available. Several of these concepts, modified by the relationship between the static deflections of the simple-span structures and the multiple-span structures, were applied to the test bridges. Due to the complexity of the problem, no correlation between measured and computed amplitudes of vibration was accomplished. However, computations based on an assumed deck contour indicate that the excessive vibration of the North Dillard Bridge can be attributed to deck roughness. With one exception the computations were made on the assumption that the test vehicle acted as a single concentrated load as it crossed the structure. This is a radical deviation from the actual case because the axle spacing or the pitching action of the load on its springs may, in some cases, contribute more to the vibration than the speed or natural vibration of the vehicle.

To determine whether the amplitude of vibration could be correlated with the frequency of application of the axle loads, the vehicle velocities required to make the application of the axle loads coincide with the frequency of the bridge were computed. The speeds at which a point of resonance is indicated by these computations are approximately 35 miles per hr for the North Dillard Bridge and 38 miles per hr for the Troutdale Bridge. As shown in Fig. 11, an increased amplitude of vibration occurs in the North Dillard Bridge at a speed near this point of resonance, but the Troutdale Bridge shows an increased vibration at a speed appreciably different than this resonant speed. The maximum amplitudes of vibration on both bridges occurred at the maximum test speed obtained. The fact that the North Dillard Bridge has an increased amplitude near the point of resonance—whereas the Troutdale Bridge does not—indicates the possibility that deck roughness may cause the application of the axle loads to act as a pulsating force on the bridge.

Assuming a smoothly rolling, concentrated load on a smooth deck, a maximum deflection of 1.11 times the static deflection was computed for a vehicle speed of 37 miles per hr for the North Dillard Bridge. For the condition of load and surface described, this value is approximately equal to the worst case in which the forced vibration is in phase and is added to the free vibration.[9] The average total deflection for the two girders at this speed was 1.32 times the static deflection (mean deflection at 5 miles per hr). The speed of 37 miles per hr was used in the computations because a maximum total deflection occurs in both girders of the North Dillard Bridge at approximately this speed. A similar analysis was made for the Troutdale Bridge at a vehicle speed of 43 miles per hr because the maximum total deflection occurred at this speed. The computed ratio of total deflection to static deflection was 1.12 and the measured ratio was 1.13 for this bridge. This approach to determining the amplitude of vibration was not considered applicable to the North Dillard Bridge because of the roughness of the deck. The computations were made for purposes of comparison only.

As shown in Fig. 9, the profiles of the wheel tracks of the two bridges indicate that the North Dillard Bridge is considerably rougher than the Troutdale Bridge. Because greater amplitudes of vibration were measured on the North Dillard Bridge, it seems probable that deck roughness is a contributing factor to vibration. It was found that the excess vibration of the rougher bridge could be quite closely accounted for by a mathematical analysis involving a number of approximations and simplifying assumptions.

The continuous profiles of the wheel tracks do not follow a pattern that can be conveniently expressed as an equation; therefore, it would be extremely difficult to base any computations on the actual deck contour. To determine whether the amplitude of vibration can be attributed to deck roughness, it was assumed that the deck contour could be represented by a sinusoidal curve. An approximate solution was obtained for the increased force on the bridge because of the wavy contour of the deck surface.[10] In arriving at a value for this increased force, it was necessary to separate the sprung part of the load from the unsprung part. For this purpose it was assumed that the unsprung weight of vehicle B was 6,000 lb. The action of the individual axles was not considered; the total weight was assumed to be a concentrated load. Assuming a wave length of 25 ft and a height of $\frac{1}{4}$ in. for the deck contour and a speed of 37 miles per hr, the computed value of the increased force on the deck was 36,600 lb, about 51% of the vehicle weight. The value of $\frac{1}{4}$ in. used for the height of the waves is less than the average height on the actual deck. However, the additional force on the structure would probably be appreciably less than this computed value because of the different axles being out of phase, the yielding of the bridge deck, and the fact that the actual roughness is variable both in length and amplitude of waves. From these computations it is conceivable that a pulsating force of 20,000 lb might exist, which is the approximate value of force required to cause the measured impact deflections. Computations for the total deflection were made assuming a 20,000-lb pulsating force was acting in addition to the 71,000-lb weight of the test vehicle. This resulted in a total deflection 31% greater than the static deflection.[11] The measured total deflection at the vehicle speed used in this analysis was 32% greater than the static deflection.

---

[9] "Vibration Problems in Engineering," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 2d Ed., 1937, p. 360, Eq. 156.

[10] *Ibid.*, pp. 238–240.

[11] *Ibid.*, pp. 356–357.

In regard to the damping characteristics of the structures, the logarithmic decrement was determined from deflectometer traces for the two obvious modes in the North Dillard Bridge and for the first symmetric mode in the Troutdale Bridge. The logarithmic decrement for the North Dillard Bridge vibrating in the first symmetric mode is approximately 0.065 and for the first asymmetric mode, approximately 0.076. The value for the Troutdale Bridge vibrating in the first symmetric mode averaged approximately 0.063. The fact that the damping in the first symmetric mode (the only mode observed at the Troutdale Bridge) was approximately the same on both bridges would seem to eliminate the difference in piers as a factor of any consequence affecting the relative amplitudes of vibration.

The instrumentation used in the tests of the deck system of the North Dillard Bridge consisted of gages at the midpoint and at one end of each stringer, and at the midpoint and at points 2 ft from each end of two adjacent floor beams. To determine the degree of fixity of the ends of the floor beams, the relative stringer stresses were used to estimate the part of the total load being applied to the floor beam at each stringer connection. Having thus established the load being applied at each quarter point, the slopes of the bending moment diagram were computed. Using these slopes and the measured strains at points 2 ft from the ends of the floor beams, the points of contraflecture were computed as 2.19 ft from the ends. From this analysis the negative moment at the ends of the floor beam was found to be 34.95 ft-kips. Under the same assumed load distribution, the negative moment for full fixity would be 92.34 ft-kips. The ratio of these values indicates that the floor-beam connection and the torsional stiffness of the girder provide 38% fixity. Continuing the analysis, the computed stress at the center of the floor beam is 4,500 lb per sq in. The mean stress at this point, computed from measured strains, is 4,030 lb per sq in.—11% lower than the foregoing value. This is considered a satisfactory agreement because the actual load distribution would vary somewhat from the values computed from stringer strains. The total truck load was applied to the three stringers; no allowance was made for the load going directly to the girders. The stress at the center, computed from the assumption of hinged end connections, is 5,775 lb per sq in. for the load distribution considered previously. This stress is approximately 43% higher than the measured value. The rigidity of the riveted floor-beam connections is apparent from this comparison.

Conclusions

The tests that were conducted on the North Dillard Bridge and the Troutdale Bridge provide much valuable information concerning the action of continuous three-span bridges under moving loads. However, no recommendations for changes in design procedures are considered warranted on the basis of this limited information. The need for additional information and verification of the findings of this project is obvious. Only through the consolidation of the results of many such tests can definite conclusions and recommendations for modifications in design practices be provided. The information derived from these bridge tests may be summarized as follows:

1. The measured stresses were generally lower than those computed on the basis of AASHO specifications. At the center of the center span of the North Dillard Bridge, the moment of inertia determined from measured strains was 9% higher than the value determined according to specifications for composite sections. For these bridges the indication is that the specifications defining the part of deck to be considered in the composite girder are conservative, but not overly so. Measured stresses at the center of the floor beams were appreciably lower than computed values. This difference is explained by the fixity of the riveted floor-beam connection and by the torsional stiffness of the girder. Although no shear keys were used between stringer and deck, measurements show that partial composite action does exist. Also, the fact that the deck is continuous over the stringers causes the part of the total load carried by the center stringer to be less than the value computed according to specifications. Because of these factors, the stress computed for the center stringer was appreciably higher than the measured value. The computed stress for the outer stringers was similar to the measured mean stress. The part of the total load carried by the outer stringers was greater than the computed value, and the degree of composite action was less for these stringers than for the center one.

2. For the stresses computed from measured strains in the North Dillard Bridge girders, the amplitude of stress oscillation (that is, the excess of the instantaneous stress over the mean) is, in some cases, greater than the allowance provided by the AASHO impact formula. However, this stress oscillation was caused by a single vehicle, and it would be unlikely that the high percentage increase would apply to the total design load for the structure. Including the specified impact allowance, the computed stresses were higher than maximum stress values obtained from measured strains in all cases, except for the less loaded girder under eccentric loading and for the negative-moment stresses in the lower girder flanges. The total stresses in the floor beams and stringers were lower than the total stresses computed according to specifications. For the Troutdale Bridge all measured stress oscillations were lower than the corresponding computed impact allowances. It was concluded that the deck roughness of the North Dillard Bridge contributed much to the impact stresses.

3. The natural frequencies of vibration of bridges of this type can be computed with reasonable accuracy by using a uniform moment of inertia that provides an equivalent internal energy. The frequency of vibration of the structures during the passage of the test vehicles remained close to the natural frequency of the bridge. No forced vibration with a frequency similar to that of the test vehicle was detected. No conclusive point of resonance is apparent from the various stress curves.

4. The greater mean deflection and mean stress and the lower natural frequency of vibration of the North Dillard Bridge can be explained by a lesser moment of inertia, which, in turn, could be attributed to the greater cracking of the deck slab of that bridge as compared with the Troutdale Bridge. Its

greater amplitude of vibration and stress oscillation appear to be largely due to the greater roughness of its deck surface.

ACKNOWLEDGMENTS

The original test program was prepared by Neil Van Eenam, M. ASCE, of the BPR, who was present during the early phases of testing and assisted in the general organization of the project. The BPR test unit was assembled by E. G. Wiles, who accompanied the unit to the test site and supervised its operation during the tests. George S. Vincent, M. ASCE, represented the BPR on the project, assisted in the field work, consulted with the writers at frequent intervals during the analyses of data, and offered helpful suggestions in the preparation of the report.

DISCUSSION

Robert K. L. Wen,[12] J. M. ASCE.—An essential point that Messrs. Edgerton and Beecroft have emphasized is the importance of deck roughness in relation to the dynamic response of the two bridges studied. If the bridge deck and approach are smooth, both experiments and mathematical analyses indicate that, within practical ranges of parameters, a sprung load or an unsprung load will seldom cause any serious dynamic effects on the structure.[13] The maximum dynamic stresses are on the order of from 10% to 15% larger than the maximum static stresses. However, field tests on highway bridges have shown consistently that the maximum dynamic deflection or stress may be expected to be 25% larger—and often more than 30% larger—than the maximum static response. Analytical studies and laboratory experiments[13,14] have also shown that large dynamic stresses can exist in a bridge if the sprung part of the vehicle has been oscillating vertically as it enters the bridge. This vertical oscillation of the vehicle may have been excited by the unevenness of the approach to the bridge.

On highway bridges there is no apparent counterpart of the periodic forcing that acts on railway bridges because of the centrifugal force of wheel unbalance in locomotives. The effect of axle spacing has been conceived by many engineers to be similar to that of a periodic forcing and to be the cause of a possible resonant state of the bridge vehicle system. However, available information, although limited,[15,16] indicates that the effect of axle spacing is much less serious. Therefore, it appears logical to attribute the major remaining source of large dynamic stresses in highway bridges directly or indirectly to road roughness or to deck roughness. By the term "indirect" it is meant that roadway roughness may induce vertical or pitching oscillations of the sprung part of the vehicle, which, in turn, may cause large dynamic effects on the bridge.

After studying much of the literature on dynamic stresses in highway bridges, the writer has received the impression that the terms "road roughness" or "deck roughness" as related to highway-bridge dynamics have not been sufficiently well defined and do not always refer to the same physical condition of the road or deck. Thus, it is desirable to examine deck roughness in some detail. The following suggested terminology is admittedly somewhat arbitrary. Nevertheless, it does describe, at least qualitatively, the different physical characteristics of a deck surface to which the vehicle and the bridge respond in different ways.

---

[12] Research Associate, Dept. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

[13] "Highway Bridge Impact Problems," by T. P. Tung, L. E. Goodman, T. Y. Chen, and N. M. Newmark, *Bulletin No. 124*, Highway Research Board, National Research Council, Washington, D. C., 1955, p. 121, Fig. 8; p. 124, Fig. 11; and p. 125, Fig. 12.

[14] "Experimental Study on a Sprung Mass with Initial Oscillations," by J. E. Taylor, thesis presented to the University of Illinois, Urbana, Ill., in 1955, in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering.

[15] "Theory of Transverse Oscillations in Girders and Its Relation to Live Load and Impact Allowances," by C. E. Inglis, *Proceedings*, Inst. C. E., London, Vol. 218, 1923–1924, p. 258, Fig. 11.

[16] "Single and Tandem Axle Dynamic Effects on a Highway Bridge Model," by R. H. Boehning, thesis presented to the University of Illinois, Urbana, Ill., in 1953, in partial fulfilment of the requirements for the degree of Master of Science, p. 40, Fig. 33.

Any deviation of a roadway surface from the assumption of a perfectly smooth and horizontal profile, an assumption usually made in a mathematical analysis of bridge vibrations, may be termed "unevenness." Unevenness may be caused by (1) obstructions, (2) roughness, and (3) profile variation.

Obstructions include any abrupt changes of appreciable magnitude in the elevation of the roadway surface. They are also characterized by their localized presence on the road. When a vehicle runs over an obstruction, impact in the literal sense occurs. Many experiments have been conducted on bridges with trucks rolling over an artificial obstruction, such as a ¾-in.-by-4-in. plank or a 2-in.-by-4-in. plank.[17,18] The resulting maximum dynamic stress can easily exceed the maximum static stress by 50% or more. Fortunately, the frequency of occurrence of obstructions on a properly maintained bridge deck is small.

The case of a vehicle passing over an obstruction on a flexible bridge deck is susceptible to mathematical analysis. Results of exploratory analytical studies[19] of this kind indicate that the order of magnitude of the dynamic stresses and deflections of the bridge is in line with those shown by field tests.

Roughness differs from obstructions by its much smaller deviation from smoothness, and also by its more distributed nature on the surface of the road or deck. It can be termed the texture of the road surface. The response of a bridge with a generally rough deck (as defined herein) to moving loads is probably similar to that of a structure to forces or disturbances of high frequencies and small magnitudes. There is experimental evidence that this situation is not serious. For example, the vibration record of a bridge with a 2½-in. grating deck[20] traversed by a heavy truck shows that the maximum excess dynamic deflection is only about 5% of the maximum static deflection. Such a deck may be considered a typical rough surface. Cracks and joints on decks, if not opened up too widely or faulted, may be considered as localized roughness.

Profile variation comprises a continuous deviation from the assumed smooth and horizontal surface. It includes the initial deflection of the bridge due to its dead load, the design grade of the deck, the inherent unevenness of the deck due to the imperfection of surface paving and leveling during construction, and any general changes in the profile of the bridge caused by subsequent exposure of the deck to traffic and weather or other factors. The deck profiles obtained by taking elevations at 10-ft intervals along the spans shown in Fig. 9 can be considered to be unevenness of this type. It may be observed from Fig. 9 that the greater part of the total deviations for the North Dillard Bridge results from the continuous upward deviation from true grade from station 3 + 00 to approximately station 3 + 80. These deviations do not represent obstructions or roughness as previously defined; they represent the difference between the profile variation and the design grade.

---

[17] "Impact in Highway Bridges," Final Report of the Special Committee, *Transactions*, ASCE, Vol. 95, 1931, p. 1090.

[18] "Test on Rolled Beam Bridges Using H20-S16 Loading," by G. M. Foster, *Research Report 14-B*, Highway Research Board, National Research Council, Washington, D. C., 1952, p. 10.

[19] "Highway Bridge Impact Investigation," *2d Progress Report, Structural Research Series No. 24*, Univ. of Illinois, Urbana, Ill., 1952, pp. 99–109.

[20] "Vibration Measurements on Simple-Span Bridges," by John M. Biggs and Herbert S. Suer, *Bulletin No. 124*, Highway Research Board ,National Research Council, Washington, D. C., 1955, p. 12, Fig. 14.

Profile variation will cause no shock or real impact on the vehicle or on the bridge. It will produce instead a continuous forcing or disturbance acting on the vehicle that reacts on the bridge. The predominant frequencies of this disturbance are lower than those caused by roughness.

The effect of profile variation is also susceptible to mathematical analysis by numerical methods, provided the variation is known.[21] However, in practice the exact profile is not available until a bridge is completed, and subsequent exposure of the deck to traffic and weather may modify it to some extent.

In view of the probability that obstructions rarely occur and that roughness need not cause great concern, research on the effect of deck unevenness might be directed to the problem of profile variation. Accumulation of records such as those shown in Fig. 9 for several representative bridges is a good beginning. In recent years, in dealing with random disturbances in general, the technique of spectral analysis has often been applied and found useful. Both this approach and statistical analyses may prove helpful in interpreting the records of profile variation, and in providing some rational basis for predicting the influence of this factor on the magnitude of the dynamic stresses and deflections in highway bridges.

Under the heading, "Vibration," Messrs. Edgerton and Beecroft present an approximate analytical solution for the increased vehicle reaction on the bridge that was caused by deck unevenness. They assume that "* * * the deck contour could be represented by a sinusoidal curve" with a wave length of 25 ft and a wave height of $\frac{1}{4}$ in., and that the vehicle speed was 37 miles per hr. They refer also to certain equations[10] given by Timoshenko. The writer has found it difficult to reproduce the value of 36,600 lb, which was given as the increased vehicle pressure due to the "wavy contour" of the deck.

It is not clear whether by "a sinusoidal curve" the authors mean a series of 25-ft sine waves, or just one single wave preceded and followed by a smooth surface. According to the first meaning, a small variation of the wave length or of the vehicle speed could produce true resonance, so that the increased pressure could theoretically approach infinity. As to the second meaning, it is difficult to conceive of the physical similarity between the actual deck contour and the assumed representation. In either case the increased vehicle pressure is directly proportional to the assumed height of the wave.

In view of the rather arbitrary nature of the assumed physical condition for the analysis and in view of the sensitiveness of the result to the variation of the assumed quantities, the writer believes that such an analysis serves only as an approximation of the order of magnitude of the dynamic responses of the bridges tested. Although this may have been the intention of the authors, the significance of this type of analysis does not seem congruous with the results of their experiments.

The analysis of dynamic stresses in highway bridges is difficult. However, a satisfactory analytical method for studying the vibration of this type of continuous bridge is still needed. In recent years substantial progress has been made in the analysis of simply supported bridges. With the aid of high-speed electronic computers, analyses can now be performed that consider the effects of such variables as axle spacing, vertical oscillation and pitching action of vehicles, vehicle springs and tires, flexibility, and the actual deck profile of the bridge.

---

[21] "Highway Bridge Impact Investigation," *2d Progress Report, Structural Research Series No. 24*, Univ. of Illinois, Urbana, Ill., 1952, pp. 111–117.

With reference to the discrepancies found in comparing the measured static and dynamic stresses with their theoretical or computed values, the study of dynamic stresses in highway bridges is still in its early stages, and research workers in this field often face the difficulty of defining the object for which they are searching. It is probably advantageous to focus attention on the relationship between the experimental static and dynamic stresses and the relationship between the theoretical static and dynamic stresses. Cross comparisons of these quantities of stress would needlessly complicate the picture at the present stage (1956). For the convenience of a narrowed-down scope of investigation and in order to obtain a proper perspective of the problem of highway-bridge dynamics, it seems desirable to separate the study of the prediction of dynamic stresses from that of static stresses, although experimental data for both can be obtained from the same test set-up.

In short, it seems that the study of bridge stresses and deflections may be pursued along two lines: (1) The study of the actual value of the static response; and (2) the study of the ratio of the dynamic response to the static response. By examining the latter ratio, the study of the dynamic aspect of bridge response can, in fact, be considered as independent of the accuracy of the prediction of static response. In order to predict the actual value of the dynamic response, any advance made by the static-response study can usually be utilized. However, the present concept of dealing with dynamic response of highway bridges does not really require an accurate prediction of its actual value. This is indicated by the manner in which provisions for bridge impact are made in the AASHO specifications. Therefore, in view of the foregoing, the fact that the measured static stresses differ appreciably from the computed values need not cause undue concern.

Roy C. Edgerton[22] and Gordon W. Beecroft,[23] Associate Members, ASCE.—Appreciation is expressed to Mr. Wen for his comments on dynamic-bridge studies.

It is agreed that a uniform terminology that would adequately describe various physical conditions of road or deck surfaces would be desirable. However, any terminology will require further explanatory material to define the degree of profile variation or roughness. For example, one could have a sizable profile variation without adverse effects on the riding quality or significant dynamic effects on the bridge deck if the variation increased slowly through an appreciable length and then decreased slowly through a similar length. Historically, roughness as applied to highways or bridges is used to indicate any surface that is not smooth. It includes surface texture, profile variations, cracks, holes, and joints. Because of this common usage it seems undesirable to limit its meaning in discussions pertaining to dynamic-bridge studies. As commonly used, roughness would be a general term synonymous with Mr. Wen's term "unevenness," with the exception that the former term does not apply to highway grade. To use and limit the meaning of a word in the vernacular would lead to confusion. The surface texture might be described as "broomed finish," "open grid," "nonskid," or any other appropriate term.

---

[22] Research Engr., Oregon State Highway Dept., Salem, Ore.

[23] Asst. Research Engr., Oregon State Highway Dept., Salem, Ore.

As stated at various times in the paper, the surface profile of the North Dillard Bridge was not as smooth as that of the Troutdale Bridge. The test results show also that the amplitude of vibration of the North Dillard Bridge was appreciably larger than that of the Troutdale Bridge. Because the superstructures of these two bridges were erected from the same plans and the same test vehicle was used in the tests of both bridges, the obvious conclusion was that the roughness of the North Dillard Bridge contributed significantly to the vibration. The computations pertaining to amplitudes of vibration were performed to determine whether or not the excessive vibration of the North Dillard Bridge could reasonably be attributed to deck roughness. In the opinion of the writers, the computations indicated that the degree of roughness of the North Dillard Bridge deck was sufficient to cause the difference in measured amplitudes of vibration between the two bridges; the values were included for this purpose. It was explicitly stated that no correlation between measured and computed amplitudes of vibration was accomplished, but computations based on an assumed deck contour indicate that the excessive vibration of the North Dillard Bridge can be attributed to deck roughness.

In connection with the assumed contour of the bridge deck and the computations pertaining thereto, reference is again made to Timoshenko's work.[10] The profile of the 160-ft center span was assumed to be represented by

$$x = \frac{h}{2}\left(1 - \cos\frac{2\,\pi\,\xi}{\lambda}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

in which x is the deflection, h denotes amplitude or wave height ($\frac{1}{4}$ in.), ξ is a horizontal distance, and λ is the wave length (25 ft). Another of Timoshenko's equations[24] was used to compute the given value for the increased force on the bridge deck. A vehicle speed of 37 miles per hr was used because a maximum measured deflection occurred in both girders at approximately this speed. The unsprung weight of the vehicle was assumed to be 6,000 lb. The average of the measured frequencies of vibration for vehicle B was 2.39 cycles per sec, giving an average period of 0.418 sec, which was used in the computations. This period of 0.418 sec and a value of 65,000 lb for the sprung weight of the vehicle were used in computing the spring constant of 38,000 lb per in. At the speed of 37 miles per hr and the wave length of 25 ft, the period of the vehicle's traverse along the wavy contour of the deck is 0.461 sec. With these values for the periods, Timoshenko's equation[24] reaches a maximum at intervals of approximately 0.44 sec. Because it was assumed that the sinusoidal curve was applicable to the center span, a maximum was selected at t = 1.42 sec, at which time the vehicle would have traveled 77 ft from the pier, or within 3 ft of midspan. By substituting $t = 1.42$ sec and the other foregoing values in Timoshenko's equation, the value of 36,600 lb was obtained for the additional pressure on the deck. As explained, the additional force would probably be appreciably less than this computed value due to the different axles being out of phase, the yielding of the bridge deck, and the fact that the actual roughness is variable both as to length and amplitude of waves. However, the computations did serve to indicate the likelihood that the roughness of the North Dillard Bridge deck is the principal reason for the marked difference in vibration between two bridges constructed from the same plans.

---

[24] "Vibration Problems in Engineering," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 2d Ed., 1937, p. 240, Eq. h.

Mr. Wen suggests that comparisons should be made between the experimental static and dynamic stresses and between the theoretical static and dynamic stresses, and that cross comparisons of these quantities cause needless complications for those engaged in dynamic studies. Attention should be brought to the fact that to the bridge engineer, whose responsibility it is to design and maintain structures that will adequately serve the needs of the traveling public at a minimum cost, the comparisons between measured and computed static stresses and between measured and computed dynamic stresses are more meaningful. A principal purpose of bridge studies of this type is to prove or disprove the adequacy of existing specifications and to acquire data from which specifications providing greater safety or greater economy may some day be drawn. Therefore, it seems that these are the logical comparisons; however, all information has been presented so that those having other interests can make other comparisons.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2922

THE VISCOUS SUBLAYER ALONG A SMOOTH BOUNDARY

BY HANS A. EINSTEIN,[1] M. ASCE, AND HUON LI[2]

WITH DISCUSSION BY MESSRS. EDWARD SILBERMAN; NICOLS N. AMBRASEYS; AND HANS A. EINSTEIN AND HUON LI

SYNOPSIS

If the sublayer is visualized as a steady, quasi-laminar flow, great difficulty is encountered in the physical description of the transition to the turbulent part of the flow. The proposed model visualizes a periodic growth and decay of the sublayer. The magnitude of the life period of the sublayer can be predicted theoretically and has been checked by experiment. As a consequence of this model, some predictions will be made herein on the generation of turbulence near the boundary.

INTRODUCTION

The viscous sublayer, in the sense of this derivation, is usually a rather thin fluid layer along the smooth boundary of a turbulent fluid flow, in which, due to the proximity of the wall, the turbulent pattern of flow cannot develop freely. At present (1957) serious students of turbulence admit the presence of this layer under the given conditions, but cannot agree on its exact physical description. However, there is general agreement that at the boundary the entire shear stress is transmitted in the fluid by viscosity (by molecular action). Outside the layer most of the shear is transmitted by eddy viscosity, or, in other words, by momentum exchange between fluid masses larger than molecular size. How the transition from molecular units to molar units takes effect is not generally agreed on, even though an ever-increasing amount of information becomes available from measurements in the proximity of frictional boundaries.

NOTE.—Published, essentially as printed here, in April, 1956, in the Journal of the Engineering Mechanics Division, as *Proceedings Paper 945*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Prof. of Hydr. Eng., Univ. of California, Berkeley, Calif.

[2] Research Engr., Odin Associates, Pasadena, Calif.

The engineering description of frictional effects in pipe and channel flow becomes consistent if turbulence is assumed to be created at the frictional boundary.[3] This assumption is supported by the observation that turbulence is created only in connection with a frictional boundary or at the interface between fluid masses of different velocity. The latter is commonly found wherever a flow separates from a solid boundary, leaving a fluid body of different velocity between the main flow and the solid boundary. A good example of such a case is found along the wake of any submerged object where the interface between main flow and wake is not stable but steadily fluctuates, constantly shedding newly created eddies into the flow if the Reynolds number is sufficiently high. The same process may also be observed in the wakes of the individual roughness elements of a hydraulically rough boundary. With the wakes located between the roughness elements, turbulence derived from a rough boundary can be said to originate at the wall, from the point of view of the flow as a whole.

If the turbulence patterns and, particularly, if the velocity distributions, which are derived from smooth boundaries, are compared with those along rough boundaries, the patterns are identical. The equation of the time-average velocity distribution near a smooth boundary can be transformed into that near a rough boundary by replacing the viscous length scale, ν/u_*, by a multiple of the roughness, k_s ($0.300\,k_s$), in which ν is the kinematic viscosity and u_* is the friction velocity. This shows that there is a basic similarity between the two processes. It may be expected that along the smooth boundary, also, the turbulence must be created predominantly at the boundary itself and not somewhere in the free flow. In addition, a satisfactory model of the viscous sublayer must explain how turbulence is created in this case. It will be shown herein that a model can be found that describes the flow system along a smooth boundary on this basis and is not contradicted by any available measurements.

THE PROPOSED MODEL

In order to attain a workable model, one must imagine first that at a given instant the turbulent flow continues all the way to the boundary with a finite main velocity at the wall. The high velocity gradient at the wall necessitates an extremely high viscous shear at that point which is not transmitted into the free fluid because of the lack of velocity gradient farther away from the wall. This shear decelerates the fluid adjacent to the boundary, creating a viscosity-controlled layer (or a viscosity-controlled sublayer), which in time grows in thickness. As the thickness of the sublayer grows, the boundary shear becomes reduced and the growth becomes increasingly slower.

This sublayer is an unsteady laminar or near-laminar flow, bounded on one side by the rigid wall and on the other side, in a rather undetermined manner, by the turbulent flow. The sublayer is comparable to a shear flow between two solid walls, which have a constant velocity difference and which continuously increase their distance, with the fluid supplied constantly through the wall representing the turbulent fluid at a rate satisfying the continuity condition. The Reynolds number of the sublayer continuously grows under these circumstances, indicating an ever-growing tendency to become unstable. It is to be expected that, under the steady effect of the disturbances from the turbulent flow outside, the sublayer flow will become unstable and turn into turbulence itself as soon as a critical Reynolds number has been reached. Turbulent mixing will occur between the previously turbulent flow and the newly created sublayer turbulence. Because turbulent mixing is much more effective than viscosity, the sublayer fluid is accelerated by the turbulent outside fluid to approximately its own velocity in a much shorter time than was required for the viscous build-up of the sublayer. The cycle is then closed, and a new cycle begins.

---

[3] "Der hydraulische oder Profil-Radius," by H. A. Einstein, *Schweizerische Bauzeitung*, Vol. 103, No. 8, February 24, 1934.

The inherently unsteady character of this flow was demonstrated by noting two flow conditions in a plastic, transparent pipe, 2 in. in diameter. For both the flows the discharge was 0.008 cu ft per sec; the average flow velocity, 0.367 ft per sec; the Reynolds number, 5,700; the Weisbach friction factor, f, 0.036; the friction velocity, u_*, 0.0246 ft per sec; and the sublayer thickness was $11.6\,\nu/u_* = 5.1 \times 10^{-3}$ ft ≈ 1.5 mm. A small hole was drilled into the bottom wall of the pipe at approximately the 6.0-cm mark of a scale, permitting the injection of dye into the boundary sublayer. The dye was introduced at a low constant rate and velocity. The dye was not expected to mix with the turbulent flow if the viscous sublayer flow was steady. Mixing occurred at approximately 3 cm downstream in one flow and almost immediately in the other, indicating the existence of sporadic mixing as proposed in the unsteady sublayer model.

The foregoing description of cyclic growth and decay of the sublayer represents the fundamental idea on which the proposed model is based. Some modifications were introduced, however, for the following mathematical description: First, the introduction of an imaginary outside boundary, which permits the passage of fluid, is not actually necessary because the distance to which viscous effects are felt regulates itself automatically if the depth of the turbulent flow is assumed to be large compared with the sublayer thickness. The displacement thickness of the sublayer may then be introduced as a measure for its thickness. Secondly, in order to receive expressions that can be integrated and otherwise handled mathematically, the eddy viscosity of the turbulent flow was assumed to be infinitely larger than the molecular viscosity of the fluid during the period of decay, resulting in a constant average velocity at all points of the turbulent flow. During the period of build-up, the effect of eddy viscosity was neglected. Thirdly, it was assumed that the area of boundary over which simultaneous growth and decay of the sublayer occurs is large in all directions compared with the sublayer thickness, reducing the sublayer generation to a one-dimensional, unsteady-flow problem.

Using a system of Cartesian coordinates (x,y,z), the generation of the sublayer can be described on the basis of the foregoing assumptions. The equation of motion for a thin layer parallel to the wall is

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial z^2} \qquad (1)$$

in which ν is the kinematic viscosity, u denotes the instantaneous sublayer velocity in the x-direction, and t is the time. The boundary conditions for a growing phase of the sublayer, beginning at $t = 0$, are for $t = 0$ and all $z \rightarrow 0$,

$$u = U_o \qquad (2a)$$

for $t > 0$ and $z = 0$,

$$u = 0 \qquad (2b)$$

and for $t > 0$ and $z \rightarrow \infty$,

$$u = U_o \qquad (2c)$$

In the given conditions, U_o is the representative velocity in the turbulent range near the sublayer. A solution for this flow is

$$u = \frac{2\,U_o}{\sqrt{\pi}} \int_0^H e^{-h^2}\,dh \qquad (3)$$

in which the parameter, H, is defined as

$$H = \frac{z}{2\sqrt{\nu\,t}} \qquad (4)$$

and h is a variable of integration.

This solution, which is the Gauss error integral, actually satisfies the boundary conditions of Eqs. 2. For $t = 0$, the limit, H, becomes infinite and the integral,

$$\frac{2}{\sqrt{\pi}} \int_0^\infty e^{-h^2}\,dh = 1 \qquad (5)$$

so that the first condition (Eq. 2a) is fulfilled. The same argument holds for $t > 0$ and $z = \infty$. For $z = 0$ and $t > 0$, the upper limit, H, becomes 0; with zero range of the integral in Eq. 3 the velocity, u, becomes 0, so that all boundary conditions are actually fulfilled.

The error integral cannot be expressed algebraically in closed form but has been tabulated[4] and made available for numerical computation.

Thickness Scale of the Sublayer.—Eq. 4 already indicates that the instantaneous, boundary-sublayer thickness should be measured by a sliding scale, such as the unit, $\sqrt{\nu\,t}$, because all distances, z, from the boundary are measured by this unit. For this purpose the so-called "displacement thickness" of the boundary layer is determined. The displacement thickness is the distance by which streamlines far away from the surface are diverted due to the velocity reduction of the incompressible fluid between the streamline and the boundary. The continuity equation for this flow indicates that the displacement thickness, δ^*, may be determined as

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U_o}\right) dz \qquad (6)$$

if an equal discharge is assumed to flow at all times between the boundary and the streamline. This interpretation is taken from the case of a boundary layer, which is visualized as long, but not infinitely long, in which the continuity may be established between a cross section 1 ahead of the boundary and a cross section 2 through the boundary (Fig. 1).

---

[4] "A Short Table of Integrals," by B. O. Peirce, Ginn and Co., New York, N. Y., 1929.

Introducing the value of u from Eq. 3 into Eq. 6, the value of δ^* is determined as

$$\delta^* = \int_{z=0}^{\infty} \left(1 - \frac{2}{\sqrt{\pi}} \int_{h=0}^{H} e^{-h^2}\, dh\right) dz \qquad (7a)$$

With the help of Eq. 4 the distance, z, is now expressed in terms of H, and Eq.

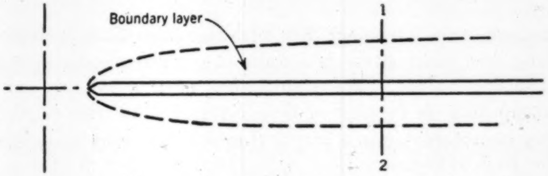


FIG. 1.—THE BOUNDARY LAYER

5 is used to expand Eq. 7a into

$$\begin{aligned}\delta^* &= 2\sqrt{\nu\, t} \int_{H=0}^{\infty} \left(\frac{2}{\sqrt{\pi}} \int_{h=0}^{\infty} e^{-h^2}\, dh - \frac{2}{\sqrt{\pi}} \int_{h=0}^{H} e^{-h^2}\, dh\right) dH \\ &= 4\sqrt{\frac{\nu\, t}{\pi}} \int_{H=0}^{\infty} \left(\int_{h=H}^{\infty} e^{-h^2}\, dh\right) dH \qquad (7b)\end{aligned}$$

The order of integrations is changed to

$$\delta^* = 4\sqrt{\frac{\nu\, t}{\pi}} \int_{h=0}^{\infty} \left(\int_{H=0}^{h} e^{-h^2}\, dH\right) dh \qquad (7c)$$

which can be integrated to

$$\delta^* = 4\sqrt{\frac{\nu\, t}{\pi}} \int_{h=0}^{\infty} e^{-h^2}\, h\, dh \qquad (7d)$$

A second integration gives

$$\delta^* = 4\sqrt{\frac{\nu\, t}{\pi}}\, \frac{1}{2} = \frac{2}{\sqrt{\pi}} \sqrt{\nu\, t} \qquad (8)$$

From Eq. 8 it may be seen that the distance scale, $\sqrt{\nu\, t}$, of Eq. 4 has the physical significance of being $\sqrt{\pi/2}$ times the displacement thickness. The result of the displacement thickness being proportional to $\sqrt{\nu\, t}$ could have been derived from dimensional considerations because $\sqrt{\nu\, t}$ is the only parameter of the problem with the dimension of a length. However, the constant of proportionality can be obtained only by integration.

Shear Stress in the Sublayer.—In order to become more familiar with the character of this flow, the shear stress may be computed. Because velocity gradients exist in the z-direction only, one may write

$$\tau = \mu \frac{\partial u}{\partial z} = \mu \frac{U_o}{\sqrt{\pi \nu t}} e^{-z^2/4\nu t} \tag{9}$$

in which τ is the shear stress and μ is the dynamic viscosity of the fluid. At the boundary, in which $z = 0$,

$$\tau_0 = \mu \frac{U_o}{\sqrt{\pi \nu t}} \tag{10}$$

At time zero, when the turbulent flow with the constant velocity still reaches the boundary, the shear stress is infinite and subsequently reduces quickly to ever-decreasing values. The significance of this infinite value may be tested by computing its impulse or time integral. An infinite, instantaneous τ_0-value may be acceptable in a physical problem as long as it has no lasting effect. The time integral is thus determined between 0 and a final time, T (total period of growth for a sublayer), as

$$\int_{t=0}^{T} \tau_0 \, dt = \int_{t=0}^{T} \mu \frac{U_o}{\sqrt{\pi \nu t}} \, dt = \frac{2 \, U_o \rho}{\sqrt{\pi}} \sqrt{\nu T} \tag{11}$$

if the dynamic viscosity is expressed as $\mu = \nu \rho$, in which ρ is the fluid density. It may be seen from Eq. 11 that the impulse for finite-time intervals, T, is always finite and approaches zero as T approaches zero, indicating the insignificance of the high τ_0-value at $t = 0$. The time-average shear, $\bar{\tau}_0$, may be computed as

$$\bar{\tau}_0 = \frac{1}{T} \int_{t=0}^{T} \tau_0 \, dt = \frac{2 \, U_o \rho}{\sqrt{\pi}} \sqrt{\frac{\nu}{T}} \tag{12}$$

and, if nomenclature is used that introduces the friction velocity, u_*, as

$$u_* = \frac{\sqrt{\bar{\tau}_0}}{\rho} \tag{13}$$

Eq. 12 may be rewritten as

$$u_*^2 = \frac{2}{\sqrt{\pi}} U_o \sqrt{\frac{\nu}{T}}$$

and a relationship between T, u_*, and U_o results in

$$T = \frac{4}{\pi} \frac{U_o^2 \nu}{u_*^4} \tag{14}$$

Eq. 14 indicates that the growing period of the sublayer may be predicted under the assumed conditions from the kinematic viscosity of the fluid, the average flow, and the shear velocities.

Decay of the Sublayer.—The decay process has been described in the foregoing in general terms. The time necessary for the decay into turbulence and for the reacceleration of the sublayer fluid to the general flow velocity, U_o, outside the sublayer was assumed to be short compared with the build-up period, T, of the sublayer. Any values averaged during the build-up period may be assumed to represent averages for the total time.

The decay process itself is unstable and highly nonlinear. At this time, any mathematical description of the decay process itself appears utterly impossible. The condition of incipient instability, on the other hand, may be studied by a stability condition of the laminar sublayer flow. A dimensional consideration indicates that this condition is characterized by a critical Reynolds number, as in the case of the corresponding laminar pipe flow or channel flow. If the Reynolds number is introduced as

$$\mathbf{R}_N{}^* = 4\,\frac{\delta_T{}^*\,U_o}{\nu} \qquad (15)$$

it may be expected to correspond to the Reynolds number for a pipe flow based on the pipe diameter. The use of the displacement thickness instead of the hydraulic radius may still change the resulting critical Reynolds number from that of a pipe, and it is quite interesting to estimate $\mathbf{R}_N{}^*$, which can be done by estimating the velocity, U_o, for a given value of u_* from the main turbulent flow.

One may assume that U_o exists at a distance, $z = n\,\delta_T{}^*$, from the boundary in the main turbulent flow. Then

$$\frac{U_o}{u_*} = 5.75 \log_{10}\left(9.05\,\frac{n\,\delta_T{}^*\,u_*}{\nu}\right) \qquad (16)$$

If Eqs. 14 and 8 are used to express $\delta_T{}^*$ in Eq. 16, U_o/u_* may be determined as 12.39 for $n = 1.0$ and from that, $\mathbf{R}_N{}^* = 780$. Similarly, values of $\mathbf{R}_N{}^*$ can be obtained for other values of n, as shown in Table 1.

TABLE 1.—CRITICAL REYNOLDS NUMBERS FOR VARIOUS ASSUMPTIONS OF U_o

U_o at	$\mathbf{R}_N{}^*$	T	$\delta_T{}^*$
$y = 0.50\,\delta_T{}^*$	525	$131\,\nu/u_*{}^2$	$12.90\,\nu/u_*$
$y = 1.0\,\delta_T{}^*$	780	$195\,\nu/u_*{}^2$	$15.75\,\nu/u_*$
$y = 2\,\delta_T{}^*$	1,073	$268\,\nu/u_*{}^2$	$18.49\,\nu/u_*$
$y = 3\,\delta_T{}^*$	1,256	$314\,\nu/u_*{}^2$	$20.00\,\nu/u_*$

It is possible to express all the variables of the problem in terms of this critical Reynolds number, as indicated in Table 1. The choice of the proper value for $z/\delta_T{}^*$ may only be made empirically from measurements, although Table 1 can be considered to cover the range within which the correct value may be found.

Momentum Transfer Through the Sublayer.—Eqs. 8 and 12 can be combined into

$$\bar{\tau}_0 = \rho\,U_o\,\frac{\delta_T{}^*}{T} \qquad (17)$$

which may be interpreted as a momentum equation. Eq. 17 indicates that the shear stress, $\bar{\tau}_0$, which is transmitted from the wall to the sublayer per unit of wall area, is used totally to reduce, $1/T$ times per second, the velocity of a mass,

$\rho\,\delta_T^*$, from U_o to zero. An equal amount of momentum is transferred using the same equation by the reacceleration of the same mass to U_o after it has become turbulent. This explanation and description of the momentum transfer from the sublayer to the turbulent flow is one of the great advantages of this model. With this model not only momentum but also fluid, dye, sediment, and heat are transferred from the streamlined flow of the sublayer to the main turbulent flow at the end of each period, T, and from the turbulent flow to the sublayer during its build-up. Such a transfer of material has been observed and cannot be explained with a basically steady flow pattern in the sublayer.

Average Velocity Profile in the Sublayer.—If the duration of the decay period is again assumed to be negligible with respect to the build-up period, the average velocity distribution in the sublayer can be determined by averaging the velocity at a given distance, z, from the boundary over the build-up period, T, only. On the basis of Eqs. 3 and 4,

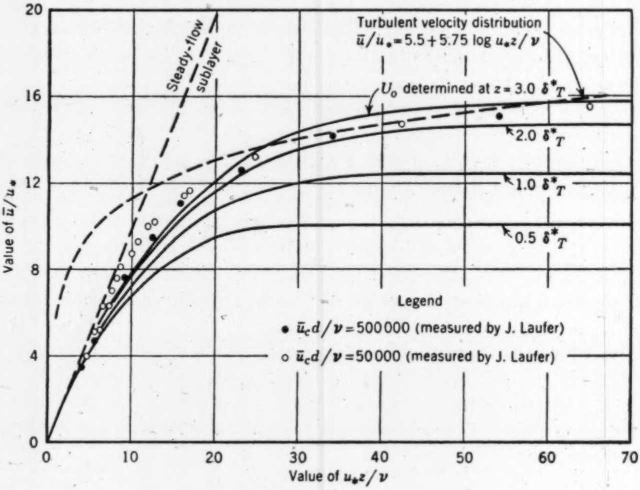


FIG. 2.—TIME-AVERAGE VELOCITY DISTRIBUTIONS IN THE SUBLAYER FOR VARIOUS VALUES OF U_o

$$\frac{\bar{u}}{U_o} = \frac{2}{\sqrt{\pi}} \int_{t/T=0}^{1} \int_{h=0}^{z/2\sqrt{\nu T}} e^{-h^2}\,dh\,d\left(\frac{t}{T}\right) \ldots\ldots\ldots\ldots (18)$$

in which $\bar{u}$ is the time-average sublayer velocity at a distance, z, from the boundary. In this form Eq. 18 shows that the proposed sublayer has a universally applicable velocity distribution if $\bar{u}/U_o$ is given in terms of $z/2\sqrt{\nu T}$. The function in Eq. 18 cannot be integrated in closed form. Numerical integration gives the curve of Fig. 2, in which values obtained by J. Laufer[5] are also shown.

---

[5] "The Structure of Turbulence in Fully Developed Pipe Flows," by J. Laufer, *Technical Note No. 2945*, National Advisory Committee for Aeronautics, Washington, D. C., June, 1953.

Root-Mean-Square Deviation of the Instantaneous Sublayer Velocity.—The root mean square of the local velocity, V_{rms}, also can only be determined numerically. The determination of V_{rms} is based on the following:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (u - \bar{u})^2\,dt} \qquad \text{(19a)}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (u^2 - 2\,u\,\bar{u} + \bar{u}^2)\,dt} \qquad \text{(19b)}$$

$$V_{rms} = \sqrt{\frac{1}{T}\left(\int_0^T u^2\,dt - 2\,\bar{u}\int_0^T \mu\,dt + \bar{u}^2\int_0^T dt\right)} \qquad \text{(19c)}$$

and

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T u^2\,dt - \bar{u}^2} \qquad \text{(19d)}$$

Eqs. 19 can be made dimensionless by dividing by u_*, as shown in Fig. 3.

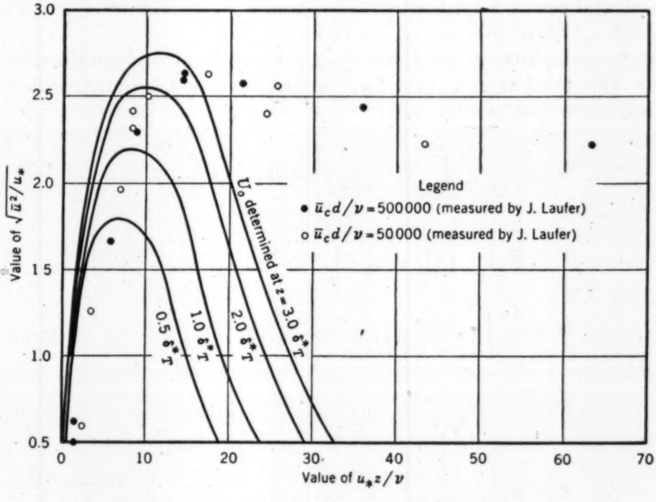


Fig. 3.—Root-Mean-Square Values of the Local Flow Velocity in the Sublayer for Various Values of U_o

Rate of Energy Dissipation in the Sublayer.—It is interesting to determine both the percentage of the total flow energy, which is dissipated into heat in the sublayer by viscous action, and the remaining percentage, which enters the main flow in the form of newly created turbulence. The rate of viscous energy dissipation, ϕ, can be directly computed per unit of time and volume as

$$\phi = \mu \left(\frac{\partial u}{\partial z} \right)^2 \quad \text{(20)}$$

The time-average dissipation, $\bar{\phi}$, per unit of time and volume is

$$\bar{\phi} = \mu \frac{1}{T} \int_0^T \left(\frac{\partial u}{\partial z} \right)^2 dt \quad \text{(21)}$$

If $\mu \, \partial u/\partial z$ is taken from Eq. 9,

$$\bar{\phi} = \frac{\mu}{T} \frac{U_o^2}{\pi \, \nu} \int_0^T \frac{1}{t} e^{-z^2/2\nu t} \, dt \quad \text{(22)}$$

and

$$S = - \frac{z^2}{2 \, \nu \, t} \quad \text{(23)}$$

then

$$\bar{\phi} = - \frac{\rho \, U_o^2}{\pi \, T} \int_{-\infty}^{-Z^2/2\nu t} \frac{1}{S} e^S \, dS \quad \text{(24)}$$

This integral can be solved numerically from tables. The result is given as a plot of $\nu^2/u_*^4 \, (\partial \bar{u}/\partial z)^2$ versus $u_* \, z/\nu$ in Fig. 4.

Of particular interest is the rate of total energy dissipation in the entire sublayer. This total rate, Φ, may be obtained by a double integration for the unit boundary area as follows:

$$\Phi = \frac{1}{T} \int_{t=0}^{T} \int_{z=0}^{\infty} \nu \, \rho \left(\frac{\partial u}{\partial z} \right)^2 dz \, dt \quad \text{(25a)}$$

$$\Phi = \frac{1}{T} \int_{t=0}^{T} \left[\int_0^{\infty} \frac{\rho}{\pi} \sqrt{\frac{2 \, \nu}{t}} \, U_o^2 \, e^{-z^2/2\nu t} \, d \left(\frac{z}{\sqrt{2 \, \nu \, t}} \right) \right] dt \quad \text{(25b)}$$

$$\Phi = \frac{1}{T} \int_{t=0}^{T} \frac{\rho \, U_o^2}{\sqrt{\pi}} \sqrt{\frac{\nu}{2 \, t}} \, dt \quad \text{(25c)}$$

$$\Phi = \rho \, U_o^2 \sqrt{\frac{2 \, \nu}{\pi \, T}} \quad \text{(25d)}$$

and

$$\Phi = \rho \frac{U_o}{\sqrt{2}} u_*^2 \quad \text{(25e)}$$

The total rate of energy, Φ_t, available in the main turbulent flow may be determined for the unit area and time interims of the average velocity, U_{avg}, of the turbulent flow as

$$\Phi_t = \bar{\tau}_0 \, U_{avg} \quad \text{(26a)}$$

and

$$\Phi_t = \rho \, U_*^2 \, U_{avg} \quad \text{(26b)}$$

from which the fraction of energy dissipation in the sublayer is

$$\frac{\Phi}{\Phi_t} = \frac{1}{\sqrt{2}} \frac{U_0}{U_{avg}} \qquad (27)$$

Application of the Model to Actual Flows.—In its application any model must describe as many of the various phases of the actual flow behavior as possible. This model has the advantage over any quasi-steady-flow sublayer description in that it explains the exchange of matter between the turbulent flow and the boundary as it has been observed to exist.

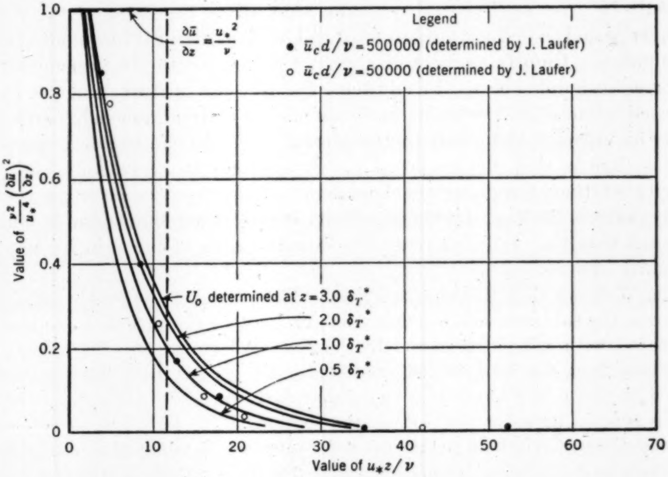


FIG. 4.—DIRECT VISCOUS-DISSIPATION RATE IN THE SUBLAYER FOR VARIOUS VALUES OF U_o

However, no model and no theory are better than the assumptions on which they are based. Although it is believed that the basic description of the sublayer cycle is correct, some of the mathematical simplifications used in the derivation and solution of the equations are somewhat questionable. The assumption of an infinitely high eddy viscosity and of the resulting constant flow velocity in the turbulent flow is definitely unrealistic. It will be necessary, therefore, to investigate the effect of this idealization of the flow, which can best be done by comparing observed velocity distributions with the prediction of the model. Fig. 2 shows several predicted velocity distributions based on various assumed U_o-values and corresponding experimental values measured by Laufer.[5] The measurement seems to confirm the contention that in a curve of $\bar{u}/u_*$ versus $u_* z/\nu$ the distribution is independent of the over-all Reynolds number of the flow. Furthermore, the graph indicates that the outside velocity, U_o, must probably be measured at a distance of between

$2\,\delta_T{}^*$ and $3\,\delta_T{}^*$ to give a velocity distribution similar to the measured distribution.

Assume that the value of $z = 3.0\,\delta_T{}^*$ indicates the point of the turbulent-velocity-distribution curve (shown in Fig. 3 as a dashed curve) in which U_o is obtained. At this point, $\bar{u}/u_* = 15.6$, which is indicated by the horizontal tangent to the computed $\bar{u}/u_*$-curve. It may be remembered that, due to the assumption of uniform turbulent velocity distribution, the flow velocities in the range of the entire sublayer are assumed to have a value of $\bar{u}/u_* = 15.6$ at the beginning of the build-up period of the viscous sublayer. This velocity distribution is assumed to have been created in the short preceding period of decay by turbulent momentum exchange with the outside turbulent flow. It would be more appropriate to assume that at the beginning of the viscous sublayer period the velocity distribution follows the extended turbulent velocity distribution. Unfortunately, this approach leads to hitherto insurmountable mathematical difficulties. In addition, the physical picture becomes rather confused because the transmission of momentum is simultaneously performed, partly by viscosity and partly by turbulence. All that can be safely concluded at this time is that the proposed theory assumes an excessively high flow velocity at time zero near the boundary. With the time-average velocity nearly correct, the final minimum velocity is probably too low, and it must be expected that the predicted root-mean-square values of the velocity near the boundary are too high.

Fig. 3 shows that this is actually so. The curves of the root-mean-square values of the sublayer velocity (Eq. 19) were plotted for various U_o-values and compared with values measured by Laufer. All curves are high along this steep rise from the wall as just predicted. They come to a sharp maximum near $u_*\,z/\nu = 6$ to 12 and drop again sharply toward zero while the measured values remain high for the rest of the section. The foregoing is reasonable because the predicted curve describes the unsteady laminar flow only, whereas the measured points additionally include the fluctuations of the turbulence. Turbulence must be expected to reach its maximum intensity just outside the sublayer so that the difference between the predicted curves and the measured values is explained for values of $u_*\,z/\nu$ larger than the maximum of the predicted root-mean-square curves.

From this comparison of the measured and predicted values of velocity measurements, it is possible to conclude that the proposed theory, although basically correct, gives only a rough approximation of the actual flow pattern. Consequently, the values for the predicted period, T, must also be expected to be larger than the actual values, which may be seen from the derivation of Eq. 17. If the range of velocity change at various distances from the wall is effectively less than assumed, the number of changes per unit of time must be higher than predicted to permit an equal shear to be transmitted by momentum exchange. Unfortunately, no measurements could be found in the literature from which the period, T, could be taken, nor is there any mention that any periodicity of the process is indicated by available measurements.

Experimental Demonstration of Periodic Character of the Sublayer.—The fact that none of the successful investigators of sublayer flow in the past noticed any periodicity is sufficient proof that the periodicity cannot be directly apparent in any velocity signal but is probably concealed by large-amplitude, random disturbances originating in the turbulent flow outside the sublayer. Therefore, it appeared most promising to locate the point of measurement as far away from the turbulent flow as possible or to measure at the frictional boundary itself. The flow velocity vanishes at that point and thus cannot be measured. However, it must be expected that the pressure reflects any such periodicity of the flow pattern. A pressure recording system was developed using a Rutishauser pressure gage as described in the Appendix. This signal was expected to contain a large percentage of random noise created by the turbulence of the main flow, calling for a special method to detect any possible periodicity. The available sound analyzers were unreliable because their band width was excessive and their frequency range insufficient for low frequencies. Continuous records were made of the pressure, and an autocorrelation curve was constructed for the records. This method has the following value: Even if the major part of the signal fluctuations is irregular and random, this part of the signal will result in a regularly dropping correlation curve. If there is a periodic component added to the random signal, this part of the signal will appear as a superimposed periodic fluctuation in the correlation curve, too, as may be seen in Fig. 5.

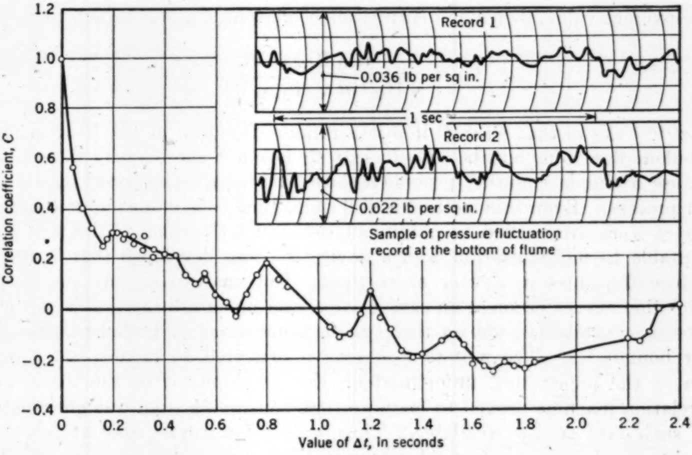


FIG. 5.—AUTOCORRELATION OF THE PRESSURE FLUCTUATION AT THE BOTTOM OF THE FLUME, RUN 1

The autocorrelation coefficient, C, for a continuous function of time, such as the boundary pressure, is

$$C = \frac{\int p_{(t)} \times p_{(t+\Delta t)}\,dt}{\sqrt{\int p_{(t)}^2\,dt}\,\sqrt{\int p_{(t+\Delta t)}^2\,dt}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

in which $p_{(t)}$ is the pressure at time t; $p_{(t+\Delta t)}$ denotes the same pressure at Δt at a later time; and the three integrals cover the same range of t. The integral values may be approximated by the sum of a large number of individual p-values. However, it is necessary to choose the number of these individual points from 200 to 1,000 to eliminate the accidental influence of individual extreme values. The determination of each individual C-value represents from 4 hr to 12 hr of work if this method is used without any special tools. Fig. 5 shows that only a large number of C-values for a wide variety of (Δt)-values are able to show any characteristic trends, such as the periodic component in Fig. 5. This direct method was abandoned as being too slow.

According to R. R. Putz, J. L. Lawson, and G. E. Uhlenbeck,[6,7,8,9] the correlation coefficient, C, can be obtained in a different way. If a new function, Π, is introduced so that Π has the value of $+1$ whenever p is larger than its time-average value and has the value of -1 whenever p is smaller than its time-average value, then a new correlation coefficient, C_1, may be determined as

$$C_1 = \frac{\int \Pi_{(t)} \, \Pi_{(t+\Delta t)} \, dt}{\sqrt{\Pi_{(t)}{}^2 \, dt} \, \sqrt{\Pi_{(t+\Delta t)}{}^2 \, dt}} \qquad (29)$$

Putz[6,7,8,9] shows that $C_1 = C$ if p is a random function of the kind in most turbulent flow components. Coefficient C_1 is much more easily determined because a simple counting process replaces the more complicated multiplication process. Even this method is still quite time consuming; approximately 2 hr of work are needed to obtain an individual C_1-value. This process is adaptable to mechanization, and a device is being developed that is able to produce C_1-values in a very short time. The analysis of any records by correlation curves becomes an easy matter, and much information is expected to become available soon on the periodic component of turbulent processes near boundaries. The writers would have preferred to include some more data in the paper, but, unfortunately, the development of the mechanical correlation machine proved to be more time consuming than was anticipated, and such data are not available. Thus, only a few curves exist at this time, which were derived by manual determination, such as those shown in Figs. 5 and 6. Although the periodic component is quite pronounced in Fig. 5, this tendency is not so clear in Fig. 6. The flow of Fig. 5 may be used to check the frequency of the fluctuations against the theoretical prediction. The flow under consideration is an open channel flow of machine oil with the characteristics as given in the Appendix. Assuming again that U_o must be measured at $3.0 \, \delta_T{}^*$, the period, T, may be determined as 1.25 sec. The measured C_1-curve shows a distinct period of 0.4 sec, or approximately one-third of the predicted value. Whether the foregoing idealization of the boundary conditions for the

---

[6] "Measurement and Analysis of Ocean Waves," by R. R. Putz, *Proceedings*, First Conference of Ships and Waves, Council of Wave Research and Soc. of Naval Archts. and Marine Engrs., 1954, Chapter 5, p. 63.

[7] "Wave Transformation: Linear Least-Square Predictions," by R. R. Putz, *Technical Report Series 29, Issue 58*, Inst. of Eng. Research, Univ. of California, Berkeley, Calif., 1954, p. 4.

[8] "Statistical Analysis of Wave Records," by R. R. Putz, *Proceedings*, Fourth Conference on Coastal Eng., Council on Wave Research, 1954, Chapter 2, p. 13.

[9] "Threshold Signals," by J. L. Lawson and G. E. Uhlenbeck, McGraw-Hill Book Co., Inc., New York, N. Y., 1950.

analytic treatment can be made responsible for such a large deviation can probably not be decided until more experience with this method of statistical analysis is available.

Boundary Sublayer and Turbulence.—Many definitions have been given for turbulence in the past, most of which stress the random character of the flow pattern in turbulence. However, the detailed description is still made by the Navier-Stokes equations.

One of the most important characteristics of turbulence can be shown by a comparison between a fluid agitated by simultaneous, complicated wave systems and one agitated by turbulence. The wave systems visualized here may be of such an amplitude that the resulting motion can be described by linearized equations. As in the case of turbulence, a motion results that appears to be random if a large area is observed and becomes absolutely continuous in the small detail; however, this motion is not turbulence. The main difference is that in the case of the wave motion any dye that is injected into the fluid will, for a long time, remain at approximately the same location without much mixing with the surrounding fluid. If dye is injected in a turbulent flow, however, it mixes intensely with the surrounding fluid and soon becomes unidentifiable. This indicates that neighboring particles will not stay close together but will soon occupy widely separated positions. This behavior is usually found in flows with strong shear motion or shear deformation of the fluid. It is true that a laminar flow also shows this deformation, but neither is it turbulent nor will it mix dye as efficiently with the surrounding fluid as will the turbulent flow. The main difference between the laminar flow and the turbulent flow seems to be that the shear motion is regular and even in the laminar flow but is irregular in the turbulent flow.

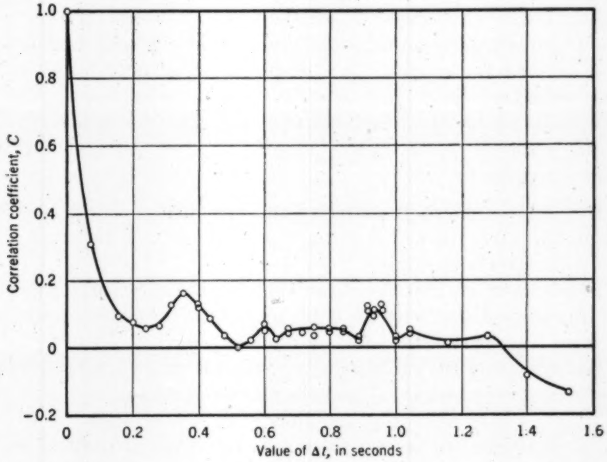


FIG. 6.—PRESSURE-FLUCTUATION AUTOCORRELATION AT THE BOTTOM OF THE FLUME, RUN 2

This difference can also be expressed as follows: Both types of motion are of the kind in which the fluid has vorticity. The difference between them is that in the laminar flow the vorticity is distributed regularly according to the main flow pattern, whereas the distribution in turbulence is in patterns that are of much smaller scale than the main flow pattern. High concentrations of vorticity appear to be characteristic for a vigorous, fresh turbulence. There is no mechanism in a turbulent flow that explains the origin of these high concentrations of vorticity in the fluid better than the proposed model of a periodically disintegrating sublayer.

No explicit expressions will be given herein for the vorticity distribution that periodically originates at the boundary to become part of the main turbulence at the end of each period, T. The distribution of vorticity at time T may be determined from the equations of the sublayer, and this distribution is probably not much changed during the process of disintegration of the sublayer. This qualitative examination may be sufficient until more data exist to prove or disprove the validity of the proposed mathematical description of the unsteady sublayer.

Possible Improvements of the Solution.—The given solution is believed to be qualitatively correct; however, the following improvements, among others, are desirable if possible:

1. It has been shown that the assumption of a constant velocity in the turbulent flow adjacent to the sublayer is artificial and has been introduced for mathematical reasons only.
2. It was assumed that the entire sublayer becomes part of the turbulent flow. It is possible that only the outer part of the sublayer becomes turbulent and that an inner part of the sublayer remains always at the boundary. Even if this steady part of the sublayer is very thin, it would change materially the constants computed for δ_T^*, T, and the other variables.
3. It is possible that the one-dimensional description of the sublayer growth is insufficient to predict the actual conditions if the real growth does not occur simultaneously over large areas of the boundary. It is conceivable that the process actually changes phase over the various parts of the boundary like a wave motion, resulting in different solutions.
4. It is desirable that a description be found for the disintegration process because sufficient time may be involved to change materially the time-average values of average velocity, velocity fluctuation, and period T.

An intense qualitative and quantitative study of the processes, as well as an extension of the proposed theoretical approach, will be necessary to answer these questions.

CONCLUSIONS

It was observed that dye and sediment particles were exchanged between a smooth boundary and the turbulent flow through the viscous sublayer, thus making the assumption of a quasi-steady sublayer impossible.

No reasonable explanation can be given for the creation of new turbulence and for the transmission of shear from a quasi-steady sublayer to the turbulent flow. The periodically growing and disintegrating sublayer pattern is proposed instead, which explains all these facts satisfactorily.

It was assumed that the period of disintegration is negligibly short compared with the growing period; average values for the entire period were derived from a simplified description of the growing period only.

The most objectionable assumption made for mathematical reasons was that of constant flow velocity in the turbulent flow outside the sublayer. It was shown that the deviations of observed patterns from the prediction of the theory can be explained qualitatively by this assumption.

No qualitative contradiction has been found to date (1957) between the proposed theory of the sublayer and flow observations.

ACKNOWLEDGMENT

The work on this problem was initially made possible by a grant of the Research Corporation, New York, N. Y. The later development of the experimental and theoretical work was sponsored by the United States Department of the Navy.

APPENDIX

Experimental Procedure Used in the Determination of Local Wall Pressures.—After the problem was conceived in 1953 a search was made for various pressure-recording devices—for example, devices that could be used to record continuously the instantaneous pressure and its fluctuations as they apply to a small part of the boundary surface of a turbulent flow. An appraisal was made of the frequencies that were expected to occur in the problem, of the magnitude of the amplitudes of the fluctuations, and of the area expected to show predominately equal pressure. On the basis of water flowing at Reynolds numbers of from approximately 10^4 to 10^5, the following specifications were drafted for the pressure gage:

Frequency response	0 to 5,000 cycles per second
Pressure sensitivity	± 0.001 pound per square inch
Maximum pressure	1 pound per square inch
Area covered by the sensitive element	0.001 foot in diameter

No commercial instrument could be found that satisfied these stringent conditions even approximately. It was therefore decided to acquire the instrument that most closely approximated the conditions. It was hoped that either the estimated conditions were too stringent or that the instrument could be improved later to adapt it to the particular purpose.

A Rutishauser transducer was used, which utilizes a membrane with a free diameter of 0.0156 ft, has a frequency response of as much as 10,600 cycles per sec, a normal sensitivity of 0.01 lb per sq in., and a maximum pressure of 1 lb per sq in. with a 0.001-in.-thick diaphragm. The membrane, together with an electrode that is 0.0016 in. behind it, represents a variable condenser, which modulates the frequency of a 10-megacycle oscillator. The output of this oscillator is then analyzed by a discriminator, resulting in a DC output signal proportional to the original motion of the diaphragm. This electric signal was amplified and recorded, or its root-mean-square value is measured. Details of the electronic system are available in a report by the junior writer.[10]

With this type of instrument it was possible to adjust the useful ranges. If the sensitivity was to be increased, for instance, it was done by replacing the diaphragm with one of a smaller thickness. Unfortunately, this change so reduces the frequency response that sensitivity can be gained by increased amplification until the unavoidable tube noises become excessive. The area of measurement can be reduced by mounting the diaphragm inside a cavity, which, itself, is connected with the free flow by a small opening. Again the reduction of measuring area is traded for frequency response, which is drastically reduced by the cavity. An explanation[10] of the theory of the cavity is also available.

An important and time-consuming phase of the experimental work was the calibration of the pressure indicator and its complex electronic amplifying and recording system. This calibration could not be made with one piece of equipment for the entire range but was divided into independent stages:

1. A static calibration was made by direct comparison with manometer readings using DC amplifiers. Only limited sensitivity could be reached with this method because of the difficulty of stabilizing high-gain, DC amplifiers.
2. A low-frequency calibration was made by using a mechanically driven excenter, which periodically compressed the air enclosed in a bellows. Frequencies of as much as 160 cycles per sec could be reached with this instrument, which automatically governed the amplitude independent of the frequency.
3. A high-frequency vibrator was used for frequencies between 130 cycles per sec and 6,000 cycles per sec. This system consisted of a loud speaker driving various resonance tubes, in the bottom of which the pressure heads were mounted.

The calibration procedure became complicated not only because different instruments were used, but also because the third method does not permit an absolute calibration but only a relative comparison of various instruments.

The calibration of the transducers with the membranes directly exposed to air was performed first. The first two methods, which permit an absolute calibration, showed that both pressure heads with all available diaphragms gave a perfectly straight response up to the maximum frequency of 160 cycles per sec. The theoretical limit of linearity can be predicted as 3,500 cycles per sec for the thinnest membrane. It is to be expected that if any deviation from the straight calibration were to occur in the range of frequencies between 160 cycles per sec and the frequencies where resonance begins to affect the calibration, such a deviation would change with the instrument and particularly with the thickness of the membrane. The calibration was expected to be still straight for both instruments as long as their signal was the same as for a given signal in the resonance tube. With this method the instruments were shown to have a straight frequency response of as much as 2,000 cycles per sec for all diaphragms in air.

---

[10] "On the Measurement of Pressure Fluctuations at a Smooth Boundary of an Incompressible Turbulent Flow," by Huon Li, *Report Series No. 65, Issue No. 1*, prepared by the Inst. of Eng. Research, Univ. of California, Berkeley, Calif., for the Office of Naval Research under contract N-onr-222(22), Project Nil 062 173, December, 1954.

The next step in the calibration was the introduction of a cavity filled with liquid with a reduced opening to the open fluid in which the pressures were measured, thus reducing the area over which the pressure is averaged. The additional drastic reduction of the frequency response that the theory of the cavity predicts was actually found to occur by this measurement. Table 2 gives the frequency response for various geometries of the cavity.

The Experiments.—When the previously enumerated values for the qualifications of the measuring device were compared with the values for characterizing the instruments by their calibration, it was found that (1) for a given opening of the measuring cavity the frequency response was reduced considerably for a significant part of the active spectrum, and (2) the sensitivity of the device was barely sufficient to separate the expected signal from unavoidable noise.

These conditions appeared to prove that the chosen instrument was unsatisfactory. Before the entire approach was changed it was decided to try at least certain partial problems, or to investigate certain ranges of conditions using the available instruments. Thus, it was asked, under what conditions is the large-scale turbulence pattern near the wall combined with low frequencies and high-pressure scales? The only possible answer was under conditions of flow with a low Reynolds number and high viscosity—that is, a flow of a highly viscous fluid. After previous attempts had already shown that it was difficult for closed-pipe systems to be free from pressure waves, which constantly travel through the system and distort the pressure records, it was decided to construct an open channel flume, circulating a flow of oil at high velocity but having a moderate Reynolds number.

TABLE 2.—FREQUENCY RESPONSE FOR VARIOUS GEOMETRIES OF THE CAVITY

Range, in pounds per square inch	Thickness, in inches	Diameter, in inches	Natural frequency in air, in cycles per second	Linearity limit in air, in cycles per second	Diameter of measured hole, in inches	Length of cavity, in inches	Predicted natural frequency in water, in cycles per second	Linearity limit in water, in cycles per second
0 to 1	0.001	3/16	10,600	3,500	0.1	1/32	3,570	1,190
						1/16	2,590	860
						1/4	1,320	440
						1	660	220
					0.01	1/32	380	126
						1/16	260	88
						1/4	132	44
						1	66	22
0 to 100	0.004	3/16	42,600	14,200	0.1	1/32	24,800	8,260
						1/16	19,100	6,360
						1/4	10,400	3,470
						1	5,320	1,770
					0.01	1/32	3,050	1,020
						1/16	2,140	714
						1/4	1,020	360
						1	540	180

The flume, with a rectangular cross section having a clear width of 11.5 in. and a depth of 8 in., was constructed of wood supported by timber bents. The oil dropped from the flume into a reservoir, from which it was pumped by an electrically driven centrifugal pump through a 10-in. pipe to the upstream end of the flume. The discharge was adjusted by bypassing part of the discharge directly into the reservoir. One of the most important features of the system was the complete mechanical separation of the flume itself from the pump and the return pipe in order to keep all vibrations away from the flume and the pressure-recording instrument, which is very sensitive to vibrations of any kind. The discharge was measured at a contraction of the return pipe, and the oil depth in the flume was measured by point gages. A continuous record was kept of the oil temperature, from which the viscosity could be determined after a set of viscosimeter tests had determined the viscosity-temperature relationship for the oil. For engineers accustomed to working with water flows the resulting flow was rather odd. With respect to turbulence, the flows were near critical, or were changing from laminar through critical to turbulent. The depth of flow was approximately 2 in., and the velocities ranged from 6 ft per sec to 7 ft per sec. Thus, the oil flowed with velocities significantly higher than wave velocity—that is, a laminar shooting flow was readily obtained.

This experimental arrangement had another great advantage. A laminar flow was first established with velocities close to critical. The pressure-recording instruments were turned on so that the noise level could be observed as it was generated by the pump and pipe flow and by other unrelated activities in the neighborhood. The flow was then slightly increased, causing turbulence to be generated, and the record continued while the flow measurements were made. It could be observed that the pressure records showed fluctuations due to turbulence of a larger order of magnitude than the general noise level.

Experimental Conditions for the Flow (*Fig. 5*).—The following flow conditions were established:

Channel, width (b), in feet	0.957
Channel slope (S_o)	0.0502
Flow depth (h), in feet	0.144
Average flow velocity (U), in feet per second	6.27
Discharge (q), in cubic feet per second	0.865
Viscosity of oil (μ), in pound-seconds/feet^{-2}	1.23×10^{-3}
Density of fluid (ρ), in slugs per cubic feet	1.73
Kinematic viscosity (ν), in square feet per second	7.15×10^{-4}

The shear was assumed to be evenly distributed over the bottom and sides; thus, the average shear stress, $\bar{\tau}_0$, was 0.313 lb per sq ft. The value of $\bar{\tau}_0/\rho$ was 0.180 sq ft per sec^2, and the shear velocity, $u_* = \sqrt{\bar{\tau}_0/\rho}$, was 0.423 ft per sec. The thickness of the viscous sublayer, conventionally defined as $\delta = 11.6\ \nu/u_*$, was 1.96×10^{-2} ft.

The period of sublayer growth and separation frequency for various values of U_o were:

	T, in seconds	n, in seconds^{-1}
U_o at 0.5 δ_T*	0.524	1.91
U_o at 1.0 δ_T*	0.780	1.28
U_o at 2.0 δ_T*	1.07	0.935
U_o at 3.0 δ_T*	1.25	0.800

Because the flow could not be maintained practicably for long, it was decided to record the pressure continuously for several minutes and to postpone the analysis of the record. A block diagram of the experimental equipment is shown in Fig. 7.

A Tektronix low-level preamplifier (type 122), which amplifies the discriminator voltage, has a noise level of as low as from 1 microvolt to 4 microvolts expressed as the input signal. The preamplifier has a maximum gain of approximately 1,000, and the frequency-response curve is essentially flat from 1 cycle per sec to 40 kilocycles per sec.

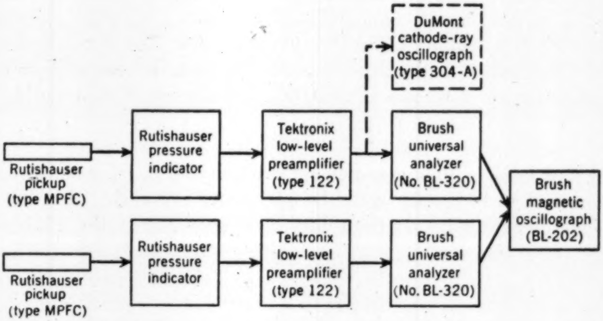

DuMont
cathode-ray
oscillograph
(type 304-A)
Rutishauser
pickup
(type MPFC)
Rutishauser
pressure
indicator
Tektronix
low-level
preamplifier
(type 122)
Brush
universal
analyzer
(No. BL-320)
Brush
magnetic
oscillograph
(BL-202)
Rutishauser
pickup
(type MPFC)
Rutishauser
pressure
indicator
Tektronix
low-level
preamplifier
(type 122)
Brush
universal
analyzer
(No. BL-320)

FIG. 7.—BLOCK DIAGRAM OF EQUIPMENT USED FOR MEASURING THE PRESSURE FLUCTUATION BETWEEN TWO POINTS

The output signal of the preamplifier was further amplified in a Brush universal analyzer and recorded by a Brush magnetic oscillograph (BL-202). The Brush universal analyzer is a DC amplifier and has a frequency response that is flat from 0 cycles per sec to 100 cycles per sec. For higher frequency signals a DuMont cathode-ray oscillograph (type 304-A) was used to replace the Brush universal analyzer in connection with a DuMont oscillograph-recording camera (type 321). A short excerpt from this record is shown in Fig. 5.

DISCUSSION

Edward Silberman,[11] A. M. ASCE.—The hypothesis of a periodic viscous sublayer in turbulent boundary-layer flow is an intriguing one. However, it would be desirable to have better experimental evidence of its existence than has been offered in the paper. A possible experiment for direct visual observation of periodicity will be described subsequently.

First, the writer suggests a modification in the theoretical development beginning with the section, "The Proposed Model: Thickness Scale of the Sublayer." Instead of using displacement thickness, let the edge of the sublayer be given by $z = \delta$ when $u/U_o = 0.99$. Then, using Eqs. 3 and 4,

$$\delta = 3.64 \sqrt{\nu\, t}$$

(The choice of $u/U_o = 0.99$ is somewhat arbitrary; if $u/U_o = 0.995$ had been used, the coefficient in Eq. 30 would be 3.98 instead of 3.64.) Let the critical Reynolds number for sublayer instability, corresponding to Eq. 15, be

$$\left.\frac{\delta\, u_*}{\nu}\right|_{\text{critical}} = C' \qquad (30)$$

in which C' is, presumably, a universal constant for boundary-layer flow. Letting $\delta = \delta_T$, when $t = T$, and using Eq. 14,

$$\frac{U_o}{u_*} = \frac{C' \sqrt{\pi}}{7.28} \qquad (31)$$

Eq. 16 is the Kármán-Prandtl equation for velocity distribution, which is ordinarily written as follows[12]:

$$\frac{u}{u_*} = 5.75 \log_{10}\left(\frac{z\, u_*}{\nu}\right) + 5.5 \qquad (32)$$

This equation was determined empirically from the completely turbulent part of the flow near a smooth wall and is apparently universally applicable to all smooth pipes, channels, and flat plates (with some room for argument about the exact values of the numerical constants). When $z = \delta_T$, then $z\, u_*/\nu = C'$ and $u = 0.99\ U_o$. Inserting these values in Eq. 32 and combining with Eq. 31, $C' = 66$ and $U_o/u_* = 16.1$ (H. Schlichting[13] gives $C' = 70$ from an inspection of J. Nikuradse's pipe data.)

The foregoing argument arrives at essentially the same value of U_o/u_* found by the authors, assuming that $\delta = 3\, \delta_T{}^*$, but there is little question as to what thickness definition to use for the sublayer. If $u/U_o = 0.995$ had been used, $C' = 73$ and $U_o/u_* = 16.25$ would have resulted.

Another conclusion can be made from the foregoing development: No boundary-layer flow in which $\delta\, u_*/\nu < 66$ can be turbulent. This conclusion has far-reaching implications in determining the minimum Reynolds number for transition in flow in ducts or channels with smooth boundaries, but it will not be developed further herein. Using this criterion, it is possible to describe a flow in which the periodic viscous sublayer, if it exists, should be clearly visible.

---

[11] Associate Prof., St. Anthony Falls Hydr. Lab., Univ. of Minnesota, Minneapolis, Minn.

[12] "Elementary Mechanics of Fluids," by H. Rouse, John Wiley & Sons, Inc., New York, N. Y., 1946, p. 194.

[13] "Boundary Layer Theory," by H. Schlichting, McGraw-Hill Book Co., Inc., New York, N. Y., 1955, p. 407.

Such an experiment would be performed on a fully developed turbulent flow in the vertex of a V-shaped channel or pipe with a small included angle. Measuring δ normal to the walls, $\delta\, u_*/\nu$ would be less than 66 until a considerable depth from the vertex is reached, and the viscous sublayer should be stretched to finite dimensions. Some measurements[14] with air flowing in a pipe of this shape with an included angle of 12° were described by E. R. G. Eckert and T. F. Irvine, who found a very deep laminar flow region and beyond that another relatively deep region with some indication of intermittent turbulence. However, no tests for periodicity in the flow were made, and periodicity was not observed. These experiments, or similar experiments with dye streams in liquid, should be repeated with the objective of seeking periodicity. In such experiments dye would have to be introduced through wall taps in order to avoid disturbing the stream. It is suggested that in a channel with a small vertex angle at the bottom, dyes of variable density slightly less than that of the flowing fluid could be introduced at the vertex so that the dye streams would rise by buoyancy into the supposedly periodic part of the boundary layer.

The third assumption in the development of the theory—that there is simultaneous growth of the sublayer over a large area of the boundary—has led the authors to expect the period in the pressure autocorrelation at a point to be the same as the computed period of the sublayer. It is more reasonable to expect the instability of the sublayer to be a function of both time and space, as was found by G. B. Schubauer and P. S. Klebanoff[15] for the laminar boundary layer in their investigation of transition. Under these circumstances, in their experiments the authors should have obtained pressure correlations from two points of variable separation along the stream direction, and should have looked for peaks in the correlation curve at intervals of $K\, U_o\, T$, in which $K\, U_o < U_o$ and represents the average rate of motion of the periodic part of the sublayer. The third assumption is still necessary and acceptable in the development of the theory.

NICOLS N. AMBRASEYS,[16] J. M. ASCE.—Although the writer does not disagree with the contentions of the authors, it is felt that the experiments might have allowed for the treatment of some additional characteristics of the turbulent flow within the sublayer. Thus, the writer could have compared the hypothesis presented by the authors with results obtained from similar experimental studies on the nonisotropic flow near a wall.

---

[14] "Simultaneous Turbulent and Laminar Flow in Ducts with Non-Circular Cross Sections," by E. R. G. Eckert and T. F. Irvine, *Journal of the Aeronautical Sciences*, January, 1955, pp. 65–66.

[15] "Contributions on the Mechanics of Boundary-Layer Transition," by G. B. Schubauer and P. S. Klebanoff, *Technical Note No. 3489*, National Advisory Committee for Aeronautics, Washington, D. C., September, 1955, p. 31.

[16] Extraordinary Asst. Prof. of Fluid Mechanics, National Technical Univ., Athens, Greece.

Some of the consequences of the results[17, 18] obtained by the writer are:

1. The thickness of the sublayer, which is defined as the thickness of the zone within which the velocity distribution does not obey von Kármán's law, extended to a dimensionless distance from the wall, equal to from 26.6 to 30.1, according to the assigned values for the universal coefficients, k and β (k varying from 0.33 to 0.4 and β, from 0.111 to 0.113).
2. Scales and microscales of turbulence exhibited a strong dependence on the local mean velocity within the sublayer.
3. Part of the energy produced within the sublayer by the pressure drop and converted into turbulent energy was influenced by basic flow characteristics but mainly by the amount of mean velocity available. Therefore, length scales in the sublayer are not position functions but depend greatly on the local mean values of the velocity. This proves that actually the hypothesis of "mixing length" does not apply for the quasi-turbulent region near the wall.
4. A simple expression was found to connect turbulent shear and local mean velocity for values of the dimensionless distance less than 27, and a new universal constant was defined.

The writer would like to learn whether the authors have observed any dependence of the turbulent shear on the mean velocity, a point of vital interest in its possible bearing on the time-average values of mean velocities and the period, T.

Hans A. Einstein,[19] M. ASCE, and Huon Li[20].—Messrs. Silberman and Ambraseys have made it clear that the problem is difficult to handle both experimentally and theoretically. The experimental difficulty is caused predominantly by the turbulence outside the sublayer, which moves past any place of observation at high velocity and causes pressure fluctuations of equal magnitude but with higher frequency than those developed in the sublayer itself. The theoretical difficulty is caused by the fact that the sublayer should constitute a transition between the still boundary and the turbulent velocity profiles, whereas a mathematical solution for the transition could be found only between the still boundary and a uniform flow of constant velocity.

There was no disagreement with the main ideas of the paper: (1) That the sublayer was interpreted as a nonsteady flow with an average flow pattern rather than as a steady flow with superimposed fluctuations; and (2) that in areas of predominantly viscous character turbulent effects may be neglected (built-up phases); and (3) that in areas of turbulent character the viscous effects are negligible (mixing with main flow). These two ideas appear to be basic and new, and may prove helpful in describing similar phenomena.

Mr. Silberman proposes the use of another definition for the effective sublayer thickness, which is allowable because the choice is completely arbitrary. The

---

[17] "Hydraulic Research," International Assn. for Hydr. Research, The Hague, Netherlands, 1954, p. 132.

[18] *Proceedings*, 6th General Meeting of the International Assn. for Hydr. Research, The Hague, Netherlands, Vol. 4, 1955, pp. (III-7)–(III-8).

[19] Prof. of Hydr. Eng., Univ. of California, Berkeley, Calif.

[20] Research Engr., Odin Associates, Pasadena, Calif

proposed use of dye was actually applied by the authors to show that at some instances a distinctly laminar flow pattern exists near the wall, whereas at some other instances turbulence appears to reach all the way to the wall. The processes are too fast for visual observation but show up well in high-speed photography.

Mr. Ambraseys' observations of the statistical flow described in the range of the sublayer are interesting. Unfortunately, no observations are available from the experiments that would permit a check of his results.

The discussers' helpful suggestions have been appreciated, and it is hoped that more measurements may become available soon that will permit a more detailed description of the sublayer with the help of the proposed theory.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2923

MOMENT-DISTRIBUTION CONSTANTS FROM MODELS

BY OTAKAR ONDRA,[1] A. M. ASCE

WITH DISCUSSION BY MESSRS. JOHN C. MURPHY; THOMAS D. Y. FOK; FRANK J. CAIN AND GERALD LUCK; AND OTAKAR ONDRA

SYNOPSIS

An experimental method of determining the moment-distribution constants for beams with variable moments of inertia is presented. The method does not rely on the use of deformeter-type gages or other special instruments. It is based on the concept of a three-dimensional solid whose properties are functions of the loads, slopes, and deflections of the statically indeterminate member which it represents. It is shown that each moment-distribution constant is a function of a ratio, J/Q, which also defines the location of the center of gravity of the solid. This ratio is evaluated experimentally by weighing the solid and by using the principles of statics. Mathematical integration or the approximate summation process are replaced by weighing, which offers simpler computations and more reliable results in comparison with standard methods of analysis. The method can be applied readily to the determination of the moment-distribution constants of arches with constant or variable moments of inertia.

Once the moment-distribution constants are known, the moments can be balanced with little difficulty.

INTRODUCTION

In applying the moment-distribution method of analysis to a given problem, several factors or constants must be determined before the process of balancing moments can be performed. In general these constants include the fixed-end moments due to loads; carry-over, stiffness, and distribution factors; shear stiffness; thrust stiffness; and flexibility. In the case of beams with a constant

---

NOTE.—Published, essentially as printed here, in September, 1956, in the Journal of the Structural Division, as *Proceedings Paper 1058*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Prof. of Civ. Eng., Manhattan College, New York, N. Y.

moment of inertia, the determination of the required factors presents no difficulties inasmuch as they are simple functions of such properties as the span, the moment of inertia, and the modulus of elasticity of the member.

For beams with variable moments of inertia and arches of either constant or variable moment of inertia, the determination of the moment-distribution constants is generally difficult. This is especially true of arches in which complex functions involving integral calculus must be evaluated. In practice graphics are used, or the approximate summation process may be substituted for mathematical integration. Although the work is simplified, the necessary tabular computations are cumbersome and time consuming. Moreover, the various methods of analysis are closely related to each other, so that it is generally impossible to check results by an algebraically independent method.

The method of structural-model analysis introduced[2] by G. E. Beggs and the general methods of determining moment-distribution constants developed[3] by William J. Eney, M. ASCE, constitute a noteworthy simplification of the problem.

The Method.—It will be shown subsequently that the moment-distribution constants for members with constant or variable moments of inertia exhibit a common parameter, J/Q, in which J and Q are the second and first moments, respectively, of the integral, $A = \int ds/E\,I$, in which ds is an infinitesimal element of length measured along the longitudinal axis of the member; E denotes the modulus of elasticity; I is the moment of inertia; and the product, $E\,I$, is its flexural stiffness.

It will be remembered that in hydrostatics a ratio analogous to J/Q defines the location of the center of pressure on a submerged plane surface exposed to hydrostatic pressure of the liquid above it. Because the hydrostatic pressure increases linearly with the depth of submersion, the relationship between the resultant hydrostatic pressure and the size and configuration of the submerged surface area may be represented by a wedge-shaped "pressure solid" whose base is the area under consideration and whose altitude varies linearly.

The pressure solid is analogous to a three-dimensional conjugate beam. The analogy, although not essential in developing the principle of the experimental method, is noted because it helps in visualizing and appraising the effect of a varying moment of inertia on the moment-distribution constants.

BEAMS

Carry-Over Factor.—By definition, the carry-over factor of a flexural member is the ratio of the moment induced at the fixed end of the member to the moment, causing rotation at the other simply supported end. In Fig. 1(*a*), a beam simply supported at a and fixed at b is subjected to a moment, M_a, at a, causing a moment, $C_{ab}\,M_a$, at b, in which C_{ab} is the carry-over factor from a to b.

In Fig. 1(*b*), the beam is simply supported at a and b. An arbitrary moment, M_a, is applied at a, causing end rotations θ_a and θ_b as shown. In Fig. 1(*c*), an additional moment load, M_b, is applied at b, inducing rotations ϕ_b and

---

[2] "An Accurate Mechanical Solution of Statically Indeterminate Structures by Use of Paper Models and Special Gages," by G. E. Beggs, *Proceedings*, A. C. I., February, 1922, pp. 58–78.

[3] "Fixed-End Moments by Cardboard Models," by W. J. Eney, *Engineering News-Record*, December 12, 1935, pp. 814–816.

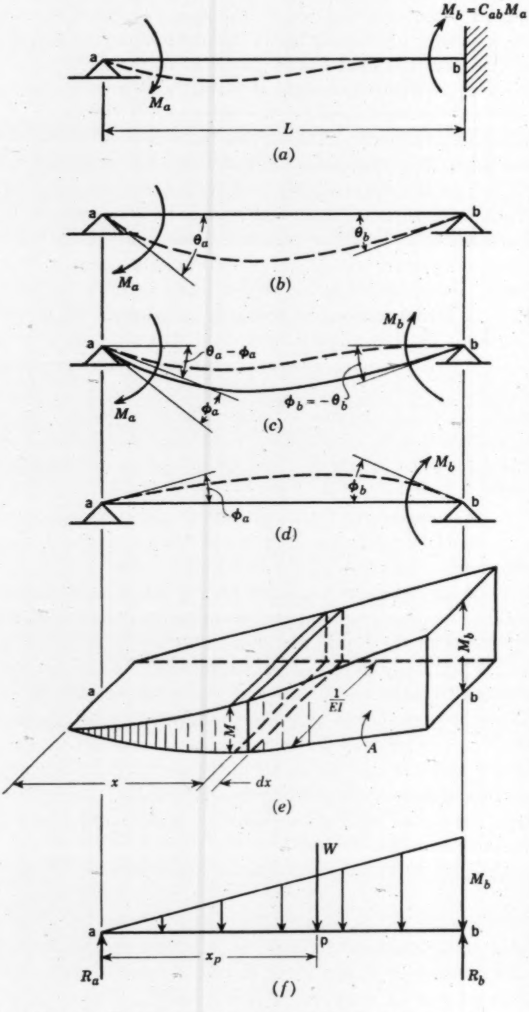

$M_b = C_{ab} M_a$
a
b
M_a
L
(a)
a
b
θ_a
θ_b
M_a
(b)
M_b
a
b
$\theta_a - \phi_a$
M_a
ϕ_a
$\phi_b = -\theta_b$
(c)
M_b
a
b
ϕ_a
ϕ_b
(d)
a
b
M_b
$\frac{1}{EI}$
M
A
x
dx
(e)
W
M_b
a
b
p
x_p
R_a
R_b
(f)

FIG. 1.—DEFINITION SKETCH FOR DETERMINATION OF CARRY-OVER FACTOR

ϕ_a at ends b and a. If the magnitude of M_b is such that $\phi_b = -\theta_b$, so that the net rotation at b caused by M_a and M_b is zero, then the total work done by moments M_a and M_b applied in the order indicated is

$$W_{ab} = \tfrac{1}{2} M_a \theta_a + \tfrac{1}{2} M_b \phi_b - M_a \phi_a \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

If the order of application of the moment loads, M_a and M_b, is reversed, the work done by M_b and M_a is

$$W_{ba} = \tfrac{1}{2} M_b \phi_b + \tfrac{1}{2} M_a \theta_a - M_b \phi_b \ldots\ldots (2)$$

Because the beam is in the same ultimate condition for either sequence of load application, the work done is the same in both cases. Therefore,

$$\frac{M_b}{M_a} = \frac{\phi_a}{\phi_b} \ldots\ldots (3)$$

The moment, M_b, prevents any rotation at b that would be induced by the moment, M_a, applied at a, and is, in effect, the fixed-end moment at b. Therefore, the ratio of the two moments is the carry-over factor from a to b,

$$C_{ab} = \frac{M_b}{M_a} = \frac{\phi_a}{\phi_b} \ldots\ldots (4)$$

It may be concluded from the definition of the angle changes, ϕ_b and ϕ_a, that if an arbitrary moment, M_b, is applied at b (Fig. 1(*d*)) the ratio, ϕ_a/ϕ_b, is the carry-over factor from a to b. The magnitude of M_b is not important inasmuch as no restrictions were placed on the magnitude of M_a at the outset of the derivation.

Rotations ϕ_a and ϕ_b correspond to shears at a and b in the conjugate beam caused by the $(M/E\,I)$-diagram being used as a load. The total load,

$$W = \int_a^b M \frac{1}{E\,I}\, dx \ldots\ldots (5)$$

is conceived of as a two-dimensional solid whose altitudes vary linearly with x—that is, $M = \frac{M_b}{L} x$—and whose base has widths equal to the $(1/E\,I)$-values. Fig. 1(*e*) is a pictorial view of the solid, and Fig. 1(*f*) is its elevation. This solid will be referred to as the pressure solid.

The pressure solid in Fig. 1(*f*) is simply supported along its edges, a and b. Its reactions, R_a and R_b, are determined, and their ratio is the carry-over factor from a to b,

$$C_{ab} = \frac{\phi_a}{\phi_b} = \frac{R_a}{R_b} \ldots\ldots (6)$$

If the moment of inertia is constant, the carry-over factor is

$$C_{ab} = \frac{\frac{1}{3} W}{\frac{2}{3} W} = \tfrac{1}{2} \ldots\ldots (7)$$

The resultant load, W, passes through the point, p, which is referred to as the center of pressure and is located at a distance, x_p, from a, as shown in Fig. 1(*f*). Taking the sum of the moments about p,

$$R_a\, x_p = R_b\,(L - x_p)$$

from which

$$\frac{R_a}{R_b} = \left(\frac{1}{x_p} L\right) - 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Equating the moment of W about a to the integral of the moments of elemental forces, dW, about the same point,

$$W\,x_p = x_p \int_a^b \left(\frac{M_b}{L} x\right)\left(\frac{1}{E\,I}\right) dx = \int_a^b \left(\frac{M_b}{L} x\right)\left(\frac{1}{E\,I}\right) x\,dx \ldots (9)$$

from which

$$x_p = \frac{\displaystyle\int_a^b \frac{dx}{E\,I} x^2}{\displaystyle\int_a^b \frac{dx}{E\,I} x} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Introducing the notation,

$$\int_a^b \frac{dx}{E\,I} = A \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11a)$$

$$\int_a^b \frac{x\,dx}{E\,I} = Q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11b)$$

and

$$\int_a^b \frac{x^2\,dx}{E\,I} = J \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11c)$$

and using this notation in Eq. 10 and Eq. 8, Eq. 6 yields

$$C_{ab} = \left(\frac{1}{x_p} L\right) - 1 = \left(\frac{Q}{J} L\right) - 1 \ldots\ldots\ldots\ldots\ldots (12)$$

Significantly, the parameter, $1/x_p = Q/J$, appears in each of the moment-distribution constants, as will be shown subsequently.

Stiffness Factor.—The stiffness factor at the simply supported end of a flexural member is defined as the moment required to rotate that end through a unit angle while the other end is held fixed. In Fig. 2(a), M_a is the stiffness factor, S_a, at a when θ_a is a unit angle in radian measure.

The bending-moment diagram, $M = M' - M''$, for the beam in Fig. 2(a) is shown in Fig. 2(b). The load on the conjugate beam, a′b′, is $\int_a^b M\ 1/E\,I\ dx$.

Taking the sum of the moments about b′ in Fig. 2(b) yields

$$-\ (1)\,L + \int_a^b \frac{M'\,dx}{E\,I}(L - x) - \int_a^b \frac{M''\,dx}{E\,I}(L - x) = 0 \ldots\ldots (13)$$

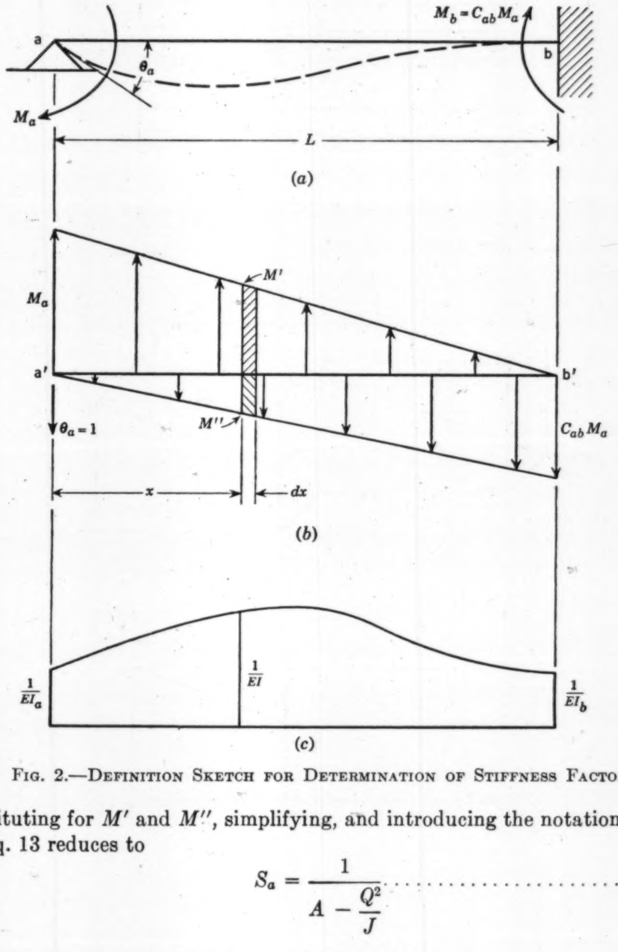

a
$M_b = C_{ab} M_a$
θ_a
b
M_a
L
(a)
M_a
M'
a'
b'
$\theta_a = 1$
M''
$C_{ab} M_a$
x
dx
(b)
$\frac{1}{EI_a}$
$\frac{1}{EI}$
$\frac{1}{EI_b}$
(c)

FIG. 2.—DEFINITION SKETCH FOR DETERMINATION OF STIFFNESS FACTOR

Substituting for M' and M'', simplifying, and introducing the notation of Eqs. 11, Eq. 13 reduces to

$$S_a = \frac{1}{A - \dfrac{Q^2}{J}} \quad \text{.......................} (14)$$

If the moment of inertia is constant, Eq. 14 yields

$$S_a = \frac{1}{\dfrac{1}{E\,I}\,L - \dfrac{\left(\dfrac{1}{E\,I}\,L\,\dfrac{L}{2}\right)^2}{\dfrac{1}{3}\,\dfrac{1}{E\,I}\,L^3}} = \frac{4\,E\,I}{L} \quad \text{...............} (15)$$

Modified Stiffness Factor.—This factor is the moment that must be applied at the end of a simply supported beam to rotate that end through a unit angle. In Fig. 1 (*d*), M_b is the modified stiffness factor, S_b', at b when ϕ_b is a unit angle.

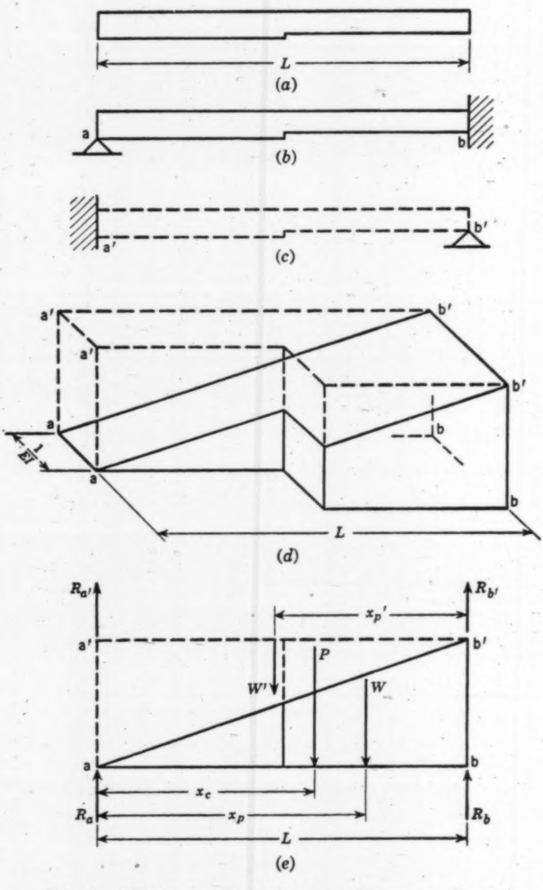

L
(a)
a
b
(b)
a'
b'
(c)
a'
b'
a'
b'
a
1/EI
a
b
b
L
(d)
$R_{a'}$
$R_{b'}$
x_p'
a'
b'
P
W'
W
a
b
x_c
R_a
x_p
R_b
L
(e)

FIG. 3.—"PRESSURE SOLIDS" FOR A DEEPENED BEAM

Taking the sum of the moments about a in Fig. 1(*f*) yields

$$x_p \int_a^b \left(\frac{S_b'}{L} x \right) \left(\frac{1}{E\,I} \right) dx - (1)\,L = 0 \qquad (16)$$

which reduces to

$$S_b' = \frac{L^2}{x_p\,Q} = \frac{L^2}{\frac{J}{Q}\,Q} = \frac{L^2}{J} \qquad (17)$$

If the beam has a constant moment of inertia, Eq. 17 yields

$$S_b' = \frac{L^2}{\frac{1}{3}\frac{1}{E\,I}L^3} = \frac{3\,E\,I}{L} = \frac{3}{4}S_b \qquad (18)$$

Relationship Between the Stiffness Factor and the Pressure Solid.—Fig. 3(*a*) represents a deepened beam, for which stiffness factors should be found. Figs. 3(*b*) and 3(*c*) show the beam supported in a manner consistent with the definition of S_a and S_b, respectively.

Fig. 3(*d*) is an oblique view of a prismatic solid, abb'a', whose base has a length equal to the span, L, of the beam and widths proportional to its $(1/E\,I)$-values. The solid is cut diagonally along the plane, aab'b'. The lower half of the cut solid is shown in continuous outline, whereas the upper half, which is volumetrically not equal to the lower half, is shown as a dashed line.

If the lower half of the original solid is simply supported along edges aa and bb, the reactions are R_a and R_b, and their sum is the weight, W (Fig. 3(*e*)). Force W passes through the center of gravity of the half-solid and also through the center of pressures in its base, located at a distance, x_p, to the right of a. An analogous notation, $R_{a'}$, $R_{b'}$, W', x_p', is applied to the upper half of the original solid.

The stiffness factor, S_a, given by Eq. 14 may be expressed as

$$S_a = \frac{1}{A - \frac{Q^2}{J}} = \frac{1}{A - \frac{Q}{x_p}} = \frac{1}{A - \frac{A\,x_c}{x_p}} \qquad (19a)$$

which, upon simplification, yields

$$S_a = \frac{x_p}{A\,(x_p - x_c)} \qquad (19b)$$

in which x_c is the distance of the centroid of the base, abba, of the solid, referred to aa. It is found by applying the principle of moments to Fig. 3(*e*) that

$$(R_a + R_{a'})\,x_c = (R_b + R_{b'})\,(L - x_c) \qquad (20a)$$

so that

$$x_c = \frac{(R_b + R_{b'})\,L}{P} \qquad (20b)$$

in which P is the sum of the four reactions, R_a, R_b, $R_{a'}$, and $R_{b'}$. Substituting this value for x_c and a value for x_p from Eq. 8, Eq. 19*b* becomes

$$S_a = \frac{R_b\,P}{A\,[R_b\,P - (R_b + R_{b'})\,W]} = \frac{R_b\,P}{A\,(R_b\,W' - R_{b'}\,W)} \qquad (21a)$$

Similarly,

$$S_b = \frac{R_{a'}\,P}{A\,(R_{a'}\,W - R_a\,W')} \qquad (21b)$$

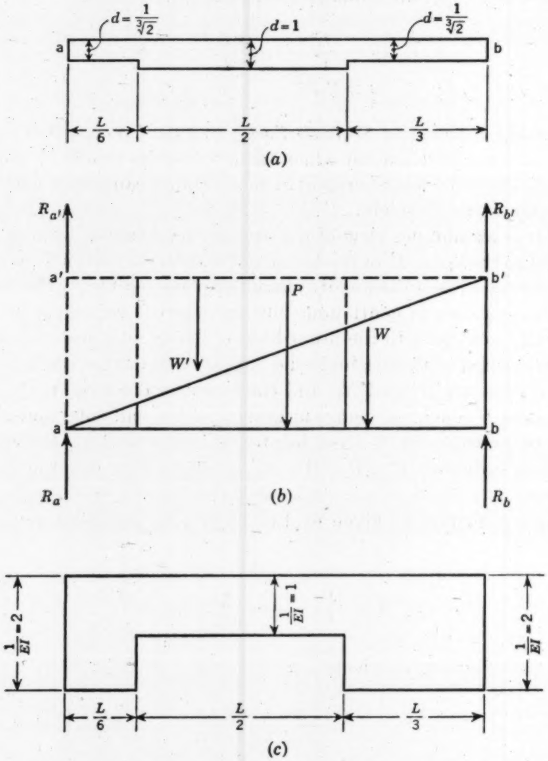

d = 1/∛2
d = 1
d = 1/∛2
a
b
L/6
L/2
L/3
(a)
R_a'
R_b'
a'
b'
P
W
W'
a
b
R_a
(b)
R_b
1/EI = 2
1/EI = 1
1/EI = 2
L/6
L/2
L/3
(c)

FIG. 4.—DEEPENED-BEAM ANALYSIS, EXAMPLE 1

The modified stiffness factor given by Eq. 17 is

$$S_b' = \frac{L^2}{J} = \frac{L^2}{\frac{J}{Q}\,Q} = \frac{L^2}{x_p\,A\,x_c} \qquad (22)$$

Substituting for x_p and x_c, Eq. 22 becomes

$$S_b' = \frac{W\,P}{A\,R_b\,(R_b + R_{b'})} \qquad (23a)$$

and, similarly,

$$S_a' = \frac{W'\,P}{A\,R_{a'}\,(R_{a'} + R_a)} \qquad (23b)$$

Eqs. 21 and 23 express the stiffness factors and the modified stiffness factors in terms of the four pressure-solid reactions, R_a, R_b, $R_{a'}$, $R_{b'}$, and area A. Referring to Eq. 6 it will be remembered that the carry-over factors are also functions of the four reactions. The determination of carry-over factors and relative-stiffness factors is illustrated in the subsequent example.

Example 1, Deepened Beam.—The subsequent data were obtained by weighing the reactions of the two half-solids shown in Fig. 4(*b*). The variation of $(1/E\,I)$-values is shown in Fig. 4(*c*).

$$\begin{aligned} R_a &= 95\text{ g} & R_{a'} &= 221\text{ g} \\ R_b &= 240\text{ g} & R_{b'} &= 97\text{ g} \\ W &= 335\text{ g} & W' &= 318\text{ g} \\ P &= 653\text{ g} & A &= \frac{3\,L}{2} \end{aligned}$$

Beams Fixed at Far End.—By use of Eq. 6,

$$C_{ab} = \frac{R_a}{R_b} = \frac{95}{240} = 0.396$$

and

$$C_{ba} = \frac{R_{b'}}{R_{a'}} = \frac{97}{221} = 0.438$$

The use of Eq. 21*a* results in

$$S_a = \frac{(240)\,(653)}{\frac{3\,L}{2}\,[(240)\,(318) - (97)\,(335)]}$$

which reduces to

$$S_a = \frac{2.38}{L}\text{ (relative)}$$

and Eq. 21*b* yields

$$S_b = \frac{(221)\,(653)}{\frac{3\,L}{2}\,[(221)\,(335) - (95)\,(318)]}$$

which reduces to

$$S_b = \frac{2.19}{L}\text{ (relative)}$$

Beams Simply Supported at Far End.—Substituting in Eq. 23*b* results in

$$S_a' = \frac{(318)\,(653)}{\frac{3\,L}{2}\,(221)\,(221 + 95)}$$

which reduces to

$$S_a' = \frac{1.98}{L}\text{ (relative)}$$

and Eq. 23*a* yields

$$S_b' = \frac{(335)\ (653)}{\frac{3\,L}{2}\ (240)\ (240 + 97)}$$

which reduces to

$$S_b' = \frac{1.80}{L}\ \text{(relative)}$$

The stiffness factors are relative because relative $(1/E\,I)$-values were used in the computation. To distribute an unbalanced moment among the members framed into a joint, relative-stiffness factors may be used providing that the widths, $(1/E\,I)$-values, of the respective solids are based on the same scale. If it is impracticable to use the same scale, or if absolute stiffness factors are required, an adjustment of the scales is necessary.

In general, if the model represents the prototype to the lineal scale of 1 in. = m-in. of prototype and if the transverse $(1/E\,I)$-scale is 1 in. = $n-1$/pound-square inches, the absolute stiffness factor for the prototype from Eq. 19*b* is

$$S_a = \frac{x_p}{A\ (x_p - x_c)}\ \frac{m}{(m\,n)\ m} = \frac{x_p}{A\ (x_p - x_c)}\ \frac{1}{m\,n} \quad\dots\dots\dots (24a)$$

which results in

$$S_{\text{prototype}} = \frac{S_{\text{model}}}{m\,n} \quad\dots\dots\dots\dots\dots\dots\dots\dots (24b)$$

If the model area, A, is evaluated in square inches and the units of m and n are as shown previously, the prototype stiffness factor is expressed in pound-inches.

Making and Weighing the Model.—It is simple to make a model—that is, a pressure solid, three-dimensional conjugate beam—capable of giving good results. The model used in example 1 was made of red oak, although any kind of seasoned wood that is reasonably free of defects is equally suitable. The material should be uniformly dry to avoid a variation in its specific weight and to prevent warping when it is cut. Plaster of paris, paraffin, and other materials that can be molded or easily cut may also be used.

The composite specimen, abb′a′, was assembled by lightly gluing onto a prismatic block of red oak (12 in. long, 3.9 in. deep, and 0.9 in. thick) two pieces of the same thickness and depth and $L/6 = 2$ in. and $L/3 = 4$ in. long, respectively. Next, the composite block was cut diagonally from a to b′.

One reaction at a time was weighed, while the other support was provided by a block of wood of suitable height. A carpenter's level was used to check the horizontal alinement of the scale platform and the supporting block at the other end. The entire experiment required only a few minutes.

Verification of Experimental Data and Results.—Errors and inaccuracies in weighing the reactions were found by weighing the solids. If discrepancies appeared between the weight of the solid and the sum of its reactions, the weighing of the reactions was repeated until a reasonably close agreement was obtained. The computed values of the four factors, C_{ab}, C_{ba}, S_a, and S_b, may be checked as described subsequently.

In Fig. 5(*a*) the moment required to cause a unit rotation at a while end b is held fixed is $M_a = S_a$. The fixed-end moment at b is $C_{ab}\, S_a$. Similarly, in Fig. 5(*b*) moment $M_b = S_b$ causes a unit rotation at b and moment $C_{ba}\, S_b$, at a. Applying the Maxwell-Mohr reciprocal theorem[4] to the equilibrated moment-load systems in Figs. 5(*a*) and 5(*b*) and rotations in Figs. 5(*b*) and 5(*a*),

$$S_a\ (\phi_a) + C_{ab}\, S_a\ (\phi_b) = C_{ba}\, S_b\ (\theta_a) + S_b\ (\theta_b) \qquad (25a)$$

Substituting for end rotations,

$$S_a\ (0) + C_{ab}\, S_a\ (1) = C_{ba}\, S_b\ (1) + S_b\ (0) \qquad (25b)$$

from which

$$\frac{C_{ab}}{C_{ba}} = \frac{S_b}{S_a} \qquad (26)$$

Thus, if any three of the four factors are known, the fourth factor may be obtained by means of Eq. 26. Because the experimental method and the necessary computations are simple, it is best to evaluate each factor separately and to use Eq. 26 as a check. If a small discrepancy appears the individual values may be adjusted graphically by the method of least squares, or by similar methods.

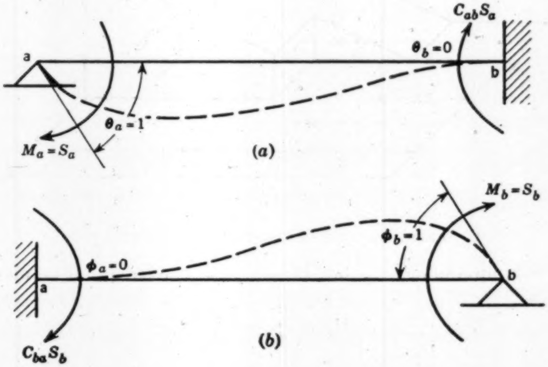


FIG. 5.—MOMENT-LOAD SYSTEMS

Substituting in Eq. 26 for three of the factors determined in example 1 leads to

$$\frac{0.396}{0.438} = \frac{S_b}{\frac{2.38}{L}}$$

from which

$$S_b = \frac{2.145}{L}$$

---

[4] "Statically Indeterminate Structures," by L. C. Maugh, John Wiley & Sons, Inc., New York, N. Y., 1951, p. 10.

This differs from the experimental value, $2.19/L$, by approximately 2% and is acceptable.

Fixed-End Moments in Beams, Concentrated Load.—Fig. 6(*a*) represents a beam with fixed ends carrying a single unit load at point k. It is desired to find the fixed-end moments, M_a and M_b.

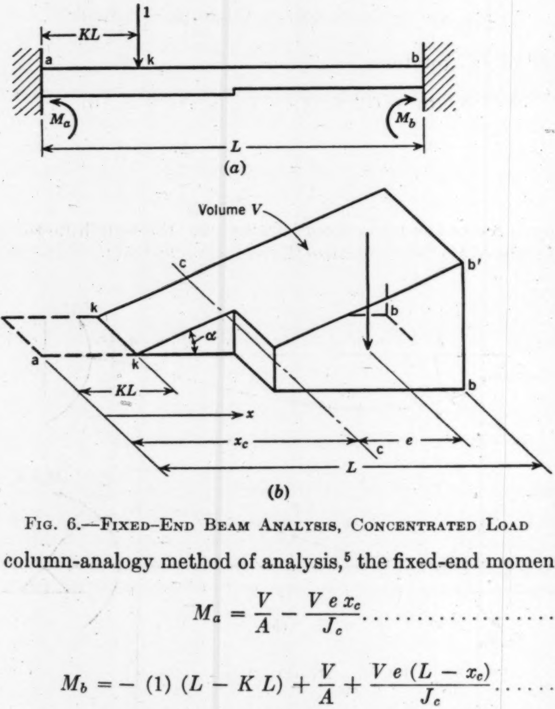


FIG. 6.—FIXED-END BEAM ANALYSIS, CONCENTRATED LOAD

By the column-analogy method of analysis,[5] the fixed-end moments are

$$M_a = \frac{V}{A} - \frac{V\,e\,x_c}{J_c} \qquad \text{(27a)}$$

and

$$M_b = -\,(1)\,(L - K\,L) + \frac{V}{A} + \frac{V\,e\,(L - x_c)}{J_c} \qquad \text{(27b)}$$

in which A is the area defined previously; x_c denotes the distance to the centroidal axis, cc, of area A measured from a; e is the eccentricity of the centroid of volume V referred to axis cc; J_c represents the second moment of area A about axis cc; and

$$V = \int_{KL}^{L} \frac{(1)\,(x - K\,L)}{E\,I}\,dx \qquad \text{(28)}$$

which is the volume of the solid shown in Fig. 6. Applying the parallel-axis theorem of second moments[6] leads to

$$J_c = J_a - A\,x_c^2 \qquad \text{(29)}$$

---

[5] "Theory of Modern Steel Structures," by L. E. Grinter, The Macmillan Co., New York, N. Y., Vol. II, 1937, p. 216.

[6] "Analytical Mechanics for Engineers," by F. B. Seely and N. E. Ensign, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., p. 415.

Rearranging Eq. 12 results in

$$\frac{J_a}{Q_a} = \frac{J_a}{A\,x_c} = x_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

or

$$J_a = A\,x_c\,x_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

Therefore,

$$J_c = A\,x_c\,(x_p - x_c) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

Substituting for J_c in Eq. 27*a* and factoring V/A, the fixed-end moment at a is

$$M_a = \frac{V}{A}\left(1 - \frac{e}{x_p - x_c}\right) \ldots\ldots\ldots\ldots\ldots\ldots (33)$$

Eq. 27*b* results in

$$M_b = -\ (1)\ (L - K\,L) + \frac{V}{A}\left[1 + \frac{e\,(L - x_c)}{x_c\,(x_p - x_c)}\right] \ldots\ldots\ldots (34)$$

Distances x_c and x_p have been encountered in the expression for the stiffness factor in Eq. 19*b*. Quantities V and e may be determined experimentally by fashioning the solid shown in Fig. 6(*b*). The altitudes represent the moment of the unit load applied at k, so that the solid again may be conceived of as a pressure solid. To avoid confusion with the pressure solid utilized in the determination of the carry-over and stiffness factors, this solid will be referred to as the "load solid."

The volume, V, of the load solid is best determined by dividing its weight, W_1, by the specific weight, w, of the material of which it is made.

The moment of the unit load applied at k at any distance, x, is given by

$$M = (1)\ (x - K\,L) \ldots\ldots (35)$$

Because the altitudes of the load solid in Fig. 6(*b*) represent the moments of the load, the slope of the upper surface of the solid is $dM/dx = 1$. If a load, P, acts at point k, the slope should be $(P)\ (1) = P$.

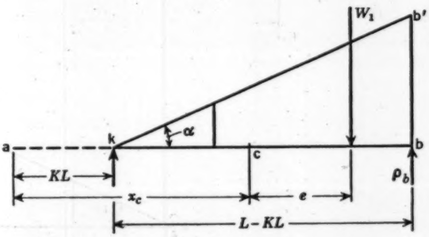
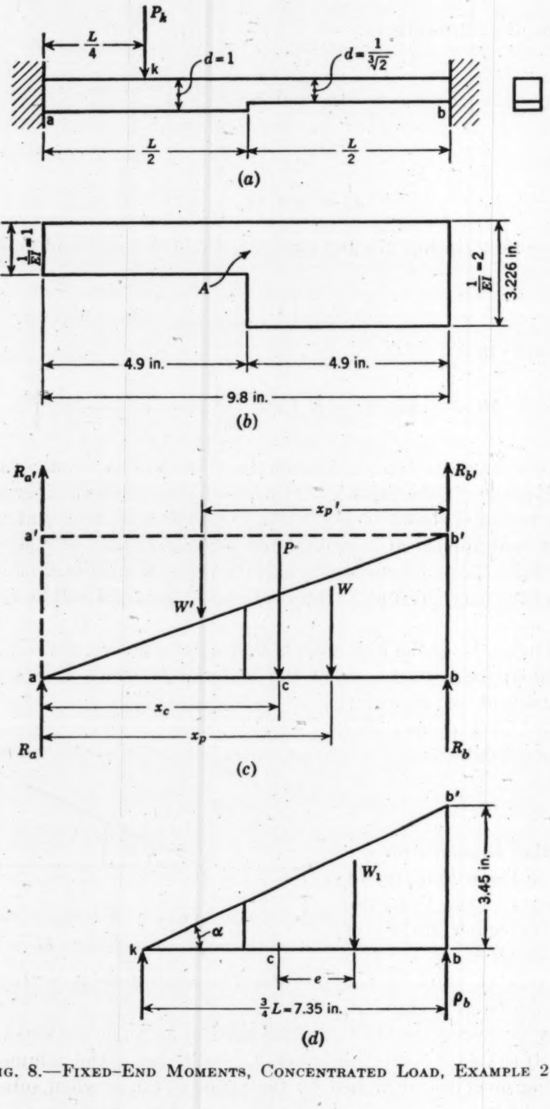


FIG. 7.—SIMPLY SUPPORTED "LOAD SOLID"

It may be impracticable to cut the solid at an angle corresponding to the load, P. If the slope is arbitrarily made equal to $\tan\alpha$, the volume determined by weighing must be multiplied by the ratio, $P/\tan\alpha$, when substituted into Eqs. 33 and 34.

The distance, e, is determined by simply supporting the load solid along its edges, k and b, and by weighing the reaction, ρ_b, at b, as illustrated in Fig. 7. Taking moments about k results in

$$W_1\,(x_c + e - K\,L) = \rho_b\,(L - K\,L) \ldots\ldots\ldots\ldots\ldots (36a)$$



P_k
$\frac{L}{4}$
k
d = 1
$d = \frac{1}{\sqrt[3]{2}}$
a
b
$\frac{L}{2}$
$\frac{L}{2}$
(a)
$\frac{1}{EI} = 1$
A
$\frac{1}{EI} = 2$
3.226 in.
4.9 in.
4.9 in.
9.8 in.
(b)
$R_{a'}$
$R_{b'}$
x_p'
a'
b'
P
W'
W
a
c
b
x_c
x_p
R_a
R_b
(c)
b'
W_1
3.45 in.
α
k
c
b
e
ρ_b
$\frac{3}{4}L = 7.35$ in.
(d)

FIG. 8.—FIXED-END MOMENTS, CONCENTRATED LOAD, EXAMPLE 2

which, when rearranged, results in

$$e = \frac{\rho_b \, (L - K\,L) - W_1 \, (x_c - K\,L)}{W_1} \quad \text{. . . . .} (36b)$$

It is not advisable to support the load solid at c and b with part kc overhanging. Although this arrangement permits a more direct evaluation of *e*, the necessity of a knife-edged support at c makes the weighing of the reaction at b more difficult because of the prolonged oscillations of the balance.

Three solids are required for the experimental evaluation of Eqs. 33 and 34. The first solid needed to find x_c has a span equal, or proportional, to the span of the fixed-end beam, a base whose widths correspond to its $(1/E\,I)$-values, and a constant height.

The second solid, termed the pressure solid, is required to find x_p by using Eq. 8. It is obtained from the first solid by cutting the first solid diagonally along the plane, ab′. The third solid, termed the load solid, which is required for the evaluation of V and e, is obtained from the second solid by cutting it along the plane, kb′, or any other inclined plane passing through k. Area A may be readily found by dividing the weight of the initial solid (prismatic solid of constant height) by its altitude, $\overline{\text{bb}'}$, and the specific weight, w, of the material.

It follows that a single solid of constant height, when cut in the proper sequence, will suffice for an experimental evaluation of both the carry-over factors and stiffness factors and the fixed-end moments for a beam.

Example 2, Deepened Beam.—Fig. 8 (*a*) shows a deepened beam fixed at both ends. The fixed-end moment at a will be determined for load P_k acting as shown. The following data were obtained by weighing:

$R_a = 128$ g
$R_b = 325$ g
$W = 453$ g
$P = 768$ g

$R_{a'} = 188.5$ g
$R_{b'} = 126.5$ g
$W' = 315$ g
$\rho_b = 251$ g
$W_1 = 361.5$ g

The following information concerns a wooden specimen, from which the load solid was made:

Dimensions	1.675 in. by 1.690 in. by 1.528 in.
Volume	4.315 cu in.
Weight	40.5 g

Specific weight, w, is w = weight/volume = 40.5/4.315 = 9.37 g per cu in. From Fig. 8 (*c*),

$$x_c = \frac{(R_b + R_{b'})\,L}{P} = \frac{(325 + 126.5)\,L}{768} = 0.588\,L$$

Similarly,

$$x_p = \frac{R_b\,L}{W} = \frac{325\,L}{453} = 0.717\,L$$

and

$$A\,(\overline{\text{bb}'})\,w = P$$

Therefore,

$$A\,(3.45)\,(9.37) = 768$$

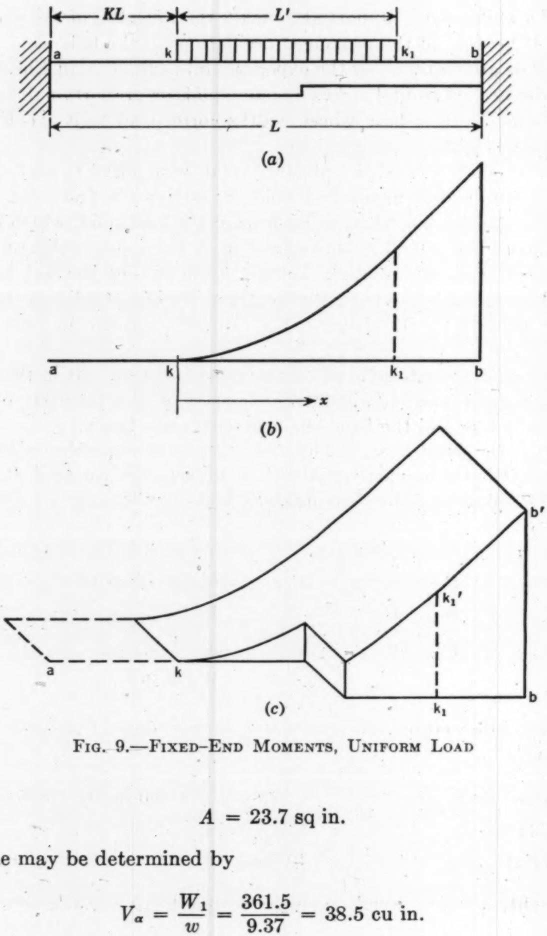


Fig. 9.—Fixed-End Moments, Uniform Load

and

$$A = 23.7 \text{ sq in.}$$

The volume may be determined by

$$V_\alpha = \frac{W_1}{w} = \frac{361.5}{9.37} = 38.5 \text{ cu in.}$$

Correcting for angle α results in

$$V = V_\alpha \frac{P_k}{\tan \alpha} = 38.5 \frac{P_k}{\frac{3.45}{7.35}} = 82\, P_k \text{ cu in.}$$

Using Eq. 36*b* leads to

$$e = \frac{251\,(0.75\,L) - 361.5\,(0.588\,L - 0.25\,L)}{361.5} = 0.1825\,L$$

Eq. 33 results in

$$M_a = \frac{82\,P_k}{23.7}\left(1 - \frac{0.1825\,L}{0.717\,L - 0.588\,L}\right) = -\,1.435\,P_k$$

In terms of any span, L, the fixed-end moment at a is

$$M_a = -\,\frac{1.435\,P_k}{9.8}\,L = -\,0.1464\,P_k\,L$$

Fixed-End Moments, Uniformly Distributed Load.—By the column-analogy method, in Fig. 9 the fixed-end moment at a, due to the distributed load of unit intensity per unit of length, can be determined from Eq. 33, in which

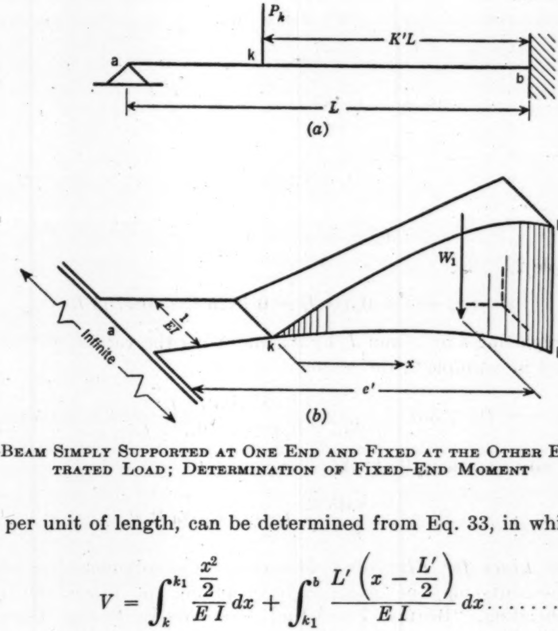


Fig. 10.—Beam Simply Supported at One End and Fixed at the Other End, Concentrated Load; Determination of Fixed-End Moment

$$V = \int_{k}^{k_1} \frac{\frac{x^2}{2}}{E\,I}\,dx + \int_{k_1}^{b} \frac{L'\left(x - \frac{L'}{2}\right)}{E\,I}\,dx \quad \text{.............} (37)$$

is the volume of the load solid shown in Fig. 9(*c*). From k to k_1, the altitudes of the solid vary as the second power of x. From k_1 to b, the variation is linear. Except for the parabolic variation of altitude between k and k_1, the principles of making and weighing the load solid remain valid.

The first of the two definite integrals in Eq. 37 may be written in the equivalent form,

$$\int_{k}^{k_1} \frac{x}{2\,E\,I}\,x\,dx$$

which may be interpreted as the volume of a solid whose base has widths equal to the ($x/2\,E\,I$)-values and whose altitudes vary linearly with x. Accordingly, the curved surface, kk_1', becomes plane, although it is not continuous with the plane surface, $k_1'b'$. If this alternate solid is used, the determination of the carry-over factor and stiffness factor, as well as the quantities, A, x_c, x_p, and J_c, in Eqs. 33 and 34, involves a separate solid because the latter must have widths proportional to the ($1/E\,I$)-values throughout its entire length.

Fixed-End Moment, Beam Fixed at One End and Simply Supported at the Other.—In Fig. 10(a), the beam is simply supported at a and fixed at b. Because a simple support or hinge has zero stiffness (($E\,I$)-value), in the column-analogy method of analysis it adds an infinite area, $dx/E\,I$, to the column section at a. As a result, the centroid of area A shifts to a.

From Eq. 27,

$$M_a = \frac{V}{\infty} - \frac{V\,e'\,(0)}{J_a} = 0 \qquad \text{(38}a\text{)}$$

and

$$M_b = -\,P_k\,K'\,L + \frac{V}{\infty} + \frac{V\,e'\,L}{J_a} \qquad \text{(38}b\text{)}$$

However,

$$\frac{J_a}{Q_a} = x_p \qquad \text{(39}a\text{)}$$

or

$$J_a = Q_a\,x_p = A\,x_c\,x_p \qquad \text{(39}b\text{)}$$

From example 2,

$$e' = x_c + e = 0.588\,L + 0.1825\,L = 0.7705\,L$$

In Eq. 27b replacing e by e' and J_c by J_a, and using the values of A, x_c, x_p, and V determined in example 2, the moment at b is

$$M_b = -\,P_k\,(7.35) + \frac{82\,P_k\,(0.7705\,L)\,L}{(23.7)\,(0.588\,L)\,(0.717\,L)} = -\,1.05\,P_k$$

In terms of any span length, L, this becomes

$$M_b = \frac{-\,1.05\,P_k}{9.8}\,L = -\,0.107\,P_k\,L$$

Influence Lines for Fixed-End Moments.—The influence-line values for fixed-end moments may be obtained by applying the procedure (presented under the heading, "Beams: Fixed-End Moments in Beams, Concentrated Load") to a series of load points, k. Starting from one end, a new load solid is cut off from the preceding one; its weight, W_1, and reaction, ρ_b, are determined; and the fixed-end moments are evaluated. This is repeated for a sufficient number of ($K\,L$)-values.

A alternate method based on the neutral-point and conjugate-beam properties[7] and utilizing the experimentally determined quantities, A, x_c, and J_c, may also be used.

---

[7] "Theory of Modern Steel Structures," by L. E. Grinter, The Macmillan Co., New York, N. Y. Vol. II, 1937, p. 210.

Moments Caused by Impressed Distortions.—The foundations of a structure may settle, spread, or rotate. Furthermore, members framed into a joint may shorten or lengthen as a result of a direct stress or temperature change, thereby causing the joint to move. These and similar distortions induce moments in the structure in addition to those caused by actual loads. The column-analogy method is the most expedient method for analyzing the effects of such distortions.

Transverse Displacement, Joint Rotation Prevented.—The effect of a relative transverse displacement, Δ, of ends a and b of the restrained member in Fig. 11 (*a*) corresponds, in the column-analogy method, to a couple equal to Δ acting about the transverse centroidal axis, cc, of the column cross section shown in Fig. 11 (*b*). The moments induced in the member correspond to the stresses in the column—that is,

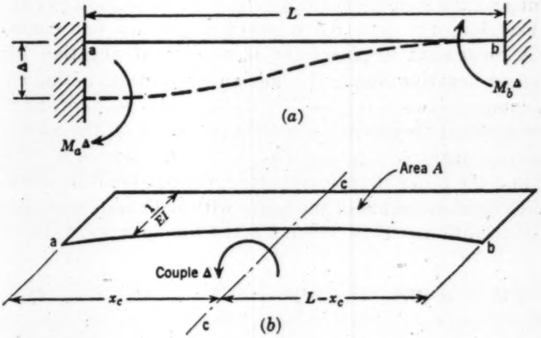


FIG. 11.—FIXED-END BEAM ANALYSIS, TRANSVERSE DISPLACEMENT

$$M_a^\Delta = \frac{\Delta}{J_c}\, x_c \qquad (40)$$

Using Eq. 29, the expression for M_a may be written as

$$M_a^\Delta = \frac{\Delta}{J_a - A\, x_c^2}\, x_c = \frac{\Delta}{Q_a\left(\frac{J_a}{Q_a} - x_c\right)}\, x_c \qquad (41)$$

Using Eq. 30, Eq. 41 leads to

$$M_a^\Delta = \frac{\Delta\, x_c}{Q_a\,(x_p - x_c)} = \frac{\Delta}{A\,(x_p - x_c)} \qquad (42)$$

In terms of the stiffness factor given by Eq. 19*b*, Eq. 42 becomes

$$M_a^\Delta = \frac{\Delta}{x_p}\, S_a \qquad (43)$$

Similarly, the moment induced at b is

$$M_b^\Delta = -\frac{\Delta}{J_c}(L - x_c) = -\frac{\Delta(L - x_c)}{A\, x_c\, (x_p - x_c)} = -\frac{\Delta(L - x_c)}{x_c\, x_p} S_a \quad \ldots (44a)$$

or

$$M_b^\Delta = -\frac{L - x_c}{x_c} M_a^\Delta \quad \ldots (44b)$$

If the member is symmetrical about the (cc)-axis, the centroidal distance, x_c, is $L/2$, and Eq. 44*b* yields

$$M_b^\Delta = -M_a^\Delta \quad \ldots (45)$$

The signs of moments M_a^Δ and M_b^Δ depend on the sense of the impressed displacement and the sign convention used. Because they can be determined readily by sketching the distorted member, a rigorous sign convention is not essential. A downward displacement is considered negative herein, and it corresponds to a negative couple (beam-sign convention) acting on the column section. A compressive stress caused by the couple is positive and is interpreted as a positive moment (beam-sign convention) acting on the member.

If a relative transverse displacement, Δ, occurs between two ends, of which one is fixed and the other simply supported, the fixed-end moment may be obtained from the end-moments of the beam with both ends fixed by the method of moment distribution. Thus, when a is the simple support, the moment at b is

$$(M_b^\Delta)' = M_b^\Delta + C_{ab}\, M_a^\Delta = -\frac{L - x_c}{x_c} M_a^\Delta + C_{ab}\, M_a^\Delta \quad \ldots (46)$$

Substituting from Eqs. 42 and 12 and reducing, Eq. 46 becomes

$$(M_b^\Delta)' = -\frac{\Delta}{A\, x_c\, x_p} L \quad \ldots (47)$$

Joint Rotation.—By definition, if moment S_a is applied at joint or abutment a, the far end remaining fixed, the rotation at a is $\theta = 1$. The moment corresponding to any rotation, θ_a, is

$$M_a^{\theta_a} = \theta_a\, S_a \quad \ldots (48a)$$

and

$$M_b^{\theta_a} = C_{ab}\, M_a = C_{ab}\, \theta_a\, S_a \quad \ldots (48b)$$

Shear Stiffness and Flexibility.—The shear stiffness, S_s, of an end-restrained flexural member, ab, is defined as the shear at the ends of the member required to cause a unit relative transverse displacement of a and b. The flexibility, F_s, is the reciprocal of shear stiffness—that is, the relative transverse displacement produced by a unit shearing force.

The shear at a in Fig. 11(*a*) is

$$S = \frac{M_a^\Delta + M_b^\Delta}{L} \quad \ldots (49)$$

Substituting for the moments from Eqs. 43 and 44*b*,

$$S = \frac{\frac{\Delta}{x_p} S_a + \frac{L - x_c}{x_c} \frac{\Delta}{x_p} S_a}{L} \qquad (50a)$$

which reduces to

$$S = \frac{\Delta}{x_c\, x_p} S_a \qquad (50b)$$

Because shear stiffness corresponds to $\Delta = 1$,

$$S_s = \frac{S_a}{x_c\, x_p} \qquad (51)$$

Combining Eqs. 40 and 43,

$$M_a^{\Delta} = \frac{1}{x_p} S_a = \frac{1}{J_c} x_c \qquad (52)$$

and substituting for S_a/x_p in Eq. 51 results in

$$S_s = \frac{x_c}{x_c\, J_c} = \frac{1}{J_c} \qquad (53)$$

and

$$F_s = \frac{1}{S_s} = J_c \qquad (54)$$

Correlation Between Model and Prototype.—The stiffness factor, S_a, in Eqs. 43, 48, and 51 is a function of area A as shown in Eq. 19*b*. Because absolute values of fixed-end moments caused by known or assumed distortions are generally required, the absolute value of the stiffness factor, or area $A = \int ds/E\,I$ must be introduced into the respective expressions.

The absolute stiffness factor for the prototype is given by Eq. 24. Using the *m*-scale and *n*-scale defined under the heading, "Beams: Relationships Between the Stiffness Factor and the Pressure Solid," the prototype moments are, from Eq. 43,

$$M_a^{\Delta} = \frac{\Delta}{A\,(x_p - x_c)} \times \frac{1}{(m\,n)\,m} \qquad (55)$$

When Δ, x_p, and x_c are expressed in inches and A is in square inches, M_a^{Δ} is expressed in pound-inches. From Eq. 48*a*,

$$M_a^{\theta_a} = \theta_a\,(S_a)_{\text{model}}\,\frac{1}{m\,n} \qquad (56)$$

When θ_a is expressed in radians and $(S_a)_{\text{model}}$ in 1/square inches, M_a is expressed in pound-inches.

Relative values of shear stiffness and flexibility given by Eqs. 51, 53, and 54 can be used to distribute shears to the members of a structure analyzed by the moment-distribution method. Therefore, no scale adjustment is necessary provided that the solids used to evaluate shear stiffness of the component members are based on the same scales. If this is not done an adjustment of scales is necessary, or the absolute shear stiffness must be found.

The absolute shear stiffness from Eq. 51 is

$$S_s = \frac{(S_a)_{\text{model}}}{x_c\, x_p} \times \frac{\dfrac{1}{m\,n}}{m^2} = \frac{(S_a)_{\text{model}}}{x_c\, x_p} \times \frac{1}{m^3\, n} \qquad (57)$$

When $(S_a)_{\text{model}}$ is expressed in 1/square inches and x_c and x_p are in inches, S_s is expressed in pounds per inch.

Example 3, Deepened Beam.—Assuming that the models in example 2 represent the prototype to the lineal scale of 1 in. = 24.5 in., the cross section of the prototype is 6.35 in. deep and 6 in. wide at b, and E is 2,000,000 lb per sq in., it is desired to determine for the prototype (Fig. 12) (*a*) carry-over factors; (*b*) absolute stiffness factors; (*c*) fixed-end moments caused by a settlement, $\Delta = 0.5$ in., at a; (*d*) fixed-end moments caused by a clockwise rotation of 0.2° at a; and (*e*) absolute shear stiffness.

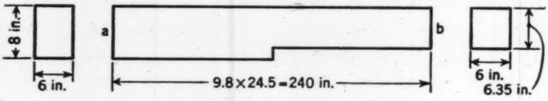


FIG. 12.—PROTOTYPE, EXAMPLE 3

It is found that

$$(E\,I_b)_{\text{prototype}} = 2{,}000{,}000 \times \frac{1}{12}\; 6 \times 6.35^3 = 2.56 \times 10^8 \text{ lb-in.}^2$$

$$\frac{1}{(E\,I_b)_{\text{model}}} = 3.226 \text{ in.}$$

$$n = \frac{1}{3.226 \times 2.56 \times 10^8} = 1.21 \times 10^{-9}$$

$$m = 24.5$$

$$x_p = 0.717\,L = 0.717 \times 9.8$$

$$x_c = 0.588\,L = 0.588 \times 9.8$$

and

$$A_{\text{model}} = 23.7 \text{ sq in.}$$

From Eq. 6,

$$C_{ab} = \frac{128}{325} = 0.394$$

and

$$C_{ba} = \frac{126.5}{188.5} = 0.671$$

Eq. 24 results in

$$S_a = \frac{0.717\ L}{23.7\ [L\ (0.717 - 0.588)]} \times \frac{10^9}{(24.5)\ (1.21)}$$

$$S_a = 7{,}900{,}000 \text{ lb-in.} = 658{,}000 \text{ lb-ft}$$

and

$$S_b = \frac{x_p'}{A\ [x_p' - (L - x_c)]} \qquad \text{(58)}$$

in which

$$x_p' = \frac{R_{a'}\ L}{W'} = \frac{188.5\ L}{315} = 0.598\ L$$

Therefore,

$$S_b = \frac{0.598\ L}{23.7\ [0.598\ L - (L - 0.588\ L)]} \times \frac{10^9}{(24.5)\ (1.21)}$$

and

$$S_b = 4{,}570{,}000 \text{ lb-in.} = 381{,}000 \text{ lb-ft}$$

Checking by Eq. 26 leads to

$$\frac{0.394}{0.671} = \frac{S_b}{658{,}000}$$

$$S_b = 386{,}000 \text{ lb-ft}$$

From Eq. 55,

$$M_a^{\Delta} = \frac{0.5}{23.7\ [9.8\ (0.717 - 0.588)]} \times \frac{10^9}{(24.5)^2\ (1.21)}$$

Therefore,

$$M_a^{\Delta} = 22{,}900 \text{ lb-in.} = 1{,}910 \text{ lb-ft} \circlearrowright$$

From Eq. 44*b*,

$$M_b^{\Delta} = -\frac{L - x_c}{x_c} M_a^{\Delta} = -\frac{L - 0.588\ L}{0.588\ L} \times 1{,}910 = 1{,}340 \text{ lb-ft} \circlearrowright$$

and from Eq. 48*a*,

$$M_a^{\theta_a} = (0.00349 \text{ radian})\ (658{,}000) = 2{,}300 \text{ lb-ft} \circlearrowright$$

Using Eq. 48*b*,

$$M_b^{\theta_a} = (0.394)\ (2{,}300) = 906 \text{ lb-ft} \circlearrowright$$

Eq. 57 results in

$$S_s = \frac{7{,}900{,}000}{(9.8)^2\ (0.588)\ (0.717)} \times \frac{1}{(24.5)^2} = 324 \text{ lb per in.}$$

Accuracy of the Method.—The experimental results obtained in examples 1, 2, and 3 are compared in Table 1 with those determined analytically.

TABLE 1.—COMPARISON OF EXPERIMENTAL RESULTS AND ANALYTICAL RESULTS

Item	Experimental	Analytical	Percentage of error
Carry-over factors			
Example 1			
C_{ab}	0.396	0.389	1.8
C_{ba}	0.438	0.456	3.9
Example 3			
C_{ab}	0.394	0.400	1.5
C_{ba}	0.671	0.667	0.6
Stiffness factors			
Example 1			
S_a	2.380	2.505	5.0
S_b	2.190	2.130	2.8
Example 3			
S_a	658,000	647,000	1.7
S_b	381,000	388,000	1.8
Fixed-end moment			
Example 2	0.1464	0.1495	2.1
Impressed distortions			
Example 3			
$M_a\Delta$	1,910	1,880	1.6
$M_b\Delta$	1,340	1,344	0.3
M_a^{θ}	2,300	2,260	1.8
M_b^{θ}	906	904	0.2
S_e	324	322	0.6

CONCLUSIONS

The experimental method of analysis described herein provides a numerically independent approach to the determination of moment-distribution constants. Although the working equations of the method are essentially the same as those used in purely analytical solutions, tedious mathematical integration or the approximate summation process are replaced by weighing.

The concept of the three-dimensional $(M/E\,I)$-solid, similar to the $(M/E\,I)$-load diagram evolved in the column-analogy method of analysis, permits a visual interpretation of the effect of the physical characteristics of the member on the various moment-distribution constants. The respective solids may be readily sketched, and their properties, such as reactions and location of centroids, may be estimated for the purposes of preliminary design. The two-dimensional moment-area method and the conjugate-beam method are not conducive to making such an estimate.

The notation, consisting of symbols such as J, Q, x_p, and x_c, permits a considerable simplification of the analytic expressions for the moment-distribution constants. For example, the stiffness factor expressed by Eq. 14 is equivalent to a textbook formula involving five integrals.[8]

The method described has been applied to the determination of moment-distribution constants of symmetrical arches with constant or variable moments of inertia.[9] The general procedure of making and weighing the solids remains the same. However, two pressure solids are required in order to determine the arch properties in the x-direction and y-direction.[9]

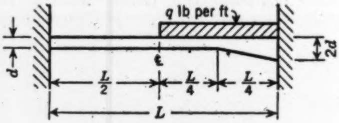
---

[8] "Analysis of Statically Indeterminate Structures," by J. I. Parcel and R. B. B. Moorman, John Wiley & Sons, Inc., New York, N. Y., 1955, p. 275.

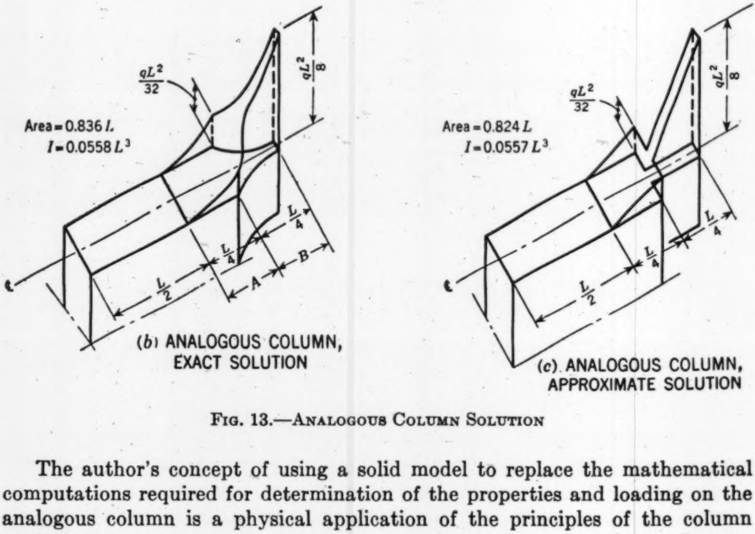
[9] "The Determination of Moment Distribution Constants of Members with a Variable Moment of Inertia," by O. Ondra, dissertation presented to Lehigh University, Bethlehem, Pa., in 1955, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

DISCUSSION

John C. Murphy,[10] J. M. ASCE.—Many numerical methods have been introduced to provide solutions for the determination of stiffness factors, carry-over factors, and fixed-end moments. The method of column analogy provides a general solution which can be applied to the majority of the problems of varying cross sections. The column analogy may be modified by taking subdivisions at convenient intervals on the analogous column to approximate values of area and moment of inertia.



(a) BEAM LOADING



(b) ANALOGOUS COLUMN, EXACT SOLUTION

(c) ANALOGOUS COLUMN, APPROXIMATE SOLUTION

Fig. 13.—Analogous Column Solution

The author's concept of using a solid model to replace the mathematical computations required for determination of the properties and loading on the analogous column is a physical application of the principles of the column analogy.

The advantages of Mr. Ondra's method are most apparent when one considers the case of a haunched beam uniformly loaded over a part or the complete span. This problem is believed to be common in the design of reinforced concrete structures.

Fig. 13(*b*) shows the pressure solid that should be considered. The volume of the pressure solid may be found for this case by integrating over the areas,

---

[10] Eng. Trainee, Pittsburgh-Des Moines Steel Co., Pittsburgh, Pa.

A and *B*. Volume *A* is a solid with a constant base width and a height that varies as the square of the distance from the center line. Volume *B* is a solid whose base width varies inversely as the cube of the distance measured from the midpoint of the beam and whose height varies in the same relationship as does the height in volume *A*.

For this type of beam and loading, the most expedient method of determining the beam constants and fixed-end moments would be to assume subdivisions of the pressure solid; these assumptions are shown in Fig. 13(*c*).

By using the properties determined by the approximate method for this particular beam, the writer found the value of the fixed-end moment at the left end of the beam for the type of loading shown was 0.0398 $q\,L^2$. The exact value of the fixed-end moment, determined by integrating for the areas and volumes on the analogous column, was found to be on the order of 0.0345 $q\,L^2$. Comparisons of these values indicate that errors of the magnitude of 15% may be involved when only the approximate method is used. The magnitude of this error is dependent on the judgment of the individual, the refinement of the method, and the complexity of the problem.

The use of the solid model when a complex pressure solid is involved will provide a less tedious method of analysis or a good check on the approximate method of solution. As Mr. Ondra has stated, two solids will be required for the determination of all the desired constants.

THOMAS D. Y. FOK,[11] A. M. ASCE.—The author is to be commended for his examination of moment-distribution-constant determinations by model-experiment procedures. With the aid of the pressure solid, the effect of a varying moment of inertia on the moment-distribution constants can be visualized easily. Indirectly, the manipulation for determining the moment-distribution constants by column analogy is more easily understood.

In appraising the experimental method, the writer wishes to stress two points: First, the accuracy of the results obtained by the experimental method compared with the exact analytical method and the approximate method ordinarily used in design-office practice; and second, the time factor as a basis for comparing the advantage of the experimental method over the other procedures. In the approximate method the properties of the analogous-column section of the member are determined by dividing the member into segments. Then the moment of inertia of each segment is assumed to be uniform, and the value is obtained by averaging the varying moments of inertia of the segment.

The degree of accuracy in the computation of moment-distribution constants is mainly determined by the type and function of the structure. In the design of a member in a structure, the values used for moment, shear, and axial stress are dependent on the physical properties of the member—that is, the modulus of elasticity, the moment of inertia, and the accuracy of the dimensions of the member. The accurate determination or estimation of the physical properties and its influence on the final results of the analysis has been the subject of debate for many years.

---

[11] Design Engr., Richardson, Gordon and Associates, Pittsburgh, Pa.

Good design practice dictates that the selection of values of the modulus of elasticity and moment of inertia should be those that give conservative design. Therefore, the accuracy of results obtained by model experiment or analytical methods can be compared on a basis of the properties assumed.

Accuracy of Results. 1.—The results obtained by model experiment are dependent on the measurement of the dimensions of the model; the homogeneity of the material used for the model; the accuracy of balancing and weighing; and the refinement of arithmetical computations. Because moment-distribution constants are functions of the three factors cited, the relative errors of the results are the sum of the relative errors of the individual factors that are independent of each other. The reduction of the relative error in one factor will proportionately reduce the relative error of the final result.

Accuracy of Results. 2.—The error of the results for moment-distribution constants obtained by an exact analytical procedure, either by the integration method or by the column-analogy method, is subject only to the error of arithmetical computation, and can be controlled by the "rounding off" of values. Therefore, accuracy can also be controlled to any degree desired. Results obtained by this method are used in making a comparison with Mr. Ondra's method.

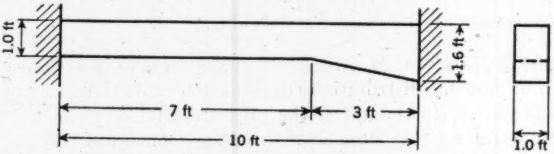


FIG. 14.—HAUNCHED BEAM

Accuracy of Results. 3.—The error occurring in the determination of moment-distribution constants by the approximate method results from the variation of moment of inertia in adjacent segments of the member being analyzed. The error can be somewhat reduced by increasing the number of segments used in the determination of the moment of inertia. The error is dependent on the type and shape of the structure and on the type of loading to which the structure is subjected; the magnitude of the error changes accordingly. Design-office practice is to use data that have been compiled in tabular or chart form and that are based on approximate methods of computation—either approximate integration or column analogy.

If errors occurring in the three items listed previously can be reduced with little effort and to a greater extent than those obtained in the approximate analytical method, the advantage of the model-experiment method is apparent with regard to comparison of accuracy.

To compare the accuracy of the determination of moment-distribution constants by the exact analytical method with the approximate analytical method, an example of a haunched beam (Fig. 14) is used.

For computation by the approximate method, the beam is divided into ten segments, each having a 1-ft length. Moment-distribution constants and fixed-end moments due to a uniform load of 1 kip per ft are determined by both the exact method and the approximate method. The results are shown in Table 2 and are compared with values obtained from data[12] published by the Portlant Cement Association, Chicago, Ill., and by column analogy, in which the properties were obtained by the approximate method.

A comparison of the errors resulting from the experimental pressure-solid method and the cardboard-model method are shown in Table 3.

In comparing the relative errors in Table 2 and Table 3, the accuracy of the results obtained by the approximate analytical method is better than that of the results obtained from the model-experiment method. It should be noted, however, that this occurs in a simply shaped member with simple loading and with only two or three different values of moment of inertia. Under such conditions the approximate analytical method furnishes more accurate results than the model-experiment procedures. With members of complex shapes, such as arches, and with complicated loadings, the accuracy of the approximate method is reduced. Use of the exact analytical method requires increased labor and becomes quite involved. In such cases the model-experiment methods will play an important role in the determination of the moment-distribution constants and will give much more accurate results with no increase in effort.

TABLE 2.—MOMENT-DISTRIBUTION CONSTANTS AND FIXED MOMENTS

Item	Method 1[a]	Method 2[b]	Difference	Percentage of error[c]
Stiffness S_A	0.448	0.458	0.010	2.23
Stiffness S_B	0.684	0.702	0.018	2.64
Carry-over factor, C_{ab}	0.704	0.716	0.012	1.71
Carry-over factor, C_{ba}	0.461	0.466	0.005	1.09
Fixed-end moment, M_{ab}, in kip-ft	6.90	−6.84	−0.060	0.87
Fixed-end moment, M_{ba}, in kip-ft	1.162	1.164	0.002	0.17

[a] By exact analytical method (Portland Cement Association "Handbook of Frame Constants"). [b] By approximate method with column analogy. [c] Percentage of error in terms of exact analytical method.

TABLE 3.—RELATIVE ERRORS BY PRESSURE-SOLID METHOD AND BY CARDBOARD-MODEL METHOD

Item	PERCENTAGE OF ERROR	
	Pressure-solid method[a]	Cardboard-model method[b]
Stiffness	5.0	−8.3
Carry-over factor	3.9	4.0
Fixed-end moments	2.1	4.5
Impressed distortions[c]	1.8	...

[a] Maximum values used. [b] Footnote reference 3. [c] No data available for cardboard-model method.

---

[12] "Handbook of Frame Constants," Portland Cement Assn., Chicago, Ill., 1947.

It is shown in Table 3 that the average accuracy of the results obtained from the cardboard models is lower than that of the results obtained from the pressure-solid method. The choice of method will ordinarily depend on the time and expense required to construct the models, test them, and make the required computations. The cardboard models can be more readily furnished than the wooden models used in the pressure-solid method. However, if plaster of paris, paraffin, or other readily molded materials can be used, the construction of the models will not necessarily govern the selection of the method used for obtaining the desired information.

It should be noted that the influence line for a fixed-end moment can be determined by one case of end rotation by using a cardboard model, whereas several pressure solids should be used in the other model method. The construction and weighing of the pressure-solid model requires more time than the measurement of a deflected curve of the cardboard model.

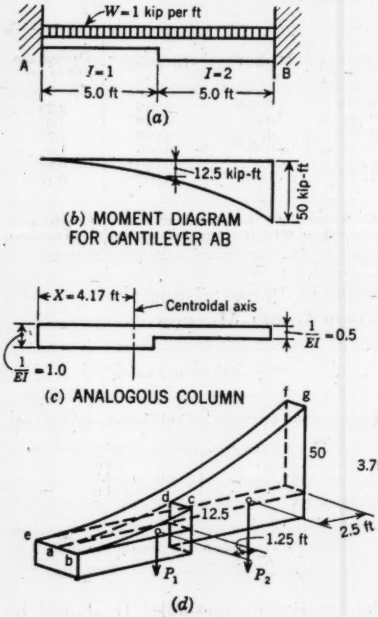

W = 1 kip per ft
A
I = 1
I = 2
B
5.0 ft
5.0 ft
(a)
12.5 kip-ft
50 kip-ft
(b) MOMENT DIAGRAM FOR CANTILEVER AB
X = 4.17 ft
Centroidal axis
1/EI = 0.5
1/EI = 1.0
(c) ANALOGOUS COLUMN
f
g
50
d
c
12.5
e
a
b
1.25 ft
2.5 ft
P1
P2
(d)

Fig. 15.—Modified Column-Analogy Method

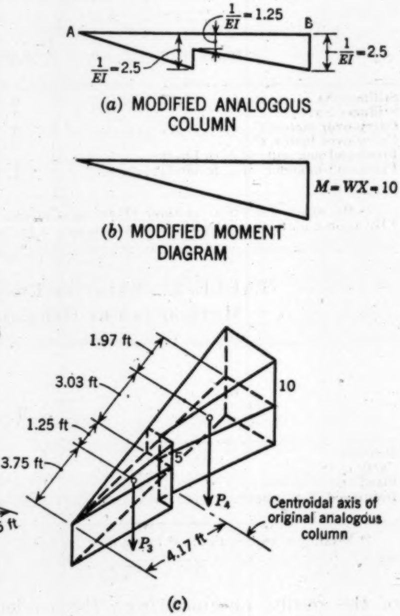

1/EI = 1.25
A
B
1/EI = 2.5
1/EI = 2.5
(a) MODIFIED ANALOGOUS COLUMN
M = WX = 10
(b) MODIFIED MOMENT DIAGRAM
1.97 ft
3.03 ft
1.25 ft
3.75 ft
10
5
P4
P3
4.17 ft
Centroidal axis of original analogous column
(c)

Fig. 16.—Prismatic Solid Solution

Time Factor in Obtaining Final Results.—The time required depends on the skill of the computer. If the type of member and loading are simple, the approximate method or the exact analytical method will give results more rapidly and accurately than those derived from the model experiments. With complicated types of structures, such as an arch with a variable cross section, time-consuming and laborious computations become necessary and there is less accuracy. If equal time is required to determine the final results by the approximate analytical method and the model method, the latter method should be used because of the greater accuracy. Time, cost, and accuracy are the governing factors in the selection of the method to be used. A satisfactory determination would probably require a rather comprehensive study.

In the determination of fixed-end moments caused by a uniform load in using the column-analogy method, the use of the moment area with a parabolic curve as a load becomes necessary. The determination of loads and moments on the analogous column is thereby complicated. In the case mentioned, a modified method can be used, as shown by Mr. Ondra under the heading, "Beams: Fixed-End Moments, Uniformly Distributed Load."

For example, the beam shown in Fig. 15(*a*), with a uniform load of 1 kip per ft, illustrates the use of the modified column section to determine the load and moment on the analogous column. In order to make the structure statically determinate for the application of the column-analogy method, the support at A is removed and the beam acts as a cantilever fixed at B. The moment diagram is shown in Fig. 15(*b*). The moment area is applied as a load on the analagous column.

The area of the analogous column, as shown in Fig. 15(*c*), is

$$1 \times 5 + 0.5 \times 5 = 7.5$$

Determining the position of the centroidal axis leads to

$$\bar{X} = \frac{5 \times 2.5 + 2.5 \times 7.5}{7.5} = 4.17 \text{ ft}$$

From Fig. 15(*d*) the load on the analogous column is

$$\begin{aligned} P &= P_1 \text{ (abcd)} + P_2 \text{ (aefg)} \\ &= \frac{1}{3} \times 12.5 \times 5 \times 0.5 + \frac{1}{3} \times 50 \times 10 \times 0.5 \\ &= 10.43 + 83.30 = 93.73 \text{ ft/ft} \end{aligned}$$

The moment, M, of the load about the center line of the analogous-column section is

$$\begin{aligned} M &= 10.43 \times -(4.17 - 3.75) + 83.30\,(7.5 - 4.17) \\ &= 277.66 - 4.38 = 273.28 \text{ ft} \end{aligned}$$

The same problem can be solved by the substitution of prismatic solids for the solid with parabolic curve, again using the modified method for the determination of loads and moments on the analogous column. The solution is shown in Fig. 16. If the modified analogous-column section is used, the moment diagram can be determined from $M = W\,X$, and the width of the analogous column is equal to $(x/2)\,(1/E\,I)$.

The load on the analogous column will be

$$\begin{aligned} P &= \text{volume } P_3 + \text{volume } P_4 \\ &= \frac{1}{3} \times 5 \times 2.5 \times 5 + \frac{5}{3}\,(1.25 \times 5 + 2.5 \times 10 + \sqrt{156.25}) \\ &= 20.80 + 73.0 = 93.80 \text{ ft/ft} \end{aligned}$$

The moment, M, of the load about the center of the original analogous column is

$$M = 20.80\,(4.17 - 3.75) + 73.00 \times 3.86 = 273.26 \text{ ft}$$

In arch analysis the member is usually one having a variable cross section. For the determination of stiffness, carry-over factors, and fixed-end moments due to concentrated or uniform loads, the approximate method may not be sufficiently accurate unless an increased number of segments are used. Because a solution by integration is laborious, either Eney's method[3] or the pressure-solid method can be used. The selection of a method will depend on the knowledge and experience of the analyst.

A disadvantage of the pressure-solid method is that, in order to determine the fixed-end moments due to concentrated loads at different points on the structure, different pressure solids must be used. This requires more time than Eney's method, in which one distortion at the end support will give influence ordinates simply by a measurement of the deflection curve of the structure.

Without considering the time required for the determination of the stiffness, carry-over factors, and fixed-end moments for indeterminate structures by the pressure-solid method, Mr. Ondra has explored a means of determining these factors by a unique experimental procedure.

FRANK J. CAIN[13] AND GERALD LUCK,[14] JUNIOR MEMBERS, ASCE.—The problem of determining the moment-distribution constants for beams with variable moments of inertia can be extremely tedious when using the classical methods of column analogy or substitution in elaborate formulas. In the past, methods have been presented to reduce the labor involved, but none are consistent with the moment-distribution method itself.

Mr. Ondra has offered a unique method, which requires the construction of a wooden model and the use of weighing apparatus to determine these constants. The theory presented and the simplicity of the model analysis are to be commended. However, in all model analysis there exist inherent errors due to poor similitude, measurements, and nonhomogeneous materials. These errors, together with the fact that the building of models is impractical for most engineering firms, might cause serious objections to the paper.

Nevertheless, in the author's conclusions it is stated that the properties of the solids may be estimated for preliminary design. The writers believe that all the criteria necessary for the application of the pressure-solid theory may be determined precisely by simple statics for use in either preliminary or final design.

The analogy of the pressure solid is reminiscent, in form at least, of a simply supported conjugate beam with an extra dimension of depth. All the properties of the solids determined by weighing may be found by assuming a unit weight and solving, by first moments of masses, for the appropriate reactions, weights, and centroidal distances.

---

[13] Graduate Asst., Dept. of Civ. Eng., Lehigh Univ., Bethlehem, Pa.

[14] Graduate Asst., Dept. of Civ. Eng., Lehigh Univ., Bethlehem, Pa.

The simplicity and rapidity of this modification will be illustrated by resolving the example used in the paper. The problem is a deepened beam with a load at a quarter point, as shown in Fig. 17(*a*).

Following the basic concepts set forth by Mr. Ondra, two othographic views (plan and elevation) are sketched, making sure that all corners are individually labeled and all dimensions clearly shown, as in Fig. 17(*b*) and Fig. 17(*c*). It may be noted that relative values may be substituted for the $(1/EI)$-values in the case of carry-over factors, which are ratios rather than absolute values.

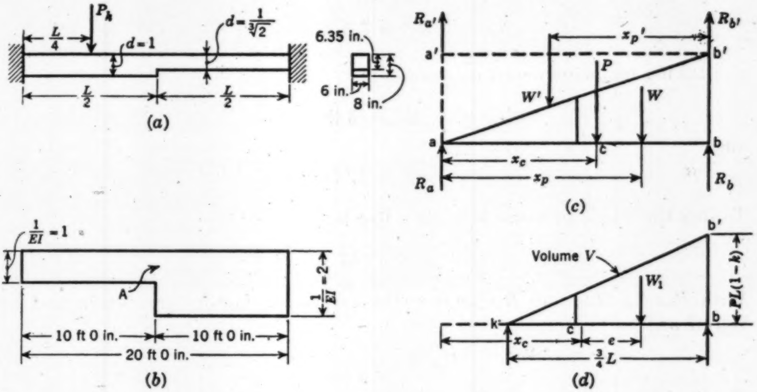


FIG. 17.—FIXED-END BEAM ANALYSIS—POINT LOAD

Carry-Over Factors.—From Eq. 6 the carry-over factors are

$$\left.\begin{aligned} C_{ab} &= \frac{R_a}{R_b} \\ C_{ba} &= \frac{R_b'}{R_a'} \end{aligned}\right\} \qquad (59)$$

To solve for carry-over, using the principles of first moments from statics, the views of the pressure solid are broken into as many sections as is convenient, remembering particularly that the solid is divided diagonally across the front view (elevation); rectangular and triangular shapes are recommended. To determine reactive weights from moments of masses, a unit weight, w, must be assumed. If w is given the value of unity throughout the problem, the final results will be consistent with the prototype beam.

For the beam in Fig. 17(*a*), $I_{ac} = I_1 = 512 \times 10^6$ in.^{4}, and $I_{cb} = I_2 = 256 \times 10^6$ in.^{4} Let $E\,I = 1$; therefore, $1/E\,I = 1$ and $1/E\,I_2 = 2$. If bb' = 1 and the unit weight equals 1, then $A = 30$ sq ft, $W = 17.5$ lb, $W' = 12.5$ lb, and $P = 30$ lb.

With the solid subdivided, take moments about the opposite corner of each separate wedge as a simply supported beam for the reactions, R_a and R_b, as well as R_a' and R_b'. Isolating wedge ab'b as a free body and taking moments about

$\overline{bb'}$ yields

$$R_a = 5 \text{ lb}$$

Moments about $\overline{aa'}$ result in

$$R_b = 12.5 \text{ lb}$$

and

$$x_p = 14.29 \text{ ft}$$

Isolating wedge aa′b′ as a free body and taking moments about $\overline{bb'}$ yields

$$R_a' = 7.5 \text{ lb}$$

and taking moments about $\overline{aa'}$ results in

$$R_b' = 5 \text{ lb}$$

and

$$x_p' = 12 \text{ ft}$$

Taking the whole pressure solid as a free body,

$$x_c = 11.67 \text{ ft}$$

With R_a, R_b, $R_{a'}$, and $R_{b'}$ known, the carry-over factors are determined by Eq. 67 as

$$C_{ab} = \frac{5}{12.5} = 0.40$$

and

$$C_{ba} = \frac{5}{7.5} = 0.667$$

Stiffness Factors.—The stiffness factors for a beam are given by Eqs. 21, which, from Eqs. 19, become

$$S_a = \frac{14.29 \times 512 \times 10^6}{30\ (2.62)\ 12} = 7{,}760{,}000 \text{ in.-lb} = 646{,}000 \text{ ft-lb}$$

$$S_b = \frac{12 \times 512 \times 10^6}{30\ [12 - (20 - 11.67)]\ 12} = 4{,}650{,}000 \text{ in.-lb} = 387{,}800 \text{ ft-lb}$$

It is suggested that relative $(1/E\,I)$-values be carried throughout the computation—that is, $E\,I_2 = (1/2)\,E\,I$.

Fixed-End Moments.—In the determination of fixed-end moments, some of the properties required are by-products of the stiffness-factor computations; these by-products are the values of x_p and x_c. To complete the fixed-end moments, only the volume and centroidal distance of the "force diagram" are required, as shown in Fig. 17(*d*). The fixed-end moments caused by a settlement of 0.5 in. at a are found to be

$$M_a^{\Delta} = \frac{\Delta S_a}{x_p} = \frac{0.5 \times 646{,}000}{14.29 \times 12} = 1{,}885 \text{ ft-lb}$$

and

$$M_b^{\Delta} = -\left(\frac{L - x_c}{x_c}\right) M_a^{\Delta} = -\left(\frac{20 - 11.67}{11.67}\right) 1{,}885 = 1{,}346 \text{ ft-lb}$$

The fixed-end moments due to clockwise rotation of 0.2° at a are

$$M_a^{\theta} = \theta_a\, S_a = 0.00349 \text{ radians} \times 646{,}000 = 2{,}252 \text{ ft-lb}$$

and

$$M_b^{\theta} = C_{ab}\, M_a^{\theta} = 0.400 \times 2{,}252 = 902 \text{ ft-lb}$$

From Fig. 17(*d*), $W_1 = 212.5$ lb; from statics, c = 3.72 ft. The fixed-end moments due to a load applied at k can be determined from

$$\left.\begin{aligned} M_a &= \frac{V}{A}\left(1 - \frac{e}{x_p - x_c}\right) \\ M_b &= -\,(L - \text{k}\, L) + \frac{V}{A}\left[1 + \frac{e\,(L - x_c)}{x_c\,(x_p - x_c)}\right] \end{aligned}\right\}\ldots\ldots\ldots\ldots(60)$$

Substituting in Eq. 60,

$$M_a = \frac{425}{60}\left(1 - \frac{3.72}{14.29 - 11.67}\right) = 2.975 \text{ ft-lb}$$

$$M_b = -\,1\,(20 - 5) + \frac{425}{60}\left[1 + \frac{3.72\,(20 + 11.67)}{11.67\,(14.29 - 11.67)}\right] = 0.36 \text{ ft-lb}$$

The author (under the heading, "Conclusions") states that "the method described has been applied to the determination of moment-distribution constants of symmetrical arches with constant or variable moments of inertia." Because the greatest problems are encountered in unsymmetrical arches, the writers are not convinced that static moments can be applied to arches as they have been applied to beams.

Otakar Ondra,[15] A. M. ASCE.—The discussion by Mr. Murphy is primarily concerned with the accuracy of column-analogy computations performed by the segment (approximate) versus integration (exact) procedures. He points out that errors on the order of 15% may be involved when the segment method is used. This would indicate that the writer's experimental method is more accurate than the numerical-segment method, which is preferred to the integration method.

It is not generally understood that the column-analogy method applied to actual design problems, such as cover-plated wide-flange beams, is likely to yield inaccurate results whether segments or integral calculus are used. This is particularly true of the fixed-end moment at the far end of the beam—that is, the end that remained fixed for the purpose of obtaining the bending-moment diagram for the member rendered statically determinate. The reason is that small differences of large numbers may be involved in the computations.

---

[15] Prof. of Civ. Eng., Manhattan College, New York, N. Y.

Although the errors depend on the position of the load, the choice of the determinate structure, and the configuration of the (1 $E\,I$)-diagram, experience has indicated that, in general, slide-rule computations are inadequate to guarantee accuracy within 15%. Occasionally, a multidigit calculating machine is a prerequisite when 5% accuracy is required regardless of the number of segments used and notwithstanding the use of integral calculus. In rigid frames and elastic arches characterized by biaxial bending of analogous-column section, a third term enters into the indeterminate moment function so that the accuracy of the arithmetical computations is further impaired.

The unreliability of slide-rule evaluation of the segment properties or integrals is not peculiar only to the column-analogy method. It also exists in the neutral-point method. Although the neutral-point reactions can be computed with a slide rule with a certain degree of accuracy because the reactions are expressed as ratios of similarly affected quantities, a potentially significant error may appear when the neutral-point reactions are transferred to the supports of the member (either a frame or an arch). The same possibility of inaccurate results is present in the solutions found by the general method for indeterminate structures, in which simultaneous equations with coefficients of a comparable order of magnitude must be solved.

Mr. Murphy states that: "The author's concept of using a solid model * * * is a physical application of the principles of the column analogy." This is true in the sense that the writer's working equations, which include Eqs. 27, 38, and 40, are borrowed from the column-analogy approach. These equations were used to avoid lengthy derivations. Actually, column analogy is not a special method of analysis in the same way in which the moment-distribution method is different from each of the other methods. It utilizes a conveniently arranged bending-moment diagram that permits the finding of the complementary moment, which is caused by the indeterminacy of structure. By coincidence this is done in a manner similar to the solution of an elementary problem of the strength of materials. It can readily be shown that the column-analogy method applied to beams is identical to the conjugate-beam method. It is only necessary to write the equation of rotational equilibrium of the conjugate beam by taking the sum of the moments of the ($M\,/\,E\,I$)-loads about the centroid of the part of the ($M\,/\,E\,I$)-diagram corresponding to one of the of the redundant fixed-end moments. If the conjugate-beam load diagram is properly prepared, the column-analogy equation can be read almost without derivation.

The conjugate-beam method has in its background the integral, $\int \frac{M\,m\,dx}{E\,I}$ (Maxwell-Mohr), or $\int \frac{M \frac{dM}{dQ}\,dx}{E\,I}$ (Castigliano), which are deflections of the fundamental strain-energy concept. Each available method of analysis, except the moment-distribution method, is a variation of the strain-energy method, in which the external work done by real or fictitious loads is equated to the internal energy stored in the structure as a result of deformation.

The writer's working formulas are not original; credit is due to James C. Maxwell and Otto Mohr and to others who have applied their own fundamental concepts. The radical difference between the existing methods of evaluation and the pressure-solid method is the replacement of summations or integrations by weighing.

According to Mr. Murphy's exact solution, the area, A, of the analogous column is 0.836 L, which differs by nearly 1% from the writer's value of 0.844 L. The small discrepancy is unimportant, but it is emphasized to show that exactness is a concept relative to the computer and his tools.

Mr. Fok has presented a thorough and valuable discussion of the accuracy and relative merits of numerical methods of analysis and the experimental methods of Eney, as well as the experimental pressure-solid method proposed by the writer. Mr. Fok is perhaps the first engineer who has applied the foregoing method outside of Lehigh University at Bethlehem (Pa.), where it was developed,[9] and Manhattan College in New York (N. Y.), where it is used by senior civil engineering students. Mr. Fok's consideration of the pressure-solid concept as an aid in visualizing the effects of a variable moment of inertia on the several moment-distribution constants applies equally to the design office and to the classroom.

From an academic point of view it is interesting to know the degree of accuracy of the moment-distribution constants obtained by the various available methods. However, in practice, the constants are seldom the ultimate objective of the computer. The moment-distribution method in which these constants are generally used has the unique feature of dissipating the effects of erroneous and even incorrect values of the carry-over factor and stiffness factor throughout the structure. This also applies to occasional arithmetical errors—for example, when a moment that is incorrect in either magnitude or sign is carried over into a joint or when an incorrect sign is written while a joint is being balanced.

An examination of the accuracy of results must go beyond the moment-distribution constants. It is possible that for certain configurations of members in the statically indeterminate structure and for certain positions and relative magnitudes of loads, the computed stresses in the structure may be sensitive to the accuracy with which the constants were computed. However, the writer believes that this is seldom the case. While analyzing continuous wide-flange beams with a variable moment of inertia effected by substantial cover plates in the midspan regions and top and bottom tie plates over the interior supports, it has been the writer's experience that it made little difference in ultimate results (reactions and bending moments) whether the variation of the moment of inertia was considered or the constants corresponding only to the respective wide-flange sections were used. These beams were actual structures and not textbook problems. The same conclusion was reached with fixed-end bents and pin-supported bents consisting of similarly built-up members. This limited experience does not imply that the relative indifference of reactions and fixed-end moments to the variation of the moment of inertia is always the case. Clearly the values under consideration are shears and bending moments only; the unit stresses are always dependent on the actual cross-section properties.

Michael J. Link repeated a part of Mr. Fok's investigation by the pressure-solid method, using a piece of fir wood 2 in. by 4 in. by 16 in. The values of stiffness and the carry-over factors are given in Table 4 together with the pertinent data taken from Mr. Fok's study.

Eq. 26, which relates the four factors, was used in conjunction with the plot of these factors so that Link's results could be adjusted. If the two stiffness factors had been considered to be correct, which is almost true of Link's values, the graphical adjustment would have yielded carry-over factors that were in extremely close agreement with the values of the frame constants. Because in an unsolved problem it would be difficult to predict which of the experimentally determined factors deviate less from the true values, an impartial graphical adjustment was performed. It is seen that the maximum percentage of error was reduced to 2.56% and that the average error in the four factors is 0.28%.

TABLE 4.—MOMENT–DISTRIBUTION CONSTANTS

	Portland Cement Association handbook values (1)	Percentage of difference between Mr. Fok's approximate values and Col. (1) (2)	Link's values by pressure-solid method (3)	Percentage of difference between Col. (3) and Col. (1) (4)	Link's adjusted values (5)	Percentage of difference between Col. (5) and Col. (1) (6)	Link's average adjusted percentage of error (7)
Stiffness, S_A	0.448	2.23	0.453	1.11	0.439	−2.00	0.28
Stiffness, S_B	0.684	2.64	0.696	1.76	0.701	2.49	
Carry-over factor, C_{AB}	0.704	1.71	0.735	4.40	0.722	2.56	
Carry-over factor, C_{BA}	0.461	1.09	0.440	−4.55	0.452	−1.95	

It is believed that it is impossible to decide whether the carry-over factor or the stiffness-factor values are more reliable for Eq. 26. The values that were experimentally and analytically determined for several beams are inconclusive. Furthermore, Eqs. 12 and 14 do not lend themselves to a qualitative analysis of relative errors. The denominator of Eq. 14 can be written as $-A\left(\frac{Q}{J}x_c - 1\right)$. The ratio, Q/J, is obtained indirectly by weighing and is therefore subject to experimental error. Because x_c is always smaller than the span length, L, the difference between the experimental values and true values of $\frac{Q}{J}x_c - 1$ in Eq. 14 will always be less than the difference between the experimental and true values of the carry-over factor expressed by Eq. 12. However, in Eq. 14 the expression in the parentheses is multiplied by A, which is determined by weighing; A may be accurate or inaccurate in value, accurate in value but inaccurate in configuration due to compensating inaccuracies in cutting, or inaccurate in both value and configuration. If the configuration is inaccurate, it affects the location of the center of pressure and, consequently, the weighed reactions of the pressure solid and the value of Q/J.

Mr. Fok's observations with regard to the time factor are essentially correct. The engineer with sufficient skill to perform the type of numerical computations and who is also familiar with the pressure-solid method may prefer the numerical method. However, it must be remembered that in actual design problems the ultimate moment of inertia and its variation are unknown. Approximate relative values may be estimated by assuming each member of the statically indeterminate structure to be fixed-ended and, thus, subjected only to the effect of the loads acting on it. After the preliminary analysis of the composite structure is made, the I-values may be revised and the variation within each span may be estimated by sketching the deflected shape of the structure or the influence lines for the support and joint moments using the Müller-Breslau principle. Several preliminary analyses must be performed before the final moments of inertia can be obtained and the design completed.

Whenever the relative moments of inertia are changed, only the computer's experience can be salvaged from the preceding computations. This is true of each of the moment-distribution constants as well as of the actual balancing of moments and sidesway correction. On the other hand, in experimental work with celluloid or pressure-solid models, the bulk of the work consists of preparing for the actual experiment—that is, mounting the deformeter gages for the given spans and cutting the model to represent both the linear geometry and the elastic flexibility of the prototype, or setting up the scale and supports for the several weighings and fashioning the wooden solids. The actual model readings and computations are quite simple and expedient. If the moments of inertia of the prototype are subsequently changed, it becomes relatively easy to remove excess model material by sawing or filing. None of the preliminary preparatory work need be repeated.

Nevertheless, it is doubtful whether experimental methods of analysis will be greatly utilized in design offices or in routine work. The chief value of these methods is in their application to complicated problems (as noted by Mr. Fok) in research, or in situations in which the utmost refinement of design is necessary because of cost, weight, and safety. These methods of analysis could effectively be applied to such situations as production-type structures, elements of machines, and airplane and submarine frames.

Perhaps the greatest merit of experimental analyses is their value as a teaching aid in developing and demonstrating various theoretical methods of analysis; in emphasizing the fact that, unlike statically determinate structures, analysis and design must be considered simultaneously because one affects the other; and in making it possible for an inexperienced engineer to predesign structures intuitively, which would be impossible without years of first-hand design experience.

The independent check of example 3 by Messrs. Cain and Luck, to which the computation of fixed-end moments caused by a concentrated load was added, agrees with the writer's values. However, the writer does not agree with the value of $M_b = 0.36$ ft-lb, which, by column analogy (and by checking their arithmetic), appears to be on the order of -0.69 ft-lb and differs from the former value by approximately 292%. This error supports the writer's contention that a numerically independent solution provided by experiment is desirable, which would be especially true of a more complicated problem than the one considered herein.

Messrs. Cain and Luck's method of determining the constants seems to be a misunderstanding of the writer's purpose. Therefore, it should be re-emphasized that the writer's primary purpose is to replace summation or integration by weighing. Actually Messrs. Cain and Luck did integrate while using familiar formulas of solid geometry that express the volume of a prism of constant width and locate its centroid. The actual pressure solids and load solids consist of two prisms, each having a rectangular base. This short cut can be used only because of the extreme simplicity of the $(1/E\,I)$-diagram. In more complicated situations Messrs. Cain and Luck could not have used existing formulas. They could only have used the segment or integration computations. The writer's working equations in terms of weights would only serve to complicate the problem and cause confusion.

For those who do not wish to use experimental methods of analysis, the writer recommends working formulas, as shown by Eqs. 12, 14, and 17 and their adaptations to support b. These equations have been reduced to their simplest possible form. The nomenclature is simple in form and concept and lends itself to diagrammatic interpretation. The well-known relationship expressed by Eq. 26 should be used to check the results. In order to evaluate the fixed-end moments, the column-analogy method will be most convenient particularly because values of A, Q_a, and J_a are already known. The value of J_c is readily obtained from J_a by the parallel-axis theorem, and x_c is the ratio, Q_a/A. All necessary computations can be conveniently arranged in tabular form. If integration is preferred to the segment method, the former may be accomplished in tabular form; the integral sign need not be shown.

The answer to Messrs. Cain and Luck's question as to whether static moments might be applied to arches is "yes." However, many summation or integration computations must also be performed.

The writer wishes to thank the discussers for their interest in his paper and their contribution to it. The apparent preference for purely numerical solutions indicates the need for continued efforts to simplify the existing methods and arrangements of computations.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2924

TRANSPORTATION PLANNING
A SYMPOSIUM

PAGE

FOREWORD 360

THE PORT, A FOCAL POINT
By Roger H. Gilman, M. ASCE 361

THE AIRPORT, A NATIONAL FACILITY
By Wilfred M. Post, Jr. 370

PROSPECTS FOR COORDINATION
By Burton W. Marsh, M. ASCE 377

FOREWORD

This group of papers is designed to focus attention on the importance of transportation and to explore the possibilities for achieving vitally needed coordination of all forms of transportation planning. There is presented the need for coordinated transportation planning in a port area in which rail, water, air, and truck transportation must be planned on the basis of interrelated land use. This is followed by a description of airport planning, financing, and operation in order to serve better the surrounding communities. Finally, the prospects for coordination of transportation are developed.

THE PORT, A FOCAL POINT

BY ROGER H. GILMAN,[1] M. ASCE

SYNOPSIS

The port has become a focal point for every form of transportation—by water, land, and air. Therefore, attention must be given to present and future trends of all forms of transportation and to integration, coordination, and cooperation among the public agencies and private interests concerned with this vital element in the national economy.

INTRODUCTION

In the field of transportation the port planner must be concerned with all modes of transport and related facilities because a port is more than simply a harbor for marine transportation.

An examination of any city or metropolitan area to which the term "port" is applied reveals that it is a focal point for transportation by water, land, and air. A port serves as a magnet attracting all modes of transportation. Thus, the many transport facilities concentrated in the port terminal area permit the direct and convenient interchange of goods and persons between various parts of the United States and the world. The result of this concentration of transportation service has been that port areas have generally experienced the greatest and most intense developments of population, industry, commerce, and trade.

Transportation by water assumes first importance as the historical reason for this concentration and development. It is significant that of the 107 cities with a population of 100,000 or more in the United States (1950 census) only 11 are not located directly on a navigable waterway, or along the borders of the Great Lakes, or along one of the coastlines. In all countries the first communities, and later the fastest growing cities, were established along the coastlines, where natural harbors and streams permitted sailing craft to serve them more easily and directly than the primitive land vehicles. In 605 A.D. the waters of the Thames River in England were described[2] as a "market for many nations repairing to them by land and sea."

As it became possible to clear forests, span rivers, and cross hills, roads were built to serve land transportation facilities. Within the past century, land transportation by rail and motor vehicle has become a reality. In the first half of the twentieth century, the third element has been added—transportation by air. Unfortunately, a significant part of the growth and expansion of transportation and the development of their related facilities had taken place during the years before the role of planning was fully recognized.

NOTE.—Published, essentially as printed here, in February, 1956, in the Journal of the City Planning Division, as *Proceedings Paper 898*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Director of Port Development, The Port of New York Authority, New York, N. Y.

[2] "The Port of London," by John Herbert, Collins, London, England, 1947.

Some of the transportation developments that have taken place in the Port of New York, New York (N. Y.), over the past three centuries serve as a typical illustration of what has happened in almost every other community. Beginning with the establishment of the first Dutch trading center at the southern tip of Manhattan Island (N. Y.), in the past three centuries waterfront facilities have been built, torn down, and rebuilt by countless private owners, as well as by the individual municipalities located along the navigable rivers and bays, as the harbor area continued to spread. The early post roads and turnpikes, which developed through the countryside at the time of the Revolutionary War, still serve to aline many highways. These older highways, as well as the more modern expressways and parkways, are the work of many different governmental agencies.

One hundred years ago, or more, the first rail tracks were built in and around the Port of New York with little, if any, thought of integration or consolidation between the trunkline railroads, of which eleven serve this area (as of 1957). Just prior to World War II, the cities of Newark (N. J.) and New York engaged in competition to win supremacy for their individual municipal airports despite the fact that these facilities served all the people, industries, and trade of the entire port district of northern New Jersey and New York.

Lack of integration and cooperation in developing these facilities has made the job for the port planner more difficult. To complicate it even further, port areas are usually the central cores of larger metropolitan regions. A port planner is therefore faced with the fact that the port's own problems are increasingly overflowing into adjoining communities, thus increasing the difficulties because each municipality or unit of government has its own responsibilities and rights. For example, within the bistate Port of New York district there are 17 counties and more than 200 municipalities, each having its own separate jurisdiction. Thus, the planner at a port, as in any local metropolitan region, must of necessity recognize and be concerned with the multiplicity of government agencies which exercise power in their respective jurisdictions.

Marine Terminals

At most ports in the United States, no single governmental agency has control over the entire waterfront. Instead, there are usually many ownerships of the shore properties. Public agencies—including cities, public authorities, states, and the federal government—operate piers, dry docks, military bases, and other special facilities alongside each other in a single port. Private industries also own and operate rail terminals, steamship piers, industrial sites, marine terminals, and many other waterfront facilities within the same harbor. Further complicating the planner's job, but at the same time making it more exciting, are recent developments in marine transportation and technology which demand new types of terminal facilities at the port.

Highways

In recent years, highways have experienced phenomenal traffic growths. This upsurge demands new vehicular arteries and terminal facilities for passenger cars as well as the buses and trucks that carry a significant percentage of the passenger and freight loads. Industry has spread to previously undeveloped sections of the metropolitan area, with the result that substantial numbers of office and plant workers now commute by car to work outside the central core of the city. As the hours of work have shortened and the hours of leisure have increased, the average family spends more of its time in the family car. The economy is permitting more and more people to own a vacation spot so that the highways now carry this additional load. The planner of a port must therefore be concerned with the need for arterial highway facilities to care for the motorist who wishes to bypass the central and congested city areas for both his business or pleasure travel; with the necessity for major tunnels and bridges to span the rivers and bays of the port; and with the challenge to his ingenuity of improving the movement of traffic on the streets within the city itself as well as providing the necessary parking and other terminal facilities in the central business districts.

RAILROADS

Of obvious importance to the complete facilities of a port are the railroads, which handle huge volumes of persons and goods moving to and from, as well as within, the metropolitan area. The port planner must be concerned with the problems that are being experienced by rail carriers throughout the United States, including losses of passenger and freight business to other forms of transportation and their financial difficulties. However, it is essential that the railroads continue to serve as one of the basic forms of transport for any metropolitan area or port in providing for mass transportation of passengers and in hauling freight. Again the planner must take into account any new developments in rail transportation as well as take advantage of what may be a new trend—the willingness of some railroads to integrate and consolidate their terminal operations and facilities.

AIR TRANSPORTATION

In air transportation the planner is not faced with long-established facilities and traditions requiring major changes to meet existing needs. There also appears to be a greater willingness on the part of the air industry to experiment and to try new methods of integrating terminal operations, which makes the planner's efforts and ingenuity more successful and satisfying. On the other hand, there are compensating difficulties that challenge the planner as he provides air transportation facilities for the growing needs of great metropolitan areas.

THE PORT OF NEW YORK

There are undoubtedly many ways in which the planning of a port as a focal point of transportation may be undertaken. However, the activities and programs being conducted in the northern New Jersey-New York metropolitan region to illustrate the types of planning performed at this great port will be presented.

The metropolitan area of New York and northern New Jersey has an extensive and complex network of land, water, and air routes, and their respective terminal facilities, which are the product of more than three hundred years of growth. The coordination of this vast transportation network has not been entrusted to any single national, state, or local agency. Instead, coordination is effected informally among the many public agencies and private companies. The region's transportation problems are too big and too complex. They involve the responsibilities of too many governmental units and the rights of too many private interests to be solved by unilateral action; they can only be solved by cooperation on a regional level. The port planner must realize that he cannot expect to "mastermind" all planning activities as he does his job. Instead, he must work with, and alongside, many others with equal responsibilities.

For example, it is significant to note that in establishing The Port of New York Authority in 1921 as their joint public agency, the states of New York and New Jersey stressed the type of cooperative and coordinated planning that should be undertaken in this area. Throughout the statute, the Port Authority's role as a catalyst in the field of planning was emphasized. In implementing its many functions and responsibilities, the Comprehensive Plan calls for the Port Authority to "request, assist, cooperate and render advice, suggestion and assistance."[3] This type of planning no doubt could be applied to any large port. Such a role is proper for any metropolitan planning agency under a governmental system with dependence on local controls.

Marine Transportation.—In most ports in the United States there is multiple ownership of waterfront properties. In establishing the Port Authority, the states of New York and New Jersey specifically required the consent of the municipality for any development by the Port Authority of marine terminal facilities in which municipal property would be utilized.

Under that assignment of limited powers, the Port Authority is presently (1957) responsible for developing and operating four marine terminals in the port—a grain terminal in Brooklyn (N. Y.), the marine terminal at Port Newark (N. J.), the Hoboken-Port Authority (N. J.) piers, and the Brooklyn-Port Authority piers. The bistate agency also plans to develop a fifth marine facility, the Elizabeth (N. J.)-Port Authority piers along Newark Bay. The balance of the waterfront is owned and operated in part by the city of New York and other municipalities; in part by the federal government with its special facilities; and in part by private industries that operate marine terminals, rail terminals, steamship facilities, as well as waterfront industrial sites.

There are approximately 340 deep-sea berths in the harbor for the handling of general-cargo vessels. In terms merely of berthing space, the port has more than enough piers and wharves. However, much remains to be done to assure that general-cargo berths have the supporting modern facilities required to handle modern ship cargoes.

As at many ports, a good many piers date from the early part of the twentieth century and even back to the nineteenth century, well before large over-the-road trucks began to haul a substantial share of import-export freight. The planner of modern general-cargo piers must provide single-deck structures with sufficient widths for truck operations and adequate floor-load capacity to permit the high piling of cargoes by modern handling equipment.

---

[3] "Comprehensive Plan Act," Laws of New York and New Jersey, 1922.

Depending on the channel location, distance between pierhead and bulkhead lines, availability of level upland areas, and accessibility by land transportation, the planner can select the type of pier best suited for a particular section of the waterfront. In some cases marginal-wharf construction parallel to the waterway is most efficient and economical, whereas at many harbors it may be advantageous to use finger-type piers.

Since 1946 substantial progress has been made in marine terminal development. In the entire United States in the period from 1945 to 1957, approximately $685,000,000 has been spent on new and modernized waterfront facilities. Of this amount, more than $155,000,000 has already been spent or committed by the Port Authority, the city of New York, and private waterfront owners at the Port of New York since World War II. The Port Authority's program alone calls for an expenditure of $150,000,000 by 1962 on self-supporting pier development in Brooklyn and New Jersey, whereas the city has announced a $130,000,000 program for the reconstruction of the municipally owned waterfront. Thus, progress on waterfront facilities will continue for many years. Fig. 1 shows the Brooklyn waterfront as it will appear in 1962.

FIG. 1.—BROOKLYN (N. Y.) WATERFRONT AS IT WILL APPEAR IN 1962

The port planner must give careful attention to new developments in marine transportation. This is particularly true for the coastwise and intercoastal trade, in which an increasing number of new types of vessels are coming into operation, such as the roll-on, roll-off ships for carrying truck trailers; sea trains which haul 100 or more rail cars; and containerships for transporting fully packed containers. All these types of vessels are designed to minimize handling costs and reduce ship turn-around time in port. As a result, there will be a need for specialized facilities at the port for berthing, loading, and unloading these vessels.

There will also be a need for more specialized facilities to accommodate growing inland-waterway operations as well as for the handling of supertankers and the increasing imports of specific commodities, such as iron ore, newsprint, foodstuffs, and bulk liquids. Careful consideration must be given to their special requirements as to channel depths, wharves, unloading devices, and backup spaces for rail cars and trucks.

Highway Transportation.—This type of transportation has progressed tremendously since 1925. In the New York-New Jersey metropolitan area alone, almost 700 miles of major arterial highways, expressways, parkways, bridges, and tunnels are in operation. The construction of this vast arterial highway program has involved an investment of approximately $2,000,000,000 as of 1957. More than $2,000,000,000 of additional projects are currently being constructed and being proposed. By 1965 a total of 850 miles of an ultimate 1,000-mile arterial highway network will be available in the New York-New Jersey metropolitan area. Approximately one-half of the arterial highway investment of more than $4,000,000,000 will have been financed by public authorities on the basis of self-supporting facilities.

The field of highway transportation, which involves every level of government—municipal, public authority, county, state, and federal—is one of the best illustrations of cooperative effort by agencies. The many responsible highway agencies can only be persuaded, rather than ordered, to develop their streets and highways to conform with an integrated over-all highway system. The job of coordinating the planning and construction of expressways, parkways, and other arterial highways within the various sections of the New York-New Jersey metropolitan area is tremendous and complex.

However, this fact has resulted in an awareness by all agencies of the essential need for integrating their activities so that their various parts will form a comprehensive and coordinated metropolitan highway system. These agencies are cooperating and are working on a day-to-day basis. There is no rigid integration of these various agencies into formal committees. Instead, the responsible public officials, executive heads, highway and traffic engineers, planning experts, and others are keeping in close contact with each other on their plans for highway development. They confer frequently, work together in the development of plans, and collaborate in the actual construction of many arterial facilities.

This coordination has been, and will continue to be, developed as the need arises. During recent years many instances could be cited in which four, five, and even six separate and independent highway agencies of various governmental units have cooperated in planning, financing, and constructing interchanges, connections, and approaches to assure the complete integration of arterial highways, tunnels, and bridges.

In 1954 the Port Authority and the Triborough Bridge and Tunnel Authority of the city of New York pooled their knowledge, experience, and past surveys in a year-long joint study of those bridges and integrated arterial approach systems required to provide effective relief of the congestion and delays at existing crossings, and to provide over-all traffic relief throughout the New York-New Jersey metropolitan area. As a result of that study, in which close contact was maintained with many other highway agencies, the two authorities in January, 1955, proposed the construction, at a cost of $400,000,000 (estimated at more than $500,000,000 as of 1957), of three major bridge projects—a twelve-lane suspension bridge between Brooklyn and Staten Island (N. Y.) and spanning the Narrows entrance to New York Harbor from the open sea; a six-lane second deck below the existing George Washington Bridge roadway (between New York and New Jersey); and a six-lane Throggs Neck Bridge joining Long Island (N. Y.) with the Bronx (N. Y.).[4] As of late 1957, construction has started on the George Washington Bridge second deck project and Throggs Neck Bridge project, and most of the necessary approvals for the Narrows Bridge and its approach highways have been granted.

Rail Transportation.—In any metropolitan region the port planner must give serious consideration to railroads as an essential form of transportation. The development of "piggy-back" operations may help the railroads to retain and increase their transporting of merchandise freight. The planner must be concerned with what these developments mean to the requirements for rail-terminal facilities and the over-all transportation picture.

The New York-New Jersey metropolitan area is giving increasing attention to the transit problem. In 1954 the two states created the Metropolitan Rapid Transit Commission as their joint agency to study the rapid transit needs of the region. In 1955 the commission, with the financial assistance of the Port Authority, initiated the most comprehensive interstate transit study ever undertaken in the area. This study was designed to develop recommendations for maintaining an adequate and feasible system of rapid transit service between New Jersey and New York.

Both agencies recognized that the development of any practical plan for rapid transit required a thorough appraisal of the future demand for mass transportation in the metropolitan area. The survey also considered the possibility of integrating rail transit lines with feeder bus lines and with private automobile travel at the railheads, as well as improved bus services where an additional investment in rail facilities was not found to be justified. The two bistate agencies agreed that no study of the rail commuter problem and a recommended solution would be realistic without developing a constitutionally sound, financially practicable, and politically feasible plan of meeting deficits and debt charges.

In May, 1957, the project director of the survey recommended to the commission the construction of a $400,000,000 bistate loop linking the Manhattan and New Jersey sides of the Hudson River, with present New Jersey commuter railroads serving as feeder lines to the loop.[5] The project director also called for the creation of a new permanent bistate transit agency to assume the responsibility of suburban transit. This agency would have local representation from the benefited areas and would receive public subsidy from tax sources to meet the substantial annual deficits.

---

[4] "Joint Study of Arterial Facilities, New York-New Jersey Metropolitan Area," The Port of New York Authority and Triborough Bridge and Tunnel Authority, January, 1955.

[5] "Metropolitan Rapid Transit Survey," Report of the Project Director to the Metropolitan Rapid Transit Comm., New York, N. Y., May, 1957.

On the basis of the various studies that were undertaken, the Metropolitan Rapid Transit Commission is expected to submit its final conclusions and recommendations to the legislatures and governors of New York and New Jersey by the end of 1957.

Air Transportation.—The need for close liaison and coordination with other public agencies and private companies also extends to the planning of metropolitan airports. This type of cooperative consultation and planning is illustrated by the Regional Airport Conference organized immediately after World War II. This group had representatives from the federal government; the state governments of New Jersey, New York, and Connecticut; and the city of New York, the Port Authority, the Regional Plan Association, and the seventeen counties of the metropolitan area. The result of the conference was the formulation of a general plan and program, which helped to crystallize the aviation planning efforts of this region.[6]

FIG. 2.—CENTRAL TERMINAL AREA, NEW YORK INTERNATIONAL AIRPORT

The foregoing study and report were important factors in the evolution of the concept under which the states of New York and New Jersey authorized their joint public agency to plan, develop, and operate major commercial airports. Under this directive, the Port Authority signed lease agreements in 1947 with the cities of Newark and New York by which it assumed the responsibility for developing their municipal airports. In its program the Port Authority is developing the four metropolitan airports—New York International, LaGuardia, Newark, and Teterboro—and a Manhattan waterfront heliport on a self-supporting basis.

The Port Authority's airport planning is based on the assumption that air transportation will become the dominant means of long-distance travel. It is also expected that the role of air transportation will expand into the short-haul travel market, with the helicopter giving increased air service to the huge short-haul market to nearby metropolitan areas and within these areas themselves. The planner should thus anticipate the terminal needs and other requirements of helicopter services to and from suburban areas. A Port Authority traffic estimate indicates that by 1965 the airports in the New York-New Jersey metropolitan area must accommodate approximately 25,000,000 passengers per yr. To provide the facilities to handle this traffic, the Port Authority has already invested almost $180,000,000 in its airports. Fig. 2 shows the ultimate development of New York International Airport. It is expected that the total investment in the metropolitan airports will exceed $700,000,000 by 1967. This regional airport program illustrates the type of planning required of today's planner, unanticipated only a generation ago.

---

[6] "Airports of Tomorrow," Regional Plan Assn., Inc., New York, N.Y., 1947.

CONCLUSIONS

It can be seen that the planner of a modern port must concern himself with every form of transportation, giving careful consideration to new developments that are increasing rapidly. Such plans require integration, coordination, and cooperation among all public agencies and private interests to the end that the port may truly become the focal point of an efficient and economical transportation network.

THE AIRPORT, A NATIONAL FACILITY

By Wilfred M. Post, Jr.[1]

Synopsis

The development and financing of a medium-sized airport are described together with its operation under a local airport authority. The legal and economic history of the airport is traced, including the creation of the airport authority following World War II.

Introduction

An airport is the "front door" to many cities. In the development of transportation, airplanes become increasingly important to the industrial and social growth of any community. A well-equipped, efficiently operated modern airport is important, as water, rail, and highway facilities have been in the past. To visitors the airport gives the first impression of a city, and citizens and public officials are striving to make this impression good. In addition, the chambers of commerce realize that in order to make effective bids for new businesses and industries adequate air-transportation facilities are an important requirement.

Although there are certain basic physical requirements for any airport, the adaptation for various communities will vary according to the geographic location, population, travel characteristics, and circumstances peculiar to each community. Planning for an airport must not only take these factors into account but the important matter of finances as well.

In addition to being a community asset, each airport plays a vital part in the air transportation of the United States. An integrated airport system and a healthy civil aviation industry are necessary for peacetime economy and wartime strength. For these reasons the federal government and most of the states actively participate in airport planning and development.

Federal aid through the years has come in different forms; in 1946 a grant-in-aid program established through the Federal Airport Act[2] provided authorization for the Civil Aeronautics Administration, United States Department of Commerce (CAA), to share construction costs with municipalities. The first few years of the program brought considerable developments because federal grants were matched by local funds and, in some states, by a combination of local funds and state funds. However, in 1954 there were no funds because of the fluctuating amounts appropriated by the United States Congress; long-range planning and financing were difficult if not impossible. As a result of

---

Note.—Published, essentially as printed here, in February, 1956, in the Journal of the City Planning Division, as *Proceedings Paper 894*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Mgr., Allentown-Bethlehem-Easton Airport, Pa.

[2] *Federal Airport Act*, Public Law 377, 79th Cong., U. S. Govt. Printing Office, Washington, D. C., 1946.

legislation[3] in 1955, the CAA will be assured of funds for a 4-yr period; longer-range planning and financing will take effect.

Public airports can become eligible for inclusion in the federal government's National Airport Plan by qualifying under either of two groups of criteria to measure aeronautical requirements. Airports presently receiving or immediately anticipating airline service may qualify under criteria based on a measurement of such activity. Those not receiving or immediately anticipating airline service may qualify under criteria based on a measurement of general aviation activity. Cities receiving airline service are classified as large hubs, medium hubs, small hubs, and non-hubs, depending on the amount of traffic they generate. The average length of the passenger haul results in classifications such as intercontinental, continental, express, and trunk or feeder. These classifications determine the length of runways; in the trunk or feeder category they are 3,500 ft long and contain a single-wheel loading of 15,000 lb; in the intercontinental category they are 8,400 ft or longer with single-wheel loadings of 100,000 lb. Such runway lengths must be increased for elevation, temperature, and gradient. Recommendations for the other elements, such as the approach standards, are specified for each type of airport. Airports not having scheduled airline service are rated on the number of based aircraft or other "aeronautical demand modifiers," such as itinerant traffic or other aviation activity important to the particular community involved.

Although most military bases are separate from commercial airports and will not be cited herein, some sites have proved feasible for joint usage especially for reserve military units and Air National Guard installations; these locations receive special consideration.

Local community finances have naturally dictated much of the planning and development. Methods of financing and subsequent operation vary widely. Some airports are operated directly by the city or county, some have airport commissions, and others have airport authorities. Certain ones, usually those of large size, are self-sustaining and have been financed from revenue bonds. Others, especially those of smaller size, must depend on local subsidy.

It is interesting to study the history of a medium-sized, trunk-line airport serving a small hub area. There are larger and finer airports than the Allentown-Bethlehem-Easton Airport in Pennsylvania; many are smaller and inferior. The following will show how one group of communities developed and planned its airport.

EARLY HISTORY

The history of the Allentown-Bethlehem-Easton Airport is a complicated one. Originally, 50 acres of land were leased from a farmer by the Department of Commerce in 1927 as an emergency landing strip for airmail pilots. In 1929 a group of public-spirited, air-minded businessmen, supported by the local Chamber of Commerce, raised \$132,000 to purchase the original 317½ acres of land, to construct a hangar, and to level the ground. In the early 1930's it became necessary to sell 50 acres to obtain the funds to continue the project. This action was later regretted because the subsequent expansion required repurchase of the land at three times the amount received for its sale. It was thus learned that the progress of aviation development can never be "sold short."

---

[3] *Public Law 211*, 84th Cong., U. S. Govt. Printing Office, Washington, D. C., 1955.

Federal funds became available in the mid-1930's, thereby making this project one of the first to construct an original runway in the United States. This work was also probably one of the most efficiently organized due to capable local direction and supervision. Three runways resulted from funds originally provided for only one runway. However, in order to qualify for these funds the airport was deeded to the municipalities for $1.00 by the private corporation mentioned previously.

FORMATION OF THE AIRPORT AUTHORITY

Toward the end of World War II, in view of expected rapid development in aviation, a joint study group appointed by the chambers of commerce recommended to the municipalities that an authority be established as the most ideal method of developing and operating a public airport. This authority was originally created by the cities of Allentown and Bethlehem, together with Lehigh County. The city of Easton and Northampton County later joined as additional sponsors. The unique venture of three cities and two counties cooperating in a common enterprise has proved most successful. Interest in the authority has done much to diminish adverse sectionalism between normally competitive municipalities. The Lehigh-Northampton Airport Authority was one of the first of its kind in Pennsylvania, and, following its success, many other municipalities have created similar authorities.

ECONOMIC ADVANTAGES

In addition to favorable publicity for the sponsors, the creation of the authority has had real economic advantages. The sound sponsorship has been a potent factor in attracting federal and state participation in the expansion of the airport. Since 1946 more than $2,000,000 have been spent (as of 1957) in its development, yet, because of the five-way division of local funds, it has required only 5 cents from each sponsor to obtain $1.00 worth of construction. The combined 25 cents supplied by the five cities has been matched by 25 cents from the state and 50 cents from the federal government. No bonded indebtedness has resulted because the five municipalities have not been called on to provide more than $20,000 in any single year. Each sponsor has been able to include such an amount in its annual budget. None of the municipalities could have individually sponsored such an undertaking. Certain projects could not be provided for in a single year, and, in such cases, the authority has been able to borrow money from local banks based on pledges of its five sponsors to provide funds over a period of years. This was done in the construction of the $1,000,000 passenger terminal building. The sponsors' share totaled $350,000, and the authority was able to borrow this money locally and repay it as the sponsors made their contributions over a period of 4 yr. This building not only provides attractive facilities for the air traveler and adequate quarters for the airlines and federal operating agencies, but it provides space for local businesses and visitors, which adds to airport revenues. Display cases in the lobby, featuring local manufacturers and their products, were installed through a self-amortizing loan at a local bank. These display cases provide substantial additional income.

PRESENT FACILITIES

The airport covers more than 500 acres of land. It has three runways; one is 5,000 ft long, equipped with high intensity lights. An instrument-landing system was commissioned in 1954 by the federal government at a cost of $110,000. There are more than 35 acres of paving, which is equivalent to 19 miles of two-lane highways; 9 miles of underground drainage pipe; and 8 miles of underground wiring.

The main hangar building provides more than 12,000 sq ft of hangar and office space for the base operator, where air taxi, aircraft sales, flight training, and other services are provided. Income from the base operators' use of these facilities is based on a percentage of gross income and averages $3,600 per yr. Because of the limited number of authority employees, the base operator, on a commission basis, collects the landing and parking fees charged to commercial airplanes. The state wing headquarters of the Civil Air Patrol and the District Safety Regulations Office of the CAA are also located in this building.

An area on the north side of the airport has been developed for housing the smaller type of airplanes owned by individuals and businesses in the area. These hangars were constructed through bank loans, which are being paid from the revenue derived. Of the original $49,000, only $14,000 remain unpaid. Rentals range from $15.00 to $30.00 per month, and thirty-five airplanes are kept in these buildings.

Two large hangars have been erected by local industries. A steel company operates two twin-engine airplanes and pays a ground rental of $2,000 per yr. A smaller hangar constructed in 1953 by a construction company produces a ground rental of $500 per yr.

Adjacent to the main hangar is a restaurant, also built on a self-amortizing loan. It is leased to a concessionaire who has also developed a children's amusement area. Rent, based on a percentage of gross income, has been increasing yearly. On the same rental basis, a golf driving range has been developed along the entranceway to the terminal building.

The Pilot's Club, located adjacent to the southwest corner of the airport on its own land, now (1956) has a membership of more than 400 and has recently completed construction of a swimming pool which, during the summer months, attracts local and visiting aviation enthusiasts.

Located in the new terminal building is the Department of Commerce Weather Bureau, which not only provides weather information and forecasts to aircraft pilots, but offers valuable weather service to farmers, manufacturers, contractors, transportation systems, the highway department, newspapers, radio and television stations, and the general public. It also issues flood warnings for the Lehigh River and Lackawaxen River in Pennsylvania.

The control tower supervises an average of 5,000 take-offs and landings each month. The operation of the instrument landing system is monitored in this tower. The Airways Communications Station is another federal agency and has been merged recently with the control tower. It is located at the intersection of five federal airways and coordinates the majority of east and westbound air traffic to and from the New York (N. Y.) metropolitan area under the direction of the Air Route Traffic Control Center in New York. More than 50,000 airplane contacts are handled each year. It is one of the busiest of such facilities in the United States. Inasmuch as federal funds were used in the construction of this building, the authority cannot charge these facilities rent. However, they pay for light, heat, and maintenance.

Scheduled Air Service

The Allentown-Bethlehem-Easton Airport is served by three airlines with fourteen daily flights. Passengers boarding and unboarding planes have increased from 9,161 in 1947 to almost 45,000 in 1954. Substantial increases have also occurred in air freight, air express, and airmail. Due to increased business, the airlines have agreed to an increase in landing fees and space rent in the terminal building. The new landing fees are computed on a weight-frequency basis, which begins at 9½ cents per 1,000 lb of maximum gross-landing weight per aircraft landing. Volume discounts of 1 cent-per-1,000-lb integrals apply after the first 150 scheduled landings per month. Ticket counter and office space in the terminal building rent at $3.50 per sq ft per yr. Parking fees are charged all aircraft remaining at the airport for more than one hour with the exception of those owned by the federal government. Itinerant commercial airplanes are charged a landing fee based on the size of the airplane. Personally owned airplanes on pleasure or training flights are not charged a landing fee. It is felt that at airports such as these this phase of flying still needs encouragement.

Other Revenue

Perimeter land owned by the authority outside the landing field is leased for farming. Several areas between the runways are also in the experimental stages of farming. Because of the underground limestone structure, alfalfa has been determined to be the best crop. Anything that requires yearly plowing or that grows too high so that visibility is impeded has not been used because of the fear of aggravating a sink-hole condition peculiar to this area. If sink holes do not develop because of improper surface drainage, more such areas will be leased to decrease maintenance expense and increase income.

Income and Expenses

The airport operates at a modest deficit, but income has risen from $27,067 in 1947 to $70,475 in 1954. However, as a result of the necessary expansion of facilities and the time required for their full utilization, expenses have increased to $85,289. The deficit is made up from allocations of the sponsors' annual contributions. Depreciation and amortization are not included in the expenses.

Zoning and Obstruction Removal

In 1955, after two years of preparation, vertical zoning resolutions were passed by Lehigh and Northampton counties. A small percentage of airports in the United States have completed such zoning programs, and this assures clear approaches to the runways and protection of the investment. In some cities runways have been closed, and the entire investment has become imperiled because of the lack of foresight in not being able to prevent the erection of obstructions. The federal and state governments participated in one project totaling $120,000, providing for clearing approaches to one end of the instrument landing runway. This project is nearly completed, and another, which will clear obstructions on the other end, has been approved for $255,000.

AIRPORT MAINTENANCE

A crew of eleven maintains all the buildings, roads, and the landing field. Snow removal in the winter and grass mowing in the summer, in addition to the maintenance of the field lights and runways, are only a few of the many tasks performed by this crew. Of great financial help in the airfield maintenance work is the safety improvement program, which is implemented by the Pennsylvania Aeronautics Commission. In this program, one-half of the state tax on aviation gasoline sold at the airport is returned for one-half of the cost of approved airfield maintenance projects. In 1954 the state's share of the tax totaled $4,000 as a result of increased gasoline sales.

Fire and crash protection is coordinated by the authority between the operating companies on a cooperative, volunteer basis and is supported by the municipal fire departments that respond to calls. In this emergency plan the airport fire truck is manned by volunteers supplied by the three airlines. Additional equipment is supplied by a steel company, a construction company, and the base operator. During 1954 equipment and personnel stood by for sixteen emergencies, but fortunately no fires or injuries resulted.

THE AUTHORITY BOARD

The greatest asset of the authority is the civic leaders who have served on the Board of Governors when asked by their municipalities. The authority is composed of a board of fifteen members. Each of the five sponsoring municipalities appoints one person every year for a term of 3 yr. The board members serve without compensation. Because of the complexity of developing and operating an airport, this is an unusually hard-working board, and considerable time is required of many of its members. Five standing committees are appointed which consist of construction, management, finance, publicity, and operations. For example, the operations committee meets at the airport weekly to see that the appearance is good and to discuss the innumerable details of maintenance. This leadership has produced substantial cooperation from industries that have given great assistance in many ways, even to the extent of assistance in acquisition of land for future expansion.

CONCLUSIONS

Much forethought and planning will be required in the future. Each community will have its own local circumstances to consider in addition to the requirements of the airplanes planned to be put into service by the operating companies. Undoubtedly, jet aircraft will require longer runways. Helicopters and convertiplanes will require no runways at all. Just what will be needed at this airport and others and what the communities can finance will take intensive study as the requirements of proposed airplanes are developed.

Each community will need a basically sound organization. The challenge of the future will be met here and throughout the United States by similar groups. Because of the foresightedness and hard work of their members, aided by competent consultants and state and federal agencies, airports will continue to become great assets.

PROSPECTS FOR COORDINATION

By Burton W. Marsh,[1] M. ASCE

Synopsis

In the past, coordination in transportation planning has been almost completely lacking. This has been due, in part, to the rapid development of the United States, traditional attitudes of competition among modes of travel, an abundance of transportation facilities, and separate regulatory agencies, with consequent separate regulatory measures being applied to different types of common carriers of passengers. These factors are, in part, responsible for the deterioration of passenger-transportation services in metropolitan areas.

Passenger-transportation conditions in metropolitan areas have become so serious that leaders in these areas recognize the need for effective cooperation if solutions to these problems are to be found and put into effect. Some groups are already beginning to attack urban-transport problems effectively as well as other problems, such as water supply, sewage disposal, smog, blighted areas, and community parking facilities.

Introduction

Appraised in terms of the past, the prospect for coordination of transportation planning is poor. However, new conditions, factors, problems, and elements have emerged. Careful consideration of these factors indicates that the prospects for coordination of transportation planning are encouraging especially where the need is most urgent—in the major metropolitan areas.

In the rapidly developing United States, each of the four major forms of transport—rail, highway, water, and air—has gone its independent way without regard to the impingement, or the effect, each one had on other forms of transportation, or on the economy and welfare of the area as a whole. There has been an uncoordinated situation to which various factors have contributed:

1. Businesses have been allowed to develop their own potential, and competitive forces have operated freely, thus resulting in a minimum of interference with the evolution and development of "the American way of life."

2. The United States has had a relative abundance of transportation in all categories, available to both shippers and travelers. This situation has been possible because of favorable conditions as to resources and high productivity with high standards of living. This abundance has contributed to a general acceptance of conditions. Had the economy been strained, it would have been necessary to give more attention to the problems involved and to the effects of free competitive evolution.

Note.—Published, essentially as printed here, in May, 1956, in the Journal of the City Planning Division, as *Proceedings Paper 988*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Director, Traffic Eng. and Safety Dept., Am. Automobile Assn., Washington, D. C.

3. Leaders of the various forms of transport have been unwilling to cooperate with each other, although there has been cooperation on some matters. Basically, they have desired to be left alone, and often they have not cooperated regarding transportation matters as a whole.

4. Another problem that has bothered certain forms of transportation more than others is lack of uniformity of regulatory practices. Regulatory measures have borne with varying weight on different forms of transport. The railroad industry, for example, feels that the regulatory measures that affect it are unduly heavy compared with those affecting its principal competitors for the movement of goods.

5. Finally, there has been a tendency to regulate separately for each form of transportation. One result has been to encourage special pleading for each type of transportation. Such special pleading applies to regulations and to the use of public funds as well.

Separate consideration of the different fields of transportation leads to undesirable results. Lacey V. Murrow, M. ASCE, gave a good example. At the conclusion of World War II, the United States sent four separate teams to China in an effort to improve the transportation situation. These groups were composed of competent engineers and specialists in their own fields. The teams worked independently, and the result was that each one attempted to solve the transport problem of China from the point of view of the particular part of transportation that that group represented. The Chinese were naturally less than completely pleased with the results; they could not afford duplicatory transportation facilities such as exist in the United States.

In the Philippines the federal government attempted to assist on transportation improvements. Instead of over-all transportation needs being considered, the needs of particular types of transportation were considered separately. One result has been a multiplicity of regulatory bodies somewhat similar to those in the United States and certainly not suited to the best interests of the Philippines.

Transportation conditions are also serious on a national scale. From the point of view of defense and over-all economy in the United States, the railroad industry is in serious trouble. Furthermore, it is very important that reasonable solutions be found so that the railroads return to, and remain in, their former state of health and strength.

Metropolitan areas are localities in which coordination of transportation planning is very important. Furthermore, such planning is no longer academic but a problem which is, or is fast becoming, urgent. Roger H. Gilman,[2] M. ASCE, has provided excellent evidence of this.

Perhaps the best example of a bad situation in metropolitan areas is in the field of transit. Basically, it is the result of the growth of the use of passenger cars as a means of transportation. Public transportation (an essential factor in metropolitan living involving an important industry) is in serious difficulties almost everywhere.

---

[2] "The Port, A Focal Point," by Roger H. Gilman, in "Transportation Planning: A Symposium," *Transactions*, ASCE, Vol. 123, 1958, pp. 361.

However, coordination of transportation planning is needed. The objective should be to achieve a reasonable balance between public transportation and passenger-car transportation, giving full recognition to the advantages and disadvantages of each.

It was found that in one of the largest cities a transportation survey was to be made. Inquiry showed that it was actually going to be limited almost entirely to a transit survey because all the attention in recent years had, it was asserted, been directed to the other elements of highway transportation and not enough to transit. However, when a community proposes to make a transportation survey and a transit survey is made instead, that is going too far in the other direction. It should not be a matter of having either one or the other. Both forms of transportation inevitably will continue, and the need is to work out a reasonable balance. Yet, this important survey seems unfortunately to be losing track of the main objective.

In contrast, in another transportation study made by a major city, transit was hardly mentioned. One would think that such a limited viewpoint would not be allowed to affect decisions.

Another case was that of an important western city. The following is a summary from its 1953 report to the legislature: "A satisfactory solution is not to be achieved by building highways and facilities for automobile transportation alone." The following should be noted: "* * * the solution can be reached only through mass rapid transit designed to move people rather than cars." Two points herein warrant comment: "only through mass transit" and "to move people rather than cars." What is intended by the latter phrase is to indicate that transit vehicles carry many more persons per vehicle than do passenger cars.

A major point in all three cases is that there are important groups in major cities dealing with a dynamic, urgent problem, but not approaching it broadly and not taking into account all related forms of transportation.

The potential of a doubling in highway traffic in just two short decades should be considered. It is natural to assume that this doubling will tend to apply to traffic in metropolitan areas if necessary steps are taken that will permit that sort of growth.

Much attention has been paid to the speed of air travel and to plans for providing still faster airplanes. Yet, one must consider the great amount of time spent getting to the airport and from it to the destination. Existing transportation facilities between the airport and the center of the city are another good example of the lack of coordination in a great many communities. Fortunately, this situation is being much improved in some communities.

The lack of proper planning in connection with the Pittsburgh (Pa.) Airport Parkway should be considered. A new divided highway was completed whose purpose was to serve that strategic airport efficiently. However, its capacity and effectiveness are being seriously reduced because local authorities do not have adequate powers of zoning and of access control. They are attempting to get the state to take over the divided highway so that access control and proper zoning can be used to avoid further deterioration of its effectiveness.

Walter Kurylo provided an interesting example with respect to the navigational clearances of bridges.[3] Under old regulations developed when conditions were greatly different, clearance requirements under bridges on navigable streams were set to safeguard small quantities of watercraft with little regard for highway transportation. High bridges are usually costly, and, if the bridges are open for ship passage, highway or railroad traffic is delayed. Before the construction of such a bridge, people in highway transportation have often indicated how unreasonable such costs and delays would be but to no avail. Instead, millions of dollars have been spent for navigational clearances for relatively unimportant watercraft. Until recently, no attempt was made to determine whether the added costs of the bridge or other overland transportation costs resulting from watercraft requirements were offset by equal or greater savings in waterway transportation costs.

To alleviate this condition the United States Department of Commerce, acting on the recommendation of the Bureau of Public Roads, brought together many agencies to make a cooperative study of this bridge clearance problem. As a result, such clearances will be established in terms of what can be justified considering the relative importance of the water, railroad, and highway transportation involved and the over-all costs. It may be important, as in many port areas, that certain bridge clearances be kept high or that movable spans be provided. There are other places in which high clearances or movable spans will be unjustified and where necessary adjustments will be expected of the small quantity of watercraft, even to the extent of hinging masts or smokestacks.

Growing Metropolitan Area Problems

Urban and metropolitan areas have been growing rapidly at the expense of rural areas. During World War II many workers came in from rural areas and never returned. This shift in population has some interesting and important aspects. Many people have been moving to or locating in the suburbs, which also has been a marked trend. However, urban redevelopment and renewal developments are making headway and are again making various centrally located urban areas attractive. This is comparatively a lesser trend than that toward suburban living. This tremendous suburban development, involving not only residences but huge shopping centers, has created severe transit problems. When cities were closely knit, transportation by streetcar or other public transit vehicles served city areas efficiently and economically. However, the dispersion of population involved in the suburban changes has made it difficult, if not impossible, to serve such gigantic areas by public transportation with frequent service and low fares. The rapid growth of highway traffic has created problems as to provision of traffic arteries and terminal facilities—both parking and loading.

Another metropolitan area characteristic is the multiplicity of governmental agencies, which constitutes one of the stumbling blocks in developing proper over-all transportation plans.

---

[3] "The Interest of the Bureau of Public Roads," by Walter Kurylo, in "Bridge Clearances: A Symposium," *Transactions*, ASCE, Vol. 123, 1958.

Because of these and other difficulties there is a great need for better coordination of transportation. Although progress will be slow, it is believed that conditions will force advances in coordination of planning of metropolitan transportation. The problem is being recognized increasingly as practical, important, and urgent.

EXAMPLES OF PROGRESS

Gilman[3] has presented the views that The Port of New York (N. Y.) Authority has taken on various matters. Many are not directly port matters, but they relate so vitally to them that The Port of New York Authority (not limited by inadequate physical boundaries) has given these matters thorough study.

There are other examples of the growing interest in over-all transportation. At Northwestern University, at Evanston (Ill.), there is a new Transportation Center. It is predicted that this will be a well-financed, important operation. The sponsors realize that there should be a better-coordinated, integrated consideration of transportation problems. The center's viewpoint is that those who will have positions of top responsibility in one form of transportation will need to know more about the other forms of transportation. They will then take a broader view, to the long-range benefit of transportation and of their own organization as well. One objective of the center is to be a major source of information where comprehensive reference resources are available to students who wish to study various transportation matters. It provides graduate training to help produce more effective transportation administrators and staff specialists. This center will have a significant impact on the whole field of transportation in the future.

In Detroit (Mich.) transit vehicle needs are considered in the planning of freeways. That may seem an ordinary example of coordination planning, but it is really an important one. In fact, there are many places where in the evolution of the freeway plan there has been no adequate consideration of transit needs of the area and how the freeway plan might help meet them. In Detroit an interesting sidelight was that the transit industry decided the bus pickup points would be off the freeway. That concept poses additional planning and design problems, but it may prove its value.

In Chicago (Ill.), it was decided that wherever future transit service appears to be warranted in connection with a projected expressway, suitable facilities for transit will be built in.

Brief consideration should be given to a few community organizational patterns that are producing improved metropolitan coordination of transportation. In Pittsburgh the Allegheny Conference on Community Development has evolved. The conference is impressed with the great importance of having a better-integrated program for transportation for the metropolitan area and is working effectively toward that end. How did this organization come into being? Leaders of the metropolitan area of Pittsburgh decided to form an organization which would rise above all differences, complications, and political jurisdictional problems. They would appraise the major needs of the over-all area, decide on priorities, and then find suitable solutions. The Allegheny Conference on Community Development was the name chosen for the resulting organization.

Parking facilities have been greatly improved as a result of work stimulated by the Allegheny Conference. Other major needs in the field of transportation have also benefited from its leadership. The procedure is to find the most competent specialist or specialists available for consultation and to devise, in cooperation with governmental authorities, the best plan and program for correction. The conference assures that persons having large financial interests in the future of the Pittsburgh area want and will support these sound improvement programs of public officials. It has encouraged and supported a wide variety of measures, including some important legislative ones.

The conference and cooperating public officials will undoubtedly produce, in time, a better-coordinated transportation situation. The greatest problem is still public transportation despite the high priority given to it. This shows that some transportation improvements are not easy to achieve.

A quite different approach has been used by the San Diego (Calif.) Metropolitan Area Transportation Study. The city of San Diego realized the growing seriousness and importance of transportation problems in the metropolitan area. A group of carefully chosen advisers competent in the field of transportation extensively studied how best to solve the many complicated transportation problems. For some time a consultant to make a thorough study and report was considered. It was finally decided to employ, on a continuing basis, a specialist in transportation who would act as a transportation coordinator for the metropolitan area. The individual selected is employed by the city of San Diego, but his job is to work solely on the metropolitan transportation situation. With numerous governmental jurisdictions in the metropolitan area, there are obviously many problems and intergovernmental complications. However, there is a growing cooperation toward solving problems of the area.

There are good reasons why progress is being made. Plans for this activity have been carefully laid with the continuing counsel of the group of specialist-advisers. There is no effort to hurry or to "railroad" ideas through without regard for the views of the many interested agencies. The transportation coordinator has a sensible pattern of objectives and plans for creating a coordinated team consisting of assigned planners, engineers, and other specialists from the various municipalities in the area, from the state, and from the armed forces.

Another part of the plan is a technical coordinating committee. This committee is composed of departmental or division heads of the city, county, and state, and also has representatives of the federal government and of related organizations, such as airport managements, parking districts, and transit companies. Plans also include keeping the public well informed and providing frequent opportunities for obtaining the ideas and reactions of interested citizens.

Such reviews indicate that there is no single established way to organize properly coordinated transportation planning. On the other hand, there are many indications that increasing numbers of persons in various metropolitan areas have concluded that a more effective mechanism is essential in order to meet growing needs. Concepts on this matter are evolving, thus providing a challenge and an opportunity for engineers to participate in developing effective answers to these problems.

A key problem is that of suitable governmental organization in a metropolitan area to produce effective action on transportation and other problems. In this connection recent developments in the metropolitan area of Toronto (Canada) are worthy of study.

Communities in this area faced various serious problems—such as water supply, sewage disposal, and transportation—which should be solved on a metropolitan basis. They realized that it would be most difficult, if not impossible, to obtain the needed kind of broad thinking, cooperation, and coordination unless some improved governmental mechanism was devised which could effectively cope with such problems on a metropolitan or regional basis.

Various ideas were proposed to solve the problem. The Ontario (Canada) Municipal Board deals with municipal problems in that province. This board considered various proposed methods for meeting the problem, acting in a semijudicial capacity. One proposal, strongly supported by the city of Toronto, called for amalgamation of the numerous local governmental agencies into a single metropolitan government. The Ontario Municipal Board ruled against this concept and decided instead that for purely local matters existing municipalities should retain their complete jurisdiction, but that the Municipality of Metropolitan Toronto would have powers concerning matters of a metropolitan nature only. The municipality taxes not the public but the constituent municipalities. As to local matters, the intent of the board was to retain the good that there is in local governments, including their closer contact with their citizenry, and to provide the additional needed power with the least possible complication of governmental structure through the creation of a metropolitan municipality with limited powers.

There has been a considerable tendency in the United States, when a particular metropolitan problem is encountered, to decide to have an area-wide authority or board or agency, giving it special powers to take care of that particular need. Thus, there exist water-supply systems, flood-control districts, park systems, and sewage-disposal agencies. Because of the many special problems of a metropolitan nature, one must anticipate having a considerable number of independent, special-purpose agencies or seek another answer. In the case of the Toronto area, the decision was to create one over-all agency that could take care of all such problems. The experience of the Municipality of Metropolitan Toronto will be well worth noting, not only from the point of view of transportation, but also from an even broader viewpoint.

CONCLUSIONS

For many years the planning survey work of state highway departments has been based on fundamental factual information, the pattern for which was developed in 1935. Although forward-looking leaders had long recognized the need for such basic facts, it took many years to get these into effective use throughout the United States. It is only now that a parallel urban fact-gathering program is being formulated although the highway transportation problem has long been most severe in these areas. Once obtained and put to use, such basic facts will help greatly not only in solving problems but in demonstrating more clearly the great need for much increased emphasis on metropolitan transportation. As more information of this sort becomes available and more attention is given thereto, it will become increasingly obvious that there must be better integration and coordination.

Developments in various parts of the United States, some of which have been cited herein, indicate that there will emerge new organizational mechanisms and new operational methods that will result in far better transportation planning and coordination of such planning in communities and metropolitan areas. Indeed, the growing problems of our mushrooming communities will force such action. Fortunately, there is a growing realization among students and leaders of the need for integrated plans and programs. There is no established pattern for coordination of transportation planning, and the evolution in each area will be the result of the ideas of the most interested persons.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

TRANSACTIONS

Paper No. 2925

EVAPORATION FROM FREE WATER SURFACES AT HIGH ALTITUDES

By Harry F. Blaney,[1] M. ASCE

With Discussion by Messrs. Carl Rohwer; Kenneth M. Turner; Irvin M. Ingerson and John W. Shannon; and Harry F. Blaney

Synopsis

In the western United States, losses due to evaporation from reservoirs and lakes at high altitudes are of importance as an element affecting the net water supply available for the irrigation of crops, the production of power, and municipal and industrial purposes. Except in unusual instances, the evaporation rate cannot be measured directly from large water areas. Thus, it is a common practice to measure the evaporation rate of water from pans, and, using coefficients, a reduction is made of the pan evaporation measurement to the lake evaporation value. At high altitudes it is seldom possible to measure the evaporation rate during the winter months because the water in the pans freezes. The paper presents data concerning evaporation of water in several western states, and develops a method of estimating monthly evaporation for the entire year from temperature measurements and other data.

Introduction

The storage of stream flow from mountain watersheds in reservoirs has made possible the development of much of the irrigated agriculture of the west. These reservoirs help to prevent floods, conserve a water supply that otherwise might be wasted, and make possible the production of power. The efficient design and later operation of a water-supply project is dependent on an awareness of the quantity of water that is lost through evaporation. There are few instances in which the evaporation rate can be measured directly from lakes or reservoirs because of the unknown elements of supply, such as losses of

---

Note.—Published, essentially as printed here, in November, 1956, in the Journal of the Irrigation and Drainage Division, as *Proceedings Paper 1104*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Prin. Irrig. Engr., Western Soil and Water Management Section, Soil and Water Conservation Research Branch, Agri. Research Service, U. S. Dept. of Agriculture, Los Angeles, Calif.

water entering or leaving the reservoir. Thus, research studies have been necessary to determine the relationships existing between evaporation of water from standard pans and climatological data, which are measurable, and that from lakes and reservoirs for which direct measurements are impossible. Most of the early studies on these relationships were made by the irrigation engineers of the United States Department of Agriculture in Colorado and California, and coefficients were developed for reducing pan records to values of lake evaporation.[2,3,4] From April, 1950, to August, 1951, a comprehensive interagency evaporation experiment was conducted by the United States Department of Commerce, the United States Department of the Interior, and the United States Department of the Navy at Lake Hefner (Okla.).[5] In 1955 the United States Weather Bureau reported the results of a study concerning evaporation of water from pans and lakes.[6]

Although evaporation-pan records at low elevations are available for many areas throughout the United States, data on the evaporation rate at high altitudes are less numerous, and measurements are limited to frost-free months because the water in the pans freezes during the winter. The evaporation losses during the winter months are needed for estimating the water supply available from reservoirs during the summer for irrigation and for municipal and power purposes. Since 1947 the irrigation engineers of the Department of Agriculture have conducted evaporation studies at high altitudes in the Huntington Lake (California) area in cooperation with an electric company. In these studies the evaporation from pans is correlated with temperature observations for the purpose of estimating the monthly loss of water due to evaporation for the entire year.

Data will be presented herein on evaporation of water from free water surfaces at high altitudes in California, Colorado, New Mexico, and Utah; a procedure for estimating the evaporation rate from climatological data is outlined for cases for which no pan records are available.

EFFECT OF ALTITUDE ON EVAPORATION

A review of the literature indicates that there is a difference of opinion as to the effect of altitude or barometric pressure on the evaporation rate of water. Field observations show that temperature decreases as altitude increases, and that the rate of evaporation is more closely related to temperature than to altitude. Direct observations of the effect of altitude on evaporation are difficult to obtain.

In 1905 the late Samuel Fortier, M. ASCE, and the Department of Agriculture, in cooperation with the state of California, conducted tests to determine the effect of altitude on the evaporation of water from a water surface by

---

[2] "Evaporation from the Surfaces of Water and Riverbed Materials," by R. B. Sleight, *Journal of Agricultural Research*, Vol. 10, No. 5, 1917.

[3] "Evaporation from Free Water Surfaces," by Carl Rohwer, *Technical Bulletin No. 271*, U. S. Dept. of Agriculture, Washington, D. C., 1931.

[4] "Evaporation from Water Surfaces in California," by Arthur A. Young, *Bulletin Nos. 54 and 54-A*, Div. of Water Resources, State of California, Sacramento, Calif., 1947 and 1948.

[5] "Water-Loss Investigations: Lake Hefner Studies," *Professional Paper No. 269*, Geological Survey, U. S. Dept. of the Interior, Vol. 1, 1954.

[6] "Evaporation from Pans and Lakes," by M. A. Kohler, T. J. Nordensen, and W. E. Fox, *Research Paper No. 38*, U. S. Weather Bureau, Washington, D. C., 1955.

measuring the depth of water that was vaporized from a series of pans whose diameters were 22 in. The pans were placed on the ground at different elevations on the eastern slope of Mt. Whitney (California).[7] The mean daily pan evaporation measurements and temperature recordings for a twenty-day period are shown in Table 1.

The results show that the decrease in evaporation was more closely related to a change in temperature than to one in barometric pressure. The close relationship of evaporation and temperatures is shown in Fig. 1. The rate of evaporation decreased uniformly from an elevation of 4,515 ft to an elevation of 8,370 ft, and then the rate decreased more rapidly to an elevation of 12,000 ft. At the summit of Mt. Whitney, the evaporation pan was exposed to winds from all directions, in contrast with those on the lower eastern slope, and, therefore, shows a slightly higher rate of evaporation.

At the Irrigation Field Laboratory established by the Department of Agriculture at Denver (Colo.) in 1915, research studies[3] were conducted on the variation in the degree of evaporation from different types of pans, and the results were extended to large water areas at an elevation of 5,200 ft. The coefficients for reducing pan evaporation measurements to lake evaporation values were determined[2,8] for sunken pans, which were 3 ft deep and whose diameters varied from 1 ft to 12 ft. These studies were interrupted in 1917 by World War I. The writer re-established the station in 1919 and continued the studies for one season as well as lysimeter studies on evapotranspiration by irrigated crops.[3]

TABLE 1.—EVAPORATION AND TEMPERATURES ON THE EAST SLOPE OF MT. WHITNEY, CALIFORNIA

Location	Elevation, in feet	Mean daily evaporation, in feet	Mean daily temperature, in degrees Fahrenheit
Soldiers Camp	4,515	0.223	82
Junction of South Fork and Lone Pine Creek	7,125	0.170	82
Hunters Camp	8,370	0.147	74
Lone Pine Lake	10,000	0.136	58
Mexican Camp	12,000	0.134	49
Summit of Mt. Whitney	14,502	0.140	48

Additional intensive studies of the evaporation problem were begun in 1923 by the Division of Irrigation of the Department of Agriculture, in cooperation with the Colorado Agricultural Experiment Station at Fort Collins, Colo. (El. 5000). In this investigation a copper-lined reservoir which was 85 ft in diameter was used instead of the 12-ft-diameter pan used in 1915. It was concluded that the evaporation of water from a sunken pan that had a diameter of 12 ft closely approximates the evaporation rate from a larger body of water.[3]

---

[7] "Evaporation Losses in Irrigation," by Samuel Fortier, *Engineering News*, Vol. 58, 1907, pp. 304–307.

[8] Discussion by R. B. Sleight of "Evaporation on United States Reclamation Projects," by Ivan E. Houk, *Transactions*, ASCE, Vol. 90, 1927.

In the period from 1928 to 1929, a study was made of the effect of altitude on evaporation by observations of daily evaporation from a pan and meteorological factors at different elevations: Imperial, Calif. (68 ft below sea level); Fort Calhoun, Nebr. (1,160 ft); Logan, Utah (4,778 ft); Fort Collins, (5,000 ft); Lake Tahoe, Calif. (6,300 ft); Victor, Colo. (10,089 ft); and Pikes Peak, Colo. (14,109 ft). A formula based on field observations was developed for computing evaporation for any altitude up to 15,000 ft above sea level, provided mean barometer readings and data on mean air and water temperatures, and mean vapor pressures and mean velocity of ground wind or water-surface wind in miles per hour are available. Detailed results of this study have been published,[3] and they indicate that for similar meteorological conditions there is a definite increase in evaporation as the altitude increases.

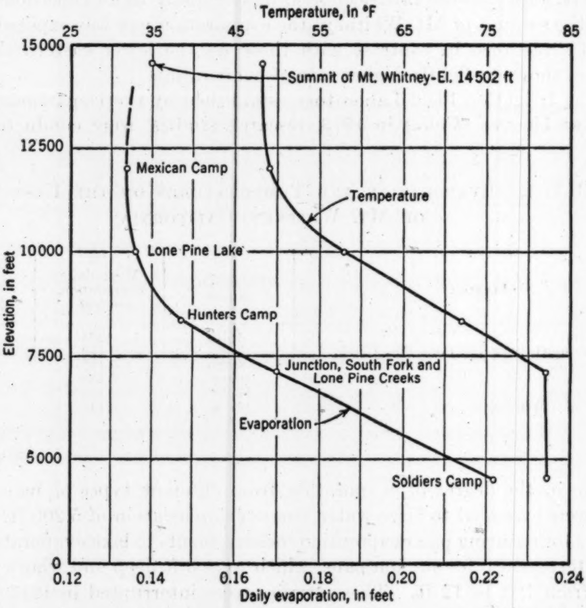


FIG. 1.—THE RELATIONSHIP BETWEEN THE ALTITUDE AND THE EVAPORATION ON THE EAST SLOPE OF MT. WHITNEY, CALIFORNIA

The results of preliminary analysis on evaporation at different elevations in California and Nevada are shown in Fig. 2. Fig. 2 also shows the mean annual evaporation from lake or reservoir surfaces at different altitudes in California and Nevada. Curve AA is a modification of a tentative graph based partly on data reported by Sidney T. Harding, M. ASCE.[9] For sites whose elevations were greater than 7,000 ft, the curve was extended to conform to the Mt. Whitney evaporation shown in Fig. 1. Curve B is the result of an analysis of approximately 60 Weather Bureau pan records and meteorological data in the southern California coastal area. In the coastal area of Los Angeles (Calif.) County the annual lake evaporation increases from about 36 in. at the coast to approximately 54 in. at 2,500 ft, as shown in Fig. 2. The annual lake evaporation begins decreasing from 54 in. at an elevation of 2,500 ft to 35 in. at 7,000 ft.

---

[9] "Evaporation from Large Water Surfaces Based on Records in California and Nevada," by S. T. Harding, *Transactions*, Am. Geophysical Union, 1935.

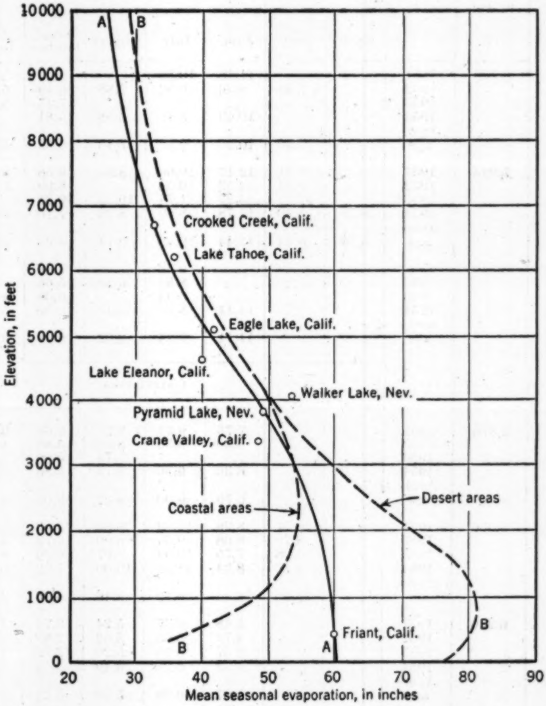


FIG. 2.—THE ESTIMATED MEAN ANNUAL EVAPORATION FROM LAKE SURFACES AT DIFFERENT ALTITUDES IN THE SIERRA NEVADA MOUNTAINS IN CALIFORNIA

EVAPORATION-PAN RECORDS

Four types of pans are used in high altitude sites—the Weather Bureau Class A pan, the Department of Agriculture screen pan, the Colorado land pan, and the Geological Survey (Department of the Interior) square floating pan. The usual coefficients for reducing evaporation from these pans to an equivalent evaporation from lakes or reservoirs are: The Weather Bureau pan, 0.70; the screen pan, 0.98; the Colorado land pan, 0.78; and the floating pan, 0.80.[3,4,10]

---

[10] "Standard Equipment for Evaporation Stations: Final Report of Subcommittee on Evaporation of the Special Committee on Irrigation Hydraulics," *Transactions*, ASCE, Vol. 99, 1934.

TABLE 2.—MONTHLY EVAPORATION FROM PANS AT HIGH ALTITUDES, UNITED STATES WEATHER BUREAU STATIONS

Location	Elevation, in feet	Year	EVAPORATION, WEATHER BUREAU PAN, IN INCHES							
			NEW MEXICO							
			April	May	June	July	August	September	October	November
El Vado Dam	6,750	1951	...	...	11.18	10.86	...	...	...	...
		1952	...	7.30	9.46	9.53	7.88	6.49	5.20	...
		1953	...	...	...	...	...	7.47	5.40	...
		1954	...	...	10.12	8.17	6.98	5.81	...	...
		Average	...	7.30	10.25	9.52	7.43	6.59	5.30	...
Santa Fe	7,045	1951	...	9.91	12.17	10.66	8.63	9.68	5.74	2.58
		1952	...	9.34	12.52	10.58	...	8.99	7.37	...
		1953	...	10.51	13.27	11.74	10.55	10.33	...	...
		1954	8.70	8.40	11.78	8.69	8.23	7.10	5.53	...
		Average	8.70	9.54	12.44	10.42	9.14	9.03	6.21	2.58
Eagle Nest	8,240	1951	...	...	...	9.32	...	7.46	...	...
		1952	...	...	...	8.84	6.59	5.66	...	...
		1953	...	...	...	6.98	...	6.03	4.30	...
		1954	...	...	11.11	5.82	5.45	6.00	5.24	...
		Average	...	...	11.11	7.74	6.02	6.29	4.77	...
			CALIFORNIA							
Shaver Lake[a]	5,376	1951	...	...	7.78	9.31	8.73	6.60	3.76	...
		1952	...	...	6.30[c]	8.33	8.69	5.80	4.40	...
		1953	...	...	...	...	7.73	5.96	...	...
		1954	...	...	6.30	9.36	8.13	5.88	3.72	...
		Average	...	...	6.79	9.00	8.32	6.06	3.96	...
Boca[b]	5,536	1951	...	6.77	8.98	11.12	8.86	7.03	3.57	...
		1952	...	8.79	8.68	9.54	9.90	6.72	4.67	...
		1953	...	5.08	7.75	10.94	9.07	6.36	3.91	...
		1954	...	8.76	8.34	11.55	10.00	7.72	6.11	...
		Average	...	7.35	8.44	10.79	9.46	6.96	4.57	...
Lake Tahoe[b]	6,230	1951	...	...	5.48	6.37	5.24	3.71	1.23[c]	...
		1952	...	...	4.71	5.65	5.62	2.89	1.37	...
		1953	...	...	4.57	5.95	5.20	2.78	1.20[c]	...
		1954	...	...	4.72	6.26	5.49	3.44	1.74[c]	...
		Average	...	...	4.87	6.06	5.39	3.21	1.39	...
Huntington Lake[a]	6,954	1951	...	...	6.91	8.05	7.80	6.15	4.05[c]	...
		1952	...	...	...	6.89[c]	7.28	4.72[c]	...	...
		1953	...	...	6.09[c]	8.25	6.86	5.65	...	...
		1954	...	...	6.14	8.42	7.09	5.54	3.94	...
		Average	...	...	6.38	7.90	7.26	5.52	4.00	...
Florence Lake[a]	7,345	1951	...	...	7.46	8.54	8.13	7.05	4.90[c]	...
		1952	...	...	...	7.05[c]	8.30	5.17	4.97[c]	...
		1953	...	...	...	8.04	8.15	6.19	4.87[c]	...
		1954	...	...	8.12	8.71	8.68	6.69	5.43[c]	...
		Average	...	...	7.79	8.09	8.32	6.28	5.04	...

[a] Records of the Agricultural Research Service in cooperation with an electric company. [b] Weather records. [c] All equipment moved 2 miles south-southwest on July 14, 1952.

(TABLE 2.—*Continued*)

Location	Elevation, in feet	Year	EVAPORATION, WEATHER BUREAU PAN, IN INCHES							
			UTAH							
			April	May	June	July	August	September	October	November
Greenriver Airway	4,063	1951	...	...	9.60	10.46	7.81	5.77	3.39	...
		1952	...	8.13	9.11	8.60	7.08	5.60	3.90	...
		1953	...	7.73	10.58	8.81	7.41	6.05	3.24	...
		1954	...	7.96	8.53	9.46	9.08	5.20	3.51	...
		Average	...	7.94	9.46	9.33	7.85	5.66	3.51	...
Utah Lake Lehi	4,497	1951	5.64	7.49	8.69	10.38	8.86	7.12	3.20	...
		1952	5.30	8.65	10.46	9.80	9.15	7.59	4.56	...
		1953	5.11	6.50	10.24	10.32	9.20	6.47	3.97	...
		1954	7.21	8.60	9.19	10.97	10.22	7.02	3.79	...
		Average	5.82	7.81	9.65	10.37	9.36	7.05	3.88	...
Logan	4,608	1951	5.65	6.56	7.28	8.61	7.86	6.36	2.48	...
		1952	...	7.29	9.03	9.19	8.17	6.32	3.88	...
		1953	3.57	4.53	7.70	8.77	9.53	5.93	3.50	...
		1954	5.39	7.13	6.56	9.61	9.34	6.11	3.22	...
		Average	4.87	6.38	7.64	9.05	8.73	6.18	3.27	...
East Porta	7,608	1951	...	6.49	6.42	8.00	5.52	6.12	2.40	...
		1952	...	...	7.79	6.96	7.06	5.51	4.30	...
		1953	...	...	9.44	8.18	6.68	5.87	2.73	...
		1954	...	...	6.98	8.07	7.53	5.17	...	...
		Average	...	6.49	7.65	7.80	6.70	5.67	3.14	...
			COLORADO							
Fort Collins	5,004	1951	3.67	4.88	4.92	7.03	5.09	5.06	3.30	...
		1952	3.83	4.60	7.83	7.05	6.06	5.46	3.42	...
		1953	3.17	5.53	6.73	6.96	6.68	6.61	3.92	...
		1954	6.48	5.69	8.35	8.95	7.20	5.63	3.58	...
		Average	4.29	5.18	6.96	7.50	6.26	5.69	3.56	...
Estes Park[d]	7,525	1951	...	7.98	6.38	9.10	7.25	7.19	3.98	...
		1952	...	6.32	10.30	8.61	6.26	5.50	3.62	...
		1953	...	4.79	8.58	7.64	5.85	6.29	3.53	...
		1954	5.54	5.32	8.78	7.99	6.73	5.19	4.88	...
		Average	5.54	6.10	8.51	8.34	6.52	6.04	4.00	...
Grand Lake[d]	8,389	1951	...	4.73	5.27	7.76	5.78	5.20	1.63	...
	8,288[e]	1952	...	4.64	7.99	8.76	5.94	6.01	...	...
		1953	...	...	8.54	8.43	6.13	7.01	3.65	...
		1954	...	...	9.05	8.60	8.53	5.87	...	...
		Average	...	4.69	7.71	8.39	6.60	6.02	2.64	...
Wagon Wheel Gap	8,500	1951	...	...	9.13	8.48	6.08	6.67	...	...
		1952	...	7.22	9.42	7.00	5.35	4.90	...	...
		1953	...	...	9.07	6.91	6.39	6.64	...	...
		1954	...	...	10.20	6.32	6.58	4.66	...	...
		Average	...	7.22	9.46	7.18	6.10	5.72	...	...
Platoro Dam[d]	9,826	1951	...	...	9.07	8.34	6.81	6.95	...	...
		1952	...	...	8.96	7.05	4.98	4.95	4.17	...
		1953	...	5.71	8.64	6.23	6.10	6.44	3.29	...
		1954	...	6.05	9.47	5.35	6.27	5.58	4.21	...
		Average	...	5.88	9.04	6.74	6.04	5.98	3.89	...

Bureau records. [e] One or more days estimated as average of measured period. [d] Bureau of Reclamation

The most commonly used pan for measuring the evaporation rate from water surfaces is the Weather Bureau Class A pan. It is 4 ft in diameter, 10 in. deep, and is set on a 6-in. wooden grillage in order to raise the water surface slightly more than 1 ft above the ground level. The coefficient ranges from 0.60 for arid areas to 0.80 for humid climates, but a value of 0.70 is usually used to reduce the pan records at high elevations to the equivalent reservoir or lake evaporation values.[3,10,11]

Observations of evaporation from the Weather Bureau pans were initiated just prior to 1905. This type was used in 1956 throughout the United States and Mexico and in other parts of the world. In the western United States, measurements are made by the Agricultural Research Service, the Bureau of Reclamation (Department of the Interior), the Weather Bureau, cities, and the state agricultural experiment stations in cooperation with other agencies, with some of the results being published by the Weather Bureau. The results of a few monthly measurements at high altitudes in California, Colorado, New Mexico, and Utah are shown in Table 2.

California Study in Huntington Lake Area

In 1946 studies of the evaporation rate at high altitudes in lakes in the Sierra Nevada Mountains near Fresno (Calif.) were begun by the division of Irrigation and Water Conservation, the Soil Conservation Service, and the Department of Agriculture in cooperation with an electric company. The studies became the responsibility of the Western Soil and Water Management Section of the Agricultural Research Service on January 1, 1954. Class A Weather Bureau evaporation stations equipped with Young screen pans were installed at Shaver Lake (El. 5376), Huntington Lake (El. 6954), Florence Lake (El. 7345), and Kaiser Pass (El. 9194) in the Upper San Joaquin River watershed. The water from the lakes, after being used several times for developing electric power, flows into a storage reservoir (Millerton Lake) created by Friant Dam, which was built by the Bureau of Reclamation and is available for irrigation by the farmers in San Joaquin Valley. The evaporation of water from a Class A Weather Bureau pan and the temperatures have been measured by the Bureau of Reclamation at Friant, Calif. (El. 380), for many years.

The purpose of the investigation was to determine the monthly and the annual evaporation losses from lakes at altitudes ranging from 4,500 ft to 9,200 ft. As indicated in Table 2, it is not possible to measure the evaporation of water from pans during the winter because of the freezing of water in the pans at these elevations. Thus, the problem of determining the quantity of evaporation during this period is difficult.

However, by correlating the measured monthly quantity of water loss due to evaporation, e, with the mean monthly temperatures, t, and the percentage of daytime hours, p, for a five-month period from June to October at Huntington Lake, the evaporation for other months may be estimated, as shown in Table 3 and Fig. 3. This procedure is a modification of a method developed by the

---

[11] "Evaporation from Water Surfaces in California," by Harry F. Blaney and Gilbert L. Corey, *Bulletin No. 54-B*, Div. of Water Resources, State of California, Sacramento, Calif., 1955.

writer during the Pecos River Joint Investigation during 1940 and 1941 from evaporation records and related meteorological data at stations in New Mexico and Texas.[12,13] By multiplying the mean monthly temperature by the monthly percentage of daytime hours of the year, a monthly water-use factor, f, is obtained. It is assumed that the monthly evaporation losses vary directly as the factor, f. Therefore, $e = k f$, in which $f = t\,p/100$ and k is the monthly empirical coefficient computed from the measured evaporation losses and temperature and from the percentage of daytime hours. The relationship between the mean monthly evaporation and the water-use factor for the period from 1947 through 1954 at Huntington Lake is shown in Fig. 3 and is based on data given in Table 3. An analysis of the data at stations in this area and for stations at lower elevations shows that the curve in Fig. 3 may be plotted as a straight line for the period from April to October, and that for the winter months the curve will be an approximate straight line at a different slope for monthly pan evaporation of 1 in. or more. The monthly evaporation rate for the period from November to May is estimated from Fig. 3 as shown in Table 3.

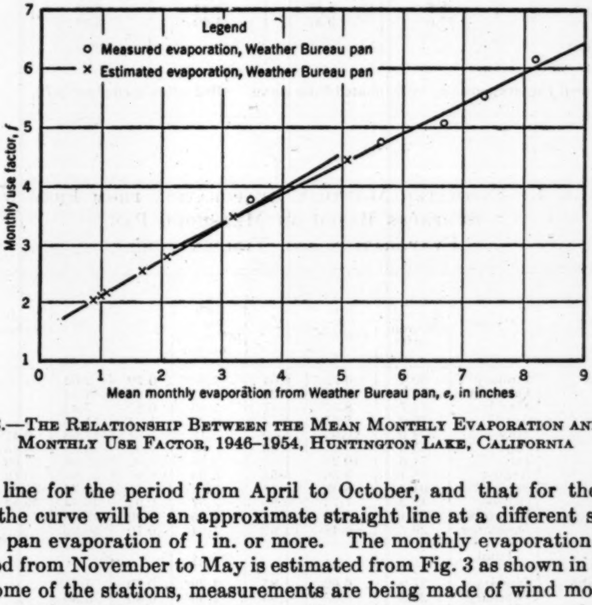


FIG. 3.—THE RELATIONSHIP BETWEEN THE MEAN MONTHLY EVAPORATION AND THE MONTHLY USE FACTOR, 1946–1954, HUNTINGTON LAKE, CALIFORNIA

At some of the stations, measurements are being made of wind movement, humidity, and water temperatures as well as of pan evaporation and air temperatures. Analyses of these data have not been completed.

ESTIMATING EVAPORATION AT HIGH ALTITUDES

A procedure similar to that described for Huntington Lake can be used to estimate evaporation during winter months when only evaporation and tem-

---

[12] "Consumptive Water Use and Requirements," by Harry F. Blaney, Paul A. Ewing, Karl V. Morin, and Wayne D. Criddle, Pt. III, Section 3, *Pecos River Joint Investigations*, National Resources Planning Board, Washington, D. C., 1942.

[13] "Consumptive Use of Water—Definitions, Methods, and Research Data," by Harry F. Blaney, *Transactions*, ASCE, Vol. 117, 1952.

TABLE 3.—MEAN MONTHLY OBSERVED AND ESTIMATED EVAPORATION FROM WEATHER BUREAU PAN AND ESTIMATED LAKE EVAPORATION, FROM 1947 TO 1954, HUNTINGTON LAKE, CALIFORNIA

Month	t, in degrees Fahrenheit	p, in percentage	f	EVAPORATION, IN INCHES	
				Pan	Lake[c]
June	51.5	9.89	5.09	6.67[a]	4.67
July	60.8	10.05	6.11	8.18[a]	5.73
August	58.3	9.44	5.50	7.33[a]	5.13
September	56.5	8.37	4.73	5.61[a]	3.93
October	48.0	7.82	3.75	3.62[a]	2.53
November	40.4	6.87	2.77	2.1[b]	1.46
December	32.6	6.72	2.19	1.1[b]	0.78
January	29.5	6.93	2.04	0.9[b]	0.63
February	30.9	6.82	2.11	1.0[b]	0.71
March	30.0	8.35	2.51	1.7[b]	1.19
April	38.8	8.87	3.44	3.2[b]	2.24
May	44.6	9.87	4.40	5.1[b]	3.56
Total				46.51	32.56

[a] Measured pan evaporation. [b] Estimated from curve. [c] Reduction coefficient, 0.70.

TABLE 4.—ESTIMATED MONTHLY EVAPORATION FROM FREE WATER SURFACES BASED ON MEASURED PAN EVAPORATION AND TEMPERATURE

Location	Month	t, in degrees Fahrenheit[a]	p, in percentage	f	e (pan evaporation, in inches)	k	Σf	Σe, in inches	Lake evaporation, in inches[b]
Fort Collins (Colo.), El. 5004	January	30.4	6.72	2.04	1.43[c]	0.70	2.04	...	1.00
	February	34.0	6.71	2.28	1.82[c]	0.80	4.32	...	1.27
	March	34.7	8.32	3.11	2.48[c]	0.80	7.43	...	1.74
	April	46.0	8.97	4.13	4.29	1.04	11.56	4.29	3.00
	May	54.9	10.05	5.52	5.18	0.94	17.08	9.47	3.63
	June	65.6	10.11	6.63	6.96	1.05	23.71	16.43	4.87
	July	71.6	10.26	7.35	7.50	1.02	31.06	23.93	5.25
	August	68.4	9.56	6.54	6.28	0.96	37.60	30.19	4.40
	September	61.3	8.39	5.14	5.71	1.11	42.74	35.88	4.00
	October	49.2	7.73	3.80	3.57	0.94	46.54	39.44	2.50
	November	36.4	6.70	2.44	1.95[c]	0.80	48.98	...	1.36
	December	29.7	6.48	1.92	1.34[c]	0.70	50.90	...	0.94
	Total				48.51				33.96
Logan (Utah), El. 4608	January	27.9	6.62	1.85	1.29[c]	0.70	1.85	...	0.90
	February	29.3	6.65	1.95	1.56[c]	0.80	3.80	...	1.09
	March	34.7	8.31	2.88	2.30[c]	0.80	6.68	...	1.61
	April	47.5	9.00	4.28	4.87	1.14	10.96	4.87	3.41
	May	54.3	10.14	5.51	6.39	1.16	16.47	11.25	4.47
	June	61.5	10.21	6.28	7.66	1.22	22.75	18.89	5.36
	July	71.1	10.35	7.36	9.05	1.23	30.11	27.94	6.33
	August	69.6	9.62	6.70	8.71	1.30	36.81	36.67	6.10
	September	61.7	8.40	5.18	6.16	1.19	41.99	42.85	4.31
	October	50.1	7.70	3.86	3.28	0.85	[illegible]	46.12	2.30
	November	37.1	6.62	2.46	1.97[c]	0.80	[illegible]	...	1.38
	December	26.4	6.38	1.68	1.18[c]	0.70	[illegible]	...	0.83
	Total				54.42				38.09

[a] t = mean monthly temperatures, from 1951 through 1954. [b] Estimated by reduction factor, 0.70. [c] Estimated value of $k\ f$.

perature records are available for summer months. Another method is to determine values of k for stations for which winter records are available and then compute monthly winter evaporation by the formula, $e = kf$. This method is illustrated in Table 4 for stations at Fort Collins and Logan, and in Fig. 4 for Fort Collins. The lower curve in Fig. 4, plotted from data given in Table 4, shows the straight-line relationship between monthly pan evaporation (e) and the use factor (f) at Fort Collins during the period from April 1 to November 1. The upper curve shows the estimated accumulated evaporation by months for the entire year.

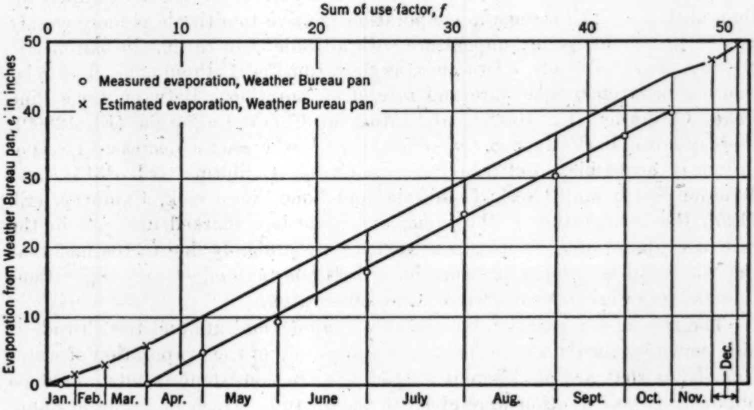


FIG. 4.—ESTIMATES OF THE CUMULATIVE MONTHLY WINTER EVAPORATION AT FORT COLLINS, COLO.

DISCUSSION

Carl Rohwer,[14] M. ASCE.—Attempts to correlate the rate of evaporation with altitude usually lead to the conclusion that other factors are more important than altitude. The change in evaporation relative to altitude is more closely related to the change in temperature with altitude than to that in barometric pressure due to altitude. However, as shown in Fig. 1, there is no close relationship between temperature and rate of evaporation. Between Lone Pine Lake, California (El. 10000), and Mexican Camp, California (El. 12000), there is a drop in the average temperature of 9°, whereas the decrease in evaporation is negligible. Between Soldiers Camp, California (El. 4515), and Junction South Fork Creek, California, and Lone Pine Creek, California (El. 7125), the temperature is the same, but there is a marked decrease in the rate of evaporation. These inconsistencies are probably due to the fact that the wind and the vapor pressure have an important effect on evaporation; these factors were not considered in the observations.

The lack of a correlation between the evaporation rate and the altitude is also shown by the data in Table 2. For the study of the evaporation of water from lakes that are at different altitudes, more consistent results would be obtained if less attention were given to the altitude, which probably has only a minor effect, and if more were given to the factors that are directly related to evaporation, such as wind velocity and difference in vapor pressure between the air and the water surface.

It is not intended to imply that changes in barometric pressure have no effect on evaporation. It is a well-known fact that the rate of evaporation is increased when the pressure is reduced. This principle is used to increase the evaporation in industrial processes that require the removal of excess liquids from products. The converse should also be true—increasing the pressure should decrease the evaporation. However, it has been difficult to apply this principle to the study of the effect of altitude on evaporation because the difference in vapor pressure, as well as the difference in wind velocity, cannot cannot be controlled in field experiments.

In 1927 and 1928 the writer conducted tests[15] on the effect of altitude on the rate of evaporation at various locations in the United States at elevations ranging from 68 ft below sea level to 14,109 ft above sea level. These tests show that when the effect of differences in wind velocity and vapor pressure was eliminated, the evaporation rate increased approximately 1.5% per 1,000-ft rise in elevation. This effect is so small that it may be ignored unless the difference in elevation between the two sites being studied is more than 5,000 ft because all evaporation observations are subject to unavoidable errors that may completely overshadow the effect of the differences in altitude.

Because evaporation observations at high altitudes are usually limited to the ice-free months, a method must be devised for making reasonable estimates of the evaporation when the pan is covered with ice. The author has shown how this can be done, and the results are given in Table 3 and Table 4. Data are available for evaluating this method for conditions at Fort Collins (El. 5004). During the period from 1887 to 1927, observations of evaporation were made on a Colorado buried pan, which had dimensions of 3 ft by 3 ft by 3 ft. The pan was set into the ground so that approximately 4 in. of the tank were above it. Observations of the evaporation rate were made during the winter at times when the ice became free from the walls of the tank. The results of these observations are shown in Table 5, Col. 8, together with the data reported by the author for the period from 1947 to 1954. In order to make a comparison possible, the Colorado pan evaporation data were converted to the equivalent Class A pan data (Col. 9) by using a conversion factor of 1.13. The agreement of the value computed by the author for January through December with the observed values (Col. 5 and Col. 9) is apparent. The lake evaporation values computed by the author and those computed by the writer, who used the Colorado pan data (Col. 7 and Col. 10), are also in agreement. Because the records are for different periods, a precise agreement should not be expected.

---

[14] Senior Irrig. Engr. (retired), Agri. Research Service, U. S. Dept. of Agriculture, Colorado State Univ., Fort Collins, Colo.

[15] "Evaporation from Free Water Surfaces," by Carl Rohwer, *Technical Bulletin No. 271*, U. S. Dept. of Agriculture, Washington, D. C., 1931.

TABLE 5.—COMPARISON OF THE ESTIMATED MONTHLY EVAPORATION WITH THE OBSERVED MONTHLY EVAPORATION AT FORT COLLINS, COLO.

Month	t, in degrees Fahrenheit[a]	p, in percentage	f	e, in inches (Class A)	k	Lake evaporation,[b] in inches	Colorado buried pan observed evaporation,[d] in inches	Class A computed evaporation,[e] in inches	Lake evaporation,[f] in inches
1	2	3	4	5	6	7	8	9	10
January	30.4	6.72	2.04	1.43[c]	0.70	1.00	1.41	1.59	1.10
February	34.0	6.71	2.28	1.82[c]	0.80	1.27	1.56	1.76	1.22
March	34.7	8.32	3.11	2.48[c]	0.80	1.74	2.62	2.96	2.04
April	46.0	8.97	4.13	4.29	1.04	3.00	4.27	4.84	3.33
May	54.9	10.05	5.52	5.18	0.94	3.63	4.98	5.64	3.88
June	65.6	10.11	6.63	6.96	1.05	4.87	5.60	6.34	4.36
July	71.6	10.26	7.35	7.50	1.02	5.25	5.79	6.55	4.51
August	68.4	9.56	6.54	6.28	0.96	4.40	5.36	6.07	4.19
September	61.3	8.39	5.14	5.71	1.11	4.00	4.41	5.00	3.44
October	49.2	7.73	3.80	3.57	0.94	2.50	3.28	3.71	2.56
November	36.4	6.70	2.44	1.95[c]	0.80	1.36	1.61	1.82	1.26
December	29.7	6.48	1.92	1.34[c]	0.70	0.94	1.30	1.47	1.01
Total				48.51		33.96	42.19	47.75	32.90

[a] Mean monthly temperatures, from 1951 to 1954 inclusive. [b] Estimated by reduction factor, 0.70. [c] Estimated kf. [d] From 1887 to 1927. [e] Conversion factor, 1.13. [f] Equivalent lake evaporation conversion factor, 0.78, for Colorado buried pan.

KENNETH M. TURNER,[16] J. M. ASCE.—A rational method for extending the seasonal pan records in order to obtain an estimate of the annual evaporation has been presented.

Parameters that are commonly used in the evaporation formulas are temperature, air movement, humidity, vapor pressure, solar radiation, and elevation. Only the temperature and air movement are measured at most of the evaporation-pan installations. When the pan record is discontinued in the fall, at the high-altitude locations the anemometer is shifted from its position at the pan, which is less than 2 ft above the ground, to a position that is several feet high.

---

[16] Associate Hydr. Engr., California Dept. of Water Resources, Sacramento, Calif.

The only parameter that is measured under uniform conditions for a full year is the temperature. Thus, the engineer is limited to the correlation of monthly evaporation with temperature measurements for the extension of individual pan records, and the correlation of reservoir evaporation with elevation at several stations to adjust the pan records to equivalent values for other locations.

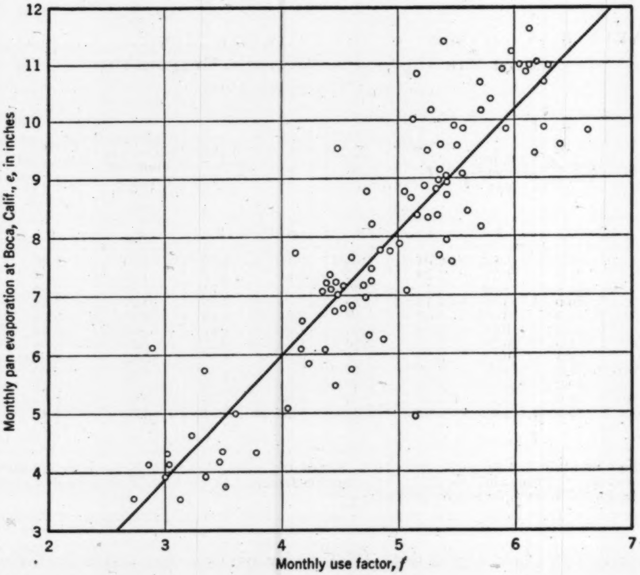


FIG. 5.—RELATIONSHIP BETWEEN THE MONTHLY PAN EVAPORATION AND THE MONTHLY USE FACTOR, FROM 1940 TO 1956, BOCA, CALIF.

The monthly mean temperature, as reported by the Weather Bureau, is the average of the recorded daily maximum temperatures and the daily minimum temperatures. Therefore, it can be described better as the average monthly median temperature. This factor must be adjusted in order to relate it to actual heat units. The author has done this by multiplying the average monthly temperature by the percentage of annual daylight hours and then by dividing by 100. This quantity, $t\,p/100$, has been termed the "monthly use factor" by the author.

The writer is engaged in a study of the hydrology of the Truckee River Basin, the Carson River Basin, and the Walker River Basin. (These rivers originate in California on the eastern slope of the Sierra Nevada Mountains and terminate in the Nevada desert.) In this study the monthly use factor has been used successfully for both the extension of the seasonal pan records and their adjustment to uniform conditions by double mass plotting. Some examples of the relationships are shown in Fig. 5 and Fig. 6.

Fig. 5, showing the correlation of monthly pan evaporation with the monthly use factor at Boca (Calif.), demonstrates that although there is a wide scattering of points there is a definite trend that may be reduced to an equation, $y = a + b x$, by the method of least squares The departure from the curve of ± 30% of the measured monthly-evaporation values would limit the use of the monthly use factors in estimating evaporation to the locations for which the summer pan evaporation rate has been recorded. At such sites the missing months represent such a small proportion of the annual evaporation that the effect of the 30% scattering is minimized.

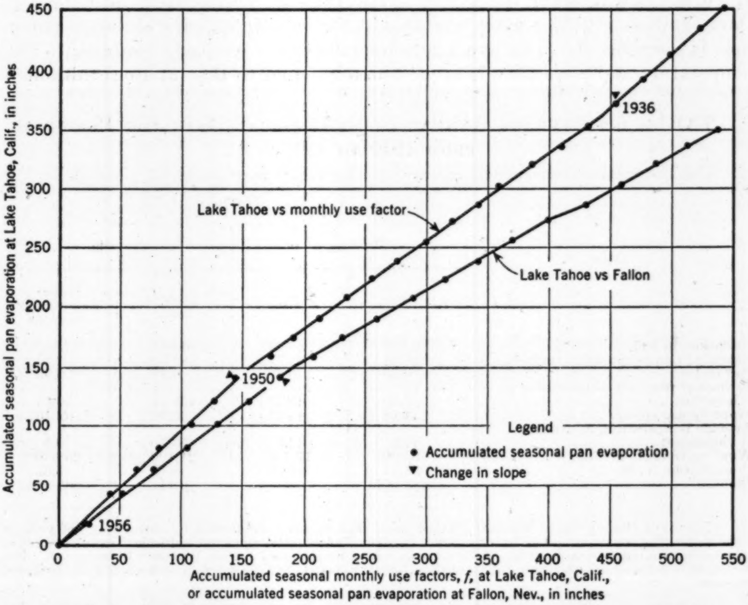


FIG. 6.—DOUBLE MASS DIAGRAM OF JUNE–SEPTEMBER PAN EVAPORATION AT LAKE TAHOE VERSUS ACCUMULATED MONTHLY USE FACTORS FOR SEASON AND JUNE–SEPTEMBER PAN EVAPORATION AT FALLON, NEV.

The evaporation and the wind-movement records are sensitive to changes in location and to exposure and changes in equipment. Thus, if it is necessary to use long-term records, it may be necessary to reduce them to a common basis by double mass plotting. Because evaporation stations are few and widely spaced, it is not always practical to plot the information against another station. In the Truckee River Basin, it was possible to plot the recorded accumulated seasonal evaporation against the accumulated monthly use factors for the same season. This method is considered valid because a minor change in location or exposure would have little effect on temperatures recorded in a standard Weather Bureau shelter. A double mass plot of the accumulated pan evaporation versus the accumulated monthly use factors at Lake Tahoe is shown in Fig. 6.

Previous investigations have indicated that both evaporation and temperature are inversely proportional to elevation. Table 6 demonstrates that this relationship is changed by other important factors. The reservoir evaporation at Boca is shown to be approximately equal to that at Fallon, Nev., which is 1,570 ft lower than Boca. The reservoir evaporation at Lahontan Dam, Nevada, is 26% greater than that at Fallon, although the elevation difference is only 235 ft. The average temperatures are similarly confused. The temperature at Tahoe City is approximately equal to that at Boca, although there is an elevation difference of 695 ft. The average temperature at Lahontan Dam is greater than that at Fallon. The only variable that explains this confusion is the humidity. The Boca and Lahontan stations are located on dry, sagebrush-covered hillsides. The stations at Tahoe and Fallon are located in relatively humid conditions created by a 193-sq-mile lake and 70,000 acres of irrigated land, respectively.

TABLE 6.—AVERAGE ANNUAL CLIMATOLOGICAL DATA FOR PERIOD FROM 1940 TO 1947

Data	Station			
	California		Nevada	
	Lake Tahoe[a]	Boca[b]	Lahontan Dam[c]	Fallon Experiment Station[d]
Elevation, in feet	6,230	5,535	4,200	3,965
Extended pan evaporation, in inches	33.98	61.71	82.04	52.25
Pan factor	0.80	0.80	0.83	0.95
Estimated reservoir evaporation, in inches	27.19	49.37	68.18	49.64
Monthly use factor	44.14	43.53	55.86	52.46
Average temperature, in degrees Fahrenheit	42.5	42.8	53.7	50.6
Wind movement, in miles	10,420	17,440	21,100	23,930
Precipitation, in inches	31.24	21.66	5.12	5.82
Climate	semihumid	arid	arid	semihumid

[a] Standard United States Weather Bureau pan. Pan factor is computed by correlating net yield of Lake Tahoe Basin with runoff of Truckee River between Tahoe outlet and the California-Nevada state line. [b] Standard United States Weather Bureau pan. [c] Standard United States Weather Bureau pan. Pan factor is computed by reservoir inflow-outflow measurements. [d] Ground pan surrounded by irrigated pasture.

A relationship should be established between the monthly use factor and the evaporation. However, the inspection of available records indicates that any empirical evaporation formula should include humidity and air movement as parameters as well as temperature. Such a formula should be developed in order to utilize the temperature, humidity, and wind records which are collected by the Weather Bureau in connection with aircraft flight weather and forest-fire, weather-forecasting programs.

The development of an empirical formula for computing reservoir evaporation from commonly measured climatological data will require additional study.

It is suggested that the reservoirs of the Lahontan basin offer an excellent opportunity to study the evaporation in relation to pan records and parameters such as temperature, humidity, elevation, and wind movement. Boca Reservoir (on the Little Truckee River), Lahontan Reservoir (on the Carson River), Topaz Lake Reservoir (on the West Walker River), and Weber Reservoir (on the Walker River) are at different elevations in an area of low precipitation and would be subject to reasonably accurate inflow-outflow measurements.

How the monthly use factors, as proposed by the author, may be used are summarized as follows:

1. To estimate data for the months for which pan records are not available where evaporation pans are operated during the irrigation season;
2. To reduce a pan evaporation record to uniform conditions by double mass plotting in cases for which the records have been affected by changes in the type of equipment of minor changes in location; and
3. To compute the evaporation rate where pan evaporation is not measured.

However, other parameters, such as humidity and air movement, should be considered in order that reliable results may be obtained. Inclusion of these parameters will necessitate further research.

IRVIN M. INGERSON,[17] M. ASCE, AND JOHN W. SHANNON.[18]—The measurement of evaporation from free water surfaces is a difficult problem, and, as the author has indicated, particularly so for sites at high altitudes. The use of pans, or of monthly mean temperatures, provides a means of estimating evaporation from the water surfaces. Whether or not the monthly mean temperatures multiplied by the percentage of daylight hours adequately integrate all the parameters affecting evaporation remains an interesting question.

The writers believe that the size and shape of the lake or reservoir have a marked effect on the evaporation rate. The exposure and the aspect of the water surface in relation to the prevailing wind direction, the vapor pressure deficit of the air mass, and the turbulence of the wind movements are also dominant factors in determining the rates of water-surface evaporation. Unquestionably, the proximity and direction of the locations of the pan or the temperature recorder in relation to the water surface would affect the correlation between water-surface and pan evaporation rates.

Studies[19] made in 1931 on Big Sage Reservoir in Modoc County near Alturas (Calif.) shows a definite relationship of the rate of evaporation to the size and shape of the water surface.

It seems logical to state that the distance over water surfaces across which air must pass influences the vapor pressure deficit of that air and, in time, decreases its absorptive powers.

---

[17] Prin. Hydr. Engr., Dept. of Water Resources, Sacramento, Calif.

[18] Land and Water Use Specialist, Dept. of Water Resources, Sacramento, Calif.

[19] "Supervision of Diversions from Pit River and Rattlesnake Creek in Hot Springs Valley, Modoc County, California, Season of 1931," by Irvin M. Ingerson, Div. of Water Resources, Dept. of Public Works, Sacramento, Calif., 1932.

It has been shown that too many experimental data and observational data have been collected in and around relatively small water-surface areas. Therefore, the results have too often been assumed to indicate that the surface area is not a substantial parameter. The results obtained from Big Sage Reservoir show that water-surface areas that are wide and expansive introduce a heretofore unused parameter that involves the average radius or the width of the water surface as measured in relation to the prevailing wind direction.

It is further noted that if the ordinarily used parameters affecting water-surface evaporation are projected into a determination of the evaporation from great bodies of water, such as the oceans, the results would indicate such extraordinary quantities and such a deluge of precipitation over the land areas that the validity of the values of the parameters used could be questioned. Therefore, in the cases of larger and more expansive reservoirs which are being developed in California, further investigations and observations should be made.

It is suggested that the reservoirs in the Lahontan area, such as Big Sage Reservoir, which lie on relatively impermeable beds, and the reservoirs or lakes in the High Sierra, which lie on solid rock in glacial cirques, provide opportunities to obtain accurate inflow-outflow measurements for determining accurate reservoir-evaporation losses.

It is believed that until such a comprehensive program of full-scale measurement of actual reservoir losses is made, determining such losses, particularly at high altitudes, will not be possible.

HARRY F. BLANEY,[20] M. ASCE.—The discussers have emphasized certain facts that have added to the limited literature available. The writer agrees with Mr. Rohwer's conclusion that other factors influencing evaporation are usually more important than altitude. In estimating the evaporation, more attention should be given to factors such as wind velocity, water temperature, and vapor pressure if these data are available. The computed annual evaporation of 32.90 in. at Fort Collins (shown in Table 5), which was based on the Colorado land pan record, is remarkably close to the 33.96-in. record computed in Table 4. This agreement indicates that the formula, $e = k f$, yields reasonable results. Corrections for the wind movement are minor when the average velocity does not exceed 3 miles per hr. The evaporation rate determined from the Weather Bureau type of pan and the air temperatures are influenced by relative humidity and apparently counterbalance the humidity factor in the formula for semiarid climates.

In Table 6 and in Fig. 5 and Fig. 6, Mr. Turner has used the monthly use factor in the writer's formula to good advantage. However, the term, "monthly use factor," shown in Table 6 should be changed to "sum of monthly use factors for the year" because the factors given are annual use factors rather than monthly use factors. The pan factor of 0.80 for the Weather Bureau pan appears high for an arid climate; a factor of 0.70 is usually used. A study made by the writer at Silver Lake, California, indicates that a factor of 0.60 should be used in converting the evaporation from a Weather Bureau pan to a reservoir-evaporation value in arid climates.[21] Mr. Turner indicates that the humidity is the only variable, which explains the confusion between the semihumid and arid evaporation records shown in Table 6. The annual wind movement varies from 10,420 miles at Lake Tahoe to 23,930 miles at Fallon. The annual wind movement should be considered in computing the reservoir-evaporation rate. The evaporation rate determined from a Weather Bureau pan temperature records automatically takes care of some of the differences in humidity. Fig. 6 shows a good correlation of the monthly use factor and the pan and evaporation at Lake Tahoe and Fallon. The writer found a similar relationship for Huntington Lake. Mr. Turner indicates that the formula, $e = k\,t\,p\,/\,100$, can be used to estimate the evaporation for the months for which pan records are not available. The method was used many times by the writer when monthly records were unavailable and when the annual evaporation rate was still needed. He does not agree with Mr. Turner's statement, " *** that any empirical evaporation formula would have to include humidity and air movement as parameters as well as temperature," when applied to semiarid climates. Mr. Rohwer has checked the writer's computation of the annual evaporation at Fort Collins with a difference of only 1.06 in. The ratio between evaporation and temperature at one station can be used to estimate the annual evaporation for another station having a similar climate. The San Francisco Bay investigation[22] has demonstrated that the formula, $u = k\,f$, can be used to determine an accurate estimate of the evaporation. Several good formulas have been developed[4,7] that include wind movement, humidity, and water temperature

---

[20] Prin. Irrig. Engr., Western Soil and Water Management Section, Soil and Water Conservation Research Branch, Agri. Research Service, U. S. Dept. of Agriculture, Los Angeles, Calif.

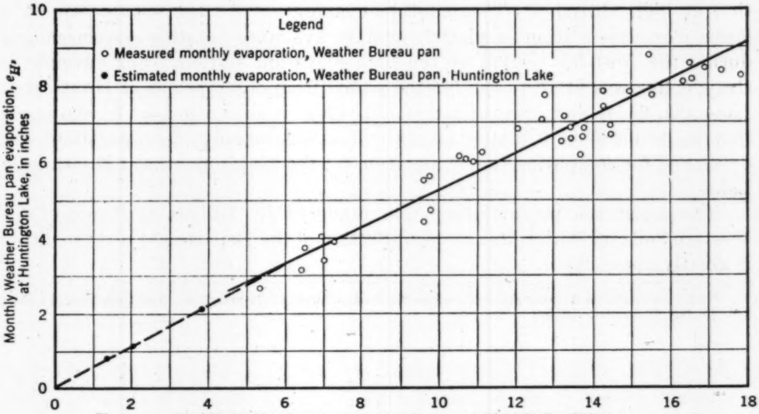


FIG. 7.—RELATIONSHIP BETWEEN MONTHLY PAN EVAPORATION AT HUNTINGTON LAKE, CALIFORNIA, AND MONTHLY PAN EVAPORATION AT FRIANT STATION, CALIFORNIA

as well as the air temperature. The values for the average humidity and the water temperatures are often not available for areas for which an estimate of the evaporation is needed.

---

[21] "Evaporation Study at Silver Lake in the Mojave Desert, California," by Harry F. Blaney, *Transactions*, Am. Geophysical Union, Vol. 38, No. 2, April, 1957.

[22] "Evaporation and Evapotranspiration Investigations in the San Francisco Bay Area," by Harry F. Blaney, *ibid.*, Vol. 36, No. 5, October, 1955.

Messrs. Ingerson and Shannon have mentioned that the size and shape of water-surface areas influence the degree of evaporation. Studies made in 1939 of the evaporation from pans located at six different sites at the Morris Reservoir, an irregularly shaped mountain reservoir in Los Angeles County, indicate a variation of 30%. A study made by the writer of evaporation at three stations adjacent to Salton Sea, California,[23] shows a variation in the pan evaporation of about 25%. This variation was due to the air that the wind moved over the pans. In one case the air coming from the Salton Sea area was moist, but air coming from the desert was dry.

The writer has used another method of estimating monthly winter evaporation at high altitudes. The monthly evaporation for the entire year at a lower elevation station is plotted against available monthly measurements during the frost-free period at the high elevation station. For example, a straight line was fitted to the plotted points from observations at Huntington Lake (El. 6954) and Friant Station (El. 400), as shown in Fig. 7. An equation ($e_H = 0.486\ e_F + 0.41$ = monthly Weather Bureau pan evaporation) was developed for computing the evaporation for the winter months at Huntington Lake.

The assistance in compiling data rendered by Gilbert L. Corey of the Western Soil and Water Management Section of the Department of Agriculture is greatly appreciated.

---

[23] "Evaporation an dStablization of Salton Sea Water Surface," *Transactions*, Am. Geophysical Union, Vol. 36, No. 4, August, 1955.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2926

AN IMPROVED DILUTION METHOD FOR FLOW MEASUREMENTS

By William A. Cawley,[1] J. M. ASCE, and Jack W. Woods[2]

Synopsis

A manganese dilution method for measuring the flow in sewers containing industrial wastes is outlined. An inexpensive technical grade of manganous sulfate was injected into the sewers, and the degree of dilution was determined quantitatively by flame spectrophotometric analysis. It is felt that this approach provides an accurate and relatively simple method for use with industrial wastes.

Introduction

The problem of measuring the flow in a sewer becomes involved when the physical construction of the sewer or the presence of industrial wastes adversely affect the use of the more common measuring devices. This becomes intensified when the classic chloride dilution method proves impracticable because of the high levels of chloride or other interfering ions in the industrial wastes.

An attempt to solve this type of problem involved finding an accurate and rapid quantitative determination that could be used as the basis for a dilution method for flow determination. The required determination was for an ion that was not present in the waste in an appreciable concentration and that could be introduced into the sewers easily and economically. Experimentation with colorimetric methods proved unsatisfactory because the procedures were too time consuming for practical application in determining flow. The literature indicated that flame spectrophotometric analysis could be applied, and subsequent experimentation established that a technical-grade manganous sulfate could be used as a trace element.

Method

The feed solution was prepared in a 55-gal drum for injection into the sewer with an air-driven barrel pump of the type used to transfer petroleum products.

Note.—Published, essentially as printed here, in October, 1956, in the Journal of the Sanitary Engineering Division, as *Proceedings Paper 1084*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] San. Engr., Rayonier Inc., New York, N. Y.

[2] Senior Analytical Chemist, Rayonier Inc., Jesup, Ga.

The colorimetric method for determining manganese was too time consuming in this study because of chloride interferences. Flame spectrophotometric analysis was used extensively and quite successfully in the quality control laboratory. The advantages that would be gained by applying this method of analysis to the problem prompted an investigation into the feasibility of its application. The equipment was first used to analyze for the sodium ion in the salt-dilution method, which proved practical, but the sodium concentration in the waste was so variable that it eliminated sodium as a trace element. The advantage of manganese as a trace element, as demonstrated in the colorimetric method, caused an investigation to be made of the possibilities of using it with flame photometry. The literature indicated that it was detectable with this equipment at the 0.1-ppm level, but experimentation showed that for practical purposes 50 ppm was the minimum detectable concentration. Field experiments established that concentrations at this level could be economically attained.

This method combined the advantages of manganese as a trace element with a flame photometer's accuracy, simplicty, and relative freedom from interferences. The flame photometer has an additional advantage in that, although the manganese and sodium may not be suitable for a trace element, it can be modified to fit any element that is detectable by flame photometry.

The results that were obtained checked closely with pumping records and the computed quantity of waste from the various processes; the reproducibility of duplicate samples was good. Because the quantity of waste from manufacturing processes, such as those that were measured, was never constant, there could be no absolute reproducibility. Disregarding the fluctuations in twelve duplicate tests, the variation in eight of the tests was less than 4.2%; the remaining four varied from 5% to 18%, with an average variation of 6.4% for the runs.

CONCLUSIONS

The use of manganese as a trace element and its detection by flame spectrophotometric analysis provide an accurate and relatively simple method of determining flows in sewers containing industrial wastes.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2927

ANCHORAGES FOR LARGE TAINTER GATES

BY ALEXANDER H. KENIGSBERG,[1] M. ASCE

WITH DISCUSSION BY MESSRS. STEPHEN WEARNE,
AND ALEXANDER H. KENIGSBERG

SYNOPSIS

Novel-type anchorages, whose distinguishing features and economy derive from (*a*) a single tie girder placed centrally within the spillway pier; (*b*) a single anchor at the upstream end of the girder; and (*c*) the downstream end, which is in a concentric grouping with the inclined side arms of adjacent tainter gates, are reviewed and compared with the conventional type.

INTRODUCTION

A novel type of anchorage for tainter gates has been developed in the Nashville (Tenn.) District by the Corps of Engineers, United States Department of the Army. It is considered to be an improvement over previous designs, and its basic features are presented herein.

In general, the versatility of tainter-type gates has been thoroughly established. Not only are the operating characteristics relatively elementary, because the basic requirement is a provision for hoisting a part of its dead weight, but the structural design of the moving parts of the gate is also relatively simple. The bearing problems that are so troublesome in vertical-lift gates do not exist at the upstream end of tainter gates. Effective sealing is also easily accomplished. However, at the downstream ends, where the considerable hydrostatic thrust of modern large gates is concentrated at a single point and must be transferred from the trunnions to the masonry piers, complex problems arise in accomplishing the load transfer. Although studies of tainter gates proper have been made, their anchorages have received little attention. The engineering problems involved will be studied, the basic methods of solving them will be reviewed, and the characteristics of the new anchorage will be compared with those of the conventional type.

NOTE.—Published, essentially as printed here, in December, 1956, in the Journal of the Waterways and Harbors Division, as *Proceedings Paper 1119*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Civ. and Structural Engr., Nashville Dist., Corps of Engrs., U. S. Dept. of the Army, Nashville, Tenn.

Basic Problems

The basic structural problem in tainter-gate-anchorage design involves an efficient method of transferring the hydrostatic load on the gates from the trunnions to its optimum location in the masonry of the spillway.

The magnitude of the forces at the gate trunnions will be examined first. The direct hydrostatic thrust is frequently 1,000,000 lb or more at each trunnion for the size of tainter gates. When the gate arms are inclined, which, incidentally, is by far the most economical and preferred arrangement, approximately one-third of this longitudinal force is converted into a lateral thrust at the trunnions.

When the loads from adjacent gates are balanced, their lateral forces are also balanced within the steelwork of the anchorage; the transfer of the balanced longitudinal loads to the masonry is a routine design procedure. However, when the water loads on adjacent gates are unbalanced, either at the end gates under all loading conditions or at intermediate piers of the spillway, an eccentric primary thrust acts on each trunnion pin. This thrust comprises a longitudinal component of approximately 1,000,000 lb and a lateral component of about 330,000 lb.

As a result of the built-in frame rigidity between the gate proper and its side arms, secondary lateral forces are induced at the trunnions that sometimes add to the effects of the primary forces. However, their magnitude is not sufficient to affect design considerations.

Although the moments developed within the anchorage steel by the primary forces at the trunnions are absorbed within the anchorage steel, their magnitude is quite impressive and has an important bearing on anchorage design and pier arrangement at the downstream end. Because trunnions are basically cantilevered supports, each 1,000,000 lb of hydrostatic thrust becomes a 3,000,000-ft-lb moment on either side of the ordinary anchorage frame, assuming a minimum distance of 3 ft from trunnion bearing to the embedded tension members.

The lateral components of the forces on the trunnions are not the governing factor in anchorage design and are best provided for as near the point of their application as practicable for any type of anchorage; therefore, they will be considered further herein. The longitudinal component of 1,000,000 lb on either side of the pier is the principal force that must be provided for in design and which is of primary concern.

There are only two basic methods of transmitting the hydrostatic thrust on the tainter gate from its trunnions to the pier masonry. One method, which uses an intermediate or transverse trunnion girder, has its longitudinal tension members, which are near each face of the pier, bonded into the concrete lying directly upstream of this girder. The other method, which may or may not use transverse girders, has its tension members coated for a considerable distance in order to prevent their being bonded to the surrounding concrete so that their stress can be delivered to anchors embedded nearer the upstream end of the spillway piers.

The first method of fully bonded side members is direct, economical, and simple to develop, but it results in applying the reactions due to gate thrust near the downstream face of the pier. At this point its contribution to dam stability is relatively small, and objectionable tensile stresses may develop within the mass concrete of the pier. For these reasons this method of anchoring is not generally considered with large gates.

There are many ways of transmitting the water thrust on the gate by using the second method, in which the side tie members do not part with their stress until some predetermined distance is reached upstream of the trunnions.

There is also a great variety of major details of design in tainter gate anchorages that have been developed and used by the Corps and by both public and private agencies. This study will be limited to two main types of anchorages, both of which have been used for large tainter gates that have been designed in the Nashville District. The ordinary type of anchorage will be termed "conventional," and the novel type will be known as the "single-girder" anchorage.

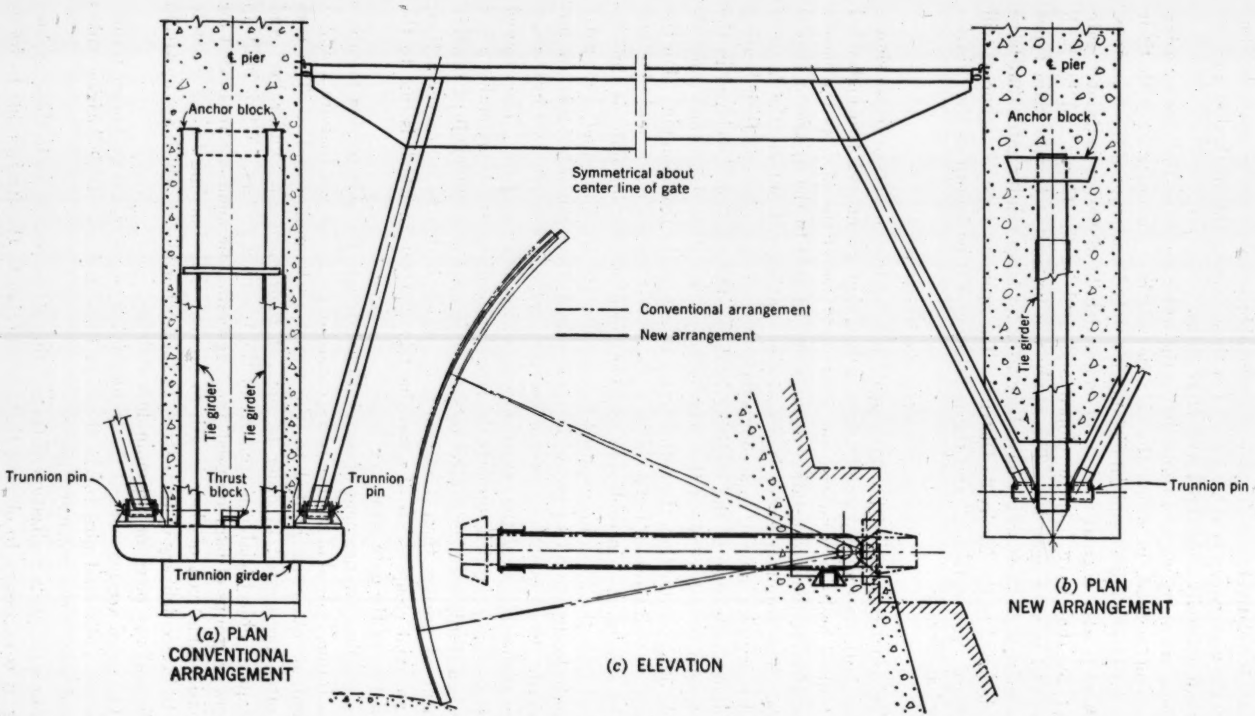
CONVENTIONAL ANCHORAGES

The usual conventional arrangement, such as the anchorage frame shown in Fig. 1(*a*), uses an exposed transverse trunnion girder and two embedded longitudinal built-up members, one near each face of the pier, as the means of transferring the hydrostatic pressures on the gate to the upstream anchor blocks. The trunnion girder is necessarily cantilevered outside the faces of the masonry pier in order to accommodate the trunnion pins and their bearing assemblies. Because of the resulting leverage, the reaction developed in the side tension members when the gate loads are unbalanced becomes approximately 50% greater than the actual water thrust on the trunnion pin. For the permissible unit stress of 18,000 lb per sq in. and a trunnion reaction of 1,000,000 lb, the area of steel in each side member, for direct tension only, is thus increased from the 55 sq in. required for carrying the trust when adjacent gates are fully loaded to 83 sq in., which is required when the gate loads are unbalanced. Consequently, a proportionate increase is also required in the size of the anchor blocks at the upstream ends of the side members. The exposed transverse girder, which supports the trunnion bearings and is preferably a box type, is a sturdy member subject to a shearing force of 1,000,000 lb; a bending moment of greater than 3,000,000 ft-lb, which is due to cantilever action; and an axial compression of approximately 330,000 lb, which is due to the inclination of the gate arms.

In another arrangement of conventional design, the side tension members of the frame, instead of single girders, consist of groups of bars that are wrapped around, or otherwise jointed to, the transverse trunnion girder. The bars are often prestressed in order to prevent their elongation under load and consequent objectionable movement of the trunnion girders. When built-up girders or groups of bars are used, the longitudinal tie members are coated for a considerable part of their length to prevent their bonding to the concrete and to insure delivery of their stress to the anchor blocks, which are upstream in the spillway piers for optimum dam stability.

The usual arrangement of the trunnions for conventional anchorage frames is to use individual pins set in trunnion hubs and adjustable bearing yokes at each end of the transverse girder.

For a complete realization of the existing stress conditions in these anchorages, it is necessary to observe that, in addition to the stresses due to frame


℄ pier
Anchor block
Tie girder
Tie girder
Trunnion pin
Thrust block
Trunnion pin
Trunnion girder
Symmetrical about center line of gate
Conventional arrangement
New arrangement
℄ pier
Anchor block
Tie girder
Trunnion pin

(a) PLAN CONVENTIONAL ARRANGEMENT

(b) PLAN NEW ARRANGEMENT

(c) ELEVATION

FIG. 1.—TAINTER-GATE ANCHORAGES

rigidity, some torsional stresses also are developed in the trunnion girders. These stresses are a result of the setting of the anchorage frame within the masonry and, especially, because of the eccentricity of the vertical trunnion reactions with respect to the trunnion girder during the period of gate operation. However, these stresses do not coincide with the maximum loading conditions and, consequently, do not govern the design.

SINGLE-GIRDER ANCHORAGES

The novel-type anchorage that was developed in the Nashville District was adopted for the 45-ft-wide-by-41-ft-high tainter gates of the Old Hickory dam (completed on March 18, 1954) on the Cumberland River upstream from Nashville.

Fig. 1(*b*) shows the new arrangement, which uses a single longitudinal tie girder with a single anchor block at its upstream end placed centrally within the spillway pier. The water thrust on the gate is transmitted directly to the center of the pier for optimum resistance for any condition of loading. It will be noted from Fig. 1(*b*) that the inclined arms of adjacent gates meet the central girder at a common point just outside its downstream end. As a result of the concentric intersection of the three members involved, their stresses are axial only under normal conditions of balanced loading, and the maximum total stress in the central longitudinal girder is never greater than that caused by the actual hydrostatic load on two halves of the adjacent gates. A relatively short pin, extending through the downstream end of the center girder, readily accommodates the trunnion hubs of the adjacent gates and delivers the water load directly from the gate arms to this girder. This method dispenses with the need for the heavy transverse trunnion girder, as well as the separate pins and their bearing yokes that are necessary in the conventional anchorage frame.

When the gate loads are unbalanced, as in the case of one gate being relieved of its hydrostatic thrust, the axial stress in the central tie girder is only one-half that of the stress that occurs when the two gates are fully loaded, whereas it is one and a half times as large in each side member of the conventional anchorage. The sectional area of the single tension member is still fully utilized because it is readily proportioned to resist either the total axial load on two gates, or the combined axial and flexural stresses due to partial loading. Consequently, no excess metal is required for any loading condition in any part of the anchorage. The outside anchorage of the end gates is identical in design with that of the intermediate gates, except that the trunnion pin protrudes only on one side of the center girder.

The lateral components of the hydrostatic thrust from the inclined gate arms will now be considered. When the components become unbalanced due to an unequal gate loading, the resulting lateral force must be transmitted from the steel anchorage to the masonry of the pier. Fig. 1(*c*) shows that this can be accomplished most effectively by using a thrust block embedded in the concrete beneath the central tie girder directly under the pin where the thrust is applied. The underside location of the thrust block develops torsional stresses in the main girder. However, as in the case of the transverse trunnion girder of the conventional anchorage, the magnitude of the torsional stresses is not important in design. Furthermore, it is resisted by the concrete fill placed in the box-type central girder after the anchorage has been set in place.

The central tie girder is prevented from bonding to the concrete for the downstream section of its length as are the side members of conventional anchorages. When elongation occurs under stress, the underside of the downstream end slides freely in a fitted groove provided for this purpose in the top of the lateral thrust block.

Fig. 1(*c*) shows a superposition of the side elevations of the two types of anchorages. At the downstream end, both the masonry pier and the steel anchorage are materially shorter for the novel design than for the conventional design. The anchorage designs shown in Fig. 1(*c*) were developed in detail for a project having tainter gates 40 ft wide and 26 ft high. The estimated weights of steel for the gate anchorages of this project are approximately 18,000 lb for each conventional type and only 9,000 lb for each single-girder anchorage.

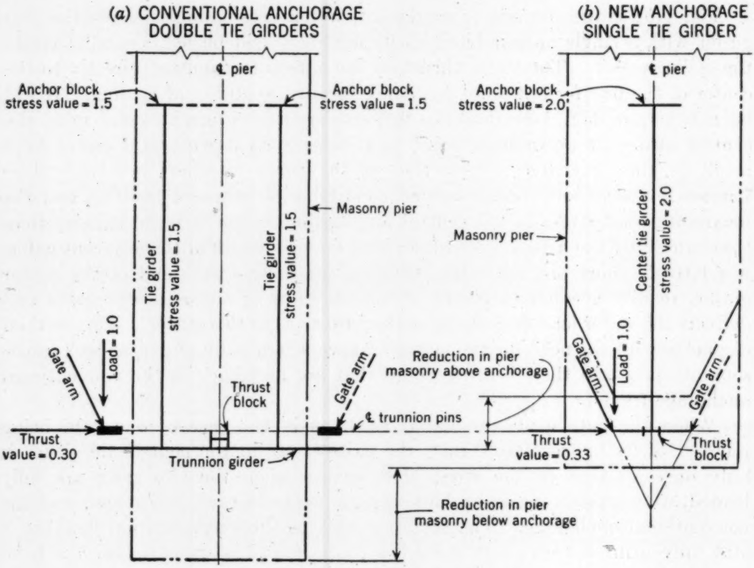

(*a*) CONVENTIONAL ANCHORAGE
DOUBLE TIE GIRDERS
(*b*) NEW ANCHORAGE
SINGLE TIE GIRDER
℄ pier
Anchor block stress value = 1.5
Anchor block stress value = 1.5
Anchor block stress value = 2.0
Masonry pier
Tie girder
stress value = 1.5
Tie girder
stress value = 1.5
Center tie girder
stress value = 2.0
Gate arm
Load = 1.0
Thrust block
℄ trunnion pins
Thrust value = 0.30
Trunnion girder
Reduction in pier masonry above anchorage
Thrust value = 0.33
Thrust block
Reduction in pier masonry below anchorage

FIG. 2.—ANCHORAGE TIE GIRDERS

The basic arrangement of a central tie girder can be adapted also to tainter gates that must have side arms parallel to and close to the faces of the spillway piers. Although the economy of materials and the simplicity of fabrication are not as great as in the case of the anchorage with inclined gate arms, the structural merits of a single tie member, single anchor block, and single thrust block located along the longitudinal center line of the pier are fully preserved.

The results of a comparative study of the two types of anchorages are summarized in Fig. 2, which illustrates schematically their fundamental engineering concepts, and in Table 1, which presents a parallel listing of several of their important features. Fig. 2 and Table 1 show that the novel-type anchorage, as compared with its conventional frame counterpart, results in an over-all simplification and improvement of structural characteristics; an economy of materials; and a compact construction with lesser costs of fabrication, handling, and installation.

TABLE 1.—COMPARATIVE DATA

Features	Conventional Anchorage	New Anchorage
	DESIGN FEATURES	
Trunnion pins	2	1
Trunnion bases	2	none
Trunnion girder	1	none
Tie girders	2 (side)	1 (center)
Anchor blocks	2	1
Thrust blocks	1	1
Type of construction	Massive, rigid frame	Compact, simple structure
	FABRICATION HANDLING AND SHIPPING	
Remarks	Cumbersome and expensive	Simple and economical
Structural steel weight ratio	1.00	0.60
Reinforcing steel, weight at anchorage ratio	1.00	0.25
	MASONRY VOLUME AND PLACEMENT	
Volume size	Large, both above and below anchorage	Smaller, both above and below anchorage
Placement	Relatively difficult due to reinforcing and side girders	Relatively simple placement at sides and center girder
	COST	
Estimated total cost ratio	1.00	0.50

TABLE 2.—TAINTER-GATE ANCHORAGE WEIGHTS

Location (1)	Gate size, h by l, in feet (2)	Hydrostatic thrust, in pounds (3)	ANCHORAGE WEIGHT Total, in pounds (4)	ANCHORAGE WEIGHT Per 1,000 lb of thrust (5)
Saluda River, South Carolina	25 by 37.5	733,000	32,000	43.6
Fontana Dam, North Carolina	35 by 35	1,340,000	72,000	53.6
South Carolina	32 by 44	1,410,000	88,000	62.5
Norfolk Dam, Arkansas	28 by 40	980,000	34,000	35.1
Wolf Creek Dam, Kentucky	37 by 50	2,140,000	68,000	31.8
Center Hill Dam, Tennessee	37 by 50	2,140,000	54,000	25.5
Cheatham Dam, Tennessee	27 by 60	1,370,000	33,000	24.1
Old Hickory Dam, Tennessee	41 by 45	2,360,000	34,000	14.4

Table 2 provides a general index of the trend in tainter-gate anchorage design as engineered by different agencies for several random projects that are listed chronologically. Some of the anchorages are essentially bonded to the concrete for the full length of their side members, whereas others have side members that are free to elongate within the masonry. The indicated weights were estimated from available drawings and are believed to be substantially correct.

In Col. 5 of Table 2, for purposes of an equitable comparison, the anchorage weights are given as a quantity of anchorage steel per 1,000 lb of hydrostatic thrust on the tainter gate. Through the years, engineers have accomplished substantial improvement, not only in the economy of materials, but in the structural functioning of tainter-gate anchorages.

Fig. 3 further demonstrates the fundamental differences between the two types of anchorages. It shows the trunnion arrangement of the novel single-girder anchorage for two adjacent gates of the Old Hickory project as viewed from the tops of the spillway piers.

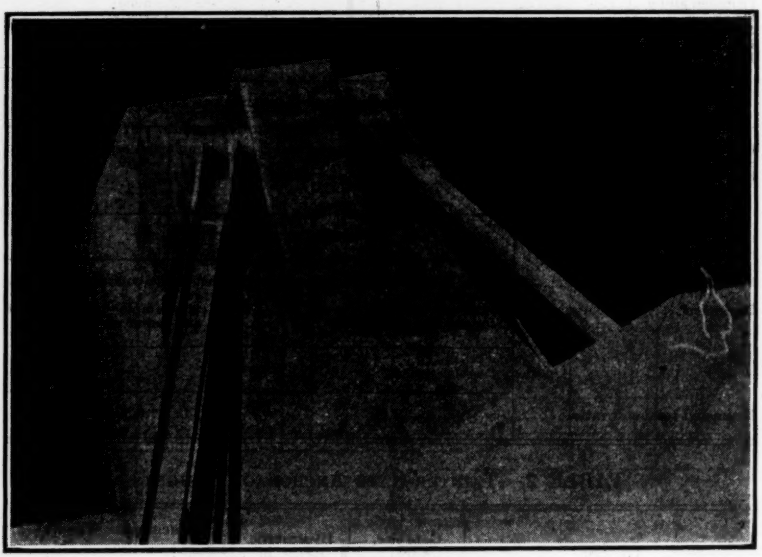


Fig. 3.—Single-Tie-Girder Anchorage

Conclusions

The over-all merits of the single-girder anchorages have been confirmed by the successful installation and operation of the six tainter gates of Old Hickory Dam, where they were subjected to full head on January 7, 1957. The estimated savings effected because of these anchorages were approximately $90,000 for this installation. When economy is added to the superior operating characteristics of tainter gates as the final consideration in determining the type of crest gate to be used for any particular project, the new anchorages materially enhance the advantage of tainter gates in regulating reservoir levels.

DISCUSSION

STEPHEN WEARNE[2].—In planning the power intakes for the Macagua No. 1 hydroelectric plant in Venezuela on the Caroní River, a paper and discussion of Dow A. Buzzell,[3] M. ASCE, concerning the greater use of tainter-type gates was of considerable interest. An intake design was developed for the Caroní River using tainter gates whose face area was 10 m by 11 m. The hydraulic development and the model tests of this intake design have been described by H. D. Morgan and J. S. Burgess.[4]

With reference to the author's paper, the anchorage design for the Macagua No. 1 project was controlled by the need for structural economy, but the whole project was scheduled in a tight and all-important program. With distribution of the 600-ton thrust on each trunnion by heavy reinforcement into the mass-concrete intake buttresses, A-frames of structural-steelwork members were adopted. The frames were to be supplied with the gates, and detailed designs were prepared by tenderers for approval. The preparation of tenders on these lines progressed so that the gate contract would be placed late in 1956; the design, construction, and erection of the first gates were to be completed in two years. It was considered essential that the large A-frames be supplied in time for casting into the buttresses during concreting because grouting of the frames into boxed-out cavities would not give the required intimate distribution of the thrust. However, this scheme was reconsidered from the standpoint of scheduling, and it was concluded that there was insufficient time to develop the design of the A-frames and order, fabricate, and ship them.

An alternate arrangement was therefore designed with the trunnions mounted on plate grillages, composed of a pair of heavy plates coupled with bolts. The plates were in the plane of the buttress faces and distributed the force to the reinforcement, which was arranged so that the grillages could be erected and grouted after the buttress concreting, if necessary.

ALEXANDER H. KENIGSBERG,[5] M. ASCE.—Without drawings of the tainter-type gates for the power intakes of the Macagua No. 1 hydroelectric scheme in Venezuela, it is difficult to comment on Mr. Wearne's examination of the gate anchorages used. It appears that the substitute arrangement that was used instead of the original anchorage design, which was similar to the conventional type, is not an optimum from the standpoint of transferring applied loads through the constituent parts of a structure to their ultimate supports.

The statement, "* * * the trunnions mounted on plate grillages," implies that the entire hydrostatic thrust of approximately 300 tons on each trunnion, instead of 600 tons, is transmitted to the grillages through bolts in shear and bending. Secondly, "* * * the plates were in the plane of the buttress faces" implies that a considerable part of the thrust is applied through the edges of the plates by bearing on the outer facing of the concrete buttresses. Thirdly, the clause, "* * * (the) grillages, composed of a pair of heavy plates coupled by bolts * * * distributed the force to the reinforcement," indicates the absence of a solid metallic member between these plates and the consequent transfer of the balance of the water thrust through the bolts by bearing on the masonry, which necessarily takes place prior to the interception of the force by the reinforcement.

---

[2] Site Engr., Nuclear Power Plant Co., Bradwell, Essex, England; formerly Caroní Section Leader, Sir William Halcrow & Partners, Cons. Engrs., London, England.

[3] "Recent Trends in Hydraulic Gate Design," by Dow A. Buzzell, *Transactions*, ASCE, Vol. 123, 1958.

[4] "Macagua No. 1 Intake Design and Model Tests," by H. D. Morgan and J. S. Burgess, Cong. of the International Assn. for Hydr. Research, Lisbon, July, 1957.

[5] Civ. and Structural Engr., Nashville Dist., Corps of Engrs., U. S. Dept. of the Army, Nashville, Tenn.

Fig. 2 shows that both the conventional anchorage and the improved single-girder anchorage transmit the water load on the gates directly to integral anchorage frames and, through their embedment, into the mass of the spillway masonry. Both types of anchorages were developed and applied to several projects by the Nashville District and have functioned successfully for many years.

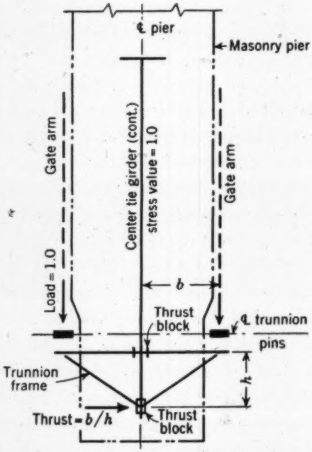


FIG. 4.—PARALLEL-GATE-ARMS ANCHORAGE

Parallel-Arms Tainter Gates.—Except for a brief reference, anchorages for parallel-arms gates were not mentioned in the paper. In order to amplify the treatment of the subject, the deleted section will be included herein.

When overflow-type tainter gates are used, or when channel operating conditions require a clear opening with the gates raised, their side arms are placed parallel and as near as practicable to the faces of the spillway piers. Fig. 4 shows an arrangement of the anchorage when such gates are involved. The axially stressed triangular frame at the downstream end of the anchorage transmits only the actual hydrostatic thrust on the gates to the central tie girder in axial stress. This procedure is similar to that shown in Fig. 2(*b*), except that in this arrangement no flexural stresses are developed in the girder even under unequal loading on adjacent gates. When such loading conditions occur, the unbalanced horizontal forces are transmitted directly into the pier masonry by the two central thrust blocks indicated on the diagram without affecting the purely tensile stress in the tie girder or its magnitude, which is the applied hydraulic thrust on the gates. In all other respects these anchorages are similar to the basic type shown in Fig. 2(*b*).

Mr. Wearne's interest in novel anchorages is appreciated, together with the opportunity his discussion has afforded to examine parallel-arms tainter gates.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2928

EFFECTS OF STORAGE IMPOUNDMENTS ON WATER QUALITY

BY MILO A. CHURCHILL,[1] A. M. ASCE

SYNOPSIS

When water in a free-flowing stream is impounded in a large storage reservoir, changes in the physical, bacteriological, sanitary-chemical, and mineral quality of the water are produced. Although most of the changes result in a generally improved water quality, certain specific qualities and downstream water uses may be adversely affected. A wide variety of observations made on reservoir waters in the Tennessee Valley are presented and summarized herein. The facts obtained and the general principles deduced therefrom should be helpful in forecasting the changes in water quality that are likely to occur in any proposed reservoir.

INTRODUCTION

Water as an essential national resource is currently (1957) receiving more study in the southeastern United States than in the past. Every state in this area is re-examining its water-rights laws to determine if changes are desirable. That the total supply of water is limited is beginning to be realized—it does not grow as the need for more water grows. This realization is accompanied by the natural impulse of every individual to take steps to obtain at least a share of the available supply. Many conflicts in water use exist, and, obviously, more conflicts will develop. A relatively new and growing use of water in this area, spray irrigation of field crops, has already produced some conflicts with other uses and potentially can produce many more. This use is largely consumptive because of evapotranspiration losses, whereas most industrial and municipal uses cause the water to pass through various processes and conduits before most of it, polluted to some degree, is returned to the water channel.

One means of increasing the available supply of water throughout the world is to prevent its waste during periods of high runoff by storing it in reservoirs. Although the storage idea is not new, it is certain to have many new advocates

---

NOTE.—Published, essentially as printed here, in February, 1957, in the Journal of the Sanitary Engineering Division, as *Proceedings Paper 1171.* Positions and titles given are those in effect when the paper was approved for publication in *Transactions.*

[1] Chief, Stream Pollution Control Section, Div. of Health and Safety, Tennessee Valley Authority, Chattanooga, Tenn.

in the immediate future. No "crystal ball" is needed to predict that the trend toward conservation of water must be accelerated.

Because storage impoundments will inevitably become more numerous and will be called on to serve more and more uses and users, the quality of the water stored in, and released from, the impoundages will be under strict scrutiny to determine if it is suitable for the varied intended uses. Intelligent planning of the conservation and development of water resources requires that the advantages and disadvantages of storage impoundments be fully understood with regard to water quality as well as water quantity.

Fortunately, impoundments result in many improvements in water quality in so far as most uses are concerned. However, some changes are detrimental for some uses. That is, reservoirs produce good and bad changes in stored water, and the net effect is not the same for all users.

TABLE 1.—PERTINENT DATA ON SELECTED IMPOUNDMENTS

Reservoir	River	Date of closure	Full Pool			
			Volume, in acre-feet	Area, in acres	Length, in miles	Depth of dam, in feet (approximately)
Cherokee	Holston	12- 5-41	1,565,400	31,100	59	150
Douglas	French Broad	2-19-43	1,514,100	31,600	43	150
Fontana	Little Tennessee	11- 7-44	1,444,300	10,670	29	440
South Holston	South Fork Holston	11-20-50	744,000	8,750	24	250
Watauga	Watauga	12- 1-48	678,800	7,200	17	330
Norris	Clinch	3- 4-36	2,567,000	40,200	77	210
Hiwassee	Hiwassee	2- 8-40	438,000	6,280	22	250
Apalachia	Hiwassee	2-14-43	58,700	1,123	10	110
Parksville	Ocoee	12-15-11	91,300	1,900	7	120
Boone	South Fork Holston	12-16-52	196,700	4,520	17	120
Fort Patrick Henry	South Fork Holston	10-27-53	27,100	893	10	80
Cheoah	Little Tennessee	12- 8-18	35,030	595	8	190
Calderwood	Little Tennessee	4-15-30	41,160	536	8	190

An examination will be presented herein of some of the changes in the physical, bacteriological, sanitary-chemical, and mineral quality of impounded waters in the Tennessee Valley that have been observed over a period of approximately 20 yr. The observed data have been obtained from various studies and investigations that have been made for a wide variety of purposes. The data presented herein have been selected to illustrate typical effects of tributary impoundments, and some unusual effects are also included. An attempt is made to draw conclusions and to state certain fundamental principles that are generally applicable for impounded waters. A section is included on reaeration in downstream river channels of water released from the lower levels of storage projects.

The eastern half of the Tennessee Valley is shown in Fig. 1. River profiles of most of the major tributaries in the eastern half of the valley are shown in Fig. 2. Reservoir volumes, areas, depths, and similar data for the projects cited herein are listed in Table 1.

OPERATION OF STORAGE PROJECTS IN TVA SYSTEM

In the Tennessee Valley Authority (TVA) system, the reservoirs are operated primarily for navigation and flood control and are used for the production

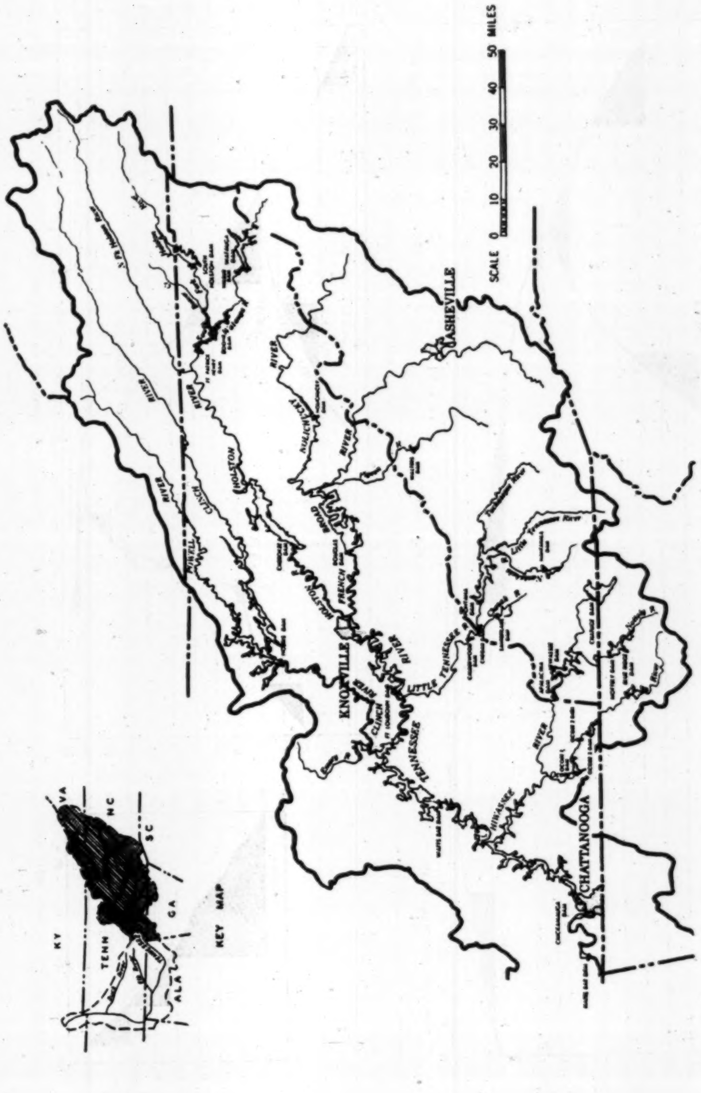

SCALE
0
10
20
30
40
50 MILES
KY
VA
TENN
NC
SC
GA
ALA
KEY MAP
KNOXVILLE
ASHEVILLE
CHATTANOOGA
N. FK. HOLSTON RIVER
POWELL RIVER
CLINCH RIVER
HOLSTON RIVER
FRENCH BROAD RIVER
NOLICHUCKY RIVER
LITTLE TENNESSEE RIVER
LITTLE RIVER
TENNESSEE RIVER
HIWASSEE RIVER
CHICKAMAUGA DAM
WATTS BAR DAM

FIG. 1.—EASTERN HALF OF THE TENNESSEE VALLEY

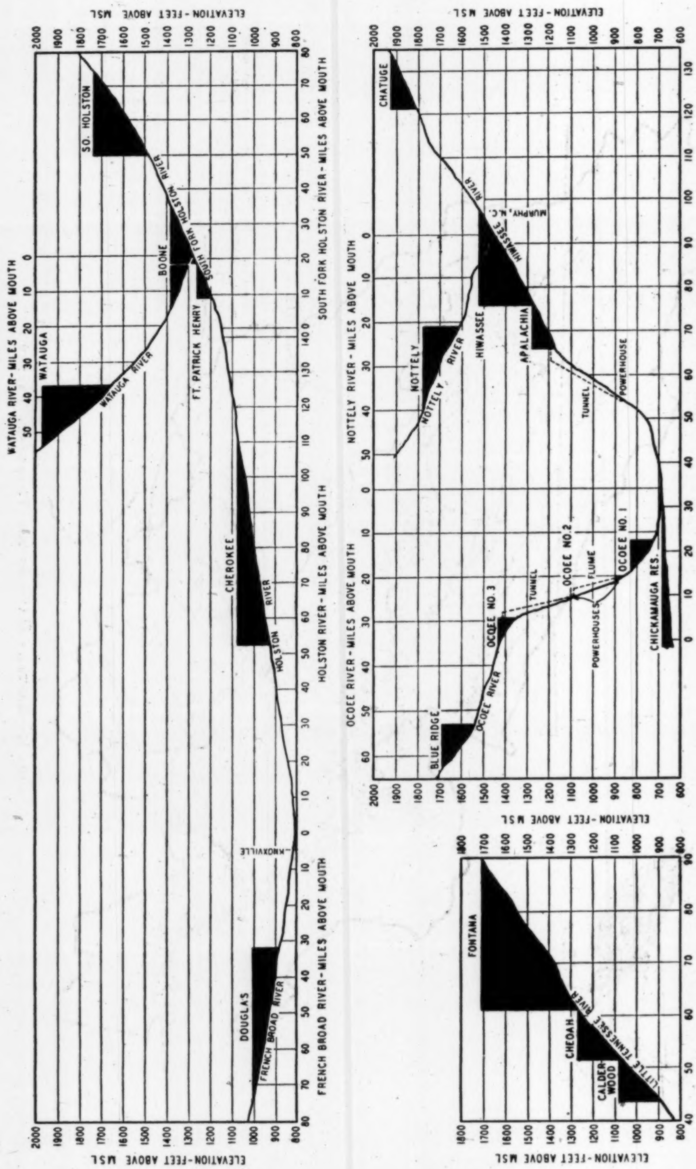


FIG. 2.—PROFILES OF SELECTED MAJOR TRIBUTARIES OF THE TENNESSEE RIVER

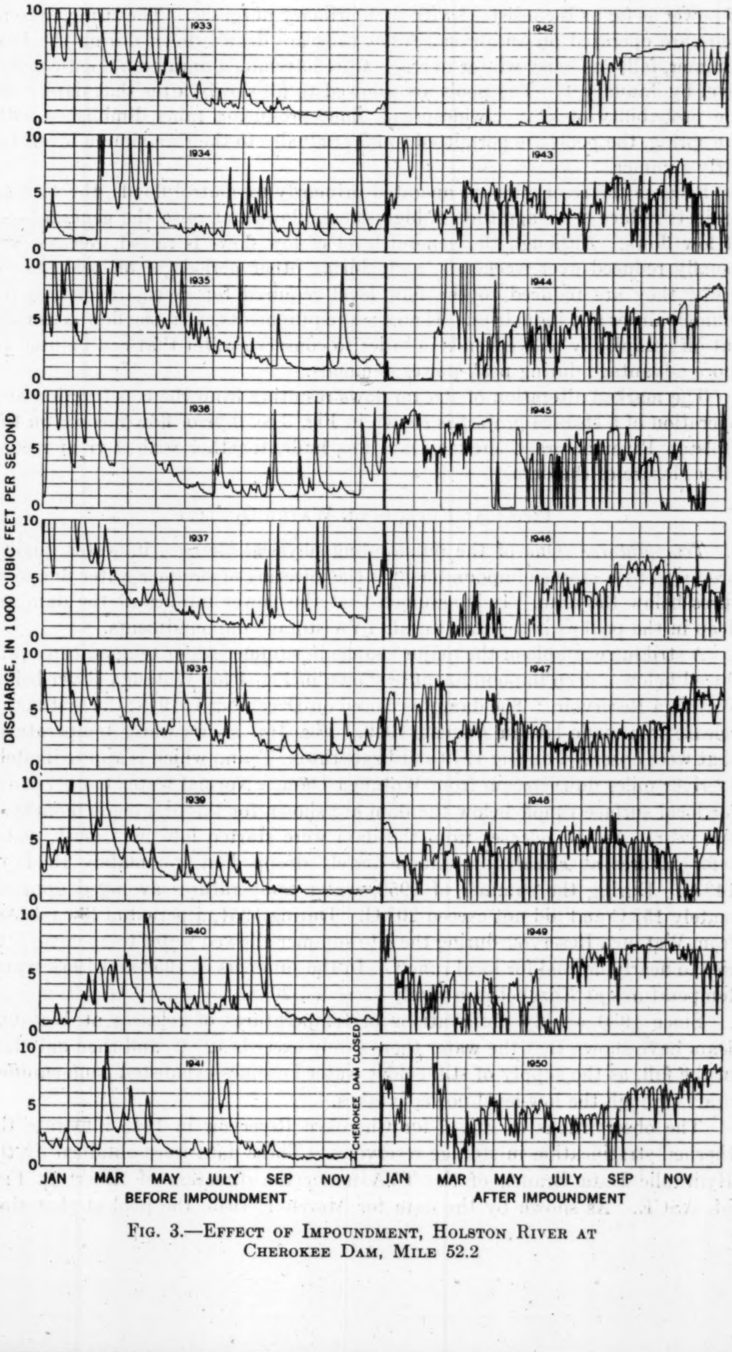


FIG. 3.—EFFECT OF IMPOUNDMENT, HOLSTON RIVER AT CHEROKEE DAM, MILE 52.2

of power as far as is consistent with such primary purposes. The storage reservoirs are operated on an annual cycle; they are drawn down during the late summer, fall, and early winter to reach a predetermined minimum flood-control level by January 1. The pools are allowed to fill slowly after that date. As the probability of heavy, widespread, flood-producing rains diminishes with the spring, the pools are permitted to fill gradually to their maximum levels by early summer.

Because the reservoirs are operated primarily to control floods, the storage projects reduce the magnitude of high stream flows and raise the general level of low flows. Although the general level of low flows is raised, releases are usually reduced over weekends, and, during other periods of off-peak power loads, they are reduced to the flow level required for downstream uses by municipalities and industries. If no such requirements exist, the flow is usually cut off completely on weekends in order to conserve water that can be used to more advantage during peak-power demands.

The marked alteration of stream flows resulting from the construction and operation of a storage project is shown in Fig. 3 by 9 yr of flow records on the Holston River before Cherokee Dam was built, together with 9 yr of record after closure of the dam.

PHYSICAL EFFECTS ON WATER QUALITY

Temperature.—One of the outstanding physical changes in water quality produced by a storage impoundment is the lowering of summer water temperatures below the dam. It is assumed that the power intakes at the dam are deep in the pool as is the case for all TVA storage impoundments.

A striking example of the major change in summer water temperatures produced below a deep impoundment is shown in Fig. 4 for Watauga River below Watauga Reservoir. The dam was closed on December 1, 1948; the first major release was made late in August, 1949. Fig. 4(*a*) shows water temperatures that were observed during 1949 at Elizabethton, Tenn., which is approximately 12 river miles downstream from Watauga Dam. Normal water temperatures for local surface runoff below the dam are shown for the first eight months of the year. As the releases from the dam were started late in August, water temperatures at Elizabethton immediately dropped approximately 13° C or 14° C. During the summer of 1950, water temperatures averaged approximately 15° C and did not exceed 20° C. During 1951, a period of low releases from Watauga Reservoir during the late summer allowed water temperatures to rise to nearly normal for local runoff. In the summers of 1952 and 1953, water temperatures at Elizabethton rarely exceeded 20° C.

Since 1950 weekly observations of temperatures of releases at Watauga Dam have shown that the water there rarely exceeds 12° C, and then only late in the fall, as the supply of still colder water becomes exhausted from summer draft through the low-level power intakes.

The observations in Fig. 5 for Cherokee Reservoir in 1945 illustrate the thermal stratification in storage reservoirs. These data were obtained by the Hydraulic Data Branch of the TVA under the direction of Albert S. Fry, M. ASCE. As shown by the data for March 1, 1945, the pool at that time

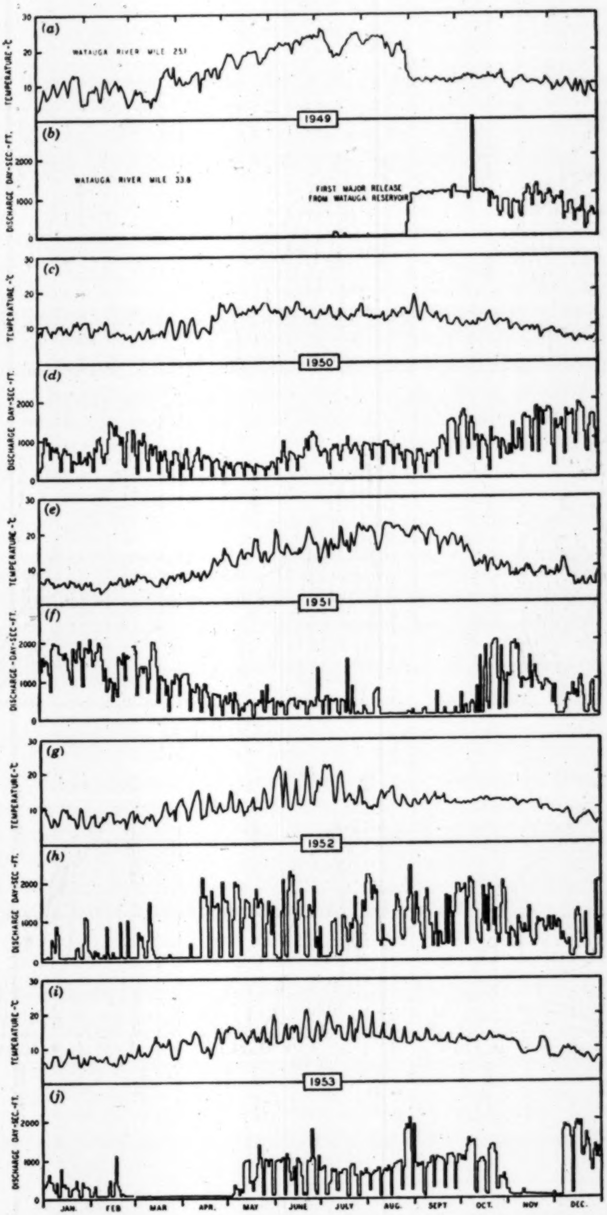


FIG. 4.—WATER TEMPERATURES BELOW WATAUGA RESERVOIR

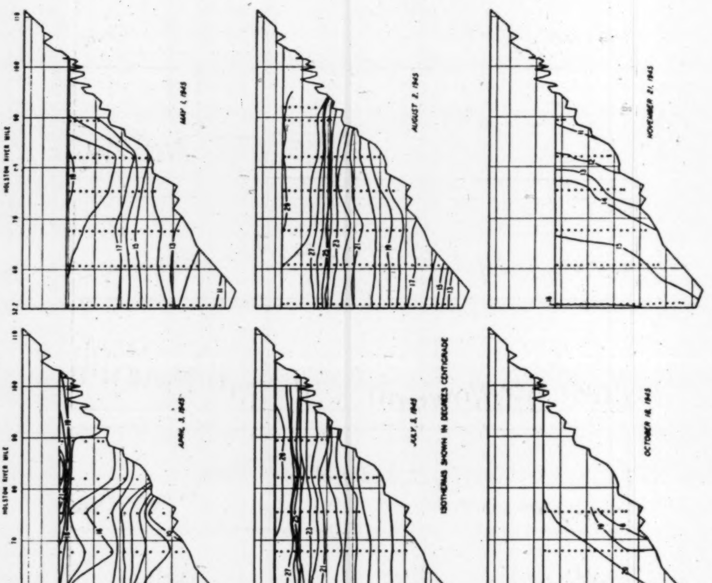


FIG. 5.—THERMAL STRATIFICATION OF CHEROKEE RESERVOIR

contained water in the narrow range of from 8° C to 11° C. There was little thermal stratification because temperatures were nearly uniform from the reservoir surface to the bottom. By April 3, the warming inflows and the direct heating of the pooled water by the sun and atmosphere had produced considerable thermal stratification. By June 6, a definite thermocline was established near El. 1020. Because stratification beginning in March or April stops the vertical circulation that exists all winter in storage pools at these latitudes, draft from the low-level power intakes removes cold water from near the level of the intakes. As the supply of cold water at the intake level is exhausted from the pool, warmer water from above sinks down to occupy the vacated space and is, in turn, discharged from the pool. By this process the discharged water gradually warms up during summer and fall. For Cherokee Reservoir in 1945, this gradual warming is illustrated in Fig. 8(*a*) to be presented subsequently. Early in April the discharged water had a temperature of 12° C. The temperature rose steadily as the supply of cold water was gradually exhausted, and by the middle of September the temperature of the discharged water had reached 25° C. The temperature profiles in Fig. 5 indicate that the water in the range of from 21° C to 24° C is at the intake level on September 8.

It is possible to predict water temperatures of releases months ahead during the summer by (*a*) obtaining a reservoir temperature "profile" such as one of those in Fig. 5, (*b*) estimating the volume of releases for succeeding months, and (*c*) by use of an elevation-volume curve for the pool to compute succeeding increments of cold-water draft from the level of the intake upward into the warmer water of the higher elevations.

From the foregoing it is evident that the extent to which the temperature of the discharged water rises in the late summer and fall is a direct function of the rate of draft on the available supply. A heavy draft on a relatively small supply of cold water will obviously exhaust it rather promptly, whereas a large supply with a light rate of draft will supply water of winter temperatures all year long.

Cooler air temperatures in the fall (beginning in September, 1945, for Cherokee Reservoir) result in a cooling of river inflows to the pool faster than the great mass of pooled water can be cooled. As a result, the inflow to the pool ceases to flow into the upper levels and begins to flow down the old river bed under the pooled water as a density current.[2] Finally, by the middle of October the entire pool is short-circuited by the density underflow. Such underflows may continue for a month or more in the fall until the pooled waters are cooled down by progressive vertical circulation to the temperature of the inflow. After that occurs, complete vertical circulation continues all winter due to diurnal temperature changes and to wind action on water that has little capacity to resist mixing.

The temperature aspects of reservoirs are treated rather fully herein inasmuch as most of the other changes in water quality are controlled to a great extent by the mechanics of storage and flow through the pool. An understand-

---

[2] "Significant Effects of Density Currents in TVA's Integrated Reservoir and River System," by A. S. Fry, M. A. Churchill, and R. A. Elder, *Proceedings*, Minnesota International Hydraulics Convention, September, 1953.

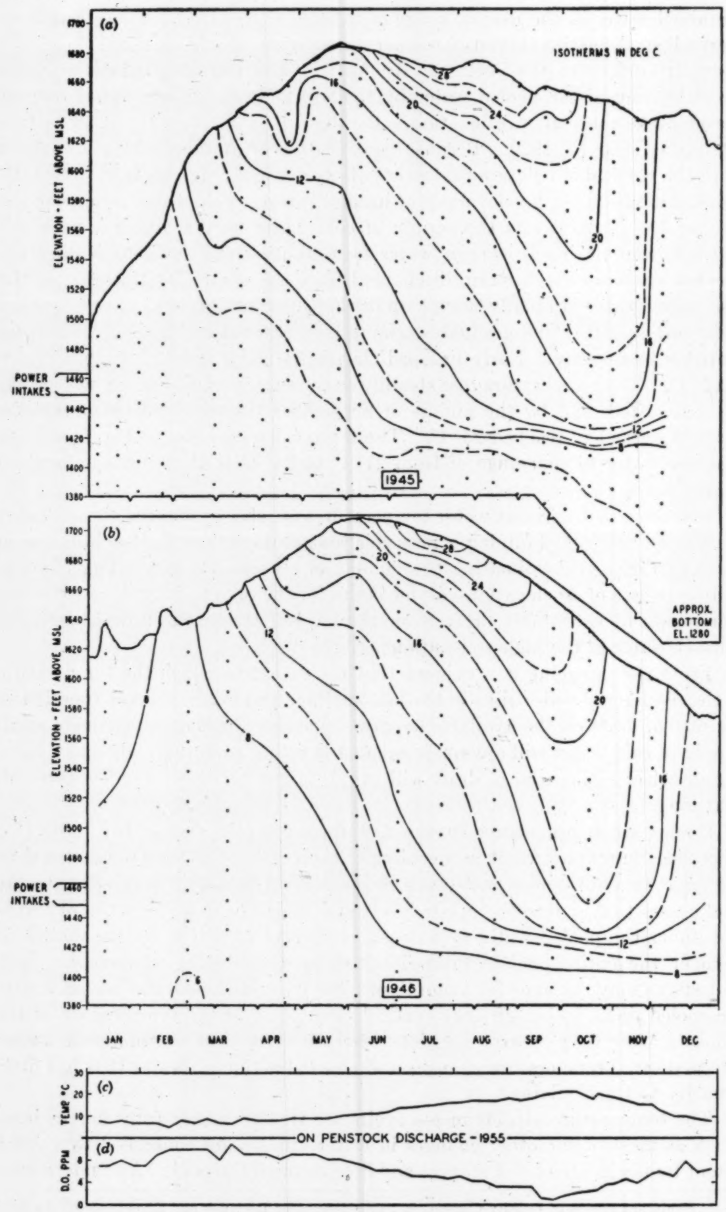


Fig. 6.—Thermal Stratification in Fontana Reservoir at Dam Showing Effects of Low-Level Power Intakes

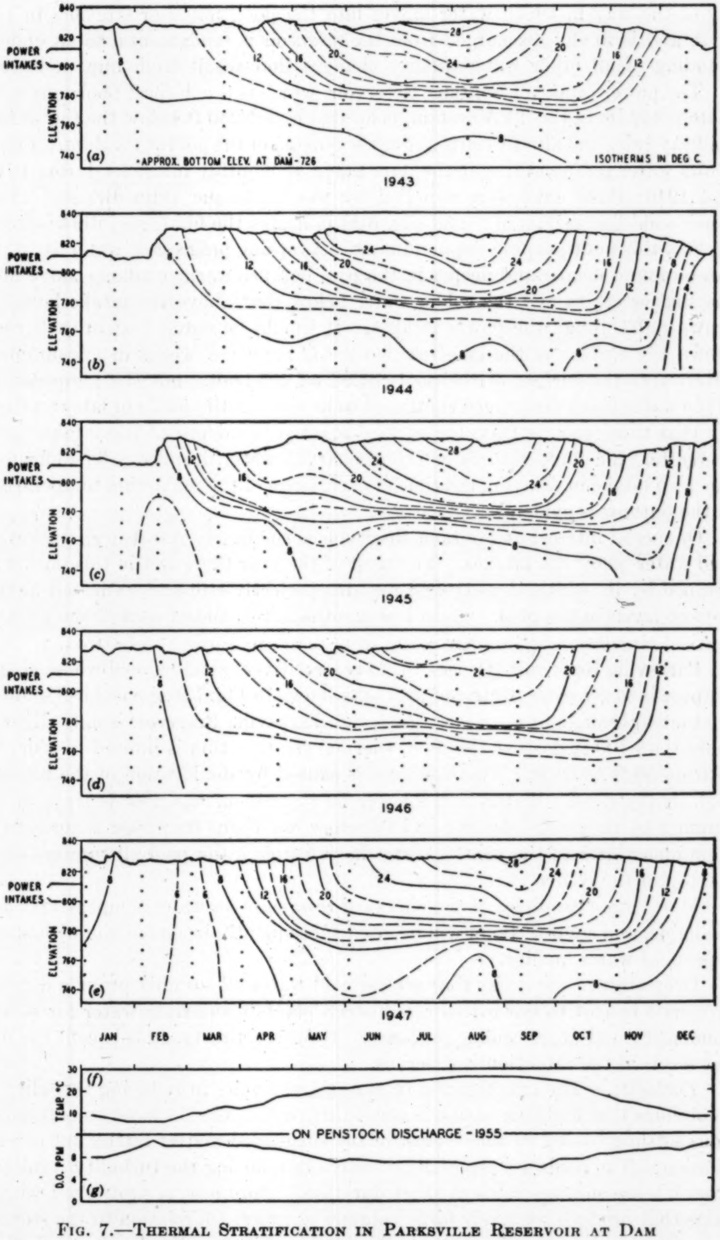


FIG. 7.—THERMAL STRATIFICATION IN PARKSVILLE RESERVOIR AT DAM SHOWING EFFECTS OF HIGH-LEVEL POWER INTAKES

ing of the way in which water moves into the pool, mixes or stratifies in the pool, and is finally discharged from the reservoir is fundamental to an understanding of all other water quality changes that result from impoundment.

The power intake of Fontana Reservoir, which is the deepest pool (approximately 440 ft) in the TVA system, is located about 250 ft below the top of the spillway gates and almost 190 ft above the bottom of the pool at the dam. Fig. 6 shows water temperatures at the dam taken at monthly intervals during 1945 and 1946; these data were supplied by the Hydraulic Data Branch. The upper solid line in Fig. 6(*a*) and Fig. 6(*b*) indicates the filling and drawn-down cycle of this multipurpose impoundment. As spring progresses, water at 10° C can be found deeper and deeper in the pool because warmer inflows move into the surface strata and the colder water below (but above the intake level) is drawn out through the power intakes. It should be noted that in each year shown the surface of the lake reached 28° C for a few weeks in midsummer. Afterwards the surface of the pool cooled off gradually, but the temperature of the water being discharged continued to increase until middle or late October. By that time, cooling from the surface reached the depth of the intake, and, thereafter, the temperatures of the discharged water dropped. The effect of the deep intake in delaying the time of occurrence of the maximum temperature in the outflow is evident.

Of special interest at Fontana Reservoir is the nearly 200-ft-deep pocket of cold water below the intakes. For most of the year the water in this pocket is trapped by its winter density and remains perfectly still and unmoved in the bottom levels of the pool. For a few months in the coldest part of the winter, vertical circulation apparently exists from reservoir surface to bottom.

Parksville Reservoir (Ocoee No. 1) is a relatively shallow pool whose single purpose is to generate electric power. As shown in Fig. 7, the pool level seldom fluctuates throughout the year. Although Parksville Reservoir is only slightly more than 100 ft deep at the dam, thermal stratification is limited to a depth of from 50 ft to 60 ft. This situation is caused by the location of the intakes high in the dam. Water in the lower 50 ft or so of the pool is trapped all summer by its greater density and therefore maintains its winter temperature from one winter to the next. Water flows through the pool all summer over the top of the colder water below.

Many more examples of the effects of reservoirs on water temperatures are available, and some of them will be cited subsequently in connection with other aspects of water quality.

Deep storage reservoirs that release cold water all summer provide a great economic benefit to downstream industrial plants using river water for steam condensing and other cooling purposes. However, this tangible benefit has not yet received the attention it deserves.

Turbidity.—The effectiveness of storage reservoirs in reducing turbidity is so obvious that little discussion is needed thereon. The pools serve as tremendous settling basins, in which most of the suspended matter settles out of suspension. The pools are especially effective in reducing the turbidity resulting from intense summer rains of short duration. Only heavy, protracted winter rains that produce relatively large volumes of runoff (in relation to the storage capacity) produce significant turbidity in the outflow. No density underflow

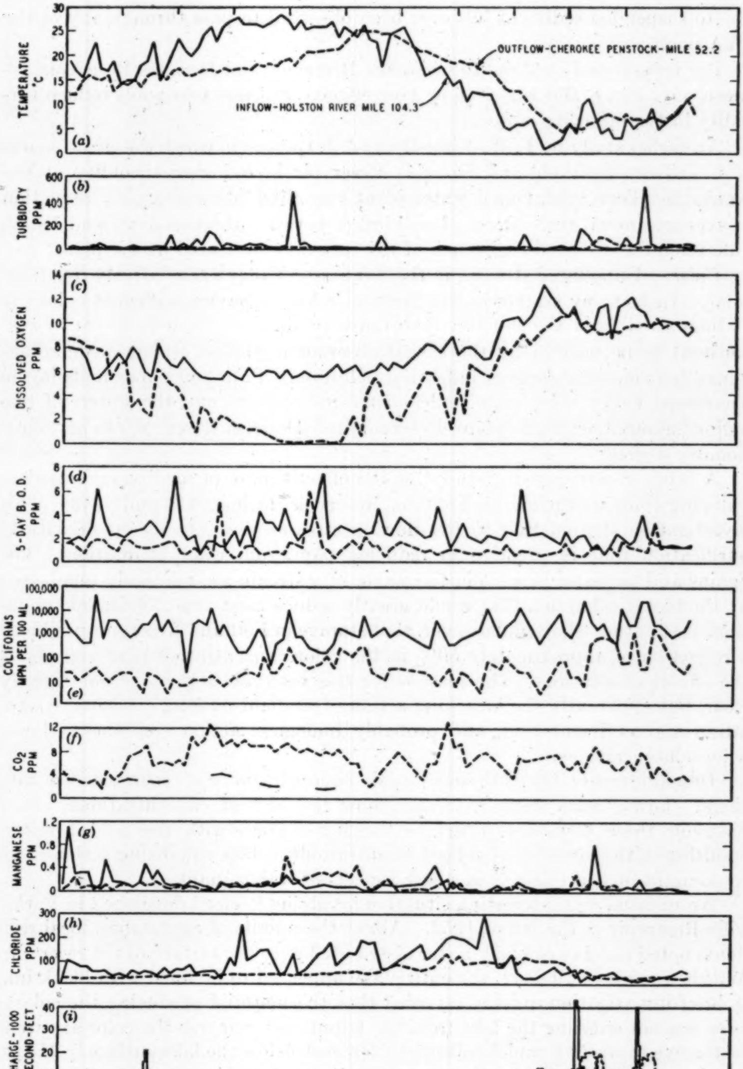


Fig. 8.—Effects of Cherokee Reservoir on Water Quality

due to suspended sediment has ever been observed to pass through any of the TVA pools.

For inflows and outflows to Cherokee Reservoir and Douglas Reservoir, respectively, Fig. 8 and Fig. 9 show how effectively these two pools reduce turbidity in the inflowing water.

An earlier study by F. W. Kittrell and J. J. Quinn,[3] in which the effectiveness of Cherokee Reservoir and Douglas Reservoir in reducing turbidity at the Knoxville (Tenn.) municipal water plant was cited, showed a 61% reduction in average annual turbidities. Local inflow between the two dams and Knoxville undoubtedly produced much of the turbidity measured at the plant.

Color.—Unpolluted streams in the Tennessee Valley are relatively free from color. In fact any stream in the Tennessee Valley having sufficient color (as distinguished from the muddy appearance produced by soil particles in suspension) to be noticed by the casual observer is almost certain to have acquired it from some form of industrial pollution. Pulp and paper mills in the Tennessee Valley color many miles of natural streams and the waters of two major impoundments (Fontana Reservoir and Douglas Reservoir) to an objectionable degree.

A fairly comprehensive study[4] of the effectiveness of an impoundment in reducing color was made on Fontana Reservoir during 1948 and 1949. This investigation showed that in free-flowing streams the only means of natural purification that is effective in reducing paper-mill color is dilution. The lignins and tannates present in the waste in a stream are relatively unaffected by the biological action that would greatly reduce most types of organic pollution. The study showed, however, that storage in Fontana Reservoir produced a reduction of approximately 50% in the color concentration over and above the effects of dilution. The reasons for this reservoir action are not entirely clear, but apparently the bleaching action of sunlight on long exposure, coagulation and sedimentation, and probably biological action combine to lower color concentrations.

Impoundments "smooth out" peak concentrations of color carried into them. Low stream flows naturally show the highest concentrations. In a reservoir these flows are mixed to varying degrees with the pooled water, resulting in the lowering of color concentrations. This equalizing action of an impoundment is important in many aspects of water quality.

An unusual and interesting situation involving "color" developed in Parksville Reservoir in the fall of 1952. About the middle of September, local residents noted patches of black water distributed over the surface of the reservoir. Within a week or 10 days, the entire lake appeared to be filled with black ink. A brief investigation quickly revealed that the material producing the coloration was not entering the lake from the tributaries, nor was the color apparent to the eye in water samples collected a few feet below the lake surface. Microscopic examination showed that the color was produced by something in suspension rather than in solution. Algae were suspected, and examinations of

---

[3] "Multi-Purpose Reservoirs Aid Downstream Water Supply," by F. W. Kittrell and J. J. Quinn, *Engineering News-Record*, May 26, 1949.

[4] "Natural Reduction of Paper Mill Color in Streams," by M. A. Churchill, *Sewage and Industrial Wastes*, Vol. 23, No. 5, 1951, p. 661.

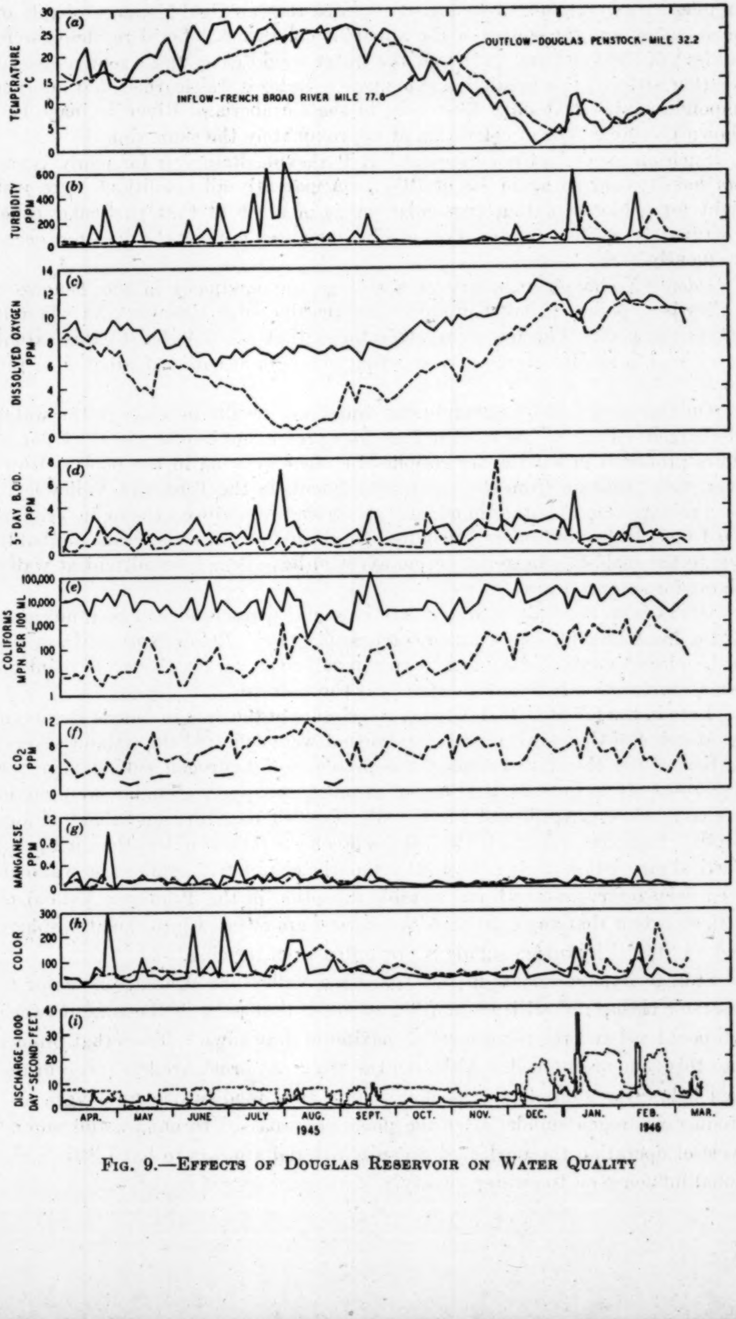


FIG. 9.—EFFECTS OF DOUGLAS RESERVOIR ON WATER QUALITY

samples by several aquatic biologists revealed that an algal bloom, probably of microcystis, was the source of the apparent coloration. Local residents were advised of the facts and were told the water would clear up as soon as cooler weather arrived; this prediction came true. Later it was learned that another impoundment, Great Falls Reservoir, in the Cumberland River drainage had shown the same type of coloration at approximately the same time.

Such an event had not happened at Parksville Reservoir for many years, nor has it occurred again (as of 1957). Apparently all conditions were just right for a bloom of that particular genus of algae at that particular time. Fortunately, such a combination of circumstances apparently does not occur frequently.

Odor.—No major tributary of a storage impoundment in the Tennessee Valley is so polluted that it produces a noticeable, objectionable odor where it enters the pool. The few minor tributaries that are polluted to this extent carry such a small volume of water that the odor becomes dissipated in the pools.

On the other hand, impoundment sometimes results in odors in the water discharged. Perhaps the most serious odor problem of impoundments is sometimes produced in local water supplies by algae growing in the pools. However, such problems from storage impoundments in the Tennessee Valley have been minor. On the other hand, in water from reservoirs on the main Tennessee River, tastes and odors attributed to algae (but, in some cases, probably due to the mold-like bacteria, actinomycetes) have been intermittent at water plants for several years.

Odors from the tailrace of a newly impounded reservoir can be a nuisance to the local area for one summer, or possibly two. Below Nottely Dam the early releases contained enough hydrogen sulfide to darken the white paint on some construction buildings located near the tailrace.

Late in the fall of 1936, following the closure of the dam in Norris Reservoir on March 4 of that year, water samples that were collected throughout several verticals near the dam revealed the presence of hydrogen sulfide near the reservoir bottom in concentrations of as much as 1 ppm. Similar sampling in January, March, April, and May, 1937, revealed no hydrogen sulfide at any depth. Samples in August, 1937, showed concentrations of less than 0.2 ppm. Vertical circulation during the winter months completely aerates the water in even deep reservoirs (with one notable exception in the Tennessee Valley) to such an extent that any existing odorous gases are released into the atmosphere and additional hydrogen sulfide is prevented from forming.

Storage impoundments in the Tennessee Valley are cleared, just prior to reservoir closure, of all trees and heavy brush that exist in the zone between full pool level and the usual level of maximum drawdown. Trees that project into this zone are topped. Although the trees and brush are left (deep in the pool below the summer thermocline) where they tend to deplete oxygen and produce hydrogen sulfide, after the pools have passed through a full annual cycle of operation the uncleared organic material appears to have little additional influence on the water quality.

BACTERIOLOGICAL EFFECTS

From the standpoint of sanitary engineering, storage impoundments are desirable barriers between upstream bacterial pollution and the downstream sites of water use. The greatest reductions in bacterial concentrations are evident in the water released during the period of thermal stratification inasmuch as released water during this period has been in storage for several months. Fig. 8 for Cherokee Reservoir shows that coliform concentrations per 100 ml in the inflow during April, May, and June, 1945, were usually in the range of from 1,000 to 10,000, whereas in the outflow concentrations were less than 50, except for one of the twenty-eight samples in this period. During July, August, and September, 1945, concentrations in the outflow were somewhat higher, reaching a maximum of 240 ppm per 100 ml. During October, November, and December, concentrations in the outflow were lower, being less than 100 ppm in all samples. During January, February, and early March, 1946, when periods of retention were much shorter and water temperatures were lower than in the stratified periods, concentrations in the outflow were frequently in the range of from 100 ppm to 1,000 ppm, and one sample in this period showed 1,600 coliforms per 100 ml.

TABLE 2.—EFFECT OF DOUGLAS RESERVOIR ON COLIFORM CONCENTRATIONS

Tributary	Drainage area at sampling station, in square miles	Number of samples	Mean coliform concentration, MPN per 100 ml
French Broad River	1,858	87	13,500
Nolichucky River	1,686	85	11,650
Pigeon River	680	88	18,500
Outflow from Douglas Penstock	4,541	87	165

The reductions noted for Cherokee Reservoir are undoubtedly much less than those that actually occurred in water passing through the pool because the untreated sewage from Morristown, Tenn., with a sewered population of 12,000, was discharged at a low elevation into Cherokee Reservoir 23 miles above the dam. The coliforms sampled in the inflow to the reservoir came principally from Kingsport, Tenn., a city with a sewered population of 18,000 located approximately 40 river miles above the head of Cherokee Reservoir; a sewage-treatment plant for Kingsport is at present (1957) under construction.

Fig. 9 for Douglas Reservoir reveals a reservoir effect on coliforms similar to that shown by Fig. 8 for Cherokee Reservoir. However, for Douglas Reservoir there are two other major tributaries, the Nolichucky River and the Pigeon River, in addition to the French Broad River. The relative sizes of the three tributaries are indicated in Table 2, together with the arithmetic mean coliform concentrations found in each river and the mean concentration in the penstock at Douglas Dam for the year covered by Fig. 9.

From Fig. 9 and Table 2 it is apparent that Douglas Reservoir reduces coliform concentration, on the average, approximately 99%. Naturally this reduction cannot be assigned to effects from the reservoir only, because a certain amount of reduction would have occurred in the open river.

A notable reduction in coliform concentrations in the raw water of the Kingsport municipal supply was immediately evident following the closure of Boone Dam 12 river miles upstream from the water-supply intake. Coliform concentrations dropped from an average level of approximately 6,000 per 100 ml for the period before the dam was closed to an average level of approximately 600 per 100 ml since closure in December, 1952. The fact that the Bristol (Tenn.-Va.) sewage-treatment plant went into operation in January, 1953, has not been overlooked. However, the reduction in coliforms at Kingsport that can be attributed to this plant is uncertain because samples near the mouth of the creek receiving the Bristol plant effluent have shown no material reduction in concentration or total numbers of coliforms. It appears that there may be a regrowth of coliforms in this stream.

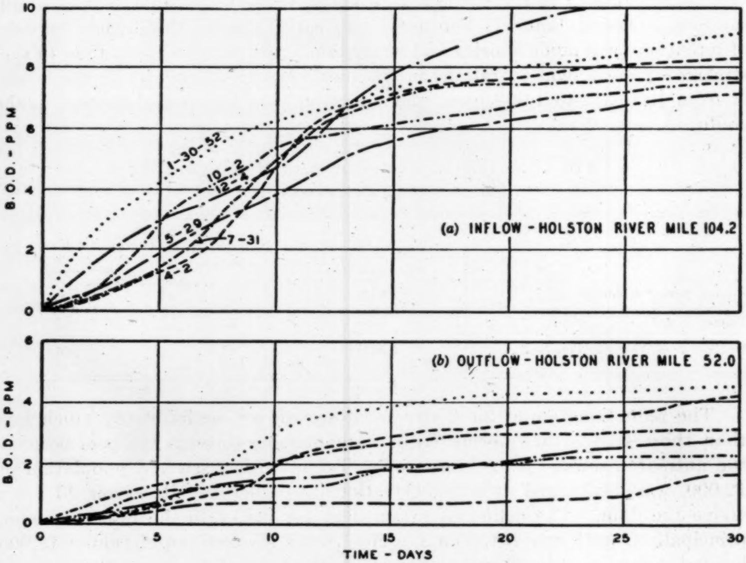


Fig. 10.—Effect of Cherokee Reservoir on Long-Term B.O.D. in 1952

Sanitary-Chemical Effects

Biochemical Oxygen Demand (B.O.D.).—When water is stored for a period of several weeks or several months, it is inevitable that the B.O.D. of the water will be substantially reduced. Fig. 10 shows a series of long-term (30-day) B.O.D.-curves for water entering and leaving Cherokee Reservoir during 1952. As indicated by the relatively low, 5-day B.O.D.-values for the inflow, the inflow to Cherokee Reservoir is not grossly polluted, but, on the other hand, there is some degree of pollution. Assuming that the inflow and outflow samples are typical, the long-term B.O.D. of the inflow is reduced approximately two-thirds by storage in the pool.

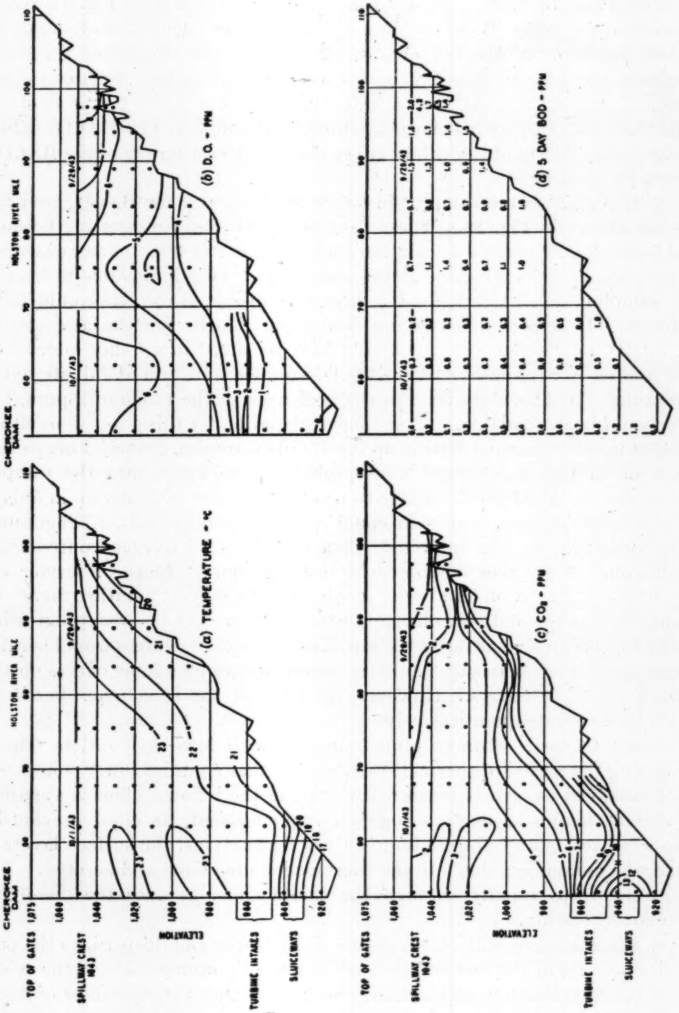


FIG. 11.—WATER QUALITY IN CHEROKEE RESERVOIR IN 1943, 2 YR AFTER CLOSURE OF DAM

Fig. 8 and Fig. 9 show reductions of 5-day B.O.D.-values by Cherokee Reservoir and Douglas Reservoir of samples collected at frequent intervals from April, 1945, to March, 1946, to be approximately 50%. Fig. 11 shows the variation in 5-day B.O.D.-values throughout the length and depth of Cherokee Reservoir in the fall of 1943, 2 yr after the closure of the dam. Throughout the downstream half of the pool, B.O.D.-values did not exceed 1.2 ppm. Because the great mass of water in the pool sampled late in September and early in October had been in storage all summer, low B.O.D.-values were expected. Many of the values are so low that the accuracy of the B.O.D.-test must be considered.

In 1946 verticals were sampled in Cherokee Reservoir for B.O.D., and the results are shown in Fig. 12. These data are particularly interesting because several 5-day B.O.D.-values deep in the pool in August and September exceeded dissolved-oxygen concentrations at the same sites. It is impossible to believe that if sampling had been delayed 5 days zero dissolved oxygen would have been found. It is probable that no detectable change (on the average) in either B.O.D. or dissolved oxygen would have occurred in a 5-day interval in August and early September. The B.O.D.-samples were run at full strength; because some B.O.D.-values deep in the pool were in the range of 2 ppm, the results should be dependable. A possible explanation for this anomalous situation is that in collecting and setting up the B.O.D.-samples, bacterial organisms not present in the water itself were probably introduced into the sample. These organisms may have been able to produce an oxygen depletion in 5 days that the original organisms present could not. According to E. J. Theriault,[5] "nitrite formation may be inhibited when the dissolved oxygen content falls below 2 ppm" in samples incubated 10 days or more. In the reservoir the water under consideration had been "incubated" for several months; therefore, undoubtedly the second, or nitrogen, stage of the B.O.D. was progressing slowly or had progressed as far as the low dissolved-oxygen values would permit.

Algae and other microscopic aquatic organisms produce some effects on the B.O.D. of reservoir waters. However, no attempt has been made herein to evaluate these effects quantitatively.

Dissolved Oxygen.—Storage impoundments with low-level outlets release water during the period of thermal stratification that contains lower concentrations of dissolved oxygen than normally exist in the inflow. This is apparent from the mechanics of flow through an impoundment, as cited previously. Due to wind action and diurnal temperature fluctuations, the surface layers of the lake, the epilimnion, down to the thermocline are mixed and aerated. The temperature drops rapidly through the thermocline, as does the dissolved-oxygen concentration.

As summer progresses the thermocline sinks deeper and deeper into the pool primarily because of the removal through low-level power intakes of the colder water of the hypolimnion, and secondly because of the summer inflow of warm water that flows into the pool above the thermocline. The epilimnion thus becomes very thick by the fall of the year and may actually extend all the way down to the level of the intakes. When the lower levels of the epilimnion

---

[5] "Detailed Instructions for the Performance of the Dissolved Oxygen and Biochemical Oxygen Demand Tests," by E. J. Theriault, *Public Health Report Supplement No. 90*, U. S. Public Health Service, Washington, D. C., 1931.

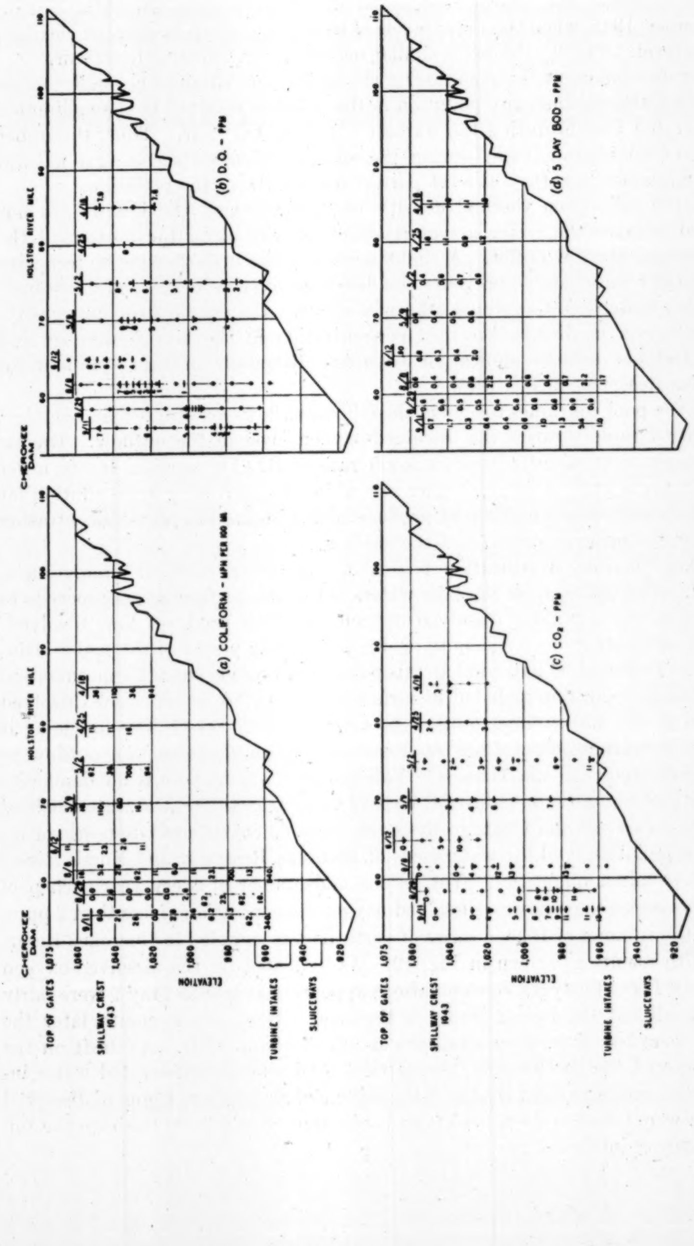


FIG. 12.—WATER QUALITY IN CHEROKEE RESERVOIR IN 1946, FIFTH SUMMER AFTER CLOSURE OF DAM

reach the intake, the dissolved-oxygen concentration in the outflow immediately increases. Fig. 5 shows the progressive lowering of the thermocline, and Fig. 8(*c*) shows the sharp increase in dissolved-oxygen concentrations in September, 1945, when the lower levels of the thickened epilimnion reached the intake level. Fig. 9(*c*) shows a similar occurrence in Douglas Reservoir.

Because inflow to the pool during the spring and summer normally enters above the thermocline, any pollution in the inflow is retained in the epilimnion and aerated therein during the warmer season of the year. Thus, the upper layers of an impoundment during thermal stratification serve as a natural aerating device to satisfy at least part of the B.O.D. of the pollution.

In the fall, when thermal density underflows exist, the inflows are not aerated because the water moves relatively slowly along the bottom of the pool toward the low outlet. A slight mixing of the underflowing waters with the warmer water above prevents the dissolved oxygen in the underflow from dropping quite as fast as would otherwise occur. Underflows probably explain the reduction in dissolved-oxygen concentration in the late September and early October releases shown for Cherokee Reservoir in Fig. 8(*c*) and for Douglas Reservoir in Fig. 9(*c*).

As the pool continues to cool in the fall, underflows are stopped and vertical circulation rapidly raises the dissolved-oxygen level in the outflow. During the winter, vertical circulation, a lower rate of B.O.D. exertion due to lower temperatures, and, usually, a higher rate of inflow (providing more dilution for upstream pollution) combine to produce high dissolved-oxygen-concentration levels in the entire reservoir and the outflow.

When thermal stratification begins in the spring, vertical circulation is halted. The pollution in the lake waters below the surface layers proceeds to "eat" into the available dissolved oxygen and thus produces low dissolved-oxygen concentrations in the outflow during the long period of thermal stratification. It should be indicated at this point that there is enough concentration of "natural" pollution included in surface runoff waters to lower the dissolved oxygen in the late-summer outflow to approximately zero. The foregoing is based on routine observations of dissolved oxygen made on the outflow of several reservoirs in the Tennessee Valley above which there is no significant quantity of domestic or industrial pollution. For example, the watersheds of Nottely Reservoir and Chatuge Reservoir have no contributed domestic or industrial pollution, and the watersheds of Watauga Reservoir and Norris Reservoir have minor quantities. Yet, in the midsummer of every year each pool produces water that shows a dissolved-oxygen concentration of less than 2 ppm.

In the summer of 1946, a series of verticals was sampled in Cherokee Reservoir with results as shown in Fig. 12. As indicated by the dissolved-oxygen values in Fig. 12, oxygen concentrations appearing as late as May 9 were fairly high in all but the lowest levels of the pool. However, a month later the values were less than 4 ppm below a depth of about 40 ft. As draft on the pool lowered the thermocline, the vertical thickness of well-aerated water increased to such an extent that near the middle of September, 4 ppm of dissolved oxygen were found as deep as 65 ft and only approximately 20 ft above the top of the power intakes.

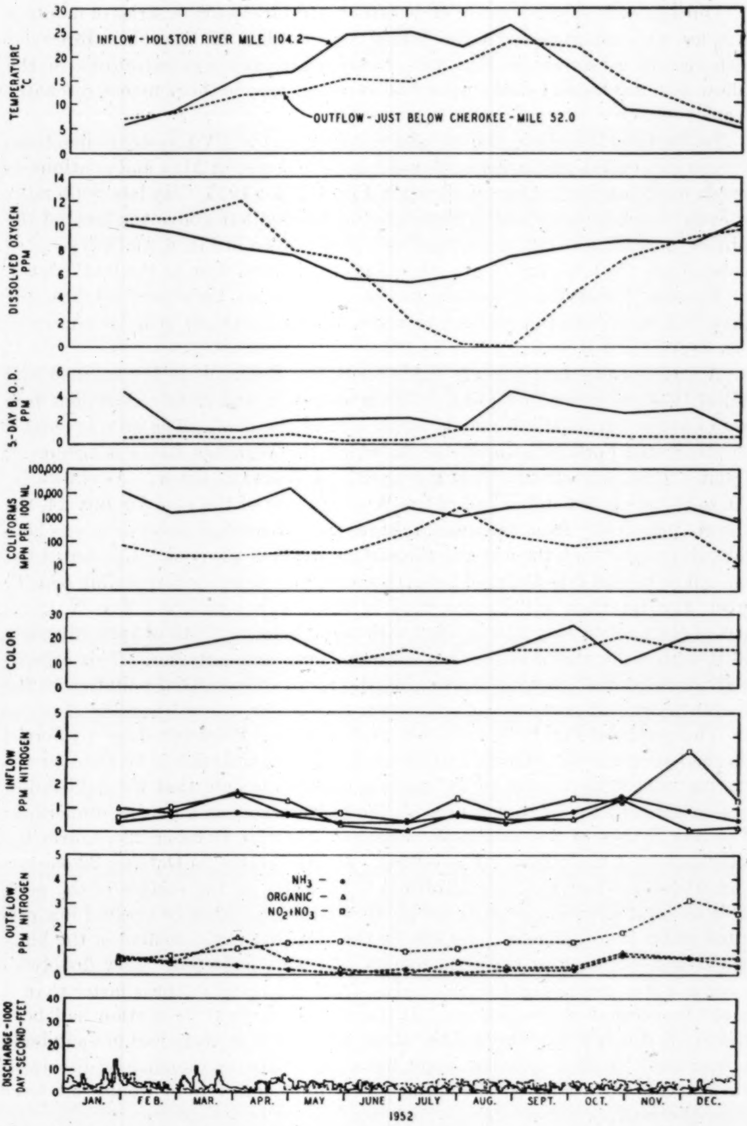


FIG. 13.—EFFECTS OF CHEROKEE RESERVOIR ON WATER QUALITY 10 YR AFTER CLOSURE OF DAM

During 1952, approximately 10 yr after reservoir closure, a series of monthly samples was collected from the inflow and outflow of Cherokee Reservoir with results as shown in Fig. 13. Dissolved-oxygen concentrations in the inflow were not found below 5 ppm, but were near zero in the penstock discharge in midsummer.

In Fontana Reservoir, the deepest reservoir in the TVA system, dissolved-oxygen concentrations in the outflow begin to decrease in May and continue to recede until late September, as shown in Fig. 6(*d*) for 1955. By late September or early October there is little thermal stratification left above the level of the intakes, according to the years, 1945 and 1946, shown in Fig. 6, and oxygenated waters from the lake surface are mixed into the lake as deep as the intake level.

Because Parksville Reservoir discharges through high-level intakes, the dissolved oxygen in the discharged water remains high all year, as shown in Fig. 7(*g*).

A series of observations was made on Boone Reservoir in the summer and fall of 1953, as shown in Fig. 14. The temperature and dissolved-oxygen data were obtained from the Watauga River arm of the pool. The dam is located on the South Fork Holston River, to which the Watauga River is tributary, about a mile downstream from the mouth of Watauga River. As shown by the isotherms in the upper end of the Watauga arm of the pool for July 14, the inflow (principally from upstream releases from Watauga Reservoir) was considerably cooler than the water in the surface strata of the pool. Consequently, the inflow passed into the pool beneath the surface at its corresponding density level; this interflow will be described in detail subsequently. The Watauga arm of the pool on July 22 was filled with water below a depth of approximately 30 ft with a low dissolved-oxygen content (zero, or nearly so). This exceedingly low dissolved oxygen is primarily the result of industrial pollution in the Watauga-arm inflow.

The graphs of Fig. 14 for October, 1953, in Boone Reservoir show a different thermal-stratification picture and dissolved-oxygen situation. In October the sloping isotherms parallel to the bottom clearly indicate that the cold inflow passed through the Watauga arm of Boone Reservoir as a density underflow. The odd shapes of the iso-dissolved-oxygen lines for October 22 confirm the location of the underflow. The bottom strata of water on October 22 showed higher dissolved-oxygen concentrations than water on the surface of the pool. This unusual situation is produced partly by the underflow of aerated but polluted water from upstream, and also by the fact that rapid cooling of the body of the lake occurred at the time the samples were collected. Low dissolved-oxygen water was brought to the surface from the middle depths faster than it could be aerated at the surface. If the rate of vertical circulation had been slower, as it would have been if the atmospheric temperature had not suddenly turned cold, surface aeration could have kept pace and such low dissolved-oxygen concentrations on the surface as 3 ppm and 4 ppm would not have been observed.

The interflow into Boone Reservoir cited previously is sufficiently important to justify a more detailed treatment. Because the interflow was actually traced into the pool by observing electrical resistance on water samples, it is necessary

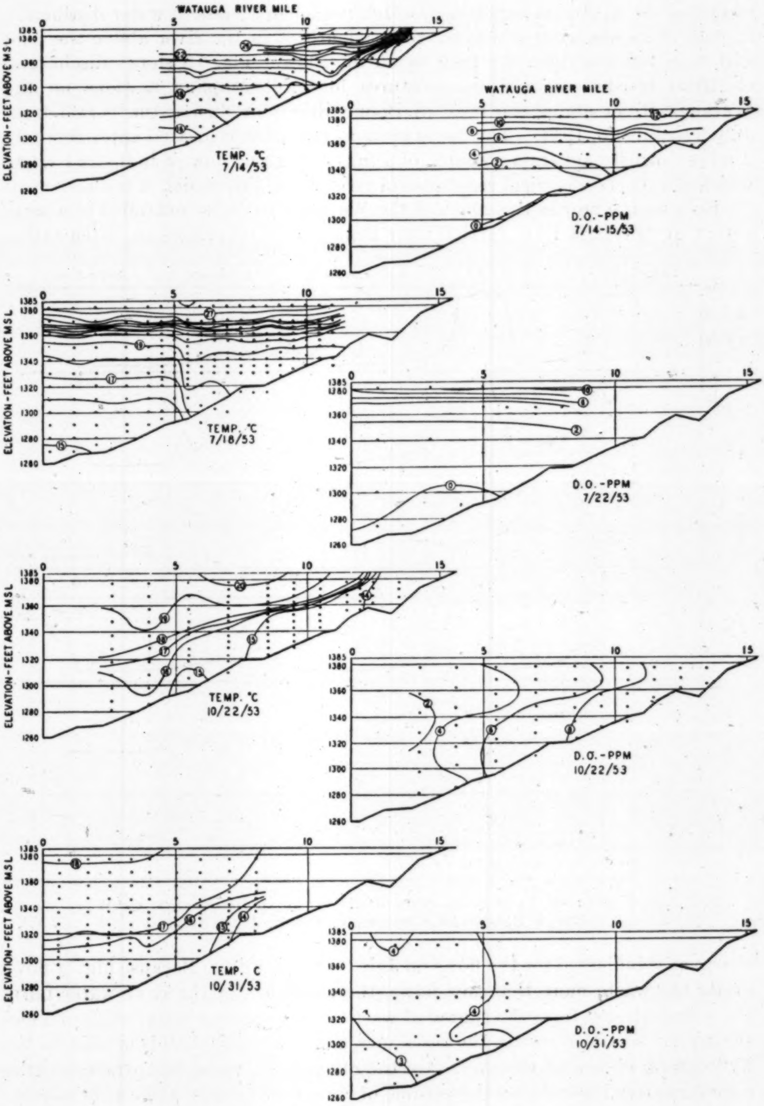


FIG. 14.—THERMAL STRATIFICATION AND DISSOLVED-OXYGEN CONCENTRATIONS IN WATAUGA RIVER ARM OF BOONE RESERVOIR, 1953

to review the upstream conditions, which produced a mass of water distinctive enough in its electrical resistance to be identified in the river above the pool and then followed into the pool as an interflow. Fig. 15 shows discharges, electrical resistances, and temperatures for two sampling stations on the Watauga River above the head of Boone Reservoir during the period from July 9 to July 13, 1953. The upper station, mile 23.0, is located approximately 2 miles downstream from a source of a fairly constant flow of industrial waste which affects the electrical resistance of river water into which it is discharged.

Because the flow in this reach of the Watauga River is controlled to a large extent by Watauga Dam (and Wilbur Dam), the fluctuations are often rather sharp, as is indicated for the flows on July 9 and July 10. Because higher flows dilute the waste more than low flows, the resistance of the river water fairly close downstream from the source of waste varies approximately as the flow of the river. For this reason the resistance graph for mile 23.0 is similar to the hydrograph of flow at this site. On the other hand, water temperature varies approximately inversely as the volume of river flow because the higher releases are heated less by the sun, local inflows, and warm industrial wastes. In travelling downstream to mile 15.5, the translatory waves of July 10 and July 12 moved quite rapidly, as indicated by the short time lag between the beginning of rise at the two stations. In the rise of July 12 the water itself did not

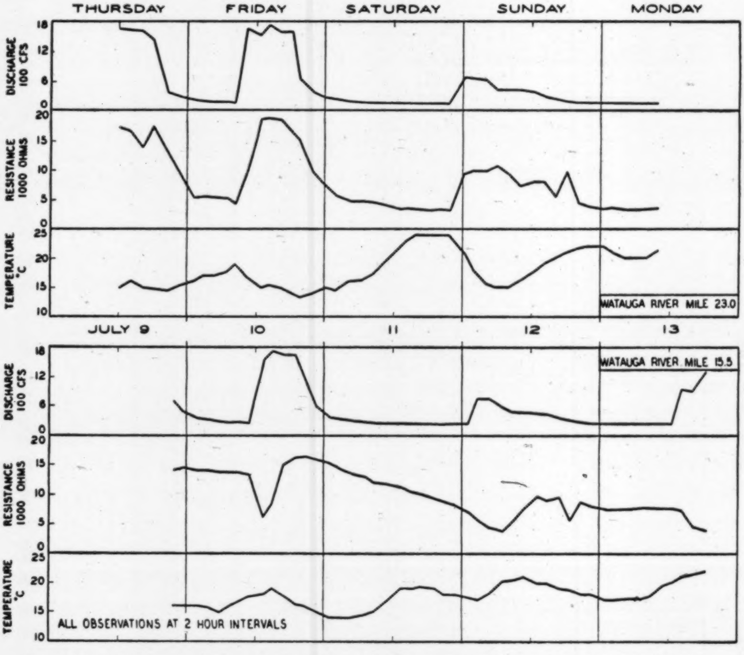
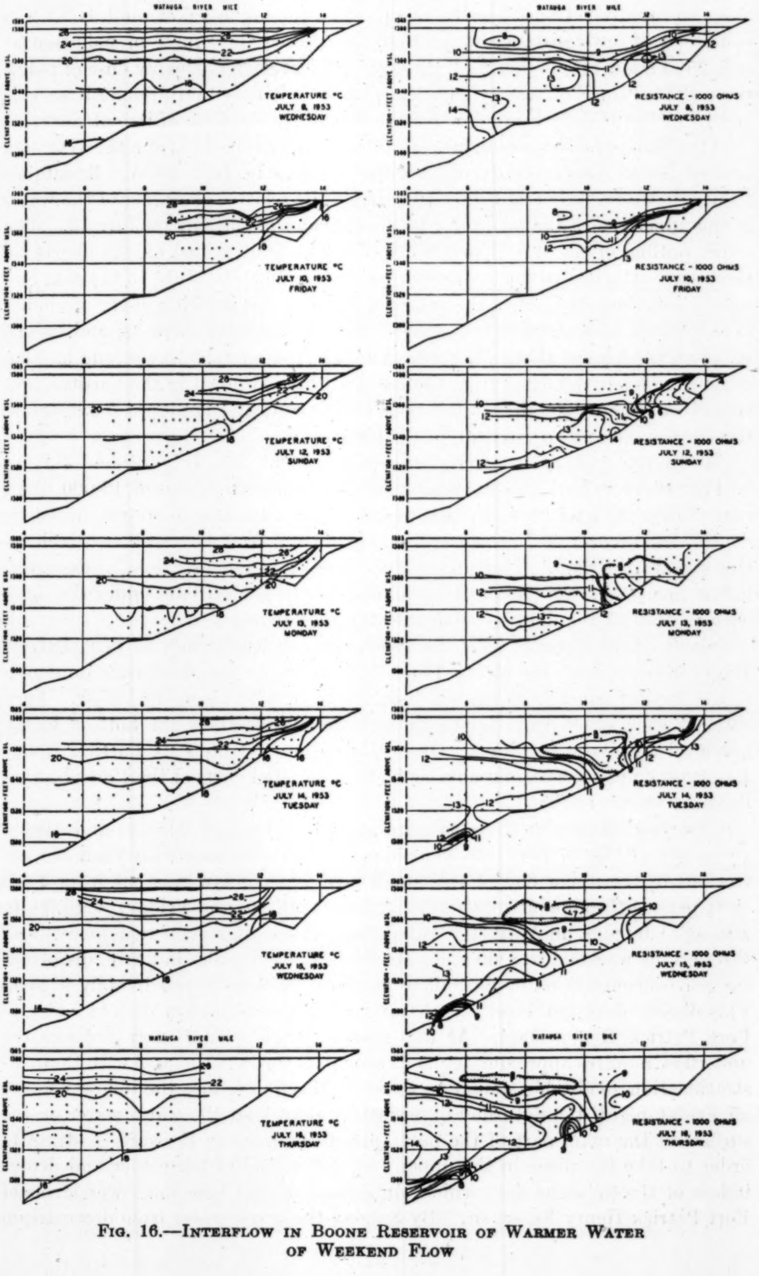


Fig. 15.—Effect of Flow Variation on Electrical Resistance of Watauga River Below Elizabethton, Tenn., July, 1953


TEMPERATURE °C JULY 8, 1953 WEDNESDAY

RESISTANCE - 1000 OHMS JULY 8, 1953 WEDNESDAY

TEMPERATURE °C JULY 10, 1953 FRIDAY

RESISTANCE - 1000 OHMS JULY 10, 1953 FRIDAY

TEMPERATURE °C JULY 12, 1953 SUNDAY

RESISTANCE - 1000 OHMS JULY 12, 1953 SUNDAY

TEMPERATURE °C JULY 13, 1953 MONDAY

RESISTANCE - 1000 OHMS JULY 13, 1953 MONDAY

TEMPERATURE °C JULY 14, 1953 TUESDAY

RESISTANCE - 1000 OHMS JULY 14, 1953 TUESDAY

TEMPERATURE °C JULY 15, 1953 WEDNESDAY

RESISTANCE - 1000 OHMS JULY 15, 1953 WEDNESDAY

TEMPERATURE °C JULY 16, 1953 THURSDAY

RESISTANCE - 1000 OHMS JULY 16, 1953 THURSDAY

WATAUGA RIVER MILE
ELEVATION-FEET ABOVE MSL
6
8
10
12
14
1385
1380
1360
1340
1320
1300
26
24
22
20
18
16
14
13
12
11
10
9
8
7

FIG. 16.—INTERFLOW IN BOONE RESERVOIR OF WARMER WATER OF WEEKEND FLOW

move so rapidly. Consequently, at mile 15.5 the highest flow on July 12 was composed of low-resistance water that had passed the source of pollution at mile 25 at a low rate of flow and, thus, received a relatively heavy dose of pollution. This "slug" of heavily polluted water was followed into Boone Reservoir and identified therein by its low resistance and by the time of its entrance.

Fig. 16 shows resistance values on water samples collected from the Watauga arm of Boone Reservoir during a series of days in July, 1953. Resistance values shown for July 8 and July 10 indicate the relative character of the water in the upper end of the pool prior to the entrance of the slug on July 12. The rough outline of the slug followed into the lake is identified on the charts by short bars attached to the 8,000-ohm resistance line. By July 13 the slug had nearly detached itself from the reservoir bottom, having hit a layer of colder, denser water at approximately mile 11 that it could not displace, and moved off down the lake at about El. 1360. On July 15 and July 16 the slug had become completely detached from the bottom, thinned down in the vertical, and spread out more laterally as the pool widens out, and it moved downstream toward the dam at approximately El. 1365.

A careful study of the resistance plots of July 10, July 12, and July 13 in Fig. 16 reveals that a mass of water having a resistance of 13,000 ohms moved into the pool, flowed down to mile 10 and El. 1340 before it found its density level (controlled primarily by temperature), and moved off at this elevation toward the dam. Thus, these data illustrate that, as would be expected, inflow to a pool seeks its density level and moves downstream under the influence of draft at the dam and inflow to the upper end.

Small impoundments with low-level power intakes, such as Fort Patrick Henry Reservoir on the South Fork Holston River, below storage impoundments are not effective in raising dissolved-oxygen levels in the inflow. Flow through such small pools during the summer passes along the bottom to the low-level outlets in the dam. Observations in 1954 on Fort Patrick Henry Reservoir emphasize certain notable factors; selected parts of the 1954 observations are shown in Fig. 17.

Dissolved-oxygen and temperature profiles of April 29, May 1, and May 2 at the top of Fig. 17 illustrate what happened to the dissolved-oxygen concentrations in the upper end of this small pool when inflow was cut from 3,400 cu ft per sec throughout most of Thursday, April 29, and Friday, April 30, to zero at 11:00 p.m. on April 30. Inflow from Boone Reservoir remained cut off through the weekend to 8:00 a.m., May 3. As the profiles indicate, dissolved-oxygen concentrations in the inflow on April 29 were approximately 4 ppm. This dissolved-oxygen level was maintained in the underflow of cold water to Fort Patrick Henry Dam. At this rate of inflow (3,400 cu ft per sec) the underflow requires approximately 3 days to pass through the pool with thermal-stratification conditions similar to those in May. Because the inflow was cut off Friday night, the Saturday observations show that the warm water on the surface of the main part of the pool moved upstream in the surface strata in order to take the place in the upper end of the pool of the colder and denser inflow of the previous day, which, on Saturday, slid into the lower levels of Fort Patrick Henry Reservoir. By Sunday the warm water from downstream

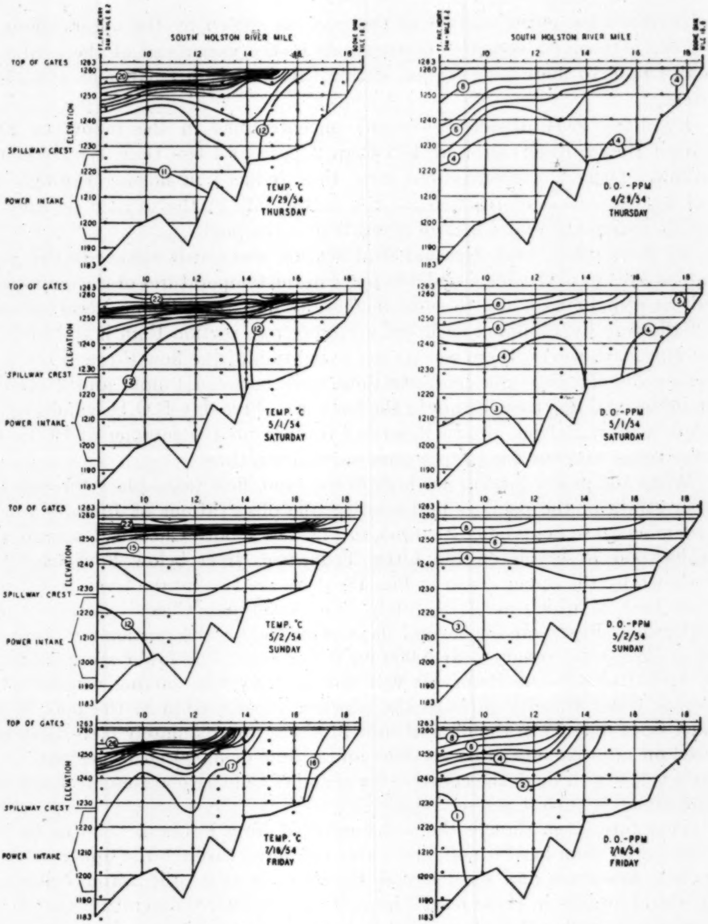


Fig. 17.—Thermal Stratification and Dissolved-Oxygen Concentrations in Fort Patrick Henry Reservoir, 1954

had covered the entire surface of the pool, as shown by the observations of May 2. Dissolved-oxygen concentrations in the upper end of the pool rose from 4 ppm to more than 6 ppm during this upstream movement of surface waters.

By July, 1954, dissolved-oxygen concentrations in the inflow to Fort Patrick Henry Reservoir were less than 2 ppm and less than 1 ppm in the outflow. Oxygen concentrations were thus reduced in flowing through the pool by the action of residual B.O.D. in the inflow, the oxygen demand of bottom sediments, and a lack of reaeration in the pool.

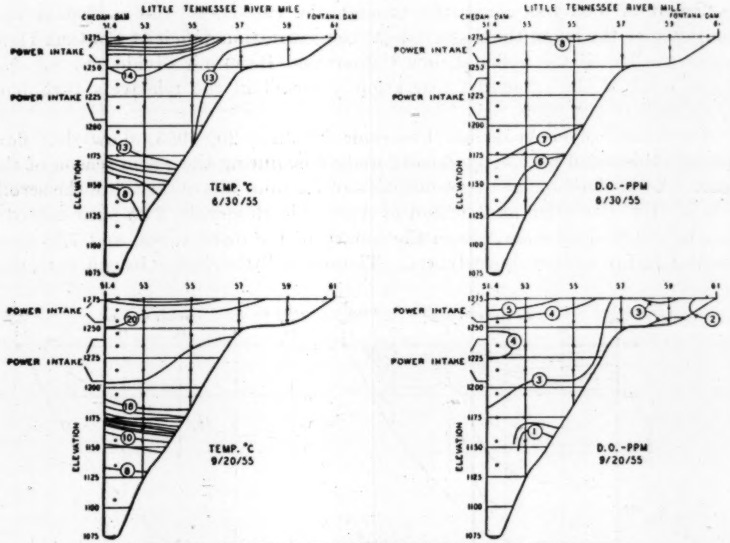
By September, 1954, thermal stratification was much weaker in the pool, and, as a result of the decreased difference in the temperatures of the epilimnion and the hypolimnion and to some increase in flow from Boone Reservoir, the underflowing hypolimnion occupied a larger cross section than it did in May and July. However, there was no net reaeration of the flow through the pool because dissolved-oxygen concentrations were between 1 ppm and 2 ppm in the inflow and less than 1 ppm in the outflow. Five-day B.O.D.-values in the inflow to Fort Patrick Henry Reservoir throughout the summer of 1954 were in the range of from 2 ppm to 4 ppm most of the time.

When the power intakes are high in the dam, flow takes place through the upper strata of the pool, as indicated by the observations of June 30 and of September 20 to September 21, 1955, in Fig. 18 for both Cheoah Reservoir and Calderwood Reservoir on the Little Tennessee River below Fontana Dam. As shown by the upper graph in Fig. 18, there are two intake levels in Cheoah Dam, both of which are fairly high. Flows through Cheoah Reservoir and Calderwood Reservoir for several days prior to the observations of June 30, 1955, were fairly steady near 5,000 cu ft per sec. The lower section (below El. 1,175) of Cheoah Reservoir was not used by the moving waters. The average water velocity through the effective cross section of the pool in the reach from mile 55 to the dam at mile 51.4 was approximately 0.15 ft per sec, based on approximate cross sections and a flow of 5,000 cu ft per sec. The mean velocity through the effective lower end of Calderwood Reservoir was also approximately 0.15 ft per sec.

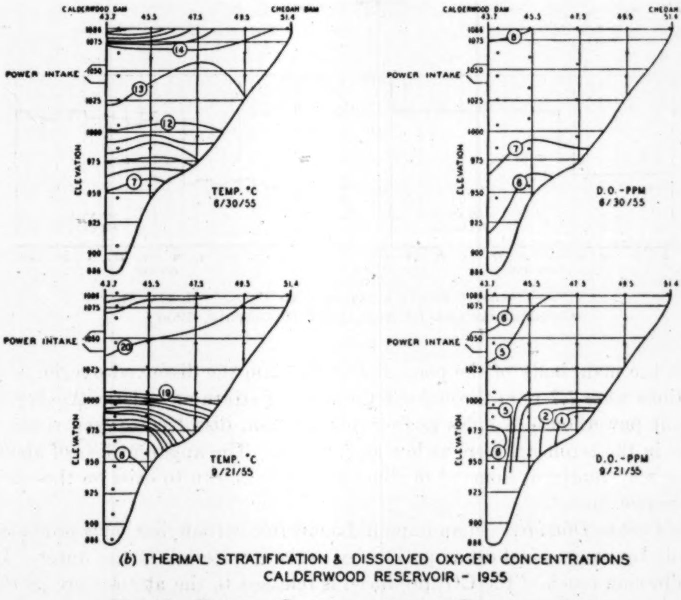
Therefore, even though the water released from Fontana Dam in June is much colder than local runoff, the water still flows through the upper levels of Cheoah Reservoir and Calderwood Reservoir because the water below the high-level intakes in these reservoirs still has a winter temperature that is the same as that below the intake level in Fontana Dam.

The dissolved-oxygen concentration in the Fontana Dam scrollcase on June 30 was 6.30 ppm, and in the tailrace, $\frac{1}{4}$ mile downstream, the concentration was only 7.20 ppm, or 72% of saturation at the existing temperature. Below Calderwood Reservoir a concentration of 7.6 ppm was observed, or 75% of saturation.

By September 20, 1955, dissolved-oxygen concentrations in the Fontana Dam outflow had decreased to 1.20 ppm in the scrollcase and 1.62 ppm in the tailrace. There was appreciable reaeration through Cheoah Reservoir and Calderwood Reservoir, and below the latter a dissolved-oxygen concentration of 5.90 ppm was observed. With the existing level of flows, it is estimated that the total time of flow through both pools was about 8 days. A reaeration



(a) THERMAL STRATIFICATION & DISSOLVED OXYGEN CONCENTRATIONS
CHEOAH RESERVOIR - 1955



(b) THERMAL STRATIFICATION & DISSOLVED OXYGEN CONCENTRATIONS
CALDERWOOD RESERVOIR - 1955

FIG. 18.—THERMAL STRATIFICATION AND DISSOLVED-OXYGEN CONCENTRATIONS IN CALDERWOOD RESERVOIR, 1955

coefficient of 0.05 was computed through the two pools; the coefficient was computed as the log of the dissolved-oxygen-saturation deficit at Fontana Dam minus the log of the deficit below Calderwood Reservoir divided by 8. No doubt most of the measured reaeration occurred in the relatively turbulent upper ends of the pools.

Observations on Apalachia Reservoir for June 29, 1955, show that flow through this small pool is by density underflow during the warm season of the year. A dissolved-oxygen concentration of 7.2 ppm was observed in the scrollcase of Hiwassee Dam at the head of Apalachia Reservoir, 7.35 ppm approximately 100 ft downstream from the mouth of the draft tubes, and 7.38 ppm about 1,100 ft farther downstream. There was little change in concentration through the main body of the pool, and at the dam the dissolved-oxygen concentrations were 7.7 ppm throughout the moving stratum. At the Apalachia Reservoir powerhouse, 8 miles farther downstream, dissolved-oxygen concentrations in the scrollcase were as low as 7.2 ppm. The apparent loss of about 0.5 ppm may represent demand of slime materials known to exist on the walls of the 8-mile tunnel.

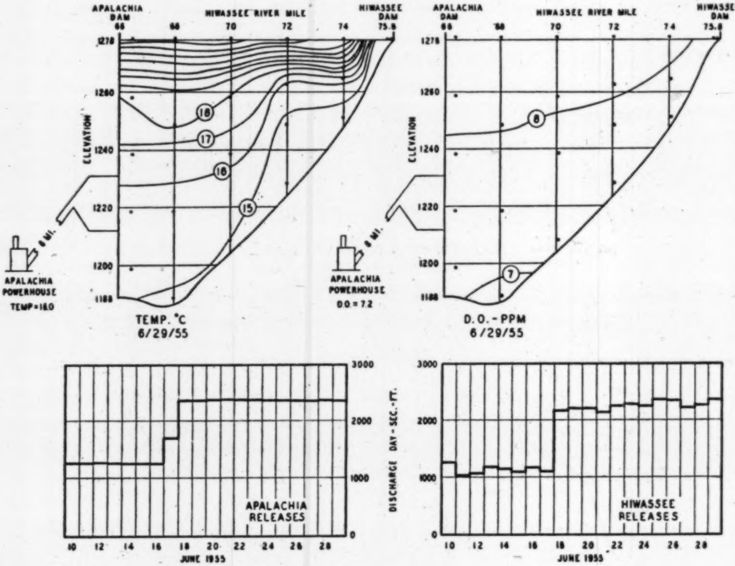


FIG. 19.—THERMAL STRATIFICATION AND DISSOLVED-OXYGEN CONCENTRATIONS IN APALACHIA RESERVOIR, 1955

Free Carbon Dioxide.—In an impoundment, free carbon dioxide is produced primarily by biochemical decomposition of organic materials in the water. In the epilimnion much of the CO_2 produced is released to the atmosphere as the water is "turned over" by diurnal temperature fluctuations and by wind action.

In the surface layers of the epilimnion, algae, through photosynthesis, are very effective in reducing concentrations of CO_2, thus raising the pH. In the hypolimnion the free CO_2 produced by bacterial metabolism is kept in solution by the greater hydrostatic pressure existing at all points below the thermocline. Because algal use of CO_2 below the thermocline is relatively minor and because little, if any, can escape, concentrations of CO_2 may increase to rather high values in the lower levels of a deep impoundment during periods of thermal stratification.

Fig. 12 is perhaps the best available illustration of the changes in the concentration of CO_2 produced for Cherokee Reservoir. In the summer of 1946 the surface layers of water, to a depth of 55 ft on August 29, showed high pH-values (from 8.0 to 8.2), a total alkalinity of approximately 80 ppm, and no free CO_2. On August 5, concentrations of CO_2 as high as 30 ppm existed near the bottom of the reservoir. Values plotted in Fig. 12 are not as high because the CO_2 was determined in the field by titration for the 1946 observations, and a certain amount of CO_2 may have escaped in the titration process. Graphic determinations[6] of CO_2-concentrations based on the pH and total alkalinity indicate that as much as 30 ppm existed, although the titration procedure indicated only 13.5 ppm as a maximum. Near the bottom, pH-values of 6.8 were observed with alkalinity values near 100 ppm.

Additional data on the effects of impoundment on CO_2 are given in Figs. 8(*f*), 9(*f*), and 11(*c*). The concentrations shown were also obtained by field titration with N / 44 sodium hydroxide to the phenolphthalein end point.

Fig. 8(*f*) shows that in the water released from Cherokee Dam at Holston River, mile 52.2, CO_2-concentrations in the summer of 1945 were in the range of from 9 ppm to 12 ppm by titration, but from 13 ppm to 18 ppm by graphic determinations. While flowing downstream to Holston River, mile 1.8, the graphically determined CO_2-concentrations were reduced to from 4 ppm to 6 ppm. Because the partial pressure of CO_2 in the open atmosphere is normally very low, most of the free CO_2 in the discharged water was released to the atmosphere. Inflow to Cherokee Dam in 1945 showed that graphically determined CO_2-concentrations were also in the range of from 4 ppm to 6 ppm.

Nitrogen.—Some interesting data on the effects of impoundments on the various forms of nitrogen were obtained from samples in, and downstream from, Boone Reservoir. The inflow to Boone Reservoir carries a heavy load of ammonia from an upstream industrial plant. The time of flow from the plant to the reservoir is so short that it precludes much conversion of ammonia to nitrites and nitrates in the open river. When this waste enters the lower levels of the pool in a density interflow, as noted previously, conversion takes place. However, when the available dissolved oxygen is exhausted, no more oxidation can occur until water is released from the Watauga arm of the pool and has an opportunity to mix with the underflowing waters of the South Fork Holston arm. These waters contain dissolved oxygen, so that conversion to nitrites and nitrates can proceed, assuming that there is a sufficient supply of the nitrifying organisms, nitrosomonas, and nitrobacter.

---

[6] "Graphic Determination of Carbon Dioxide and the Three Forms of Alkalinity," by E. W. Moore, *Journal*, A.W.W.A., Vol. 31, No. 1, 1939, p. 51.

An unusual aspect of the foregoing situation in the Watauga arm of Boone Reservoir is that, although dissolved-oxygen concentrations in the lower levels were zero in 1953, the water in samples from the lower levels did not have a hydrogen-sulfide odor. This condition apparently exists because the carbonaceous demand of the organic pollution was satisfied by the available dissolved oxygen, and the nitrogeneous demand was biochemically unable to reduce sulfates to sulfides. The possibility also exists that if the available dissolved oxygen in the inflowing waters was not sufficient to supply the carbonaceous demand, reduction of the nitrates that were present (by upstream oxidation) would supply the necessary oxygen and thus prevent odors.

It is apparent that an ammonia waste can pass through a reservoir and still have a considerable oxygen demand, which would reduce the rate of net gain in oxygen concentrations in the open river below the dam. Any oxygen in the nitrates and nitrites of the outflow would be available to control odors that might be produced by downstream organic pollution.

Fig. 13 demonstrates that low concentrations of ammonia and organic nitrogen passed through Cherokee Reservoir in 1952. There was some conversion to nitrites and nitrates but not as much as might be expected. The concentration of nitrifying organisms may have been too low for any more conversion. However, no data are available to indicate the existing concentrations of such bacteria.

Mineral Quality Effects

Iron and Manganese.—A question that is frequently raised regarding the effects of impoundments on downstream water quality concerns the presence of iron and manganese. The answer is that, with one important exception, there have been no significant difficulties in downstream water supplies below any of the TVA storage projects.[7]

Iron and manganese in the soluble (ferrous and manganous) forms cause staining in municipal water systems. When these forms are oxidized to the ferric and manganic states prior to filtration, the resulting particulate matter can be filtered out. In an uncontrolled open river, any soluble iron and manganese brought into the stream in ground water is oxidized by the dissolved oxygen of the stream and is thereby converted to an insoluble state. If the stream flows into a deep reservoir, the oxidized forms of these two minerals may be reduced to a soluble state. All iron and manganese in the water that is caught below the thermocline in the spring, when stratification begins, is subject to being put into solution even though the particulate matter may have settled to the bottom of the pool. Furthermore, any oxidized iron and manganese that is brought into the pool in the warm season may settle out of the epilimnion into the hypolimnion and thus be subject to solution.

Fortunately for the water supplies of the upper Tennessee Valley area, significant commercial deposits of manganese are apparently limited to the northeastern section of the valley,[8] in the drainages of the Holston River and Nolichucky River. These major deposits occur along the western front of the

---

[7] "Effects of River System Development on Water Quality in the Tennessee Valley," by F. W. Kittrell and F. W. Thomas, *Journal*, A.W.W.A., Vol. 41, September, 1949, p. 777.

[8] "Industrial Resources of Tennessee," Tennessee State Planning Comm., Nashville, Tenn., 1945.

Unaka Mountains. Several other areas have minor deposits, but they are less significant. Iron deposits, however, are fairly widespread over the valley.

It might be expected from the distribution of manganese deposits and the locations of manganese mining and ore-washing operations that the most serious water-supply problem due to manganese would occur below South Holston Reservoir, with perhaps less difficulty below Boone Reservoir, Fort Patrick Henry Reservoir, Cherokee Reservoir, and Douglas Reservoir. To date (1957) the only significant problem has developed at the Bristol (Tenn.) water plant. This plant has its intake on the South Fork Holston River, 1 mile below South Holston Dam and its powerhouse. South Holston Reservoir, when full, is 250 ft deep at the dam (Table 1). The power intake comes off the pool at approximately 150 ft below full-pool level, and 100 ft above the bottom of the original river channel at the dam. During normal operations at South Holston Dam, water released through the single power unit supplies municipal use at Bristol. To provide for a continuity of supply at Bristol when the turbine must be unwatered for routine inspection and maintenance work, a small fixed-dispersion cone valve was installed at the bottom of the dam in the diversion tunnel used during dam construction to bypass the river.

The dam was closed in November, 1950, and the low-level outlet was not needed until February 8, 1952, for the first routine inspection of the turbine. The water released was satisfactory for conventional treatment by the Bristol plant. The second inspection was made during the period from October 25, 1954, to November 1, 1954. Operating experiences at the Bristol filter plant are summarized from a letter by the filter-plant operator, Odell W. Gray:

> "1. Saturday, October 23. The filter plant operators were notified by TVA that water from South Holston Reservoir would be shut off Monday, October 25, to Friday, October 29. The operators were asked to call TVA for water in case of water shortage at the raw water intake and to call when the intake pool was full in order not to waste water.
>
> "2. Wednesday, October 27. Water was low at raw water intake, pumps were pulling air. Pumps were shut off four hours. TVA was called for water. The water received was high in color, turbidity, taste and odor. The chlorine demand was 15 ppm. It was necessary to increase alum and chlorine rate of feed and start the application of activated carbon.
>
> "3. (*a*) Friday, October 29. Water was low at raw water intake. TVA was called for water.
>
> (*b*) At 9:30 p.m. evidence of presence of manganese in raw water noted by operator on duty. Six ppm manganese, and 15.0 ppm iron were present in raw water. Concentrations in finished water reached a high of 2.0 ppm manganese and 0.1 ppm iron. The operators realized the chlorine feed control apparatus could not supply the demand and so obtained a diffuser with which they were able to feed an additional five pounds per hour by keeping cylinder warm. By the use of the additional chlorine for oxidizing and by throttling the raw water pumps to 2 MGD in order to obtain a rate of 30 ppm chlorine applied, they were able to reduce the manganese concentration from 2.0 ppm to 0.8 ppm in the finished water.
>
> "4. Sunday, October 31. Water was low at raw water intake. TVA was called for water. At 4:30 p.m. manganese 5.0 ppm, iron 15.0 ppm in raw water. Manganese 0.8 ppm, iron 0.1 ppm in finished water.
>
> "5. Monday, November 1. At 2:00 p.m., manganese was 6.0 ppm (off color), and iron was 3.0 ppm in raw water. Manganese 0.0 ppm, iron 0.0

> ppm in finished water. The turbine at South Holston Dam was operated for short period Monday p.m.
>
> "6. Tuesday, November 2. At 2:00 p.m., manganese was 0.6 ppm in raw water, 0.0 ppm in finished water. At 8:30 p.m. manganese 0.5 ppm in raw water, 0.0 ppm in finished water.
>
> "From Tuesday, November 2, to November 11, the operators had a normal raw water with manganese concentrations of 0.2–0.3 ppm and iron concentrations of 0.1–0.2 ppm.
>
> "During the above period the operators had from 75 to 100 complaints and inquiries concerning the water with variations of from a mere comment to threats to sue for damages. Many people suffered damages to wearing apparel, linens, etc. from the manganese stain which brought threats to sue. The City of Bristol, Tennessee, suffered considerable damage due to the additional chemicals used in treating the water. The experience proved to be most embarrassing to the City Commissioners as well as to the entire water department personnel."

When the situation at South Holston Reservoir was investigated, a series of water samples was collected from the pool just upstream from the dam. The data on these samples are shown in Fig. 20(*a*). The extremely high concentrations of 50 ppm of iron and 18 ppm of manganese at the bottom of the pool were at the same level as the low-level outlet in the dam.

At the elevation of the power intakes, 100 ft above the bottom of the pool, the iron and manganese concentrations were very low, as indicated in Fig. 20(*a*) and Fig. 20(*b*). It is not known why the dissolved-oxygen concentrations were so low in the region of the thermocline and somewhat higher just below. Dead algae and other decaying aquatic organisms, existing at least temporarily at this level of sharply increased density, may have been the cause, or a density interflow of muddy water a week prior to the sampling may have been the reason.

The lowest strata in the pool were almost devoid of oxygen, as might be expected in the layer of soluble iron and manganese. The water samples collected from near the bottom of the pool were murky but not colored. However, after being handled and after standing in the laboratory for a few days, the bottom samples acquired a typical, dense iron color.

Although there were no more difficulties at the Bristol plant after normal discharges were resumed through the power intake, it was expected that as the lake began to circulate vertically to the bottom there might be a noticeable increase in iron and manganese at the Bristol water plant; such an increase was not reported. More samples were collected from the reservoir on March 10, 1955, the results of which are shown in Fig. 20(*b*). Oxygen concentrations were still zero at the bottom and were low for almost 20 ft above the bottom. Correspondingly, high iron and manganese concentrations were still present in the bottom waters.

A total solids determination was run on all samples to determine why vertical circulation was not complete at this time of the year. The bottom samples were materially higher in mineral content. The increased mineralization in the bottom water, together with its relatively low temperature, produced a stratum on the bottom that was more dense than it was possible for the upper waters to attain even at the maximum density temperature of 4°C. Hence, the lake failed to turn over completely.

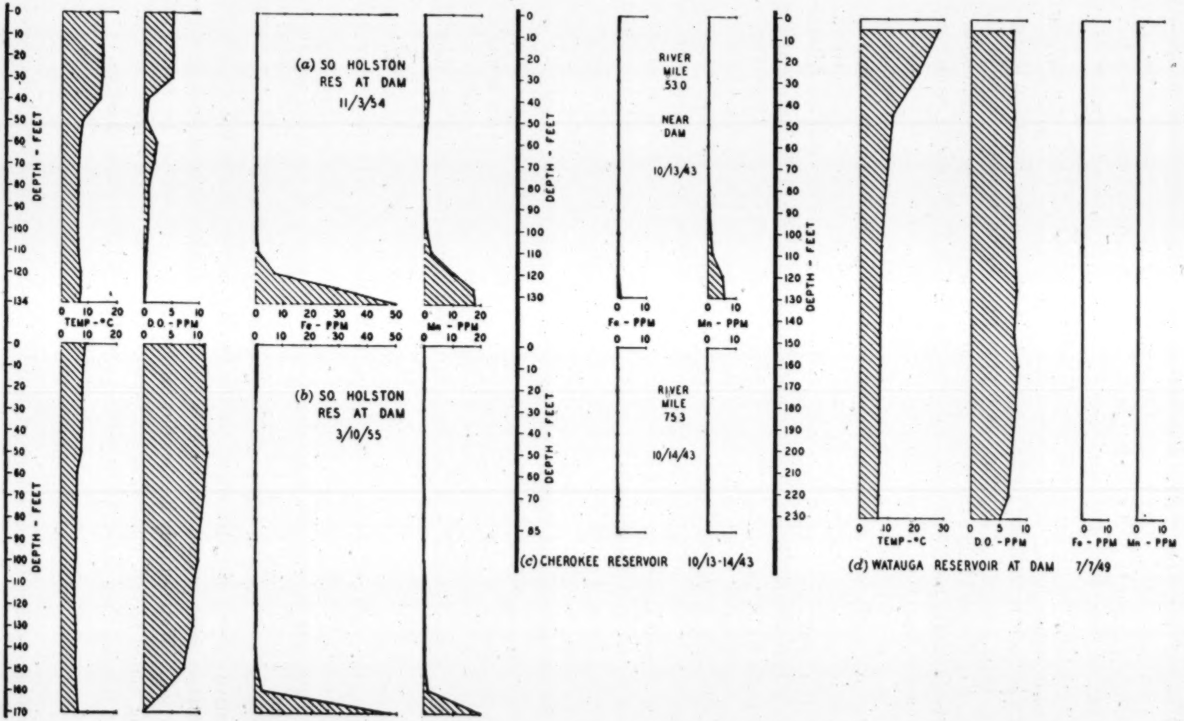


FIG. 20.—IRON AND MANGANESE IN SOUTH HOLSTON RESERVOIR, CHEROKEE RESERVOIR, AND WATAUGA RESERVOIR

Routine water-temperature observations by the Hydraulic Data Branch of the TVA have consistently revealed that the water at the bottom of the lake is slightly warmer than that higher in the pool. Because all water has a temperature of more than 4°C, the increased density due to dissolved minerals is probably the reason for the warmer water staying at the bottom.

An investigation of the presence of iron and manganese in Cherokee Reservoir was made in October, 1943 (Fig. 20(*c*)). Little iron was found, but manganese concentrations of as much as 6 ppm were found near the dam in the bottom of the pool below the level of the turbine intakes. The manganese was probably slowly mixed with the upper waters during the following winter and discharged from the pool, and did not affect the downstream water supply.

Fig. 8(*g*) and Fig. 9(*g*) show one year's experience with manganese in the inflow and outflow of Cherokee Reservoir and Douglas Reservoir during the period from April, 1945, to March, 1946. These results show the concentration of total manganese and do not indicate how much is in the soluble reduced form. Kittrell and Quinn[7] have indicated that breakpoint chlorination at the Knoxville water plant, below both Cherokee Dam and Douglas Dam, apparently precipitated some, if not all, of the manganese on the filter sand.

Although Watauga Reservoir is geographically close to South Holston Reservoir, it is geologically well separated because there are no significant deposits of manganese on its watershed. On July 7, 1949, a series of samples taken throughout the entire depth of 230 ft revealed low concentrations of iron and manganese (Fig. 20(*d*)). An industry requiring no manganese in its process water is located a few miles downstream from Watauga Dam. No difficulties due to iron and manganese have been reported.

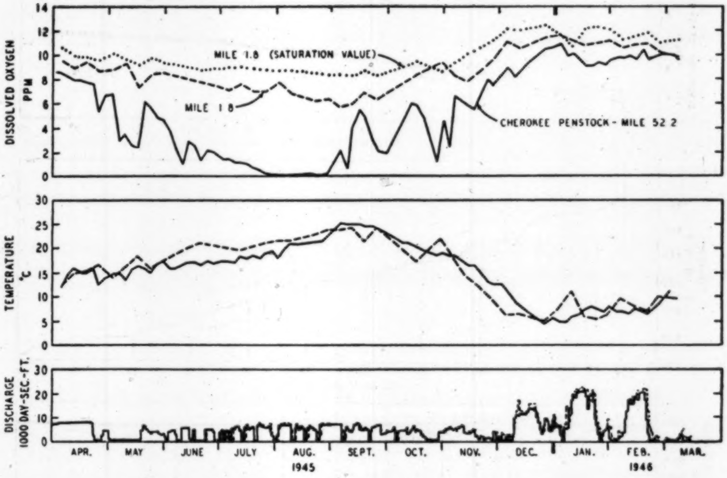
General.—A storage reservoir has an equalizing, or smoothing, effect on mineral quality in general. Peaks and valleys of concentrations are smoothed out so that water of more uniform mineral quality results downstream.

Fig. 8(*h*) shows a good example furnished by the variation in chloride content in the inflow and outflow of Cherokee Reservoir. Industrial pollution upstream from the pool produced peak concentrations of as much as 300 ppm during the period of observations. The peak concentration in the outflow was 136 ppm in October when there were underflows.

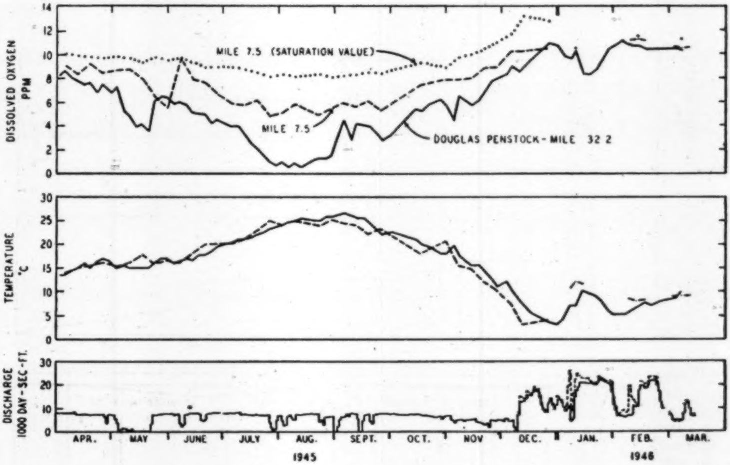
There are substantial beneficial effects of a reservoir on water hardness. Runoff during above-normal flows is low in hardness, as it is in all dissolved minerals, and much of this water is stored in the pool for later release. The relative volume of water stored during the low-flow period of the year, when relatively hard ground water makes up almost all the inflow, is small by comparison. Water-supply intakes downstream from the reservoir are spared the long period of high mineralization that usually occurs in the summer and fall, and instead they are supplied with water of much lower mineralization.

Reaeration Below Storage Impoundments

There is little factual information available on the rate at which deoxygenated water released from impoundments becomes reaerated in river channels downstream. Because the released water ordinarily has a low B.O.D., the rate



(a) REAERATION IN 50 MILES OF HOLSTON RIVER BELOW CHEROKEE DAM



(b) REAERATION IN 25 MILES OF FRENCH BROAD RIVER BELOW DOUGLAS DAM

FIG. 21.—REAERATION IN 25 MILES OF FRENCH BROAD RIVER BELOW DOUGLAS DAM

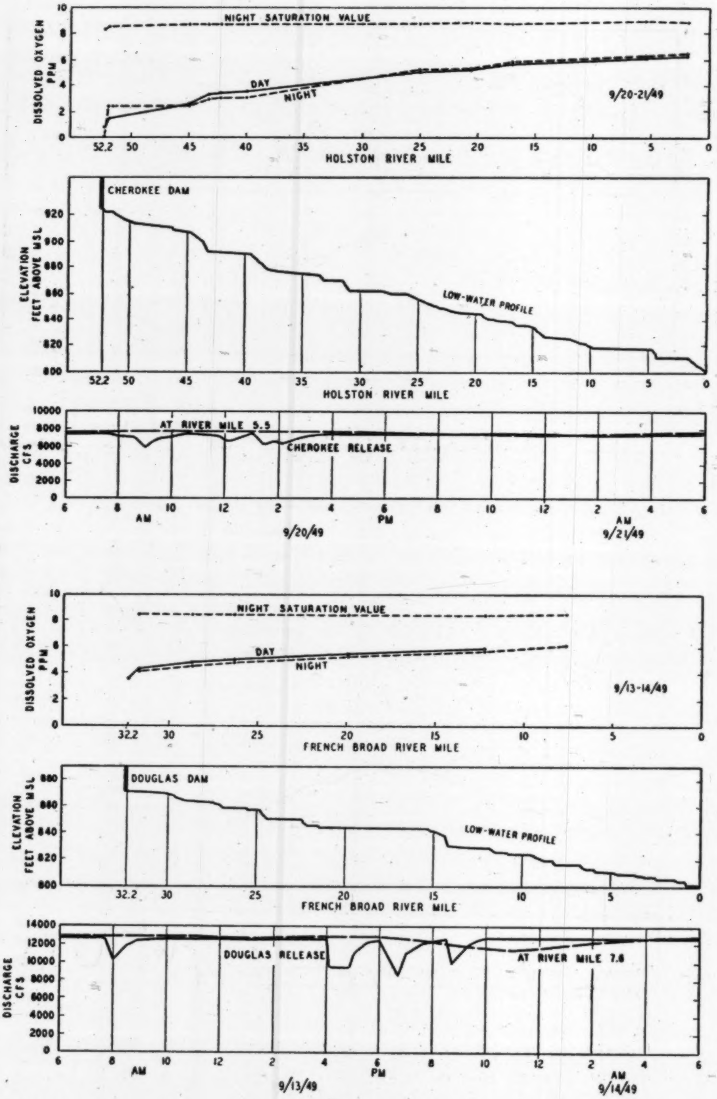


FIG. 22.—REAERATION IN LOWER HOLSTON RIVER AND LOWER FRENCH BROAD RIVER IN SEPTEMBER, 1949

of reaeration is not retarded, nor its measurement confused, by the demand for oxygen of upstream pollution.

Fig. 21 shows observed reaeration in the lower Holston River and the lower French Broad River in 50 miles and 25 miles, respectively, of open-river channel. Local inflow is small into both river reaches. No significant quantities of pollution are added in the local inflow. The B.O.D.-values are indicated in Fig. 8(*d*) and Fig. 9(*d*) to be very low, being ordinarily less than 2 ppm in 5 days. However, little information is available on the time of flow through these river reaches. It is known from actual observations[9] that water flowing at 7,900 cu ft per sec requires 24.5 hr to traverse this 50-mile reach of the Holston River at an average velocity of 3 ft per sec. During the period of greatest dissolved-oxygen deficit at the dam (Fig. 21), the extent of reaeration was approximately 6 ppm in the 50-mile reach, starting near zero at the dam. Although the pickup was 6 ppm, or slightly more, the water still lacked 2 ppm of reaching saturation for the temperatures existing at the lower end of the reach.

The extent of oxygen pickup in the lower French Broad River during August, 1945, was approximately 4 ppm, starting with 1 ppm at the dam, and saturation was about 8 ppm. Thus, there was a deficit of 3 ppm (almost 37% of saturation) still remaining at the lower end of the 25-mile reach.

Fig. 22 shows low-water profiles of the lower Holston River and lower French Broad River. The slope of both rivers in these reaches is approximately the same—about 2.2 ft per mile.

Reaeration at 7,400 cu ft per sec in the lower Holston River from September 20 to September 21, 1949, progressed rather uniformly from a site 0.2 miles below the dam (mile 52.0) to mile 1.8, increasing from near 2 ppm below the dam to 6.4 ppm at the lower end of the reach. The exact time of travel is not known, but it is assumed to be only slightly longer than that for 7,900 cu ft per sec, or approximately 25 hr. The reaeration coefficient, C_r, for the entire reach is computed as 0.48 from

$$k_2 = \frac{\log D_{doA} - \log D_{doB}}{t_{AB}} \qquad \text{(1)}$$

in which D_{doA} is the deficit in the dissolved-oxygen concentration at A; D_{doB} is the deficit in the dissolved-oxygen concentration at B; and t_{AB} is the time, in days, for water to travel from A to B.

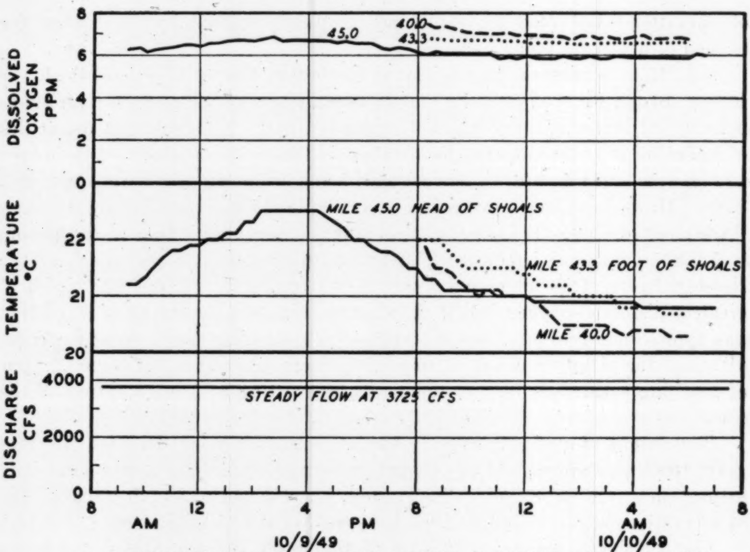
In the lower French Broad River more details of the time of water travel are available, thus permitting the computation of the reaeration coefficient for the night observations of September 14, 1949 shown in Table 3.

Apparently the long, flat pools in the two reaches below Hodges Ferry, even though they are connected by sharp shoals at low flow, hold the reaeration coefficient to below that observed for the upper two reaches, between the Douglas Reservoir bridge and Hodges Ferry.

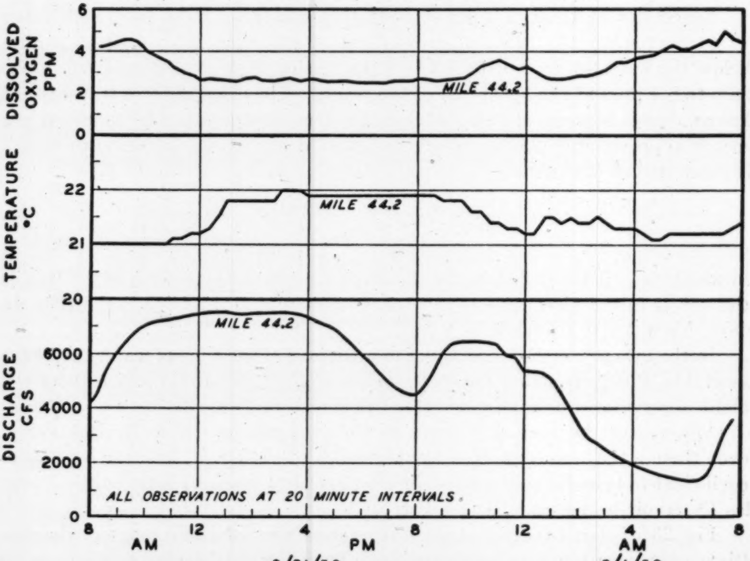
Fig. 23(*a*) shows the variation in reaeration observed at a point in Holston River approximately 7 miles downstream from the dam during a 24-hr period

---

[9] Discussion by M. A. Churchill of "Translatory Waves in Natural Channels," by J. H. Wilkinson, *Transactions*, ASCE, Vol. 110, 1945, p. 1229.

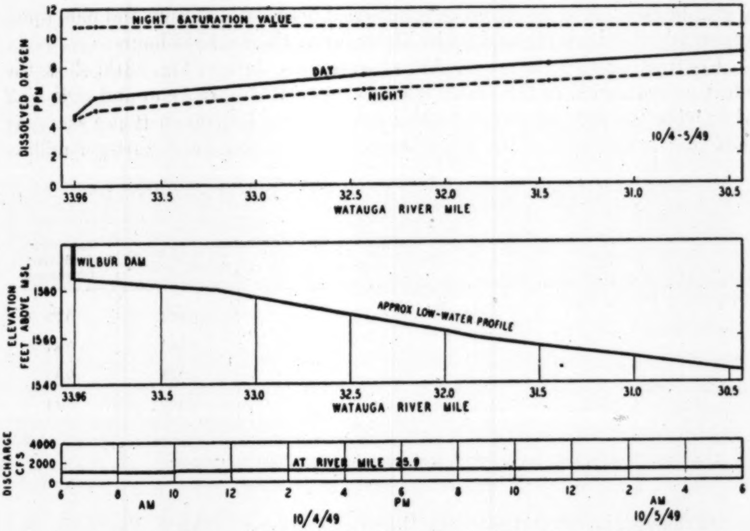


(*a*) VARIATION IN REAERATION DURING 24-HOUR STEADY FLOW HOLSTON RIVER - OCTOBER 1949

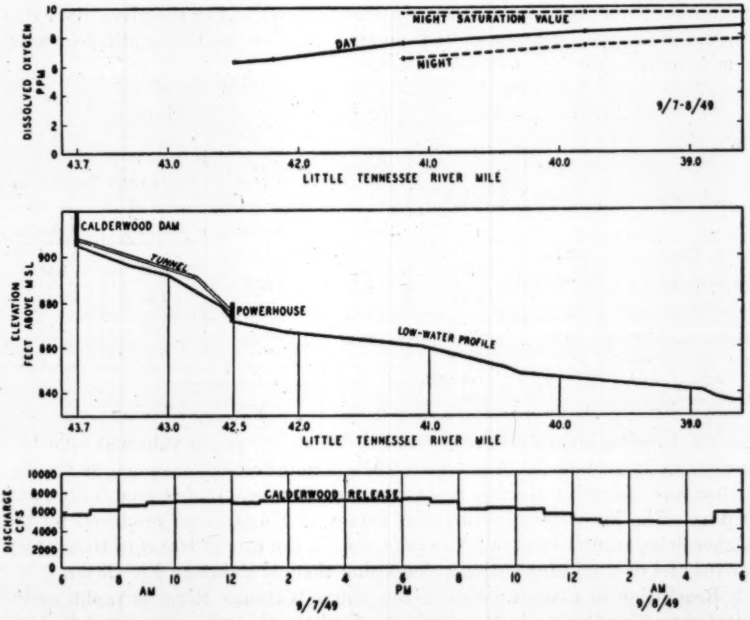


(*b*) VARIATION IN REAERATION DURING 24-HOUR UNSTEADY FLOW HOLSTON RIVER - AUGUST 1950

FIG. 23.—REAERATION VARIATION IN HOLSTON RIVER



(a) REAERATION IN WATAUGA RIVER BELOW WILBUR DAM - OCTOBER 1949



(b) REAERATION IN LITTLE TENNESSEE RIVER BELOW CALDERWOOD DAM - SEPT 1949

FIG. 24.—REAERATION IN WATAUGA RIVER AND IN LITTLE TENNESSEE RIVER BELOW CALDERWOOD DAM IN SEPTEMBER, 1949

of steady flow. The dissolved oxygen varied between 5.85 ppm and 6.90 ppm, apparently due to oxygen added by algae during the daylight hours.

Contrasting with the steady-flow observations, data in Fig. 23(*b*) show the effect on reaeration in the 8-mile reach between Cherokee Dam and mile 44.2 of varying the flow between 1,300 cu ft per sec and 7,500 cu ft per sec. As indicated in the curves, low flows obtain more oxygen (on a part-per-million basis) than higher flows traversing the same distance. This is due to the fact that the lower flows require a longer time and have less depth, and therefore less volume per unit of surface area, and that a given particle of water can make more round trips between water surface and stream bed in the given river distance. Relative roughness of the channel increases as the depth decreases, thus increasing mixing at shallower depths.

TABLE 3.—REAERATION IN LOWER FRENCH BROAD RIVER [a]

Location	Mile	Dissolved oxygen, in parts per million	Time of water travel, in days	Reaeration coefficient (darkness)
Bridge below Douglas Reservoir	31.7	4.08	0.053	0.79
Kiker Ferry	28.7	4.48	0.043	0.84
Hodges Ferry	26.3	4.80	0.128	0.48
Dumplin Creek	19.9	5.29	0.167	0.44
Huffaker Ferry	12.2	5.72		

[a] On September 14, 1949, for a river flow of approximately 12,000 cu ft per sec.

TABLE 4.—REAERATION IN UPPER WATAUGA RIVER [a]

Location	Mile	Time of water travel, in days	DISSOLVED OXYGEN, IN PARTS PER MILLION		REAERATION COEFFICIENT	
			Daylight	Darkness	Daylight	Darkness
Below Wilbur	33.85	0.033	5.90	5.05	6.02	3.53
Above Siam	32.43	0.027	7.38	6.37	4.80	2.40
Siam Bridge	31.45	0.038	8.12	6.90	4.60	2.04
Near Dugan Mill	30.43		8.84	7.52		

[a] From October 4 to October 5, 1949, with a river flow of 1,000 cu ft per sec.

The time lag shown by the increase in dissolved-oxygen values at mile 44.2 beginning at 10 p.m. on August 31, 1950 is noteworthy inasmuch as this increase was caused by the low flows occurring at mile 44.2 between 6 p.m. and 9 p.m. The lower dissolved-oxygen values at 1 a.m. were produced by the higher flows around 10 p.m. The difference in the rate of travel of translatory waves and of the water initially composing them is shown in Fig. 23(*b*).

Reaeration in a short reach of the upper Watauga River is rapid, as indicated in Fig. 24(*a*). In the reach observed the slope is approximately 11 ft per mile. During the observation period in October, 1949, the river flow was quite steady near 1,000 cu ft per sec. The rapid rate of reaeration and the increase in rate during daylight hours due to the addition of oxygen by algae are demonstrated in Table 4. Although the dissolved-oxygen concentration at the lower end of the reach was fairly high, with water at 12°C, there was still a deficit below saturation of approximately 14%.

The foregoing illustrates the rapid reaeration produced by a turbulent stream flowing at a velocity of 2.6 ft per sec in the upper reach, 2.2 ft per sec in the middle reach, and 1.6 ft per sec in the lower reaches. The night reaeration coefficients are roughly proportional to these velocities.

Still higher reaeration coefficients have been observed on the Little Tennessee River below the Calderwood Dam powerhouse, as shown in Table 5. The slope of the river is not quite as steep as for the upper Watauga River, being about 9 ft per mile through the reach studied, but the river is wide and shallow even at the observed flows of 7,000 cu ft per sec. The width varies between 600 ft and 1,200 ft, and the mean depth at 7,000 cu ft per sec is on the order of 3.5 ft in the narrow sections and 1.5 ft in the wider reaches. Water velocities vary from 3.2 ft per sec in the deeper reaches to 4.2 ft per sec on the broad, shallow shoals.

TABLE 5.—REAERATION IN LOWER LITTLE TENNESSEE RIVER[a]

Location	Mile	Time of water travel, in days	Dissolved oxygen, in parts per million, during daylight	Reaeration coefficient, during daylight
1,400 ft below powerhouse	42.25		6.48	
		0.019		9.7
Gaging station	41.25		7.42	
		0.020		11.9
	39.85		8.19	
		0.022		9.4
	38.60		8.58	

[a] From September 7 to September 8, 1949, with a river flow of 7,000 cu ft per sec.

Although these coefficients are very high, the shallow, turbulent river is an ideal aerator. With such high coefficients a stream could leave a powerhouse with 1 ppm of dissolved oxygen (a deficit of 8.6 ppm at 18°C) and become reaerated to within 1 ppm of saturation within approximately 2½ hr, or in a distance of 6.3 miles at 3.7 ft per sec.

CONCLUSIONS

Water released through low-level power intakes from deep storage impoundments during the summer months is considerably cooler than that flowing in local unregulated streams. This cool water is of great economic benefit for steam condensing and other cooling purposes. The released water is less turbid and usually has less color than that in unregulated streams of the area. Odors caused by algae are practically nonexistent.

Bacterial concentrations in the released water are normally less than 10% of concentrations in the inflow. This great reduction is of significant benefit to downstream domestic water supplies.

Dissolved-oxygen concentrations in the outflow during the summer months are normally far below saturation. The B.O.D. of the released water is also low; therefore, the rate of reaeration below the dam is relatively rapid, being unretarded by B.O.D. Low rates of released flow are reaerated in relatively short distances downstream from the dam, whereas higher discharges require many miles of open-channel flow before oxygen saturation is reached. Single-purpose impoundments having power intakes, or any other single-purpose intakes, high on the face of the dam discharge water having high oxygen concentrations all year long.

Deep, thermally stratified impoundments located downstream from significant deposits of iron ore or manganese ore may release, through deep outlets, water having relatively high concentrations of these minerals in soluble, unoxidized forms.

The equalizing effects of a reservoir on water quality are quite beneficial to downstream municipal and industrial water supplies because the range of variation and the rate of change in almost all water characteristics is significantly reduced, thus facilitating water treatment.

ACKNOWLEDGMENTS

The data presented herein cover a span of 20 yr and part of the work of several divisions of the TVA. However, the contributions of Kittrell, Fry, and Guy R. Scott, M. ASCE, have been especially valuable.

The paper was written under the direction of F. E. Gartrell and C. M. Davidson, and R. A. Buckingham supervised the initial preparation of the illustrations. The writer is indebted to Kittrell and to Thomas M. Riddick, M. ASCE, for reviewing the original manuscript and for many helpful suggestions.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2929

7

CONTROL OF HIGHWAY ACCESS
A SYMPOSIUM

PAGE

FOREWORD
By Curtis J. Hooper, M. ASCE 466

EXPERIENCES IN NEW YORK
By Bertram D. Tallamy, M. ASCE 467

ECONOMIC EFFECTS OF THE GULF FREEWAY
By Dewitt C. Greer, M. ASCE 472

EXPERIENCES IN INDIANA
By Carl E. Vogelgesang, M. ASCE 477

EXPERIENCES IN LOS ANGELES
By Hugo H. Winter, M. ASCE 486

USER BENEFITS IN CALIFORNIA
By Ralph A. Moyer, A.M. ASCE 499

FOREWORD

BY CURTIS J. HOOPER,[1] M. ASCE

The limitation or control of access to public highways is probably the major feature that distinguishes modern highways from those built earlier. Also, to provide for the ever-increasing numbers of moving motor vehicles, it is necessary that separating grades at intersections, dividing highways to separate traffic moving in opposite directions, and providing for extra lanes to accommodate heavy traffic flows be embodied in the design of highways currently being constructed. However, it is believed that the denial of the rights of those abutting public highways to enter on them is the fundamental change in the concept of a modern highway. This concept of "planned access" has been slow to evolve, and many states are not legally authorized to construct highways that include access prohibitions. Other states have had such favorable acceptance of this principle that they exercise access control whenever land is purchased for highway relocations. A few states curtail roadside development by declaring certain existing highways to be freeways.

The purpose in arranging this symposium was to present information and to encourage a greater use of controlled highway access in order to reduce the obsolescent aspects of the highways needed for motor vehicle economy.

---

[1] Associate, Wilbur Smith & Associates, New Haven, Conn.

EXPERIENCES IN NEW YORK

By Bertram D. Tallamy,[1] M. ASCE

Synopsis

Experience with the New York State Thruway has shown how limited-access highways can foster safety, save travel time, reduce wear and tear on vehicles and drivers, relieve traffic on parallel roads, and promote great economic development of the area through which an expressway runs. Conservative estimates credit the New York State Thruway with attracting at least $150,000,000 in new or expanded enterprises, with an annual payroll of more than $100,000,000. The fact that more superhighways are needed throughout the United States has been realized by the United States Congress, and legislation has been passed that provides for interstate highway construction.

Introduction

A subject that has intrigued highway engineers has been the necessity for controlling highway access. However, much of the thought has been largely theoretical because the mileage of modern controlled-access roads is small when compared with the total mileage of the various highway systems.

Despite this somewhat limited field of observation, a few conclusions as to the desirability of access control were made. It was found, for example, that limited-access divided highways promote safety, eliminate wasteful stop-and-go driving, save time, and reduce the wear and tear on the vehicle as well as on the driver. There are many other advantages of the control of access, which have been the subject of conversations and papers.

The world's longest controlled-access highway, the New York State Thruway, has been in operation since the first toll booth opened in June, 1954. Since that time the actual effects that the Thruway has exerted on the state's economy, as well as its effects on traffic operations, have been observed.

Background

Some of the factors that have resulted in the construction of the Thruway and that have made its operation so valuable to New York State should be considered. New York has the greatest population of the forty-eight states and is second only to California in the number of motor vehicle registrations. It has a diversified economy with a tremendous concentration of industry, commerce, and agriculture. It ranks second of all the states in the production of milk and apples; third in vegetable growing; and sixth in hay production. There are 50,000 manufacturing firms of all sizes and types plus a tourist business that is worth $2,000,000,000 annually.

Note.—Published, essentially as printed here, in the Journal of the Highway Division, in January, 1956, as *Proceedings Paper 871*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Federal Highway Administrator, Bureau of Public Roads, U. S. Dept. of Commerce, Washington, D. C.; formerly Chairman, New York State Thruway Authority, Albany, N. Y.

These facts result in an acute traffic problem. Motor vehicle registrations in New York increased 25% between 1949 and 1954, and there was an even greater increase in the use of the vehicles. In October, 1955, there were 14,385 miles of New York State highways, of which more than 8,000 miles were rated as less than tolerable. The Thruway is not included in this evaluation because it connects with all the principal routes and forms the trunk line for the swift movement of people and goods across the state.

The principal east-west highways are Route 5 and Route 20, and the major north-south roads are Route 9 and Route 9-W. Much work has been done on these main arteries in the past. However, it became more and more obvious in New York, as in other states, that no program of widening and reconstructing existing routes could ever provide the necessary traffic relief. Not only was the cost prohibitive, but it offered no permanent solution. Along these principal routes, mushrooming roadside developments increased to a point where the highways efficiently carried only a fraction of their original traffic capacity. Whatever safety features the highways originally had were dissipated by multicolored electric signs and accident-breeding marginal developments. During 1945, New York State officials began a long-range program of planning and building a system of arterial routes into and adjacent to cities to relieve the urban traffic problems. At the same time, the Thruway was planned in order to link the two largest cities, New York and Buffalo, by way of Albany, Schenectady, Utica, Syracuse, and Rochester.

The Thruway

As was the case in several other states, New York was forced to resort to toll financing to construct the New York State Thruway. This expressway extends from Buffalo eastward to Albany and south to Nyack, crosses the Hudson River at the 3-mile Tappan Zee Bridge, and extends southward through Westchester County to the fringe of New York City.

During the period from June, 1954 to October, 1955, motorists and truckers traveled 862,700,000 miles, thus presenting a solid basis for observing and analyzing the effects of limited-access highways.

It has been established, for example, that the control of access reduces accidents, particularly those that are fatal. The fatality rate in 1955 was 2.55 fatalities per 100,000,000 vehicle-miles compared with the 1954 national average of 6.5 fatalities per 100,000,000 vehicle-miles and a New York State average of 5.3 fatalities per 100,000,000 vehicle-miles. Moreover, not one fatal accident was due in any way to the design or construction of the highway itself. Unfortunately, one cannot correct faulty judgment or human error. Motorists and truckers traveled 96,000,000 miles before the first fatal accident on the Thruway was recorded.

Of the hundreds of letters received by the New York State Thruway Authority mentioning the advantages of Thruway travel, the majority listed advantages such as freedom from irritation and fatigue and savings in driving time. Apparently motorists considered that the reduced nervous strain that the Thruway offers is one of its greatest attributes. They observed that they arrived at their destinations more relaxed and in a better physical and mental condition than when they used the existing parallel highway system.

Also, motorists can plan to travel on the Thruway at the adopted speed limits—namely, 60 miles per hr for passenger cars and 50 miles per hr for all other vehicles. Except for the time taken for refueling, rest, and meals, these maximum speeds have become average. These rates are considerably more than the average of from 30 miles per hr to 45 miles per hr that results when the existing parallel highway system is used.

The New York State Thruway has aided industry as well as the individual motorist. A large industrial company has planned its operations so that the new expressway serves as a modern connecting artery among the firm's various plants over which parts and assemblies are channeled. A series of independent tests staged by the company's transportation experts to test the operating economies for its trucks on the Thruway demonstrates this firm's interest in the expressway.

On a round trip between Buffalo and Schenectady, the test showed that the firm's trucks averaged better time (10.64 miles per hr) on the Thruway than on parallel highways. Furthermore, it was discovered that during this trip the trucks saved 14.2 gal of fuel, made 298 fewer gear shifts, and 69 fewer full stops. Such statistics promise substantial savings in transportation costs for the company's large trucking operations.

One of the effects of a new limited-access highway is the relief of traffic pressures on parallel routes. This has been demonstrated graphically in New York since the opening of the Thruway. For example, traffic volume on a section of parallel Route 20 south of Utica decreased 40% in the first six months after the Thruway was completed in that area. Similar effects have been noted, although usually to a lesser degree, along other sections of Route 20.

This siphoning of through traffic from existing routes has not been considered favorably by everyone. The merchants and small enterprises along the older highways have complained of a sharp decline in business. However, the Thruway performs a basic function—relieving traffic on parallel routes and reducing the annual cost of maintaining them, thus making it possible for these parallel routes to serve the local or regional traffic adequately. This fact, combined with the accessibility and the new economic climate of the region being created by the cross-state Thruway, is already replacing this business with new commerce, which is certain to exceed the lost business in the near future.

THE THRUWAY'S ECONOMIC EFFECTS

The most amazing benefits of the Thruway observed to date (October, 1955) have been to the over-all economy of the state.

During the planning stages and the construction period, the New York State Thruway Authority was confronted by contentions that loss of taxable property taken for Thruway purposes would impose a financial burden on the communities through which it ran.

However, it appeared certain to the Authority that this loss would be more than compensated for by the increased value of the remaining property because of its accessibility to the Thruway. The Authority has been observing this trend and has found that in every case in which data are available the increases have far exceeded the taxable losses. The greatest gains have been evident on property immediately adjacent to the Thruway interchanges, with lesser increases being noted in rough proportion to the distance from the interchanges. This influence is radial, and its extent is dependent on the accessibility to the Thruway by way of the existing road network.

Although it is too soon to assess the full impact of the Thruway on real estate values, and although there is no formula for measuring its impact, a few typical examples of the trends are:

1. On the north side of the Thruway at a Syracuse interchange, there is a 23-acre piece of property. No water, sewer, or gas facilities are available. Before the Thruway was located in this area, the land was appraised at $100 per acre. In March, 1955, this plot was sold to a machinery company for $46,000, or $2,000 per acre.
2. In Tarrytown a local merchant who had originally paid $55,000 for a 4-acre corner site near the Thruway in 1952 resold this property in 1954 for approximately twice that amount to an operator who plans a shopping center.
3. The *Wall Street Journal* made a detailed study of the effect of the Thruway on the state's economy. As a result of this study it was found that a 21-acre property along the Thruway in Syracuse was sold for $15,000 in 1951, but in 1955, 12½ acres of the same land were sold for $150,000. The survey revealed that the value of the land near the Thruway in the Buffalo area had jumped from a few hundred dollars an acre to $5,000 an acre.
4. The village of Fultonville (population 900) was bisected by the Thruway, thus initially creating concern among the residents. However, after careful consideration of the benefits of the Thruway, a more favorable attitude was adopted with satisfactory results. Fultonville now has a larger assessed valuation than it had prior to the construction of the Thruway, and the village debt has been retired. Because of new enterprises, the tax rate has decreased from $33.00 per $1,000 of assessed value to $19.00 per $1,000 of assessed value.

Until industrial growth along the Thruway is taken into consideration, the full economic impact of controlled-access highways cannot be made evident. When a state highway is built through undeveloped property, the property undergoes an increase in value because of its accessibility to the new facility. Almost overnight the highway attracts gasoline stations, restaurants, tourist cabins, and other concession-type developments.

Although some of this type of development is evident in the immediate vicinity of the Thruway interchanges, the commercial expansion has been dominated by the construction of industrial plants of various sizes.

In October, 1955, it was estimated conservatively that more than $150,000,000 worth of new industrial plants had been erected adjacent to the Thruway. These plants employ approximately 30,000 persons with an annual payroll of more than $100,000,000.

Perhaps the most vigorous growth attributable to the Thruway has been in the Syracuse area, where several large new facilities, each costing many millions of dollars, have been erected. Some of the vast new industrial sites have direct access to the expressway. One sector of Syracuse Industrial Park, a 1,000-acre district on the Thruway, is occupied by a dozen projects, or more.

These projects include steel-supply warehouses; storage and retail-chain depots; a bank; various offices; a radio-broadcasting concern; and metals, machinery, and other manufacturers.

In other areas of the state, both large and small firms have also been aware of the Thruway's potential. In the Hudson Valley the superhighway was one element resulting in the decision of a large industrial corporation to erect one of its giant factories near the Kingston interchange. The Thruway was also a factor that induced another manufacturer to locate in Mountainville in Orange County. In announcing an expansion at Canajoharie in the Mohawk Valley, a packing company stated that the Thruway provided a means for rapid delivery to New York City. In Westchester County the availability of a large plot at the major interchange with the Cross County Parkway was a major reason for the location at that site of the $30,000,000 Cross County Center, one of the largest shopping centers in the United States. The same pattern prevails across the state, and the New York State Thruway Authority believes that the economic revolution has just begun. Modern expressways benefit agriculture, industry, commerce, labor, and the tourist business, and they provide a basis for an amazing variety of new enterprises.

THE FUTURE

Some of the effects that the Thruway has had on New York State have been cited; other effects will be observed as the Thruway continues serving the state.

New and effectively located controlled-access highways are urgently required elsewhere in the state as well as across the United States.

The tremendous toll that the outmoded highways exact in lives, property damage, human misery, and economic waste compensates for the cost of providing an adequate highway system. The average driver is paying as much as an extra cent a mile to travel on the existing outmoded free highway system. These facts are known, but a better recognition of the sweeping economic improvements that express highways create is needed.

The cost of constructing modern highways must be balanced against many factors, most important of which are the economic growth and development that have become synonymous with new expressways and major highway-construction programs. These benefits have been underemphasized in spite of the fact that they are much greater than the cost of providing the highways themselves.

Adequate highways are the foundation of progress in the United States. They are absolute necessities, not only for traffic relief but also for the continued expansion of the economy.

ECONOMIC EFFECTS OF THE GULF FREEWAY

BY DEWITT C. GREER,[1] M. ASCE

SYNOPSIS

The conclusions drawn from investigations of the economic effects of limited-access facilities on adjacent properties are reviewed. A detailed account is also presented of the method of approach and conclusions of an economic survey made in 1951 of the Gulf Freeway in Houston (Tex.). The increase in land values of adjacent properties after a period of 5 yr is evaluated.

LIMITED-ACCESS FACILITIES

The conditions governing the economic effect of the limited-access projects vary. There appears to be no single set of values that might apply to all projects.

Appraisers have actively investigated the effect of limited-access facilities. Adrian F. McDonald reported[2] that

> "Observation and study over a period of several years appear to warrant certain general conclusions. Sections of Connecticut suitable for residential use and with easy access to the Merritt and Wilbur Cross Highway systems have increased in desirability, rate of marketability and value. Today it is easier to sell properties within a mile of the Connecticut Expressway routes than ever before, and at better prices."

With reference to towns in Fairfield County (Conn.), McDonald states that

> "The results were so clear cut as to be remarkable in their consistency. Sales of land adjoining the Merritt Parkway and in the vicinity showed a substantially higher level of prices than comparable land some distance removed. Particularly noticeable was the high level of prices of those properties which were at a higher elevation than the highway surface. The only cases where lower prices were found were those involving adjacent land below the traveled portion of the road. Inquiry developed the fact that in these cases noise actually seemed to roll down to envelop these properties and the buying public recognized this phenomenon."

In connection with the Wilbur Cross Highway in Connecticut, McDonald reveals that:

> "In Greenwich and Darien in Fairfield County, sales for the past five years show absolutely no adverse effects from extremely heavy truck traffic on U. S. No. 1 even where the rear lines of properties may be within 100 feet to 150 feet of the traffic."

Concerning property adjoining, but without direct access to, high-speed express highways, he states that

> "In cases where such properties are bordered by intersecting side roads or exit ramps from the expressway, there are innumerable instances in Connec-

---

NOTE.—Published, essentially as printed here, in January, 1956, in the Journal of the Highway Division, as *Proceedings Paper 872*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Highway Engr., State Highway Dept., Austin, Tex.

[2] "The Review," Soc. of Residential Appraisers, Chicago, Ill., February, 1953, pp. 15–16.

ticut of sales for business purposes at fabulous prices. This also is true of properties which are actually off the expressway proper but which are accessible to it by reason of access and exit ramps."

He summarizes—

> **"No general conclusion can be drawn except that there is no general rule applicable in all cases. The appraiser must analyze the problem case in the light of his own experience and market reaction. However, in Connecticut it seems certain generalities are true. Limited access expressways have drawn people as well as industrial plants from the city to the country. That has helped to curb (but not eliminate) increase in city residential values. On the other hand, both rate of marketability and values in outlying areas have tended to rise, many times in substantial degree."**

Frank C. Balfour has reported many studies made of the California developments. In reference to expressways, Balfour states[3]:

> **"The Expressway, the Freeway, and the various types of toll roads are designed primarily to meet the needs of automotive transportation. The underlying principle of this type of highway design is to move the greatest volume of traffic with maximum safety and efficiency. The secondary consideration of the expressway is to serve the abutting property. However, we can demonstrate that the modern limited access highway facility's effect on both abutting properties and properties in the entire area of its influence is so great, both from a monetary and psychological standpoint, that it far overshadows the immediately apparent benefits of the conventional highways."**

Balfour cites numerous examples of increase in land value because of access to the freeway, including industrial, commercial, and residential development.

There are many other authoritative reports on studies of limited-access facilities. The report by McDonald was mentioned because it indicates some results influenced by construction features. The report by Balfour expresses in broad terms the findings that appear to be usual in all reported cases. An examination follows of the study of the Gulf Freeway, which represents experience in Texas.

THE GULF FREEWAY

In May, 1950, the Texas Highway Department, in cooperation with the Bureau of Public Roads, United States Department of Commerce (BPR), initiated a study of the effect of the Gulf Freeway in Houston (Tex.) on adjacent land values and land use. Ensuing conferences between personnel of the BPR, the city of Houston, and the Texas Highway Department outlined a general method of procedure and resulted in an agreement on the objective of the study. These preliminary conferences emphasized the fact that the success of the study depended on the data that could be obtained, and that an extensive examination should be made before any definite procedure could be outlined. In January, 1951, negotiations culminated in a contract based on a generalized outline of the proposed work.

The section of the Gulf Freeway in the city of Houston that was studied was 7.8 miles long, extending from Main Street adjacent to, and southwest of,

---

[3] **"America's Highway Problems and Their Likely Effect on Real Estate Markets," by Frank C. Balfour, *Appraisal Journal*, October, 1954, p. 499.**

the central business district in a southeasterly direction and continuing as a rural expressway to the city of Galveston (Tex.). The first mile southeast of Main Street forms a four-street dispersal system with cross streets at grade. This section is provided with a traffic-light control for speeds of 30 miles per hr. The extension within the city is classified as a freeway.

Work on surveys and plans of the Gulf Freeway began in 1943, which dates the earliest realization of the facility. Construction of the first 3 miles from the center of the city southeastward began early in 1946, and the section was opened to traffic in October, 1948. In October, 1951, approximately 6½ miles of the Gulf Freeway were in operation. The remainder had not been placed in operation during the period of this study and, in fact, was not opened until August, 1952. The opening of the Gulf Freeway by increments had an effect on the transfer of ownership and development of properties, and property sales reflect this tendency. The greater increases in value appeared to follow after the facility was placed in use, which may be partly due to speculation.

The change in the use of the land can be shown best by "before-and-after" photographs. There is evidence that the change is steadily proceeding adjacent to the freeway as well as increasing in width. It will probably be several years before the activity becomes stabilized. The property values will increase as the ownership of the property is changed and as it is put to new, higher, and better use because of the favorable position of the land in regard to the transportation facility. In this early interval the principal development was in unimproved areas or in areas with meager improvements.

The basic concept of the land-value study was a comparison of land values reflected by the actual sales prices of land areas along the freeway, with land areas at other locations in the city equally well served by the street system prior to the freeway construction and equidistant from the civic center, and of comparable environment and similar development and usage. The change in land value because of its position was chosen for measurement. The comparison would have been simple in application if all lands had been vacant, or less complicated if all lands had been occupied by similar improvements. The land along the freeway varied from large undeveloped areas through the stages of areas of low-type residential housing; decadent neighborhoods of once the best, but now obsolete, residential properties in the process of conversion to rooming houses and commercial properties; neighborhoods of modern, medium-income residential housing; areas of modern apartment housing; and areas of business properties, all varying within themselves. To determine land values as reflected by sales prices, a deduction of the sales price of improvements was required. In the final analysis the price of land per square foot was used for comparisons, frontage conditions being disregarded because of the complications.

Investigations revealed that the assessments in Houston were based on the 1940 appraisals at 70% of the appraisal value. The assessments thus indicated the appraised value of the improvements on each parcel of land that was transferred, and these values were accepted as being the best information available. This premise emphasized the effect of the varying purchasing power of the dollar over a period of years, and was responsible for the conversion of sales prices applied to improvements into three categories: (1) The dollar with no adjustment; (2) the dollar adjusted to the Consumers' Index; and (3) the dollar adjusted to the Construction Index.

One of the major financial institutions of Houston made available its records of property sales throughout the city, which were complete and maintained over a long period of years. These sales records, together with city assessment records, were the sources of data pertinent to the land-value study.

In order that definite comparisons might be made, the length of the Gulf Freeway was divided into six sections dependent on the nature of improvements and environment. These sections were divided further to delineate the section immediately adjacent to and parallel to the facility and designated as group 1, and a secondary zone bordering, and slightly more distant than, the primary zone was designated as group 2. Areas in all respects comparable to each of the sections of group 1, but in various sections of the city in which no influence of the Gulf Freeway might be realized, were selected for direct comparison of land sales prices and were denoted as group 4. Group 3, composed of a selection of areas at a considerable distance from the freeway but in the same quadrant of the city, was chosen and analyzed to check on comparative land values in the directional area of the freeway.

All bona fide property sales in all sections for the periods from 1939 to 1941, 1945 to 1946, and 1949 to 1951 were examined. In the first period there were 858 sales; in the second, 1,041 sales; and in the third, 397 sales, thus totaling 2,296 sales, in which 199 repeat sales appeared. Repeat sales were separately analyzed because of interim improvements and the opportunity of enlarging the scope of the findings of the survey.

The route of the freeway was well established in 1945, and construction was begun in 1946. In comparing the land values, the period from 1939 to 1941 was used as the 5-yr interval before the freeway became a reality. The period from 1949 to 1951 was taken as representing the 5-yr interval after the freeway became a reality.

The most outstanding conclusions made from an analysis of all sales are:

1. During the 5-yr interval after the Gulf Freeway became a potential factor of influence on land values, properties adjacent or very close to the facility increased to a greater extent than similar property located in any other section of the city. The properties in the secondary zone of influence increased in market value to a greater extent than did the properties farther removed from the facility.

2. During this interval all methods of analysis used in the study have the same directional trend, indicating that the facility definitely influenced the increase in market values of properties in the areas through which the facility passed.

3. During the 5-yr period immediately preceding the period of potential influence of the facility, the increase in market value of properties adjacent to the future route of the Gulf Freeway closely approximated the median of increases in all area groups studied.

The analysis of repeat sales of identical properties sold and resold after the freeway became a potential influence in the community indicated that a property owner in the primary zone of influence had approximately twice as many opportunities to resell as did the owner in zones beyond this influence. The gain on investment was substantially greater than that of the owner beyond the influence of the facility.

A comparison of the change in land values of the group of areas adjacent to the freeway, with this group chosen to represent similar properties in areas not influenced by the freeway, illustrates the similarity of the trend. The close agreement in the extent of change in land value due to the freeway, regardless of the method used for the deduction of the price of improvements from the sales price of the property, is also demonstrated. By expressing the relationship of changes in land values between groups as the difference in percentages of value changes, the effect of a variation in the purchasing power of the dollar is minimized and a true comparison results. A valid conclusion may be drawn for the relationship of land-value changes for the 5-yr interval between the time when the freeway became a source of influence and the date of this survey. The percentage increase in land values along the freeway as compared with the land values of the group of similar properties not influenced by the freeway was determined as 81% for the group in which the improvements were deducted without regard to a price index; 95% for those areas in which improvements were deducted in proportion to the Consumers' Price Index; 111% for the land for which the improvements were deducted in proportion to the Construction Index; and 92% for repeat sales with improvements included. The reasonably close agreement in these percentages is convincing.

The Texas Highway Planning Survey made an analysis of the study with regard to the justification of the right-of-way cost of this facility, which was borne by the city of Houston. This analysis involved the assessed evaluation of each section of each group with relative percentage increases in total value of each section in the group along the freeway as compared with similar sections not influenced by the freeway. It was determined that, if assessed and taxed proportionately, the increase in market value along the freeway, in addition to the increase in market value of the similar lands not so situated, would yield revenue sufficient to defray the right-of-way cost in 18 yr. Allowing for the reasonable continuance of the trend, the increase should double within a few years. When this potential is accomplished, increased tax revenues of 9 yr would serve to defray the right-of-way expense to the city.

No two cities or conditions could be identical, but the fact that the construction of freeways tends to increase the value of the adjacent land is believed to be established by this study, and the variance should be in degree only.

EXPERIENCES IN INDIANA

BY CARL E. VOGELGESANG,[1] M. ASCE

SYNOPSIS

The experience leading to the adoption and the use of control of access in Indiana is presented. The evolution described involves the measures considered in an effort to provide satisfactorily for the traffic on the Indiana State Highway System through the construction of bypasses and dual-lane pavements; the subsequent ribbon-type development, which eventually led to the enactment of permissive controlled access legislation; and, finally, legislation that requires that all newly constructed bypasses must be of the limited-access type. Some of the more important aspects of the legislative acts are quoted. The benefits of controlled access are enumerated as well as the conclusions that were made following a study of the experiences in Indiana.

INTRODUCTION

Indiana's highway system had its inception in 1917; however, due to legal difficulties, the department was not formed as a permanent organization until November, 1919. The greatest problem that the department had was that of creating highways at the earliest possible date where none had existed before.

In the succeeding years this feat was accomplished. However, the volumes and speed of the traffic had rendered the major routes substantially obsolete. The ideal section, which was 1.36 miles long and was constructed in 1922 and 1923 between Dyer and Schererville on U. S. 30, was a bold attempt to insert higher standards into the highway program. It consisted of a 40-ft pavement, wide shoulders, provision for walks, and lighting facilities together with several landscape features. The ideal section was widely advertised and conceived of as a pattern for all highways in the future. However, with respect to the Indiana program, the ideal section never fulfilled its expectations.

In 1935 the divided-lane highway at grade, except for separations at railroads, was created. As first conceived, a minimum right of way of 200 ft was established with two 22-ft pavements separated by a 44-ft median strip. The first section, which was opened in 1937, was on U.S. 30 from U.S. 41 to Cline Avenue, which is a distance of 2.23 miles. Since that date 308 miles of dual-lane highways throughout the state have been completed.

Bypasses have also played an important part in the Indiana highway system. At one period, they were regarded as the answer to the alleviation of traffic congestion in urban areas. From 1922 to 1954, 40 bypasses were constructed in Indiana as grade facilities. Because access was not controlled, a ribbon development of residential and business structures sprang up along the

---

NOTE.—Published, essentially as printed here, in January, 1956, in the Journal of the Highway Division, as *Proceedings Paper 873.* Positions and titles given are those in effect when the paper was approved for publication in *Transactions.*

[1] Chief Engr., State Highway Dept. of Indiana, Indianapolis, Ind.

bypasses serving the larger urban areas to such an extent that they have become little more than congested city streets. As an example, the bypass at Lafayette, which was built in 1937 and which presently (1955) carries volumes in excess of 11,000 vehicles, has been so solidly hemmed in during the intervening years by commercial and residential developments that, as a part of the interstate system, it will require relocation approximately one-half mile farther away from the city.

The control of access was first considered in 1944, at which time the first origin and destination traffic survey was conducted in Fort Wayne. The recommendations of this survey, presented in a 1946 report,[2] called for an expressway system as a long-range solution of Fort Wayne's traffic problem.

This system was to have been developed over a 10-yr period at an estimated cost of $26,000,000. The highway department and the city officials of Fort Wayne entered into an agreement to proceed with the construction of the expressway system, provided that it was acted on favorably in a referendum vote to be held in the fall of 1947. In spite of a concerted publicity effort by the officials of the city, a sufficient number of citizens voted against the project.

This action had such an adverse effect on the highway department that the consideration of the use of expressways within urban areas was ruled to be not applicable to the other studies pending at that time.

However, in 1948 the first contracts were awarded on the Tri-State Highway in Lake County (in the Chicago, Ill., area) from the Indiana-Illinois state line to a point approximately 2 miles east. This facility, designed as a limited-access highway, was Indiana's continuation of the Illinois section of the Tri-State Highway whose construction was begun during that period. The first section was completed and opened to traffic in 1951. Since that date, an additional 4.6 miles has been opened.

Other examples of full or partial limited-access facilities include:

Route	Location	Approximate distance, in miles
U.S. 31-E	in Jeffersonville	2.2 (from Ohio River to Kopp Lane)
U.S. 31	in Indianapolis	2.3 (Madison Avenue)
S.R. 100	in Indianapolis	3.0 (from U.S. 40 to U.S. 421)
S.R. 37	from Indianapolis to Noblesville	19.0
S.R. 62	at Evansville	8.5 (from Second Avenue west to county line)
U.S. 30	in Fort Wayne	1.2 (from Beuter Road east to intersection with existing U.S. 30)
U.S. 52	Templeton bypass	2.0
U.S. 41	Morocco bypass	4.0
U.S. 41	Schneider bypass	5.2
Total		47.4

This record of limited-access facilities, constructed in Indiana prior to October, 1955, reveals that much work remains to be done. The essential need for a more extensive use of limited-access standards has been uppermost in the plans of the Indiana State Highway Department; only the lack of adequate funds and long-range policy have prevented their adoption.

---

[2] "Highway Plan For Ft. Wayne," Pt. 2 of *Metropolitan Area Traffic Survey, Fort Wayne, Indiana*, State Highway Dept. of Indiana, Indianapolis, Ind., 1946.

HIGHWAY PERFORMANCE

The situation in Indiana has been described briefly in the Introduction, showing that, with few exceptions, the state highway system was developed with no control of access. That is, unless some extenuating hazardous condition existed, a property owner could obtain permission to build a driveway onto any highway. Therefore, as the volume of traffic increased and as the distance to an urban area decreased, an increase in the number of driveways was observed. On every important route entering the urban centers, this ribbon type of development is found, which, with regard to traffic movements, makes the route similar to a congested city street.

The expectation that bypasses, because of their construction on new locations free from built-up conditions, would provide a large measure of relief to traffic has not proved entirely satisfactory in the larger urban areas. It has been demonstrated conclusively that, because accesses were not controlled, residential and commercial developers have immediately taken advantage of the ideal location opportunities of these facilities to proceed with construction in an unrestricted manner.

Examples of how extensive this build-up has been on several bypasses will illustrate the seriousness of continuing on an unlimited policy. On a 4.2-mile section of the Marion bypass, built in 1940–1942, there are 138 access points, one occurring every 150 ft. On the Kokomo bypass, built in 1948–1950, there are 159 access points in the 7-mile section, or one every 235 ft. The Lebanon bypass, which was built in 1951, contains 57 access points in the 5-mile length, or one every 465 ft. On the Fort Wayne bypass, built in 1948, there are 110 access points in a 3-mile length, or one every 143 ft. Bypasses located at the smaller urban centers, where extensive building has not taken place, have served, and are serving, traffic effectively.

Wayne Kellams, in a newspaper article, stated that a driver using the 156-mile Indiana east-west toll road instead of U.S. 20 would avoid 74 stoplights, 17 railroad crossings, 1 drawbridge, 15 school zones, 88 heavy traffic entrances and exits, 20 major highway crossings, and 603 minor traffic entrances and exits. The other major trunk highways in Indiana would show similar statistics and, in some cases, even more access points, indicating the need to give serious consideration to the control of access.

The 308 miles of dual-lane highways in the state have proved to be an effective facility in the handling of traffic. Two hundred and twenty-one miles of this mileage are on the interstate system, which, as preliminary studies have indicated, can be improved with regard to limited-access standards. Built-up conditions on this mileage are substantially limited to the intersections, which will offer the most difficult problems in the separation of grades and the construction of interchanges.

INTERSTATE HIGHWAY SYSTEM

A pattern was set by the issuance on August 4, 1954, of a memorandum[3] by the Bureau of Public Roads, United States Department of Commerce (BPR), to the states defining the standards applicable to the interstate highway system. The memorandum states that federal interstate funds can be approved only on the basis that such routes be designed and constructed to freeway standards.

This pronouncement was initially met with scepticism. It was considered to be a constructive move, but doubt was expressed as to how funds could be obtained to meet the extreme costs of such a program. However, legislation[4] passed by the United States Congress in 1956 provided a basis for the interstate highway building program.

Since the memorandum was published, the state highway department of Indiana has taken steps to conform to the new federal policy. The Indiana interstate system consists of approximately 1,100 miles on nine different routes. In accordance with a directive from the highway commission, a comprehensive study[5] of the system was made, so that, when the federal program was approved, the state could proceed without delay.

The study involved the evaluation of the present interstate highways in order to determine whether they could be improved in accordance with limited-access standards on the existing alinement, or whether complete relocation would be required. By eliminating all direct access to the new facilities, the problem of providing adequate service roads would be paramount particularly where the existing alinement was to be followed.

The study of the Indiana interstate highway system will require approximately 2 yr to complete. An aerial survey was made of the entire system in the spring of 1955, with the widths of coverage ranging from 1 mile to 4 miles. On several routes it already appears from preliminary investigations that reflying will be necessary because of the advantage of relocating greater distances from the existing alinement than was originally anticipated. These aerial views not only provide the basis of the detailed studies, but the final recommended alinement and major design elements thereof will be depicted on strip maps developed from mosaics of the aerial contact prints.

BYPASSES

During the early part of 1954 the Indiana Highway Commission adopted a policy of building bypasses to limited-access standards. Previous commissions had been urged to consider this plan because of the growing volume of cases in which the constructed bypasses were definitely headed for obsoleteness. Four bypasses under construction at the time of the adoption of the new policy were, prior to their opening or during construction, declared limited-access facilities, except access points included in the contract plans. The state has 9 bypasses

---

[3] "Policy and Procedure Memorandum No. 20-4," Bureau of Public Roads, U. S. Dept. of Commerce, Washington, D. C., 1954.

[4] *Federal Aid Highway Act*, 1956.

[5] "Interstate Highway System in Indiana " State Highway Dept. of Indiana, Indianapolis, Ind.

(2 completed and 7 still to be placed under contract) that are now in various stages of planning and that will be built to limited-access standards.

LIMITED-ACCESS LEGISLATION

Definition.—The first limited-access legislation[6] in the state of Indiana was enacted on March 7, 1945. Following are important excerpts from the law:

> "A 'limited access facility' is defined as a highway or street especially designed for through traffic, and over, from, and to which owners or occupants of abutting land or other persons have no right or easement or only a limited right or easement of direct access, light, air, or view by reason of the fact that their property abuts upon such limited access facility or for any other reason. Such highways or streets may be parkways, from which trucks, busses, and other commercial vehicles shall be excluded; or they may be free ways open to use by all customary forms of street and highway traffic."

Local Agency Cooperation.—

> "The highway authorities of the state, counties, cities, and towns, acting alone or in cooperation with each other or with any federal, state, or local agency of any other state having authority to participate in the construction and maintenance of highways, are hereby authorized to plan, designate, establish, regulate, vacate, alter, improve, maintain, and provide limited access facilities for public use whenever such authority or authorities are of the opinion that traffic conditions, present or future, will justify such special facilities—Provided, That within cities and towns such authority shall be subject to such municipal consent as may be provided by law. * * * Said units may regulate, restrict, or prohibit the use of such limited access facilities by the various classes of vehicles or traffic in a manner consistent with this act."

Traffic Regulation.—

> "The State Highway Commission of Indiana and the proper authorities of any county, city, or town having charge of any highway or street affected by this act, are authorized to design any limited access facility and to regulate, restrict or prohibit access as to best serve the traffic for which such facility is intended. In this connection such authorities are authorized to divide and separate any limited access facility into separate roadways by the construction of raised curbings, central dividing sections, or other physical separations, or by designating such separate roadways by signs, markers, stripes, and the proper lane for such traffic by appropriate signs, markers, stripes and other devices. No person shall have any right of ingress or egress to, from, or across limited access facilities to or from abutting lands, except at such designated points at which access may be permitted, upon such terms and conditions as may be specified from time to time by rules and regulations adopted and promulgated as by law provided."

Property Acquisition.—

> "For the purposes of this act, such authorities of the State, counties, cities, or towns, may acquire private or public property and property rights for limited access facilities and service roads, including rights of access, air, view, and light, by gift, devise, purchase or condemnation in the same

---

[6] *Acts 1945*, Chapter 245, Sec. 36, 3101, Burns Repl., 1949.

manner as is now or hereafter may be provided by law to acquire such property or property rights for the laying out, widening or improvement of highways and streets within their respective jurisdictions. * * * The rights of all property owners who may claim damages, as provided by the Constitution of the State of Indiana, are preserved herein and may be enforced under the present laws of the State of Indiana.

"Court proceedings necessary to acquire property or property rights for purposes of this act shall take precedence over all other causes not involving the public interest in all courts, to the end that the provisions of limited access facilities may be expedited."

Powers of State and Local Authorities.—

"The highway authorities of the state, county, city, or town, may designate and establish limited access highways as new and additional facilities or may designate and establish an existing street or highway as included within a limited access facility. The state or any of its subdivisions or municipalities shall have authority to provide for the elimination of intersections at grade of limited access facilities with existing state and county roads, and city and town streets, by grade separation or service road, or by closing off such roads and streets at the right-of-way boundary line of such limited access facility; and after the establishment of any limited access facility, no highway or street which is not part of said facility shall intersect the same at grade. No city or town street, county or state highway, or other public way shall be opened into or connected with any such limited access facility without the consent and previous approval of the proper authorities of the state, county, city, or town having jurisdiction over such limited access facility. Such consent and approval shall be given only if the public interest shall be served thereby."

Financing.—

"The State Highway Commission of Indiana may enter into agreements with the respective counties, cities and towns of the state or with the federal government, and such cities and towns may enter into agreements with the federal government, respecting the financing, planning, establishing, improvement, maintenance, use, regulation, or vacation of limited access facilities or other public ways in their respective jurisdictions, under this act or under any state and/or federal laws authorizing such cooperation, to carry out and facilitate the purposes of this act."

Location and Vacation of Roads and Streets.—

"In the carrying out of the purposes of this act and in the development of any limited access facility the State Highway Commission of Indiana and the several counties, cities and towns of the state are authorized to plan, designate, establish, use, regulate, alter, improve, maintain, and vacate local service roads and streets or to designate as local service roads and streets any existing road or street, and to exercise jurisdiction over service roads in the same manner as is authorized over limited access facilities under the terms of this act, if in their opinion, such local service roads and streets are necessary or desirable."

Traffic Regulation Penalities.—

"It is unlawful for any person (1) to drive a vehicle over, upon, or across any curb, central dividing section, or other separation or dividing line on limited access facilities; (2) to make a left turn, a semi-circular, or U-turn

> except through an opening provided for that purpose in the dividing curb section, separation, or line; (3) to drive any vehicle except in the proper lane provided for that purpose and in the proper direction and to the right of the central dividing curb, separation section, or line; (4) to drive any vehicle into the limited access facility from a local service road except through an opening provided for that purpose in the dividing curb or dividing section or dividing line which separates such service road from the limited access facility proper. Any person who violates any of the provisions of this section is guilty of a misdemeanor and upon arrest and conviction therefor shall be punished by a fine of not less than one dollar ($1.00) nor more than fifty dollars ($50.00) or by imprisonment in the city or county jail for not less than five days nor more than ninety days, or by both such fine and imprisonment."

Amendment.—An amendment[7] to the 1945 law was passed, specifically directing the highway commission to designate and construct all bypasses to limited-access standards. The law states that

> "Whenever the State Highway Commission constructs a by-pass highway around any city or town, the Commission shall be required to designate and establish such by-pass highways as limited access highways."

From the foregoing it can be seen that broad powers have been granted for establishing limited-access facilities.

LIMITED-ACCESS RESOLUTIONS

In order to protect the dual-lane sections in service on the interstate system, resolutions declaring that the respective sections be limited-access highways, except those access points existing at the time of the field inventory, have been prepared and approved by the commission. Limited-access signs, which are 36 in. square, have been placed on all routes covered by the resolutions and are located on alternate sides of the highway at 2-mile intervals at the right-of-way line and facing the highway. The following message is carried—"Limited Access Facility, Direct Access Will Not Be Granted—Before Planning Improvement on Adjoining Property—Contact Indiana State Highway Dept.—102 North Senate Ave., Indianapolis, Ind."

Prior to the erection of the signs along the routes, newspaper and radio publicity was provided. In most cases the newspapers printed the message as front page material. Directives were sent to the six district offices instructing them to deny all permits to those sections covered by the resolution. In those cases in which the applicant insisted that the driveway was essential and in which no other access outlets were available, the case was referred to the Central Office of the Indiana State Highway Department.

To date, these resolutions have proved effective as a major deterrent to the further granting of access permits. The cooperation of property owners and commercial interests has been unusually fine; following a thorough understanding of the purpose of these resolutions, they have indicated their intention of either abandoning their application request or have accepted an alternate solution offered by the commission.

---

[7] *Acts 1955*, Chapter 197, approved March 7, 1955.

BENEFITS

The benefits of limited-access highways may be divided into two classifications: First, those benefits that directly accrue to the highway user and second, those benefits that the state and its citizens receive in the form of (*a*) reduced highway residential and business construction costs and (*b*) losses due to the permanence of limited-access highways.

The direct benefits to the highway user are:

1. The uniform speed that can be maintained over a limited-access route results in time savings to the motorist, depending on the length of the trip. The translation of this time saved to a monetary basis results in large savings, particularly to commercial interests.
2. Fuel consumption is reduced by the elimination of stops and slowdowns. Due to a uniform travel speed that is void of stops, maintenance costs on the vehicle in the form of tire wear and mechanical repairs are reduced.
3. An important benefit lies in the reduction of accidents, which statistics show will be approximately from 10% to 50% of those on an uncontrolled-access highway. For example, in California the death rate per 100,000,000 vehicle-miles was 2.12 on all freeways and 9.39 on all rural highways. In Maine the death rate per 100,000,000 vehicle-miles was 2.8 on the Maine Turnpike, and on parallel U.S. 1 the rate was 22.3 deaths per 100,000,000 vehicle-miles. In Michigan there were 6.7 deaths per 100,000,000 vehicle-miles on the Detroit Industrial Expressway, and on parallel U.S. 112 the death rate per 100,000,000 vehicle-miles was 15.0.[8]

The indirect benefits, which are not as tangible but are important, are concerned with eliminating highway obsoleteness. The limited-access highway has all the attributes necessary to fix it as a permanent improvement. The alinement is fixed permanently because of the control of all access, and its service to traffic continues unabated. As the volume of traffic increases, sufficient right of way is available to add the needed additional lanes. Residential and commercial construction are thus provided with a fixed location for which an orderly development can be planned without the future risk of losing investments because of a relocation.

CONCLUSIONS

The adoption of the controlled-access highway by the Indiana State Highway Department for the interstate system, the major state routes, and all bypasses is the most significant step ever taken in the history of the department. Widespread publicity has been given to the new policy, with a most encouraging response from all parts of the state acclaiming the wisdom and advantages of the new type of highway.

Property owners and business interests have set up, and are trying to set up, their programs of development in order that they will fit into the future facilities. The state is cooperating with these individuals so that the change to the new standards will cause a minimum of disruption.

---

[8] "Let's Build Safety Into Our Highways," by Carl Fritts, *The Highway Magazine*, Armco Metal Products, Inc., Middletown, Ohio, September, 1955.

The enactment of the 1956 Federal Highway Law[9] made it possible for the state to set its program into action. Additional legislation[10] (which became effective in 1957) was required by the state to provide adequate matching funds for such new grants. There is every reason to believe that the vehicle owners will respond to higher taxes if they are definitely assured limited-access facilities will be provided.

Highway building will not be limited merely to reconstructing the interstate system, but will also include a great many major traffic arteries in the state.

---

[9] *Federal Aid Highway Act of 1956.*

[10] *Acts 1957*, Chapter 51, March 15, 1957.

EXPERIENCES IN LOS ANGELES

BY HUGO H. WINTER,[1] M. ASCE

SYNOPSIS

The history of the freeway system in the Los Angeles (Calif.) metropolitan area is presented. The construction of the units of that system, the direct and indirect benefits already derived from the completed units, and the benefits to be obtained from a completed, integrated system of freeways are reviewed.

INTRODUCTION

Los Angeles (Calif.) has long been an advocate of controlled-access highways or freeways. Having an area of 453 sq miles in the hub of a metropolitan area of more than 1,200 sq miles, Los Angeles has become increasingly dependent on automotive transportation. The increased volumes of traffic and the relatively long travel distances have demanded that highways be designed for a great capacity and high over-all speeds with safety. Urban freeways that are built to modern standards probably will provide the rapid transit elements of an effective urban transit system.

Between 1937 and 1958, 152.3 miles of freeways were completed and in operation in Los Angeles County, and at the close of 1957, 28 miles of freeways were under construction. Routes have been adopted for 230 additional miles on which the right of way is being protected or acquired and for which plans are being prepared.

As of January 1, 1958, the freeway program represented the expenditure of about $535,000,000. More than $100,000,000 per yr are being expended in Los Angeles County on approved freeway projects.[2] There are 620 miles of freeways, including those already listed, in the approved master plan of freeways.

Fig. 1 shows the freeway units that were completed or under construction and those that were budgeted or projected in the freeway resolution as of January 1, 1958.

By analyzing the completed units and by projecting their demonstrated benefits to projects that are proposed for construction, it is possible to show that the Los Angeles area has received, and will continue to receive, tremendous dividends from its freeways.

EARLY HISTORY OF THE FREEWAY SYSTEM

Growing street traffic congestion and mass transit inadequacies finally resulted in a major action taken by the city of Los Angeles in 1937. The mayor,

---

NOTE.—Published, essentially as printed here, in January, 1956, in the Journal of the Highway Division, as *Proceedings Paper 874*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Asst. Engr. of Design, Bureau of Eng., Los Angeles, Calif.

[2] "District VII Freeways," by P. O. Harding, *California Highways and Public Works*, January-February, 1955.

with the support of a Citizens' Transportation Committee, appointed a Transportation Engineering Board to study the problem and submit definite recommentations. The board was composed of Lloyd Aldrich, M. ASCE, the chairman, K. Charles Bean, and Rudolph Weber.

An analysis of an extensive traffic and transportation survey, in which the Federal Works Progress Administration participated, resulted in the factual report,[3] and, in December, 1939, a second report[4] outlined the general findings and recommendations of the board.

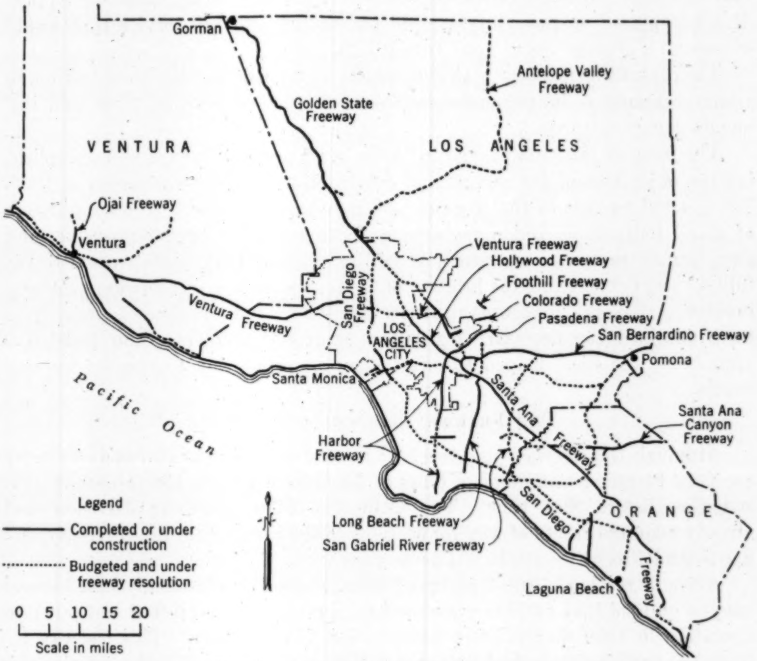

Gorman
Antelope Valley Freeway
Golden State Freeway
VENTURA
LOS ANGELES
Ojai Freeway
Ventura
Ventura Freeway
Hollywood Freeway
Foothill Freeway
Colorado Freeway
Pasadena Freeway
San Bernardino Freeway
Pomona
San Diego Freeway
LOS ANGELES CITY
Ventura Freeway
Santa Monica
Pacific Ocean
Santa Ana Freeway
Santa Ana Canyon Freeway
Harbor Freeway
Long Beach Freeway
San Gabriel River Freeway
San Diego Freeway
ORANGE
Laguna Beach
Legend
Completed or under construction
Budgeted and under freeway resolution
0 5 10 15 20
Scale in miles

Fig. 1.—District VII Freeways

The program of the board was essentially as follows:

1. The adoption of a comprehensive freeway plan designed to serve the needs of the entire metropolitan area;

2. The progressive construction of the units of the freeway plan as rapidly as financing could be provided;

3. The use of the freeways by express buses and the provision of bus-loading facilities adjacent to the freeway roadways;

---

[3] "Report of Traffic and Transportation Survey," The Citizens Transportation Survey Committee, Los Angeles, Calif., July, 1940.

[4] "A Transit Program for the Los Angeles Metropolitan Area," The Transportation Eng. Board, Los Angeles, Calif., December, 1939.

4. The provision of wide medians on the freeways, where indicated, for future rail rapid transit use;

5. The coordination of surface transit operations with the express bus and rapid transit routes along the freeways; and

6. The eventual construction of standard-type, rail rapid transit facilities in the central business district and elsewhere to supplement the routes along the freeways in accordance with the growth of the area.

The program was proposed recognizing the predominant use of the private automobile as a transportation medium. However, as much emphasis as possible was placed on providing a rapid transit bus and rail service that would encourage a greater use of mass transit facilities.

The report was received with enthusiasm by officials and the public although accomplishment of its recommendation would involve years of effort and tremendous expenditures.

The freeway program proved to be the most practical means of accomplishing the objectives of the report, and construction of units was begun in 1939. The general success of the freeway program has been due to the cooperation of many individuals and organizations. Officials have been responsive, and city, county, and state engineers, as well as planners, have collaborated to the fullest extent concerning the location of the routes and the engineering problems involved. Several civic organizations have defended the freeway principle; have obtained the necessary legislation, increased financing, and legislative approvals; and have helped to offset vigorous opposition to specific freeway routes.

The Freeway System on its Way

Although the freeway plan received its first official approval in in 1941 by the City Planning Commission, units of the system were in the process of construction before that time. The California State Highway Division had already acquired rights of way for sections of the San Bernardino Freeway and Santa Ana Freeway outside of Los Angeles City.

A 5-mile section of the Pasadena Freeway, constructed mostly in a natural canyon on land that had been acquired for a park, was completed and put into operation in 1939 shortly after presentation of the report. This freeway replaced an existing state highway route, Figueroa Street, which passed through the business and commercial area of the Highland Park district of the city of Los Angeles. The construction of the freeway was vigorously opposed by the businessmen whose enterprises were located on Figueroa Street. These concerns believed that the removal of the through traffic would seriously affect business; the same merchants are at present enthusiastic in their approval of this freeway. The removal of the heavy traffic from Figueroa Street has greatly improved accessibility to their establishments by both vehicluar and pedestrian traffic. In addition, it has resulted in a general growth and betterment of the entire adjoining district.

Simultaneously, a 1.3-mile section of the Hollywood Freeway through Cahuenga Pass, a natural traffic bottleneck and major connecting route between Los Angeles proper and its San Fernando Valley section, was opened to traffic. Greater ease of travel increased the daily traffic volume through this pass from 40,000 cars to 60,000 cars shortly after the freeway was opened. The traffic volume has increased to more than 185,000 cars per day.

The first completely urban freeway passing through a highly developed area of the city to be considered for construction was the combination of the Hollywood Freeway from Vermont Avenue to the Los Angeles Civic Center, and the Santa Ana Freeway from the civic center to the east city boundary at Indiana Street. These freeways with their subsequent extensions were adopted as the permanent routing for State Route 2 (U. S. 101), an exceedingly important route traversing the city from northwest to southeast. No particular opposition was encountered in adopting the first section, but the section that went through the Hollywood district of the city of Los Angeles was bitterly opposed. This opposition delayed approval of the freeway agreement between the city and state for more than a year, during which time, and even thereafter, numerous hearings were held before the City Council, the State Highway Commission, and even the State Legislature Interim Committees.

A characteristic of almost every freeway unit has been that their proposed location and construction have been vigorously protested. This has been the case even though public sentiment is generally favorable toward freeway construction. It was believed that as more freeways were opened for operation and their benefits were realized the opposition to the freeways would decrease. This has not been the case. The cooperative and independent studies by city and state engineers of available routes, with their advantages and disadvantages, have proved invaluable in obtaining final decisions. It has also been generally characterisitic that once the routing has been approved most of the opposition decreases and there is a general demand for quick action to get the freeway into operation.

The early freeway accomplishments have been followed successively and at a constantly increasing rate by the adoption of routes and the acquisition and construction of freeways included in the master plan. These freeways have been constructed as units of the state highway system using state gasoline tax and motor vehicle funds, including federal aid funds, which were made available for this purpose. The gasoline tax was increased 1½ cents in 1947, and another equal increase occurred in 1953, resulting in a rate of freeway acquisition and construction of from $60,000,000 per yr to $100,000,000 per yr since 1955.

Probably the most helpful measure that was ever passed to assure the eventual construction of the freeway system at realistic right-of-way prices was the act (Chapter 20, 1952, Second Extraordinary Session, California State Legislature, as amended to date) permitting the advance purchases of vacant properties along adopted freeway routes. This act provides for a $30,000,000 revolving fund, reimbursable from the regular highway funds when allocated for right-of-way acquisition along a freeway project. It was estimated that in October, 1955, the local state highway district had, by the expenditure of $20,300,000 of these funds, effected a saving of $133,655,000 in ultimate right-of-way costs in this rapidly growing area. This saving is a real dividend, collectible because an approved freeway system exists. The act has greatly encouraged the official establishment of routes well in advance of construction and, thereafter, the protection of the right of way by use of this revolving fund.

Other advantages of the advance establishment of routes are the assurance and stability thus given to the districts traversed. Properties can then be developed with confidence and a knowledge of what will happen when the freeway is constructed. It is the uncertainty of not knowing just where the freeway is going that creates unrest, hampers development, and actually lowers property values. Therefore, route adoption is an intangible asset of considerable value.

All financing of the freeways has been on a "pay-as-you-go" basis. This method insures that the highway funds are directly used in acquisition and construction and that there is no loss in interest charges. However, it automatically limits the rate of construction to the amount of available funds, which at the present time have been insufficient. It is believed by many that the benefits of the freeways are so great that they would more than offset the interest payments involved if the remaining badly needed freeways were to be constructed at a much more rapid rate by bond financing.

The demonstrated benefits, which already have been observed of the freeways in actual operation, will be examined subsequently.

DEMONSTRATED BENEFITS OF THE FREEWAYS

Safety.—In spite of the general official approval of the freeway plan and the acceptance of the freeway idea by the public, there was a growing criticism by the press and the public of the alleged "high accident" rate and the "high speeds" of the traffic using the freeways. Serious recommendations were made for limiting the speeds on the Pasadena Freeway to 45 miles per hr in an attempt to reduce the supposed great accident rate.

Although the Bureau of Engineering is primarily a design organization, it appeared that, in defense of the freeway principle, it would be necessary to assemble the facts and to make this information available to the public to counteract some of the growing opposition and unfounded criticism. The Bureau of Engineering therefore undertook a study resulting in a report[5] that has been given widespread publicity.

In this study, for the purpose of a comparison with the existing freeways, the following surface streets were selected:

1. Wilshire Boulevard, a major east-west traffic artery from downtown Los Angeles to Beverly Hills, with a length of 6.98 miles; and
2. Figueroa Street, a north-south state highway from downtown Los Angeles to the Imperial Highway, with a length of 8.29 miles.

The selected routes are well-controlled boulevards having heavy traffic loads under conditions normal to city streets. In a 3-yr period chosen for comparison purposes (from 1948 to 1950 inclusive), the travel on these arteries totaled 600,000,000 vehicle-miles.

Compared with these arteries were the following freeways which were in operation during the same 3-yr period:

---

[5] "Comparison of Parkways (Freeway) with Surface Streets in Capacity, Time Saving, and Safety," by Lloyd Aldrich, report to the Los Angeles Board of Public Works, Los Angeles, Calif., November, 1951.

1. The Pasadena Freeway, which goes from Glen Arm Street, Pasadena, to College Street, which is a distance of 6.12 miles;
2. The San Bernardino Freeway, which is 2.3 miles long, going from Vignes Street to Indiana Street;
3. The Santa Ana Freeway, going from Vignes Street to Ditman Street, which is a distance of 3.8 miles; and
4. The Hollywood Freeway, which goes from Odin Street to Vineland Avenue, a distance of 3.07 miles.

The travel on the Pasadena Freeway totaled 426,000,000 vehicle-miles, and a combination of the four freeways accounted for 776,100,000 vehicle-miles. The comparisons are therefore proportional to the volumes of traffic.

At the time that the comparison was made (1948–1950), the Pasadena Freeway, one of the first local freeways constructed, had been in operation for approximately 10 yr. Therefore, the Pasadena Freeway is not constructed to the high standards of the more recent freeways. The roadway does not have the continuous shoulders that are now provided on all freeways. However, to compensate partly for this deficiency, occasional parking bays were added in 1955. The freeway has narrow median strips, curve radii that are as short as 515 ft, inadequate or no accelerating and decelerating lanes at ramp connections, and insufficient side clearances to piers and abutments. The San Bernadino Freeway, following an existing surface boulevard, is also of lower design standards. The abrupt transition from freeway conditions to surface street conditions at the several temporary terminals then existing created a relatively high accident potential, and, in fact, most of the recorded accidents occurred at these sites.

TABLE 1.—Recorded Accidents

Facility	Rates per 100,000,000 Vehicle-Miles for 1948 to 1950 (3 Yr)	
	Fatality Accident	Combined Fatality and Injury Accident
Pasadena Freeway	0.7	46.5
Hollywood, San Bernardino, and Santa Ana freeways	2.9	60.0
Combined freeways	1.7	53.9
Combined Figueroa Street and Wilshire Boulevard	4.2	259.8

No records were available at that time for the newly opened sections of the Hollywood Freeway, which is in fact the first local freeway to be constructed in conformity with modern design standards. In view of these facts the demonstrated safety advantages of the previously listed freeways as compared with the surface arteries are all the more remarkable.

Table 1 presents the recorded accidents on the surface arteries and freeways for the 3-yr period between 1948 and 1950.

An examination of this table indicates that the combined accident rate of the surface arteries for the 3-yr period was 5.6 times that of the Pasadena Freeway and 4.8 times that of the combined freeways. The fatality rate of the surface arteries is 6 times that of the Pasadena Freeway and 2½ times that of the combined freeways.

A summary of the fatal freeway accidents through 1954 indicates that the Hollywood Freeway fatalities per 100,000,000 vehicle-miles was 1.44 based on 1,179,000 vehicle-miles of travel, confirming the more favorable effect of modern design on the fatality rate. The combined experience on the freeways through 1954 was 2.25 fatalities per 100,000,000 vehicle-miles. This compares with 5 fatalities per 100,000,000 vehicle-miles for surface streets.

The injury accident rate in 1954 for all surface streets in Los Angeles was 175 accidents per 100,000,000 vehicle-miles as compared with 57 accidents per 100,000,000 vehicle-miles occuring on freeways.

The statistics indicating a most favorable record for the freeways fully justify the assertion that there is a decided advantage from a safety standpoint in freeway travel over that of a highly regulated, heavily used urban surface artery.

Although figures show freeway travel to have decided advantages, engineers, highway builders, and administrators cannot ignore the fact that accidents occur on freeways and that more should be known about them.

By analyzing the accidents as to location, type, and severity, it can be determined whether there are accident-prone locations. Sometimes the contributing condition is easily seen. At other times the cause is not so obvious. It is particularly valuable to know whether geometric design features have contributed to the more severe freeway accidents.

A brief summary of the 46 accidents resulting in 56 deaths on the freeways in Los Angeles, through October, 1955, is of interest. Eleven of the drivers involved in the 46 accidents were classified by the police as being drunk. Two additional drivers were known to have been drinking. In only 2 of the 11 accidents involving drunken drivers was anyone other than the driver or an occupant of the car killed. Five of the deaths were pedestrians, one of whom had been drinking; another was a flagman who was struck by a vehicle that was traveling at a rate of only 15 miles per hr. Only 1 accident was attributed to a mechanical failure resulting in a car crossing the center strip.

Drivers who were 25 yr or younger accounted for 20 of the 46 accidents. In 29 accidents only 1 vehicle was involved. Sixty-nine percent of the accidents occurred during darkness although only 30% of the traffic is at night.

Inadequate freeway design standards appeared to be a minor contributing cause. In those cases in which inadequate design standards were the cause, the design was usually inferior to the present standards. Four of the 5 accidents on the San Bernardino Freeway appear to be in this category. In three of these cases the narrow dividing strip was crossed on curves with less than the existing minimum standards. The fourth occurred at a site at which the oncoming traffic is introduced into the high-speed lane. Nine accidents occurred at ramps leading off the freeway; 2 of these accidents involved drunken drivers and 2 cases could be attributed to inferior design.

The summary of fatal accident causes clearly shows that a freeway that has been constructed in accordance with the present high standards of design is inherently a facility of outstanding traffic safety when it is properly used. It can be expected that with the aid of the improved operating and regulating techniques that are being studied and developed by the traffic engineers and enforcement officers, the safety record can be greatly improved in the future.

Capacity.—A comparison was made of the relative capacity of the freeway as compared with the capacity of the urban surface traffic artery. To secure the maximum use of the traveled surface streets in the urban area, from four to eight traffic signals per mile are necessary. Many of the methods known to traffic engineers are also used, including parking bans, prohibiting left turns, and off-center laning. Although all these methods are necessary for maximum capacity and safety, they penalize the individual driver at some point in his journey, as well as the lesser streams of traffic crossing the major arteries. The result is that the most favored surface arteries carry a maximum of about 625 vehicles per lane per hr. This rate is comparable to many of the freeway lanes for which the measured traffic is 1,800 vehicles per lane per hr and which, when they operate under pressure, carry as many as 2,200 vehicles per lane per hr over sustained periods.

Time Savings.—That high speeds are common on the freeways also was not sustained by facts. More than 10,000 radar readings on the Pasadena Freeway have established an average speed of 46.4 miles per hr between the hours of 7:00 a.m. and 6:00 p.m.

Six test runs were made on surface streets paralleling the Pasadena Freeway and its proposed extension via the Harbor Freeway. Seventy-three signals were encountered in a distance of 19.2 miles, and an average speed of 20.8 miles per hr was recorded. On two trips at different locations, the travel speed was reduced to 6 miles per hr for a distance of 1½ miles.

Similar test runs paralleling the San Bernardino-Hollywood-Santa Monica Freeways for a distance of 30 miles showed an average speed of 23.2 miles per hr. One hundred and fifteen signals were encountered, and speeds were reduced to less than 10 miles per hr in many places, the lowest speed recorded for any half mile being less than 5 miles per hr.

Summary of Physical Benefits.—The 1951 study showed that, in comparison with a surface traffic artery, the freeway moves 3 times as much traffic in one-half the time and with approximately one-fifth the risk of an accident.

Direct Economic Benefits Resulting from Freeway Use

Report on "Economy of Freeway".—The great costs of the right of way and the construction resulted in a criticism of the expenditure of the vast sums of money allocated to the freeway projects. The freeways were termed "gold plated," and there was a tendency to emphasize the improvement of the surface arteries at a considerably smaller first cost. Having demonstrated the physical benefits of the freeways, it was considered necessary to measure their monetary value. The motorists should be informed of the advantages of the freeway in terms of the financial cost. The legislative and public support for the freeway program to continue must be gained and held. In constructing freeways the motorists' money was invested in a facility returning some substantial dividends.

This led to the publishing of the second report,[6] in which every effort was made to be conservative and to establish values that could be listed as minimum benefits that could not effectively be challenged.

Operating Economies.—By the use of speed and delay studies and fuel savings tests, which are fully described in the report, it was possible to establish a minimum saving of 0.33 cent in gasoline to motorists using the freeways and 0.24 cent in maintenance savings per vehicle-mile. Other estimates in the report are based on research and reasoning rather than on experiment. The American Association of State Highway Officials (AASHO) in a report[7] agreed closely with the figures previously mentioned that were obtained by the Bureau of Engineering of the city of Los Angeles.

Accident Savings.—The assignment of an economic value to the accident savings was probably a new idea at the time of the report. The local freeways, while operating under the then unfavorable conditions of short lengths and overloading, consistently maintained an accident record that is many times better than that of the surface traffic arteries of Los Angeles, which are the best controlled and most efficient of surface boulevards. Therefore, it seems reasonable to include the reduction of accidents as a fixed characteristic of modern freeways and to enter it as an economic credit. The value applied to accident savings was determined by using the records of the California Insurance Commission; the auto insurance premium costs, excluding theft, fire, and comprehensive coverage; and from other records concerning the total gasoline consumption. This information resulted in the determination of an accident savings of 0.56 cent per vehicle-mile due to the use of the freeways.

The Value of Time.—Time savings are of considerable and actual worth, but it appeared difficult to assign a value to them. In the case of a commercial vehicle, a saving in the driver's wages and an increase in the number of trips per day can be measured in terms of money. However, it is not so easy to assign a value to pleasure time or to time spent in going to and from work.

Therefore, because the report was conservative, the saving in time for pleasure vehicles was completely ignored although this omission is probably debatable. However, the time value of trucks was distributed to all the vehicles in the proportion of trucks to the total traffic, and an average value for time savings of 0.87 cent per mile was obtained.

Summary of Report.—A summation of operating savings, accident savings, and time savings establishes a total of 2 cents per mile as the minimum direct benefits to the motorists using the freeways. Applying this total to the conservative estimate of 776,100,000 vehicle-miles of freeway travel over 16.57 miles of freeways in operation for a 3-yr period results in an indicated saving of $15,522,000. The original cost of the 16.57 miles of freeways that were studied was $42,027,000. If the savings could have been applied to the payment of the freeways, their original cost would have been amortized in about 3 yr.

This agrees closely with a later study,[8] which states that, based on test runs by engineers other than those employed by the Los Angeles Bureau of Engineering, Los Angeles motorists save $50,000,000 per yr on 45.1 miles of completed freeways estimated at $143,000,000. On this basis the freeways would also pay for themselves in less than 3 yr.

---

[6] "The Economy of Freeways," by Lloyd Aldrich, report to the Los Angeles Board of Public Works, Los Angeles, Calif., June, 1953.

[7] "Road User Benefit Analyses for Highway Improvements," A.A.S.H.O., 1952.

[8] "An Appraisal of Freeways vs. Surface Streets in the Los Angeles Metropolitan Area," Automobile Club of Southern California, Los Angeles, Calif., 1954.

The economic figures developed by the Bureau of Engineering, the Automobile Club of Southern California, and AASHO receive wide recognition because they clearly show the enormous value of the freeways. Additional reports have indicated by various studies the direct benefits of the freeway system in the Los Angeles area.

Indirect Benefits.—The additional indirect benefits of the freeways cannot be evaluated easily in monetary terms. The 1952 AASHO report[7] contains no value for traffic accident savings, but it does include a definite value for comfort and convenience. Other benefits are (*a*) the stabilization or enhancement of property value; (*b*) the ability to plan properly for the land use and the zoning of adjacent areas to provide the most effective development of the area; (*c*) the relief of existing overburdened surface arteries; (*d*) the doubling of the practical radius of real estate development on a travel time basis; (*e*) the increased access to recreational or cultural facilities; (*f*) the increased mobility in times of disaster emergencies; (*g*) the increased tourist travel; and (*h*) all other advantages of the betterment of transportation. The benefits to the large quantities of traffic that continue to use formerly heavily congested surface arteries after the freeway system is built are not usually visualized. Before-and-after surveys have shown that the removal of through traffic from surface arteries to the freeways is of benefit to community business, property values, surface travel time, and safety on the surface system. In addition, the intended use of the freeways by express buses will greatly increase the economic value of the freeways to the general public, of which motorists are a large part. The monetary value of these benefits could be great because they are so vitally integrated with the general financial health and progress of the region. If there were no freeways the region would suffer losses. This thought is amplified in an article[9] enumerating the many benefits of the freeways constructed in the Los Angeles area.

The rate of both residential and industrial growth has been greatly spurred in the areas adjacent to the San Bernardino Freeway and the Santa Ana Freeway. When they were planned, these freeways crossed lands that were almost wholly devoted to agriculture. These areas have since experienced a residential and industrial development of amazing proportions.

Attention should be directed to before-and-after construction studies that have been made by the California Division of Highways of the effects of a freeway on the districts traversed. These studies supply authentic, well-documented evidence of the direct benefits that have accrued to various districts of California from the freeways serving them.

EVALUATING THE BENEFITS OF A FREEWAY SYSTEM

General.—It seemed desirable to carry the studies somewhat further in trying to establish the benefits of a freeway system as compared with the abstract and general benefits derived from a previous report.[9] This was done in a more recent report.[10]

---

[9] "Revealing Direct Benefits from Freeway Development," by Paul Harding, California State Div. of Highways, Sacramento, Calif., July–August, 1955

The value of a freeway increases as it becomes integrated in a freeway system. Temporary terminals with their resultant bottlenecks are eliminated, and a more flexible use of the freeways to reduce surface-artery congestion is made possible.

Furthermore, it is difficult to use a single freeway most effectively for express bus service. Express bus operation is dependent on surface feeder lines, which cannot be established effectively until express bus service is available on an integrated freeway system.

The failure to construct the freeway units of a system rapidly enough has resulted in an overloading of the freeways that have been completed, thus reducing their efficiency. For example, the Hollywood Freeway presently handles 202,000 cars per day in its maximum traffic section. This is a result of an undue diversion of traffic from parallel surface arteries. The fact that 40% and 48% of the traffic on Sunset Boulevard and Temple Street, respectively, was diverted is understandable because these streets closely parallel the freeway. However, a traffic reduction of from 20% to 30% on streets such as Olympic Boulevard and Riverside Drive, which are located as far as 2 miles from the freeway, indicates the need of the Olympic and Golden State Freeways, which parallel these streets. Although the Hollywood Freeway carries this heavy load well, with peak hour flows of as great as 9,000 cars per hr in one direction on a four-lane roadway, it does so at reduced speeds and with a potential congestion that can be caused by the slightest accident or emergency.

There is little doubt that this or any area would be of benefit if the proposed freeway system could be built as rapidly as possible. However, the construction of freeways is permitted only as rapidly as the funds become available. Although the freeways in Los Angeles are at present being constructed at the rate of approximately $100,000,000 per yr, this amount is not adequate to provide the freeways as rapidly as they are needed. The freeways demanded by the present traffic flows greatly exceed the present operating freeway system.

Two major points to be considered are:

1. If it is necessary to continue to construct the freeway system on the basis of one unit at a time as funds permit, those giving the greatest return for the money invested should be built first.
2. If freeway benefits are as great as is shown by the analyses, an attempt should be made to expedite their early construction by a method of financing other than, or supplemental to, the pay-as-you-go method.

Determination of Freeway Priorities.—In considering the first major point previously mentioned, it seemed necessary to determine a ratio between the freeway costs and the benefits for different conditions of traffic demand, terrain, and local development. This cost-benefit ratio has two purposes: One is to determine if a freeway is justified as compared with the improvement or reconstruction of surface routes. The other purpose is to permit a proper comparison of one freeway route with another, or to establish the correct priority for constructing the individual freeways of a planned system.

---

[10] "A Study of Freeway System Benefits," by Lloyd Aldrich, report to the Los Angeles Board of Public Works, Los Angeles, Calif., September, 1954.

In justifying the first cost of a freeway, it is desirable to be extremely conservative, as was the case in a previously mentioned report,[6] and to avoid using any controversial values. However, in comparing the value of two proposed freeways, an important factor is the proportion of truck traffic to the estimated total traffic. If a time value is used for trucks only, the comparison is in favor of the route that will carry the greatest quantity of trucks even when the largest demand and the greatest traffic relief favor another route. For situations in which a motorist is willing to drive additional miles in an attempt to save time, or to pay a toll for the same purpose, some measure is given of the value he himself puts on his time.

Therefore, in the most recent report[10] of the Bureau of Engineering, values have been tabulated for passenger car benefits, with and without time savings, and benefits for pickups and trucks using the freeways. Average benefits computed for a large mileage of the proposed freeways are shown in Table 2.

For computing truck benefits for the individual freeway routes, the California Public Utility Commission's constructive mileage ratios have been used. This mileage is used for rate-making purposes in the transportation of property. It is designed to compensate for adverse physical conditions of the highway, such as grades, poor roadway surfaces, curvatures, traffic signals, congestion, and other factors that add to the cost of truck operation in excess of the normal expense.

TABLE 2.—AVERAGE BENEFIT PER VEHICLE-MILE

Vehicle	Average annual vehicle-miles, in millions	Average annual benefit, in millions of dollars	Average benefit per vehicle-mile, in cents
Passenger cars with time value	3,726.8	138.9	3.73
Passenger cars without time value	3,726.8	41.6	1.12
Trucks	257.8	25.6	9.93
Pickups	208.2	9.7	4.06
Weighted average vehicle			4.16

In order to estimate the benefits of a particular freeway route, it is necessary to estimate the anticipated traffic volumes at the time the freeway is put into operation and the anticipated traffic growth in 20 yr. These volumes must be divided into percentages of passenger car traffic, pickup traffic, and truck traffic. From these volumes and from known lengths of the freeways, an estimate of vehicle-miles of travel can be determined. From the percentages and constructive mileage ratios, a weighted average benefit per vehicle-mile can be determined. From these two factors the total estimated benefits can be computed.

Estimates of the first costs of the freeway units can be made by standardized methods of computing the right-of-way costs and the construction costs.

Thus, a measure of the value of the freeway is determined, and this value can be compared with the value of the old or surface route; by relating the costs to the benefits a cost-benefit ratio can be obtained. The cost-benefit ratio is the measure of the value of the freeway as compared with other freeway routes. It is also a measure of the relative value or priority of the individual freeway routes.

In order to demonstrate this method and its results, computations were made of the cost-benefit ratios of four typical proposed freeway units of the Los Angeles freeway plan. Two routes, the Industrial Freeway and the Golden State Freeway, will carry a heavy percentage of trucks and pickups, and, thus, were found to have cost-benefit ratios of 4.5 and 3.4, respectively. One of the routes, the Olympic Freeway, will carry an exceedingly heavy volume of passenger cars, with a low percentage of trucks and pickups. However, the cost-benefit ratio was also 3.4, which was equal to that of the Golden State Freeway, which has heavy truck and pickup use. The Colorado Freeway, the fourth route to be compared, shows a smaller total volume of traffic and a relatively low percentage of trucks and pickups, resulting in a cost-benefit ratio of 9.5.

Benefits from a Completed System of Freeways.—The approved 1955 master plan of freeways included a total of 620 miles, including those freeways that were already built, those that were being constructed, and those that were being authorized. To demonstrate the tremendous benefits of even a partly completed system of freeways, a study was made of a group of representative freeway projects in the Los Angeles metropolitan area, not including those projects already completed or budgeted. These projected freeways totaled 185 miles, which, when added to the freeways already assured, would increase the total to approximately one-half of the system contemplated by the master plan. The estimated cost of the 185 miles of freeways was $775,000,000.

The reduction of annual benefits to a present value, with an interest rate of 3% compounded annually, indicates that the benefits would equal the initial cost of the 185 miles in less than 7 yr (passenger car time savings included), or in 15 yr (if passenger car time savings are not included). In addition, for a 20-yr period the benefits would be more than 3 times the original cost, with an interest rate of 3% and including passenger car time savings.

Such facts as have been demonstrated by the findings for the Los Angeles freeway system, and which are applicable to any freeway-type highway or system wherever properly proposed, raise a question as to the adequacy of existing financing methods. These facts provide justification for considering bond financing.

To justify such an expedited freeway construction program, it is necessary to demonstrate that (*a*) the design and planning can be sufficiently expedited; (*b*) the purchase and the clearing of the rights of way can be accomplished; and (*c*) the contractors have supplied, or can supply, the needed construction equipment and services to complete the accelerated program. It has been shown clearly that each of the foregoing requirements can be met.

CONCLUSIONS

The freeway, or controlled-access highway, is of benefit not only to the highway user but to the community as well. The planning and construction of freeways should therefore be accelerated to meet this outstanding local and national need.

USER BENEFITS IN CALIFORNIA

By Ralph A. Moyer,[1] A. M. ASCE

Synopsis

An analysis is given of the highway-user benefits resulting from the early completion of the freeways with full control of access that comprised the 1,912 miles of rural and urban highways in California on the National System of Interstate Highways in 1955. The analysis is important in determining the economic justification for the $2,300,000,000 needed to complete the California Interstate System during a 10-yr period.

It is shown that the estimated user benefits in the first 10-yr period totaled $1,000,000,000, which is greater than the total interest on the investment for a period of 30 yr. The estimated user benefits for a 30-yr period approximated $8,000,000,000, which is more than twice the amount of the total interest and the principal that would be required to finance the Interstate System.

Introduction

Control of access is the primary factor which makes it possible for freeways to carry exceptionally large traffic volumes and at the same time provide savings and benefits to users that fully justify the right-of-way costs and the construction costs of the freeways on this system.

The analysis presented herein indicates the magnitude of the funds involved in the construction of freeways and in the operation of the traffic on them. Such an analysis is of major importance in evaluating the "Grand Plan" proposed by President Eisenhower in 1954 for an accelerated, integrated, national highway program.

During 1954 and 1955 the foregoing plan stimulated a greater interest in the national highway program than any previous plan in the history of highway development in the United States. The plan encompassed all highway needs (as of the period from 1954 to 1955) and the projected needs during the next three decades. It involved a total expenditure of $101,000,000,000 between 1955 and 1964 to improve the roads and streets in all road systems. Top priority was given to the modernization of the 40,000-mile National System of Interstate Highways, which, it was proposed, should be financed mostly with federal funds.

Doubt was expressed that the federal government and the states would agree on a plan to finance a $101,000,000,000 program. However, the highway situation was so critical that only a bold plan, such as the one proposed by the President, could resolve highway-traffic problems. It was important that all the factual data pertaining to the financing and the economic justification for

---

Note.—Published, essentially as printed here, in January, 1956, in the Journal of the Highway Division, as *Proceedings Paper 875*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Prof. of Civ. Eng. and Research Engr., Inst. of Transportation and Traffic Eng., Univ. of California, Berkeley, Calif.

embarking on such a tremendous public-works program be fully explored before the United States Congress and the states reached a decision either for the program or against it.

Although the analysis of highway-user benefits is an important factor in determining the economic justification for the expenditure of highway funds, it is only a part of a much larger study that should be made in developing a sound highway program for each state. Studies such as the state-wide highway-planning surveys and the engineering investigations, in which practically all states have cooperated with the Bureau of Public Roads, United States Department of Commerce (BPR), since 1935, have great value and importance. These surveys and investigations provide the basic facts for the formulation and evaluation of the President's program, but their value and importance are not generally understood. Therefore, it seems appropriate to relate the user-benefit analysis to the broader features of the highway-planning surveys, and to review briefly the development and functions performed by state-wide highway-planning surveys.

Importance and Significance of the State-Wide Highway-Planning Surveys

The large-scale highway program proposed by the President represents the culmination of 20 yr of intensive fact-finding through the state-wide planning surveys and engineering investigations. The latter were begun under the provisions of the Hayden-Cartwright Act of 1934,[2] which authorized the expenditure of not more than 1½% of the amount of federal-aid highway funds apportioned for any year to any state for highway-planning surveys and engineering investigations.

At the time the highway-planning surveys were initiated, they were described[3] as follows:

> "They consist of a number of related studies that seek to determine the present state of the whole rural highway system; to rate the service rendered by the numerous parts; to prepare the way for a selection of that part of the whole system which, by reason of its relative importance and absolute utility, so far as we may now see, merits inclusion in future improvement plans; to assemble the facts necessary for an estimate of the ultimate cost of owning and maintaining the economically necessary improved system; all to the end that a definite economically and socially defensible, integrated highway improvement program may be established and the future of highway transportation may be protected from the hazards inherent in short-sighted and shifting public policy."

It is significant that the state-wide highway-planning surveys, which were begun in the 1930's, have been conducted in all states during the past 20 yr in cooperation with the BPR in accordance with the broad policy outlined previously and have yielded valuable factual data. There have been many important changes in the pattern of highway transportation since the surveys were initiated in 1935. The full impact and the significance of these changes can be determined because the basic traffic and highway fact data have been carefully studied during the past 20 yr.

---

[2] *Public Law 393*, 73rd Cong., U. S. Govt. Printing Office, Washington, D. C., June 18, 1934.

[3] "Highway Practice in the United States of America," Public Roads Administration, Washington, D. C., 1949, pp. 49–50.

The Interstate System in California

One of the important results of the planning surveys was the establishment of the 40,000-mile National System of Interstate Highways. Traffic surveys show that this system carries more than 14% of all highway traffic in the United States, although it represents only 1.2% of the total road mileage. In Table 1 the mileage and traffic data for the California Interstate System (as of 1954) are given. It is interesting to note that the urban mileage, which totaled 245.5 miles or one-eighth of the total mileage on the California Interstate System, carried one-third of all traffic on the system, and that the traffic on the 1,912.6 miles of this system comprised approximately one-seventh of all highway traffic on all streets and highways in the state. Truck traffic, including buses, represented 12% of the total traffic. The remaining 88% consisted of passenger car traffic, which included panel trucks, pickup trucks, and four-tired delivery vehicles.

TABLE 1.— Mileage and Traffic Data for the California Interstate System

Classification	Miles in 1954	Traffic In 1953	
		Average Daily Vehicle-Miles	Annual Vehicle-Miles
Rural interstate	1,667.1	12,503,000	4,564,000,000
Urban interstate	245.5	6,554,000	2,393,000,000
Total	1,912.6	19,057,000	6,957,000,000

TABLE 2.— Costs for Right of Way and Construction of the California Interstate System in a 10-Yr Program

Item	Rural	Urban
Right-of-way cost, from 1955 to 1964	$124,807,000	$1,022,180,000
Total cost of 10-yr program	$592,318,000	$1,729,235,000
Total cost per mile	$355,200	$7,044,000

The costs of a 10-yr program for constructing the Interstate System to full freeway standards and to specifications that will be adequate for traffic 20 yr hence, according to estimates prepared by the California Division of Highways, are given in Table 2. For the rural section of the Interstate System, the right-of-way costs are approximately one-fifth of the total cost of the rural section, whereas for the urban section the right-of-way costs are more than one-half of the total cost of that section. In addition, the total cost of the urban section is approximately three times that of the rural section. The total cost for a 10-yr program for right of way and for construction of the California Interstate

System is estimated at $2,321,553,000, or nearly 10% of the estimated cost for the National System of Interstate Highways in all states.

ANALYSIS OF HIGHWAY-USER BENEFITS

In making this study, highway user-benefit data were used as well as other data developed by the writer through research studies conducted at Iowa State College, at Ames, and at the Institute of Transportation and Traffic Engineering of the University of California at Berkeley. One of the best sources of information for a user-benefit analysis is the report[4] by the Committee on Planning and Design Policies of the American Association of State Highway Officials (AASHO). Another useful source is the report[5] on freeway benefits by the engineering department of Los Angeles. Both reports deal primarily with an analysis of the benefits in the operation of passenger cars; they provide only limited data concerning commercial vehicles, such as buses and trucks in various weight classifications, which are such an important part of traffic. Buses and trucks complicate the study of user benefits because they strongly influence such factors in vehicle operation as speeds, time savings, accidents, and costs. The use of ratios in relating truck and bus operating costs to passenger car operating costs provides the best available solution. However, this method is not accurate for user-benefit analysis of individual projects, for which the effects of grades, traffic stops, time savings, and accident savings may vary greatly depending on local conditions. The importance of developing more reliable and complete information with regard to operating costs for many different conditions for all major classes of commercial vehicles is apparent when it is recognized that approximations, when applied to the benefits predicted for a project life of from 20 yr to 30 yr, may result in errors of millions of dollars on a single major project.

The need for highway improvements and the benefits derived from them has been so great that highway engineers have seldom considered it necessary to make a careful analysis of user benefits and to demonstrate the economic justification for the expenditure of public funds for building roads and streets. However, when a $101,000,000,000 program is considered, engineering responsibility becomes greater than in the past. As a result of this program, a fixed pattern of transportation, notably in metropolitan areas, probably will be established for the next 40 or 50 yr. The investment of $100,000,000,000 in highways would be so costly that it would possess a degree of permanence that could threaten progress in planning future transportation facilities, especially especially for large metropolitan areas. Therefore, highway engineers should plan wisely for the future, not only by using the facts that are furnished by the highway-planning surveys, but also by developing new facts that are urgently needed to plan transportation facilities for large metropolitan areas.

Highway engineers should consider carefully certain practices recommended by the Subcommittee on Benefits and Cost of the United States Federal Inter-Agency River Basin Committee.[6] The subcommittee reviewed the current benefit-cost practices of various federal agencies and formulated mutually acceptable principles and procedures for determining benefits and costs for all water-resource projects in the United States.

---

[4] "Road User Benefit in Rural Areas," Committee on Planning and Design Policies, A.A.S.H.O., Washington, D. C., 1955.

[5] "A Study of Freeway System Benefits," Street and Parkway Design Div., Eng. Dept., Los Angeles, Calif., 1954.

[6] "Proposed Practices for Economic Analysis of River Basin Projects," Subcommittee on Benefits and Costs, U. S. Federal Inter-Agency River Basin Committee, Washington, D. C., 1950.

The purpose of the procedures of economic analysis set forth by the subcommittee was to determine and evaluate the most effective use of the economic resources of the United States in developing and programming water-resource projects. The comprehensive nature of the proposed procedures is shown by the following partial list of items that are examined in the report[6]:

1. Standards for the measurement of benefits and costs;
2. Classification into primary (direct) and secondary (indirect or induced) benefits and costs;
3. Effect of the price levels (present and future);
4. Interest rates, discount rates, and risk allowances;
5. Period of the analysis—project life and economic life;
6. Adjustment for varying levels of employment and economic activity;
7. Effect on the land values;
8. Effect on production as determined by the amount of, and value placed on, goods and services;
9. Consequential damages; and
10. Treatment of taxes.

Some state highway departments are considering the need for economic analyses of highway projects. However, the practices with regard to the procedures used in making these analyses vary greatly in different states. Frequently, none are made, or if they are made they are often sketchy and oversimplified.

The analysis of benefits for the California Interstate System presented herein considers two major items—the user benefits and the project costs. Analyzing on a system basis, rather than on a project-by-project basis, may oversimplify the solution. However, the solution presented indicates the magnitude and the importance of user benefits in determining the economic justification for a program in a given state. Furthermore, it may stimulate a greater interest in developing standard procedures and the necessary factual data elsewhere.

USER BENEFITS ON THE URBAN FREEWAYS IN THE CALIFORNIA INTERSTATE SYSTEM

User benefits on urban freeways consist of savings in time costs, vehicle operating costs (fuel, oil, tires, and repairs), and traffic-accident costs. It is assumed that full freeways will be built on all parts of the urban portion of the California Interstate System, which will provide full control of access and the complete elimination of intersections at grade, vehicular traffic conflicts, and pedestrian traffic. Geometric design features will be used to expedite the safe movement of the large volumes of through traffic that are characteristic of arterial highways in the urban areas.

The weighted average benefit of 4.16 cents per vehicle-mile on the urban freeways in the Interstate System represents the average benefit or saving resulting from the operation of cars, trucks, and buses on full freeways instead of on the state highway routes on major city streets (for which there are many intersections at grade controlled with traffic signals that require frequent stops).

The time-cost savings of $1.35 per hr for passenger cars and $3.00 per hr for trucks and buses are the values used by the California Division of Highways and the Los Angeles engineering department for highway-user benefit studies. The AASHO Committee on Planning and Design Policies also adopted[4] a time value of $1.35 per hr for passenger cars as being "representative of current opinion for a logical and practical value."

It is difficult to establish a time value for passenger cars, but there are three factors that should be considered:

1. The highway-planning surveys in California and in other states have shown that from 60% to 70% of the total passenger car travel is for business trips.
2. In many studies the time saved is the most reliable factor to be considered in assigning traffic to freeways. More than one-half of the trips on the freeways show a saving in time when compared with trips that could be made by the same cars using the nearest alternate route.
3. Motorists have been willing to pay a toll to travel on turnpikes where the principal user benefit is the time saved; recent studies made by the BPR have shown that a time-saving advantage is shown in more than 80% of the trips by passenger cars using the Maine Turnpike and the Pennsylvania turnpike.[7,8] The time-cost savings per vehicle-mile for cars is 2.61 cents; the corresponding value for trucks and buses is 3.18 cents per vehicle-mile; and 2.68 cents per vehicle-mile represents the time-cost savings for all vehicles.

The vehicle-operating-cost savings adopted for this study are the same as those used by the Los Angeles engineering department in its studies of freeway benefits. The savings in operating costs are due primarily to the elimination of traffic stops on the freeways, which permits all classes of vehicles to operate on the freeways at uniform speeds. The operating costs for various types of vehicles can be measured for any given project and are, therefore, costs that can be established with a greater degree of accuracy than the time-cost savings and accident-cost savings. The operating-cost savings adopted are average values considered to be representative of the benefits in operating cost provided by full freeways in urban areas. For passenger cars the vehicle-operating-cost saving is 0.52 cents per vehicle-mile, and for trucks and buses this value is 3.18 cents per vehicle-mile. The vehicle-operating-cost savings for all vehicles is 2.68 cents per vehicle-mile.

The accident-cost saving of 0.6 cents per vehicle-mile is the same value as that used in the freeway-benefit study by the Los Angeles engineering department, although a slightly different procedure was used in computing the accident-cost saving.

The California highway safety record on full freeways from 1941 to 1954 was outstanding and fully justified the accident-cost saving of 0.6 cents per

---

[7] "Pennsylvania Turnpike Traffic Analysis," by Daniel O'Flaherty, *Public Roads*, Vol. 28, October, 1955, pp. 203–223.

[8] "Traffic Usage of Maine Turnpike," by Glenn E. Brokke, *ibid.*, pp. 224–230.

vehicle-mile of the city of Los Angeles. In Table 3 the traffic and accident data on state highways are given for 1954, together with the traffic and accident data on full freeways for the 14-yr period in which the full freeways have been in operation. The average daily traffic for the given mileage of full freeways for the period from 1941 to 1954 was 41,822 vehicles. The low fatality rate on the full freeways indicates best the accident-cost saving provided by full freeways.

The city of Los Angles computed the average accident cost per vehicle-mile as 0.7 cents, based on the monthly reports received from the California Insurance Commissioner of automobile insurance premiums (excluding theft, fire, and comprehensive coverage). By using the BPR estimate of total highway travel per year and the determination of the annual cost of highway accidents of the National Safety Council, Chicago, Ill., an average accident cost of approximately 0.7 cents per vehicle-mile was also obtained.

TABLE 3.—1954 Traffic and Accident Data on Rural State Highways and on Rural and Urban Freeways in California[a]

Number of lanes	Miles	Average daily traffic	Accidents per million vehicle-miles	Fatal and injury accidents per million vehicle-miles	Fatalities per 100,000,000 vehicle-miles
2	10,846	2,359	2.21	0.98	9.49
3	143	12,300	2.65	1.03	9.68
4 undivided	181	21,301	3.54	1.29	6.47
4 divided	220	15,712	3.11	1.16	8.18
4 divided expressways	667	11,723	1.71	0.74	6.45
Full freeways—1954, rural and urban	131[b]	45,194	1.27	0.53	1.92
Full freeways from 1941 to 1954—rural and urban[c]	421[d]	41,822	1.37	0.58	2.15

[a] California Division of Highways traffic and accident data. [b] Total traffic on the 131 miles of freeways opened to traffic in 1954 was 2,161,000,000 vehicle-miles. [c] Total traffic on freeways was 6,420,000,000 vehicle-miles from 1941 to 1954. [d] Year-miles.

The Los Angeles studies of traffic accidents showed that travel on freeways in that area involved only one-fifth of the accident risk that prevails on typical surface routes. With four-fifths of the accident risk removed, the Los Angeles engineering department computed the accident-cost saving on the freeways to be four-fifths of the 0.7 cents per vehicle-mile, or 0.6 cents per vehicle-mile.

In this study the accident records of the California Division of Highways were used to determine the value of 0.6 cents per vehicle-mile for accident-cost savings. The best indication of accident costs was the fatality record. From 1949 to 1954 the fatality rate for the rural state highways in California was 9.6 per 100,000,000 vehicle-miles, and on the major city streets comprising the state highway system the fatality rate exceeded 10.0 per 100,000,000 vehicle-miles. The average fatality rate on all streets and highways in the state was 7.0 per 100,000,000 vehicle-miles. On the full freeways during a 14-yr period, the fatality rate (Table 3) was 2.15 per 100,000,000 vehicle-miles. On the basis of these data, a conservative estimate of 0.9 cents per vehicle-mile has been used in this study for the accident costs on the major city surface streets on the Interstate System and 0.3 cents per vehicle-mile on full freeways.

The difference in these two costs, totaling 0.6 cents per vehicle-mile, represents a conservative estimate of the saving in accident costs provided by full freeways.

USER BENEFITS ON THE RURAL FREEWAYS IN THE CALIFORNIA INTERSTATE SYSTEM

The highway-user benefits on the rural portion of the California Interstate System consist of savings in time costs, vehicle operating costs, and traffic-accident costs, which are the same as those that were used on the urban portion of the Interstate System but on a greatly reduced basis. Not all the mileage on the rural portion of the Interstate System will consist of the full freeway-type facility. The present highways on the rural portion of the Interstate System consist of two-lane, three-lane, and a considerable mileage of the four-lane divided and undivided highway and expressway classifications listed in Table 3. A large part of the present mileage consists of freeways that have partial control of access but have intersections at grade. There are fewer traffic stops, and the traffic volumes are lower than on urban expressways. There is less congestion, and the speeds of vehicles on the existing highways on the rural portion of the Interstate System are greater than the speeds of vehicles on the urban portion. All these factors contribute to a reduction in the user benefits for the rural freeways on the Interstate System when compared with the benefits previously listed for the urban freeways on the Interstate System.

On the rural freeways in the California Interstate System, it is estimated that the average speed of passenger cars will be approximately 60 miles per hr, based on speed surveys on existing freeways in rural sections. The average speeds on the existing rural highways on the Interstate System range from 40 miles per hr to 50 miles per hr. Although the proposed rural freeways for the Interstate System will provide improved alinement, a reduction of grades, and the elimination of traffic stops, the most important improvements of this type have already been made on more than 1,400 miles of the rural portion of the California Interstate System. Test runs with passenger cars on a considerable mileage of highways in the rural portion of the Interstate System have indicated that the vehicle operating costs at the lower speeds of from 40 miles per hr to 50 miles per hr on the existing mileage will average approximately 0.5 cents per mile to 1.0 cents per mile less than the corresponding cost for vehicles operating at a 60-miles-per-hr average speed on the rural freeways. Using the AASHO time-saving cost of $1.35 per hr for passenger cars results in a time saving of 0.75 cent per mile while driving at an average speed of 60 miles per hr instead of an average speed of 45 miles per hr. Thus, it is assumed that the time-saving benefits on the rural freeways are almost offset by the increased vehicle operating costs. It is also assumed that the only saving or benefit to passenger cars is the accident-cost saving, which the California accident statistics indicate should be approximately the same on rural freeways on the Interstate System as they are on the urban freeways, or 0.6 cent per vehicle-mile for both cars and trucks.

The benefits for trucks on the rural freeways on the Interstate System are also greatly reduced as compared with the benefits on the urban freeways on the Interstate System. The reduction in grades and curvature and the elimination of traffic stops will provide greater advantages in reduced operating costs for trucks than for cars. The increased speed of trucks on the rural freeways of the Interstate System will increase the fuel, oil, and tire costs. However, on the basis of the available test data, the savings in operating costs for trucks provided by highway improvements on the rural freeways of the Interstate System will approximately equal the increased operating costs due to the operation at greater speeds on the freeways.

The time-cost saving for trucks was computed using an average truck speed of 35 miles per hr on the existing rural highways and 45 miles per hr on the rural freeways. Using the value for truck time costs of $3.00 per hr results in a time-cost saving of 1.9 cents per vehicle-mile.

The accident-cost saving for trucks on the rural freeways in the Interstate System is assumed to be the same as on the urban freeways in the Interstate System, or 0.6 cents per vehicle-mile. The total benefit for trucks on the rural freeways in the Interstate System, therefore, totals 2.5 cents per vehicle-mile.

The speed characteristics of bus traffic more closely approximate those for passenger car traffic on the rural portion of the Interstate System; however, the vehicle-operating-cost characteristics for buses are somewhat similar to the truck operating costs. The costs for bus traffic, which comprises less than 1% of the total traffic on the Interstate System, are assumed to be the same as for truck traffic. With trucks and buses comprising 12% of the total traffic on the rural portion of the Interstate System, the weighted average benefit per vehicle-mile for all traffic on this system will total 0.83 cents per vehicle-mile, which is only approximately one-fifth of the benefit of 4.16 cents per vehicle-mile for the traffic on the urban portion of the Interstate System in California.

TRAFFIC ON THE INTERSTATE SYSTEM

R. M. Zettel[9] has estimated annual increases of 3.8% per yr in the California motor-vehicle registration and 4.5% per yr in the California motor-vehicle travel. An average rate of 4.0% per yr is used in this study as a conservative estimate of the traffic increase during the next 30 yr on the California Interstate System.

The 1953 traffic count on the Interstate System, when used as a basis for computing future traffic, results in a 4% increase per year, yielding expansion factors of 1.601 for the 1965 traffic, 2.3699 for the 1975 traffic, and 3.5081 for the 1985 traffic. The estimated traffic volumes on the California Interstate System for 1955 through 1985 are given in Table 4. The traffic volumes are believed to be conservative because careful studies have not been made to determine the extent to which new traffic will be generated or induced by an accelerated program of construction on the Interstate System.

An expansion factor of 3.5 for the 1985 traffic, together with the projected traffic volumes on the urban freeways in the Interstate System that exceed 8,000,000,000 vehicle-miles, raises the question—can urban freeways be built in a city such as Los Angeles to carry the projected traffic volumes in 1985 when it is a well-known fact that the present freeways in that city are overcrowded during the peak hours? The answer is that as of 1955 only approximately one-eighth of the freeway mileage planned for the Los Angeles area had been built. If the remaining mileage of planned urban freeways in this area could be built as a part of an accelerated program, much of the congestion on the present freeways would be eliminated. The projected traffic volumes on the urban freeways in the Interstate System, which are 3½ times the present volumes, indicate that the pressures for developing a satisfactory solution to the mass transportation problems in large metropolitan areas will be so great that a solution may be forthcoming. It will not be possible to expand indefinitely the space occupied by urban freeways. More efficient use may have to be made of the freeways as presently planned by moving more people instead of more cars. An analysis of user benefits beyond 1985 has not been made, but it is evident from the preceding information that, in the near future, long-range studies should be begun to investigate the transportation requirements and the type of facilities that should be developed after all the space that may reasonably be allotted to freeways is occupied.

---

[9] "Financing California's Highways; Part 1, Highway Investment and Pricing," by R. M. Zettel, Joint Fact-Finding Committee on Highways of the California Legislature, Sacramento, Calif., 1952.

TABLE 4.—Estimated Annual Traffic Volumes on the California Interstate System, 30-Yr Period

Classification	Estimated Annual Traffic Volumes in 1,000,000 Vehicle-Miles			
	1955	1965	1975	1985
Rural	4,936	7,307	10,816	16,011
Urban	2,588	3,831	5,671	8,395
Total	7,524	11,138	16,487	24,406

TABLE 5—Computed User Benefits of the California Interstate System Based on a 10-Yr Construction Program (1956-1965)

Part of system	First 10-yr benefits, from 1956 to 1965	Second 10-yr benefits, from 1966 to 1975	Third 10-yr benefits, from 1976 to 1985	Total 30-yr benefits, from 1956 to 1985
Rural	$ 298,700,000	$ 757,200,000	$1,121,000,000	$2,176,900,000
Urban	784,800,000	1,990,000,000	2,945,000,000	5,719,800,000
Total	$1,083,500,000	$2,747,200,000	$4,066,000,000	$7,896,700,000

Summary of User Benefits of the Interstate System

In computing the user benefits of the Interstate System, which are given in Table 5, it was assumed that the program of improvement would be completed in a 10-yr period and would proceed at a uniform rate. It was also assumed that the user benefits would be maintained at approximately the same rate over the 30-yr period, during which time the traffic would be expected to pay for the entire cost of the freeways built during the 10-yr period. When viewed on the basis of the highway-operating-cost trends of the past 10 yr, the preceding assumptions appear reasonable.

It was assumed that if the construction of the Interstate System were to be completed in a 10-yr period, the maximum benefit would accrue over the entire system only during the eleventh year and the period following that year. The benefit during the first 10-yr period was computed on a proportionate basis in terms of the percentage of total mileage that would be completed during each year, and by using the benefited traffic volume estimated for that year. Thus, by the end of the fifth year, one-half of the system would be assumed to be completed, but the benefits for the fifth year would total the average percentage of the system improved for the fifth year, or 45% of the benefit for a fully completed Interstate System.

Traffic to which the benefits are related in the first 10-yr period has been considered herein as the average of the traffic in the first and last years of that period. The 10-yr benefits that were obtained are slightly greater than they would have been had they been obtained by totaling the benefits computed separately for each year. However, the slight difference is more than compensated for by an important consideration in the accelerated program—namely, that projects will most likely be scheduled in the order of their importance as measured by the return of the benefits to the traffic. Thus, the projects with large traffic volumes, which, after their completion, will return the greatest benefits to users, are most likely to be scheduled early in the program. This would result in greater benefits during the first 10-yr period than if the projects were built on a random schedule with no regard for the early completion of the high-priority projects.

TABLE 6.—COMPUTED COSTS FOR THE CALIFORNIA INTERSTATE SYSTEM USING 30-YR SERIAL BONDS WITH INTEREST AT 3% (CONSTRUCTION PERIOD, 1956-1965)

Part of system	Total interest for 30 yr	Total principal	Total interest plus principal
Rural	$ 275,430,000	$ 592,318,000	$ 867,748,000
Urban	804,100,000	1,729,235,000	2,533,335,000
Total	$1,079,530,000	$2,321,553,000	$3,401,083,000

The benefits in the second and third 10-yr periods were determined by using the computed traffic volumes for each year based on the expansion factor for that year, assuming a 4% annual increase in the traffic during the 30-yr period. For the second and third 10-yr periods, therefore, the full user benefits were assigned on the basis of a fully completed Interstate System built during the first 10-yr period. The traffic volumes on the rural and urban sections of the system and the user benefits were computed for each year instead of using the averaging procedure adopted in computing user benefits for the first 10-yr period.

In Table 6 are listed the computed interest charges and the principal required for a completed Interstate System in California constructed over a 10-yr period. It is assumed that 30-yr serial bonds will be used with an interest rate of 3%. In comparing the user benefits given in Table 5 with the investment costs in Table 6, it is interesting to note that the user benefits during the first 10-yr period are sufficient to pay all the interest charges for 30 yr for the the entire investment of $2,321,553,000. The benefits during the first and second 10-yr periods from 1956 to 1975 are great enough to pay for both the interest and principal and leave a balance of approximately $500,000,000. Finally, during a 30-yr period corresponding to the life of the bonds, the ratio of benefits to costs is 2.3 to 1. This clearly indicates the economic justification for the early completion of the Interstate System in California.

Of greatest concern to highway engineers is the urban section of the system, for which the average cost per mile is estimated to be $7,044,000 per mile as compared with $355,200 per mile for the rural section. During a 30-yr period the benefits on the urban section exceed the cost by a sum greater than $3,000,000,000, and the ratio of benefits to costs is 2.3 to 1. This great margin of benefits over cost is even more significant when it is recognized that more than $1,000,000,000 of the cost of the urban freeways in the Interstate System is for right of way. It is questionable if this part of the cost of the Interstate System should be amortized in 30 yr because it represents a value of property that is not likely to be depreciated with time but may retain its investment value or purchase price indefinitely.

CONCLUSIONS

The preceding analysis of user benefits of the Interstate System in California provides substantial evidence of the soundness of the President's proposal for an accelerated program for the construction of the Interstate System and the use of federal funds to insure its early completion. Furthermore, it should be recognized that the benefits will be shared not only by the highway users but also by all people in the United States. However, there are many government agencies competing for public funds, and, for this reason, the analysis of highway-user benefits should be refined and should be supplemented by as much additional information as can be obtained. The adoption of such a procedure would permit appraising and evaluating competing public-works projects on the basis of an economic analysis, which should provide for the most effective use of national resources and for the promotion of the general welfare of all the people of the United States.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2930

PLASTICS: ENGINEERING MATERIALS

By C. Howard Adams[1]

Synopsis

This paper is concerned solely with the engineering characteristics of plastics. A description of the reaction of these materials to the various environmental factors of force, time, temperature, and humidity is presented. The magnitude of the subject requires a generalized treatment, such as that which would be used in a description of the engineering characteristics of metals. The subject of plastics as engineering materials will be developed through a series of comparisons of plastics with the older materials used in civil engineering; a series of comparisons of plastics by generic groups; a selected set of quantitative comparisons with other materials; and, finally, an analysis of certain pertinent design considerations.

The Civil Engineering Field

To establish the materials and properties of interest to the civil engineer, reference was made to the Encyclopedia Britannica (1),[2] which, quoting from the 1828 Charter of the Institution of Civil Engineers (London), defines civil engineering as:

> "(The) art of directing the great sources of power in nature for the use and convenience of man, as the means of production and traffic in states, both for external and internal trade as applied in the construction of roads, bridges, aqueducts, canals, river navigation, and docks for internal intercourse and exchange, and in the construction of ports, harbours, moles, breakwaters, and lighthouses and in the art of navigation by artificial power for the purposes of commerce and in the construction and adaptation of machinery and in the drainage of cities and towns."

In the early nineteenth century the civil engineer was the only professionally recognized civilian engineer. He operated in many fields, some of which

---

Note.—Published, essentially as printed here, in October, 1956, in the Journal of the Engineering Mechanics Division, as *Proceedings Paper 1072.* Positions and titles given are those in effect when the paper was approved for publication in *Transactions.*

[1] Mgr., Plastic Product Development, Research and Eng. Div., Monsanto Chemical Co., St. Louis, Mo.

[2] Numerals in parentheses—thus, (1)—refer to corresponding references in the Bibliography (see Appendix I).

are considered outside the province of the profession. The modern civil engineer designs and builds heavy, durable structures, including roads, bridges, dams, canals, and drainage systems for cities and towns. He is concerned with strength, stiffness, and durability.

PLASTICS AND THE ESTABLISHED MATERIALS OF CIVIL ENGINEERING

In designing and building roadways, bridges, or other structures, the civil engineer uses many different types of materials. Four that are generally used in such structures will be compared with plastics in order to bracket qualitatively the engineering characteristics of plastics. The data presented are on the essentially unoriented form of the material except for wood, which is a directional product of nature.

The short-time or static fracture strengths of steel (2), wood (3), concrete (2), and glass (4) are compared with those of plastics (5) in Fig. 1(*a*). Data are from tensile tests except those for concrete, which are from compression tests. The strength range of plastic materials is greater than that of any one of the other materials. Depending on the type of plastic composition and the reinforcement used, it is currently possible to cover greater than a two-hundredfold strength range. Most plastics have less strength than steel and the stronger woods. However, certain reinforced plastic compositions approach steel in strength.

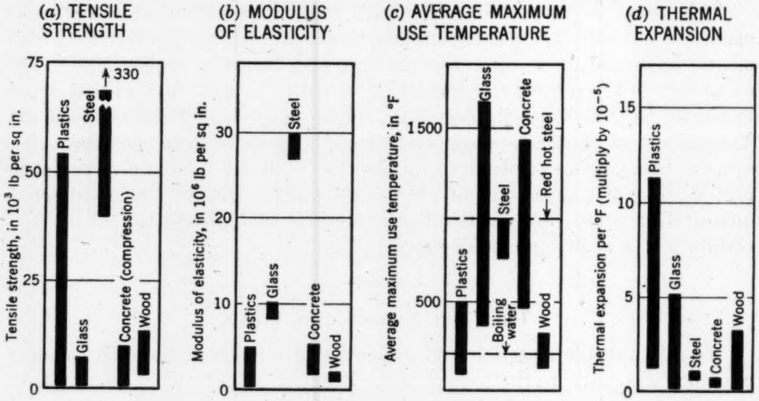


FIG. 1.—PHYSICAL PROPERTIES OF ENGINEERING MATERIALS

The elastic modulus of plastics, as shown in Fig. 1(*b*), is less than the corresponding value for steel and glass. Plastics can be as flexible as low-modulus rubber or as stiff as concrete or wood. The plastics exhibiting the highest modulus of elasticity values—approximately one-fifth the value of steel—are reinforced with glass fibers.

A comparison of the ductility of plastic materials with that of glass, concrete, and wood shows that even the most brittle plastics are generally more ductile than the other materials. The elongation of plastics at break in a tensile test may be from less than 1% to greater than 800%.

Plastics are lighter than most conventional materials (2). Depending on physical form, type of plastic material, and the reinforcing system used, plastics can weigh from as little as 2 lb per cu ft to greater than 130 lb per cu ft. The base polymers in solid form may have a specific gravity from between 0.9, or lighter than water, to 1.5, or heavier than water.

The useful temperature range of the majority of plastic materials is below the wood char point of approximately 380° F (Fig. 1(*c*)). Many materials in the thermoplastic or heat-softening group, even in nonload bearing uses, must be designed to function at a temperature that is less than the boiling point of water. The thermosetting, or heat-hardening plastics can be used under the right circumstances at temperatures above the boiling point of water, and even up to the wood char point previously mentioned. One or two types of plastics can be used with a small load at 500° F.

Flammability is of significant importance in the engineering use of plastics. In general, plastics burn in a manner similar to wood. This characteristic can be controlled to a substantial degree by composition modification. In considering whether the burning characteristics of a plastics material present a problem (7), the engineer should carefully weigh other factors, such as light weight, shatter hazard, and softening temperature.

The experience with the use of plastics outdoors is rather limited. However, one that is satisfactory for outdoor service has a background of approximately twenty years of experience (8). Plastic materials must be protected from the weather either by the incorporation of pigments in the material itself, or by some type of surface treatment (9).

The change in dimensions of a structure and its components relative to a change in temperature is an important design factor to be considered by the engineer. Fig. 1(*d*) shows that plastics exhibit a wide range of thermal expansion behavior, much wider than for the other materials that were charted (2, 3, 10, 11).

The preceding description of plastics has shown that properties such as strength, stiffness, weight, and ductility can be varied. It has also been indicated that the strongest plastic is comparable to a structural steel, and that the weakest has a strength less than that of wood. It has been shown that further study is needed concerning the weather resistance of plastics and that the maximum use temperature of plastics is less than that of the materials cited in the comparison. The information presented thus far should be considered only as an introduction to the general subject. It is recognized as being of limited value to the designer.

The Generic Group of Plastics

Having examined plastics as a class of materials, it is necessary to explore the various groups, and determine to what extent the great range of properties can be explained by variation in the base plastic. The terms used to identify each group may appear rather formidable, although representing a considerable simplification of the correct chemical name. The designations represent a compromise between euphony and accuracy in the interest of public acceptance. Before examining the various properties of the generic groups of plastics, it is necessary again to indicate that plastics are of two major types—thermosetting or thermoplastic. Generally, the thermoset materials are more heat resistant and stronger than the thermoplastics. The former are most widely used in producing what is known to the trade as reinforced plastics. The data charted are from single point determinations. The ranges are due to composition or formulation variation and do not reflect the temperature, time, or other variables on a given plastic. These will be examined subsequently.

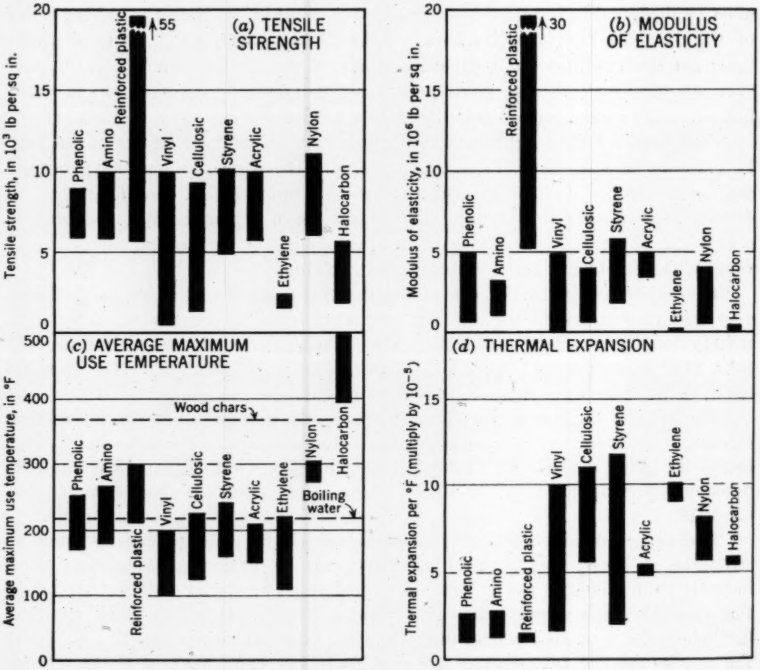


FIG. 2.—PHYSICAL PROPERTIES OF THE MAJOR TYPES OF PLASTICS

The short-time or statistic tensile strength (5) of the major types of plastics are compared in Fig. 2(*a*). The three materials on the left in Fig. 2(*a*) belong in the thermosetting category; the remaining materials are thermoplastic. Thermosetting resins combined with glass fibers give a high-strength, reinforced plastic, breaking at a stress of 55,000 lb per sq in. Thermoplastic resins such as vinyl are weakened by adding softening agents known as plasticizers.

The stiffness characteristics (5) of plastics are indicated in Fig. 2(*b*). As was the case with strength, the greatest stiffness is attained through reinforcement.

The thermosets are less ductile (5) than the thermoplastic materials as measured by elongation at fracture. Ethylene and plasticized vinyl plastics show great extension at fracture, ranging from 300% to a maximum on the order of 700% as compared with a value of 0.5% for a typical phenolic molding material.

The ductility of styrene materials as measured by elongation at break will range from approximately 1% for unmodified styrene to as great as from 35% to 40% in the case of styrene plastic blended with rubber compounds. There are indications that ductility should be estimated from data other than the elongation at break in a static test. Because of the time-dependent nature of the plastic materials, rigid vinyl if tested at a slow rate of speed will show a high elongation at fracture, whereas if tested rapidly the material will break at an elongation of less than 1%.

Ethylene is the least dense (5) and the lightest plastic commercially available, whereas vinyl is the heaviest. The observations are based on the unmodified plastic. It is possible through foaming to make lightweight material from most plastics, ranging from a low of 2 lb per cu ft to the density of the material in a solid form.

The average maximum use temperature data (5) shown in Fig. 2(*c*) clearly indicate the difference between thermosetting and thermoplastic materials. As with other properties, time and load dependence must be recognized in a particular design situation.

The most heat resistant of the unmodified plastics are found in the halocarbon family. Members of this group, some of which are thermoplastic, may be used under 0 load up to 500° F. Nylon under 0 load is the most heat resistant of the commercial thermoplastics.

The linear thermal-expansion characteristics of plastics are given in Fig. 2(*d*). As a type the thermoplastics change more readily with temperature than do the thermosets. In practice this has meant that metallic inserts are more readily handled in phenolic and amino moldings than in thermoplastic moldings.

SPECIFIC ENGINEERING DATA COMPARED

Having examined the relationship of plastics to the established materials of civil engineering, and having highlighted the characteristic properties of the several members of the plastics group, a consideration of some specific engineering data is in order.

The stress-strain curve, usually an engineer's introduction to load-deformation behavior, provides a useful picture of the behavior of materials. Fig. 3 compares the stress-strain behavior determined in standard American Society of Testing Materials (ASTM) tests of mild steel (12) and Douglas fir wood (low moisture) (6) with three types of plastics. Because of the wide range of load or stress and strain behavior represented, the information was plotted on a logarithmic scale. The curves show the superiority in strength and stiffness of steel as compared with selected plastics. The curve also indicates the similarity in modulus and strength between a common wood used in construction and a laminated plastic material. The strength and stiffness of

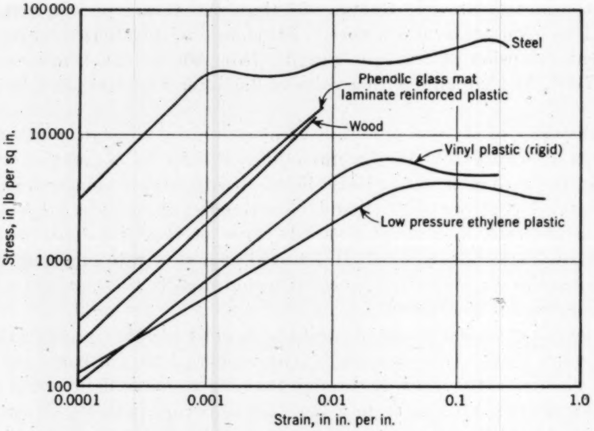


FIG. 3.—STRESS-STRAIN CHARACTERISTICS OF PLASTIC, STEEL, AND WOOD AT ROOM TEMPERATURE

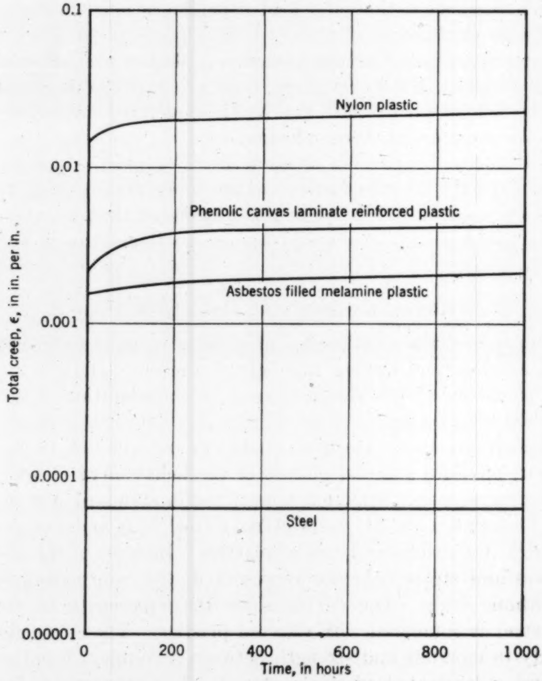


FIG. 4.—CREEP CHARACTERISTICS OF PLASTIC AND STEEL AT ROOM TEMPERATURE AND A TENSION OF 2,000 LB PER SQ IN.

vinyl plastic (rigid) and a low-pressure ethylene plastic are less than the corresponding values for laminated plastic material.

Creep data for certain plastics (13, 14, 15) and a typical mild steel are compared in the semilogarithmic plot of Fig. 4. Common stress and temperature conditions provide a basis for grading the load-carrying capacity of the materials that are being compared. It is apparent that in designing a part for a structure using any one of these materials, more specific data on creep behavior would be needed. As was the case in previous comparisons of load-deformation behavior, Fig. 4 shows that, when creep is the design criterion, plastic materials must be designed to carry stresses of a low order as compared with steel.

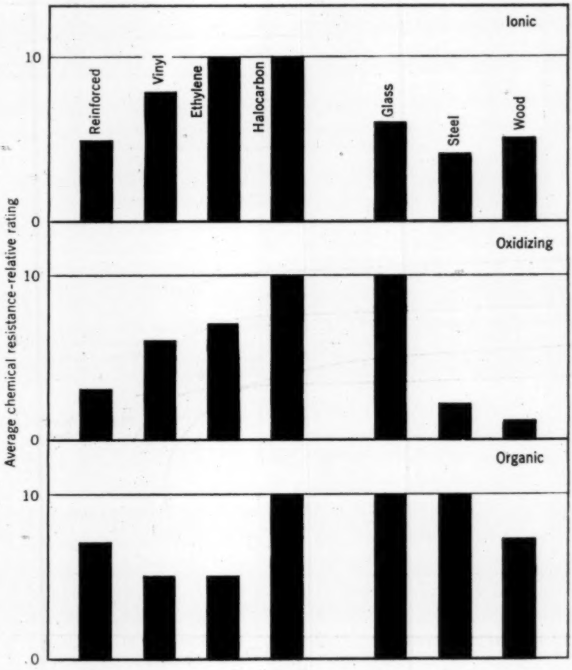


FIG. 5.—AVERAGE CHEMICAL RESISTANCE—RELATIVE RATING 0–10

Plastics materials have applications in buildings and fluid handling systems in which corrosion resistance is a problem. Fig. 5 (5, 16) shows that plastics are generally superior to steel and wood for situations in which corrosive liquids and corrosive atmospheres exist. The steel cited in Fig. 5 is of the ordinary structural variety. It is recognized that alloy types will perform better than the material shown in the chart. A comparison of this nature should always be subject to critical review because of the specific nature of the reaction of chemicals with materials. However, the generalities presented should be helpful in leading the engineer to further investigation of a particular plastic type when confronted with a corrosion problem. Certain plastics have applications in pipelines that are used for corrosive liquids and have performed better than any other material with respect to maintenance. Also, the smooth surface of the plastic results in a lower pressure drop in moving the liquid through the pipe. Glass is an excellent material for resisting chemical attack, and of the four plastics shown, only the halocarbons are more resistant. However, glass, which is a heavy, brittle material, frequently requires support and protection in the form of a metallic backing. Glass tends to complicate jointing and valving problems, whereas plastics are light, minimize jointing and valving problems, and offer no shatter hazard. Although a given plastic type is more resistant to a narrower range of chemicals than glass, this factor is counterbalanced by the range covered by all types of plastics.

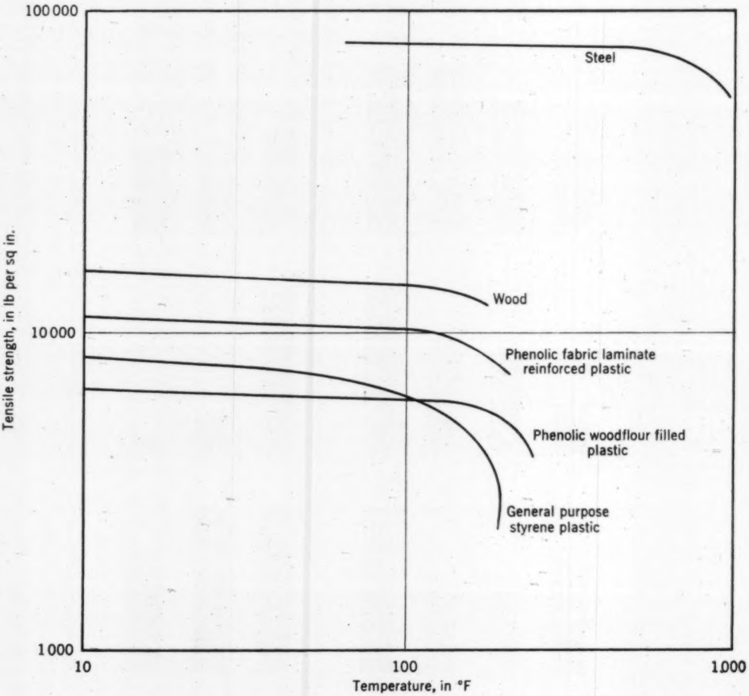


Fig. 6.—Temperature Effect on Tensile Strength of Plastic, Steel, and Wood

Plastics are more sensitive to temperature than steel or concrete. Fig. 6 (3, 17) compares the short-time tensile strength of several plastics with that of low-alloy steel and Douglas fir wood (low moisture) as a function of temperature. Plastic materials show a temperature dependence similar to that of wood, and, when compared with Douglas fir, most of the thermoplastic materials—styrene being an example—show a more rapid loss of strength as the temperature is increased. Steel is in a completely different category. It begins to show a substantial decrease in strength at a level approximately 400° greater than that of the most heat-resistant plastic material. A large part of the research dollar of the plastics industry is spent in an effort to improve strength characteristics at elevated temperatures.

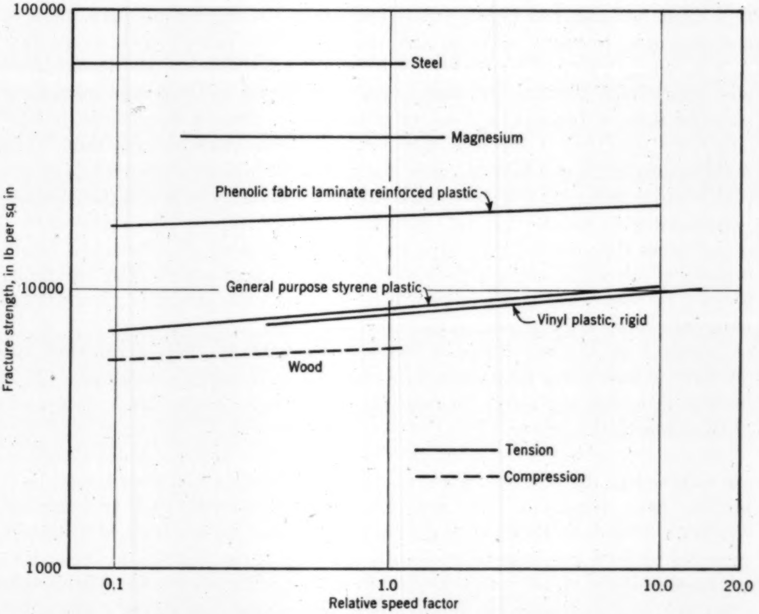


FIG. 7.—TIME EFFECT ON FRACTURE STRENGTH OF PLASTICS, MAGNESIUM, STEEL, AND WOOD

The time dependence of plastics has been mentioned and is shown graphically in Fig. 4. Fig. 7 shows the effect of testing speed on the fracture strength for SAE 1020 steel (18), die-cast magnesium (19), Douglas fir wood that was tested parallel to the grain (20), and three types of plastics (21). The log-log plot tends to minimize the greater effect of this variable on plastic materials. However, a careful examination of the data will show that the metals are relatively insensitive to speed in the range that was used in the tests. Wood shows a time or speed dependency similar to that of plastics.

The data shown in the charts have come from many sources, as indicated in the Bibliography herein. The most comprehensive source material on plastics is contained in a book by the Manufacturing Chemists' Association (5), a publication by the Society of the Plastics Industry (22), the publications of the ASTM, and the various technical journals and trade journals that are devoted to materials and their behavior. A particularly useful source of specific engineering data is the *Proceedings* of the ASTM.

DESIGN CONSIDERATIONS

Having briefly surveyed the characteristics of the important plastics groups, the question of how to approach design with regard to plastic materials being used in engineering applications can now be considered. First, fabrication must be examined because it is important to understand how plastics are converted to useful objects, and the effect this has on their properties. As the term "plastics" indicates, high polymers, which are the base materials from which plastics are made, can be softened by heat and moved by pressure into or through a form of some kind. They subsequently harden either by cooling or by the advancment of a chemical reaction. The molding operation can take many forms, not all of which involve allowing the plastic to become hard in the mold cavity. Certain plastics in the thermoplastic category are formed by extrusion. In this process the rotation of a screw forces the material through a warm chamber, where it softens and finally issues through a die having a fixed shape onto a cooling table. This technique is used to coat wire, make large sheets, and manufacture thin film. Sheet material prepared by the extrusion process can be highly directional in terms of physical properties. Thus, anisotropy becomes another factor that the design engineer must be aware of in working with plastic materials. It is present to a varying degree in many items fabricated from plastic regardless of the method of forming.

Injection molding is another technique in which thermoplastic materials are softened to the semifluid state in a heated chamber and then forced by a plunger into a cold mold. The rapid cooling of the material leads to a strained condition similar to that existing in rapidly cooled glass. This condition is superimposed on the anisotropy resulting from the flow of the material into the mold. If the strain is too great, the plastic part is subject to failure under thermal shock conditions as well as under mechanical conditions. The effect can be minimized by control of molding times, temperatures, and, in some instances, by postannealing the molded part.

Compression and transfer molding used to form thermosetting materials leads to some anisotropy that is usually different than that encountered with thermoplastic materials. In the latter the directional effect is usually related to the flow forces orienting the filler materials. In the manufacture of reinforced plastics, anisotropy is developed and at the same time controlled by the lay-up of the reinforcing fibers or sheets. This effect is shown in Fig. 8 (23), which shows the effect of the direction of weave of a reinforcing fabric on the elastic constants of a plastic sheet. The symbols used in Fig. 8 are listed in Appendix II.

Plastics are lightweight materials, a fact that has led some individuals to conclude that plastics are the optimum materials for use in lightweight structures. This inference has been drawn in instances in which the strength-weight ratio—or the strength of a plastic divided by its weight—indicates that pound for pound the plastic material will give a stronger structure than steel in some cases. The fallacy of such a generalized concept has been put in its proper perspective by several investigators. The findings of a group of these investigators are summarized in Table 1 (24). In this work, considera-

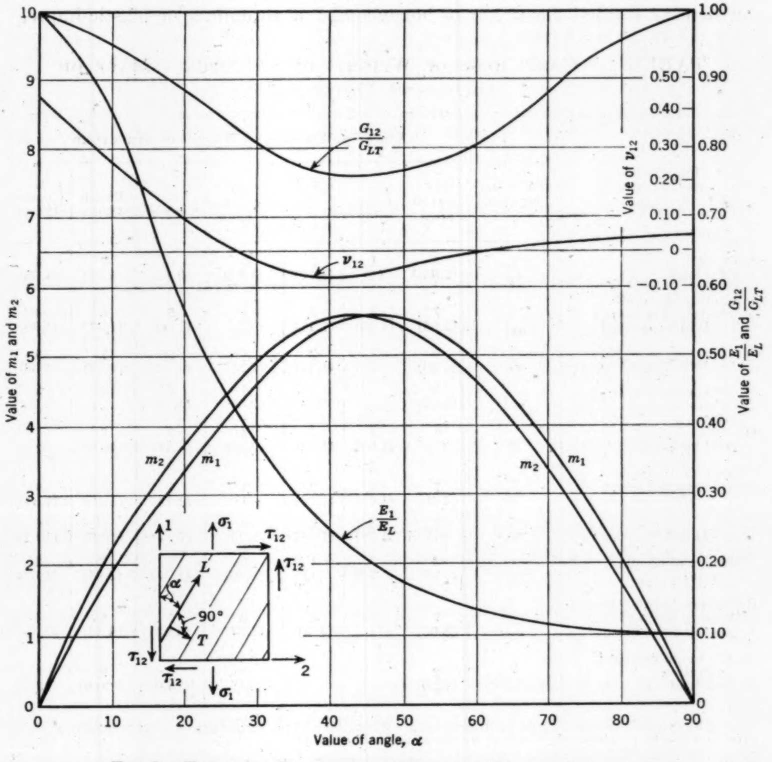


FIG. 8.—EFFECT OF DIRECTION OF WEAVE ON ELASTIC CONSTANTS OF REINFORCED PLASTICS AND LAMINATES

tion was given to the problem of designing plastic structures for their possible advantage of being lightweight. The study showed that the simple ratio of strength to weight is not always satisfactory for predicting the minimum weight of a structure. In turn, it emphasized the fact that structures perform many different functions. Some structures are designed primarily for strength; some are designed for rigidity or stiffness, and other are designed to damp vibrations. Depending on the particular design consideration that is superimposed on the main objective of a lightweight structure, a material that is heavier than a plastic may be used. Given the choices of type of loading and geometrical configuration of the structural member, the use of plastics in flexure as sandwich panels is favored. The sandwich-panel design is based on the same principles of mechanics as the familiar and widely used structural component, the I-beam. The sandwich panel is an I-beam with a light web and high-strength flanges. The web or core of a sandwich panel is made of a low-density foamed plastic, or of honeycombs of aluminum or plastic-impregnated paper bonded with phenolic or other plastic adhesives. The flange or face is made of reinforced plastic, metal, or other material, and is bonded to the core with plastic adhesives. This procedure illustrates the important point that plastics in combination with other materials and in various forms give the engineer a new degree of design freedom.

TABLE 1.—COMPARISON OF WEIGHTS OF STRUCTURES MADE OF DIFFERENT MATERIALS

Functional specification	Criterion for minimum weight[a]	ORDER OF INCREASING WEIGHT OF STRUCTURE						
		SAE ×4130 steel	Titanium alloy	75S-T aluminum alloy	Paper laminate, phenolic	Cast phenolic	Tough styrene	Crystal styrene
Strength Axial load	$\frac{\gamma}{\sigma_y}$	4 (2.83)[b]	1 (1)	2 (1.12)	3 (1.81)	6 (5.12)	7 (9.18)	5 (4.68)
Bending of simple rectangular beam Width specified	$\frac{\gamma}{\sigma_y^{1/2}}$	7 (2.94)	3 (1.33)	2 (1.11)	1 (1)	5 (1.63)	6 (1.97)	4 (1.39)
Depth specified	$\frac{\gamma}{\sigma_y}$	4 (2.83)	1 (1)	2 (1.12)	3 (1.81)	6 (5.12)	7 (9.18)	5 (4.68)
Torsion of circular shaft	$\frac{\gamma}{\tau_y^{2/3}}$	5 (2.62)	. . . . . .	1 (1)	4 (2.04)	3 (1.39)	. . . . . .	2 (1.20)
Stiffness Axial load	$\frac{\gamma}{E}$	1 (1)	3 (1.08)	2 (1.04)	4 (2.25)	5 (7.17)	7 (10.6)	6 (10.1)
Bending of simple rectangular beam Width specified	$\frac{\gamma}{E^{1/3}}$	7 (2.40)	6 (1.70)	2 (1.22)	1 (1)	5 (1.41)	4 (1.40)	3 (1.35)
Depth specified	$\frac{\gamma}{E}$	1 (1)	3 (1.08)	2 (1.04)	4 (2.25)	5 (7.17)	7 (10.6)	6 (10.1)
Torsion of circular shaft	$\frac{\gamma}{G^{1/2}}$	5 (1.65)	. . . (1.30)	1 (1)	5 (1.66)	6 (1.77)	7 (2.24)	3 (1.62)
Resistance to impact Bending of simple rectangular beam	$\frac{\gamma E}{\sigma_y^2}$	7 (8.60)	1 (1)	2 (1.30)	3 (1.57)	5 (3.93)	6 (8.53)	4 (2.33)
Resistance to resonant vibration Axial load	$\frac{\gamma}{C E \sigma_y^{n-1}}$	2 (22.0)	. . . . . .	. . . . . .	1 (1)	4 (75.0)	5 (120.9)	3 (73.2)

[a] The symbols used are listed in Appendix II. [b] Numbers in parenthesis give relative weight of structure for material concerned in terms of weight of lightest structure.

The use of plastics as light-transmitting materials in aircraft, automotive, and construction applications was pioneered by the construction of the Model T. The reasons for using plastic in the side curtains of the Model T were primarily economic and not subject to critical engineering analysis. However, with the development of the acrylic aircraft canopy prior to and during World War II, plastics entered the light-transmitting field for the following reasons: The plastic was lighter than materials that were conventionally used, it could be formed readily into complex shapes, and was free of the shatter hazard of conventional glazing materials. The use of plastic allowed the aircraft designers considerable freedom and made possible a clean aerodynamic shape. The safety glass that is found on all cars is safe because of a plastic interlayer between the glass surfaces. This contribution to safety began in the late nineteen twenties with the use of a cellulosic sheet that yellowed quickly and did not bond the glass faces satisfactorily. Continued research has led to improved vinyl-type plastic interlayers. Panels of safety glass made of this type, which were exposed outdoors for periods of more than ten years, show almost no color change and excellent adhesion to the glass faces. The use of plastics as automotive glazing materials is limited to the rear windows in convertiles because of their poor-to-fair scratch resistance. The convertible rear window has given the automotive designer more freedom than was possible with glass sewn into the canvas back. At the same time, this use has improved the safety factor by introducing a lightweight flexible material that is completely resistant to shock.

Other possible uses of plastics that the engineer must deal with are in fluid-flow machinery, building components, machine components, and electrical power transmission equipment. It is hoped that the properties data and engineering analysis of the behavior of plastics presented herein will prove helpful in problems involving material selection in one or more fields.

CONCLUSIONS

The foregoing data indicate that plastic materials have many useful properties that warrant their consideration for engineering applications. Among these are ease of fabrication, light weight, adaptability to combination with other materials in achieving new structural effects, and a capacity to vary many properties over a wide range to suit the needs of a specific application.

APPENDIX I. BIBLIOGRAPHY

(1) **Encyclopedia Britannica,** Vol. V, 1952, p. 734.

(2) **"Textbook of the Materials of Engineering,"** by H. F. Moore and M. B. Moore, McGraw-Hill Book Co., Inc., New York, N. Y., 8th Ed., 1953.

(3) **"Wood Handbook,"** *Agriculture Handbook No. 72*, Forest Products Lab., U. S. Dept. of Agriculture, Madison, Wis., 1955.

(4) **"Encyclopedia of Chemical Technology,"** edited by Raymond E. Kirk and Donald F. Othmer, Interscience Publishers, Inc., New York, N. Y., 1951.

(5) **"Technical Data on Plastics,"** Manufacturing Chemists Assn., Washington, D. C., October, 1952.

(6) **"Stress-Strain Relations in Timber Beams (Douglas Fir),"** by A. G. H. Dietz, *Bulletin No. 118,* A.S.T.M., October, 1942, p. 19.

(7) **"Building Code Regulation of Plastic Building Materials,"** by F. J. Rarig, *Plastics in Building, Publication 337,* Building Research Inst., National Academy of Sciences, National Research Council, Washington, D. C., April, 1955, p. 34.

(8) **"Glazing and Interior Illumination,"** by O. L. Pierson, *ibid.*

(9) **"Weather Aging of Styrene and Phenolic Plastics,"** by J. R. Taylor and C. H. Adams, *Mechanical Engineering,* October, 1954, p. 803.

(10) **"Materials and Methods 37,"** Reinhold Publishing Corp., New York, N. Y., Vol. 130, No. 5, 1953.

(11) **"Portfolio of Material Engineering File Facts,"** *Materials and Methods,* Reinhold Publishing Corp., New York, N. Y., 3rd Ed., 1951.

(12) **"Principles of High Polymer Theory and Practise,"** by A. X. Schmidt and C. A. Marlies, McGraw-Hill Book Co., Inc., New York, N. Y., 1948.

(13) **"The Effect of Temperature on the Creep of Two Laminated Plastics as Interpreted by the Hyperbolic-Sine Law and Activation Energy Theory,"** by W. N. Findley, C. H. Adams, and W. J. Worley, *Proceedings,* A.S.T.M., Vol. 48, 1948, p. 1217.

(14) **"Creep-Time Relations for Nylon in Tension, Compression and Bending,"** by J. Marin, A. C. Webber, and G. F. Weissman, *ibid.*, Vol. 54, 1954, p. 1313.

(15) **"Creep, Long Time Tensile and Flexural Fatigue Properties of Melamine, Phenolic Plastics,"** by D. Telfair, C. H. Adams, and H. W. Mohrman, *Modern Plastics,* May, 1947, p. 151.

(16) **"Practical Indices for Chemical Resistant Plastics,"** by R. B. Seymour, *Journal,* Soc. of Plastics Engrs., Vol. 10, No. 1, 1954, p. 15.

(17) **"Low Temperature Behavior of Organic Plastics,"** by H. K. Nason, T. S. Carswell, and C. H. Adams, *Special Technical Publication No. 78,* A.S.T.M., 1949.

(18) **"An Investigation of Rate of Strain on the Results of Tension Tests of Metals,"** by P. G. Jones and H. F. Moore, *Proceedings,* A.S.T.M., Vol. 40, 1940, p. 610.

(19) **"The Effect of Speed of Testing on Magnesium Base Alloys,"** by A. A. Moore, *Proceedings,* A.S.T.M., Vol. 48, 1948, p. 1133.

(20) **"Speed of Testing of Wood: Factors in its Control and its Effect on Strength,"** by L. J. Markwardt and J. A. Liska, *ibid.*, p. 1139.

(21) **"Effect of Speed of Test Upon Strength Properties of Plastics,"** by A. G. H. Dietz, W. J. Gailus, and S. Yurenlea, *ibid.*, p. 1160.

(22) **"Plastics Engineering Handbook,"** The Soc. of the Plastics Industry, Inc., Reinhold Publishing Corp., New York, N. Y., 1954.

(23) **"Physical and Engineering Properties of Plastics,"** by A. G. H. Dietz, *Plastics in Building, Publication 337,* Building Research Inst., National Academy of Sciences, National Research Council, Washington, D. C., April, 1955, p. 11.

(24) **"Consideration in the Design of Plastic Structures for Light Weight,"** by C. H. Adams, W. N. Findley, and F. D. Stockton, *Modern Plastics*, August, 1955, p. 139.

APPENDIX II. NOTATION

C = stress coefficient of damping capacity $\left(\frac{\text{(pounds per square inch)}^{1-n}}{\text{cycles}}\right)$;
E = Young's modulus of elasticity;
E_1 = Young's modulus of elasticity, direction 1;
E_L = Young's modulus of elasticity, longitudinal direction;
G = shear modulus of elasticity;
G_{12} = shear modulus of elasticity, directions 1 and 2;
G_{LT} = shear modulus of elasticity associated with longitudinal and transverse directions;
L = longitudinal direction;
m_1 = distortion factor, accounts for tensile and shear strains, direction 1;
m_2 = distortion factor, accounts for tensile and shear strains, direction 2;
n = empirical dimensionless number;
α = angle to longitudinal direction;
γ = specific weight;
ν_{12} = Poisson's ratio associated with directions 1 and 2;
σ = tensile strength;
σ_y = tensile yield strength;
τ = shear stress; and
τ_y = shear ultimate strength.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2931

MECHANICS OF STREAMS WITH MOVABLE BEDS OF FINE SAND

By Norman H. Brooks,[1] J. M. ASCE

With Discussion by Messrs. Thomas Blench; James R. Barton; Hans A. Einstein and Ning Chien; Enos J. Carlson and M. Gamal Mostafa; Hsin-Kuan Liu; Pin-Nam Lin; and Norman H. Brooks

Synopsis

A laboratory study was made of the characteristics of streams flowing over a loose bed of fine sand in order to determine what factors govern the equilibrium rate of transportation of fine sand in suspension. Twenty-two experimental runs were performed in a 40-ft tilting flume for various conditions with bed sand of two different sizes (0.10 mm and 0.16 mm). Each run represented a uniform open-channel flow in equilibrium with the sand bed.

Because of the extreme variability of channel roughness, the transportation rate could not be expressed as a unique function of the bed shear stress, the channel geometry, and the properties of the sand. This is contrary to all previous theories for the equilibrium transportation rate of suspended load. At low velocities, the large irregular dunes which formed on the stream bed made the bed friction factor more than six times larger than the friction factor for the smooth sand beds obtained at higher flow rates.

By using the mean velocity and the depth (or the water discharge and sediment discharge) as independent variables and the slope as a dependent variable, an orderly qualitative relationship between the pertinent variables was obtained.

Introduction

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically, for convenience of reference, in the Appendix.

General.—The transportation of sediment in suspension in a stream is caused

---

Note.—Published, essentially as printed here, in April, 1955, as *Proceedings-Separate No. 668.* Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions.*

[1] Asst. Prof. of Civ. Eng., California Inst. of Technology, Pasadena, Calif.

by the turbulent diffusion of material upward from the bed. The distribution of the concentration over the depth may be derived by assuming similarity between the suspended-sediment transfer and the momentum transfer and by then using the logarithmic velocity law given by Theodor von Kármán,[2] Hon. M. ASCE, to obtain the distribution of the diffusion coefficient. The resulting expression, which is called the suspended-load equation, may be written as follows, according to Hassan M. Ismail,[3] A.M. ASCE:

$$\frac{c}{c_a} = \left(\frac{d-y}{y}\,\frac{a}{d-a}\right)^z \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which c is the point concentration of suspended sediment at a distance, y, above the bed; c_a is the concentration at some reference level, $y = a$; d denotes the total depth; and z, the exponent, is given by

$$z = \frac{w}{\beta\, k\, u_*} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which w is the settling velocity of the particles, β denotes the ratio of the diffusion coefficient for sediment to the kinematic eddy viscosity, k represents the von Kármán universal constant, and u_* is the shear or friction velocity. The derivation of Eq. 1 was first published by Hunter Rouse,[4] M. ASCE, with the simplification $\beta = 1$ and was verified by the experimental work of Vito A. Vanoni,[5] M. ASCE.

This suspended-load equation is satisfactory, except near the boundaries where the assumptions under which the equation was derived break down. Indeed, from the equation itself it is easily seen that as y approaches 0, c approaches infinity, a physically impossible situation. Consequently, it is not a simple matter to supply a boundary condition, and Eq. 1 contains an unpredictable quantity, c_a, at an arbitrary reference level, $y = a$. Thus, Eq. 1 yields only the relative distribution within the stream, whereas the absolute concentrations depend on the diffusion mechanisms at the sand bed—that is, at the source of the suspended material. Little is known about the interactions between a turbulent stream and a movable sand bed. Obviously, if only the relative concentrations are known, it is impossible to find the sediment discharge by integration.

To show how limited the development of suspended-sediment transportation theories has been, the problem might be roughly compared with the problem of turbulent flow in pipes. It would be as if only the relative velocity distribution were known for a given pipe size and hydraulic gradient, without any knowledge of how the total discharge is related to the other variables.

The purpose of this paper is to report some experimentally determined relationships between the sediment discharge and the hydraulic characteristics of an open-channel flow with a movable bed of fine sand. No attempt is made to derive a general method for obtaining a boundary condition for the suspended-load equation, but the shortcomings of two existing theories will be mentioned.

---

[2] "Some Aspects of the Turbulence Problem," by Theodor von Kármán, *Proceedings*, 4th International Cong. of Applied Mechanics, 1934, pp. 54–91.

[3] "Turbulent Transfer Mechanism and Suspended Sediment in Closed Channels," by Hassan M. Ismail, *Transactions*, ASCE, Vol. 117, 1952, pp. 409–446.

[4] "Modern Conceptions of the Mechanics of Fluid Turbulence," by Hunter Rouse, *ibid.*, Vol. 102, 1937, p. 534.

[5] "Transportation of Suspended Sediment by Water," by Vito A. Vanoni, *ibid.*, Vol. 111, 1946, pp. 67–133.

Apparatus and Procedure

Tilting Flume.—The experiments were performed in the 40-ft, closed-circuit tilting flume shown in Fig. 1. The water was recirculated with its entire sediment load so that it was not difficult to maintain a stable equilibrium condition for hours at a time. Because the velocity in the return pipe was always high enough to prevent any significant deposition of sand, the layer of sand covering the bottom of the flume contained practically all the sand in the system. The inside surfaces of the flume were painted and were found to be hydrodynamically smooth; no artificial roughness was used.

Because it was found that temperature had an appreciable effect on the transportation rate, four 1,000-watt immersion heaters were installed to regulate the temperature of the water for the second series of runs (runs 21 through 30). After run 21, damping screens were used at the inlet to reduce the large-scale turbulence generated in the transition section and elbows. The distance required in the flume for the establishment of uniform conditions was thereby shortened to generally less than 6 ft. A comparison of runs 21 and 21a (without and with screens, respectively) showed that the final equilibrium established was not affected by the screens.

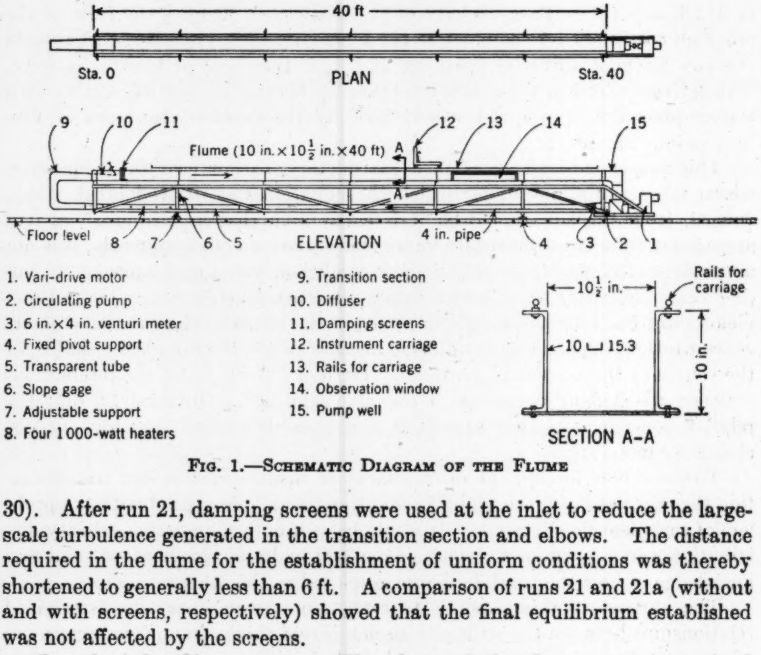

40 ft
Sta. 0
PLAN
Sta. 40
9
10
11
12
13
14
15
Flume (10 in. × 10½ in. × 40 ft)
A
A
Floor level
8
7
6
5
ELEVATION
4 in. pipe
4
3
2
1
1. Vari-drive motor
2. Circulating pump
3. 6 in. × 4 in. venturi meter
4. Fixed pivot support
5. Transparent tube
6. Slope gage
7. Adjustable support
8. Four 1000-watt heaters
9. Transition section
10. Diffuser
11. Damping screens
12. Instrument carriage
13. Rails for carriage
14. Observation window
15. Pump well
10½ in.
Rails for carriage
10 ⌴ 15.3
10 in.
SECTION A-A

Fig. 1.—Schematic Diagram of the Flume

Procedure for Measurements.—The slope of the flume was determined directly from a gage mounted on the truss (near the jack used for tilting the flume). A point gage mounted on a carriage was used to measure water surface and bed elevations relative to the flume at a number of stations along the flume. When the water surface was wavy, the mean elevation was determined by averaging the point gage readings on adjacent crests and troughs. In order to determine the average elevation of the surface of the sand bed, the bed configuration for the entire flume (starting with run 12) was leveled in reaches of from 2 ft to 4 ft with a specially built scraper, after stopping the flow of water at the end of each run. The mean depth, *d*, was found by averaging the difference between the water surface and bed elevations over the section of the flume in which the flow was in equilibrium. This procedure is illustrated in Fig. 2 for run 27. For run 5 and runs 8 through 13, the probable error in the measured mean depth was approximately 0.005 ft, whereas in all the other runs it was of the order of 0.001 ft.

The slope of the energy line relative to the flume was computed from the bed and water surface measurements and was added to the slope of the flume to obtain the true energy gradient. The flume was adjusted to keep the relative slope 0.0001 or less. Fig. 2 also shows the computed energy line for run 27.

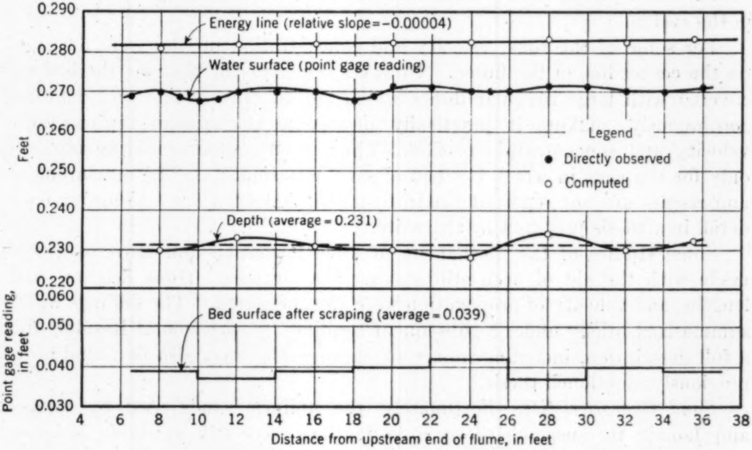


Fig. 2.—Typical Flume Profiles for a Run with Dunes on the Bed and Small Ripples on the Water Surface (Run 27—Flume Slope = 0.00241)

The discharge was determined from the pressure drop across a standard, bell-shaped, 6-in. by 4-in. pipe reducer immediately downstream from the pump (Fig. 1). Because the location was not a desirable one, the meter was calibrated with detailed point velocity measurements in the open channel, both with and without sediment, and the venturi coefficient was found to be 0.95. The probable error in the measured discharges is believed to be 1% or less.

The total sediment discharge was measured by sampling the flow in the end tank over the pump with a siphon. The sampler was a vertical copper tube of 0.302-in. inside diameter, with a 180° turn on the bottom end. The entire load was in suspension and well mixed at the sampling position; nevertheless, the sampler tip was moved around gently during sampling to assure the collection of representative samples. The head on the siphon was adjusted to make the velocity in the sampler tip equal to the mean velocity of flow. The fall velocity of the sand was much smaller than the vertical flow velocity in the end tank. In addition, precautions were taken to avoid errors due to sand storage in the sampling tube. For each run a number of 1-liter samples were taken, usually in groups of three, and the concentration for each sample was determined by filtering the sample and drying and weighing the residue of sand. For simplicity the concentrations are all reported in grams of dry sand per liter of mixture.

The sediment discharge concentration, $\bar{C}$, measured in this way, is thus the total rate of sediment transportation (including bed load) divided by the discharge and is not necessarily the same as the average concentration of sediment in suspension in the open channel.

For each liter withdrawn from the flume, a liter of clear water was added at the same time to keep the depth of flow constant. As the flow circuit was closed, the depth in the open channel was governed by the quantity of water in the system.

For some of the runs, velocity and concentration profiles were measured on the center line of the flume. However, for many of the runs the bed was covered with large irregular dunes which moved downstream and deformed continuously, making it practically impossible to measure representative velocity and concentration profiles. Thus, good profiles could be obtained only for the runs in which the bed of sand was smooth. The measurements and results are not reported herein owing to lack of space but are given in detail in a thesis prepared by the writer.[6]

Some studies of the mechanism of dune formation and movement were made with the aid of both still and motion pictures. Dune heights, wave lengths, and velocity of propagation were also measured. The findings will be summarized briefly under a subsequent heading, "Experimental Results," but a full description, including numerous photographs, may also be found in the previously mentioned thesis.[6]

Sand Characteristics.—Fortunately, the uniform sands used by Vanoni[5] and Ismail[3] for suspended-load investigations were still available and were used in these experiments, making it unnecessary to prepare new sand. Two different uniform quartz sands were used: sand No. 1, with a mean sedimentation diameter of 0.16 mm for runs 2 to 13; and sand No. 2, with a mean sedimentation diameter of 0.10 mm for runs 21 to 30. The sieve analyses of the two sands are shown in Fig. 3, with the geometric mean sieve size, D_g, and the geometric standard deviation, σ_g, indicated for each. At 25° C the mean fall velocities are 0.060 ft per sec and 0.030 ft per sec, respectively. Approximately 145 lb of sand (dry weight) were used in the flume for each of the two series, which was equivalent to a uniform layer slightly more than a $\frac{1}{2}$ in. thick on the bed of the flume.

Establishing Uniform Flow in Equilibrium; Reproducibility.—Before making the final measurements for a run it was first necessary to establish uniform flow in equilibrium with the sand bed. With a movable bed this was not easy because of the changing configurations of the bed (smooth or rippled) and the often observed tendency for the sand to spread itself unevenly throughout the length of the flume. Equilibrium, as used hereinafter, implies (1) that the mean depth of flow was constant over a working section of the flume and (2) that the pattern and distribution of sand on the bed had stopped changing.

---

[6] "Laboratory Studies of the Mechanics of Streams Flowing over a Movable Bed of Fine Sand," by Norman H. Brooks, thesis presented to the California Institute of Technology, at Pasadena, in 1954, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Originally, it was believed that the rate of transport and the flow velocity would be uniquely determined by the size of sediment, the size of channel, and the shear stress on the bed, which in turn depends simply on the depth, width, and slope of the stream. Although this belief is held by many investigators of sediment transportation phenomena, it was found, nevertheless, that the relationship is not unique.

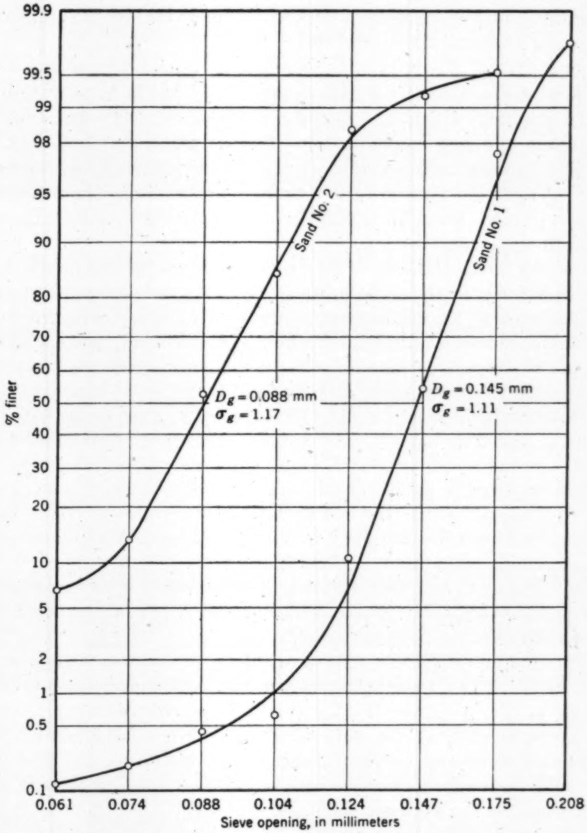


FIG. 3.—SIEVE ANALYSES OF THE SANDS

This false assumption led to immediate difficulty in programming the runs to be made in the flume. The three main factors that could be regulated were the depth, *d*, the discharge, *Q*, and the slope, *S*. The width was fixed and the sediment load could not be controlled directly. At first, an attempt was made to select the depth and slope for each run and then to adjust the discharge to obtain equilibrium; but in the course of the experiments it was found that for some depth-slope combinations there are two, or maybe even more, different equilibrium discharges and sediment loads. These are possible because of the large variations in the roughness of the channel due to the appearance of different configurations of the sand bed.

Another source of difficulty with choosing *d* and *S* first was the sensitivity of the flow to changes in *Q*. Small changes in *Q*, made to adjust to a given slope, often produced significant changes in the other variables, such as sediment discharge and bed configuration. Sometimes it was not even possible to determine immediately whether or not an adjustment of discharge was toward or away from equilibrium. Consequently, many hours could have been spent trying to obtain an equilibrium by adjusting the discharge to match a given depth-slope combination.

However, by fixing the rate of flow and the depth, it was possible to make the final adjustment in the slope quite simply. If the first approximation for the slope was in error, the adjustment required could be estimated by measuring the slope of the energy line relative to the flume. The adjustment of the slope of the flume would then cause only slight changes in the equilibrium. The slope of the energy line was really already fixed by the flow; in effect the inclination of the flume itself would simply be adjusted to this slope, with the only net change to the flow being a very minor redistribution of depths over the length of the flume.

The time required to reach equilibrium depended on the sediment load and the amount of adjustment in the bed configuration required. With high loads, less than one-half hour was required, but with light loads, up to four hours were sometimes needed. The final configuration of the bed was found to be independent of the initial bed configuration, whether flat or in dunes, evenly or unevenly distributed in the flume. The initial condition affected only the amount of time required to reach equilibrium.

The reproducibility of the runs was excellent. This was well demonstrated in run 28, in which two water surfaces and two bed profiles were taken on two different days. Scraping the bed for the first bed-elevation profile erased the dunes and ripples so that the dunes had to be regenerated and equilibrium re-established before any other measurements, including the second set of profiles, could be made. The two sets of profiles were practically identical, indicating that the equilibrium achieved was stable, well defined, and reproducible. The computed mean depths, velocities, and slopes for these cases were practically identical, as shown by the following tabulation:

	First day	Second day
Depth, *d*, in feet	0.285	0.284
Mean velocity, *U*, in feet per second	1.31	1.32
Slope, *S*	0.00245	0.00242

Other runs gave similar evidence of being very stable.

EXPERIMENTAL RESULTS

Each run represented a stable uniform flow in equilibrium with its movable sand bed. Runs were established and the measurements were made according to procedures previously outlined under the heading, "Apparatus and Procedure."

The sand was so fine that the rate of suspended-load transport was much greater than the rate of bed-load transport, except possibly in runs 10 and 26 where the total transportation rate was very small. This does not mean that an insignificant amount of material was in motion on the bed; on the contrary, several layers of sand grains were almost always sliding along the bed, but at a relatively low rate compared with the velocity of the water. If, for example, the same amount of material was in suspension, it would have yielded a much larger transportation rate (expressed as weight per unit time passing any cross section) because the sand then traveled at the full speed of the water. However, the distinction between bed load and suspended load is quite nebulous, and no attempt was made here to make an artificial separation. All the values of transportation rate and discharge concentration reported in this section refer to the entire sediment discharge, bed load included.

For purposes of comparison, a few runs (C–1, C–2, C–3, C–4, C–6, and C–7) were made without any sediment.

Summary of Data.—The most important measured and computed quantities for each flow are tabulated in Table 1. In Table 1, r is the hydraulic radius equal to $b\,d/(b + 2\,d)$, in which b is the width equal to 0.875 ft for all runs; U_* is the shear velocity for the whole channel equal to $\sqrt{g\,r\,S}$; U is the average velocity of flow for the entire channel equal to $Q/b\,d$; f is the Darcy-Weisbach friction factor for the whole channel equal to $8\,(U_*/U)^2$; r_b is the hydraulic radius for the bed section, computed from the side-wall correction procedure; U_{*b} is the shear velocity for the bed section equal to $\sqrt{g\,r_b\,S}$, in which g is the acceleration due to gravity; f_b is the friction factor for the bed alone, equal to $8\,(U_{*b}/U)^2$; and F is the Froude number equal to $U/\sqrt{g\,d}$. The side-wall corrections were made by the basic procedure of Joe W. Johnson,[7] M. ASCE, with a few minor modifications to make the computations direct instead of by trial and error. His method employs f and the Kármán-Prandtl resistance equation for hydrodynamically smooth pipes. The derivations and equations used, as well as a sample computation, are reported in detail in the writer's thesis.[6]

In the subheadings, D_s is the mean sedimentation diameter of the bed material. For the sediment load, the mean sedimentation diameter is usually slightly less because of the sorting.

Where entries are missing in Table 1, either the item does not apply or the data were not obtained. Whenever there is a reasonable basis for making good estimates, when the data are missing, estimated values have been entered and designated by a footnote.

For all the entries in Table 1 only the significant figures are reported. The accuracy of the values is thus indicated directly; for example, the discharge is

---

[7] "The Importance of Side-Wall Friction in Bed-Load Investigations," by Joe W. Johnson, *Civil Engineering*, Vol. 12, 1942, pp. 329–331.

TABLE 1.—SUMMARY OF EXPERIMENTAL RESULTS

Run No.	Discharge, Q (cu ft per sec)	Depth, d (ft)	Hydraulic radius, r (ft)	Slope, S	Shear velocity, U_* (ft per sec)	Average velocity, U (ft per sec)	Friction factor, f	Water temperature, T (° C)	Bed hydraulic radius, r_b (ft)	Bed shear velocity, U_{*b} (ft per sec)	Bed friction factor, f_b	Number of sediment-discharge samples	Sediment discharge concentration, $\overline{C}$ (g per liter)	Total sediment discharge, G (lb per min)	Froude number, $\mathbf{F}$	Water surface condition	Bed condition
								CLEAR WATER									
C-1	0.695	0.232	0.152	0.0050	0.157	3.44	0.0167	18	0.156	0.159	0.017	...	...	...	1.27	Smooth	No sand
C-2	0.45[c]	0.171	0.123	0.0050	0.141	3.0[c]	...	18.5	...	...	0.0185[c]	...	...	...	1.3[c]	Smooth	No sand
C-3	0.24[c]	0.115	0.091	0.0050	0.121	2.4[c]	...	18	...	...	0.0205[c]	...	...	...	1.3[c]	Smooth	No sand
C-4	0.32[c]	0.225	0.148	0.00125	0.077	1.6[c]	...	14.5	...	...	0.0195[c]	...	...	...	0.6[c]	Smooth	No sand
C-6	0.47	0.179	0.127	0.0049	0.142	3.00	0.0180	27	0.131	0.145	0.0185	...	...	...	1.25	Smooth	No sand
C-7	0.435	0.241	0.155	0.00195	0.099	2.06	0.0183	17	0.157	0.099	0.0185	...	...	...	0.74	Small waves	No sand
								SAND BED (D_s = 0.16 MM)									
2	0.54[c]	0.284	0.172	0.0018	0.100	2.15[c]	0.017[c]	17	0.174[c]	0.100	0.018[c]	0	...	...	0.71[c]	Small waves	Smooth
3	0.435	0.243	0.156	0.0025	0.112	2.04	0.024	22	0.178	0.120	0.0275	5[a]	1.95[a]	3.1	0.73	Waves	Smooth
4	0.43	0.236	0.153	0.0024	0.108	2.08	0.022	12.5	0.164	0.112	0.023	5	2.45	3.9	0.75	Waves	Smooth
5	0.28	0.18	0.13	0.0031	0.115	1.8	0.033	26	0.145	0.12	0.038	5	1.9	2.0	0.74	Large ripples	Meanders
6	0.345	0.195	0.135	0.0024	0.103	2.00	0.021	21	0.143	0.106	0.0225	6	2.45	3.1	0.80	Large waves	Smooth
7	0.435	0.243	0.156	0.0021	0.103	2.04	0.020	31.5	0.170	0.107	0.0225	7[b]	2.15[b]	3.5	0.73	Waves	Smooth
8	0.375	0.24	0.155	0.0023	0.11	1.75	0.030	27.5	0.185	0.12	0.036	6	1.5	2.1	0.63	Ripples	Meanders
9	0.285	0.245	0.155	0.0026	0.115	1.35	0.059	27.5	0.21	0.13	0.079	12	1.1	1.2	0.47	Ripples	Dunes
10	0.20	0.25	0.16	0.0020	0.10	0.93	0.095	24	0.225	0.12	0.135	4	0.2	0.15	0.33	Smooth	Dunes
11	0.205	0.155	0.115	0.0033	0.11	1.5	0.043	26	0.135	0.12	0.050	6	2.7	2.1	0.67	Large ripples	Meanders
12[d]	0.37	0.30	0.178	0.0022	0.111	1.40	0.050	26	0.248	0.131	0.070	6	0.72	1.0	0.45	Ripples	Dunes
13	0.215	0.197	0.136	0.0035	0.124	1.25	0.078	26.5	0.178	0.142	0.102	15	1.2	0.95	0.50	Large ripples	Dunes
								SAND BED (D_s = 0.10 MM)									
21	0.435	0.236	0.154	0.00225	0.106	2.10	0.0205	25.0	0.166	0.110	0.022	6	4.85	7.9	0.76	Waves	Smooth
21a	0.435	0.236	0.154	0.0022	0.104	2.10	0.020	25.0	0.165	0.108	0.0215	6	4.9	8.0	0.76	Large waves	Smooth
23	0.325	0.189	0.132	0.00245	0.102	1.96	0.0215	25.0	0.141	0.106	0.023	12	5.1	6.2	0.79	Large waves	Smooth
24[e]	0.265	0.226	0.149	0.0028	0.116	1.34	0.060	25.0	0.197	0.133	0.079	6	4	4	0.50	Ripples	Dunes
24[e]	0.265	0.17	0.12	...	...	1.8	...	25.0	...	...	...	6	7	7	0.76	Small waves	Low ripples
25	0.20	0.187	0.131	0.0033	0.118	1.23	0.074	25.0	0.168	0.134	0.095	10	5.3	4.0	0.50	Large ripples	Dunes
26	0.20	0.279	0.170	0.0013	0.084	0.82	0.084	25.0	0.245	0.101	0.12	6	0.19	0.14	0.27	Smooth	Dunes
27	0.20	0.231	0.151	0.00235	0.107	0.99	0.094	25.0	0.207	0.126	0.13	11	1.35	1.0	0.36	Small ripples	Dunes
28	0.33	0.284	0.172	0.0024	0.116	1.32	0.062	25.0	0.244	0.138	0.088	12	3.6	4.45	0.44	Ripples	Dunes
29	0.52	0.280	0.171	0.00185	0.101	2.13	0.0180	25.2	0.178	0.103	0.019	15	3.45	6.7	0.71	Smooth	Smooth
30	0.265	0.281	0.171	0.00215	0.108	1.08	0.080	25.0	0.246	0.130	0.115	12	1.75	1.7	0.36	Small ripples	Dunes

[a] Also five samples at 17.5° C, $\overline{C}$ = 2.1 g per liter, and five at 12.5° C, $\overline{C}$ = 2.35 g per liter. [b] Also seven samples at 28° C, $\overline{C}$ = 1.8 g per liter; and 10 at 26° C, $\overline{C}$ = 1.6 g per liter. [c] Estimated. [d] Long, flat sand wave present, but data pertain only to rugged dune section. [e] Dual equilibrium due to a long, flat sand wave.

reported to the nearest 0.005 cu ft per sec. Of all the runs, 5, 8, 9, 10, and 11 were the least accurate, as shown by the entries for d, r, U_*, r_b, and U_{*b}, which are given only to the nearest 5 units in the third place. The second series, runs 21 to 30, is more reliable because of improvements in experimental technique gained by experience with the earlier runs. However, there is no significant difference in the conclusions which may be drawn from each of the two series of runs.

Because of the limitations of the flow system, it was not possible to cover a wider range of conditions. Discharges less than 0.20 cu ft per sec were not used because sand storage in the return pipe could have caused large errors in the discharge measurement and a shortage of sand in the flume section. Depths greater than 0.30 ft were not used because the effect of the walls becomes too large, and difficulty was encountered with the inlet condition. In addition, velocities giving Froude numbers in excess of 0.80 caused such high standing surface waves that the flow was completely unmanageable. Nonetheless, in spite of these three restrictions, enough conditions were covered to lead to some valuable conclusions, which are believed to be qualitatively valid over a much wider range of conditions.

No attempt was made to derive an empirical formula for the transportation rate on the basis of these flume data. However, some qualitative relationships will be examined subsequently under the heading, "Analysis of Experimental Results."

Observations of Water Surface and Bed Configuration.—When the mean velocity and the transportation rate were low, the dunes which formed were fairly large and haphazardly placed; the friction factors were as large as 0.13. As the velocity of flow was increased, the dune wave length increased slightly, the pattern of the dunes became more regular, the dunes moved faster, and the friction factor decreased slightly. The height of the fully developed dunes did not change appreciably, possibly because of insufficient sand in the system to develop higher dunes.

The dune height in the central part of the channel was usually from 0.5 in. to 0.6 in. (average) and up to 1.5 in. (maximum). The dune velocity ranged from 1/5,000 to 1/500 of the mean stream velocity. The average wave length varied from 4.3 in. to 5.6 in.

A typical dune configuration is shown in Fig. 4 for run 30, which had a transportation rate intermediate between the highest and lowest of the runs with dunes.

As the velocity of flow was progressively increased, a point was reached at which the sand tended to collect in a single long flat wave, which traveled perpetually through the system without changing its size or shape. The wave was a long, thick deposit of sand with a smooth flat top over which the water flowed with reduced depth and increased velocity and sediment load. In the remainder of the flume, the bed would still be covered with rugged dunes for which the friction factor remained large. Pronounced sand waves were present in runs 12 and 24, but, because the flow was in equilibrium both on the sand wave and away from it, the data in Table 1 pertain either to the dune section or the sand wave separately. It was not apparent why these solitary sand waves were formed.

Meandering of the bed was another puzzling phenomenon that was observed for several runs with the 0.16-mm sand, although not with the smaller sand. The sand distributed itself nonuniformly across the channel by forming a ridge which weaved back and forth in the channel. The bed surface was covered with small sand ripples, and the bed friction factor was considerably less than for the runs with rugged dunes.

With a further increase in velocity (and transportation rate), the sand again spread itself uniformly throughout the flume, with the surface of the bed being quite smooth except for some small ripples near the wall.

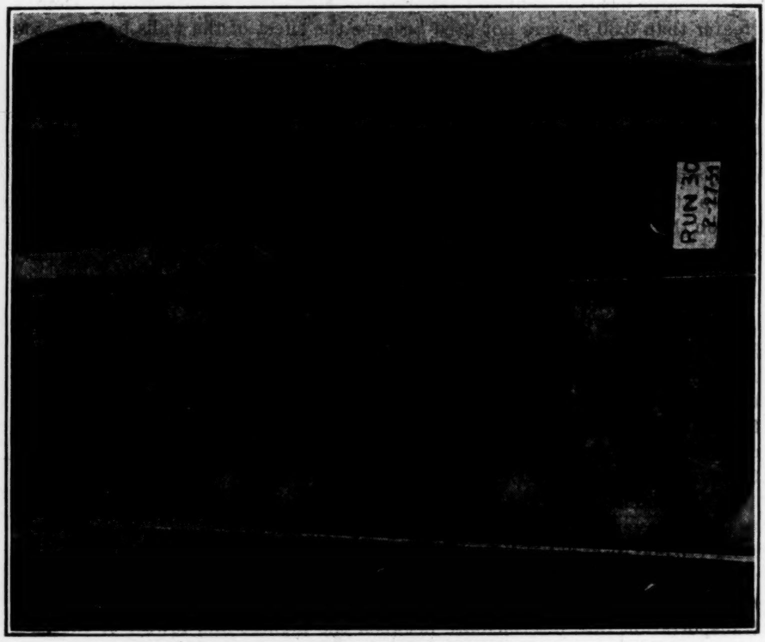


FIG. 4.—SIDE AND TOP VIEWS OF A TYPICAL DUNE FORMATION (DOWNSTREAM IS TO THE RIGHT)

Up to a Froude number of **F** equal to from 0.7 to 0.75, the water surface was relatively calm. At the lowest Froude number (**F** = 0.27), the surface had a glassy smoothness in spite of large bed irregularities. At higher Froude numbers before the dunes disappeared, the surface became quite rippled as a result of the disturbance to the flow by the bed. However, when **F** reached some critical value between 0.7 and 0.75, large surface waves with wave lengths of from 10 in. to 12 in. developed. A typical wave train would move slowly upstream and finally disappear without breaking; new waves would form and repeat the process. In the presence of these surface waves, the sand bed was always smooth in the center of the channel, except for very slight undulations of the same wave length as the surface waves, when the waves were unusually large.

At still higher Froude numbers, the waves become extremely high, sometimes as high as the average depth of flow, and long, low, rapidly shifting antidunes are formed. No runs were performed at this extreme condition because it was impossible to ascertain whether the flow was in equilibrium, and most of the usual measurements were impossible. However, it was also noted that, even at velocities substantially above critical ($\mathsf{F} > 1$), the water surface never flattened out again as it would have for clear water flow; it continued to be intensely wavy as long as there was a sand bed.

ANALYSIS OF EXPERIMENTAL RESULTS

The experimental data given in Table 1 have been plotted in several different ways in Figs. 5 to 7 in order to show how the total sediment discharge is related to the other characteristics of the laboratory streams. Because the bed-load discharge is small, the total sediment discharge may be considered equivalent to the suspended-load discharge for all practical purposes.

Impossibility of Taking Slope or Shear As an Independent Variable.—From Fig. 5 it appears that the mean velocity, U, and the sediment discharge concentration, $\bar{C}$, are not uniquely determined by the depth, d, and the slope, S. For example, in Fig. 5(a) the points, $U = 2.04$, $\bar{C} = 1.95$, and $U = 1.35$, $\bar{C} = 1.1$, have practically the same d-value and S-value but different velocity and concentration values. This result is completely contrary to present concepts of open-channel hydraulics; it is commonly believed that the velocity of flow and the sediment transportation rate can be uniquely determined if the depth, width, and slope, and hence the shear, are known, in addition to the size of material.

In applying any flow formula (such as the Manning equation or the Darcy-Weisbach formula) to a flow over a movable bed of fine sand, the usual assumption of approximately constant roughness leads to a gross misconception about the relationship of velocity to depth and slope. Actually, the "constant" roughness can change more than any of the other variables because of the changing bed configuration. For example, increasing the slope of a stream with depth constant is supposed to increase its velocity, but from Fig. 5 it is apparent that higher slopes are frequently associated with lower velocities for a given depth. Similarly, if the depth is increased, holding the slope constant, the velocity should increase. The experimental results, however, show that this is not always the case.

Because the walls have a significant and variable effect on the flow, it is perhaps preferable to compare the data on the basis of the hydraulic radius for the bed, r_b, and the bed shear, obtained from the side-wall correction computations (Table 1). If U and $\bar{C}$ are plotted again against S and r_b, instead of against S and d, and lines of constant shear (or simply constant $r_b S$) are drawn, the same conclusions may be drawn. If anything, one might even be led to conclude that the higher concentrations and velocities are associated with the smaller shears, a result completely contrary to widely held beliefs about sediment transportation.

In the range of experiments conducted, it may be noted from Table 1 that the shear velocity for the bed, U_{*b}, has remarkably little variation compared with the velocity or transportation rate. It may indicate that in the range of sediment loads encountered, the bed configuration tends to adjust itself in a way which greatly reduces the variation of shear that might ordinarily be expected. This being the case, it is improbable that the suspended-load transport of a stream can ever be expressed directly in terms of the shear.

Depth and Velocity As Independent Variables.—Because the velocity and the sediment discharge concentration could not be expressed uniquely as a function of the slope and either the depth or the bed hydraulic radius, an attempt was made to discover which quantities could be logically used as independent variables. It was found that any two variables representing two of the following three groups could be considered as independent: (*a*) d or r_b; (*b*) U or Q; and (*c*) $\bar{C}$ or G. For Fig. 6, d and U were used as coordinates, and for Fig. 7, Q and G were used. These two particular combinations of independent variables seemed most logical and made the results the easiest to interpret.

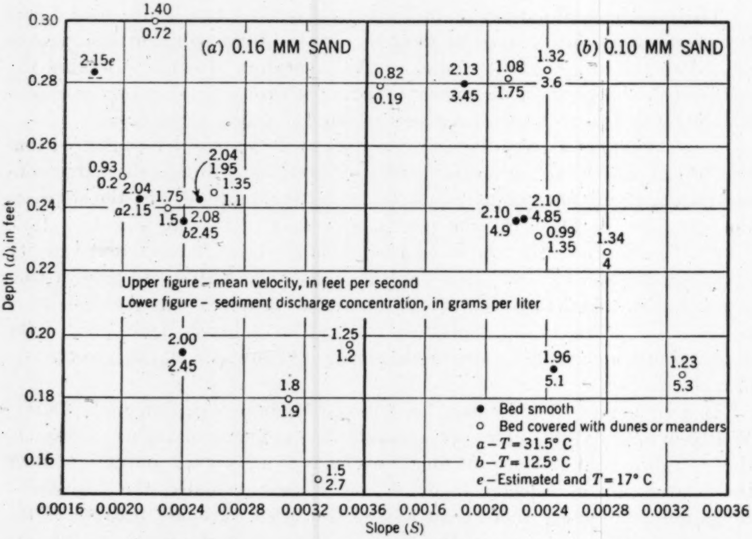


FIG. 5.—VELOCITY AND CONCENTRATION VERSUS DEPTH AND SLOPE

The sand size is, of course, a very significant independent variable, but, because only two sizes of sand were used, little could be deduced about the effect of the sand size on the sediment load. No information was obtained on the behavior of a graded sand.

Although temperature is a variable of secondary importance, it was still found advisable to regulate it at 25° C for the second series of runs (21 to 30) to avoid any possible confusion resulting from too many variables changing at once. For the earlier runs (2 to 13) with 0.16-mm sand, the range in temperature was from 21° C to 27.5° C, except for three runs; these three cases are specially labeled in all the figures. When the temperature rises, the settling velocity of the sand increases, and there is a tendency for the sediment discharge to decrease, other things being equal (Table 1, runs 3 and 4).

The following fairly definite qualitative conclusions may be drawn from Fig. 6, in which U and d are taken as the independent variables, and two of the dependent variables, f_b and $\bar{C}$, are supplied for each of the plotted points.

a. The variables f_b and $\bar{C}$ are uniquely determined by U and d for the flume. From these, all the other quantities listed in Table 1 may be determined so that the character of the flow is completely determined by a selection of U and d. This was also borne out by general experience in operating the flume; by choosing the depth and mean velocity, one, and only one, definite equilibrium flow could be obtained.

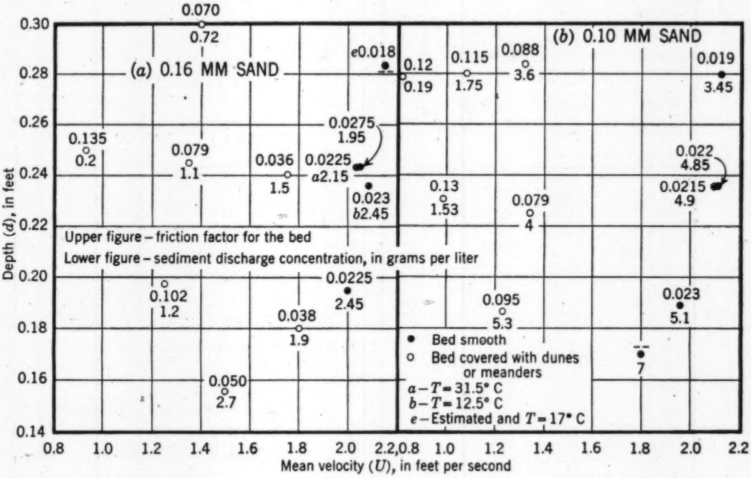


FIG. 6.—BED FRICTION FACTOR AND CONCENTRATION VERSUS DEPTH AND VELOCITY

b. For a fixed depth, $\bar{C}$ increases as U increases in most cases. Also, f_b decreases markedly as U increases because of the changes in the bed configuration. For example, in the case of the 0.16-mm sand, Fig. 6(*a*) shows that increasing the velocity from 1.25 ft per sec to 2.0 ft per sec for $d \doteq 0.195$ ft results in a drop in f_b from 0.102 to 0.0225, because the dunes are washed away and the sand bed becomes flat. At the same time, the discharge concentration, $\bar{C}$, is increased from 1.2 g per liter to 2.45 g per liter.

c. For a fixed velocity, $\bar{C}$ decreases slightly as d increases, but f_b does not appear to depend much on the depth in the limited range studied.

d. The foregoing conclusions apply equally well for both sand sizes.

e. For a given value of d and U, the concentration of sediment discharge, $\bar{C}$, for the 0.10-mm sand is approximately from 2 to 4 times as much as that for the 0.16-mm sand in the range of conditions covered.

f. The friction factor, f_b, appears to be numerically the same for both sand sizes for any given value of d and U. Because f_b may be influenced by the amount of sediment load as well as the dune configuration, no further conclusions may be drawn here regarding the factors governing f_b.

In Fig. 6(*b*) it may be noted that there is a substantial gap between the points for runs with smooth bed and dune-covered bed. During the investigation an attempt was made to establish some points in the gap, but it appeared to be impossible. A sand wave would always form, thereby dividing the flume into a reach of higher velocity and smooth bed and another reach of lower velocity and rough bed. For the larger sand size (Fig. 6(*a*)), three intermediate points were established, two at a velocity of 1.8 ft per sec and the other at 1.5 ft per sec; these were the three runs for which meanders were observed. However, for slightly lower velocities, the sand waves would probably interfere in the same way as for the 0.10-mm sand. It is not known whether this is a

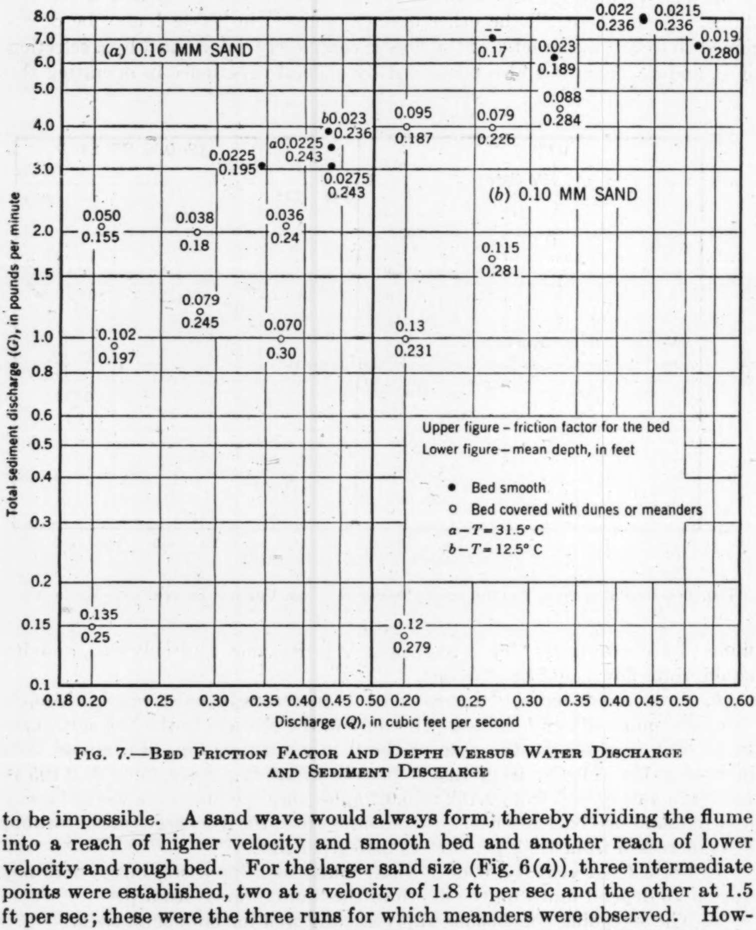


FIG. 7.—BED FRICTION FACTOR AND DEPTH VERSUS WATER DISCHARGE AND SEDIMENT DISCHARGE

system effect, or whether there actually are impossible combinations of depth and velocity for these sand sizes.

Water Discharge and Sediment Discharge As Independent Variables.—Some corollaries of the foregoing conclusions can be deduced by replotting the data (Fig. 7) with the discharge, Q, and the total sediment transportation rate, G, used as the independent variables, and the values of f_b and d recorded opposite the plotted points. The scales are logarithmic and the length of the cycle in the Q-scale is just twice that of the G-scale. Additional conclusions based on Fig. 7 are as follows:

a. All the variables, including f_b and d, can be uniquely determined from Q and G. For example, to transport 1.5 lb per min of 0.16-mm sand with a discharge of 0.25 cu ft per sec in the laboratory flume, it may be estimated from the neighboring points in Fig. 7(*a*) that the required depth will be approximately 0.19 ft and $f_b = 0.07$. For the same load of 0.10-mm sand, there follows from Fig. 7(*b*) that $d = 0.27$ ft and $f_b = 0.12$, with $Q = 0.25$ cu ft per sec as before. The required slope in each case can be obtained by working back through the side-wall correction equations. It may be noted that in a long, open-circuit flume, Q and G are the only two variables which are independent and can be directly controlled. Everything else adjusts to these values.

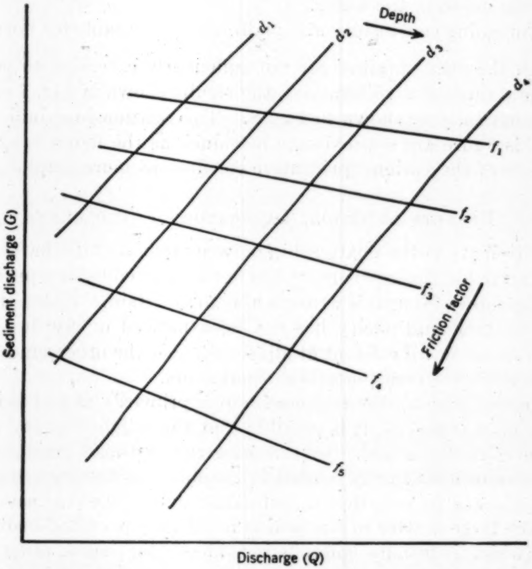


FIG. 8.—SCHEMATIC REPRESENTATION OF THE RELATIONSHIPS OF SUSPENDED-SEDIMENT TRANSPORTATION FOR THE FLUME (INCREASE IN d AND f IS THE DIRECTION OF THE ARROWS)

b. Maintaining a constant, G, while Q is increased, requires that the depth, d, increase, although not as much as Q. The velocity, U, also increases.

c. For a constant, Q, an increase in G requires a decrease in d. This is a significant conclusion because it implies that the depth of flow depends on the sediment load for any given discharge; hence, a unique stage-discharge relationship did not exist for the laboratory flume. For example, run 23 and run 28 had practically identical slopes and discharges (Table 1) but two different depths, 0.189 ft and 0.284 ft, respectively. The transportation rate for the former was 6.2 lb per min and for the latter 4.45 lb per min.

d. For a given value of Q, the largest f_b-values are associated with the smallest sediment transportation rates. From Fig. 7(*b*) it appears that f_b reaches an upper limit of about 0.12 or 0.13 (very large values), and a further decrease in G does not increase f_b much more. Physically, there must be a limit to the roughness of the channel because the height of the dunes is certainly limited by the depth of the water.

e. The foregoing conclusions are qualitatively the same for both sand sizes.

Although the data obtained are not sufficiently extensive to permit establishing general quantitative formulas, the results shown in Fig. 7 can be represented schematically as shown in Fig. 8. The relationships indicated by the curves in this figure are approximate inasmuch as the figure is intended only to make some of the various qualitative conclusions more graphic.

RESULTS OF OTHER LABORATORY EXPERIMENTS

The multiplicity of the relationship between sediment discharge, depth, and slope encountered in the experiments has not been cited by any previous laboratory investigators so far as this writer is able to ascertain. Either the roughness effect causing this multiplicity has not been noticed or else it has not been nearly so pronounced (if existent at all) for some of the other sand sizes heretofore used in sediment transportation experiments.

With coarse material, the sediment moves primarily as bed load; although dunes have been observed, it is possible that the roughness may not undergo nearly such radical changes. In this case the bed-load transportation rate would be more or less directly related to the shear, as first suggested by P. Du Boys,[8] and checked by countless investigators during the past several decades. However, the large scatter of the points in all graphs of bed-load transportation rates (which is usually ignored) is evidence that some other effect, such as roughness, has not been provided for. In some bed-load formulas, notably that of the Waterways Experiment Station at Vicksburg, Miss., the Manning roughness coefficient, n, has been included in the formula as an independent variable; however, a method for predicting n is then needed. The bed-load theory of Hans A. Einstein,[9] M. ASCE, considers variable channel roughness and includes a method for determining it. His method will be reviewed subsequently in connection with suspended-load theories.

---

[8] "Le Rhône et les rivières à lit affouillable," by P. Du Boys, *Annales des ponts et chaussées*, Series 5, Vol. 18, 1879, pp. 141-195.

[9] "The Bed-Load Function for Sediment Transportation in Open Channel Flows," by H. A. Einstein, *Technical Bulletin No. 1026*, S.C.S., U. S. Dept. of Agriculture, Washington, D. C., September, 1950.

It is doubtful whether changes in roughness play a very significant part in the mechanics of streams with beds of silt or clay, or the finely ground silica flour used by Anton A. Kalinske, M. ASCE, and C. H. Hsia.[10] It seems that dunes may never become very large in this material because it is moved so easily by flowing water even at very low velocities. For nine different flows over a bed covered with this extremely fine material, Hsia found that the Manning coefficient, *n*, varied only from 0.0119 to 0.0098, with one of the highest values being for the run with the smallest concentration of sediment discharge (0.64%) and the smallest *n* for the run with the largest concentration (11.1%). He found that the transportation rate did have a direct relation to the bed shear, increasing when the shear increased.

Consequently, it may be that the multiple relationship between sediment transportation rate and bed shear occurs only for some intermediate range of sediment sizes under ordinary circumstances, or possibly it would occur only at unusually large shears for coarse material and at extremely small shears for fine materials.

FIELD OBSERVATIONS OF NATURAL STREAMS

Some recent observations by the Missouri River Division of the Corps of Engineers[11] (United States Department of the Army) showed how rugged and irregular the bed of a river can be. In a reach selected for sediment transportation studies on the Missouri River near Omaha, Nebr., periodic soundings of the sandy bottom showed that the character of the bed surface was quite changeable, being smooth at some times or places and rugged and irregular (much like the bed of the flume) at other times or places. A number of examples showed that the elevation of the bottom of the stream changed by as much as 6 ft or 7 ft in distances of less than 50 ft in a flow from 10 ft to 15 ft deep on the average.

Although it was not possible to make a thorough analysis of large river dunes, the evidence indicates that the size of dunes in fine sand is more or less proportional to the depth of flow. Because the dune size is relative, the Darcy-Weisbach *f*, which is a function of the relative roughness, should be used to characterize a dune configuration instead of the Manning *n*, which is directly related to the actual size of the roughness elements. If the channel roughness for the laboratory runs had been reported as the Manning *n*, the values obtained would seem small compared with values obtained for natural rivers; a bed surface covered with dunes 0.5 in. high in a flow 3 in. deep is very rough, whereas in a flow several feet deep the same absolute bed configuration would be considered relatively smooth.

For large natural rivers, Luna B. Leopold, A.M. ASCE, and Thomas Maddock,[12,13] M. ASCE, have found that the roughness, and hence the depth and velocity of flow, are greatly affected by the amount of sediment load the stream must carry. Their observations agree well with the flume data from which the same conclusion was deduced.

---

[10] "Study of Transportation of Fine Sediments by Flowing Water," by A. A. Kalinske and C. H. Hsia, *Bulletin No. 29*, Univ. of Iowa Studies in Eng., Iowa City, Iowa, 1945.

[11] "Sediment Transportation Characteristics Study," by Missouri River Div., Corps of Engrs., U. S. Dept. of the Army, First Interim Report, Appendix D, Topographic Maps, Omaha, Nebr., February, 1952.

[12] "The Hydraulic Geometry of Stream Channels and Some Physiographic Implications," by L. B. Leopold and T. Maddock, Jr., *Professional Paper 252*, Geological Survey, U. S. Dept. of the Interior, Washington, D. C., 1953.

[13] "Relation of Suspended-Sediment Concentration to Channel Scour and Fill," by L. B. Leopold and T. Maddock, Jr., *Proceedings*, 5th Hydraulics Conference, *Bulletin No. 34*, Univ. of Iowa Studies in Eng., Iowa City, Iowa, 1952, pp. 159–178.

For example, Leopold and Maddock in a figure[14] give the following information. When Hoover Dam was built on the Colorado River, the sediment load below Hoover Dam was sharply reduced because of the desilting action of the reservoir. At Yuma, Ariz., the annual discharge of suspended sediment after the dam closure was only one-tenth of what it had been previously. For a typical flow of 12,000 cu ft per sec, the depth jumped from approximately 5 ft to 9 ft, the velocity decreased from about 5 ft per sec to 3 ft per sec, and the Manning n increased from approximately 0.013 to 0.030 (which corresponds to a five-fold increase in the Darcy-Weisbach f). The slope did not change, although the stream bed elevation dropped approximately 8 ft in the first ten years after construction of the dam. This is an example in which for a given discharge and slope the stream responded quite differently when the suspended load was artificially reduced. Qualitatively, the changes which have occurred are in perfect agreement with the conclusions based on the laboratory data plotted in Fig. 7.

Leopold and Maddock suggest that the increase in roughness was an effect of the decrease in suspended load. However, it is doubtful if the effect is direct; probably the bed of the stream has become covered with irregular dunes giving increased resistance to the flow. Actually, it is fortunate that the stream can adjust its bed, change the hydraulics of the stream, and thus greatly reduce its transporting capacity. Otherwise, the degradation at Yuma would undoubtedly have been much more severe.

Another example is given by Leopold and Maddock in a figure [15,16] showing the changes in the hydraulic characteristics of the Colorado River at Grand Canyon, Ariz., occurring during the annual flood of December, 1940, to June, 1941. On the rising stage, the suspended load was very large, and the stream bed aggraded; on the falling stage the load was much less, and the bed was scoured. For two approximately equal discharges, the following data may be read from two sets of points on their curves:

	Rising stage	Falling stage
Discharge, in cubic feet per second	13,000	12,000
Suspended-load discharge, in tons per day	500,000	47,000
Depth, in feet	9.5	16.5
Velocity, in feet per second	5	2.6
Gage height, in feet	8	7.5
Stream bed elevation, in feet	−1	−9
Width, in feet	290	290

---

[14] "The Hydraulic Geometry of Stream Channels and Some Physiographic Implications," by L. B. Leopold and T. Maddock, Jr., *Professional Paper 252*, Geological Survey, U. S. Dept. of the Interior, Washington, D. C., 1953, p. 38, Fig. 28.

[15] *Ibid.*, p. 32, Fig. 23.

[16] "Relation of Suspended-Sediment Concentration to Channel Scour and Fill," by L. B. Leopold and T. Maddock, Jr., *Proceedings*, 5th Hydraulics Conference, *Bulletin No. 34*, Univ. of Iowa Studies in Eng. Iowa City, Iowa, 1952, p. 164, Fig. 4.

Presumably the slope of the river was approximately the same in both cases as the stage was not changing rapidly. Assuming a constant slope, it may readily be shown that the Darcy-Weisbach friction factor, *f*, was 6.5 times as large on the falling stage as on the rising stage and that, on the falling stage, when the transportation rate was small, the bed shear was actually much larger.

A further examination of the figure[15] of Leopold and Maddock reveals that for a given depth the discharge depends on the amount of load. For example, when the depth was 16.5 ft, the discharge on the falling stage was 12,000 cu ft per sec, as indicated in the foregoing tabulation, whereas it was 40,000 cu ft per sec by interpolation on the rising stage when the sediment discharge was large.

In this second example also, the observed changes in the Colorado River during the passage of a flood agree qualitatively with the conclusions drawn from the flume experiments. The Colorado River is an especially good example for illustrating these effects because the sediment load, carried mostly in suspension, is large and variable, and the bed is predominantly of fine sand.

On another point, however, there is an apparent contradiction between the observations of Leopold and Maddock and the flume data. From a massive collection of data on natural rivers, they conclude[17] that "* * * a wide river having a particular velocity is observed to carry a smaller suspended load than a narrow river having the same velocity and discharge." In other words, for a fixed velocity the suspended-load discharge concentration increases when the depth is increased. This is just the opposite of what has been concluded from the flume experiments where the sand size is held fixed.

However, their statement applies to streams as found in their natural condition; it does not mean then that, if any given stream were artificially made narrower and deeper, the sediment discharge would increase. Because Leopold and Maddock compare a number of different streams, the size of the bed material is a variable which is not taken into account. Perhaps it might be construed on the basis of these writers' conclusions and the results of the flume experiments that the narrow streams carry more load only because they are typically cut in finer material than the wider streams. When the bed material is the same, however, there is flume evidence to show that a wide shallow stream would carry more suspended load at a given velocity.

When the sediment size is essentially fixed, the writer's conclusion regarding the effect of depth on concentration is supported by the field observations of Raymond A. Hill,[18] M. ASCE, published in 1926 for the Colorado River at Yuma. A curve given by him shows that as the depth increases the average concentration decreases with the velocity remaining constant.

THEORIES OF SUSPENDED-LOAD DISCHARGE

From the foregoing discussion of both flume and river data, it is evident that any workable theory of suspended-sediment transportation must include a method for determining the stream roughness because it is not adequate to assume that it is constant. In fact, the use of the ordinary fixed-bed, open-channel hydraulics for alluvial streams can lead to gross errors because in any typical flow equation the roughness can change as much, if not more than, all the other variables.

---

[17] "The Hydraulic Geometry of Stream Channels and Some Physiographic Implications," by L. B. Leopold and T. Maddock, Jr., *Professional Paper 252*, Geological Survey, U. S. Dept. of the Interior, Washington, D. C., 1953, p. 24.

[18] Discussion by R. A. Hill of "Permissible Canal Velocities," by Samuel Fortier and Fred C. Scobey *Transactions*, ASCE, Vol. 89, 1926, pp. 961–964.

The bed-pickup theory advanced by Emory W. Lane, M. ASCE, and Kalinske[19] is simplified by consideration of the pickup mechanism from a flat bed surface only, whereas it has been observed in the flume that the existence of dunes greatly facilitates the suspension of bed material. Thus, a more general theory is needed which takes account of the changing bed configuration and its tremendous effect on the pickup mechanism. Furthermore, the original analysis of Lane and Kalinske, and subsequent modifications thereof,[10] were based on the common supposition that the shear could be used as an independent variable.

Einstein and Nicholas L. Barbarossa,[20] M. ASCE, have made a significant attempt to analyze river channel roughness. A basic implication of their approach also is that there is a definite stage-discharge relationship for a stream and that there will be a particular roughness associated with each point on the rating curve. This is equivalent to assuming that there is a unique relation between discharge, depth, and slope, irrespective of the sediment load. There is now strong evidence both from the field and the laboratory, indicating that this is not a basically correct physical law. It may appear true for some natural streams, inasmuch as it is not easy to isolate the effects of the various variables until something unusual happens to the stream, such as damming.

The analysis of Einstein and Barbarossa appears consistent because their basic curve for finding the channel roughness is based on stream data computed from a stage-discharge curve for each stream. Consequently, their analysis will probably be found adequate when applied to the natural streams from which it was derived but of limited help in predicting what changes would occur in the stream whose equilibrium is drastically upset by man-made works.

Einstein's basic transportation theory[9] is based on this analysis of roughness. An examination of his theory will reveal that for a given bed material there is only one equilibrium rate of flow and sediment transportation rate corresponding to each combination of bed hydraulic radius and slope. Experiments performed by this writer show that this is certainly not true for fine sand in the laboratory flume and, with some substantiating evidence from Leopold and Maddock[12,13] that this assumption is not always correct for natural rivers either. Therefore, it is believed that Einstein's basic approach to the determination of channel friction and suspended-load discharge will have to be modified.

Thomas Blench,[21] M. ASCE, in his regime theory for canals has avoided consideration of basic quantities such as the shear. Because his analysis is purely empirical, it may be that he has arrived at some results which are just as reasonable as those of other investigators who have assumed that the transportation rate of suspended sediment depended on the shear. However, it is difficult to evaluate his work because he has not reported any of the basic data. but only some of the constants in various empirical equations.

---

[19] "The Relation of Suspended to Bed Material in Rivers," by E. W. Lane and A. A. Kalinske, *Transactions*, Am. Geophysical Union, Pt. IV, Vol. 20, 1939, pp. 637–641.

[20] "River Channel Roughness," by Hans A. Einstein and Nicholas L. Barbarossa, *Transactions*, ASCE, Vol. 117, 1952, pp. 1121–1146.

[21] "Regime Theory for Self-Formed Sediment-Bearing Channels," by Thomas Blench, *ibid.* pp. 383–408.

In view of the complexity of suspended-load phenomena, it is no wonder that there has been considerable difficulty in finding a reasonable theory for the determination of the suspended-load transporting capacity of streams flowing over movable beds. This writer has no basic theory to propose and has sought only to establish some basic facts with which to evaluate existing theory. At present, there is a great need for more information on exactly what happens in such streams; without it sediment transportation theory will remain a jumble of approximate theories based on a great variety of different assumptions.

Conclusions

The principal conclusions may be summarized as follows:

1. For the laboratory flume it was found that neither the velocity nor the sediment discharge concentration could be expressed as a single-valued function of the bed shear stress, or any combination of depth and slope, or bed hydraulic radius and slope. This finding is contrary to the almost universally held assumption that the suspended-sediment transporting capacity of a stream can be uniquely related to the geometry of the stream cross section, the slope of the channel, and the size of the bed material. Two present theories,[9,19] based on this erroneous assumption, were found to be inadequate.

2. The difficulty in relating the velocity and concentration to the bed shear arises because the changeable bed configuration caused extremely large variations in the channel roughness. For the 0.10-mm sand, the Darcy-Weisbach friction factor for the bed varied from 0.019 for a run for which the velocity was high and the sand bed was swept smooth, to 0.13 for a run for which the velocity was low and the bed became covered with a stable pattern of large irregular dunes.

3. When either mean velocity and depth or water discharge and total sediment discharge are used as the pair of independent variables for the flume data, all other quantities are uniquely determined by what appears to be an orderly and logical relationship among the variables.

4. From the data obtained in the laboratory flume with 0.10-mm sand and 0.16-mm sand, the following qualitative relationships were found:

a. For constant discharge, Q, an increase in the sediment discharge, G, requires a decrease in the depth, d. Field observations indicate that this relationship is also true for some large rivers.

b. For a given slope and discharge, it was found that two depths of flow were possible. When the sediment discharge was small, the depth was large, the velocity was small, and the bed was rough, and when the sediment discharge was large, the depth was small and the bed of the laboratory channel was smooth. Field data show that this conclusion applies to natural streams as well.

c. If the water discharge, Q, is to be increased without changing the total sediment discharge, G, an increase in d would be necessary, although this increase is relatively less than in Q.

d. For a given value of Q, the largest bed friction factors are associated with the lowest values of G.

e. When the mean velocity of the stream, U, is increased, with no change in the depth, d, the bed friction factor, f_b, is decreased, and the sediment discharge concentration, $\bar{C}$, generally increases.

f. When d is increased, holding U constant, $\bar{C}$ decreases slightly, and f_b does not change appreciably in the range of conditions covered. Field evidence supports this conclusion.

g. For a given value of d and of U, f_b seemed to be numerically the same for both sand sizes used. The bed configurations also appeared to be the same.

h. For a given value of d and of U, the discharge concentration, $\bar{C}$, for the 0.10-mm sand was from two to four times as large as $\bar{C}$ for the 0.16-mm sand.

ACKNOWLEDGMENTS

The writer is grateful to Vito A. Vanoni for his supervision and generous assistance during the course of the research. The research was performed with the aid of a National Science Foundation Predoctoral Fellowship.

APPENDIX. NOTATION

The following symbols have been adopted for use in the paper and are presented herewith for the guidance of discussers:

a = distance from stream bed to point of reference for suspended-load equation (Eq. 1);
b = width of flume (rectangular channel);
$\bar{C}$ = sediment discharge concentration, or concentration of sediment in a sample of the discharge = G/Q (weight per unit volume);
c = point concentration at the distance y from the bed;
c_a = point concentration at $y = a$;
D_g = geometric mean sieve diameter;
D_s = mean sedimentation diameter;
d = mean depth of flow;
F = Froude number = $U/\sqrt{g\,d}$;
f = Darcy-Weisbach friction factor for channel = $8(U_*/U)^2$;
f_b = friction factor for bed alone, calculated from side-wall correction procedure[7] = $8\,(U_{*b}/U)^2$;
G = total sediment discharge = $\bar{C}\,Q$;
g = gravitational acceleration;
k = von Kármán universal constant;
Q = discharge;
r = hydraulic radius = $b\,d/(b + 2d)$;
r_b = hydraulic radius for bed section of flow, obtained by side-wall correction procedure[7];
S = slope of energy grade line;
T = temperature of water;

U = mean velocity = $Q/b\,d$;
U_* = shear velocity for whole channel = $\sqrt{\tau_o/\rho} = \sqrt{g\,r\,S}$;
U_{*b} = shear velocity for bed only = $\sqrt{g\,r_b\,S}$;
u_* = shear velocity for two-dimensional flow = $\sqrt{\tau_o/\rho} = \sqrt{g\,d\,S}$;
w = settling velocity of sand grains;
y = vertical distance from the bed of the stream;
z = exponent in suspended-load equation (Eq. 2);
β = ratio of turbulent-diffusion coefficients for sediment and momentum;
ρ = mass density of water;
σ_g = geometric standard deviation of sand-size distribution; and
τ_o = mean shear stress on the boundary of a channel.

DISCUSSION

Thomas Blench,[22] M. ASCE.—From the description of experiments given in the paper, the writer believes that a steady state of sediment movement was not reached and that, even if it had been reached, no quantitatively useful results could have been obtained with the suspended load and bed-load measured as one. The test for equilibrium used in the author's experiments (about which the author makes the following statement: "The two sets of profiles were practically identical, indicating that the equilibrium achieved was stable, well defined, and reproducible") seems to be no more than a test of the fact that the same conditions, applied for the same time, produce the same degree of disequilibrium. It is not surprising therefore that "the data obtained are not sufficiently extensive to permit establishing quantitative formulas." Why they should be considered sufficient to destroy two well-established theories, of which the regime theory is one, is not clear.

The writer wishes to draw attention to several misunderstandings of the regime theory: First, that it is his own, whereas pains have always been taken to attribute it to Gerald Lacey. Second, that it avoids consideration of shear, whereas, in the way Lacey presents it, $g\ d\ S$ is brought explicitly into the regime slope formula. Third, that basic data were not made available, whereas the writer's paper gives its own references[21] abounding with data. Fourth, that it is based on an erroneous assumption (in the "Conclusions") concerning suspended load, whereas it makes no assumption about suspended load and offers no means for dealing with it; it is essentially a bed-load theory. Actually, the theory consists of explaining observed facts of canal self-adjustment that were found to be represented by certain formulas; the formulas do not depend on the theory.

James R. Barton[23].—In the field of sedimentation, where controlled and reliable data are rather scarce, it is encouraging to learn of new results. The author's measurements appear to be fairly complete, and the care with which they were taken should make them reliable.

Recent studies at Colorado State University made for the Missouri River Division of the Corps of Engineers indicate that most of the author's conclusions are valid. Conclusions 4*a* and 4*d* agree very well with the findings at Colorado State University. The changing pattern of bed roughness found in these studies is also similar to that reported by the author.

The sediment used in the Colorado State University studies had a median diameter of 0.180 mm and a standard deviation of 0.037 mm. The bed roughness was found to vary systematically with the mean velocity of flow, as shown in Fig. 9 In Fig. 9, C is the Chezy coefficient. For velocities less than 1.8 ft per sec the bed was covered with dunes similar to those shown in Fig. 4. Between velocities of 1.8 ft per sec and 2.4 ft per sec, sand bars occurred on the bed, and the water surface was undulating so that uniform flow could not be established. The flume used was 4 ft wide and 70 ft long, and the sand bars were generally from 6 ft to 10 ft long with the crest of the sand bars always diagonal to the direction of flow. Each sand bar generally extended across the entire width of the flume. This sand-bar phase seems to coincide with the author's meandering bed condition. For velocities greater than 2.4 ft per sec, the sand bars were swept away and the bed became free of all dunes and ripples. This phase has been named by the writer as a plane bed as contrasted to the author's smooth bed. The term "smooth" was avoided because, although no ripples of any size existed on the bed, the sand grain roughness was sufficiently large so that the bed was not hydraulically smooth. Computation of the thickness of the laminar sublayer indicated that the median grain diameter and the laminar sublayer were approximately the same size, leaving many of the larger grains projecting above the laminar sublayer. Further increase in velocity greater than 4.0 ft per sec resulted in the formation of antidunes, although no data were gathered for the antidune condition.

---

[22] Prof. of Civ. Eng., Univ. of Alberta, Edmonton, Alberta, Canada, and Cons. Engr.

[23] Dept. of Civ. Eng., Colorado State Univ., Fort Collins, Colo.

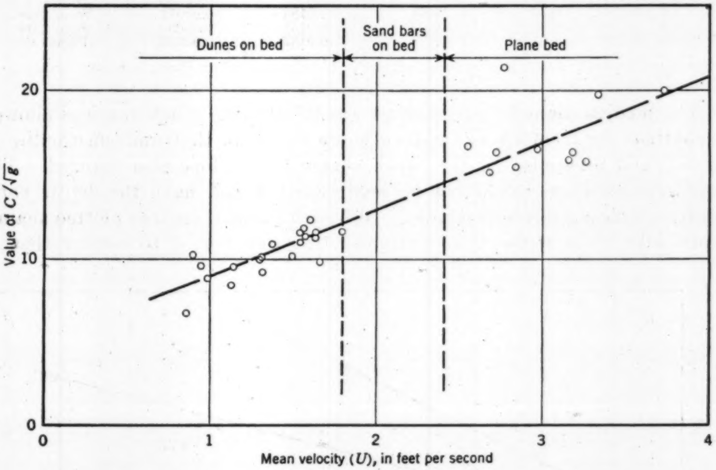


Fig. 9.—Relationship Between Mean Velocity and Roughness Coefficient

During many of the experiments, the depth did vary during the period of establishment of uniform flow. The variation of depth was often caused by nonuniform movement of the sediment through the system. However, after the flow reached a stable uniform stage, the depth also became stable.

The author seems to have assumed too much in arriving at conclusion 4*b*. In fact, the writer is somewhat confused by conclusion 4*b* after reading conclusion 3. The confusion probably arises from the fact that slope is not mentioned in 3, although it is a definite part of 4*b*, and therefore it is difficult to visualize the exact interrelationship. However, under any circumstances, data gathered at Colorado State University do not substantiate conclusion 4*b*. Although the writer will not attempt to explain conclusion 4*b*, he does feel that more data from several different flumes should be gathered and analyzed before 4*b* is accepted as a reality. Because no other experimenters have reported such a finding, it may be a characteristic of the flume used which produced such unusual results.

In the 4-ft flume at Colorado State University, recent data tend to challenge the conclusion that for a given slope and discharge two depths are possible. One series of tests is presented in Table 2.

TABLE 2.—RESULTS OF TESTS IN A 4-FT FLUME

Discharge, Q (cu ft per sec)	Mean depth, d (ft)	Mean velocity, $U = Q/A$ (ft per sec)	Manning coefficient, n	Slope, S	Bed condition
5.8	0.94	1.54	0.0216	0.00056	Dunes
5.8	0.85	1.70	0.0204	0.00068	Dunes
5.8	0.79	1.84	0.0183	0.00071	Sand bars
5.8	0.66	2.20	0.0154	0.00090	Sand bars
5.8	0.61	2.38	0.0137	0.00093	Plane bed

The median diameter of the sand was 0.180 mm. Each run was allowed to continue for from six to eighteen hours to assure that uniform conditions existed, and invariably, if the depth changed, the slope also changed. The slope and discharge could not be held constant and have the depth vary. Additional data are recorded in Fig. 10, which shows discharge plotted against depth with slope as the third variable. It seems logical to assume that, if more than one depth can exist for a given slope and discharge, the data should exhibit more scatter than the normal scatter pattern shown in Fig. 10. Some of the scatter may be explained by a variation in temperature. The temperature varied from 15° C to 23° C for curve A, with most of the runs being made at a temperature of 19° C. The extreme temperatures are shown next to the points involved. All runs for curve B were made at temperatures between 20° C and 22° C except one which was made at 27° C, as shown on the graph.

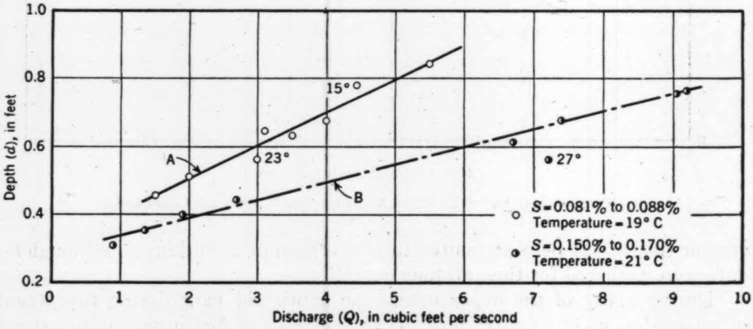


FIG. 10.—RELATIONSHIP BETWEEN DISCHARGE AND DEPTH

Observations of the experiments and an analysis of the data from the experiments made at Colorado State University both fail to support conclusion 4*b*. Therefore, the writer feels that conclusions 1 and 4*b* should not be generally accepted until further evidence, obtained from more than one experimental source, has verified the same findings as those of the author.

HANS A. EINSTEIN,[24] M. ASCE, AND NING CHIEN,[25] A.M. ASCE.—The author has presented some very interesting results and on the basis of these has expressed some doubts on the generality of existing methods in describing the roughness and sediment transport of alluvial channels. According to his findings, the transportation rate cannot always be expressed as a unique function of the bed shear stress, the channel geometry, and the properties of the sediment because of the extreme variability of the channel roughness. The modern theories on sediment transport[9,26] have fully recognized the important role played by the bar resistance in defining the alluvial channel flow. At low and medium rates of bed-load motion the bar resistance is a major part of the total frictional resistance of fine sand beds, and yet the shear transmitted to the bed through the shape resistance of the sand bars has only a minor effect on the bed-load transport. Therefore, the sediment transport has been related only to the other part of the bed shear—namely, the grain resistance of the bed material. Thus, the conventional methods permit unique determination for most flows of any two of the following four variables—unit discharge, unit sediment load, depth, and slope—if the other two are known.[27] Brooks has claimed that, although the depth and slope of the alluvial channel are uniquely determined by the discharge and the sediment load, the reverse is not true. This is due to the fact that, for the same depth, slope, and bed material, in one case the bar resistance may be large and the sediment load and discharge small, and in another case the bed may become quite smooth and be able to carry a larger flow and sediment load. This discussion attempts to reassess the author's findings as follows:

1. It will be shown that the behavior of bar resistance in the author's experiments differs from the findings of past river observation, and it is precisely this difference which results in the complicated flow pattern as observed in the author's flume.
2. It will be shown that some of the phenomena which the author has observed in his flume, and which he introduces as new and as contradictory to past experience, are actually in full agreement with previous theories and are believed to apply to rivers within very limited ranges of flow and sediment conditions.
3. It will be shown that the field observations quoted by Brooks as proof of his findings do not fit the case that he intended to illustrate and can actually be fully explained by the past methods of description.

---

[24] Prof. of Hydr. Eng., Univ. of California, Berkeley, Calif.

[25] Associate Prof. of Hydraulics, Tsing Hwa Univ., Peiping, China.

[26] "Formulas for Bed-Load Transport," by E. Meyer-Peter and R. Müller, International Assn. for Hydr. Research, 2d Meeting, Stockholm, 1948, pp. 39–65.

[27] "Graphic Design of Alluvial Channels," by Ning Chien, *Transactions*, ASCE, Vol. 121, 1956, p. 1267.

4. An attempt will be made to explain the deviations of Brooks' results from the river data by an interpretation of his and other similar results in the form of a bar-resistance graph as proposed by the senior writer and Barbarossa.[20]

In 1951 a paper[20] was published indicating that the resistance of a movable bed can be divided into two additive parts, the first of which is related to the roughness of the surface, as defined by the representative grain size in the bed. It may be considered as the roughness obtained if the grains are arranged in a perfectly regular manner so as to create a perfectly flat, rough surface. An additional frictional effect is obtained if the same particles are arranged in a more irregular fashion—in such a way that they protrude irregularly from an average bed surface or they actually arrange themselves in the form of bars or other irregularities. This latter part of the friction was shown in the paper to be a function of the bed-load transport rate, which itself is a function of a parameter, ψ'. The functional relationship between U/u_*'' and ψ' is shown in Fig. 11 by curve N; U is the average velocity and $u_*'' = \sqrt{S\, r_b''\, g}$ is the shear velocity corresponding to the bar resistance. The author's data are analyzed in terms of the same parameters and are shown in Fig. 11 as open circles for the case, $D_s = 0.16$ mm. It can be seen that at low stages the bars behave the same way in the author's flume as they do in rivers. At higher flows the experimental points begin to deviate systematically from the natural-stream curve and line up along a higher curve, indicating that sand bars disappear sooner than in natural streams. It will be shown subsequently that, if the bars behave this way in the author's flume, the flow relationships become extremely complicated indeed.

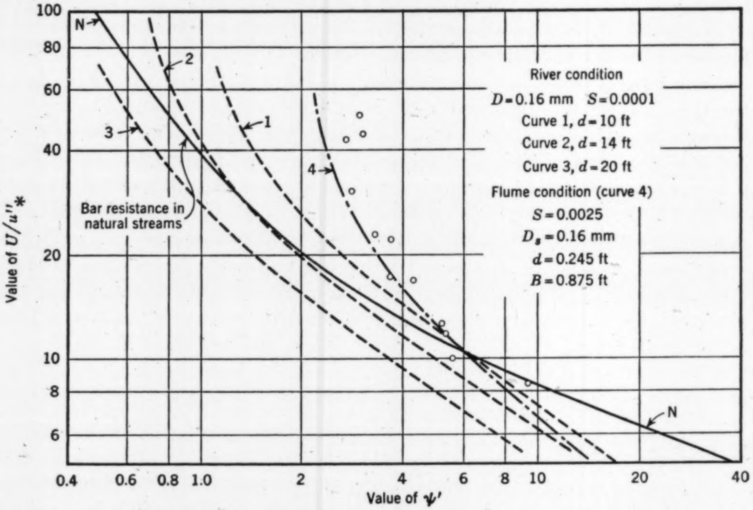


FIG. 11.—BAR RESISTANCE FOR NATURAL STREAMS, FOR BROOKS' EXPERIMENTS (CIRCLES), AND FOR FOUR HYPOTHETICAL CASES IN WHICH THE SAME DEPTH, SLOPE, AND BED MATERIAL ARE ASSUMED TO BE ABLE TO CARRY DIFFERENT FLOWS AND LOADS

In order to demonstrate whether different flows and sediment loads can be carried by the same channel at equal slope, depth, and bed material, the subsequent argument will be conducted on the basis of curves which show the various aspects of the relationship between the bar resistance and the flow intensity. Three hypothetical curves, 1, 2, and 3, were introduced in Fig. 11, each giving all possible combinations of ψ' and U/u_*'' for a given combination of bed-material size, slope, and depth, assuming no relationship to exist between ψ' and U/u_*''. No bank friction is introduced in this derivation. Curve 1 is obtained as follows: For each point of the curve, the depth is arbitrarily divided into r_b' and r_b''; for example, $r_b' = 6$ ft and $r_b'' = 4$ ft. Now ψ' can be computed as

$$\psi' = \frac{\rho_s - \rho_f}{\rho_f} \frac{D_{35}}{S\, r_b'} \dots\dots\dots\dots\dots\dots\dots\dots (3)$$

and U/u_*'' is obtained, using the logarithmic friction formula (or any other such formula) for U, as

$$\frac{U}{u_*''} = \frac{5.75 \sqrt{r_b'} \log (12.27\, x\, r_b'/D)}{\sqrt{r_b''}} \dots\dots\dots\dots\dots (4)$$

in which ρ_s is the density of the solids, ρ_f that of the fluid, and x is a parameter defining the transition between friction on smooth and rough boundaries. The three curves represent all possible combinations of r_b' and r_b'' for the three depths. If curve N applied to these flows, one may conclude that its intersection with the three curves gives the actual solutions. One would read thus from Fig. 11 that the 10-ft depth has only one solution with $U/u_*'' = 10.3$, a well-defined bar condition. Curve 3 does not intersect curve N, indicating that such an intersection must occur at a very high value of U/u_*''—with the bed flat and the bar resistance negligible. Curve 2, however, follows curve N for a short distance, indicating that any one of these points is a possible solution. This is a case in which various discharges and sediment rates are possible at the same combination of depth, slope, and grain size. Here, the old prediction method[20] shows the exact behavior which Brooks claims to have newly discovered. To explain this point further, the stage-discharge curve for this particular channel has been constructed according to methods previously published[20] and is shown in Fig. 12. The bar resistance is the dominating part of the total frictional resistance at low stages, resulting in a slow increase of discharge with the rise of stage, as shown by part AB of the rating curve. After the depth reaches 13.5 ft, a further increase in flow actually makes the bed smoother. That means that an increase in r_b' is accompanied by a decrease in r_b''. The changes in both hydraulic radii are of the same order of magnitude, resulting in an almost constant water depth of 14 ft in that range of flows (part BC of Fig. 12). At still higher stages, the bar resistance becomes insignificant, and the discharge increases rapidly with the stage, as shown by part CD of the rating curve. It is immediately evident that any attempt to correlate the measured discharge and sediment load with the channel characteristics in range BC of the flows will become a hopeless task, considering the uncertainties and errors involved in field measurements. The existence of such uncertainty in predicting discharge and sediment load in very limited ranges of conditions was known previously, but not much significance was attached to this fact because of the rather rare occurrences of these cases. Thus, the statement that more than one solution of discharge and bed-load rate is possible for a given depth, slope, and grain size does not in itself prove that the old approach of determining channel roughness is not applicable to this case.

For a better understanding of Brooks' findings, curve 4, which is similar to curves 1 to 3, was drawn in Fig. 11 for the conditions existing in the first part of his experiments. It represents a hypothetical bar-resistance curve if various flows and loads are possible in the flume at a depth of 0.245 ft and at a slope of 0.0025. Over a large range of conditions the line follows the experimental points, justifying the author's claim that in his experiments the mean velocity and sediment concentration were not uniquely determined by the channel and flow geometry. A further analysis of the experimental data reveals that the results are even more complicated. If the same depth and slope can carry different discharges, the question arises as to whether the same discharge can be carried by different depths at a given slope. To ascertain this point several hypothetical bar-resistance curves are shown in Fig. 13 for both natural streams and conditions existing in the author's experiment. Each of these lines represents, in this case, the bar-resistance relationship if the same discharge can be carried by different depths at the same slope. For natural streams such a condition is not possible because each of the hypothetical lines intercepts the actual bar-resistance curve only at one point, and yet it is quite possible in the author's flume because part of the hypothetical line again goes through several experimental points. Indeed, if the author's flume study had been conducted in such a way that a rating curve with reasonable range could have been obtained, it would have followed one of several possibilities, as shown diagrammatically in

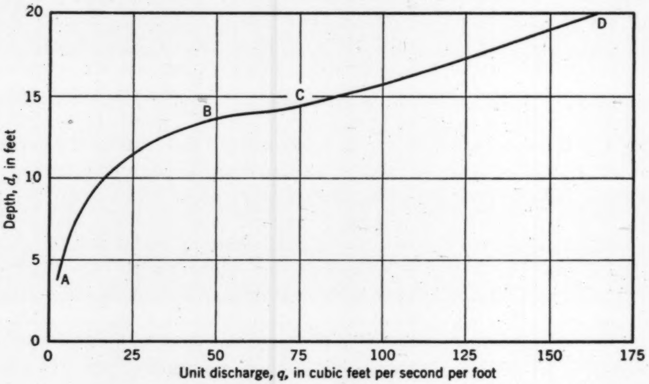


FIG. 12.—RATING CURVE OF A CHANNEL WITH A BED-MATERIAL SIZE OF 0.16 MM AND A SLOPE OF 0.0001, COMPUTED BY THE EINSTEIN-BARBAROSSA METHOD

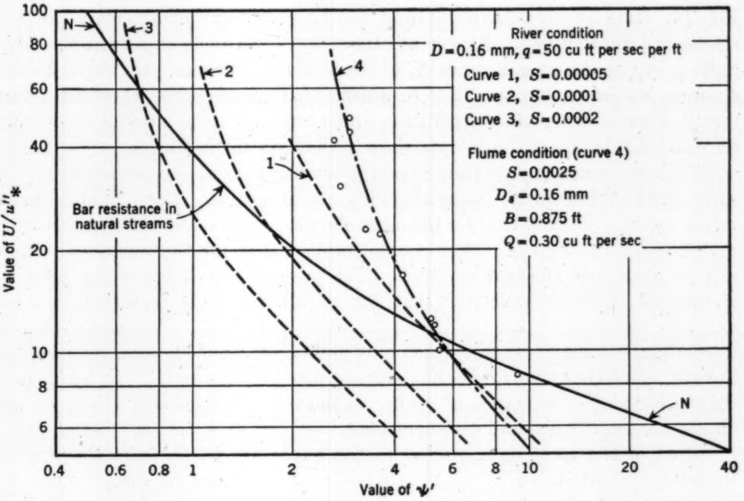


FIG. 13.—BAR RESISTANCE FOR NATURAL STREAMS, FOR BROOKS' EXPERIMENTS, (CIRCLES), AND FOR FOUR HYPOTHETICAL CASES IN WHICH THE SAME DISCHARGE IS ASSUMED TO BE CARRIED AT DIFFERENT DEPTHS WITH THE BED MATERIAL AND SLOPE GIVEN

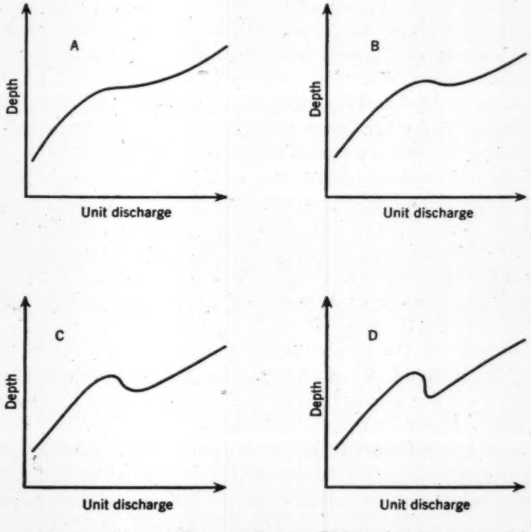

A
Depth
Unit discharge
B
Depth
Unit discharge
C
Depth
Unit discharge
D
Depth
Unit discharge

FIG. 14.—VARIOUS POSSIBLE RATING CURVES ACCORDING TO THE RESULTS OF BROOKS' FLUME STUDIES

Fig. 14. It may even have followed one type of curve at a certain range of stages and another type of curve at other stages. Among those various possibilities, in one extreme is curve A, which is also sometimes possible in natural streams, as shown by Fig. 12. In another extreme is curve D, which occurs rarely, if ever, in nature. Operating under such intricate circumstances, it is no wonder that the author's data show a wide scatter in Fig. 5.

The author has quoted from several river observations to demonstrate that what he has found in his flume also happens in nature. These field measurements seem to indicate that for the same discharge and slope, the depth changes with the sediment load. Brooks also implied that for these examples there was no change in the bed material. The present knowledge of the behavior of alluvial channels indicates that the depth and unit sediment load are uniquely determined by the slope and unit discharge. The apparent contradiction between these two statements must be reconciled either by accepting the argument that rivers do behave as suggested by the author, or by showing that either slope, discharge, or grain size actually changed sufficiently to produce a changed depth and sediment load.

Most of the river data quoted in the paper are taken from the work of Leopold and Maddock[12,13] in which the characteristics of the channel, the discharge, and the suspended load are correlated quantitatively. Fully recognizing the difficulties involved in sampling the bed load, the analysis, nevertheless, suffered by the fact that the sediment load includes not only bed-material load which controls the stability of the channel. It has been demonstrated[28] conclusively that under the same flow conditions the transport rate of wash-load may change drastically, depending on the availability of the material from the upstream watershed. During the dry period of the year much of the sedimentary material is made available in the watershed by weathering or other processes. The first rain or snow melt erodes these loose materials accumulated at the ground surface, bringing a large amount of sediment into the stream channels. After the loose material is removed from the surface, the erosion caused by the surface runoff is drastically reduced. This accounts for the fact that in the Colorado River at Grand Canyon the suspended-sediment load, in which the wash-load constitutes the major part, is ten times larger in the rising stage than it is in the falling stage at a comparable discharge. Indeed, even if the change in suspended load actually reflects a change in bed-material load, the river will not respond to this change by filling up or scouring the bed to a depth of as much as 8 ft in a matter of days. What actually occurred in the Colorado River at Grand Canyon for that particular occasion was a transient change of the local river bed during a flood, as stated[29] by Lane and Whitney M. Borland, A. M. ASCE. It should not be interpreted as an adjustment of river regimen in response to the change of independent variables imposed on the channel by the watershed.

The example of the Colorado River at Yuma, on the other hand, does reflect a change in river regimen after the completion of a series of dams (Hoover Dam in 1935, Parker Dam and Imperial Dam in 1938) along the upstream channel.

---

[28] "Can the Rate of Wash Load Be Predicted from the Bed-Load Function?" by H. A. Einstein and Ning Chien, *Transactions*, Am. Geophysical Union, 1953, pp. 876–882.

[29] "River-Bed Scour During Floods," by E. W. Lane and W. M. Borland, *Transactions*, ASCE, Vol. 119, 1954, pp. 1069–1089.

The data indicate that, for the same slope and for a typical flow of 12,000 cu ft per sec, the depth jumped from approximately 5 ft to 9 ft, the velocity decreased from approximately 5 ft per sec to 3 ft per sec, and the Manning *n* increased from 0.013 to 0.030, due to the reduction of incoming sediment load. The pertinent point in interpreting the data revolves around the question of whether there was a change in bed-material size caused by the upstream dams. Immediately after the closure of a dam, the clear water released by the dam will erode the downstream channel, causing a rapid degradation immediately below the dam.[30] The sediment removed from the bed by clear water consists, to a large extent, of fine particles, causing the bed material to become gradually coarser. As the bed degrades, gravel strata or clay lenses which were covered before become exposed to the flow. These two types of material may actually prevent excessive degradation. A detailed mapping of the bed material of the Missouri River downstream from Fort Peck Dam in Montana[31] indicates the exposure of such gravel lenses and clay strata which are scattered along the channel. However, at the places where the underlying layers are not exposed, the change of the bed-material size may remain small. Measurements made in the Colorado River[32] also indicate that the bed-material size increased from 0.175 mm to 33 mm at a distance of 12.8 miles below Hoover Dam, and from 0.18 mm to 7.0 mm at a distance 17.4 miles below Parker Dam. Both changes took place in a period of thirteen years. The measuring section at Yuma is located 15.8 miles downstream of Imperial Dam. Between January, 1940, and September, 1951, the total volume of bed material removed between Imperial Dam and mile 15.6 is estimated to be 60,000,000 cu yd. One should expect that with the degradation of the channel bed a coarsening of bed material in the downstream channel took place, even if Imperial Dam is of much smaller capacity than Hoover Dam or Parker Dam. A survey of the bed-material samples taken at Yuma[32] indicates that the size varies from year to year in a rather unsystematic manner. The following extreme values were selected from the data:

D_{50} = 0.125 mm in March 24, 1940
0.140 mm in November 8, 1951
0.320 mm in January 12, 1946
0.390 mm in April 13, 1947

and hydraulic computations[9] have been made for two cases, taking the representative bed-material size as 0.14 mm and 0.35 mm, respectively. Table 3 gives the result of the computations. The agreement between the measured and the computed values is excellent. It is rather surprising that a small change in bed-material size results in such a significant change in Manning's *n*. This is caused by the fact that as the bed material becomes coarser, the same flow is able to move a much smaller load, resulting in a much larger bed undulation.

---

[30] "Retrogression on the Lower Colorado River After 1935," by J. W. Stanley, *Transactions*, ASCE, Vol. 116, 1951, pp. 943–957.

[31] "Degradation Observations of Missouri River Downstream from Fort Peck Dam," Fort Peck Dist., Missouri River Div., Corps of Engrs., U. S. Dept. of the Army, Omaha, Nebr., June, 1951.

[32] "River Control Work and Investigations, Lower Colorado River Basin, Calendar Years 1950 and 1951," Office of River Control, Bureau of Reclamation, U. S. Dept. of the Interior, Washington, D. C., 1952.

The soundings in the Missouri River near Omaha, Nebr., as cited by the author, revealed that the channel bed is deeper and rougher at one side and shallower and smoother at the other. From June to August, 1951, when the Missouri River was flowing at a constant discharge of about 65,000 cu ft per sec, a large number of bed samples were taken by the Corps of Engineers, the mechanical analysis of which indicated that the average bed-material size is 0.15 mm in the flat area and 0.24 mm in the rough area. The erratic behavior of the Missouri River bed can be explained, at least qualitatively, by the difference in the material which composes that part of the bed.

TABLE 3.—CHANGE IN REGIMEN OF COLORADO RIVER AT YUMA AFTER THE CLOSURE OF IMPERIAL DAM

Time, with respect to the closure of the dam	DATA USED IN THE COMPUTATIONS				COMPARISON BETWEEN COMPUTED (C) AND MEASURED (M) VALUES					
	Q (cu ft per sec)	b (ft)	S (ft per mile)	D_s (mm)	d (ft)		U (ft per sec)		Manning n	
					C	M	C	M	C	M
Before........	12,000	480	1.27	0.14	5.06	5.00	4.91	5.20	0.0139	0.0125
After.........	12,000	430	1.50	0.35	8.36	8.70	3.37	3.10	0.0306	0.0310

Knowing that the behaviors of natural streams can be fully explained by the conventional methods without resorting to any drastic new interpretations, as proposed by the author, one must ask why the observations made by the author in his flume have never before been revealed. The writers, after reading the author's voluminous thesis,[6] are very much impressed by his carefulness and thoroughness in conducting the experiments. It is extremely unlikely that the data may contain any serious errors in measurement or in interpretation. In order to find an explanation of the results, a survey of available data on other flume studies was made and showed that there exist at least two other sets of measurements in which fine sand with comparable flows were used. The first one was conducted by the senior writer in 1944–1946 at the California Institute

TABLE 4.—RANGE OF VARIABLES USED IN THREE FLUME STUDIES

Investigator	FLUME		SEDIMENT			FLOW		
	Width (ft)	Length (ft)	D (mm)	S_o[a]	Transport rate (lb per sec per ft)	d (ft)	U (ft per sec)	S
Brooks..........	0.875	40	0.16 0.10	1.075 1.110	0.003 to 0.075 0.0025 to 0.15	0.15 to 0.30 0.17 to 0.28	0.93 to 2.15 0.83 to 2.13	0.0018 to 0.0035 0.0013 to 0.0033
Barton and Lin...	4	70	0.18	1.17	0.0006 to 0.45	0.30 to 1.38	0.90 to 3.74	0.00044 to 0.0021
Einstein.........	0.875	40	0.106 to 0.120	1.19 to 1.22	0.115 to 1.34	0.43 to 0.44	1.76 to 3.13	0.0017 to 0.0039

[a] S_o = sorting coefficient = $\sqrt{D_{75}/D_{25}}$ = unity for a uniform material.

of Technology, at Pasadena, and a more recent one was published by Barton and Pin-Nam Lin[33] at Colorado State University. The range of the main variables used in these studies is listed in Table 4. The data are analyzed again in terms of the bar resistance, U/u_*'', and the flow intensity, ψ', and are plotted in Fig. 15. Most interesting is the fact that the results of each flume study are consistent in themselves and yet deviate from the river curve and from the other curves. In both the experiments of Brooks and Barton and Lin the sand bars disappear sooner than those in nature. However, this happens at different flow intensities for the different series. Proceeding from low to high flows, the beginning of deviation from the river curve follows the following sequence:

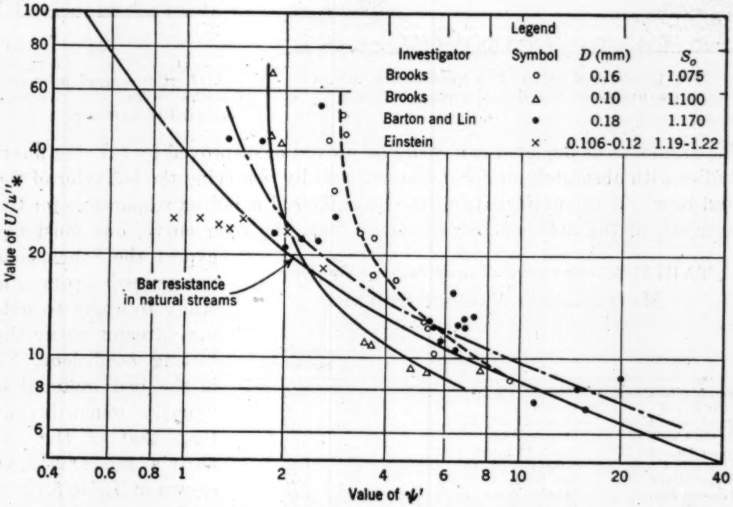


FIG. 15.—SAND-BAR RESISTANCE IN FLUMES COMPARED WITH THAT IN NATURAL STREAMS

Brooks................$D = 0.16$ mm; $S_o = 1.075$
Brooks................$D = 0.10$ mm; $S_o = 1.11$
Barton and Lin........$D = 0.18$ mm; $S_o = 1.17$

The results obtained by the senior writer, although limited in range, fall more or less along the river curve even at comparatively small values of ψ' and have a sorting coefficient, $S_o = 1.19$ to 1.22. The difference in the behavior of the bar resistance in these studies must reflect in one way or another the difference in conditions employed in these experiments. As far as the writers are able to ascertain (Table 4), the only significant parameter seems to be the sorting coefficient of the bed material. Up to a sorting coefficient of 1.2, the more uniform the bed material is, the less persistent are the sand bars. Such a conclusion is not altogether unexpected. Local segregation of bed material takes place if the gradation is large. Fig. 16 is a sketch of such a bar. At the crest of a sand bar where the velocity is higher, the material is usually finer; the coarsest material is found at the trough. The changes in velocity and grain size seem to compensate each other, resulting in a more uniform shear over the bed than in the case of uniform bed material. Under such circumstances the sand bars are naturally more stable for mixtures with higher sorting coefficients.

---

[33] "A Study of the Sediment Transport in Alluvial Channels," by James R. Barton and Pin-Nam Lin, *Report No. 55JRB2*, Colorado State Univ., Fort Collins, Colo., 1955.

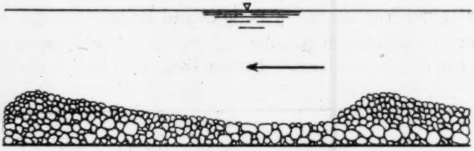


FIG. 16.—SKETCH OF THE BED SEGREGATION IF THE GRADATION OF THE BED MATERIAL IS LARGE

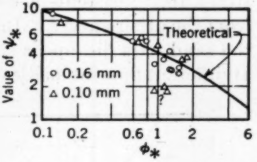


FIG. 17.—COMPARISON OF BROOKS' RESULTS WITH THE WRITERS' ($\phi_* - \psi_*$)-RELATIONSHIP

The argument as depicted can easily be proved or disproved by running flume studies with absolutely uniform material and by observing the behavior of the sand bars. If the uniformity of the bed material is indeed responsible for the deviation of the author's bar resistance from the river curve, one must not expect the conclusions determined from his study to apply to natural streams where the sorting coefficient, S_o, of the bed material is usually much larger than that of the author's material, as shown in Table 5.

TABLE 5.—SORTING COEFFICIENTS OF BED MATERIAL FOR VARIOUS STREAMS

Stream	Location	Sorting coefficient
Brandywine Creek	Georgia, Pa.	3.02
Waar River	Holland	2.48
Brandywine Creek	Chades Ford, Pa.	2.08
Cherry Creek	Denver, Colo.	1.73
Upper Danube	Gÿor, Hungary	1.73
Mountain Creek	Greenville, S. C.	1.57
Salinas River	San Lucas, Calif.	1.57
Missouri River	Omaha, Nebr.	1.56
Rhine River	Brugger Bridge[a]	1.53
Colorado River	Yuma, Ariz.	1.49
Rio Grande River	Bernardo, N. Mex.	1.47
Mississippi River	Near Old River[b]	1.45
Rio Grande River	Bernalillo, N. Mex.	1.43
Atchafalaya River	Barbre Landing-Krotz Springs	1.42
Middle Loup River	Dunning, Nebr.	1.40
Boise River	Notus, Idaho	1.37
Niobrara River	Cody, Nebr.	1.36
Rio Grande River	San Acacia, N. Mex.	1.36
Missouri River	Fort Randall	1.33
Bear River	Western Bridge, Utah	1.28
Bear River	Trenton Bridge, Utah	1.24
Fraser River	Canada	1.24
Tiber River	Rome, Italy	1.23
Mississippi River	Baton Rouge, La.	1.23
Atchafalaya River	Simmesport-Melville	1.17

[a] 2 miles upstream from Lake of Constance. [b] 300 miles upstream from Head of Passes.

In conclusion, the writers wish to present the relationship of ϕ_* and ψ_* derived from the author's experiments as compared with the theoretical curve proposed by the senior writer[9] (Fig. 17). In spite of the great variability of the channel roughness in the author's experiment, the agreement is quite satisfactory except for the results of three runs. This again indicates that the concept in linking the bed-load transport with the grain resistance of only the bed material is basically sound.

Acknowledgment.—The analysis made in preparing this discussion represents part of the research conducted for the Corps of Engineers, Missouri River Division, at Omaha, under contract with the University of California.

Enos J. Carlson,[34] M. ASCE, and **M. Gamal Mostafa**,[35] A. M. ASCE.—Some of the conclusions drawn by Brooks from his limited experimental data seem to contradict well-established theories of sediment hydraulics.

Several of the runs gave higher mean velocities or higher sediment transport for lower boundary shears, and from these runs the author concluded that it is impossible to take the shear or the slope as independent variables. What these runs may actually show is the known fact that when there is sediment in transport, the rate of energy dissipation does not depend only on the rate of flow, water section, and sediment size, but also on the rate of sediment motion and the mode of bed formation, both of which are, in turn, dependent on the hydraulic conditions and sediment characteristics.

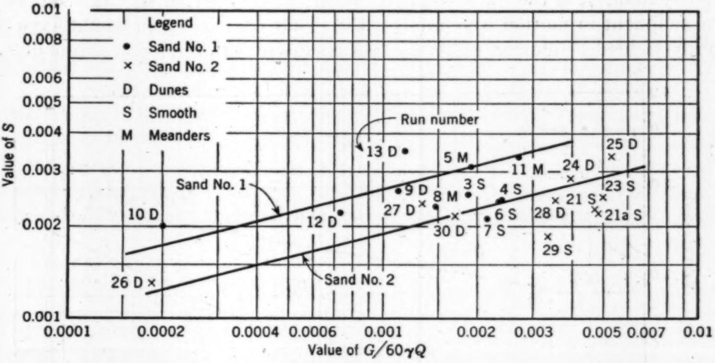


Fig. 18.—Effect of Slope on the Ratio of Sediment to Flow Discharge

In an attempt to analyze the presented data, two points of interest are raised:

1. The total weight of sand used in the flume was 145 lb, which is extremely small, giving a depth of sediment on the flume bottom of approximately ½ in. The author mentions the possibility of that being the reason that fully developed dunes did not change appreciably in height. This could mean that for some runs, due to the shortage of sand, the friction was forced to be less than what would naturally occur if the sand were 2 in. or 3 in. deep. Ten of the twenty-two runs given in the data are reported to have shown sand dunes. For some, if not all, of these runs, one cannot be certain about the validity of deriving any conclusions as regards the relation of discharge to depth and slope because the friction was more or less restricted.

2. The total weight of sand in motion was approximately 0% to 5% of the total weight of sand in the flume. The sand particles in the bed were not of uniform size but had a certain gradation, as shown in Fig. 3. The particles that were picked up by the flow, particularly after dunes had formed, were certainly the smallest in size and, therefore, should not have been rep-

---

[34] Hydr. Engr., Hydr. Lab., Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo.

[35] Associate Director of Works, Hydr. Research Station, Delta Barrage, Egypt.

resented by the mean sedimentation size of the bed. Determining the characteristics of the sediment in motion in each run by analyzing the dry sand from the samples that were taken for determining concentration would probably show an increase in mean size with an increase in sediment load.

However, the author's data should still show some trend if plotted nondimensionally (except for the runs in which friction may have been restricted by the exposure of a part of the flume bottom) because all of the variables of flow, rate of transport of sediment, and bed formation, and the characteristics of both the sediment in motion and in the bed should be interdependent. If $G/60\,\gamma\,Q$ is plotted against slope (Fig. 18) different functions for different sand beds would be expected; however, the data have a wide scatter. The symbol, γ, is the unit weight of water, 62.4 lb per cu ft. If a sample of the sediment in motion were analyzed in every run, each run would have certainly yielded a different mean size, always lower than the mean sedimentation size, because the amount of sediment in motion is relatively very small and would necessarily be finer than the average bed material. The data plotted in Figs. 18 and 19 would probably fit a group of lines for different sediment sizes if the actual size of the sediment in motion were known in each case.

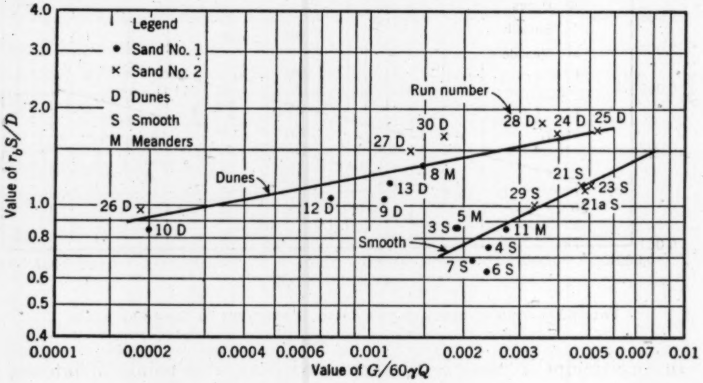


FIG. 19.—NONDIMENSIONAL PLOT SHOWING THE TREND OF DUNES AND SMOOTH BEDS

In Fig. 19, $(r_b\,S)/D$ is plotted against $G/60\,\gamma\,Q$, both being nondimensional ratios. All the runs giving dunes follow a certain function, whereas those giving a smooth bed follow another function, with the runs having "meanders" appearing in both functions. It follows that for the same hydraulic conditions as given by r_b, S, and Q two different ratios of sediment to water discharge might have been produced, one with a smooth bed and the other with a dune formation. However it is doubtful that this could have actually resulted if every run was carried to a sediment equilibrium condition under a uniform flow of water.

If a run was started with a small depth of flow and then the depth was gradually increased, dunes which form at small depths might not have had enough time to be removed. Also, if a run was started with a large depth of water and a smooth bed and then the depth was gradually decreased, dunes might not have had enough time to develop. Nonuniformity of the flow instability of the flow conditions, or nonequilibrium of sediment discharges —or any of these factors combined—would necessarily produce errors that might have seriously affected the results, especially in such a narrow flume and within such a small range of slope and depth variation.

Hsin-Kuan Liu,[36] A.M. ASCE.—The author has made some very interesting observations and has drawn certain conclusions which, if proved to be true, are very important contributions to the study of sediment transport.

The method of determining the depth of flow used by the author is questionable although no better method is known. The true, effective depth of flow is not known because of the excessive dune movement which affects the velocity distribution and the channel roughness. Whether Brooks let the suspended material deposit or let the flow wash the bed when he stopped the flow was not mentioned. It seems that either one of these two possibilities must have occurred because the flow was stopped. Consequently, the measured depth could not accurately represent the true depth of flow. A high concentration of suspended load would result in heavy deposition, thereby reducing the depth of flow more than a low concentration of suspended load would.

The author limited the depth of flow to a maximum of 0.3 ft because of the narrow flume. Such a small depth, in comparison with the large height of the sand dunes (for example, 0.3 ft is the flow depth and 0.12 ft is the height of sand dunes), makes one wonder if the average depth, as determined by the author, had any significance. In a flume 40 ft long by 10 in. wide, 145 lb of sand seems to be inadequate. Because it was possible for the maximum dune height to reach 1.5 in., the ½-in. sand layer could be entirely removed in some places, leaving the bottom uncovered. How much this could affect the mechanics of sediment transport, caused by the change of boundary conditions in such a case, is uncertain, and therefore the question arises as to the justification of applying some of the author's conclusions to other investigators' data.

As no bed-load sample was taken, it is doubtful that "* * * the layer of sand covering the bottom of the flume contained practically all the sand in the system," as stated by the author under the heading, "Apparatus and Procedure: Tilting Flume." The composition of the bed material may also affect the mechanism of sediment transportation.

When the flow is in equilibrium, the depth of flow and the pattern and distribution of sand on the bed remain constant, and the energy gradient stays constant and equal to the water-surface slope and bed-surface slope. The formula,

$$\tau_o = \gamma \, r_b \, S \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which τ_o is the shear along the bed, γ is the specific weight of the fluid, r_b is the hydraulic radius of the bed, and S is the slope of the bed, is true only for steady, uniform flow conditions. Fig. 2 indicates that the flow was not uniform and that the bed did not have a constant slope. For the same average depth of flow in a reach, a converging flow acts differently from a diverging flow, neither of which can be represented by Eq. 5.

---

[36] Asst. Prof., Dept. of Civ. Eng., Colorado State Univ., Fort Collins, Colo.

According to the writer's experience, if discharge and sediment load are used as independent variables in two-dimensional flow, and depth, slope, and bed roughness as dependent variables, the following relationships exist for a given sediment:

$$d = \phi_1 (q, q_s) \qquad (6)$$

$$S = \phi_2 (q, q_s) \qquad (7)$$

and

$$f_b = \phi_3 (q, q_s) \qquad (8)$$

in which q is the unit flow discharge; q_s denotes the unit sediment discharge; and ϕ_1, ϕ_2, and ϕ_3 indicate functions. Eqs. 6, 7, and 8 are related through the use of the continuity equation and open-channel flow formula,

$$q = U\,d = \sqrt{\frac{8\,g}{f_b}}\,d^{3/2}\,S^{1/2} \qquad (9)$$

Therefore, only two of the three equations need to be determined.

In the case of the natural river the independent variables are q and S instead of q and q_s because the bed slope is usually approximately constant. As the river discharge changes, the flow depth, channel roughness, and bed-material load also change; that is,

$$d = \phi_4 (q, S) \qquad (10a)$$

$$f_b = \phi_5 (q, S) \qquad (10b)$$

and

$$q_s = \phi_6 (q, S) \qquad (10c)$$

If Eq. 10*b* can be found, Eq. 10*a* can be replaced by Eq. 9, which is known. Consequently, for an alluvial stream there are primarily two difficult problems to be solved—namely, Eq. 10*b*, the law of channel roughness, and Eq. 10*c*, the law of sediment transport.

As stated under the heading, "Apparatus and Procedure: Establishing Uniform Flow in Equilibrium; Reproducibility," Brooks found that "* * * for some depth-slope combinations there are two, or maybe even more, different equilibrium discharges and sediment loads." Under the heading, "Conclusions," in paragraph 4*b*, it was stated that "for a given slope and discharge, it was found that two depths of flow were possible." In his examination of results, the author stated that it is impossible to take slope as an independent variable. However, it is generally known that the slope is an independent variable in most natural alluvial streams because as the flow discharge changes, the flow depth changes more readily than the slope does. According to the author's statement, "for a given slope and discharge, it was found that two depths of flow were possible"; this means that if depth is plotted against discharge (known as a rating curve) for a constant slope, the rating curve is not a single-valued curve, or such a curve may not even be possible. The foregoing conclusion based on the author's findings is, as a rule, contrary to engineering experience. Further research of using the slope as an independent variable is therefore needed. It is suggested that the study be conducted in a complete recirculatory flume system. The slope of the water surface should be maintained at a constant value, and a series of tests should be made by varying the discharge and measuring the corresponding normal depth and the corresponding total sediment. A rating curve covering a wide range of discharge against the normal depth and a sediment rating curve covering a wide range of discharge and sediment load can be helpful in understanding this problem.

Brooks' procedure of using d and S as independent variables to find the corresponding Q-value seems to be inadvisable to the writer. Although it is not impossible, it is very time consuming. For example, when the flow has a rigid boundary, it is easier to determine the normal depth of flow by letting a certain discharge pass through a flume having a constant, uniform slope than it is to find the corresponding discharge for any predetermined normal depth and slope. It is even more difficult and time consuming to adjust the flume slope experimentally for a given discharge and normal depth. However, when the experimental relationship,

$$d = \phi' \, (q, S, \text{boundary}) \qquad \text{(11)}$$

is found for a flow having a rigid boundary, any two variables, such as d and S or d and q, can be used as independent variables to find the third one. For the same reason, the inadvisability of choosing d and S as independent variables to find discharge prevails for flow over an alluvial bed. This might explain why for given values of d and S, the author found that there were two or more equilibrium discharges.

His finding in this case is in contradiction to his statement (in conclusion *a* under the heading, "Analysis of Experimental Results: Depth and Velocity As Independent Variables") that "* * * f_b and $\bar{C}$ are uniquely determined by U and d * * *," which means that the slope is also uniquely determined by U and d by using Eq. 9. The question then arises as to how Brooks could obtain two different equilibrium discharges by maintaining the same slope and depth because the slope is self-adjustable to the discharge and depth of flow. The writer is concerned about whether Brooks waited long enough for true equilibrium conditions to be reached especially because it was difficult to do.

The author explained that it is possible to have two (or even more) different equilibrium discharges and sediment loads because of variations in the channel roughness caused by dunes. Such an explanation is doubtful because sand dunes do not constitute an independent variable. Although the determination of their characteristics and channel roughness is still far from satisfactory, they must depend uniquely on flow conditions. For a given flow condition, if it is assumed that there exists a unique value of channel roughness, it is impossible to have more than one equilibrium discharge.

Another possible explanation for the strange condition found by the author might have been caused by the particular experimental setup used. It is noticeable that run 3 and run 9 did not have the same water surface condition, which may have affected the flow condition for a flow having a depth of 0.245 ft. More data from other independent tests are needed to verify the author's conclusions before they can be accepted.

The use of field data to verify laboratory conclusions needs careful examination because, in general, the conditions are quite different. The sediment in a natural stream is seldom uniform in size distribution, and consequently the sorting mechanism might be important. That the degradation rate slows down below a dam as the years pass is caused in part by the increase in bed-material size by sorting.

Depth and velocity were favored by the author to be used as independent variables instead of depth and slope, yet in some cases the author could not vary these two independent variables (in conclusion *f* under the heading, "Analysis of Experimental Results: Depth and Velocity As Independent Variables"). This proves that independent variables are chosen with a view to facilitating the experimentation, and not because the variables are in themselves independent or dependent.

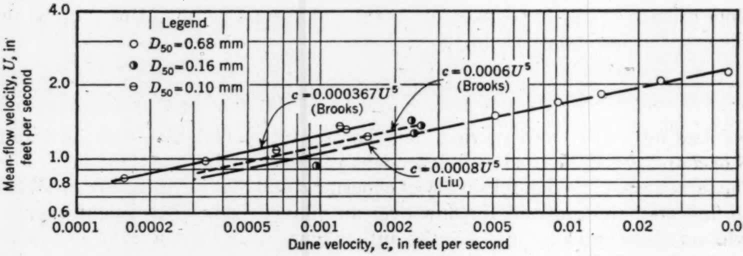


FIG. 20.—RELATIONSHIP BETWEEN DUNE VELOCITY AND MEAN FLOW VELOCITY

From the writer's experiments on Ottawa sand[37] (between U. S. Standard sieves No. 20 and No. 30)bed-load movement is usually in the form of sand wave motion because ripples or sand dunes appeared on the bed shortly after the beginning of the bed-load movement. The author's statement (under the heading, "Experimental Results") that "* * * several layers of sand grains were almost always sliding along the bed" is therefore doubtful. In conclusion 4*d* the author stated: "For a given value of *Q*, the largest bed friction factors are associated with the lowest values of *G*." This statement is valid only after the formation of dunes. Before sand dunes appear, the sediment transport rate is extremely small, the bed is plane, and the bed roughness can be expressed by Manning's n, which is proportional to $D_{50}^{1/6}$. The corresponding value of f_b is usually smaller than it is after the formation of sand dunes. Therefore, at this stage it is not true that the largest f_b-values are associated with the smallest sediment transportation rate.

---

[37] Discussion by Hsin-Kuan Liu of "The Present Status of Research on Sediment Transport," by Ning Chien, *Transactions*, ASCE, Vol. 121, 1956, p. 877.

The dune velocity (c) measured by the author[6] is plotted against the mean flow velocity (U) in Fig. 20. Data[6] of $D_{50} = 0.10$ mm indicate a relationship between dune velocity and mean flow velocity that is similar to the writer's experiments.[37] The writer's data result in $c = 0.0008\ U^5$ for $D_{50} = 0.69$ mm; the author's data result in $c = 0.000367\ U^5$ for $D_{50} = 0.10$ mm; and an approximate equation can be written for $D_{50} = 0.16$ mm as $c = 0.0006\ U^5$.

In conclusion, the writer wishes to mention that the author's conclusions 1, 2, 4*b*, and 4*d* seem to be doubtful.

Pin-Nam Lin[38].—The principal issue raised in the paper is whether there is only one equilibrium rate of flow and sediment transportation rate corresponding to each combination of bed hydraulic radius and slope. This statement should be read with the understanding that, among other factors, a given temperature, channel, and sediment are implied; otherwise the statement will be trivial. On the basis of Fig. 5, the author concluded that more than one equilibrium condition may exist for given values of depth and slope. He specifically cited run 3 and run 9 to support his contention. However, Table 6, prepared from the author's data, shows that runs 3 and 9 were made at different temperatures. The effect of temperature may be clearly seen by comparing runs 3 and 7. The slope of run 3, when corrected for the effect of water temperature (by linear interpolation), might be as low as 0.0023, and one would have been tempted to dismiss Brooks' argument. Inspection of Table 5, however, shows that, when corrected for temperature, run 3 could have practically the same values of d and S as run 8. From Fig. 5 it is difficult to determine whether there are any other runs similar to runs 3 and 8, except for possibly run 21 ($U = 2.10$ and $\bar{C} = 4.85$) and run 27 ($U = 0.99$ and $\bar{C} = 1.35$). Therefore, it appears that not more than four runs plainly support the author's contention. Of these four runs, run 8 and run 27 were for the case of dunes on the bed, and run 8 involves a "meandering" bed which is not really in equilibrium.

TABLE 6.—Analysis of Brooks' Data

Run No.	U (ft per sec)	$\bar{C}$ (g per liter)	d (ft)	S	Temperature (°C)
3	2.04	1.95	0.243	0.0025	22
7	2.04	2.15	0.243	0.0021	31.5
8	1.75	1.5	0.24	0.0023	27.5
9	1.35	1.1	0.245	0.0026	27.5

In view of the large errors normally encountered in the experimental study of sediment transport, it is not safe to draw firm conclusions—even if only qualitative—on the basis of a few runs. A better procedure to arrive at any conclusion would be to search for a method by which the data may be presented to indicate general trends. Such trends, if existing, would then be representative of the entire group of data. Any conclusion drawn on the basis of these general trends of behavior would be more significant than that drawn on the basis of a few runs. Considerations of the effect of density gradient on vertical mixing have led the writer to the use of the following family of dimensionless factors in the analysis of sediment data: $\frac{w\,\bar{C}}{U\,\mathsf{F}^2}$; $\frac{w\,\bar{C}_p}{U\,\mathsf{F}^2}$; $\frac{w\,\bar{C}}{U_*\,S}$; and $\frac{w\,\bar{C}_p}{U_*\,S}$, in which $\bar{C}_p$ is the sediment concentration at a chosen point. The detailed reasoning leading to the formulation of such parameters has been given in a report by Barton and the writer.[33] When the slopes of the author's data presented in Fig. 5 are plotted against $w\,\bar{C}/U\,\mathsf{F}^2$ (Fig. 21), orderly trends in the data can be seen.

---

[38] Dept. of Concrete and Hydraulics, Corbett, Tinghir & Co., New York, N. Y.

As shown in Fig. 21, the data for each size of sand may be represented by two curves that have a tendency to converge or intersect, as evidenced by the data for 0.16-mm sand. Thus, it appears that for each size of sand there exists a critical value of $W\,\bar{C}/U\,\mathsf{F}^2$ or S, above which the data for each sand are definitely represented by two curves.

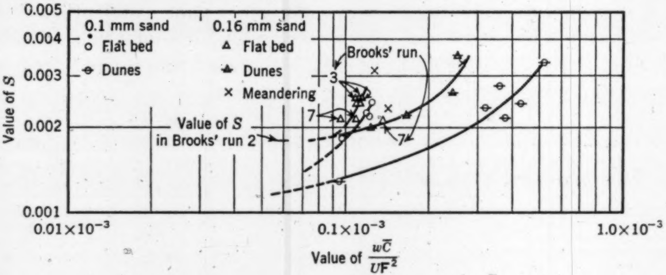


FIG. 21.—A PLOT OF BROOKS' DATA

In terms of symbols, the following functions of the most essential and mutually independent variables may be written as

$$S = f_1(U, d, \mu, \rho, g, w) \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

and

$$\bar{C} = f_2(U, d, \mu, \rho, g, w) \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which μ and ρ are the dynamic viscosity and density of water, respectively, and g is the gravitational acceleration. From Eq. 11 and Eq. 12,

$$S = f_3(U, d, \rho, \bar{C}, g, w) \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Because the function, f_3, is to be determined by Fig. 21, the uniqueness or multiplicity of the function, f_3, is determined by the uniqueness or multiplicity of the relationship between S and $w\,\bar{C}_p/U\,\mathsf{F}^2$. According to Fig. 21, f_3 is unique only for a given material and a given type of bed configuration. Thus, in the range of the author's data, f_3 is not unique, no matter which six of the pertinent variables are chosen as independent. In other words, for the same size of sand, there will be two forms of f_3, one each for the case of flat bed and the case of a bed covered with dunes, respectively. To be more specific, one may write,

$$S = f_4\left(\frac{w\,\bar{C}}{U\,\mathsf{F}^2}, \frac{w}{U}, \text{bed configuration}\right) \ldots\ldots\ldots\ldots (14)$$

This is so because a horizontal or vertical line drawn in the region of branching curves may have two intersections with the curves for the same size of sediment. Thus, in this region of branching curves, the author's conclusion 1, with due modification to take other essential variables into account, is supported by the author's own data. Conclusion 3 is not compatible with his data.

Another important point made by the author is that the transportation rate could not be expressed as a unique function of the bed shear stress, the channel geometry, and properties of sands. A similar finding was published by E. Meyer-Peter and R. Müller[26] in 1948. They found that an experimental formula, relating the rate of sediment transport and the shearing stress for the case of a flat bed, always predicted too high a rate of transport when applied to a bed covered with dunes. Although the study was primarily intended for the so-called bed-load transport, the fundamental mechanism, as far as the rate of sediment transport is concerned, should be comparable. From a practical point of view the problem of uniqueness here is perhaps not so important as the problem of existence of any function for the bed-material load discharge expressed in terms of bed shear stress and other pertinent variables. The study by Meyer-Peter and Müller of bed-load transport furnishes definite proof that such functional relationships do exist. Data from Colorado State University[39] for 0.18-mm sand show a definite functional relationship between $\bar{C}$ and $U/\sqrt{\tau_o/\rho}$ in approximately two-dimensional flow carrying heavy suspended load (τ_o is the bed shear stress). With the aid of Brooks' data, it now appears that two relationships between $\bar{C}$ and $U/\sqrt{\tau_o/\rho}$ may be obtained.

According to Fig. 21, the author's data indicate that for each size of sediment and each type of bed configuration,

$$S = f_5\left(\frac{w\,\bar{C}}{U\,\mathsf{F}^2}\right) \ldots\ldots\ldots\ldots (15)$$

In a study by Barton and the writer[40] it is shown that, for each type of bed configuration, one may expect that

$$\frac{U}{U_*} = f_6\left(\frac{w\,\bar{C}}{U\,\mathsf{F}^2}\right) \ldots\ldots\ldots\ldots (16)$$

An interesting deduction then is

$$\frac{U}{U_*} = f_7\,(S, \text{bed configuration}) \ldots\ldots\ldots\ldots (17)$$

---

[39] "A Study of the Sediment Transport in Alluvial Channels," by J. R. Barton and P. N. Lin, Dept. of Civ. Eng., Colorado State Univ., Fort Collins, Colo., 1955, Fig. 20.

[40] *Ibid.*, Fig. 15.

The writer regards Eq. 17 as indicating a very interesting property of the resistance coefficient of an alluvial channel and believes that further investigations of these simplifying expressions may prove worth while.

Acknowledgment.—In the course of discussion, an error in the original version of Fig. 21 was noted by the author. As a result, the figure and parts of the discussion as originally submitted have been modified.

NORMAN H. BROOKS,[41] J.M. ASCE.—The writer certainly welcomes the vigorous discussion that has been presented. The critical nature of the discussions has demonstrated that his findings and ideas are not generally recognized and accepted by various engineers.

In presenting this material the writer realized that the scope of the experimental investigation was far too limited to derive general quantitative relationships between the various characteristics of alluvial streams that carry an appreciable quantity of bed material in suspension. The data were presented in as simple a form as possible, and no numerical or dimensional analysis was attempted. In fact, it was felt that it would be helpful for the profession to examine the basic data in order to determine some of the general interrelationships. Before dimensional analysis can be successfully applied to any problem, one should know which variables are pertinent and which functional relationships it would be most worth while to seek.

The writer does not claim to have discovered that dunes affect channel roughness; this fact has been known for a long time. However, the writer disagrees with the generally accepted assumption that the total roughness can be uniquely determined from the grain-size distribution of the bed material, the water cross section, and the slope (or the bed shear). For example, even though Einstein and Barbarossa[20] separate the bed shear into two parts—one for the actual grain roughness and the other for the roughness of the dunes or bars—they still imply that the friction factor and discharge can be determined uniquely from the variables mentioned.

Several of the discussers seemed to have misinterpreted the meaning of uniqueness or single-valuedness of a functional relationship. As an example, consider the simple relationship, $y = \sin x$. Certainly y is a unique function of x because for any value of x there is only one possible value of y. However, if $x = \sin^{-1} y$, the value of x is not uniquely determined by the value of y. Similarly, the observation that the slope is a unique function of the depth and the velocity (or discharge) does not mean that velocity must, therefore, be a unique function of the depth and the slope. As in the example described, several combinations of depth and velocity may yield identical values of the slope. In this case, if the consideration begins with a given depth and slope, it is reasonable to expect that there may be more than one possible velocity. Obviously, in analyzing problems in alluvial hydraulics it would be helpful to have relationships which are unique so that one will not be troubled with a multiplicity of solutions. Therefore, it is suggested that one should avoid thinking of depth and slope as independent variables because of the possible multiple-valued nature of the relationships which result.

---

[41] Asst. Prof. of Civ. Eng., California Inst. of Technology, Pasadena, Calif.

The organization of this closing discussion is similar to that of the original paper. Questions regarding the apparatus and procedure will be answered first and some pertinent new data will be presented. The main part of this discussion will be concerned with the discussers' interpretations of the laboratory data. Finally points relating to field observations will be presented.

Apparatus and Procedure.—Blench and Carlson and Mostafa doubted that true equilibrium (for a run) had been established in the flume when the data were taken. During the runs a check of the stability was made by determining the water-surface profiles frequently. After the initial period required to reach equilibrium—which sometimes was as short as one-half hour—the water surface elevations never changed significantly. Furthermore, complete water surface profiles and bed surface profiles taken after running for different lengths of time showed identical results. This, then, would rule out the possibility suggested by Blench that "* * * the same conditions, applied for the same time, produce the same degree of disequilibrium." Equilibrium for both the water and sediment discharge was achieved for each run, with the possible exceptions of runs 12 and 24, for which there were long, flat sand waves, and both flat and dune-covered sections occurred simultaneously in the flume.

Carlson and Mostafa also questioned the regulation of the depth of flow during the establishment of a run. As all the experiments were performed in a closed-circuit flume (Fig. 1), the average depth was governed solely by the quantity of water put into the system. Thus, from the very beginning of a run, the depth was necessarily maintained at a correct average value, although there was a possibility for a slight, transient nonuniformity.

Liu has questioned the uniformity of flow conditions represented by Fig. 2 for run 27. By inspection it is found that the maximum variation of the depth is ±0.003 ft, with an average value of 0.231 ft. In the bed elevation, the deviations are from +0.003 ft to −0.002 ft (after leveling in 4-ft reaches), and the fluctuations in the water surface readings are even less. These variations are exceedingly small for flume experiments, and the large number of water surface readings taken for each run should guarantee that the deviations are no larger. Furthermore, considering that the slope S in Eq. 5 should properly be the slope of the energy grade line, instead of the bed slope as specified by Liu, it is apparent that the slight variations in bed and water surface elevations are of no consequence.

In regard to a further question by Liu concerning the procedure for stopping the flume, it may be stated that the pump was stopped abruptly. At this instant a large wave was formed at the downstream end, which traveled upstream to the inlet. The passage of this large wave displaced only a relatively few grains of sand off the dune crests and into the troughs. Hence, the effect of the disturbance due to the stopping of the flow is considered to be nil as far as the mean depth of the sand bed is concerned. It should be remembered that the system was closed so that there was no drainage of water off the bed at the time that a run was stopped.

When the flow was stopped, the sediment in suspension settled to the bed, thereby necessitating a correction in the total depth, as pointed out by Liu. The depth of deposition of the suspended sediment was computed and found to

TABLE 7.—SUMMARY OF EXPERIMENTS BY

Run No.	Discharge, Q (cu ft per sec)	Depth, d (ft)	Hydraulic radius, r (ft)	Slope, S	Shear velocity, U_* (ft per sec)	Average velocity, U (ft per sec)	Friction factor, f	Water temperature, T (°C)	Bed hydraulic radius, r_b (ft)
SERIES I, SAND No. 3									
A	0.435	0.241	0.156	0.0021	0.103	2.06	0.020	25.0	0.166
B	0.269	0.242	0.156	0.0027	0.116	1.27	0.068	25.0	0.187
C	0.193	0.241	0.156	0.0021	0.103	0.91	0.101	25.0	0.218
SERIES III, SAND No. 4									
H-2	0.436	0.233	0.152	0.0025	0.111	2.13	0.0215	24.3	0.167
H-3	0.387	0.223	0.148	0.00225	0.104	1.97	0.022	24.0	0.162
H-7	0.293	0.237	0.154	0.00275	0.117	1.41	0.054	24.1	0.199
H-7a	0.293	0.243	0.156	0.00275	0.118	1.38	0.059	24.6	0.210
SERIES II, SAND No. 5									
2b	0.170	0.241	0.156	0.0020	0.100	0.80	0.124	26.0	0.220
2a	0.180	0.241	0.156	0.0021	0.102	0.85	0.115	25.6	0.219
2	0.193	0.241	0.156	0.0024	0.110	0.91	0.115	25.5	0.220
3a	0.207	0.241	0.156	0.0026	0.114	0.98	0.108	25.0	0.219
3	0.219	0.241	0.156	0.00275	0.118	1.04	0.103	25.0	0.220
4	0.253	0.241	0.156	0.0027	0.116	1.20	0.075	25.0	0.214
5	0.292	0.241	0.156	0.0024	0.109	1.38	0.050	25.0	0.203
6	0.327	0.241	0.156	0.00225	0.106	1.55	0.038	25.0	0.195
7	0.358	0.241	0.156	0.0021	0.102	1.69	0.029	25.0	0.185
8	0.387	0.241	0.156	0.0020	0.100	1.83	0.024	25.0	0.176
1	0.435	0.241	0.156	0.00225	0.106	2.06	0.0215	25.0	0.171
9	0.561	0.241	0.156	0.0039	0.140	2.66	0.022	25.0	0.177

[a] Data for run H-3 apply only to the flow over the flat section of the sand wave, which covered major

be less than 0.001 ft in all but two or three cases, for which corrections of 0.001 ft were made.

The writer concurs with Carlson and Mostafa and Liu in the opinion that it would have been advisable to use more sand in the flume. Under some conditions small bare spots did appear on the bottom. However, had there been more sand in the flume, the roughness for some of the runs with dunes might have been even higher, tending thereby to enforce the writer's conclusions rather than to weaken them. In subsequent experiments more sand has been used, yet these experiments lead to the same general conclusions.

Barton has raised a valid criticism of the writer's use of the term "smooth." Because smooth already has a special meaning in hydrodynamics, the use of "plane" or "flat" would have been better to describe the condition of the bed in the absence of dunes. The writer prefers the term "flat" because it does not imply the preciseness of "plane" and is already commonly used in describing natural topography of a large scale.

New Experimental Results.—The experimental runs reported in the paper were performed during 1953–1954. Since then, further investigations along these same lines have been conducted by Vanoni and the writer. Some of the results of this research are presented in Tables 7 and 8.

NOMICOS IN THE 10.5-IN. FLUME (1956)

Bed shear velocity, U_{*b} (ft per sec)	Bed friction factor, f_b	Number of sediment discharge samples	Sediment discharge concentration, $\bar{C}$ (g per liter)	Total sediment discharge, G (lb per min)	ANALYSIS OF SEDIMENT LOAD		Froude number, **F**	Bed condition
					D_g (mm)	σ_g		
(D_g = 0.145 MM, σ_g = 1.30)								
0.106	0.021	9	1.85	3.0	..	..	0.74	Flat
0.136	0.092	10	1.2	1.2	..	..	0.45	Dunes
0.121	0.141	5	0.23	0.17	..	..	0.33	Dunes
(D_g = 0.137 MM, σ_g = 1.38)								
0.116	0.0235	7	2.3	3.8	0.155	1.35	0.78	Flat
0.108	0.024	12	3.3	4.7	0.096	1.60	0.73	Flat[a]
0.133	0.071	12	2.0	2.2	0.093	1.56	0.51	Sand wave[b]
0.136	0.078	..	..	..	..	..	0.49	Dunes
(D_g = 0.152 MM, σ_g = 1.76)								
0.119	0.175	6	0.30	0.19	0.070	1.7	0.29	Dunes
0.122	0.162	6	0.59	0.40	0.072	1.9	0.31	Dunes
0.130	0.163	12	0.82	0.59	0.069	2.1	0.33	Dunes
0.136	0.153	7	1.15	0.88	0.065	1.7	0.35	Dunes
0.140	0.145	7	1.8	1.5	0.061	1.9	0.37	Dunes
0.136	0.103	7	2.5	2.4	0.062	1.6	0.43	Dunes
0.125	0.065	14	3.4	3.8	0.064	1.7	0.50	Dunes
0.119	0.047	14	2.9	3.5	0.065	1.7	0.55	Sand wave[b]
0.112	0.035	14	3.3	4.4	0.071	1.8	0.61	Sand wave[b]
0.106	0.027	7	3.2	4.7	0.073	1.8	0.66	Flat
0.112	0.0235	6	3.4	5.6	0.085	2.3	0.74	Flat
0.149	0.025	7	5.6	12	0.163	1.7	0.95	Flat

part of flume. [b] Data for runs H-7, 6, and 7 are composites for flat and dune-covered sections together.

The scope of the original investigation has been expanded in essentially two ways by (1) using sands with a wider gradation of grain sizes and (2) making similar experiments in a wider and longer flume. The distribution of sieve sizes for the three new sands used, designated Nos. 3, 4, and 5, respectively, are shown in Fig. 22. The characteristics of the size distribution of all five sands are summarized in Table 9. Although the geometric mean sieve diameters (D_g) for sand Nos. 3, 4, and 5 are close to that for sand No. 1, the geometric standard deviations (σ_g) are considerably larger. Values of the mean sedimentation diameter are presented in Table 9 only for sand Nos. 1 and 2 for which σ_g was small. For the other sands the spread in the grain sizes is so large that a mean sedimentation diameter has little meaning.

Sand Nos. 3 and 4 were obtained from a foundry supply company and were not further processed except for washing. Sand No. 5 was a synthetic preparation designed to give a logarithmically normal size distribution with $D_g = 0.15$ mm and $\sigma_g = 1.8$. Under microscopic analysis, sand No. 4 was found to contain more than 99% quartz grains of subrounded shape. The other sands are believed to have been similar in composition.

Because the apparatus and procedure used in the more recent experiments were practically identical with those previously used, only the changes will be

TABLE 8.—SUMMARY OF EXPERIMENTS BY VANONI

Run No.	Discharge, Q (cu ft per sec)	Depth, d (ft)	Hydraulic radius, r (ft)	Slope, S	Shear velocity, U_* (ft per sec)	Average velocity, U (ft per sec)	Friction factor, f	Water temperature, T (°C)	Bed hydraulic radius, r_b (ft)
SAND No. 4, $D_g = 0.137$ MM, $\sigma_g = 1.38$;									
2-9	0.510	0.238	0.203	0.00141	0.096	0.77	0.124	23.4	0.230
2-3	0.615	0.243	0.207	0.00204	0.117	0.90	0.133	24.5	0.238
2-8	0.715	0.240	0.205	0.00280	0.136	1.07	0.129	25.2	0.233
2-1	0.855	0.240	0.205	0.00278	0.135	1.28	0.090	25.5	0.231
2-7	0.930	0.237	0.203	0.00277	0.134	1.40	0.074	22.4	0.227
2-6	1.00	0.249	0.211	0.00246	0.129	1.44	0.064	27.4	0.236
2-17D[a]	1.17	0.302	0.248	0.00201	0.127	1.39	0.067	18.9	0.284
2-17F[b]	1.17	0.203	0.177	0.00276	0.125	2.07	0.029	18.9	0.186
2-2	1.38	0.233	0.200	0.00205	0.115	2.13	0.0235	23.5	0.208
SAND No. 4, $D_g = 0.137$ MM, $\sigma_g = 1.38$;									
2-12	1.21	0.541	0.390	0.00039	0.070	0.80	0.061	24.6	0.488
2-5	1.54	0.528	0.383	0.00070	0.092	1.04	0.063	23.4	0.480
2-10	1.87	0.549	0.394	0.00105	0.116	1.22	0.072	21.9	0.505
2-11	2.23	0.536	0.387	0.00122	0.123	1.49	0.055	25.2	0.485
2-13D[a]	2.65	0.553	0.396	0.00102	0.119	1.72	0.038	20.7	0.482
2-16F[b]	3.50	0.524	0.381	0.00107	0.115	2.39	0.0185	16.5	0.408
2-4	3.84	0.544	0.391	0.00107	0.116	2.53	0.0170	24.9	0.416

[a] Dune section in runs with a long sand wave. [b] Flat section

noted herein. The experiments listed in Table 7 were performed by George Nomicos in the 10.5-in. flume. Prior to his work, a new venturi meter with a 3-in. throat was installed just upstream of the transparent tube (Fig. 1). The total-load sampler was also improved by using a smaller tube at the bottom of the loop to prevent any possibility of sand storage in the sampler.

The procedure followed by Nomicos in making his experiments was practically identical to that followed by the writer. At least 300 lb of sand were used, or enough to cover the bed to a depth of approximately 1 in.—which was twice as deep as in the writer's experiments. In presenting his results in those cases where a long, flat sand wave appeared, Nomicos did not report separate data for both the rough and the smooth section but gave only composite averages for the entire flume (except for run H-3, as noted in Table 7).

The runs listed in Table 8 were performed by Vanoni and the writer in a flume, 33.5 in. wide and 60 ft long. Hydraulically, it is the flume that Vanoni[5] used in his early experiments on transportation of suspended sediment, but structurally it has been rebuilt to facilitate the adjustment of slope. Schematically it is similar to the 10.5-in. flume. This structural modification has greatly facilitated making experiments of the type presented herein because the slope can be easily adjusted even with the water flowing.

The procedure used in the larger flume was again practically the same as that previously reported on. One important difference was that there was no temperature control. Even though the variation of temperature during each run was slight, owing to the large volume of water in the system, the run tempera-

AND BROOKS IN THE 33.5–IN. FLUME (1956)

Bed shear velocity, U_{*b} (ft per sec)	Bed friction factor, f_b	Number of sediment discharge samples	Sediment discharge concentration, $\overline{C}$ (g per liter)	Total sediment discharge, G (lb per min)	ANALYSIS OF SEDIMENT LOAD		Froude number, F	Bed condition
					D_g (mm)	σ_g		
DEPTH RANGE: 0.203 TO 0.302 FT								
0.102	0.140	20	0.037	0.071	0.086	1.44	0.28	Dunes
0.125	0.153	13	0.24	0.54	0.094	1.53	0.32	Dunes
0.145	0.147	16	1.15	3.1	0.096	1.49	0.38	Dunes
0.144	0.101	12	1.9	6.0	0.086	1.56	0.46	Dunes
0.142	0.083	20	2.2	7.6	0.086	1.54	0.51	Dunes
0.137	0.072	8	1.4	5.3	0.086	1.64	0.51	Dunes[e]
0.136	0.077	4	2.2	9.8	0.092	1.49	0.44	Dunes[e]
0.133	0.031	8	3.0	13	0.097	1.46	0.81	Flat[e]
0.117	0.024	22	2.5	13	0.095	1.60	0.78	Flat[e]
DEPTH RANGE: 0.524 TO 0.553 FT								
0.078	0.076	16	0.0033	0.015	0.097	1.66	0.19	Dunes
0.104	0.079	16	0.068	0.39	0.085	1.60	0.25	Dunes
0.131	0.092	16	0.21	1.5	0.084	1.49	0.29	Dunes
0.138	0.069	16	0.67	5.6	0.078	1.52	0.36	Dunes
0.126	0.046	4	1.45	14	0.088	1.50	0.41	Dunes[e]
0.119	0.0195	..	..	..	..	..	0.59	Flat[e]
0.120	0.0180	16	1.15	16.5	0.124	1.45	0.60	Flat

in runs with a long sand wave. [e] Sand wave in system.

tures ranged from 16.5°C to 27.4°C. The flume was charged with 2,500 lb of sand No. 4 which was enough to cover the entire bed to a depth of 0.15 ft or nearly 2 in. With this depth of sand there was no difficulty with bare spots on the bed except in extraordinary cases, such as in front of a large sand bar or sand wave. As with the 10.5-in. flume, the sediment samples were withdrawn from the vertical section of pipe a few feet above the pump. Three different samplers were used having two, three, or four sampling tips connected to a manifold, the choice of samplers used depending on the total rate of flow.

The arrangement of the data in Tables 7 and 8 is identical to that of Table 1 except that the column entitled "Water surface condition" has been replaced by two columns giving the geometric mean sieve diameter (D_g) and the geometric standard deviation (σ_g) for the composite of the sediment discharge samples for each run. As before, the size distributions were determined by dry sieve analysis. The value of D_g was practically identical to the median diameter (D_{50}) because the sand sizes were very nearly logarithmically normally distributed. In cases where there is a slight difference between the two quantities, D_g has been determined by the method suggested by George H. Otto.[42]

The terminology used in Tables 7 and 8 to describe the bed condition is the same as used previously except that the word "flat" is used in place of "smooth," in line with Barton's suggestion. The term "sand wave" is used for the condition (described under the heading, "Experimental Results") in which the sand spreads itself nonuniformly in the flume, creating a thick flat section in one part and a thinner dune section in the other. Barton has used the term "sand bar" to describe the same phenomenon observed in his flume studies. A typical front of a sand wave is shown for run 2–17 in Fig. 23. In the direction of flow this is the point of transition from flat bed (small depth) to dune-covered bed (larger depth); the transition from dunes to flat bed is always gradual. As Barton pointed out, the sand front was always observed to be diagonal to the flow for some unknown reason. For some of the runs in the large flume, several secondary sand fronts or sand bars were observed in addition to the main one, and usually followed behind it.

---

[42] "A Modified Logarithmic Probability Graph for Interpretation of Mechanical Analysis of Sediment," by George H. Otto, *Journal of Sedimentary Petrology*, Vol. 9, No. 2, 1939, pp. 62–76.

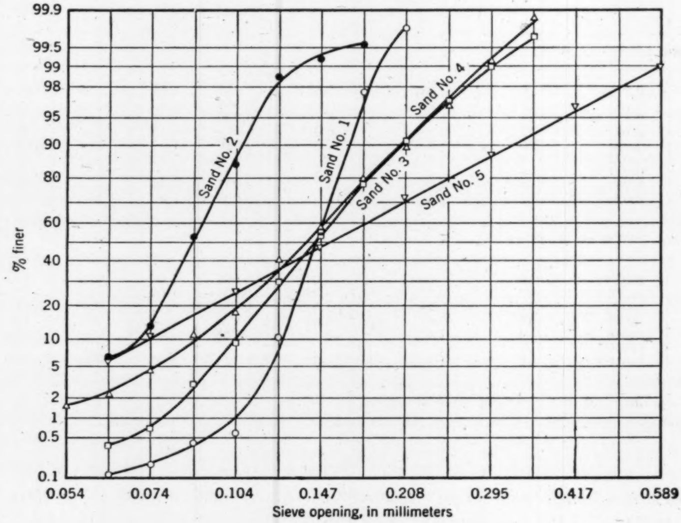


FIG. 22.—SIEVE ANALYSES OF SANDS

TABLE 9.—SUMMARY OF SAND SIZE DISTRIBUTIONS

Sand No.	D_g (mm)[a]	Geometric Standard Deviation,[b] σ_g	D_{35} (mm)[c]	D_{65} (mm)[d]	D_s (mm)
1	0.145	1.11	0.140	0.151	0.16
2	0.088	1.17	0.084	0.094	0.10
3	0.145	1.30	0.130	0.159	..
4	0.137	1.38	0.123	0.155	..
5	0.152	1.76	0.123	0.191	..

[a] $D_g = \sqrt{D_{84} D_{16}}$. [b] $\sigma_g = \sqrt{D_{84}/D_{16}}$. [c] 35% finer. [d] 65% finer.

The data in Tables 7 and 8 are presented graphically in Figs. 24 through 27 in a manner slightly different from that used previously. All the experiments made by Nomicos were at the constant depth of 0.24 ft, and all the runs made by Vanoni and the writer were intended to have depths of 0.24 ft or 0.54 ft. With the depth thus limited to two selected values, more attention could be focused on exactly what changes take place as the velocity increases with depth constant. In effect, a horizontal cross section is being taken through Fig. 6(*a*).

Fig. 24 shows how the slope (S), the shear velocity for the bed (U_{*b}), and the bed friction factor (f_b) changed with velocity for a nominal depth of 0.24 ft for runs made with sand Nos. 1, 3, and 4 in either of the two flumes. It should be noted that runs 3, 8, 9, and 10 of the original data are also included for comparison with the new data. It is noteworthy that the points from three different investigations in the two different-sized flumes on three sands with different geometric standard deviations may easily be fitted with common curves for S and f_b.

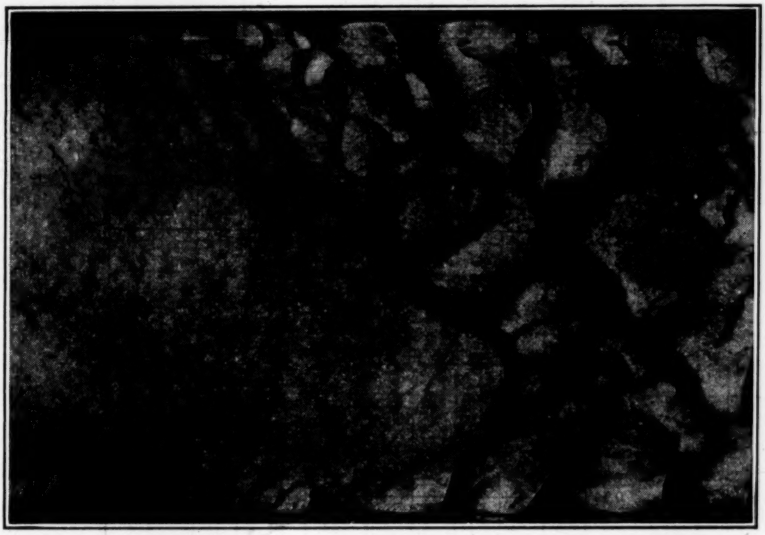


Fig. 23.—Typical Front of a Sand Wave (Run 2-17, Full Width of 33.5-In. Flume, Downstream to the Right)

Similar curves for d = 0.54 ft (sand No. 4 in the 33.5-in. flume) and d = 0.24 ft (sand No. 5 in the 10.5-in. flume) are shown in Figs. 25(*a*) and 25(*b*), respectively. In Fig. 26 are presented the concentrations measured for the runs in each of the three categories mentioned. Points for some of the new runs (2-17D, 2-17F, and H-3) are not included in Figs. 24 through 26 because the depths lie outside the range used for the figures.

The graph in Fig. 27 is a composite of all the data for sand Nos. 1, 3, 4, and 5 and is similar to Fig. 7. The water discharge (Q) and the sediment discharge

(G) have been replaced by q and q_s, the same quantities, respectively, per unit width. As before, the logarithmic cycles in the abscissa are twice as long as those in the ordinate; the depth is shown next to each plotted point. The friction factors, which are also labeled adjacent to the points in Fig. 7, have been omitted because they do not appear to define a simple pattern of contours. On the other hand, contour lines for constant depths of 0.24 ft and 0.54 ft have been drawn, and they show the same general tendency as the constant-depth lines in Fig. 8. Because of the different sands used for a depth of 0.24 ft, it was necessary to plot three slightly different contours for this depth. In these experiments the unit sediment discharge (q_s) increases when σ_g increases, with Q, d, and D_g staying nearly constant. Unfortunately, Fig. 27 is not free from temperature effects as the range covered is from 16.5°C to 27.5°C. It

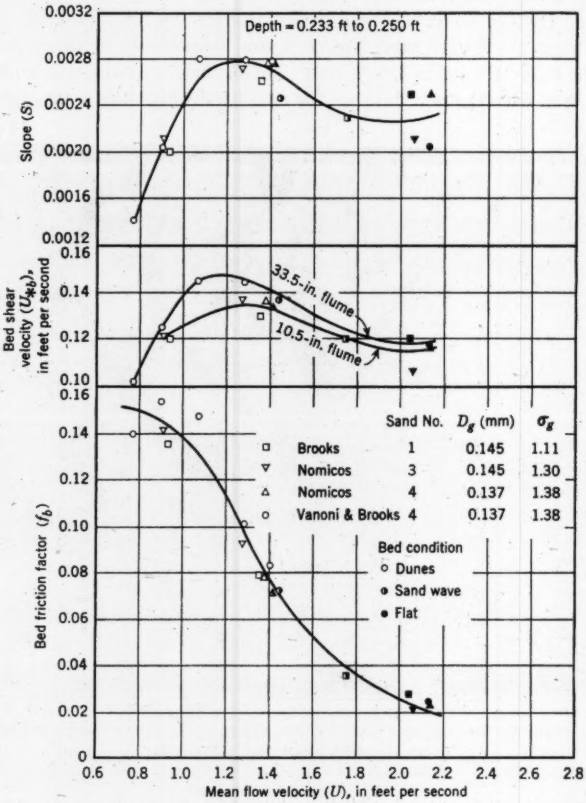


FIG. 24.—VARIATION OF S, U_{*b}, AND f_b WITH U AT A NOMINAL DEPTH OF 0.24 FT IN THE 10.5-IN. FLUME (SQUARES AND TRIANGLES) AND THE 33.5-IN. FLUME (CIRCLES)

is expected that the scatter would be less if the temperature were constant, as in Nomicos' Series II (sand No. 5) in which $T = 25.0°C$ to $26.0°C$.

More complete and detailed information and analysis of this work and related investigations are contained in two final research reports.[43,44]

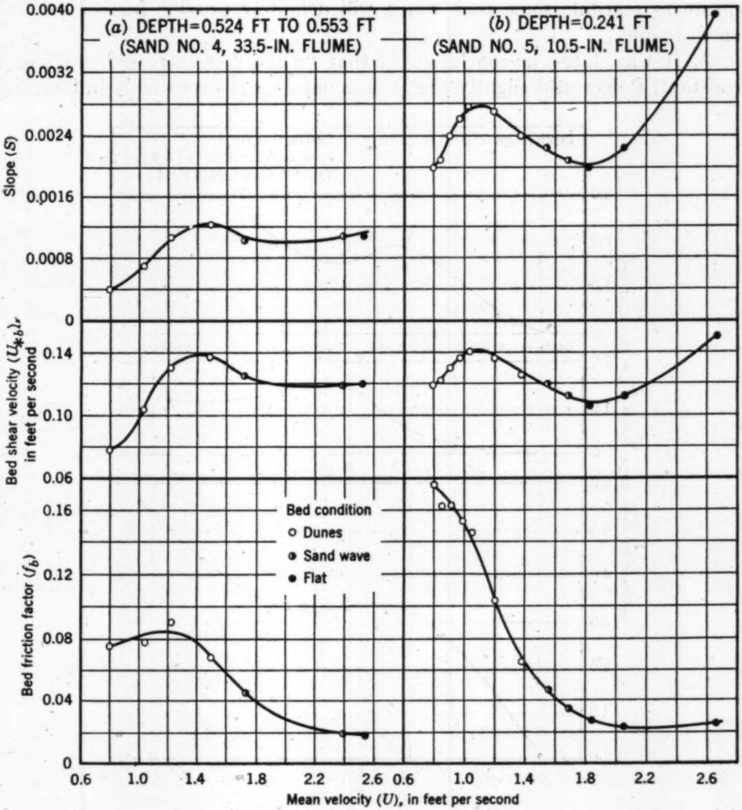


FIG. 25.—VARIATION OF S, U_{*b}, AND f_b WITH U AT CONSTANT DEPTHS

Discussion of Experimental Results.—For brevity and to avoid duplication the new results will not be discussed in detail; careful study will determine how well the new data support the previously stated conclusions. It can be shown that the foregoing data support all the previous conclusions of the writer with the following exceptions:

---

[43] "Laboratory Studies of the Roughness and Suspended Load of Alluvial Streams," by Vito A. Vanoni and Norman H. Brooks, *Report No. E68*, Sedimentation Lab., California Inst. of Technology, Pasadena, Calif., 1957.

[44] "Laboratory Studies of Dunes in Streams," by Norman H. Brooks and Vito A. Vanoni, *Report No. NSF1709*, Sedimentation Lab., California Inst. of Technology, Pasadena, Calif. (manuscript in process).

a. In conclusion 4*b*, the writer stated: "For a given slope and discharge, it was found that two depths of flow were possible." Because of the manner in which the runs were scheduled in the recent experiments, no new evidence on this conclusion was obtained. However, good evidence presented by Barton indicates that this conclusion may not be generally true. It is hoped in the future to schedule some runs which will definitely provide further evidence on this point.

b. It was stated in conclusion 4*f* that "when d is increased, holding U constant, $\bar{C}$ decreases slightly, and f_b does not change appreciably in the range of conditions covered." With the wider range of depth now obtained, it was found that f_b is substantially lower for the larger depth for a given velocity in those cases where the bed was covered with dunes; this result is obvious from Figs. 24 and 25(*a*).

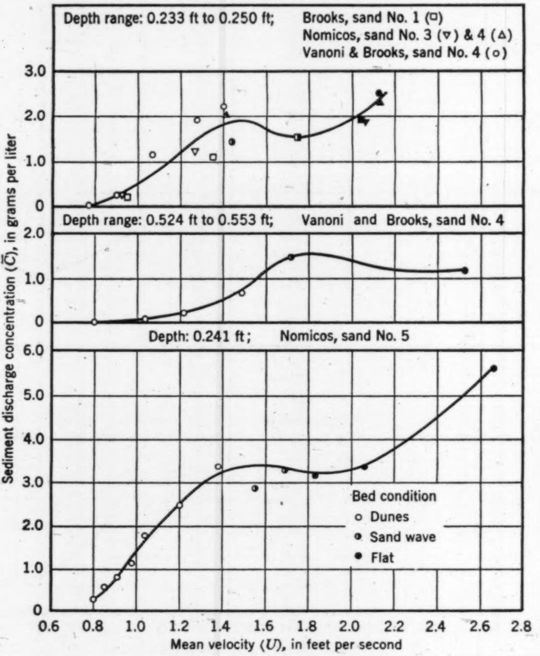


FIG. 26.—VARIATION OF $\bar{C}$ WITH U FOR VARIOUS SANDS AT CONSTANT DEPTHS

In conclusions 4*g* and 4*h* the results for two different sand sizes are compared, and, therefore, they cannot be checked against the new data presented herein.

Without exception, the discussers challenged the writer's basic conclusion (Conclusion 1) which states (in part):

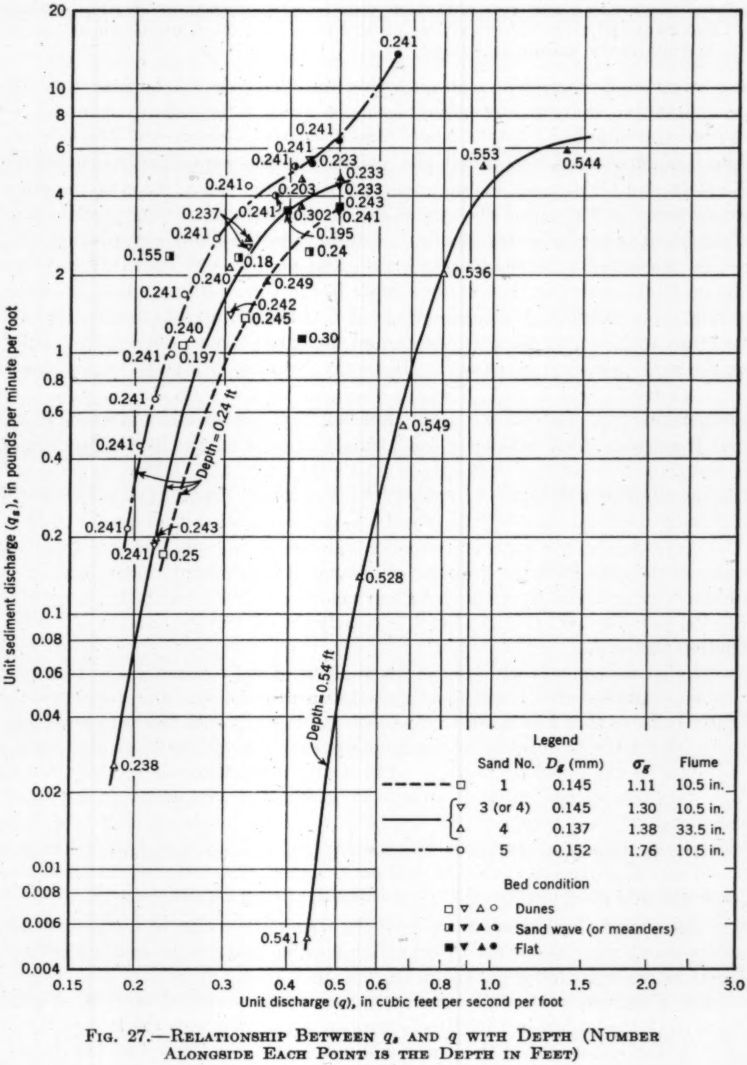


FIG. 27.—RELATIONSHIP BETWEEN q_s AND q WITH DEPTH (NUMBER ALONGSIDE EACH POINT IS THE DEPTH IN FEET)

> "For the laboratory flume it was found that neither the velocity nor the sediment discharge concentration could be expressed as a single-valued function of the bed shear stress, or any combination of depth and slope, or bed hydraulic radius and slope."

Lin is certainly correct in stating: "This statement should be read with the understanding that, among other factors, a given temperature, channel, and sediment are implied * * *." Among the discussers, Carlson and Mostafa and Liu, Lin, and Blench doubted the validity of the experiments themselves. Einstein and Chien and Barton apparently accepted the findings of the flume experiments but doubted the generality of the findings, feeling perhaps that some peculiar condition resulted in the findings in the flume experiments.

To the criticism of the lack of a sufficient number of runs to substantiate the original conclusion, the writer's reply lies in the additional data presented herein. For example, Lin has pointed out that the writer did not actually have two runs with exactly the same depth and slope, but having different velocities, at the same temperature as well. However, Nomicos has performed a pair of runs (Series I, runs A and C) which have exactly the same depth, slope, and temperature; yet the velocity in run A was 2.06 ft per sec while in run C it was 0.91 ft per sec. The validity of conclusion 1, at least for the laboratory flumes, is certainly conclusively demonstrated by Figs. 24 and 25. The curves representing slope as a function of velocity for a constant depth have very definite inflections in them for $d = 0.24$ ft and a long horizontal section for $d = 0.54$ ft. Therefore, one cannot expect to choose a value of S and only one corresponding value of velocity from the graphs. Similarly, it is apparent that U cannot be uniquely determined from the bed shear velocity. The cause of the surprising results, as indicated in conclusion 2, is clearly the extreme variations of the bed friction factor, f_b.

The writer concurs with Liu in his analysis of the resistance and transport problem starting with Eq. 6 (leading only up through Eq. 9). However, the writer believes that Liu in one of his subsequent paragraphs has reasoned incorrectly about the uniqueness of the functional relationship between the various variables as explained previously. The writer must vigorously disagree with Liu, even on logical grounds, when he makes the statement (referring to conclusion 1):

> "His finding in this case is in contradiction to his statement * * * that 'f_b and $\bar{C}$ are uniquely determined by U and d,' which means that the slope is also uniquely determined by U and d by using Eq. 9."

The same kind of misinterpretation of the writer's meaning of uniqueness and independent variables leads to other implausible statements in the first and fourth paragraphs following the paragraph just quoted from.

Liu disagreed also with conclusion 4*d* which states that "for a given value of Q, the largest bed friction factors are associated with the lowest values of G." Liu believes that the dunes will not develop at the lowest transportation rates, although he has found experimentally[45,37] that ripples start forming on a smooth bed practically as soon as the sediment starts to move. It has been the writer's experience also that if there is any transportation at all,

---

[45] "The Present Status of Research on Sediment Transport," by Ning Chien, *Transactions*, ASCE, Vol. 121, 1956, p. 840.

large dunes will grow in fine sand if sufficient time is allowed. For this reason conclusion 4*d* should not be unreasonable. However, from Fig. 25(*a*) it may be noted that for the runs with $d = 0.54$ ft the friction factor (f_b) at the lowest velocity is slightly less than the maximum (compare this with conclusion 4*e*). For run 2-12 ($d = 0.541$ ft and $U = 0.80$ ft per sec), the friction factor, f_b, was 0.076, which is still a large value considering that G was only 0.015 lb per min (or $\bar{C} = 3.3$ ppm). For this run the flume was operated continuously for 54 hr; starting from a flat bed, it took about 24 hr to reach equilibrium, and during the remaining 30 hr the equilibrium appeared to be stable.

The writer welcomes Liu's brief analysis of the dune-velocity data. Fig. 20 confirms the writer's observation[6] that the velocity of advance of the dunes actually decreases as the grain size gets smaller and presumably the sediment load gets larger.

It is encouraging to know that Barton has made experiments at Colorado State University which are similar to the ones performed by the writer and that his results are in many ways similar to those reported herein. The data that he has presented in Fig. 9 certainly demonstrate the same general behavior of the friction factor that has been noted by the writer. In comparing data for different depths of flow it was found that the region which Mr. Barton has labeled "sand bars on bed" (Fig. 9) moves to the left as the depth decreases. It has been puzzling to the writer to note that even in comparing the 10.5-in. flume and 33.5-in. flume with the same depth of flow that the interval of velocity giving sand bars or sand waves is not exactly the same in the two flumes, being slightly higher for the wider flume.

TABLE 10

Run No.[33]	Q (cu ft per sec)	d (ft)	S	Temperature (°C)	Bed
29	5.8	0.60	0.00121	25	Flat
31	4.2	0.41	0.00123	26	Flat

Fig. 10 perhaps implies that the relationship between depth, slope, and discharge is simpler than it really is, even for the flume used. If all the data published by Barton and Lin[33] are plotted on a curve similar to Fig. 10, the lines of constant slope cannot be drawn as straight lines such as lines A and B of that figure. In particular, along line B for $S = 0.0015$ to 0.0017, the slope actually drops to 0.0012 or less in the gap between discharges of 3 cu ft per sec and 6 cu ft per sec. This is indicated by the runs in Table 10. Run 29 falls on line B whereas run 31 falls below; on the other hand, one is led to expect from Fig. 10 that these points would fall between curves A and B on the basis of their observed slopes.

In spite of this deviation, the writer concurs with Barton that the Colorado State University data do not support conclusion 4*b*. However, some of their results indicate that for a given depth and slope more than one equilibrium discharge could probably be found in the flume at Colorado State University.

From a study of the runs in Table 11 it is seen that when the depth is constant and the velocity decreases, the slope actually increases because the friction factor for the bed increases so rapidly. As illustrated in Fig. 24, this must be a temporary trend in S; if the velocity were further reduced below 1.19 ft per sec (Barton's run 16), the slope must eventually drop also, thus repeating some slopes already obtained for higher velocities. Therefore, although no two of their runs have exactly the same depth and slope with two different discharges, it is seen from the foregoing data that such a result would undoubtedly have occurred had a complete sequence of runs been made at a depth of 0.40 ft in the flume at Colorado State University.

The analysis presented by Carlson and Mostafa appears to be oversimplified. In Fig. 18 the slope is plotted against concentration (expressed as $G/60\,\gamma\,Q$). The discussers have given no logical reason why the slope should be so simply related to the concentration, and the scatter of the points indicates this oversimplification. Fig. 19 shows how the observations for dunes and those for flat beds plot along two different lines in a curve of $r_b\,S/D$ versus concentration. A curve such as this does not solve the problem of whether there will be dunes or not in a given case; the engineer has no basis for choosing from which curve to read the answer. The further inadequacy of Fig. 19 is illustrated by the discussers' conclusion:

TABLE 11

Run No.	Q (cu ft per sec)	U (ft per sec)	d (ft)	S	U_{*b} (ft per sec)[a]	f_b[a]	Temperature (°C)	Bed
31	4.2	2.56	0.41	0.00123	0.118	0.0169	26	Flat
27	2.1	1.31	0.40	0.00140	0.131	0.080	24	Dunes
16	1.9	1.19	0.40	0.00158	0.140	0.110	22	Dunes

[a] Computed by the writer; other data from "A Study of the Sediment Transport in Alluvial Channels," by J. R. Barton and P. N. Lin, Dept. of Civ. Eng., Colorado State Univ., Fort Collins, Colo., 1955.

> "It follows that for the same hydraulic conditions as given by r_b, S, and Q two different ratios of sediment to water discharge might have been produced, one with a smooth bed and the other with a dune formation."

If the analysis leading to the curve in Fig. 19 were sound, this would appear to be a logical conclusion. However, the fact that this conclusion is completely contrary to the observed results does not cast doubts on the writer's conclusions but on the analysis instead. When values of r_b, S, and Q are selected, the friction factor is automatically determined; it follows then that it is impossible to produce either a smooth bed or a dune-covered bed when the friction factor is restricted to one value.

The writer does not believe that the shortcomings of the curves of Carlson and Mostafa can be remedied by information on the grain sizes of the sediment load. Nonetheless, because this was an omission of the original paper, the available information on the size distributions of the sediment samples is given in Table 12. Unfortunately, the data are incomplete, but the extreme range of transportation rates is covered so that the missing values could be expected to lie within the range of the values reported therein. The observed variations in D_g and σ_g between runs is believed to be of little consequence.

Another dimensional analysis presented by Lin is based largely on a report[33] which was issued after the writer's original paper had been submitted for publication. Because Lin presents only the main results of his theoretical analysis without the underlying reasoning, a detailed discussion of the analysis would not be appropriate here; hence, the following comments are confined to material presented in his discussion.

Lin has plotted the writer's data according to his analysis in Fig. 21. The usefulness of his analysis is restricted, however (as with Fig. 19), because there is no way to predetermine which of the two curves (one being for dunes and the other for flat bed) applies in any given case.

The writer concurs with Eqs. 11 and 12 but not with Eq. 13. To obtain Eq. 13, it is necessary to rewrite Eq. 12 to make the viscosity, μ, a function of U, d, ρ, $\bar{C}$, g, and w and substitute it into Eq. 11. In effect $\bar{C}$ is therefore made an independent variable and μ, a dependent one. Because μ is a property of the fluid it is hardly appropriate for a dependent variable. Furthermore, the writer is of the opinion that it is impossible to take U, d, and $\bar{C}$ all independently for a given material (with a given settling velocity, w) because many combinations will be impossible regardless of what the viscosity of the water is.

TABLE 12.—ANALYSIS OF SEDIMENT LOAD FOR RUNS IN TABLE 1

Run No.	$\bar{C}$ (g per liter)	D_g (mm)	σ_g
	Sand No. 1		
3	1.95	0.147	1.10
4	2.45	0.141	1.13
6	2.45	0.145	1.11
7	2.15	0.149	1.09
10	0.2	0.146	1.10
12	0.72	0.128	1.16
	Sand No. 2		
21a	4.9	0.079	1.22
24	4	0.070	1.28
26	0.19	0.073	1.36
30	1.75	0.069	1.30

Lin states that the writer's "conclusion 3 is not compatible with his data." On the contrary, from examination of the data in Figs. 6 and 7 and in Figs. 24 through 27, it seems to be an inescapable conclusion; the fact that Lin, on the basis of Fig. 21, arrives at the opposite conclusion is more likely to indicate that the analysis is faulty. In fact, his statement that "* * * in the range of the author's data, f_3 is not unique, no matter which six of the pertinent variables are chosen as independent" appears illogical by the same line of reasoning as given for the previously presented mathematical examples, $y = \sin x$ and $x = \sin^{-1} y$. A relationship between several variables may not be unique when written in one form but still be unique when written in a different way.

Two of the discussions (those by Einstein and Chien and Lin) refer to the work of Meyer-Peter and Müller.[26] The latter have divided the energy gradient into two parts—that due to the grain roughness of the surface and that resulting from the "topography" of the bed. Like Einstein, they have theorized that the rate of bed-load transport is dependent only on friction associated with the grain roughness. Their final formula for the rate of bed-load transportation shows that the transportation rate depends on the shear with a correction factor $(k_s/k_r)^{\frac{3}{2}}$. Meyer-Peter and Müller have not suggested how one is to predict what this factor should be, nor have they indicated that there might possibly be any difficulty with multiple solutions. That is, for a given value of shear there may be more than one possible value of this correction factor. Therefore, the writer would like to dispute the statement by Lin to the effect that Meyer-Peter and Müller have published a finding similar to the writer's conclusion 1.

The writer wishes to thank Blench for pointing out the misunderstandings in regard to regime theory. From Blench's paper[21] it is seen that the regime theory certainly does include the shear; on the other hand, from the form of the equations comprising generalized regime theory, as reported by Blench, there is no possibility for nonuniqueness of the type observed by the writer in the laboratory. The writer certainly does concur with Blench, however, when he implies that the discharge and the grain size and other characteristics of the alluvium are to be considered independent variables, and the depth, width, and slope of a stream are considered dependent variables.[46]

In regard to the last misunderstanding enumerated by Blench, it should be pointed out that the "two present theories" referred to under conclusion 1 were those of Lane and Kalinske,[19] and Einstein and Barbarossa[20]. It was not the intention of the writer to dispute the regime theory, which does not deal explicitly with sediment load.

Einstein and Chien have presented a very thorough and thought-provoking analysis of the writer's paper. Their analysis is based primarily on an interpretation of the experimental results on the basis of the theory for river-channel roughness of Einstein and Barbarossa[20]. The discussers demonstrate how the bar-resistance curve of Einstein and Barbarossa can actually lead to multiple solutions for certain values of the depth, slope, and grain size. They state:

> "The existence of such uncertainty in predicting discharge and sediment load in very limited ranges of conditions was known previously, but not much significance was attached to this fact because of the rather rare occurrences of these cases."

Inasmuch as the discussers did not offer any reference in connection with this statement, it may be inferred that it was previously known by only a few people and that whether or not these are rare occurrences remains to be demonstrated. If such cases are not rare, the usefulness of the proposed bar-resistance curve will be limited by the fact that one will not know which of two or more possible solutions to choose as the correct answer.

The writer thinks that the uncertainty of solutions derived from the bar-resistance curve may be more widespread than indicated by the curve itself because of the large range of scatter of the original river data on which the bar-resistance graph is based.[47] The tremendous effects of this scatter on a typical, computed rating curve have already been ably demonstrated with numerical examples by James J. Doland, M. ASCE, and Ven Te Chow,[48] A.M. ASCE, and Sir Claude Inglis,[49] M. ASCE, in discussing the paper by Einstein and Barbarossa. In Fig. 28 the bar-resistance curve has been plotted as a solid line, with two dashed lines on each side indicating the approximate range of scatter as shown in the original derivation of the curve. In addition, Fig. 28 shows curves 1, 2, and 3 from Fig. 11. Allowing that possible solutions may be anywhere within this band of scatter, and not necessarily precisely on the bar-resistance curve, one finds a wide range of possible solutions for curves 1 and 2. For example, curve 2 lies within the zone of scatter from $U/u_*'' = 9$ to $U/u_*'' = 50$. Therefore, the writer does not believe that the discussers can argue that the occurrences of such uncertainty are rare, merely on the basis of the bar-resistance curve.

---

[46] "Regime Theory for Self-Formed Sediment-Bearing Channels," by Thomas Blench, *Transactions*, ASCE, Vol. 117, 1952, p. 395.

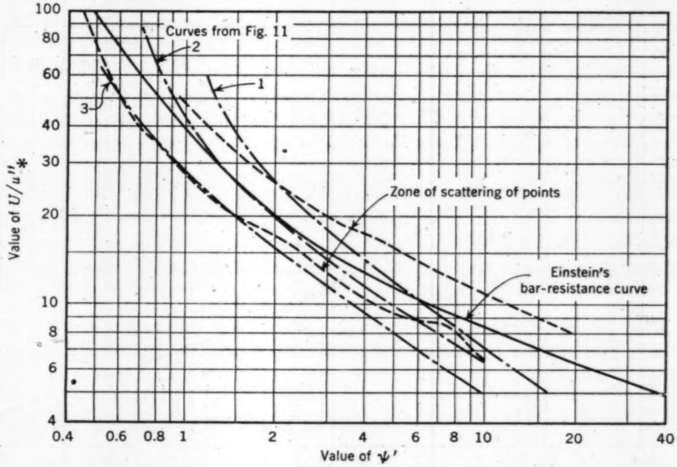


FIG. 28.—RANGE OF SCATTER OF POINTS DEFINING THE BAR-RESISTANCE CURVE

Einstein and Chien have noted that the flume values of U/u_{*b}'' and ψ' do not follow the bar-resistance curve for natural rivers (Fig. 15). They have suggested that the discrepancy is due to the unnatural uniformity of the material used in the flume. Following this suggestion, experiments were made with sands of varying geometric standard deviation (Fig. 22 and Table 9) with the results as given in Tables 7 and 8. The discussers' sorting coefficient,

---

[47] "River Channel Roughness," by Hans A. Einstein and Nicholas L. Barbarossa, *Transactions*, ASCE, Vol. 117, 1952, p. 1127, Fig. 3.

[48] Discussion by James J. Doland and Ven Te Chow of "River Channel Roughness," by Hans A. Einstein and Nicholas L. Barbarossa, *ibid*, p. 1134.

[49] Discussion by Sir Claude Inglis of "River Channel Roughness," by Hans A. Einstein and Nicholas L. Barbarossa, *ibid.*, p. 1142.

S_o, is related to the more commonly used geometric standard deviation, σ_g, in the following way for the logarithmic normal distribution:

$$S_o = \sqrt{\frac{D_{75}}{D_{25}}} = \left(\sqrt{\frac{D_{84}}{D_{16}}}\right)^{0.67} = \sigma_g^{0.67} \ldots\ldots\ldots\ldots\ldots (18)$$

Determinations of ψ' and U/u_*'' were made for all the runs reported in Tables 7 and 8 as well as for all the data presented in Table 1. In making these computations the procedure suggested by Einstein and Barbarossa[20] was used, without simplifying assumptions. By a change of parameters and specially derived working graphs, the writer was able to solve Eq. 4 for r_b' and r_b'' without resorting to trial and error.[43] In Eq. 4, D was replaced by D_{65} to take account of the nonuniformity of the sediment.[20] The values of ψ' were computed by use of Eq. 3. The computed values for the writer's flume data are slightly different from those of Einstein and Chien because a distinction was made between D_s, and D_{65} and D_{35}. All these data are plotted together in Fig. 29. The surprising result is that the sorting of the material makes practically no difference in the bar-resistance relationships for the flume data. Even runs in Series II ($\sigma_g = 1.76$) behave almost identically to the writer's runs 2 through 13 ($\sigma_g = 1.11$). For $\sigma_g = 1.76$, Einstein's sorting coefficient, S_o, is 1.46, which is greater than the majority of the values listed for natural rivers in Table 5.

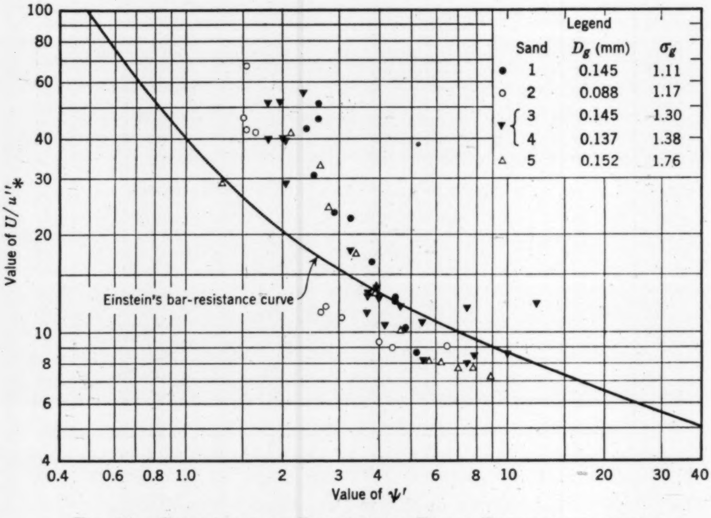


FIG. 29.—COMPARISON OF POINTS FROM FLUME EXPERIMENTS WITH BAR-RESISTANCE CURVE

A possible explanation for the apparent inconsistency between flume and river data is the wall effect. Whereas in the laboratory flumes the maximum width-to-depth ratio was only about 12, natural rivers commonly have width-to-depth ratios of the order of 100. In the laboratory the stream is confined to a straight channel, and the walls are usually smooth. The side-wall correction procedure[7] is supposed to eliminate the wall effect as far as the computation of the bed friction is concerned, but there is no known correction for the effect of the walls on the sediment transportation and dune configuration.

Roughness of alluvial channels will have to be expressed basically in terms of a velocity parameter instead of a shear parameter such as ψ' used by Einstein and Barbarossa. Because large changes in roughness have been observed to occur with almost negligible changes in the bed shear (or U_{*b}), it appears that the latter is a poor choice of independent variable. Said M. Ali and Maurice L Albertson,[50] M. ASCE, have presented an interesting analysis of roughness of alluvial streams based on the Reynolds number, which is a velocity parameter.

In the concluding paragraph of the discussion by Einstein and Chien, the writer's results are compared with Einstein's ($\phi_* - \psi_*$) relationship.[9] As the derived points fit Einstein's curve reasonably well in most cases, the discussers are led to conclude that "* * * the concept in linking the bed-load transport with the grain resistance of only the bed material is basically sound." In considering this deduction it is well to realize some of the assumptions that were made in deriving the plotted points from the writer's data presented. In the first place, the functional relationship given in Fig. 17 is basically for bed-load transport. In order to evaluate the bed-load discharge from the writer's measured values of total load discharge, the computations involve finding the ratio of suspended-load discharge to bed-load discharge. The suspended-load discharge is computed on the basis of the following principal assumptions:

1. The concentration of suspended material at a distance equal to two grain diameters above the bed ($y = 2\,D$) may be considered a boundary condition for the suspended-load equation (Eq. 1). This concentration is found by dividing the bed-load transport by $2\,D\,(11.6\,u_*')$ in which u_*' is the shear velocity associated with the grain resistance only.

2. The velocity distribution is expressed in terms of $u_*'/0.40$, thus implying that (*a*) the von Kármán constant $k = 0.40$ and (*b*) only the shear due to grain resistance affects the velocity distribution in the vertical. (Actually, u_*' introduced by assumption 1 cancels the same quantity arising from assumption 2 in the final computations in Einstein's method.)

3. The exponent, z, of the suspended-load equation is given by

$$z = \frac{w}{0.40\,u_*'} \quad \text{(19)}$$

in which w is the settling velocity and 0.40 is $\beta\,k$ (compare Eq. 2). It is implied by this relationship that the upward diffusion of the suspended load depends only on u_*' and not on the over-all shear velocity, U_* (or U_{*b}).

Using these assumptions, the product of concentration and velocity may be integrated from $y = 2\,D$ to $y = d$ to yield the rate of suspended-load transport as a certain multiple of the bed-load transport. Knowing this, the bed load may be computed from the measured total sediment discharge.

---

[50] "Some Aspects of Roughness in Alluvial Channels," by Said M. Ali and Maurice L Albertson, Dept. of Civ. Eng., Colorado State Univ., Fort Collins, Colo., August, 1956.

Now, each one of the three basic assumptions mentioned is open to serious question especially for runs with dune-covered beds. In the first place it is unreasonable to apply the suspended-load equation down to an evaluation of 2 D when the height of the dunes is of the order of several hundred grain diameters. In the second place, it should be noted that many investigators[45] have shown that the von Kármán constant, k, for sediment-laden streams is substantially less than the assumed value of 0.4. Data reported by Barton and Lin[33] also show that assumption 2 is poorly supported in fact for runs with dune-covered beds. In regard to assumption 3, the experiments of Barton and Lin further indicate that the theoretical exponent, z, based even on the total shear velocity, U_*, is still larger than the observed exponent when the bed is covered with dunes. As it is not uncommon to have $u_*' < 0.5\, U_*$, the error in the shape of the suspended-load distribution—resulting from using a value of z which may be more than twice the actual value—can be considerable.

Although the gross errors in each of the assumptions may tend to cancel out in the interlocking of the assumptions in the computations, the writer still believes that one is not justified in making definite conclusions about the relation of bed load to suspended load from results of such a tortuous calculation. In fact, the only supporting evidence for Einstein's analysis of the suspended load[9] is an analysis similar to that described previously for experiments made by Einstein in the same 10.5-in. flume that the writer used at the California Institute of Technology.

Field Observations.—Einstein and Chien have correctly pointed out some of the weaknesses of the field examples cited by the writer. There are very few field data which are sufficiently detailed to verify conclusively or disprove the conclusions reached in a laboratory flume. The main difficulties with the example of the Colorado River at the Grand Canyon are that there is no information on the division of the total suspended load between wash-load and bed-material load, and there are no data on slope or grain sizes in the bed. The changes that took place during the spring flood of December, 1940, to June, 1941, probably did not occur in a matter of days, as indicated by Einstein and Chien, but in a matter of weeks and months. It seems as though this should be sufficient time for scouring the bed and changing the roughness. In this connection it has been observed in a laboratory flume that a system of dunes can be wiped out in less than 1 min by a flow of a higher velocity which is in equilibrium only with a flat bed.

The writer recognizes that the scour of the river channel at the Grand Canyon may not be generally true of the entire river system. As noted by the discussers, Lane and Borland[29] found that on the Rio Grande the narrow sections of the river tended to scour out during floods, whereas the wider sections tended to aggrade. Although it may well be that the over-all regime of the river is not changed, the changes that occur in any given reach must be in response to the changes in the discharge and sediment load entering that reach. Even though the changes may not proceed all the way to a new equilibrium, the alterations of the stream characteristics are certainly in that direction.

The situation at Yuma is certainly complicated by the location of Imperial Dam, 15.8 miles upstream. Not only is there general degradation taking place below Imperial Dam, as mentioned by Einstein and Chien, but there is also local choking of the river channel between Imperial Dam and Laguna Dam (about 6 miles downstream) due to the sludge returned from the desilting works.[51] By the diversion of essentially clear water from the river to the All-American Canal the remaining water has to carry a higher concentration of sediment below Imperial Dam. In order to move this material down the river and preserve the channel below Imperial Dam it has been necessary to sluice the channel for several hours every week or two with a relatively high discharge (20,000 cu ft per sec or more). It is perhaps the nature of this sluicing operation and the varying nature of the sludge from the desilting works that gives the apparently erratic changes in the median grain size of the river bed at Yuma, without any clear-cut coarsening of the bed sediment there.

While the writer believes that a coarsening of the bed material is not necessary to produce an increase in the roughness when the sediment load decreases, Einstein and Chien demonstrate by an example (Table 3) that a change in the grain size from 0.14 mm to 0.35 mm can make a significant change in the Manning *n* for the same discharge. It is presumed that the source of their data marked "measured" in Table 3 is Leopold and Maddock[14] as no new reference has been given. However, the discussers' use of 0.14 mm as the typical grain size before closure of Hoover Dam and 0.35 mm for after closure is not at all consistent with the observation of $D_{50} = 0.140$ mm on November 8, 1951, cited by them; in fact, both $D_{50} = 0.14$ mm and $D_{50} = 0.35$ mm have occurred after closure. Thus, the discussers' deduction that the change of the roughness at Yuma from before closure to after is due entirely to a change in median grain size does not appear sound. The writer believes that the main contributing factor to the increase in the roughness is the great reduction in the bed-material load whether the bed becomes slightly coarser or not.

In his discussion, Liu has stated, "* * * it is generally known that the slope is an independent variable in most natural alluvial streams because as the flow discharge changes, the flow depth changes more readily than the slope does." Although the slope S cannot adjust rapidly over a long reach of a river, nevertheless the slope of a graded stream[52] is, in the long run, just that slope which is required for transporting the water and bed-material load supplied to the reach. In this way the discharge Q and sediment transport rate G may be considered the independent variables, and the river will in the course of time work toward the slope which is required to achieve equilibrium with Q and G.

In the short run, of course, there will be large fluctuations in Q and G. But also in the short run there may be local scour or fill, indicating temporary non-equilibrium because the river cannot adjust instantaneously to the equilibrium slope required for the given Q and G. However, the writer's hypothesis that several combinations of Q and G can be in equilibrium with the same slope really allows a river greater flexibility because large changes in Q, G, and stage may take place without requiring any significant changes in S, which is by far the slowest to change. Hence, there appears to be no special reason why several combinations of values of Q and G cannot occur at the same slope in a natural river as well as in a laboratory flume.

---

[51] "River Channel Conditions Imperial Dam to Laguna Dam," Colorado River Front Work and Levee System, Bureau of Reclamation, U. S. Dept. of the Interior, Region 3, Boulder City, Nev., August, 1954.

[52] "Concept of the Graded River," by J. Hoover Mackin, *Bulletin*, Geological Soc. of America, Vol. 59, 1948, pp. 463–512.

Summary.—In general, this closure has (1) attempted to explain the logic of uniqueness and nonuniqueness of various functional relationships; (2) answered several questions about procedure; (3) introduced more flume data in support of the writer's basic conclusions; (4) pointed out some of the difficulties in the analyses presented by the discussers; and (5) given further opinions on field examples. For the reasons detailed, the writer still adheres to all the conclusions originally presented with the possible exception of 4*b*.

The writer appreciates all the efforts of the discussers in presenting so much valuable material on these controversial problems. Although all the discussers have disagreed with all or part of the writer's conclusions, the writer is unable to agree substantially with any of the discussers even after careful restudy of the available facts and of the discussions presented. It is hoped that further information from both the laboratory and the field will resolve some of the problems presented in this paper.

Acknowledgments.—The experiments and the analysis of the data presented herein are part of the research sponsored by the Missouri River Division of the Corps of Engineers and the National Science Foundation, Washington, D. C., at the California Institute of Technology.

The writer is indebted to George Nomicos for performing the experiments in the 10.5-in. flume; to Hugh S. Bell, Jr., for his assistance in making the experiments in the 33.5-in. flume and for analyzing all the data; and to Vito A. Vanoni for his supervision of the sponsored research and for offering many valuable suggestions and criticisms.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2932

MUNICIPAL ORDINANCES FOR INDUSTRIAL WASTES

BY JULIAN R. FLEMING,[1] M. ASCE

SYNOPSIS

In 1944 the officials of Knoxville (Tenn.) began the engineering design of a system of intercepting sewers, pumping stations, and sewage-treatment plants, the construction of which was completed in 1955. Presented herein are the main features of the ordinances and regulations that were set up by the city officials concerning the reception of industrial wastes into the system and treatment plants. The principal features that are described are the system of charges and collections, aspects of the ordinance that protect the system and the employees from toxic wastes and other types of harmful by-products, and several case histories of the specific liquid wastes that have received special study. Included among the case histories are those of the soft-drink industry, metal-plating works, milk-processing plants, meat-packing plants, fertilizer plants, and paper manufacturers.

THE KNOXVILLE ORDINANCE

Principal consideration will be given herein to the ordinances adopted by the city of Knoxville (Tenn.), with limited attention being given to the more general aspects of the problem. The ordinances that are now in effect in Knoxville were passed by the city council in 1954 and 1955.

Historically, the work began in 1944 when the city engaged two firms of consulting sanitary engineers to study the city's need for a system of intercepting sewers and one or more sewage-treatment plants, and to prepare an engineering report based on their findings. The report was submitted, and at a later date one of the firms was employed to design the interceptors and treatment plants.

In 1953 bids were taken, and the work was begun at a bid cost of approximately $6,000,000 to be financed entirely by revenue bonds. The bonds are

---

NOTE.—Published, essentially as printed here, in October, 1956, in the Journal of the Sanitary Engineering Division, as *Proceedings Paper 1085*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Director, Div. of San. Eng., Tennessee Dept. of Public Health, Nashville, Tenn.

to be retired over a period of approximately thirty-five years, and interest, operating expenses, and other costs are to be compensated for by a sewer service charge for all customers (within the city) using the sewer system. The basis of the service charge is that each customer pays a percentage of his water bill for the charge. Table 1 shows the schedule for all classes of customers. Billing and collecting is handled by the Knoxville Utilities Board, a municipally owned organization that distributes electricity, gas, and water.

A description follows of a single bracket of another type of billing structure that is available to those using great quantities of water. In this type of billing, a one-year contract is agreed on between the city and the customer. If the contract agreement is for a use range of between 0 cu ft per month and 1,000,000 cu ft per month, the customer pays a base rate of $175 per month plus 4 cents per 100 cu ft for all the water used (that is, all that is returned to the sanitary-sewer system) to as much as 1,000,000 cu ft. If the usage is beyond the contract figure, the customer pays 5 cents per 100 cu ft for all the water used in excess of 1,000,000 cu ft. Other brackets range to as much as 6,000,000 cu ft per month in steps of 1,000,000 cu ft per month. In the previously described rate structure, the bill is increased 50% for service beyond the previously described city limits, and a 10% penalty is applied for overdue bills.

TABLE 1.—SEWER SERVICE CHARGES[a] FOR KNOXVILLE, TENN.

Water used, in cubic feet per month	Charge Per 100 Cu Ft, in Cents	
	Inside city[b]	Outside city[c]
First 500 (minimum)	13.0	20.0
Next 3,500	16.0	24.0
Next 6,000	14.0	21.0
Next 10,000	11.0	16.5
Next 10,000	10.0	15.0
Next 10,000	9.0	13.5
Next 10,000	8.0	12.0
Next 20,000	7.0	10.5
More than 70,000	6.0	9.0

[a] Net rates; after ten days past due date of bill, 10% should be added. [b] Minimum bill inside city is 65 cents. [c] Minimum bill outside city is $1.00.

The writer feels that an ordinance applying to the reception and handling of liquid wastes within a sanitary-sewer system should consider the following principal factors:

1. Protection is required of the sewer system from substances that are harmful to the pipes, jointing material, and manholes. Workmen should be protected from gases or chemical poisons that may be discharged into the sewer. Flammable or explosive substances of all kinds should also be excluded.

2. The sewage-treatment plant should be protected from substances that may seriously interfere with its normal operation, such as overloading by slugs of concentrated material, chemical poisons, liquid wastes that are extremely high in biochemical oxygen demand (B. O. D.), suspended matter, color, or chlorine demand.

3. There should be a fair system of charges for industry, so that each firm pays its proper share of the cost of handling and treating the waste.

Before presenting the more specific provisions of the ordinance and experiences with its use, the position of industries with respect to the utilization of the city sewer system and treatment plant should be mentioned briefly. Most, but not all, of the industries within the city of Knoxville discharge their liquid wastes into the city sanitary-sewer system, including washroom, toilet, drinking fountain, and miscellaneous sanitary wastes, in addition to the liquid wastes from manufacturing processes. Provided that the wastes are pretreated, thus producing an effluent acceptable to the city health department or to the state health department (depending on which has jurisdiction in the particular case), industries can either place all liquid wastes into the city system, or treat them before their final disposal into a water course. In almost all the cases that arose in Knoxville prior to 1956, industries elected to discharge all polluted matter into the city system rather than to set up treatment facilities of their own. This is the more economical policy in most cases, especially for those in which the particular industry is in a medium-sized or large city.

Two other possible conditions may also arise. One is that industries may be requested to do limited pretreatment or recovery, or both, prior to the acceptance of their liquid wastes into the city system. In this case also, it will usually be to their economic advantage to use the city system.

In the latter case, in which a part of or all the particular industrial waste will not be accepted by the city in its raw state, or in any state of pretreatment that the industry can reasonably supply, it may be necessary to give complete treatment to all industrial wastes. In addition, substantial changes may have to be made in the operating or recovery features of the processes, or in extreme cases it may be necessary for the plant to be moved to a more favorable location.

Some of the major items included in the Knoxville Ordinance are:

a. All customers shall have approved meters on all water supplies that are ultimately discharged into the sanitary-sewer system, or else shall meter the liquid wastes. For home owners this means that in almost all cases the customer pays the sewer service charge on all water used through his meter. In the cases of commercial users and industrial users who may utilize large quantities of unpolluted cooling water, and in the case of some industries which (1) evaporate part of their water, (2) return some of it in an unpolluted state to the storm sewer, or (3) use it in their product, one or more secondary meters may be used so that the customer can receive credit for that part of the water used but not returned to the sanitary sewer.

b. A rate beyond that given in Table 1 may be charged to industrial users if the B.O.D., suspended solids, chlorine demand, or other properties of the waste are such that handling the waste creates a great expense in the treatment plant. These properties are based on periodic laboratory examination of the particular waste. At present (1958) such a penalty is not being applied. All customers are billed at a uniform rate per unit volume of waste contributed according to the standard billing policy. If in the future the city decides that a surcharge should be imposed on certain customers, each case will be considered separately, and the billing policy recommended by the consulting engineers will be passed on by the city council. The writer believes that such a surcharge should have been imposed on all industries from the beginning of the operation of the treatment plant, based on B.O.D., suspended solids, and chlorine demand if any of these characteristics exceeded an established allowable limit for normal waste.

c. Within eighteen months after the passage of the industrial-waste ordinance, all industrial customers must file certain information with the city. Each industrial customer shall express intention within twenty-one months of accepting admission to the city system, when pretreatment is required or a surcharge is to be levied by the city. Within twenty-four months all industrial customers who must pretreat their wastes are required to submit plans to the city showing the proposed treatment facilities. The industry must show evidence within twenty-seven months that a contract has been awarded for construction when pretreatment is necessary. All the preceding time periods are dated from the passage of the ordinance, and in special cases a time extension may be granted.

d. Certain substances that may be present in industrial wastes are excluded from the sewer system either in whole or in part. In some cases a limited quantity is permissible, and this may be based either on the individual plant or on the concentration of this material at the treatment plant from the entire system. Some typical examples of materials that are excluded are:

(1) Liquids with a temperature greater than 150° F.;

(2) Grease, oil, or other substances that will solidify or become viscous in the sewer;

(3) Gasoline or similar liquids or gases that are flammable or explosive;

(4) Substances that tend to settle out in the sewers and cause stoppage or obstruction to flow;

(5) Liquids that are corrosive, highly acid, or highly alkaline (pH between 5.5 and 9.5);

(6) Toxic or poisonous substances;

(7) Cyanides; and

(8) Toxic radioisotopes.

Highly colored wastes must be omitted or subjected to special review. Slugs of liquid wastes that may cause temporary overloads on the sewers or the treatment plant are excluded unless they are discharged more uniformly. Iron, chromium, copper, and zinc are limited at the treatment plant to 15 ppm, 5 ppm, 3 ppm, and 2 ppm, respectively.

There are specific provisions for certain types of industries, such as packing houses, poultry-killing plants, milk-products plants, and plants producing oil wastes. There are special regulations for plants whose wastes require pretreatment or recovery before admission to the sewer system.

Some of the foregoing provisions may seem to impose hardships on the industries. However, as long as the problem is approached cooperatively by both city and industry, so that the best possible solution is achieved with the least hardship and with the acceptance of all wastes that can reasonably be handled at the treatment plant, the ordinance is reasonable and essential to the protection of the collection system and the treatment works. There may be occasional cases in which it will be necessary to use the ordinance to the fullest possible extent, but, in general, the most liberal possible use of the treatment facilities will be made available to industry.

During 1954 and 1955 the writer was employed by the city of Knoxville as a special consultant on industrial wastes and on various other types of work connected with the interceptor sewer system and treatment plants. Samples from various industrial plants were collected, and laboratory examinations were made of them. Reports were prepared from the data obtained, and recommendations were made to the city concerning the acceptance of wastes into the system with or without pretreatment. It was also necessary to consider, not only the effect of a particular industry on the sewer system and treatment plant, but also the collective effect of all industrial plants of a given type on the operation of the treatment plant, such as the milk-product and the meatpacking groups, the metal-plating industry, ando thers. Considerable time and effort was spent in determining what wastes or parts of wastes might go into the storm sewer and what into the sanitary sewer. The program was undertaken in advance of the completion of the main sewage-treatment plant because this plant did not begin operating until the fall of 1955.

Knoxville has a sizable soft-drink bottling industry, which includes one of the largest companies as well as several others. Most plants discharged their process wastes into storm sewers or directly into surface streams. An analysis of the wastes indicated that there was considerable pollutional material in the typical waste from a soft-drink plant. For example, a series of samples from one composite-plant waste (excluding sanitary sewage) showed an average pH of 10.0, total solids of 1,200 ppm, dissolved solids of 1,150 ppm, and a 5-day B.O.D. of 570 ppm. It was obvious that this waste should not be discharged to the storm sewer or to a surface stream. Further investigation revealed that nearly all the organic pollution resulted from the small quantity of drink residue in the returned bottles. The elimination of the first part of the bottle rinse generally permitted the remainder of the plant wastes, except sanitary sewage, to be discharged into the storm sewer, and, therefore, the plant received credit on this part of the water used.

Because of the toxic nature of their liquid wastes, metal-plating works are always potentially dangerous to sewage-treatment plants. Four plants of this type were found in the city of Knoxville. One was a small specialty shop. Another was rather small, but handled a considerable quantity of chromium and other plating on a semiassembly-line basis. Another plant handled plating of outdoor electrical fixtures with cadmium for the most part. The fourth was a large plant, in which many types of metal processing were performed, including several plating operations. One plant discharged its wastes into a closed storm sewer, which later opened into a ditch; another, into one of the creeks of the city a certain distance upstream of the river; and the large plant, into a major creek a short distance from the mouth of the creek. The logical solution for these industries seems to be to take all the wastes from the small plants into the city system and make a thorough investigation of the large plant to ascertain if the concentration of all important toxic materials can be kept within allowable limits at the treatment plant. If such control cannot be achieved without pretreatment, then such a procedure will be required prior to the reception of this waste into the collection system.

The city contains several milk-processing plants, but only three were considered to be of special importance to the operation of the sewage-treatment plant. All manufacture cottage cheese, and at the end of each batch process all the whey and several rinses are released rapidly into the sewer system. A detailed study of the largest plant was made, including sampling of several weeks' duration. Operating practices were excellent, and, in general, the wastes contained negligible quantities of milk and milk products. However, the slug dumping of the cottage-cheese-whey vat imposed an extremely heavy B.O.D.-load for a short time. Various tests of the B.O.D. of the whey showed values ranging from 26,000 ppm to 48,000 ppm, or an average of approximately 36,000 ppm (all based on a 5-day B.O.D.). The whey increased the B.O.D.-population equivalent of the plant from 660 ppm without the whey to 2,800 ppm with the whey. With the whey load concentrated into less than 30 min, the slug effect for the short time period was that of a population equivalent of approximately 100,000 ppm. The effect on the treatment plant could be disastrous. All the plants were advised to remove the whey from the city sewer system completely, although using a 24-hr holding tank to release this by-product gradually into the system would have improved the situation.

Knoxville has two medium-sized slaughter houses, which are combined with meat-packing plants. One has been connected to the city sewer system for years. The other is near the Tennessee River and has used a private sewer to the river in the past. In constructing the interceptor system, a provision was made to take this plant into the city system. In both cases the packing houses were asked to remove blood, hair, and all similar material, and to install a dual unit vibrating screen to remove most of the suspended material from the paunch manure. They also were required to install an adequate and properly operated grease trap in order to exclude greasy and oily substances from the system. Except as noted previously, all liquid wastes are accepted on the same basis as other wastes.

A fertilizer plant some distance from the city system asked permission to build a private sewer to reach the system, but this request was refused on the basis of the low pH of the waste and its high total acidity. Laboratory tests have shown a total acidity of 35,000 ppm and a pH of about 1.0. The theoretical quantity of water required to raise the pH to 6.0 would be 100,000 parts of water with a pH of 7.0 to one part of the waste.

A difficult problem is that caused by a company that manufactures brown paper for making cardboard boxes. The plant uses the neutral sulfite process. In addition to approximately 1,250,000 gal per day of general wastes, the plant produces approximately 50,000 gal of waste cooking liquor every 24 hr. The writer's analysis of this liquor showed a 5-day B.O.D. of approximately 19,000 ppm and a quantity of total solids of 220,000 ppm. A 4-hr composite sample taken during normal plant operation showed that the B.O.D. of the over-all waste was approximately 930 ppm, and the total solids were 9,600 ppm. The over-all waste is dark brown, has a medicinal odor, and responds poorly to primary sewage treatment. Mixed with approximately 20×10^6 gal per day of city sewage, the waste imparts a distinct brown color to the entire mixture, and the same color goes through the sewage-treatment plant and into the Tennessee River. At low flows in the river there is noticeable color along the north bank for hundreds of yards below the outfall. In addition, the waste has a toxic effect on the normal bacteria in the sewage and may be harmful to the operation of the digesters at the treatment plant. At present, the waste is taken through the plant on a trial basis, but, in so far as removal of color and solids is concerned, the results have proved little except that the plant cannot handle this type of waste. It is probable that the removal of the waste cooking liquor, combined with a more careful operation of the paper plant, will produce a waste that the city plant can handle successfully and without the existing serious color problem.

Many other problems were investigated prior to the completion of the main sewage-treatment plant, and some have arisen since the plant began operation in 1955. An attempt was made to use foresight rather than hindsight in solving those problems specifically relating to industrial waste. No two city systems are the same, and, in all probability, it will be necessary for each city to derive its own solutions during the various construction phases of interceptors and treatment plants. The cases that have been presented are typical of those arising in Knoxville and include those that have received the most serious consideration. The city contains several textile mills, but, except for some minor color problems, no serious trouble is expected from these sources because of the sizable dilution available from the remainder of the city wastes.

CONCLUSIONS

The experiences that the city of Knoxville has had with industrial wastes in the city sewer system and treatment plant show that an adequate ordinance regulating such wastes is essential to the system. Model ordinances, such as that published[3] by the Federation of Sewage and Industrial Wastes, can be used as a general guide in the adoption of a local ordinance. Nevertheless, special local conditions must be considered, and for small cities a short, simple ordinance is usually preferable to an extensive one. In all cases in which industrial wastes are a major problem in the sewerage system, an engineering consultant should be employed by the city to sample the individual wastes, make analyses, and prepare reports on the proper treatment of each type of waste. Experiences such as those described herein are only a general guide for other cities in the handling of the problem.

---

[3] "Municipal Sewer Ordinances", *Manual of Practice No. 3*, Federation of Sewage and Industrial Wastes, 1949.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2933

SIMPLIFICATION OF DESIGN BY ULTIMATE STRENGTH PROCEDURES

By Phil M. Ferguson,[1] M. ASCE

With Discussion by Messrs. Tung Au; Herman S. Schick; Zdeněk Sobotka; Milan Spanovich; Sylvester M. Ulicny; and Phil M. Ferguson

Synopsis

Ultimate strength design procedures for beams and columns are developed for use with a rectangular stress block. Although the use of design charts is recommended, it is shown that most computations are simple and practical without such aids. Design procedures are simpler than present working stress methods. Numerical examples are presented.

Introduction

For the first time designers in the United States have a choice between long-established methods and the more recent method of ultimate strength design. In the new edition of the Building Code of the American Concrete Institute (ACI), design by this method is permitted as an alternate procedure. Furthermore, the "Report of the ASCE-ACI Joint Committee on Ultimate Strength Design"[2,3] (hereafter referred to as the report) is a milestone in reinforced concrete design, providing a sound basis for ultimate strength design.

The use of ultimate strength methods results in designs that are better balanced than those previously attained. Nevertheless, some designers, noting the lengthy formulas in the report, are concerned about design methods which might prove to be more involved and time consuming. The ultimate strength theory for flexure, or for combined flexure and axial load, can be simple, and ultimate strength design can also be a simple procedure. Such design is

---

Note.—Published, essentially as printed here, in July, 1956, in the Journal of the Structural Division, as *Proceedings Paper 1022*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Prof. of Civ. Eng., Univ. of Texas, Austin, Tex.

[2] "Report of ASCE-ACI Joint Committee on Ultimate Strength Design," *Proceedings Paper 809*, ASCE, October, 1955.

[3] "Ultimate Strength Design," by ACI-ASCE Committee 327, *Journal*, A.C.I. Vol. 27, January, 1956; *Proceedings*, Vol. 52, p. 505.

fundamentally less complex and more rapid than conventional straight-line theory.

The report does not suggest ultimate strength procedures for diagonal tension and bond. Thus, design for shear should still be governed by working stress conditions.

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically for convenience of reference in Appendix II.

Assumed Compression Stress Block.—On casual inspection, the report appears complex for two basic reasons: First, the committee did not feel that it was desirable to standardize on a single diagram the stress distribution in compression. "A rectangle, trapezoid, parabola, or any other shape which results in ultimate strength in reasonable agreement with tests" may be assumed. Secondly, formulas are given for permissible eccentric column loads. Although these formulas appear complicated, similar relationships in the case of working stress procedures are not even feasible. The fact that a direct relationship can now be written is evidence of the greater simplicity of ultimate strength relationships.

A designer must choose a specific distribution for compressive stresses. The writer believes that the rectangular stress block is best adapted to design at this time and that it is the simplest form. Other distributions may have an advantage for the analysis of research results, but their advantage for design remains to be demonstrated. Either the parabola, trapezoid, or rectangle seems to give the necessary accuracy for design. The presentation herein is restricted to the rectangular stress block pioneered in the United States by Charles S. Whitney,[4] M. ASCE.

Load Factors.—A proper choice of the load factors is important because ultimate strength design must be based on moments and shears large enough to provide an adequate safety factor. The recommended load factors also have a second purpose—assuring that tension cracking under working loads will not be excessive. Although six expressions for ultimate strength design are given, the designer can easily establish which govern specific cases. For instance, for a beam the first two relationships are

$$U = 1.2\,B + 2.4\,L \qquad (1)$$

and

$$U = K\,(B + L) = 1.8\,(B + L) \qquad (2)$$

in which U is the ultimate strength of the section; B denotes the effect of a basic load, consisting of the dead load plus volume change due to plastic and elastic actions, shrinkage, and temperature; L is the effect of the live load plus impact; and K represents the load factor, which is equal to 1.8 for beams subject to bending only. The second relationship gives a larger design ultimate on beams only when B is greater than L.

BEAM DESIGN

Rectangular Beams.—Under the provisions of the report, the possibility of a compression failure is entirely removed by limiting the percentage of tension steel to 0.40 f_c'/f_y, in which f_c' is the concrete strength and f_y represents the yield point stress of steel. This is approximately 12% less than balanced steel by the Whitney theory using the rectangular stress block. For a value of f_c' greater than 5,000 lb per sq in., this limiting percentage will be further reduced by 0.025 for each 1,000 lb per sq in. of excess. All beam design thus becomes design for tension.

---

[4] "Plastic Theory of Reinforced Concrete Design," by Charles S. Whitney, *Transactions*, ASCE, Vol. 107, 1942, p. 251.

A tension failure will always occur when the tension steel reaches the yield stress, f_y—that is, when $T = A_s f_y$. An arbitrary limit on the value of f_y used in computations is placed at 60,000 lb per sq in. in order to reduce crack size at working loads. The compressive stress block shown in Fig. 1, with a uniform stress of 0.85 f_c', is considered to be deep enough to make C, the total compression in the beam or the column carrying moment, equal to T, which is the total tension. From Fig. 1, $0.85\, f_c'\, b\, a = A_s\, f_y$, rearranging

$$a = \frac{A_s\, f_y}{0.85\, f_c'\, b} \quad \text{(3)}$$

Therefore, $M_u = T\, j\, d = A_s\, f_y \left(d - \frac{a}{2}\right) = A_s\, f_y\, d \left(1 - \frac{a}{2\, d}\right)$, in which b represents the width of the member; d is the depth measured from the compression face to the tension steel; a is the depth of the rectangular stress block; A_s denotes the area of the steel being used; M_u represents the ultimate bending moment; and j is the ratio of the distance between resultants of compressive stress and tension stress to depth d. Eq. 3 may be rewritten in the form of Eq. 4a and Eq. 4b, as in the report:

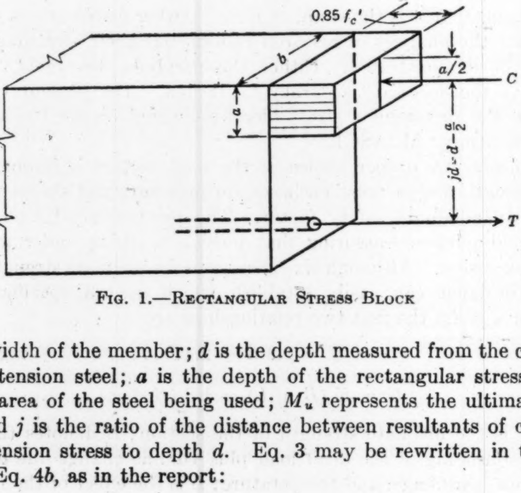


FIG. 1.—RECTANGULAR STRESS BLOCK

$$M_u = A_s\, f_y\, d \left(1 - \frac{0.59\, p\, f_y}{f_c'}\right) \quad \text{(4a)}$$

and

$$M_u = f_c'\, b\, d^2\, q\, (1 - 0.59\, q) \quad \text{(4b)}$$

in which $q = \frac{p\, f_y}{f_c'}$ and $p = \frac{A_s}{b\, d}$. The foregoing equations are limited to q-values not greater than 0.40, so that

$$\text{Maximum } p = 0.40\,\frac{f_c'}{f_y} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

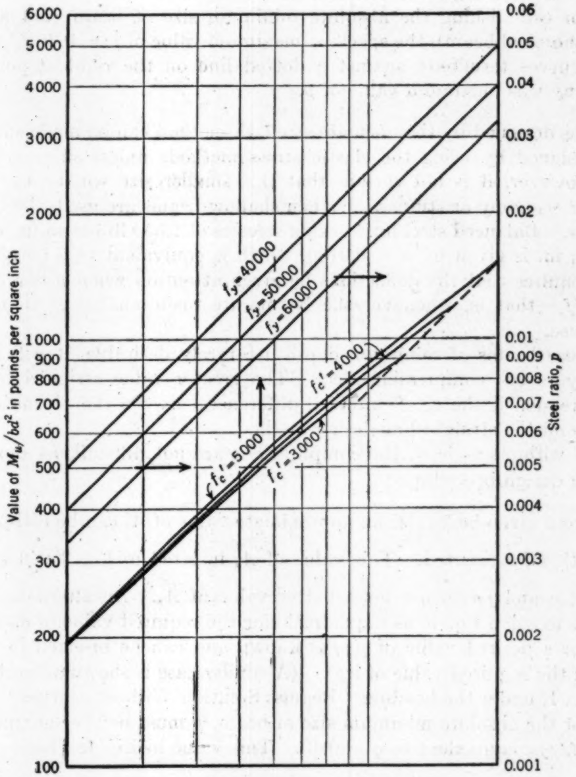


FIG. 2.—RECTANGULAR BEAM CURVES

The design chart of Fig. 2, reproduced from the report,[5] simplifies the use of these formulas as follows:

1. For a given beam size the chart is entered at the known value of $\frac{M_u}{b\,d^2}$, proceeding horizontally to the f_c'-curve, thence vertically to the f_y-curve, and horizontally again, yielding the required value of p, which represents $\frac{A_s}{b\,d}$.

---

[5] "Ultimate Strength Design," by ACI-ASCE Committee 327, *Journal*, A.C.I., Vol. 27, January, 1956; *Proceedings*, Vol. 52, p. 516, Fig. 4.

2. For a desired value of p, this sequence is reversed, reading the required $\left(\frac{M_u}{b\,d^2}\right)$-value to establish the beam size (problem 10 in Appendix I under the heading, "Beams: Solution with Curves").

3. For establishing the absolute minimum size of beam (not always the most economical beam), the specified maximum value of $p = 0.40\,f_c'/f_y$ is used. The f_c'-curves terminate against a dotted line on the right at points which accompany this maximum value of p.

Beams designed by the ultimate strength method can be made smaller than those designed by using the elastic stress methods unless shear controls the size. However, it is not certain that this smaller size will be the optimum either for economy or stiffness. When shallow beams are used, the deflections are larger. Balanced steel for working stresses of 1,350 lb per sq in. and 20,000 lb per sq in. is given by $p = 0.0136$, which is equivalent to $0.18\,f_c'/f_y$. The report requires that the deflections be given attention when p is greater than $0.18\,f_c'/f_y$—that is, whenever the beams are made shallower than those in current use.

Because beams of minimum depth are rarely desirable, it will seldom be necessary to use compression steel. The greater value assigned to concrete in compression is the chief practical difference between the ultimate strength method and the straight-line design method.

Even without a chart, the computations are not difficult, as shown in the following design procedures:

a. For a given beam size an approximate value of A_s can be estimated from $\frac{M_u}{f_y\,j\,d}$ with $j\,d$ estimated. This value of A_s inserted in Eq. 3 will give better values of a and $j\,d$ to use for a better value of A_s. An alternate procedure would be to solve Eq. 4*b* as a quadratic for the required value of q.

b. For a desired value of p, q is known and can be inserted in Eq. 4*b* to establish the required value of $b\,d^2$. (A similar case is shown as problem 10 in Appendix I, under the heading, "Beams: Solution Without Curves.")

c. For the absolute minimum size of beam, p must be the maximum—that is, $0.40\,f_c'/f_y$, equivalent to $q = 0.40$. This value in Eq. 4*b* yields

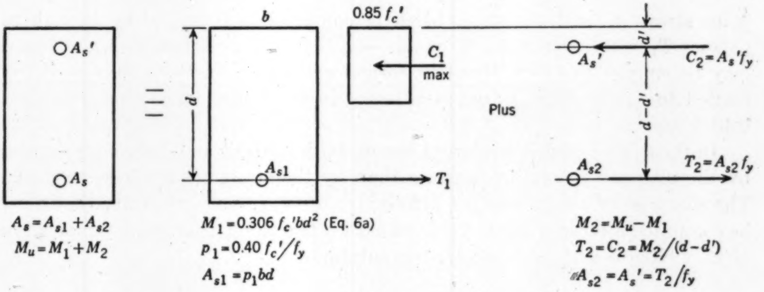
$$M_u = 0.306\,f_c'\,b\,d^2 \qquad (6a)$$

and, in Eq. 4*a*,

$$M_u = A_s\,f_y\,d\,[1 - (0.59 \times 0.40)] = A_s\,f_y\,(0.764\,d) \qquad (6b)$$

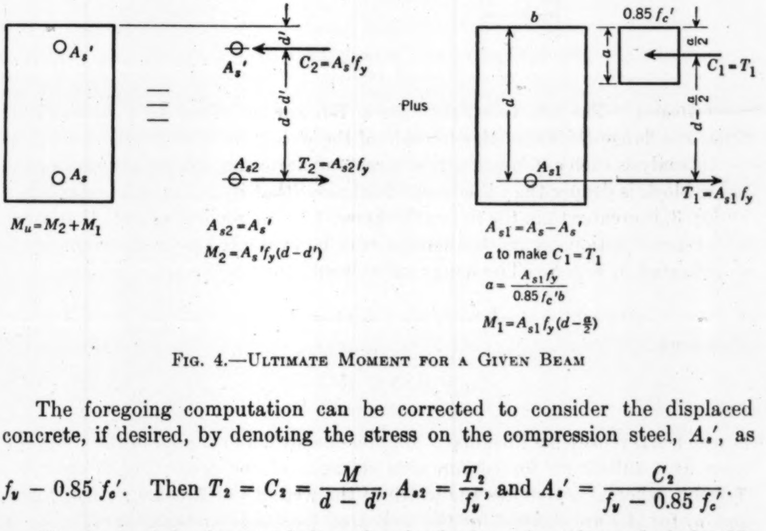
Compared with the straight-line stress procedures, the analysis of given beams is simpler by the ultimate strength theory. Design by ultimate strength methods is certainly no more difficult. Only one design chart is needed instead of one for each value of n.

Rectangular Beams with Compression Reinforcement.—Double reinforced beams seldom will be needed for strength (although compression steel can be used effectively in reducing the added deflection due to creep of concrete). These beams can be designed quickly by considering the total M_u subdivided into two bending couples—M_1, which can be resisted without compression steel and which is equal to Eq 6*a*, and M_2, which can be resisted by additional compression and tension steel, as shown in Fig. 3. The steel, A_{s1}, for the moment, M_1, is found from the maximum $p = 0.40\ f_c'/f_y$. The remaining moment, $M_2 = M_u - M_1$, is resisted by C_2 and T_2 as a couple with an arm, $d - d'$, and a steel stress equal to f_y. If the displaced compression concrete is neglected, then $A_{s2} = A_s' = \dfrac{M_2}{f_y\ (d - d')}$. The total, A_s, is then the sum of A_{s1} and A_{s2}.



A_s'
A_s
$A_s = A_{s1} + A_{s2}$
$M_u = M_1 + M_2$
b
d
A_{s1}
T_1
$0.85\ f_c'$
C_1 max
$M_1 = 0.306\ f_c'bd^2$ (Eq. 6a)
$p_1 = 0.40\ f_c'/f_y$
$A_{s1} = p_1 bd$
Plus
d'
A_s'
$C_2 = A_s'f_y$
$d - d'$
A_{s2}
$T_2 = A_{s2}f_y$
$M_2 = M_u - M_1$
$T_2 = C_2 = M_2/(d - d')$
$A_{s2} = A_s' = T_2/f_y$

FIG. 3.—DESIGN OF A DOUBLE REINFORCED BEAM



A_s'
A_s
$M_u = M_2 + M_1$
d'
A_s'
$C_2 = A_s'f_y$
$d - d'$
A_{s2}
$T_2 = A_{s2}f_y$
$A_{s2} = A_s'$
$M_2 = A_s'f_y(d - d')$
Plus
b
d
A_{s1}
$0.85\ f_c'$
a
$\frac{a}{2}$
$C_1 = T_1$
$d - \frac{a}{2}$
$T_1 = A_{s1}f_y$
$A_{s1} = A_s - A_s'$
a to make $C_1 = T_1$
$a = \frac{A_{s1}f_y}{0.85\ f_c'b}$
$M_1 = A_{s1}f_y(d - \frac{a}{2})$

FIG. 4.—ULTIMATE MOMENT FOR A GIVEN BEAM

The foregoing computation can be corrected to consider the displaced concrete, if desired, by denoting the stress on the compression steel, A_s', as $f_y - 0.85\ f_c'$. Then $T_2 = C_2 = \dfrac{M}{d - d'}$, $A_{s2} = \dfrac{T_2}{f_y}$, and $A_s' = \dfrac{C_2}{f_y - 0.85\ f_c'}$.

For given reinforcing, not balanced as in the foregoing, the analysis for M_u shown in Fig. 4 is recommended, which is in accordance with Eq. 4 of the report,

$$M_u = (A_s - A_s') f_y d \left(1 - \frac{0.59 (p - p') f_y}{f_c'}\right) + A_s' f_y (d - d') \ldots (7)$$

In Eq. 7, p' is the symbol for $\frac{A_s'}{b d}$. Both A_s' and A_s are assumed at the yield point stress, f_y, and the stress block in compression is only deep enough to balance T_1, which equals $A_{s1} f_y$, or $(A_s - A_s') f_y$. The pattern shown in Fig. 4 may be simpler to follow than the formal equation. In this instance, A_{s1} is limited to $p_1 = 0.40 f_c'/f_y$, or, as is stated in the report, maximum $p - p' = 0.40 f_c'/f_y$.

In the case of double reinforced beams, both analysis and design are simpler by the ultimate strength procedure than by the straight-line stress methods. The compression steel stress is a definite quantity, f_y, not a quantity that must be computed after locating the neutral axis. The same rectangular beam curve already cited is useful for the M_1-computation.

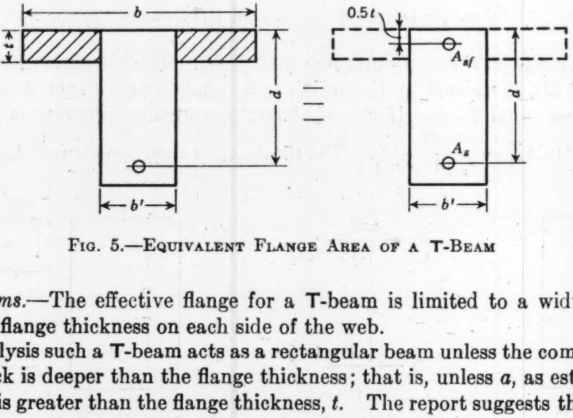


FIG. 5.—EQUIVALENT FLANGE AREA OF A T-BEAM

T-*Beams.*—The effective flange for a T-beam is limited to a width of six times the flange thickness on each side of the web.

In analysis such a T-beam acts as a rectangular beam unless the compression stress block is deeper than the flange thickness; that is, unless a, as established by Eq. 3, is greater than the flange thickness, t. The report suggests that when real T-beam action occurs, the flanges may be evaluated as compression steel, as indicated in Fig. 5. The flange value would then be

$$0.85 f_c' (b - b') t = A_{sf} f_y$$

Therefore,

$$A_{sf} = 0.85 f_c' (b - b') \frac{t}{f_y} \ldots\ldots\ldots\ldots (8)$$

in which A_{sf} represents imaginary compression steel and is used in the computations as a substitute for compressive strength of the projecting flanges of a T-beam, whereas b' denotes the width of the web of the T-beam. If p_w for A_s and p_f for A_{sf} are defined for the web area (rectangular beam area) as $p_w = \frac{A_s}{b' d}$ and $p_f = \frac{A_{sf}}{b' d}$, and if A_{sf} is inserted at midflange depth, then Eq. 7 can be adapted directly to give the report equation,

$$M_u = (A_s - A_{sf}) f_y d \left[1 - \frac{0.59 (p_w - p_f) f_y}{f_c'}\right] + A_{sf} f_y (d - 0.5\,t) \,..\, (9)$$

In designing steel for a given T-beam under a given value of M, a simple procedure would consist of:

1. Assuming $j\,d = d - 0.5\,t$ and computing an approximate value of $A_s = \frac{M_u}{f_y (d - 0.5\,t)}$;
2. Computing an approximate value of $a = \frac{A_s f_y}{0.85 f_c' b}$;
3. If $a > t$, computing A_{sf} from Eq. 8, providing a balancing tension area, $A_{s2} = A_{sf}$, and computing $M_2 = A_{sf} f_y (d - 0.5\,t)$; if $a < t$, see item 6;
4. Computing $M_1 = M_u - M_2$ and designing the steel, A_{s1}, for this moment, considering the web only as a rectangular beam (and noting the report limit on A_{s1}, as for a rectangular beam, to a maximum percentage of $0.40\,f_c'/f_y$, based on a web area, $b'\,d$); and
5. Obtaining the total tension steel as A_{s1} plus A_{s2}; and
6. If $a < t$, computing a value of A_s as for a rectangular beam of width b.

Such methods avoid the complex problem of locating a resultant C-value or determining $j\,d$ by elastic methods. Only one rectangular beam curve is needed to speed up this design.

COLUMNS

Concentrically Loaded Columns.—The report uses the theoretical concentric load capacity established by column tests reported[6] in 1933:

$$P_0 = 0.85\,f_c' (A_g - A_{st}) + A_{st} f_y \ldots\ldots\ldots\ldots\ldots (10)$$

in which A_g is the gross area of the column and A_{st} is the cross-sectional area of all the longitudinal bars. The report goes much further and recommends that every column be designed for eccentricity, a minimum of 0.05 times the depth for a spiral column and a minimum of 0.10 times the depth for a tied column. Hence, no column should be designed as axially loaded, and the foregoing value of P_0 is only a theoretical limiting capacity. The values of the ultimate load, P_u, for a short, eccentrically loaded column will be examined subsequently.

For a column with an unsupported length, h, greater than fifteen times its least lateral dimension, t, the maximum axial load, P_u', should not be greater than

$$P_u' = P_0 \left(1.6 - 0.04\,\frac{h}{t}\right) \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

in which P_0 is the maximum concentric load capacity of the section with $h/t < 15$. For large eccentricities, P_u (which is independent of h/t) will often be smaller than P_u' and will govern for either short or long columns.

---

[6] "Reinforced Concrete Column Investigation—Tentative Final Report of Committee 105, F. E. Richart, Chairman," *Journal*, A.C.I., Vol. 4, February, 1933; *Proceedings*, Vol. 29, p. 275.

Combined Direct Stress and Bending.—In combined direct stress and bending, the simple relationship, $T = C$, used for beams must be replaced by the slightly more complex summation of axial forces equal to zero. In Fig. 6(*a*), in which C_s represents the total force on A_s' and C_c denotes the resultant of the compression on the concrete,

$$P_u + T - C_c - C_s = 0 \qquad (12a)$$

Rearranging Eq. 12*a* yields

$$P_u = C_c + C_s - T \qquad (12b)$$

The report, using the notation of Fig. 6(*b*), gives this expression as

$$P_u = 0.85\, f_c'\, b\, d\, k_u\, k_1 + A_s'\, f_y - A_s\, f_y \qquad (12c)$$

It will be noted that *e* can be so small that no tension is caused, which requires a different form for the foregoing equation.

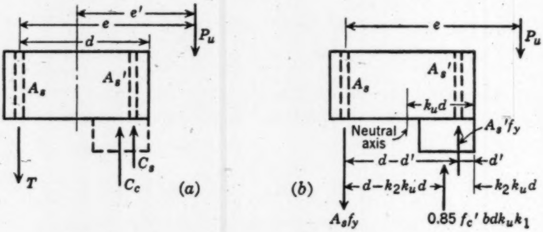


FIG. 6.—COMBINED DIRECT STRESS AND BENDING

Those cases which can fail in tension and those which must fail in compression should be studied separately. The report defines the boundary between the two cases in terms of what can be termed the balanced load, P_b, which is equally likely to cause failure in tension or compression,

$$P_b = 0.72 \left(\frac{90{,}000}{90{,}000 + f_y} \right) f_c'\, b\, d + A_s'\, f_y - A_s\, f_y \qquad (13)$$

If the ultimate load, $P_u < P_b$, only a tension failure is possible. If $P_u > P_b$, only a compression failure is possible.

Tension Failures—Basic Theory.—When the tension steel yields, the neutral axis shifts toward the compression face and a secondary compression failure occurs. At failure the compression steel is assumed at yield stress, and the final compression stress block is considered to be reduced to the minimum consistent with the compression required by Eq. 12*b*.

The report uses the summation of the moments about the tension steel for the second equilibrium equation,

$$P_u\, e = C_c\, (d - k_2\, k_u\, d) + C_s\, (d - d') \qquad (14a)$$

and

$$P_u e = 0.85 f_c' b d^2 k_u k_1 (1 - k_2 k_u) + A_s' f_y (d - d') \ldots\ldots (14b)$$

In Eq. 14*b* the eccentricity of the axial load, which is measured from the centroid of the tension steel, is denoted by e; k_1 is the ratio of the average compressive stress to $0.85 f_c'$; and k_2 represents the ratio of distance between the extreme fiber and the resultant of compressive stresses to $k_u d$ (Fig. 6(*b*)). The ratio of the distance between the extreme fiber and the neutral axis to depth d is denoted by k_u. For Eq. 12*b* and Eq. 14, k_2/k_1 should not be taken as less than 0.5, and k_1 should not be taken greater than 0.85.

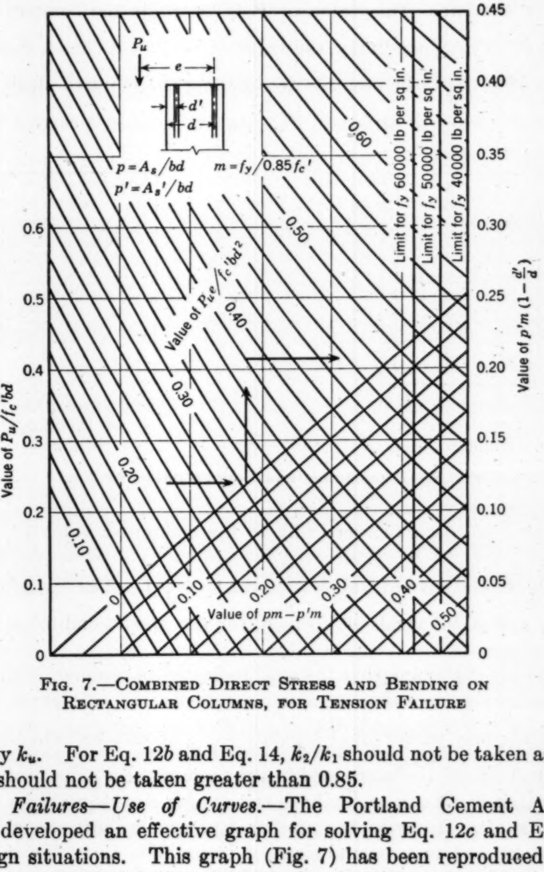
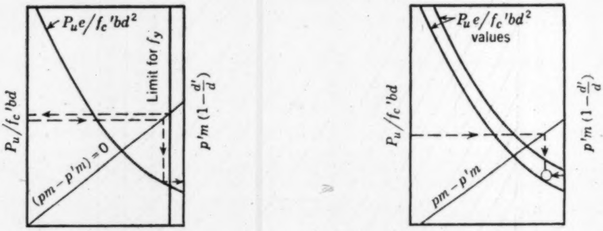


FIG. 7.—COMBINED DIRECT STRESS AND BENDING ON RECTANGULAR COLUMNS, FOR TENSION FAILURE

Tension Failures—Use of Curves.—The Portland Cement Association (PCA) has developed an effective graph for solving Eq. 12*c* and Eqs. 14 for several design situations. This graph (Fig. 7) has been reproduced from the report, and an error in labeling the right-hand ordinate has been corrected. The percentage of steel is taken separately for tension and compression, and each percentage is based on the area, $b\,d$, rather than on the gross column area. This graph was developed on the basis of a trapezoidal stress distribution, but the results are equally applicable to the rectangular stress block. The simplicity of the solutions for several practical cases are demonstrated as follows:

Problem 1.—The determination of the minimum column size and steel is required for a given load and eccentricity, using symmetrical steel.

Because $m = \frac{f_y}{0.85 f_c'}$, the value of m is known. Symmetrical steel yields $p\,m - p'\,m = 0$. The intersection in Fig. 8(*a*) of the line marked "$p\,m - p'\,m = 0$" with the vertical line marked "limit for f_y" establishes the proper ordinate for $\frac{P_u}{f_c'\,b\,d}$ on the left. This value of P_u/f_c' permits the computation of the minimum $b\,d$. Correspondingly, the value of $b\,d$ that is chosen determines the exact value of $\frac{P_u}{f_c'\,b\,d}$ to be used for determining the required steel. As shown by the dotted lines with arrows in Fig. 8(*a*), this value is carried horizontally to the $(p\,m - p'\,m)$-curve; vertically to the computed value of $\frac{P_u\,e}{f_c'\,b\,d^2}$; and horizontally to the needed value of $p'\,m\left(1 - \frac{d'}{d}\right)$. The value of p' can be computed, and $p'\,b\,d$ yields A_s', which, for the given conditions, requires an equal A_s-value.



(*a*) DETERMINATION OF COLUMN SIZE AND REQUIRED STEEL

(*b*) DETERMINATION OF ULTIMATE MOMENT AND *e*

FIG. 8.—PROCEDURES FOR USING CURVES OF FIG. 7

Problem 2.—Assuming that the steel indicated in problem 1 is more than is desirable to use, or more than the specifications permit, a larger column should be considered; new values of $\frac{P_u}{f_c'\,b\,d}$ and $\frac{P_u\,e}{f_c'\,b\,d^2}$ should be computed; and the required steel should be determined in the same manner as in problem 1. Except by trial, it is difficult to establish the column size required for a particular desired value of p. However, the preceding process is short enough to permit a quick solution by trying several sizes, as in problem 12 in Appendix I, under the heading, "Columns: Solution Using Curves."

Problem 3.—Unsymmetrical steel for any given value of $p\,m - p'\,m$ involves no change in procedure except for the use of the appropriate $(p\,m - p'\,m)$-curve.

Problem 4.—The permissible ultimate moment, M_u, with a given value of P_u on a given column can be found by beginning with $\frac{P_u}{f_c'\,b\,d}$ on the left and $p'\,m\left(1 - \frac{d'}{d}\right)$ on the right, as shown in Fig. 8(*b*). The circled point establishes $\frac{P_u\,e}{f_c'\,b\,d^2}$, from which e and $M_u = P_u\,e'$ can be found.

Problem 5.—The ultimate load, P_u, at a given e-distance for a given column requires multiple trials and can be obtained more easily from Eq. 9 of the report:

$$P_u = 0.85\,f_c'\,b\,d\left(p'\,m' - p\,m + 1 - \frac{e}{d}\right)$$

$$+ \sqrt{\left(1 - \frac{e}{d}\right)^2 + 2\,\frac{e}{d}\,(p\,m - p'\,m') + p'\,m'\left(1 - \frac{d'}{d}\right)} \quad \ldots (15)$$

in which $m' = m - 1$.

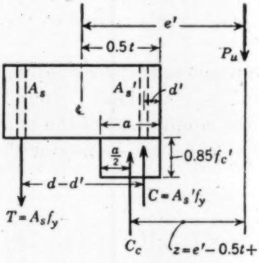


FIG. 9.—FORCES IN EQUILIBRIUM, TENSION FAILURE OF COLUMN

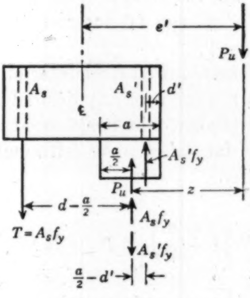


FIG. 10.—EQUILIBRIUM COUPLES WITH $A_s' \neq A_s$

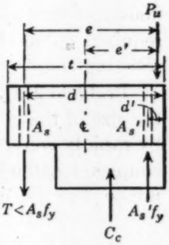


FIG. 11.—FORCES IN EQUILIBRIUM COMPRESSION FAILURE OF COLUMN

Tension Failures—Design Without Curves.—A better understanding of tension failures is obtained by considering solutions based on simple equilibrium equations. For symmetrical steel and a rectangular stress block, such solutions are so simple that there is little need for curves and formulas.

The fundamental relationships will be established from Fig. 9. If $A_s = A_s'$, then $T = C_s$, and the summation of vertical forces leads to $C_c = P_u$. For an average value of $f_c = 0.85\,f_c'$, the depth, a, of the stress block must be such that

$$C_c = 0.85\,f_c'\,b\,a = P_u \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

The report states that the average concrete stress is limited to k_1 $(0.85\,f_c')$, with k_1 not exceeding 0.85. However, k_1 always appears as a part of the product, $0.85\,f_c'\,b\,d\,k_u\,k_1$. For design it is simpler to consider the average concrete stress as $0.84\,f_c'$ and to define the depth of stress block in terms of $a = k_1\,k_u\,d$. The result is mathematically identical.

Using the moment arm, z, from Fig. 11, the couple, $C_c\,z$ (or $P_u\,z$), must be balanced by the couple, $C_s\,(d - d')$. These two relationships are simple to use either in analysis or design, which will be illustrated by the following examples, which are similar to those already outlined for solution by curves.

Problem 1(a).—The determination of the minimum column size and steel is required for a given load and eccentricity, using symmetrical steel.

Eq. 13 gives the report value of P_b, which is the maximum load that can accompany a tension failure. With $A_s = A_s'$ and with P_u used for P_b, the minimum value of $b\,d$ can be found as follows because all other terms are known:

$$P_u = 0.72\left(\frac{90{,}000}{90{,}000 + f_y}\right) f_c'\,b\,d \ldots\ldots\ldots\ldots\ldots (17)$$

For this value of $b\,d$, or any other value of $b\,d$, the depth of the stress block, a, can be determined from $C_c = 0.85\,f_c'\,b\,a = P_u$. Because C_c is located in the middle of the stress block, the arm, z, of the (C_c and P_u)-couple is known. The required value of $A_s' = A_s$ can be determined by equating this couple to the resisting couple, $P_u\,z = A_s'\,f_y\,(d - d')$.

Problem 2(a).—If the required steel determined in problem 1(a) is excessive, the size of the column required for any desired percentage of steel can be found by rapidly converging trials. The basic equation is the equilibrium of the two couples, written herein for $A_s' = p'\,b\,t$ (although $A_s' = p'\,b\,d$ can be used with minor changes, if desired),

$$P_u\,z = P_u\left(e' - 0.5\,t + \frac{a}{2}\right) = p'\,b\,t\,f_y\,(d - d') = p'\,t^2\,f_y\,(t - 2\,d') \ldots (18)$$

in which e' represents the eccentricity of the axial load measured from the centroid of the member. For a given value of e, with d' and p' known, any reasonable estimate for a leads to an equation that can be solved for the required value of t. For this value of t, a can be computed from $C_c = P_u = 0.85\,f_c'\,t\,a$. If necessary, t can be recomputed, or z can be computed and used to establish A_s'—$P_u\,z = A_s'\,f_y\,(d - d')$. Problem 12 in Appendix I (under the heading, "Columns: Solution Without Curves") is a numerical example.

Problem 3(a).—The case of unsymmetrical steel is more involved, but the designer may find that Fig. 10 leads to a simpler evaluation, in terms of couples, than does the formal equation of the moments about the steel. For a value of $A_s > A_s'$,

$$P_u = C_c + C_s - T \ldots\ldots\ldots\ldots\ldots (19)$$

$$P_u = C_c + A_s'\,f_y - A_s\,f_y \ldots\ldots\ldots\ldots\ldots (20)$$

and

$$= P_u + A_s\,f_y - A_s'\,f_y \ldots\ldots\ldots\ldots\ldots (21)$$

When C_c is shown vectorially, as in Fig. 10, the three couples in equilibrium are $P_u\,z$, clockwise; $A_s\,f_y\left(d - \frac{a}{2}\right)$, counterclockwise; and $A_s'\,f_y\left(\frac{a}{2} - d'\right)$, counterclockwise. Thus, it becomes apparent that the moment effectiveness of A_s' is relatively small because of its small arm. From the point of view of design, A_s' does little good unless C_c has already developed a large depth of stress block a.

Problem 4(a).—The permissible ultimate moment, M_u, for a given value of P_u on a given symmetrical column can be found directly. Because $P_u = C_c$, the depth of the stress block is established. In the same manner, the steel resisting couple equal to $P_u\,z$ establishes z and e'.

Problem 5(a).—The ultimate load, P_u, at a given distance, e, for a given symmetrical column can also be determined directly. The steel resisting couple, $A_s'\,f_y\,(d - d')$, establishes the magnitude of the couple, $P_u\,z$,

$$z = e' - 0.5\,t + 0.5\,a$$

The stress block depth, a, is defined by

$$\frac{P_u}{0.85\,f_c'\,b} \qquad \text{(22)}$$

yielding

$$A_s'\,f_y\,(d - d') = P_u\,z = P_u\left(e' - 0.5\,t + \frac{1}{2}\,\frac{P_u}{0.85\,f_c'\,b}\right) \qquad \text{(23)}$$

The only unknown is P_u.

Compression Failures—Basic Theory.—The first term on the right-hand side of Eq. 13 establishes the maximum compression that can exist on the concrete along with a tension failure. The corresponding maximum depth of stress block a for this condition can be established from

$$0.72\left(\frac{90{,}000}{90{,}000 + f_y}\right) f_c'\,b\,d = 0.85\,f_c'\,b\,a \qquad \text{(24)}$$

Those who are familiar with Whitney's analysis will note that this limiting value of a is not his fixed value of $0.537\,d$ but rather a function of f_y. For $A_s' = A_s$, the limiting value of e for this boundary condition can be established from

$$P_u\,z = A_s'\,f_y\,(d - d') \qquad \text{(25)}$$

For a lesser value of e that is equivalent to a value of $P_u > P_b$, the tension steel stress is less than f_y. It may become zero or may even become compression. In this range, unit deformations must be considered. Analyses based on deformations lead to the conclusion that over this range, from tension failure to zero eccentricity, an approximate linear relationship exists between the axial load and the moment. The report recommends that either of two relationships for such compression failures be used, one of which (shown in Eq. 13 of the report) is

$$P_u = \frac{A_s' f_y}{\dfrac{e}{d - d'}} + \frac{b\,t\,f_c'}{\dfrac{3\,t\,(e - 0.5\,t + d')}{d^2} + 1.18} \qquad (26)$$

in which P_u is the axial load on the section; e is the eccentricity of the axial load measured from the centroid of the tensile reinforcement; and b is the width of the section. Fig. 11 shows d, d', and t. This expression is identical to Whitney's semiempirical equation,[4] except that it is written in terms of e instead of e' and the term, d', is defined herein as concrete cover.

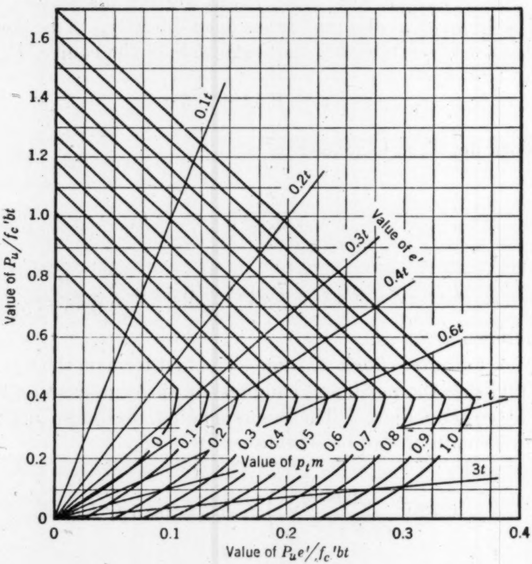


FIG. 12.—DESIGN CURVES FOR RECTANGULAR COLUMNS ($d = 0.8\,t$) WITH COMBINED DIRECT STRESS AND BENDING

Compression Failures—Design with Curves.—On the basis of Eq. 26, Whitney has prepared a series of design charts,[7] which are published in the report in the form of interaction diagrams usable for either tension or compression failures. Because of the spacing of (e'/t)-lines, as shown in Fig. 12,[8] these charts are most suitable for design for values of e'/t that are 1.0 or less. This was the reason the PCA curve of Fig. 7 was recommended for design involving tension failures. The use of Whitney's curves will be illustrated.

---

[7] "Application of Plastic Theory to the Design of Modern Reinforced Concrete Structures," by Charles S. Whitney, *Journal*, Boston Soc. of Civ. Engrs., Vol. 35, January, 1948, p. 29.

[8] "Ultimate Strength Design," by A. C. I.-ASCE Committee 327, *Journal*, A. C. I., Vol. 27, January, 1956; *Proceedings*, Vol. 52, p. 519, Fig. 6(a).

Problem 6.—The design of a column for a given value of P_u, an approximate total steel value of p_t equal to 2%, and the minimum eccentricity, e', of 0.1 t is desired.

These curves use p as a percentage of the gross column area, $b\,t$. For given materials, $p\,m$ is known. The intersection of the line, $e' = 0.1\,t$, with the curve for the desired value of $p_t\,m$ gives a point that can be projected horizontally to the left (Fig. 12) for a required value of $\frac{P_u}{f_c'\,b\,t}$ and, thus, the required value of $b\,t$. The only successive approximation element involves the cover over the steel—that is, d in terms of t. Curves are available for $d = 0.80\,t, 0.85\,t, 0.90\,t,$ and $0.95\,t$, and the extreme range of values of $\frac{P_u}{f_c'\,b\,t}$ for these four cases with $e = 0.1\,t$ seems to be only approximately 10%. A numerical example is shown as problem 11 in Appendix I (under the heading, "Columns: Solution Using Curves").

Problem 7.—An analysis of a given column to obtain the maximum value of P_u for a given value of e' (or the reverse) is directly accomplished. Entering the chart of Fig. 12 with a value of e'/t and $p_t\,m$ and proceeding to an intersection establishes $\frac{P_u}{f_c'\,b\,t}$ (as in problem 6), and P_u can be computed from this quantity.

For a given value of P_u, entering the curve at a given value of $\frac{P_u}{f_c'\,b\,t}$, then moving horizontally to the proper $(p_t\,m)$-curve, establishes the permissible value of e'/t, from which the permissible value of e' may be determined.

Problem 8.—To design a column for given values of P_u and e', as well as for an approximate fixed value of p_t, requires successive approximations. The values of d/t must be estimated to select the correct chart. The value of e'/t must be estimated in order to enter the chart and find the intersection with the desired $(p_t\,m)$-curve. The intersection fixes a required value of $\frac{P_u}{f_c'\,b\,t}$ as the ordinate, or of $\frac{P_u\,e'}{f_c'\,b\,t^2}$ as the abscissa. Either value establishes the column size, which is subject to further correction if either d/t or e'/t has been estimated incorrectly.

Compression Failures—Design Without Curves.—The use of Eq. 26 directly for design proves to be simpler than might be expected. The sequence in the subsequent paragraph is suggested for symmetrical steel.

An assumption is made regarding the column size, t. The known concrete cover leads to a known value of $d - d'$, and $e = e' + \frac{d - d'}{2}$, in which e' measured from the center line may be given in inches or as 0.1 t. The value of A_s' can be expressed as $p'\,t^2$, in which p' is the desired steel ratio for the steel on one face. Substitution of these values in Eq. 26, with t^2 in the numerators left as a symbol, leads to P_u = (constant) (t^2) and a desired value of t, which is not very sensitive to the original assumed value. If desired, a second computation can be made on the basis of the new t-value. When t is established, the original form of Eq. 26 can be used to establish the exact required value of A_s', which is the only remaining unknown. This procedure is described in problem 11 in Appendix I (under the heading, "Columns: Solution Without Curves").

If the value of eccentricity indicates the possibility of a tension failure, it is advisable to compare P_b from Eq. 13 with the given value of P_u. This analysis is valid only if $P_u \geqq P_b$—that is, for symmetrical steel if

$$P_u \geqq P_b = 0.72 \left(\frac{90{,}000}{90{,}000 + f_y} \right) f_c' \, b \, d \ldots\ldots\ldots\ldots (27)$$

If $P_u < P_b$, then tension governs the design.

A second design procedure is given herein, which shows more clearly what is involved in the use of Eq. 26, even though it does not seem to shorten the computations.

If Eq. 26 is plotted with axes P_u and $M_u = P_u \, e'$, it is almost a straight line, as shown in Fig. 13(*a*). The approximate line should pass through the point representing the limiting case of P_b for tension failure. However, with many values of d' and f_y, it misses the point by a small margin. This discrepancy is negligible and can be ignored. Only the upper part of this line is significant. It is convenient to define and use the intercepts on both axes, namely, P_0 and M_0, the latter being the computed ultimate bending-moment resistance, ignoring any weakness in tension—

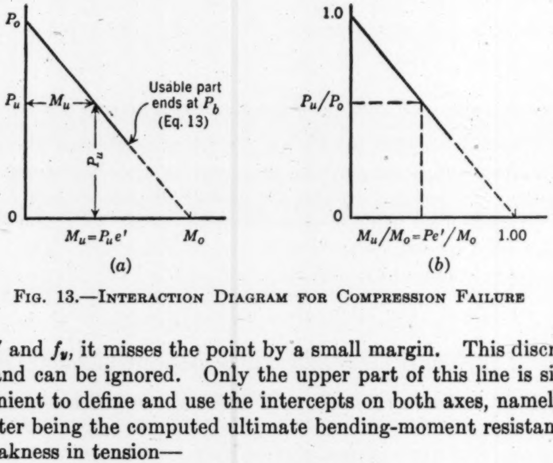


FIG. 13.—INTERACTION DIAGRAM FOR COMPRESSION FAILURE

$$e' = 0, M_u = 0, P_0 = 0.85 \, f_c' \, b \, t + 2 \, A_s' \, f_y \ldots\ldots\ldots\ldots (28)$$

and

$$e' = \infty, P_u = 0, M_0 = A_s' \, f_y \, (d - d') + \frac{1}{3} f_c' \, b \, d^2 \ldots\ldots\ldots (29)$$

Eq. 28 is the same as the value of P_0 in Eq. 10, except that it ignores the concrete displaced by the steel and shows $2 \, A_s'$ instead of the total A_{st}. The foregoing differences are negligible. Therefore, it is recommended that P_0 be taken as the value of Eq. 10 in cases of symmetrical steel,

$$P_0 = 0.85 \, f_c' \, (A_g - A_{st}) + 2 \, A_s' \, f_y \ldots\ldots\ldots\ldots\ldots (30)$$

When the curve of Fig. 13(*a*) is replotted in dimensionless form, as shown in Fig. 13(*b*),

$$\frac{P_u}{P_0} + \frac{M_u}{M_0} = 1 \qquad (31)$$

or

$$P_0 = \frac{P_u}{1 - \dfrac{M_u}{M_0}} \qquad (32)$$

or

$$P_u = P_0 \left(1 - \frac{M_u}{M_0}\right) \qquad (33)$$

Eq. 32 is desirable because it defines what might be termed the equivalent axial load. It can be solved by trial—that is, by assuming the ratio, M_u/M_0, computing P_0, picking a column, and computing the value of M_0 of the column to provide a better (M_u/M_0)-ratio for use in another trial. Eq. 33 is an alternate form that is useful for checking a given column. Both equations are limited to values of $P_u \underset{>}{\equiv} P_b$ of Eq. 13.

Circular Columns.—To the writer's knowledge, there are no design aids published for the circular column case (however, such curves are presented in footnote reference 21 from the closing discussion), or for the square column with a circular pattern of steel. The report contains equations for P_u, for both tension and compression failures, which indicate that graphs such as those already cited can be made for these forms of columns.

The report equation for compression failure of a circular column is

$$P_u = \frac{A_{st}\, f_y}{\dfrac{3\, e'}{d} + 1} + \frac{A_g\, f_c'}{\dfrac{9.6\, D\, e'}{(0.8\, D + 0.67\, d)^2} + 1.18} \qquad (34)$$

in which D is the diameter of the column and d is the diameter of the circle circumscribing the reinforcement. Eq. 34 can also be used for design, as outlined for the use of Eq. 26 with rectangular columns.

If e' is large the column capacity could be limited by tension, for which

$$P_u = 0.85\, D^2 f_c' \left[\sqrt{\left(\frac{0.85\, e'}{D} - 0.377\right)^2 + \frac{p_t\, m\, d}{2.5\, D}} - \left(\frac{0.85\, e'}{D} - 0.377\right)\right] \qquad (35)$$

in which p_t equals $\dfrac{A_{st}}{b'\, d}$. When tension governs, Eq. 35 can be solved quickly for a permissible value of P_u at any given value of e' by using a trial value of D and the desired $(p_t\, m)$-value. A proper value of D can be established by a few trials, after which the equation can be solved for the needed value of $p_t\, m$.

Columns with Bending About Both Axes.—The use of a uniform stress of $0.85\, f_c'$ over the compression area makes possible a simpler solution for loads hat are eccentric about both major column axes than the elastic analysis solution. Hermann Craemer has presented a straightforward solution[9] of this problem.

CONCLUSIONS

It has been shown how a great variety of reinforced concrete members can be designed with only the aids contained in the report, or even without such aids. No single case of design procedure is more difficult than that required by the present working stress methods, and most cases are substantially simpler. Appendix I includes several numerical solutions that apply these methods.

If ultimate strength design is used to produce shallower beams, more attention must be given to deflections. However, ultimate strength design need not lead at once to shallower members. It is the designer's responsibility to choose as he sees fit.

In time more design aids undoubtedly will be available to shorten design procedures. The writer believes that ultimate strength design is not only better but that it is also simpler.

APPENDIX I. TYPICAL PROBLEMS

BEAMS

Problem 9.—Compute the ultimate moment capacity of a rectangular beam with $b = 15$ in., $d = 24$ in., $f_c' = 3{,}000$ lb per sq in., $A_s = 3.00$ sq in., and $f_y = 50{,}000$ lb per sq in.

Solution Using Curves.—Entering Fig. 2 on the right with $p = 300/15 \times 24 = 0.00833$, proceeding horizontally to $f_y = 50{,}000$, thence vertically to $f_c' = 3{,}000$, then horizontally to the left yields $\frac{M_u}{b\,d^2}$ as approximately 370 lb per sq in. Therefore,

$$M_u = 370 \times 15 \left(\frac{\overline{24^2}}{12}\right) = 267{,}000 \text{ ft-lb}$$

Solution Without Curves.—The maximum value of p is

$$p = 0.40 \frac{f_c'}{f_y} = 0.40 \left(\frac{3{,}000}{50{,}000}\right) = 0.0240$$

The actual $A_s <$ maximum; therefore, the depth of stress block a is

$$a = \frac{A_s\,f_y}{0.85\,f_c'\,b} = \frac{3.0\ (50{,}000)}{(0.85)\ (3{,}000)\ (15)} = 3.92 \text{ in.}$$

The arm of the couple is

$$24 - \frac{3.92}{2} = 22.04 \text{ in.}$$

---

[9] "Skew Bending in Concrete Computed by Plasticity," by Hermann Craemer, *Journal*, A.C.I., Vol. 23, February, 1952, p. 516.

Therefore,

$$M_u = 3.0\ (50{,}000)\ \frac{22.04}{12} = 276{,}000 \text{ ft-lb}$$

The agreement would be closer if a curve with a finer scale were available.

Problem 10.—Design a rectangular beam to carry an ultimate moment of 200 kip-ft with $f_c' = 3{,}000$ lb per sq in. and $f_y = 50{,}000$ lb per sq in.

Solution.—The absolute minimum size would be obtained by using

$$p = 0.40 \left(\frac{f_c'}{f_y} \right) = 0.40 \left(\frac{3{,}000}{50{,}000} \right) = 0.0240$$

Because this would require a careful analysis of deflection, p will be limited to the value of $0.18\ f_c'/f_y$ suggested by the report as the break point at which such special analysis is necessary. Therefore,

$$p = 0.18 \left(\frac{3{,}000}{50{,}000} \right) = 0.0108$$

Solution with Curves.—Entering Fig. 2 with $p = 0.0108$, as in problem 9, and reading $\frac{M_u}{b\,d^2}$ as approximately 490, the required $(b\,d^2)$-value is

$$b\,d^2 = \frac{M_u}{490} = \frac{200{,}000\ (12)}{490} = 4{,}900$$

If $b = 12$ in., then $d = 20.2$ in. Therefore, use $b = 12$ in. and $d = 21$ in. The actual value of $\frac{M_u}{b\,d^2}$ becomes

$$\frac{200{,}000\ (12)}{12\ (\overline{21})^2} = 453$$

Entering Fig. 2 with this value, proceeding horizontally to $f_c' = 3{,}000$ lb per sq in., thence vertically to $f_y = 50{,}000$ lb per sq in., and horizontally yields $p = 0.0098$. The required value of $A_s = p\,b\,d = 0.0098\ (12)\ (21) = 2.47$ sq in.

Solution Without Curves.—The minimum possible beam size would be obtained by computing $b\,d^2$ from Eq. 6*a*. If, as previously stated, $p = 0.18\ f_c'/f_y$ is used to avoid special deflection problems, then $q = p\,f_y/f_c' = 0.18$. This value in Eq. 4*b* yields

$$M_u = f_c'\,b\,d^2\,q\,(1 - 0.59\,q)$$

Therefore,

$$200{,}000\ (12) = 3{,}000\ b\,d^2\ (0.18)\ (1 - 0.59 \times 0.18) = 482\ b\,d^2$$

and

$$b\,d^2 = 4{,}980 \text{ cu in.}$$

Then use $b = 12$ in.; therefore, $d = 20.3$ in., or 21 in. Although p will be slightly reduced when d is increased to 21 in. $p = 0.18 \times f_c'/f_y$ will be satisfactory to use in Eq. 4*a*:

$$M_u = A_s f_y d \left(1 - 0.59 \frac{p f_y}{f_c'}\right)$$

$$200{,}000\ (12) = A_s\ (50{,}000)\ (21)\ (1 - 0.59 \times 0.18) = 938{,}000\, A_s$$

$$A_s = 2.56 \text{ sq in.}$$

The foregoing value is probably more accurate than a value that can be read from curves, but it can be refined by using $p = \frac{2.56}{(12)\ (21)} = 0.0101$ in Eq. 4*a*. The result is $A_s = 2.54$ sq in.

Columns

Problem 11.—Design a square tied column for an ultimate axial load of 400 kips using $f_c' = 3{,}000$ lb per sq in. and approximately 2% (p_t) of steel of intermediate grade ($f_y = 40{,}000$ lb per sq in.). The report recommends that all columns be designed for a minimum eccentricity of 0.10 t for tied columns.

Solution Using Curves.—The chart of Fig. 12, which assumes $d = 0.8\,t$, will be utilized. Enter with the evaluation of

$$p_t\, m = \frac{p_t f_y}{0.85 f_c'} = \frac{0.02\ (40{,}000)}{0.85\ (3{,}000)} = 0.314$$

and with $e' = 0.10\,t$ to determine an intersection, which shows that $\frac{P_u}{f_c'\, b\, t} = 0.81$. Hence,

$$b\, t = \frac{P_u}{0.81 f_c'} = \frac{400{,}000}{(0.81)\ (3{,}000)} = 165 \text{ sq in.}$$

Because $b = t = 12.8$ in., use a 13-in.-by-13-in. column. For a 1.5-in. cover, 0.25-in. ties, and No. 8 bars, $d = 13 - 1.5 - 0.25 - 0.5 = 10.75 \text{ in.} = 0.825\,t$. Precision requires interpolation between charts for $d = 0.8\,t$ and $d = 0.85\,t$, but it would be safe to use the chart in Fig. 12. The actual value of $\frac{P_u}{f_c'\, b\, t}$ becomes

$$\frac{P_u}{f_c'\, b\, t} = \frac{400{,}000}{3{,}000\ (13)\ (13)} = 0.79$$

A horizontal line through this point intersects $e' = 0.1\,t$ at $p_t\, m = 0.28$. Hence,

$$p_t = \frac{0.28\ (0.85)\ (3{,}000)}{40{,}000} = 0.0178$$

and

$$A_s = 0.0178\ (13)\ (13) = 3.00 \text{ sq in.}$$

Therefore, use four No. 8 bars with an area of 3.16 sq in.

Solution Without Curves.—The design will be based on Eq. 16,

$$P_u = \frac{A_s' f_y}{\dfrac{e}{d - d'}} + \frac{b\, t\, f_c'}{\dfrac{3\, t\, (e - 0.5\, t + d')}{d^2} + 1.18}$$

The expression $e - 0.5\, t + d'$, represents the eccentricity, e', about the center of the column $= 0.1\, t$. Assume $t = 16$ in.; then $d' = 1.5$-in. cover $+$ 0.25-in. ties $+$ 0.5 in. (for half-bar diameter) $= 2.25$ in. Therefore,

$$d - d' = 16 - 2.25 - 2.25 = 11.5 \text{ in.}$$

and

$$e = e' + \frac{d - d'}{2} = 0.1\, t + \frac{11.5}{2} = 1.60 + 5.75 = 7.35 \text{ in.}$$

Because $p'\, t^2$ is equal to $\frac{p_t\, t^2}{2}$, which equals $0.01\, t^2$, replace A_s' with $0.01\, t^2$ and substitute t for b because this is a square column. Therefore,

$$\begin{aligned} 400{,}000 &= \frac{0.01\, t^2\, (40{,}000)}{\dfrac{7.35}{11.50}} + \frac{t^2\, (3{,}000)}{\dfrac{3\,(16)\,(0.1)\,(16)}{(16 - 2.25)^2} + 1.18} \\ &= 625\, t^2 + 1{,}890\, t^2 = 2{,}515\, t^2 \end{aligned}$$

and

$$t = 12.6 \text{ in.}$$

Try $t = 13$ in.; then $d' = 2.25$ in., $d = 10.75$ in., $d - d' = 8.50$ in., and $e = 1.3 + \frac{8.50}{2} = 5.55$ in. Therefore,

$$\begin{aligned} 400{,}000 &= \frac{0.1\, t^2\, (40{,}000)}{\dfrac{5.55}{8.50}} + \frac{t^2\, (3{,}000)}{\dfrac{3\,(13)\,(0.1)\,(13)}{10.75^2} + 1.18} \\ &= 613\, t^2 + 1{,}850\, t^2 = 2{,}463\, t^2 \end{aligned}$$

Because this yields $t = 12.7$ in., use a 13-in.-by-13-in. column. This computation is insensitive to a poor initial guess for t. The same equation solved for A_s' results in

$$400{,}000 = \frac{A_s'\, (40{,}000)}{\dfrac{5.55}{8.50}} + 1{,}850 \times \overline{13}^2 = 61{,}300\, A_s' + 313{,}000$$

Therefore, $A_s' = 1.42$ sq in.; and, because $A_{st} = 2\, A_s' = 2.84$ sq in., use four No. 8 bars with an area of 3.16 sq in. The computed steel is slightly less than that from the chart because the actual value of d is slightly more than the value of $0.8\, t$ on which the chart is based.

The same procedure is available when the eccentricity, e', is known in inches. However, if e' indicates that the load is much outside the kern point, the load must be compared with P_b from Eq. 13. The foregoing process assumes a compression failure, which will not occur if the actual value of P_u is less than P_b.

Problem 12.—Design a square tied column for an ultimate load of 400 kips at an eccentricity of 11 in., using $f_c' = 3{,}000$ lb per sq in., and p_g, approximately 2% of intermediate-grade steel ($f_y = 40{,}000$ lb per sq in.).

Solution Using Curves.—Although the chart in Fig. 12 may be used, the one in Fig. 7 is better for designs that involve a possible tension failure. For symmetrical steel, $p\,m - p'\,m = 0$. The smallest possible column is given by the intersection of the line for $p\,m - p'\,m = 0$, with the vertical line on the right for $f_y = 40{,}000$ lb per sq in. This corresponds to $\dfrac{P_u}{f_c'\,b\,d} = 0.505$ and $p'\,m\,[1 - (d'/d)] = 0.253$. Because

$$m = \frac{f_y}{0.85\,f_c'} = \frac{40{,}000}{0.85\ (3{,}000)} = 15.7$$

and d'/d is near 0.2, thus making d near 0.8 t, the required value of p' is

$$\frac{0.253}{0.8\ (15.7)} = 0.0201$$

which is somewhat greater than the desired value of 0.01. The corresponding column size, $b\,d$, is

$$b\,d = \frac{P_u}{f_c'\ (0.505)} = \frac{400{,}000}{3{,}000\ (0.505)} = 264 \text{ sq in.}$$

For $b = t$ and $d = 0.8\,t$, $t^2 = \dfrac{264}{0.8} = 331$ sq in.; therefore, $t = 18.1$ in. By trial, select a larger column in order to reduce the steel requirement.

Try $t = 20$ in. and $d' = 1.5$ in. + 0.37-in. ties + 0.5 in. (for half-bar diameter) = 2.37 in. Then $d = 17.63$ in. and

$$\frac{P_u}{f_c'\,b\,d} = \frac{400{,}000}{3{,}000\ (20)\ (17.63)} = 0.378$$

Because the value of e from tension steel is

$$e = e' = \frac{d - d'}{2} = 11 + \frac{15.25}{2} = 18.62 \text{ in.}$$

therefore,

$$\frac{P_u\,e}{f_c'\,b\,d^2} = \frac{400{,}000\ (18.62)}{3{,}000\ (20)\ (17.63^2)} = 0.400$$

To find p', proceed horizontally from $\dfrac{P_u}{f_c'\,b\,d} = 0.378$ on the left to intersect the curve of $p\,m - p'\,m = 0$, thence vertically downward to intersect the curve of $\dfrac{P_u\,e}{f_c'\,b\,d^2} = 0.400$, and finally horizontally to yield $p'\,m\,[1 - (d'/d)]$

= 0.130 on the right. This yields

$$p' = \frac{0.130}{\left(1 - \frac{2.37}{17.63}\right) 15.7} = 0.00958$$

which is equivalent to a value of p_g of

$$\frac{2\ (0.00958)\ (20)\ (17.63)}{20\ (20)} = 0.0169$$

which is probably acceptable. However, an additional trial may give a value that is closer to the desired value of p_g.

Try $t = 19$ in. and $d' = 2.37$ in.; then $d = 16.63$ in., $d - d' = 14.25$ in., and

$$\frac{P_u}{f_c'\ b\ d} = \frac{400{,}000}{3{,}000\ (19)\ (16.63)} = 0.422$$

Furthermore, $e = 11 + \frac{14.25}{2} = 18.12$ in. and

$$\frac{P_u\ e}{f_c'\ b\ d^2} = \frac{400{,}000\ (18.12)}{3{,}000\ (19)\ (\overline{16.63^2})} = 0.460$$

These values in the chart lead to $p'\ m\ [1 - (d'/d)] = 0.173$ and

$$p' = \frac{0.173}{\left(1 - \frac{2.37}{16.63}\right) 15.7} = 0.0128$$

Use a 19-in.-by-19-in. column. This chart defines p' as $\frac{A_s'}{b\ d}$ rather than in terms of the total column area. The required value of A_{st} is

$$A_{st} = 2\ (0.0128)\ (19)\ (16.63) = 8.07 \text{ sq in.}$$

Eight No. 9 bars with an area of 8.00 sq in. are satisfactory, considering the possible accuracy in reading curves, with four bars on the compression face and four bars on the tension face.

Solution Without Curves.—The smallest possible column would be given when the value of P_b from Eq. 13 is made equal to the known value of P_u of 400 kips. With symmetrical steel the last two terms are equal and opposite, thus yielding

$$P_b = 0.72 \left(\frac{90{,}000}{90{,}000 + f_y}\right) f_c'\ b\ d = 0.72 \left(\frac{90{,}000}{90{,}000 + 40{,}000}\right) 3{,}000\ b\ d$$

$$= 1{,}500\ b\ d = 400{,}000$$

The minimum value of $b\ d = 267$, or for $b = t$ and $d = 0.85\ t$, $t = 17.7$ in. Try $t = 18$ in.; then $d' = 2.37$ in., $d = 15.63$ in., and $d - d' = 13.25$ in. Because the tension on A_s balances the compression on A_s', the compression on the concrete, C_c, equals P_u. Hence, the depth of stress block is

$$a = \frac{P_u}{0.85 f_c' b} = \frac{400{,}000}{0.85\ (3{,}000)\ (18)} = 8.74 \text{ in.}$$

Fig. 9 shows that the lever arm for the couple formed by C_c and P_u is

$$z = e' - 0.5\, t + \frac{a}{2} = 11 - 9 + 4.37 = 6.37 \text{ in.}$$

This arm gives a moment of 400 (6.37) = 2,548 kip-in. The moment must be balanced by the couple made up of A_s and A_s', acting at a stress of f_y with an arm equal to the quantity, $d - d'$. Therefore,

$$2{,}548 = A_s'\ (40)\ (13.25)$$
$$\text{Required } A_s' = 4.82 \text{ sq in.}$$

Because this considerably exceeds the desired 1%, which would be 3.24 sq in., try $t = 19$ in.; then $d' = 2.37$ in., $d = 16.63$ in., and $d - d' = 14.25$ in. Therefore,

$$a = \frac{400{,}000}{(0.85)\ (3{,}000)\ (19)} = 8.25 \text{ in.}$$

$$z = 11 - 9.5 + 4.13 = 5.63 \text{ in.}$$

$$P_u z = 400\ (5.63) = 2{,}250 \text{ kip-in.} = A_s' f_y\ (d - d') = A_s\ (40)\ (14.25)$$

$$A_s' = 3.95 \text{ sq in.}$$

Therefore, $p_g = 2\ (3.95)/\overline{19}^2 = 0.0128$, which is acceptable. Use a 19-in.-by-19-in. column with four No. 9 bars on the compression face and four No. 9 bars on the tension face with $A_{st} = 8.00$ sq in.

When using symmetrical steel, the solution without curves is as rapid as the procedure used in the solution with curves.

APPENDIX II. NOTATION

A_c = total concrete area;
A_g = gross column area;
A_s = steel area;
A_{s1}, A_{s2} = parts of tension steel area;
A_s' = area of compression steel;
A_{sf} = imaginary compression steel area used in computations as a substitute for compressive strength of projecting flanges of a T-beam;
A_{st} = total column steel area;
a = depth of rectangular stress block;
B = effect of basic loads in computing needed ultimate strength;
b = width of rectangular beam or column; total width of T-beam flange;
b' = width of web of T-beam;

C = total compression in beam or in column carrying moment;

C_1, C_2 = parts of C;

C_c = total compression on concrete;

C_s = total compression on A_s';

D = diameter of circular column;

d = depth of beam or column from compression face to tension steel; also, in a circular column, the diameter of a circle circumscribing the reinforcement;

d' = cover over steel measured from center of bar;

e = eccentricity of axial load measured from the centroid of tension steel;

e' = eccentricity of axial load measured from the centroid of member;

f_c' = concrete cylinder strength, in pounds per square inch;

f_s = steel stress in tension;

f_y = yield point stress of steel;

h = unsupported length;

j = ratio of distance between resultants of compressive stress and tension stress to depth d;

K = load factor in computing ultimate strength;

k_1 = ratio of average compressive stress to $0.85\,f_c'$, used also as ratio of depth of stress block a to $k_u\,d$;

k_2 = ratio of distance between extreme fiber and resultant of compressive stresses to $k_u\,d$; or ratio of the same distance to the depth of equivalent stress block;

k_u = ratio of distance between extreme fiber and neutral axis to depth d (to steel);

L = effect of live load plus impact, in computation of needed ultimate strength;

M_0 = computed ultimate bending-moment resistance, ignoring any weakness in tension;

M_u = ultimate bending moment;

M_1, M_2 = parts of total moment;

m = symbol for $\dfrac{f_y}{0.85\,f_c'}$;

m' = symbol for $m - 1$;

n = ratio of modulus of elasticity of steel to that of concrete;

P_b = ultimate (eccentric) load capacity of a column when failure of both tension steel and compression area occurs simultaneously;

P_0 = ultimate load capacity of column under concentric load;

P_u = ultimate load capacity of column under eccentric load;

P_u' = limitation on ultimate load capacity of a long column;

p = symbol for $\dfrac{A_s}{b\,d}$;

p_1 = a part of total p corresponding to A_{s1};

p_g, p_t = symbols for $\dfrac{A_s}{b\,t}$ for column;

p_f = symbol for $\dfrac{A_{sf}}{b'\,d}$;

p_w = symbol for $\frac{A_s}{b' d}$;

p' = symbol for $\frac{A_s'}{b d}$;

q = symbol for $\frac{p f_y}{f_c'}$;

T = total tension;

T_1, T_2 = parts of total tension;

t = slab thickness; also column thickness or diameter;

U = ultimate strength of section; and

z = a moment arm (Fig. 10).

DISCUSSION

Tung Au,[10] A.M. ASCE.—Simple and concise explanations have been presented of the procedures for the design of reinforced concrete members by the ultimate strength method, based on the recommendations of the ASCE-ACI report.[2] The paper demonstrates that the ultimate strength method can be taught in the first course of reinforced concrete design as easily as the conventional straight-line method.

Although the ultimate strength method is consistently more logical, it also has its weaknesses in the equations for computing the ultimate strength of eccentrically loaded columns. The equations recommended by the report are based primarily on the theory advanced by Whitney.[4,7] The equations for tension failures were developed on a rational basis, assuming an equivalent uniform stress distribution, whereas the equations for compression failures were developed on a semiempirical basis. Whitney further suggested the following:

1. Add a small additional eccentricity to e' to allow for deflection in the case of a tension failure; and
2. Adjust the constants in the formulas for compression failures so that, when e' is zero, P_u will be two and one-half times the value for an axially loaded column as specified in the 1947 ACI Building Code.[11]

These suggestions are not included in the final recommendations for eccentrically loaded columns of any type in the report or in the latest edition of the ACI Building Code.[12] However, in Appendix C of the ASCE-ACI report, in which the derivations of formulas are given, the suggestions are included in the formulas for both square and circular columns with round cores. This inconsistency seems to be an oversight, and the adjusted values should not be used.

The ultimate strength of a circular section with a round core subject to combined bending and axial load is given by Eq. 16 and Eq. 17 in the ASCE-ACI report. However, the ultimate strength of a square section with a round core has not been given explicitly in the final recommendations. Referring to Appendix C of the report, formulas for a square section with a round core can be obtained easily from the formulas for a rectangular tied column by assuming that half of the steel is effective on each side of the section, and by using 0.67 d as the effective distance for the steel. Thus, when tension controls,

$$P_u = 0.85\, f_c'\, D^2 \left[\sqrt{\left(\frac{e'}{t} - 0.5\right)^2 + \frac{0.67\, d}{t}\, p_t\, m} - \left(\frac{e'}{t} - 0.5\right) \right] \quad .. (36)$$

---

[10] Visiting Associate Prof. of Civ. Eng., Carnegie Inst. of Technology, Pittsburgh, Pa.

[11] "Building Code Requirements for Reinforced Concrete (ACI 318-47)," *Proceedings*, A.C.I., Vol. 44, September, 1947, pp. 1-64.

[12] "Building Code Requirements for Reinforced Concrete (ACI 318-56)," *ibid.*, Vol. 52, May, 1956, pp. 917-980.

and, when compression controls,

$$P_u = \frac{A_{st} f_y}{\frac{3\,e'}{d} + 1} + \frac{A_g f_c'}{\frac{12\,t\,e'}{(t + 0.67\,d)^2} + 1.18} \ldots\ldots\ldots\ldots\ldots (37)$$

It should also be noted that the balanced load, P_b, which is equally likely to cause failure in tension or in compression, has not been given for either a circular or square section with a round core, but it can easily be determined by substituting proper values of t, D, and d into the similar equation for a rectangular tied column with symmetrical reinforcement. Hence, for a square column with a round core,

$$P_b = 0.72 \left(\frac{90{,}000}{90{,}000 + f_y} \right) \left(\frac{t + 0.67\,d}{2} \right) t\, f_c' \ldots\ldots\ldots\ldots (38)$$

and, for a circular column with a round core,

$$P_b = 0.72 \left(\frac{90{,}000}{90{,}000 + f_y} \right) \left(\frac{0.8\,D + 0.67\,d}{2} \right) D\, f_c' \ldots\ldots\ldots (39)$$

With these additional equations, the solution of square or circular columns with round cores can be performed completely.

Because a trial-and-error procedure is quite often involved in the design of eccentrically loaded columns, curves such as those shown in Fig. 6 and Fig. 7 in the ASCE-ACI report are helpful in simplifying the design. Similar charts for circular and square sections with round cores have been presented by Whitney and Edward Cohen,[13] J. M. ASCE, and by the writer.[14]

In examining Fig. 7 of the ASCE-ACI report, as well as the curves by Whitney and Cohen,[13] it is noted that the intersection of the curve representing tension failure and the curve representing compression failure does not fall exactly on the balanced load, P_b. The author has indicated that, with many values of d and f_y, the intersection is missed by a small margin for a rectangular tied column. The discrepancies seem to vary also with the cross sections of the columns. For a square column with a round core, the intersections of curves are close to the corresponding values of P_b at $f_y = 40{,}000$ lb per sq in. for all values of d. At greater values of f_y, the values of P_b are slightly less. For a circular column the discrepancies are larger, depending on f_y and d. Closer agreement may be obtained by using a factor of $\frac{1}{2}$ $(0.8\,D + 0.75\,d)$ instead of $\frac{1}{2}$ $(0.8\,D + 0.67\,d)$ in Eq. 39 as suggested by the writer in another paper.[14]

Herman S. Schick,[15] A.M. ASCE.—A valuable service has been rendered in emphasizing the facility of design by the principles of ultimate strength. Fig. 2, which is reproduced from the ACI report,[2] can be changed easily to be more useful for most design work by using ordinary rectangular Cartesian coordinates rather than the logarithmic units used in the ACI diagram. The ratio, f_c'/f_y, occurs frequently in the formulas and is fundamental, so that, by using the ratio instead of the f_y-values, the p-values become straight lines. Most values of $\frac{M_u}{b\,d^2}$ in ordinary work will fall between 100 lb per sq in. and 900

---

[13] "Guide for Ultimate Strength Design of Reinforced Concrete," by C. S. Whitney and E. Cohen, *Proceedings*, A.C.I., Vol. 53, November, 1956, pp. 455–490.

[14] "Ultimate Strength Design Charts for Columns Controlled by Tension," by Tung Au, *ibid.*, Vol. 54, December, 1957, pp. 471–480.

[15] Structural Engr., Edwin A. Keeble, Archt., Nashville, Tenn.

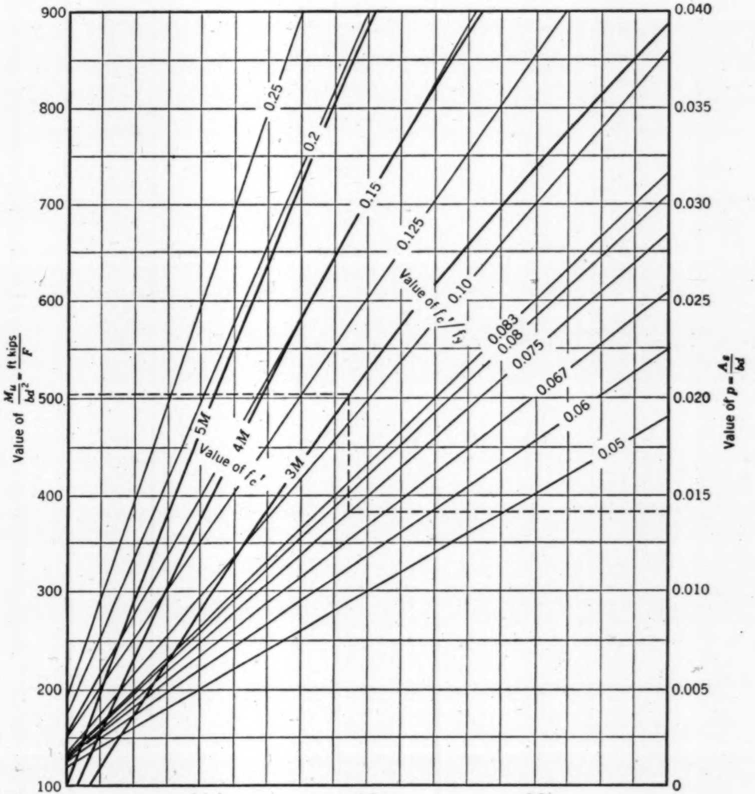


FIG. 14.—RECTANGULAR BEAM DIAGRAM

lb per sq in., and the most important q-values lie between 0.1 and 0.2, with the value of 0.18 having special significance. By using appropriate scales for these values, the curves for values of f_c' of 3 M, 4 M, and 5 M are easily plotted (Fig. 14) to show an adequate distinction of q-values for the desired strength of concrete to be used. The value of M_u will most conveniently be in units of ft-kips. Therefore, it is more convenient to use the constant, $F = b\,d^2/12{,}000$. To facilitate the computation, a nomogram (Fig. 15) has been prepared to show the values of $b\,d^2$ and F for the most common values of b and d. Eq. 3 can be rewritten as $a = (A_s/b)(f_y/0.85f_c')$, and, in this form, a nomogram (Fig. 16) shows the values of a for the usual values of A_s/b and f_c'/f_y.

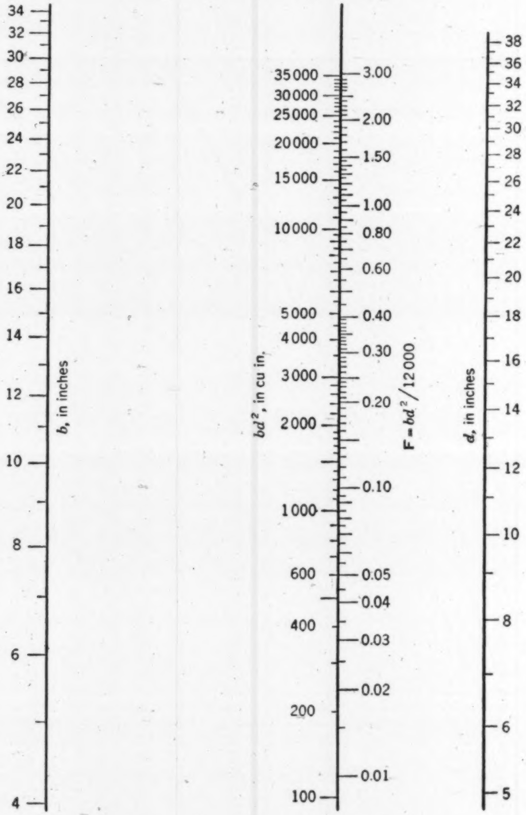

34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
b, in inches
35000
30000
25000
20000
15000
10000
5000
4000
3000
2000
1000
600
400
200
100
bd², in cu in.
3.00
2.00
1.50
1.00
0.80
0.60
0.40
0.30
0.20
0.10
0.05
0.04
0.03
0.02
0.01
F = bd²/12000
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
5
d, in inches

FIG. 15.—NOMOGRAM FOR $b\,d^2$ AND $F = b\,d^2/12{,}000$

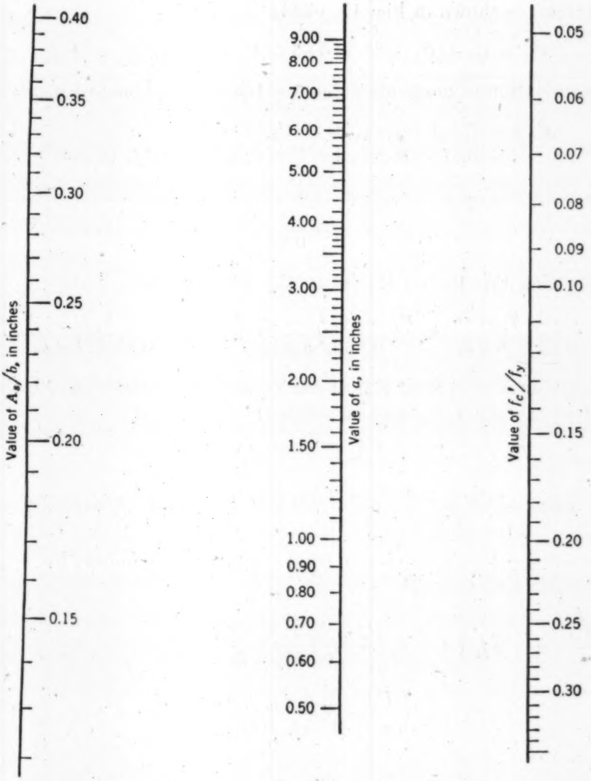


FIG. 16.—NOMOGRAM FOR DEPTH OF STRESS BLOCK a

ZDENĚK SOBOTKA[16].—The procedures that prove that the ultimate strength design of reinforced concrete can be much simpler and more time saving than the design method based on the straight-line theory have been presented excellently.

The solution of a special problem in the design of eccentrically loaded columns with double unsymmetrical reinforcement (which fails in tension) will be shown herein. The distribution of the reinforcement between the stretched and compressed zone is such that the total steel area, A_{st}, is at a minimum. The notation listed in Appendix II is used.

The total steel area,

$$A_{st} = A_s + A_s' \qquad (40)$$

is assumed to be a minimum in a column with the cross-sectional dimensions b and t. Following the author's procedure in Eq. 12a and Eq. 12b, the summation of axial forces, as shown in Fig. 17, yields

---

[16] Docent of Theory of Elasticity and Strength of Materials, The Technical Univ., Prague, Czechoslovakia.

$$P_u = C_c + C_s - T = 0.85\, f_c'\, b\, c\, k_1 + A_s'\, f_y' - A_s\, f_y \tag{41}$$

and the summation of moments about the tension steel, as in Eq. 14, results in

$$\begin{aligned} P_u\, e &= C_c\, (d - k_2\, k_u\, d) + C_s\, (d - d') \\ &= 0.85\, f_c'\, b\, d^2\, k_u\, k_1\, (1 - k_2\, k_u) + A_s'\, f_y\, (d - d') \end{aligned} \tag{42}$$

in which

$$k_u = \frac{c}{d} = \frac{C_c}{0.85\, f_c'\, b\, d\, k_1} \tag{43a}$$

Substituting Eq. 40 and Eq. 41 in Eq. 43*a* leads to

$$k_u = \frac{P_u + T - C_s}{0.85\, f_c'\, b\, d\, k_1} = \frac{P_u + A_s\, f_y - A_s'\, f_y}{0.85\, f_c'\, b\, d\, k_1} = \frac{P_u + 2\, A_s\, f_y - A_{st}\, f_y}{0.85\, f_c'\, b\, d\, k_1} \tag{43b}$$

Placing values of C_c from Eq. 41 and k_u from Eq. 43*b* in Eq. 42, the following function of A_s and A_{st} may be written:

$$\begin{aligned} F\, (A_{s1}\, A_{st}) &= P_u\, e - (P_u + 2\, A_s\, f_y - A_{st}\, f_y) \\ &\times \left(d - k_2\, \frac{P_u + 2\, A_s\, f_y - A_{st}\, f_y}{0.85\, f_c'\, b\, k_1}\right) - (A_{st} - A_s)\, f_y\, (d - d') = 0 \end{aligned} \tag{44}$$

The condition for the minimum total steel area, A_{st}, follows by equating the derivative, $\partial F/\partial A_s$, to zero:

$$\begin{aligned} \frac{\partial F\, (A_s\, A_{st})}{\partial A_s} &= -\, 2\, f_y \left(d - k_2\, \frac{P_u + 2\, A_s\, f_y - A_{st}\, f_y}{0.85\, f_c'\, b\, k_1}\right) \\ &\quad + 2\, f_y\, k_2 \left(\frac{P_u + 2\, A_s\, f_y - A_{st}\, f_y}{0.85\, f_c'\, b\, k_1}\right) + (d - d')\, f_y = 0 \end{aligned} \tag{45a}$$

Therefore,

$$4\, k_2\, \frac{P_u + 2\, A_s\, f_y - A_{st}\, f_y}{0.85\, f_c'\, b\, k_1} - d - d' = 0 \tag{45b}$$

or, after substituting Eq. 40 and Eq. 41,

$$\frac{4\, k_2\, C_c}{0.85\, f_c'\, b\, k_1} - d - d' = 0 \tag{46}$$

From Eq. 46 the distance between extreme compressed fiber and neutral axis for the minimum total steel area is

$$c = k_u\, d = \frac{C_c}{0.85\, f_c'\, b\, k_1} = \frac{d + d'}{4\, k_2} \tag{47}$$

If the cover over tension and compression steel is the same—that is, if $d + d' = t$—Eq. 47 becomes

$$c = k_u\, d = \frac{t}{4\, k_2} \tag{48}$$

The summation of the moments about the tension steel and the substitution of Eq. 48 yields

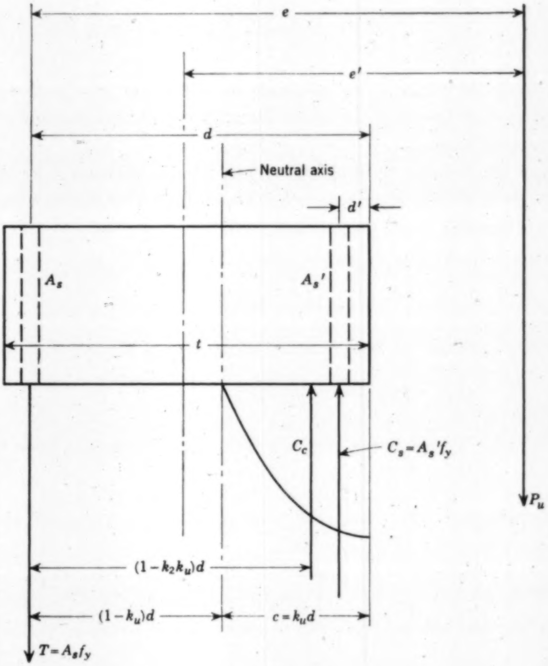


Fig. 17.—Representation of Axial Forces

$$P_u\, e - 0.85 f_c'\, b\, (d + d')\, (3\, d - d')\, \frac{k_1}{16\, k_2} - A_s' f_y\, (d - d') = 0 \ldots (49)$$

The summation of the moments about the compression steel, as shown in Fig. 17, leads to

$$P_u\, (e' - 0.5\, t + d') + 0.85 f_c'\, b\, (d + d') \times (d - 3\, d')\, \frac{k_1}{16\, k_2} - A_s f_y\, (d - d') = 0 \,..\, (50)$$

From the foregoing equations, formulas for the tension and compression steel area in columns with given cross-sectional dimensions may be obtained as follows:

$$A_s' = \frac{1}{f_y\, (d - d')} \left[P_u\, e - 0.85 f_c'\, b\, (d + d')\, (3\, d - d')\, \frac{k_1}{16\, k_2} \right] \ldots (51)$$

$$A_s = \frac{1}{f_y\,(d - d')}\left[P_u\,(e' - 0.5\,t + d') + 0.85\,f_c'\,b\,(d + d')\,(d - 3\,d')\,\frac{k_1}{16\,k_2}\right] \ldots (52)$$

The value of the ratio, k_1/k_2, depends on the shape of the stress diagram. Thus, the preceding formulas are valid for all stress diagrams, not only for the rectangular stress block.

The other problem is to find the depth of the rectangular-column cross section for a given width, b, and the total percentage of steel, p_t, when the total steel ratio is a minimum.

Adding Eq. 51 and Eq. 52 and letting

$$A_{st} = p_t\,b\,t \ldots\ldots\ldots\ldots\ldots (53)$$

leads to

$$p_t\,b\,t = \frac{1}{f_y\,(d - d')}\left[P_u\,(e + e' - 0.5\,t + d') - 0.85\,f_c'\,b\,(d + d')^2\,\frac{k_1}{8\,k_2}\right] \ldots (54)$$

For symmetrical cover over the steel—that is, for $d + d' = t$—Eq. 54 becomes

$$p_t\,b\,t = \frac{1}{f_y\,(t - 2\,d')}\left[2\,P_u\,e' - 0.85\,f_c'\,b\,t^2\left(\frac{k_1}{8\,k_2}\right)\right] \ldots\ldots (55a)$$

and

$$\left[p_t\,f_y + 0.85\,f_c'\left(\frac{k_1}{8\,k_2}\right)\right] b\,t^2 - 2\,p_t\,f_y\,b\,d'\,t - 2\,P_u\,e' = 0 \ldots (55b)$$

This is the quadratic equation with the unknown, t, from which

$$t = \frac{p_t\,f_y\,d'}{p_t\,f_y + 0.85\,f_c'\left(\dfrac{k_1}{8\,k_2}\right)} \times \left\{1 + \sqrt{1 + \frac{2\,P_u\,e'}{b\,(d')^2\,p_t^2\,f_y^2}\left[p_t\,f_y + 0.85\,f_c'\left(\frac{k_1}{8\,k_2}\right)\right]}\right\} \ldots (56)$$

After computing t, A_s follows from Eq. 45*b*,

$$A_s = 0.85\,f_c'\,b\,(d + d')\,\frac{k_1}{8\,k_2\,f_y} + \tfrac{1}{2}\left(A_{st} - \frac{P_u}{f_y}\right) \ldots\ldots (57)$$

and A_s', from Eq. 40,

$$A_s' = A_{st} - A_s \ldots\ldots\ldots\ldots\ldots (58)$$

The depth, t, of the rectangular cross section for a given ratio, $h = b/t$, and for a given total percentage of steel, p_t, follows from

$$t^3 - \frac{2\, p_t\, f_y\, d'\, t^2}{p_t\, f_y + 0.85\, f_c' \left(\frac{k_1}{8\, k_2}\right)} - \frac{2\, P_u\, e'}{h \left[p_t\, f_y + 0.85\, f_c' \left(\frac{k_1}{8\, k_2}\right)\right]} = 0 \quad \text{..(59)}$$

Introducing a new unknown,

$$x = \frac{t}{2\, d'\, p_t\, f_y} \left[p_t\, f_y + 0.85\, f_c' \left(\frac{k_1}{8\, k_2}\right)\right]$$

yields

$$x^2\,(x - 1) = \frac{P_u\, e'}{8\,(d')^3\, h\, p_t^3\, f_y^3} \left[p_t\, f_y + 0.85\, f_c' \left(\frac{k_1}{8\, k_2}\right)\right]^2 \quad \text{.......(60)}$$

in which x may be tabulated for different values of the expression on the right side. Then A_s follows from Eq. 57 as in the preceding case, and A_s' follows from Eq. 40. Similarly, the solution of these problems for columns with other forms of cross section and for eccentric tension may be obtained.

Milan Spanovich,[17] J. M. ASCE.—Basing all his computations on an assumed rectangular stress block, Mr. Ferguson has illustrated some interesting simplifications in design by ultimate strength procedures. It appears that the degree of effectivenes of the simplifications in practical design problems is yet to be proved, because the design criteria for shear, bond, and diagonal tension are still governed by working stress conditions.

With respect to selecting a stress block, it may be assumed that the ultimate strength procedure is as versatile as rectangular, trapezoidal, parabolic, or other compression distributions in reasonable agreement with tests. Experience should bring the more adaptable assumption into specifications.

In reviewing the tables of the report, it was noted that the trapezoidal stress block produces values that are more closely related to test results than those values obtained by either parabolic or rectangular distributions. The comparative values of ultimate strength were all greater by stress blocks than by tests. The average degrees of variation were as follows: Trapezoidal block, 0.13%; rectangular block, 0.19%; and parabolic block, 0.22%.

Keeping all failures in tension and carrying a constant compressive yield stress simplifies basic beam design, resulting in easier tabulations that are readily adapted to a charting system.

Sylvester M. Ulicny,[18] A.M. ASCE.—Theory and design procedures are presented systematically and clearly so that an otherwise complex analysis can be understood.

The validity of this design procedure is illustrated in the ASCE-ACI Joint Committee test results.[2] The apparent deviations in the ratio of the test sample strengths, as compared with the computed values by this method, are probably due to a difference in the actual shape of the stress-strain curves of the different concretes used. Also, as noted in other tests,[19] the behavior of concrete in flexure is different from the behavior of the test cylinder in compression.

---

[17] Graduate Research Asst., Civ. Eng. Dept., Carnegie Inst. of Technology, Pittsburgh, Pa.

[18] Structural Engr., Highway Div., Swindell-Dressler Corp., Pittsburgh, Pa.

In examining Craemer's method[9] of analyzing a column subject to bending about each of its major axes, it is seen that this problem is approached analytically by assuming a shape for the compression area of the cracked section and by using iteration to solve the equations. A trial-and-error method for tension failure design of this problem is proposed herein, which enables the designer to visualize the physical behavior of the column.

In analyzing a short, rectangular column for bending about its major axes a scale drawing (Fig. 18) should be used, which shows the applied load in its correct position relative to the column section and which also shows the location of the reinforcing steel. A trial cracked section is then assumed with a compression area, A_c. The center of gravity of the tension steel and the compression area may be found by using a steel stress of f_y in tension and $f_y - 0.85 f_c'$ in compression, and $0.85 f_c'$ for the compression concrete stress. The solution is obtained by satisfying the equations of statics, $P_u = C_s + C_c - T$ from Eq. 10*a* and $P_u e \doteq (C_s + C_c) z$.

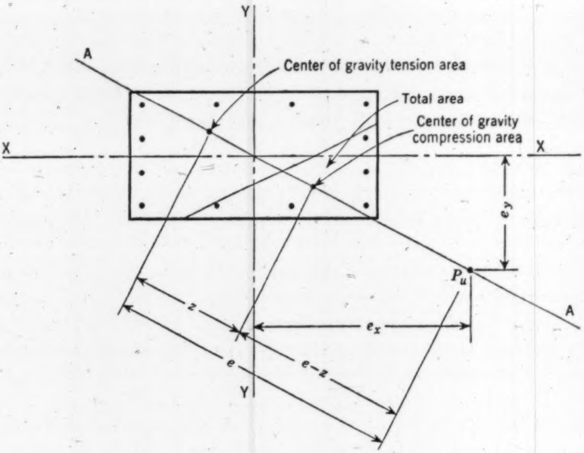


FIG. 18.—LOCATION OF CENTERS OF GRAVITY

A good first approximation of the cracked section can be obtained by approximating the locations of the centers of gravity of the tension steel and the compression area. These centers of gravity must be in line with the applied load, as shown in Fig. 18 on $\overline{AA}$.

---

[19] "Ultimate Theory in Flexure by Exponential Function," by G. M. Smith and L. E. Young, *Journal*, A.C.I., Vol. 26, November, 1955.

The writer is aware that a narrow band parallel to the neutral axis exists in which the steel is not stressed to its yield point. However, comparisons of the results by this method with test results are needed in order to arrive at a practical method for locating the band.

PHIL M. FERGUSON,[20] M. ASCE.—Each discusser has attempted to extend the practical field of ultimate strength design or to make such design still simpler. A paper by Whitney and Cohen[13] should also be noted because it has added design curves for circular columns and square columns with circular cores, both of which were unavailable in the past.

Mr. Au has presented formulas for the balanced load, P_b, for circular columns and square columns with circular cores. These formulas are helpful and have not, to the writer's knowledge, been presented elsewhere. This study of the differences between the formula value of P_b and the intersection of the tension and compression failure lines as given by Fig. 13 deserves further consideration as ultimate strength design procedures are standardized.

The curves and nomograms prepared by Mr. Schick are examples of how beam design charts may be adapted to the thinking of the individual designer. The writer regrets that Mr. Schick did not include specific examples of the use of his curves, which would thereby have demonstrated their advantages. There was a scarcity of subdivisions on the ordinates for percentage of steel in the chart in Fig. 2 of the report. Chart 1 of Whitney and Cohen[13] has corrected this condition.

Mr. Sobotka's study of the economical arrangement of unsymmetrical steel in columns is somewhat more advanced than the simplified approach adopted in the paper. As engineers use ultimate strength design, it is certain that many such studies will follow and indicate economical procedures. The inherent simplicity of ultimate strength design makes such studies more feasible.

The comparison between the various stress blocks presented by Mr. Spanovich indicates that the differences are negligible from the standpoint of practical design. It is probable that research on eccentrically loaded columns can be correlated better on the basis of a trapezoidal or parabolic stress block. However, practical design is simplified by the use of the rectangular stress block, and the resulting accuracy appears to be more than adequate.

The writer feels that he may have misunderstood Mr. Spanovich's statement about the possible effect of design criteria for shear, bond, and diagonal tension on the simplicity of design by ultimate strength methods. Design methods for shear need re-evaluation and, possibly, an extensive revision. The impact of these changes appear to be identical on working stress methods and ultimate strength design.

Whether design is by working stress methods or by the ultimate strength theory, certain members are more critical in shear than in moment. If ultimate strength design leads to smaller members, then shear has more chance to control these members. However, ultimate strength design for reinforcing steel in members of a given size is not made more complex. After section size is established from shear, the steel is simply established from moment.

---

[20] Prof. of Civ. Eng., Univ. of Texas, Austin, Tex.

The semigraphical solution of the column with the load eccentric about both axes, as suggested by Mr. Ulicny, is simpler than the iteration process of Craemer. Whitney and Cohen[13] present a similar procedure. The writer has not used either method sufficiently to choose between them. It may be that the choice depends on an engineer's general attitude toward graphical solutions.

The change to ultimate strength design will probably not be sudden nor complete in the immediate future. However, because better balanced designs will result with some eventual economies, all efforts toward a better understanding of the inherent simplicity of ultimate strength design should be encouraged.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2934

MECHANISM OF REAERATION IN NATURAL STREAMS

By Donald J. O'Connor,[1] J. M. ASCE, and William E. Dobbins,[2] M. ASCE

With Discussion by Messrs. Thomas R. Camp; Alexander N. Diachishin; Aale Pasveer; Myrton C. Rand; and Donald J. O'Connor and William E. Dobbins

Synopsis

The basic theory of turbulent flow has been applied successfully to explain many phenomena occurring in natural and artificial waterways. In this study, turbulent-flow theory, both isotropic and nonisotropic, has been utilized to formulate the theory of reaeration in natural streams that have been subjected to deaeration by the biological oxidation of organic material. The general differential equation of the oxygen balance in a river is accepted as the basis for the analysis, and a theoretical derivation of the reaeration coefficient is presented. Experimental work concerning reaeration under turbulent conditions was conducted to verify the theoretical development. Field data from sanitary river surveys are presented to show the comparison between the theoretically computed values and the observed values of the reaeration coefficients. This comparison, as well as the experimental work, substantiates the derived formulas.

Introduction

The discharge of organic impurities, such as municipal sewage and industrial wastes, into a body of water presents an important problem in the field of sanitary engineering. The decomposition of this organic material by bacteria in their metabolic processes results in the utilization of the dissolved oxygen. The replacement of the oxygen by reaeration occurs through the water surfaces

---

Note.—Published, essentially as printed here; in December, 1956, in the Journal of the Sanitary Engineering Division, as *Proceedings Paper 1115*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Asst. Prof., Dept. of Civ. Eng., Manhattan College, New York, N. Y.

[2] Associate Prof. of San. Eng., College of Eng., New York Univ., New York, N. Y.

exposed to the atmosphere. An increase in the concentration of organic material stimulates the growth of bacteria, and oxidation proceeds at an accelerated rate. The concentration can be so great that a condition results in which the receiving water body is completely devoid of dissolved oxygen. Every stream is limited in its capacity to assimilate organic wastes. Within this limit there is no more economical method of waste disposal. Therefore, an evaluation of the natural purification capacity of a steam is of fundamental and practical value. The knowledge of the rates of deoxygenation and reaeration is essential to this evaluation. A mathematical development of a theory describing reaeration in natural streams is presented herein.

REVIEW OF PREVIOUS WORK

Notation.—The letter symbols adopted for use in this paper are defined where they first appear and are arranged alphabetically, for convenience of reference, in Appendix II.

General.—The relationship between the deoxygenation of polluted water and the reoxygenation, or reaeration, of the water establishes a pattern in the dissolved oxygen concentration known as "the dissolved oxygen sag." The theoretical analysis of this relationship has been presented by N. W. Streeter and E. B. Phelps (1).[3] The differential equation, which describes the combined action of deoxygenation and reoxygenation, is as follows:

$$\frac{dD}{dt} = K_1 L - K_2 D \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Eq. 1 states that the net rate of change in the dissolved oxygen saturation deficit is equal to the algebraic sum of two independent rates. The first rate is that of the oxygen-utilization reaction caused by the bacteria in the stabilization of the organic material. The reaction increases the oxygen deficit, D, at a rate that is proportional to the concentration of the organic material, as expressed by the biochemical oxygen demand, L. The proportionality factor, K_1, is a temperature function. The second rate is that of reaeration, which replenishes the oxygen that was utilized by the organic material. The reaeration is proportional to the dissolved oxygen deficit, and the negative sign indicates that it decreases the deficit. The proportionality factor, K_2, is also a temperature function, but it is a function of the turbulence of the stream. The integration of Eq. 1 results in

$$D_t = \frac{k_1 L_0}{k_2 - k_1} (10^{-k_1 t} - 10^{-k_2 t}) + D_0 (10)^{-k_2 t} \ldots\ldots\ldots\ldots (2)$$

In Eq. 2, L_0 and D_0 are the initial biochemical oxygen demand and the initial dissolved oxygen deficit, respectively, and k_1 and k_2 are the proportionality constants based on common logarithms. The value of D_t is the dissolved oxygen deficit at time t. A fundamental derivation of an equation defining the reaeration coefficient, k_2, will be attempted specifically herein.

---

[3] Numerals in parenthesis—thus: (1)—refer to corresponding items in the Bibliography (see Appendix I).

If there is no utilization of the dissolved oxygen within the body of water, the rate of reaeration may be expressed as

$$\frac{dD}{dt} = -K_2 D \qquad (3)$$

which when integrated results in

$$D_t = D_0 (10)^{-k_2 t} \qquad (4)$$

in which $K_2 = 2.31\, k_2$.

It has been found (1) that the reaeration coefficient, k_2, was influenced by the hydraulic and the physical characteristics of the river channel. The relationship between the coefficient and the hydraulic and physical characteristics was formulated empirically as follows:

$$k_2 = \frac{E\, U^{n_1}}{H^2} \qquad (5)$$

in which U represents the velocity of the flow, in feet per second; H designates the depth above low water, in feet; and E and n_1 represent the empirical constants, depending on the physical and the hydraulic conditions of the channel. The constant, n_1, was related to the function, "mean velocity increase per 5-ft increase in the river stage." The constant, E, was related to the slope of the channel and to an "irregularity factor," which is a measure of the relative roughness of the channel bottom.

At approximately the same time that the results of the Ohio River studies were presented, a two-film theory of mass transfer was proposed (2). In the absorption of a gas by a liquid, the gas must diffuse from the gas phase to the liquid phase. The theory is based on the assumption that there are two films, located at the interface, through which the gas must pass by molecular diffusion and below which the concentration of the dissolved gas is uniform.

For a gas of low solubility, such as oxygen in water, it has been shown that the resistance of the gas film is negligible in comparison with that of the liquid film. For this case the Lewis and Whitman equation may be expressed as follows:

$$N = \frac{1}{A}\frac{dm}{dt} = \frac{V}{A}\frac{dc}{dt} = K_L (C_s - c) \qquad (6)$$

in which N represents the rate of mass transfer per unit time per unit area; A equals the area through which diffusion occurs; V designates the volume of the liquid; K_L is the liquid-film coefficient; C_s represents the concentration at the gas liquid interface; and c is the concentration of the liquid in the body.

Extensive work has been performed and reported in the literature on film coefficients (3, 4, 5), and several empirical relationships have been proposed relating these coefficients to the turbulence factors—that is, the power input of impellers, the peripheral speeds, and the superficial air velocity. The validity of the two-film concept has not been tested adequately, but it has provided a convenient method of analysis and a means of reporting data on gas absorption.

From Eq. 3 and Eq. 6 it follows that

$$K_2 = K_L \frac{A}{V} \qquad (7a)$$

or

$$K_2 = \frac{K_L}{2.31\, h_a} \qquad (7b)$$

in which h_a is the average depth of the liquid.

It has been assumed in Eq. 3 and Eq. 6 that the concentration of the dissolved oxygen throughout the depth of the liquid is uniform at any particular time, and that the diffusion through the depth does not control the over-all rate of oxygen transfer.

In the case of a stagnant body of liquid, the rate of absorption is governed by the rate at which the oxygen is transferred by molecular diffusion throughout the depth, as given by Fick's law of diffusion—

$$\frac{\partial m}{\partial t} = - D_L\, A \frac{\partial c}{\partial y} \qquad (8)$$

in which D_L represents the coefficient of the molecular diffusion (diffusivity); $\partial m/\partial t$ designates the time rate of the mass transfer; and $\partial c/\partial y$ is the concentration gradient in the direction of diffusion.

From Eq. 8 it may be shown that

$$\frac{\partial c}{\partial t} = D_L \frac{\partial^2 c}{\partial y^2} \qquad (9)$$

Eq. 9 states that the rate of change of the concentration of the dissolved gas occurring at any point y is proportional to the differential of the concentration gradient.

Eq. 9 has been used (6) in a mathematical analysis, assuming the following boundary conditions:

$$c = c_0 \text{ when } y > 0 \text{ and } t = 0$$
$$c = C_s \text{ when } y = 0 \text{ and } t > 0$$

and

$$\frac{dc}{dy} = 0 \text{ when } y = h \text{ and } t > 0$$

in which c represents the dissolved oxygen concentration, and the axis is such that $y = 0$ at the surface.

Using these conditions, a formula was presented for the average concentration of oxygen in a column of water, h feet deep, which, when expressed in terms of the dissolved oxygen deficit, is

$$D_t = 0.811\, \pi\, D_0 \left(e^{-Q} + \frac{e^{-9Q}}{9} + \frac{e^{-25Q}}{25} \right) \qquad (10)$$

in which $Q = \frac{\pi D_L t}{4 h^2}$ and t represents the time of exposure.

On the basis of this equation and experimental determination, a value of 1.42 sq cm per hr at 20° C was determined for the coefficient of molecular diffusion.

The International Critical Tables (7) give a value of the diffusion coefficient of oxygen in water as 0.0713 sq cm per hour at 18° C. Other investigators (8) have indicated that in the determination of the value of 1.42 sq cm per hr at 20° C apparently the combined effects of molecular and eddy diffusion were measured. In order to apply Eq. 10 to turbulent flow in a river, the assumption was made (9) that

> "the actual existing conditions of turbulence can be replaced by an equivalent condition composed of successive periods of perfect quiescence between which there is imposed instantaneous and complete mixing. The length of the quiescent period then becomes the time of exposure in the diffusion formula, * * *"

as shown in Eq. 10. An empirical determination was made (10) indicating that the logarithm of the time of exposure is a linear function of the forward-flow velocity and that the time increases with the depth of flow. The rate of change in the dissolved oxygen deficit may be determined by a stepwise procedure.

Eq. 9 may be integrated, assuming different boundary conditions:

$$c = c_0 \text{ when } y > 0 \text{ and } t = 0$$
$$c = C_s \text{ when } y = 0 \text{ and } t > 0$$

and

$$c = c_0 \text{ when } y = \infty \text{ and } t > 0$$

Using these conditions, it can be shown (11) that the rate of mass transfer across the surface is

$$N_m = (C_s - c_0)\left(\frac{D_L t}{\pi}\right)^{\frac{1}{2}} \qquad (11)$$

in which N_m is the rate of mass transfer per unit time per unit area.

Eq. 11 expresses the rate of the absorption of a gas at the surface of a stagnant liquid of infinite depth as a function of the time.

Eq. 10 and Eq. 11 are based on the assumption that the absorption of oxygen by a body of liquid is purely by molecular diffusion throughout the entire depth of the body. However, in any practical case the mixing, which is caused by convection and density currents or by turbulence, invalidates the assumption of perfect quiescence. In order to apply these equations, it has been necessary to use a physical model that is not truly representative of the actual coalitions. The usual assumption has been that there were successive periods of perfect quiescence between which there was imposed complete and instantaneous mixing. This assumption has necessitated an empirical determination of a fictional period of quiescence, whereas the true physical picture is one of continuous turbulent mixing. However, Eq. 3 and Eq. 6 are applicable in cases in which the fluid turbulence is a significant factor. The only condition necessary for their application is that the concentration of the dissolved gas should be uniform in the body of the liquid at any particular instant. However, the advantages that these equations gain through simplicity are lost through generality because the coefficients encompass all the variables that influence the process. No analysis has been made in which the rate of reaeration was expressed as a function of the variables involved in turbulent flow. The coefficients must be determined experimentally before the equations can be applied.

The following theoretical development presents a mathematical amplification of the formulas, defining the reaeration coefficient for turbulent-flow conditions in a natural stream. Modern developments in fluid mechanics have resulted in an understanding of the phenomenon of turbulence, which has permitted a rational analysis to be made of the reaeration problem. Therefore, it is pertinent to present some of the basic concepts of fluid turbulence that are applicable to the process of reaeration in turbulent flow.

Basic Theory of Fluid Turbulence

Turbulent flow consists of a complex secondary motion, which is superimposed on the primary motion of translation. Turbulence is characterized by eddies, which transport particles of fluid from one layer to another with varying velocities. The eddy motion, which is erratic and seemingly unpredictable, can only be defined in a probability sense. Thus, the principles of statistics are used in order to define quantitatively the parameters of turbulence, such as the size of the eddies and the velocity fluctuations.

The velocity at any point in turbulent flow varies in magnitude and in direction. It is convenient to represent the velocity vector by velocity components along the three rectangular axes. The velocity in the direction of flow, which is taken along the x-axis, is represented as $U \pm u$, in which U is the mean velocity and u represents the fluctuation. The fluctuation of the velocities along the other two axes, y and z, is represented by v and w, respectively. The fluctuations, u, v, and w, vary with time, and, by definition, the arithmetic means must necessarily be zero. It has been shown that these fluctuations follow the normal error law, thus indicating their random nature. A significant average is the root-mean-square or the standard deviation of the component, v, from its mean value of zero.

Two types of turbulence can be designated, depending on the interrelationship of u, v, and w. If there is a complete lack of correlation of the velocity fluctuations in the different directions, the turbulence is referred to as isotropic. This type of turbulence, in which there is neither a shearing stress nor a velocity gradient, is approached in the flow downstream from the screens, in a hydraulic jump, and in the center of a deep and wide-open channel. Nonisotropic turbulence is characterized by a significant correlation between the velocity fluctuations and by a velocity gradient and shearing stress. Nonisotropic turbulence is shown in flow in pipes and in comparatively shallow open channels.

Although the velocity fluctuations define the intensity of turbulence, some linear measure is required to define the scale of turbulence. In this regard the mixing-length hypothesis has been presented by L. Prandtl (12). The velocity component, v, is related to the forward-flow velocity gradient by assuming that the vertical flow produced by the eddy is effective over a distance, l, in the y-direction. This relationship may be expressed by

$$\overline{|v|} = l \frac{dV}{dy} \qquad \text{(12)}$$

The mixing length, l, resembles the mean free path in the kinetic theory of gases. It signifies a distance that a particle moves from its point of departure from the mean motion to its point of remixing with the main body of the fluid. It is doubtful whether it is possible to assign a more definite physical meaning to the mixing length, but this length is a measure of the average size of the eddies that are responsible for the fluid mixing. Eq. 12 applies to the turbulent condition in which a velocity gradient exists. Because it is possible for the turbulence in a natural river to approach isotropy, in which case the velocity gradient approaches zero, it is appropriate to consider the approach taken by Geoffrey I. Taylor (13) and developed further by him (14). The approach is based on the principles of statistics and leads to a more precise definition of the mixing length. Taylor has suggested

> "It is clear that whatever we mean by the diameter of an eddy (scale of turbulence), a high degree of correlation must exist between the velocities at two points which are close together when compared with this diameter. On the other hand, the correlation is likely to be small between two points situated many eddy diameters apart. If, therefore, we imagine that the correlation R_y between the values of the speed at two points distant y apart in the direction of the y coordinate has been determined for various values of y, we may plot a curve of R_y against y, and the curve will represent the statistical distribution of the velocity fluctuation along the y-axis. If R_y falls to zero at y equal Y, then a length can be defined such that
>
> $$l = \int_0^\infty R_y\, dy = \int_0^Y R_y\, dy$$
>
> This length * * * may be a possible definition of the average size of the eddies."

The preceding analysis can apply to both isotropic turbulence and nonisotropic turbulence, although the available research indicates that the analysis is used primarily in the former case. Prandtl's mixing-length hypothesis refers only to nonisotropic turbulence. The relationship between the shearing stress and the velocity gradient is as follows:

$$\tau = \epsilon \rho \frac{dU}{dy} \quad \text{.........................} \quad (13)$$

in which τ is the shearing stress, dU/dy represents the velocity gradient, ρ is the density, and ϵ designates the kinematic eddy viscosity. The quantity ϵ is analogous to the coefficient of viscosity in the expression for shear in laminar flow. Just as the molecular viscosity depends on the velocity and mean free path of the molecules, so does the eddy viscosity depend on the velocity and mean free path of fluid particles. The relationship is

$$\epsilon = \overline{|v|}\, l \quad \text{.........................} \quad (14)$$

The eddy viscosity varies from point to point in a fluid, in accordance with the variation of the mixing length and the velocity fluctuation as indicated by Eq. 14. In an actual river the eddy viscosity varies with the depth and the width of the channel. If the flow is steady and uniform and is limited to the case of a wide stream, the eddy viscosity will be a function of the depth only. In such a case the variation of the eddy viscosity with the depth is as follows:

$$\epsilon = \kappa\, y \left(\frac{\tau_0}{\rho}\right)^{\frac{1}{2}} \left(1 - \frac{y}{H}\right) \qquad (15)$$

in which τ_0 is the bottom shear and κ is the von Kármán universal constant having the value of approximately 0.4. In this case the axis, $y = 0$, is taken at the invert of the channel. Eq. 15 indicates that the value of the eddy viscosity is a maximum at a point located at a distance equal to one-half of the depth from the surface, and the eddy viscosity is zero at the surface and at the bed. It is probable that its value is not absolutely zero at the surface, particularly in cases in which surface agitation is evident. However, experimental measurements (14) indicate that the value of the eddy viscosity follows the functional trend of Eq. 15 as close to the surface as it was possible to make the measurements.

The simultaneous transport of fluid particles between the adjacent layers involves the transport of any inherent characteristic of the fluid, such as the heat content and the concentration of dissolved material. This process is usually referred to as eddy diffusion or eddy transport. The time rate of change of this characteristic at any point may be expressed as follows:

$$N_c = -\, D_\epsilon \frac{dj}{dy} \qquad (16)$$

in which N_c represents the rate of transport of the characteristic in the y-direction, D_ϵ denotes the diffusion coefficient, and j represents the concentration of the characteristic. The negative sign indicates that the transport is in the direction of decreasing concentration. The diffusion coefficient represents a combination of the velocity and the length which describes the mixing or diffusion process. In a turbulent condition of flow, the eddy-diffusion coefficient, D_ϵ, is approximately equal to the eddy viscosity, ϵ. The approximate equality has been indicated (15) by studies that have been made of the diffusion of droplets of dye (16) having the same density as the water in which the diffusion took place.

The basic concepts of fluid turbulence and the fundamental laws of reaeration are used in the analysis and the solution of the problem of reaeration under turbulent-flow conditions in a river.

MATHEMATICAL DEVELOPMENT OF THE THEORY OF REAERATION

The passage of oxygen from the atmosphere to a body of turbulent fluid may be described in the most complete and general manner by the two-film concept. The concept, when expressed in differential form similar to Fick's law of hydrodiffusion, leads to

$$\frac{\partial m}{\partial t} = D_G \left(\frac{\partial c}{\partial y}\right)_1 \qquad (17a)$$

$$\frac{\partial m}{\partial t} = D_L \left(\frac{\partial c}{\partial y}\right)_2 \qquad (17b)$$

and

$$\frac{\partial m}{\partial t} = D_\epsilon \left(\frac{\partial c}{\partial y}\right)_3 \dots\dots\dots\dots\dots\dots\dots (17c)$$

in which $(\partial c/\partial y)_1$ is the concentration gradient through the gas film; $(\partial c/\partial y)_2$ represents the concentration gradient through the liquid film; $(\partial c/\partial y)_3$ designates the concentration gradient in the body of the liquid below the liquid film; D_G denotes the molecular diffusivity of the gas through the gas film; D_L represents the molecular diffusivity of the gas through the liquid film; and D_ϵ is the eddy-diffusion coefficient of the gas in the body of the liquid.

In addition to the molecular diffusion through the gas and liquid films, Eqs. 17 also include a factor for the eddy diffusion through the body of the liquid. In the majority of cases of natural rivers, turbulent flow prevails. The value of the eddy-diffusion coefficient is of the same order of magnitude as that of the eddy viscosity of the fluid as previously described. That the control for the entire process is the liquid film may be seen from the following approximate values of the coefficients:

$$D_G = 0.70 \text{ sq ft per hr}$$
$$D_L = 0.00008 \text{ sq ft per hr}$$

and

$$D_\epsilon = 50 \text{ sq ft per hr}$$

The oxygen diffusivities through the gas film and the liquid film are temperature functions, and the indicated values are those at temperature conditions of 18° C (7). The magnitude of the eddy viscosity depends on the physical and hydraulic characteristics of the channel as given by Eq. 15. An average value for the eddy viscosity may be computed from Eq. 15 as follows:

$$\epsilon = \frac{\kappa H}{6} \left(\frac{\tau_0}{\rho}\right)^{\frac{1}{2}} \dots\dots\dots\dots\dots\dots\dots (18a)$$

and

$$\epsilon = \frac{\kappa H}{6} (H g S)^{\frac{1}{2}} \dots\dots\dots\dots\dots\dots\dots (18b)$$

By using Eq. 18*b*, the mean value of the eddy viscosity was computed for each river, which will be referred to subsequently. The value of 50 sq ft per hr for D_ϵ is the minimum of the values thus computed. Because the eddy diffusivity is so great in comparison with the diffusivity through the liquid film, it follows from Eqs. 17 that the concentration gradient throughout the depth of the liquid is extremely small compared with the gradient across the liquid film. In all practical cases the concentration of dissolved oxygen throughout the depth of the liquid body may be considered as being uniform. This fact has been substantiated by actual measurements of dissolved oxygen gradients (17). It may also be concluded that Eq. 3 and Eq. 6 can be applied to the case of reaeration in a river because the only condition necessary for their application is that of the uniform concentration of dissolved oxygen throughout the liquid depth at any particular time. Eq. 10 is not applicable because it fundamentally defines a case of molecular diffusion throughout the depth of the liquid body. Except in the vicinity of the surface, turbulent transport is so great compared with molecular diffusion that the latter effect is insignificant. Molecular diffusion is the controlling factor at a point close to the surface where the eddy viscosity approaches zero or is equal to zero. However, if the eddy viscosity has a finite value at the surface, it is possible to conceive of the existence of a liquid film, but with the elements of this film being continuously replaced by the turbulence of the fluid below. In either case Eq. 11 may possibly apply because it defines the rate of gas absorption at the surface of the body of water. However, this equation was developed on the basis of an infinitely deep liquid, and its application to a liquid film of small thickness is doubtful. The conclusions concerning reaeration for turbulent flow, which may be made based on the preceding examination, can be summarized as follows:

1. The liquid film at the water surface is the controlling factor in the process.
2. The concentration of the dissolved oxygen may be considered as being uniform throughout the liquid body.
3. The effect of turbulence on the liquid film at the surface must be evaluated.

For turbulent conditions the particles at the surface are continuously exchanged for liquid from the main body. Any particular part of the surface may be replaced at any time after its creation. At that instant the concentration of oxygen is equal to that of the main body. The concentration becomes saturation immediately, and the transfer of oxygen across the surface is accomplished by molecular diffusion. In accordance with Fick's law the rate of transfer is governed by the concentration gradient, which is a function of time. The rate of transfer across this part of the surface will depend on the length of time that the surface has been exposed to the atmosphere. The over-all rate of transfer must therefore depend on the distribution of ages of the surface elements, which, in turn, is controlled by the state of turbulence. The function defining this age distribution has been considered to be that presented by P. V. Danckwerts (11). The analysis can be applied to the case of an open channel. For a unit surface area in a wide-open channel in which the flow is turbulent, the average rate of absorption is assumed to be uniform over the area. Liquid elements at the surface, which have been exposed to the atmosphere for varying lengths of time, are replaced by those arising from the turbulent motion of the body of the fluid. The mean rate of surface renewal, r, is assumed to be constant. Because of the random nature of the turbulence, the chance of any part of the surface being replaced is independent of its time of exposure.

For any given unit area of surface at any particular instant, there is a distribution of ages ranging from zero (just replaced) to infinity. The distribution is defined by a function, $f\,t$, in which $f\,t\,dt$ is the relative part of the area having ages t and $t + dt$.

It has been shown that

$$f\,t = r\,e^{-rt} \qquad \text{(19)}$$

Eq. 19 is the function defining the distribution of surface ages.

By substituting Eq. 19 for the area in Eq. 8, the rate of transfer of oxygen across a unit area of surface may be expressed as follows:

$$\frac{\partial m}{\partial t} = - D_L r \int_0^\infty e^{-rt} \left(\frac{dc}{dy}\right)_{y=0} dt \quad \text{(20a)}$$

and

$$\frac{\partial m}{\partial t} = - D_L r \left(\frac{d\bar{c}}{dy}\right)_{y=0} \quad \text{(20b)}$$

in which $\bar{c}$ is the Laplace transform of c.

As shown previously, the fundamental equation of transfer for the unsteady state is

$$\frac{\partial c}{\partial t} = - D_L \frac{\partial^2 c}{\partial y^2} \quad \text{(21)}$$

The boundary conditions which may apply during the time when the transfer occurs are as follows:

$$c = c_0 \text{ when } t = 0 \text{ and } 0 < y < Y_L$$
$$c = C_s \text{ when } t > 0 \text{ and } y = 0$$

and

$$c = c_0 \text{ when } t > 0 \text{ and } y = Y_L$$

In the distance from the surface, $y = 0$ to $y = Y_L$, the diffusion is molecular, as predicated by the basic equation of hydrodiffusion. Below this distance, eddy diffusion controls the rate of transfer.

If Eq. 22 is multiplied by e^{-rt} and integrated with respect to t between the limits of zero and infinity, the result is

$$\frac{\partial \bar{c}}{\partial t} = D_L \frac{\partial^2 \bar{c}}{\partial y^2} \quad \text{(22)}$$

Applying the boundary conditions, it has been shown (18) that

$$\frac{dm}{dt} = (C_s - c_0)\ (D_L\, r)^{\frac{1}{2}} \coth \left(\frac{r\, Y_L^2}{D_L}\right)^{\frac{1}{2}} \quad \text{(23)}$$

By comparing Eq. 23 with Eq. 6,

$$K_L = (D_L\, r)^{\frac{1}{2}} \coth \left(\frac{r\, Y_L^2}{D_L}\right)^{\frac{1}{2}} \quad \text{(24)}$$

The term, r, in Eq. 23 and Eq. 24 is the average rate of replacement of the surface and has the dimensions of t^{-1}.

The existence of a laminar film at the surface of a stream, as implied previously, results in a mathematical model that seems to be a good representation of the physical conditions. The concentration of the dissolved oxygen can be less than the saturation value at a point near the surface, although it would seem that at the interface the concentration must be that of saturation. A representative value of r, as will be shown, equals 1 per 10 sec, and an average value of D_L equals 0.00008 sq ft per hr. The value of Y_L equals 0.04 cm when the coth term in Eq. 25 equals 1.01. If the film has a thickness greater than 0.04 cm, the value of the coth term is closer to unity. It is doubtful that the exact magnitude of the film thickness can be determined. However, reported values (4) approximated from experimental data may be compared with the value of the film thickness as defined previously. For a comparable condition the thickness of the film was 0.08 cm. For most practical cases it may be assumed that the value of the coth term is close to unity.

If a laminar film does not exist at the surface of a stream, an alternate mathematical model may be used to develop a relationship defining the reaeration coefficient. A body of water of average depth, H, beneath any part of the surface area is assumed to remain quiescent for intervals of time during which oxygen is absorbed by molecular diffusion. At various times, defined by r, the body is assumed to be mixed instantaneously from top to bottom. The differential equation for this is the same as Eq. 23, but the boundary conditions are as follows:

$$\begin{array}{llll} & c = c_0 & t = 0 & 0 < y < H \\ & c = C_s & t > 0 & y = 0 \\ \text{and} & \dfrac{dc}{dy} = 0 & t > 0 & y = H \end{array}$$

The third equation must apply because there can be no transfer across the bottom. From a mathematical development similar to that of Eq. 24,

$$K_L = (D_L\, r)^{\frac{1}{2}} \tanh \left(\frac{r\, H^2}{D_L} \right)^{\frac{1}{2}} \qquad (25)$$

The foregoing boundary conditions in Eq. 25 are identical with those used previously (6). However, in the present development, the rate of the surface renewal has been considered. When the value of $\left(\frac{r\, H^2}{D_L} \right)^{\frac{1}{2}}$ is 2.65, the tanh term equals 0.99, and when $\left(\frac{r\, H^2}{D_L} \right)^{\frac{1}{2}}$ increases, the value of tanh approaches unity. Based on the same conditions for r and D_L as used previously, the value of H equals 0.04 cm. In other words, the average depth of the stream must be equal to 0.04 cm for the tanh term to have a minimum value of 0.99. It is obvious that for all practical cases the tanh term may be taken as unity. It can be concluded that

$$K_L = (D_L\, r)^{\frac{1}{2}} \qquad (26)$$

or

$$k_2 = \left(\frac{D_L\, r}{2.31\, H} \right)^{\frac{1}{2}} \qquad (27)$$

With the same boundary conditions used in the development of Eq. 11, the equality indicated by Eq. 26 was also developed. Two assumptions are implied in this equality. First, the concentration is uniform throughout the depth, and second, the concentration does not change with time. It has been shown that the first assumption is valid for turbulent flow in a river.

The second assumption is fulfilled in any reaeration process in which the time of renewal is sufficiently short that the concentration can be assumed to remain constant within that time. It will be shown that the time of renewal ranges from 1 sec to 100 sec, which is of sufficiently short duration that the assumption of a constant concentration is valid.

Before Eq. 26 or Eq. 27 can be applied, the rate of surface renewal, r, must be related to the turbulence of a river and expressed in terms of measurable parameters.

DETERMINATION OF THE RATE OF SURFACE RENEWAL

In turbulent flow, momentum, mass, heat, or any inherent characteristic of the fluid can be transferred from one layer of fluid to another. Therefore, the basic concepts of turbulence can be used to determine the rate at which particles at the surface layer can be replaced by particles arising from the turbulent motion in the body of the fluid. The intensity of turbulence can be defined by some mean measure of the velocity fluctuations, such as $\overline{|v|}$, and the scale of turbulence can be defined by the mixing length, l. The latter signifies a distance a particle moves from its point of departure from the mean forward motion to the location at which it mixes again with the main body of the fluid. Therefore, only particles within a zone defined by a mixing length from the surface will affect the renewal of this surface. Furthermore, any particle located at a distance greater than the mixing length from the surface will be deflected from its vertical path before reaching the surface. It can be reasoned that vertical flow exhibiting small length and great velocity characteristics will cause a greater rate of surface renewal than a flow of great length and less velocity. Therefore, the particles at the surface are replaced at a rate directly proportional to the intensity of turbulence and inversely proportional to the scale of turbulence. Surface renewal can be considered to take place in a period of time defined by

$$t = \frac{l}{\overline{|v|}} = \frac{1}{r} \qquad (28)$$

for which the values of l and $\overline{|v|}$ are those that prevail in the vicinity of the surface. At any point, the time period defined by Eq. 28 will vary due to the random nature of the variables involved. However, because the length and the velocity are average values, t is the average time during which the surface renewal takes place. It can be shown that according to the Prandtl theory and by actual measurements the values of the parameters in the vicinity of the surface vary approximately in a linear fashion with the depth, and the values approach zero as the distance to the surface is decreased. As in the case of the values of the eddy viscosity, it is probable that the values of the parameters are not absolutely zero at the surface. If the values of the parameters are zero, their ratio is not necessarily zero. Because of the linear variation of the parameters with depth, their ratio is approximately constant within a distance defined by a mixing length from the water surface. The mixing length and the vertical-velocity fluctuation or their ratio must now be defined in terms of measurable dimensions or characteristics of the channel.

NONISOTROPIC TURBULENCE

The development of an expression for the rate of surface renewal is confined in this case to a condition of a marked velocity gradient which is characteristic of nonisotropic turbulence. Rearranging the basic relationship stated in Eq. 12, it follows that

$$\frac{\overline{|v|}}{l} = \frac{dU}{dy} \quad \ldots\ldots\ldots\ldots (29)$$

The ratio between the velocity fluctuation and the mixing length has been assumed to define the rate of surface renewal. Therefore,

$$r = \frac{dU}{dy} \quad \ldots\ldots\ldots\ldots (30)$$

Eq. 30 states that the rate of the surface renewal equals the velocity gradient at the surface. The rate at which the surface layer moves with respect to the adjacent layer is accompanied by a shearing action, causing a vertical displacement of fluid particles, which, in turn, is responsible for the rate at which the particles at the surface are displaced.

It has been shown (19, 20) that the von Kármán universal logarithmic velocity law for pipes can be applied to the case of uniform two-dimensional flow in an open channel. The velocity as a function of the depth can be expressed as follows:

$$U = U_m + \frac{1}{\kappa}\left(\frac{\tau_0}{\rho}\right)^{\frac{1}{2}}\left(1 + \log_e \frac{y}{H}\right) \quad \ldots\ldots\ldots\ldots (31)$$

In the foregoing equation, U_m is the mean velocity. Differentiating this equation with respect to y, an expression for the velocity gradient is obtained as follows:

$$\frac{dU}{dy} = \frac{1}{\kappa\, y}\left(\frac{\tau_0}{\rho}\right)^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots (32)$$

or

$$\frac{dU}{dy} = \frac{(H\, g\, S)^{\frac{1}{2}}}{\kappa\, y} \quad \ldots\ldots\ldots\ldots (33)$$

Substituting Eq. 33 in Eq. 30 yields

$$r = \frac{(H\, g\, S)^{\frac{1}{2}}}{\kappa\, y} \quad \ldots\ldots\ldots\ldots (34)$$

It has been reasoned that only the particles within a distance to the surface equal to the mixing length affect the rate of the renewal and, therefore, y may be taken equal to H without significant error. Hence,

$$r = \frac{(H\, g\, S)^{\frac{1}{2}}}{\kappa\, H} \quad \ldots\ldots\ldots\ldots (35)$$

The value of the von Kármán universal constant, κ, is generally taken as equal to 0.4. Actual measurements in both the field and the laboratory indicate that the value of κ may be less than 0.4 (21, 22, 23). The condition is particularly evident in streams carrying a sediment load and in streams in which the height of the roughness elements is of the same order of magnitude as the depth of the flow. This effect is indicated further by extensions of the mixing-length theory, such as that presented by C. G. Rossby and R. B. Montgomery (24), in which a measure of the surface roughness has been included. Therefore, any theory that considers the roughness elements is more appropriate for natural rivers. However, because of the difficulty of establishing the roughness parameter in any given stream, κ has been considered to equal 0.4 in the case of nonisotropic turbulence.

A functional formula expressing the reaeration coefficient, k_2, in terms of physically measurable parameters can be developed directly. Substituting the value of r, as indicated by Eq. 35, in Eq. 27 and accounting for the dimensional constants results in

$$k_2 = \frac{480\, D_L^{\frac{1}{2}}\, S^{\frac{1}{4}}}{H^{5/4}} \quad \text{(36)}$$

Eq. 36 defines the reaeration coefficient for the case of nonisotropic turbulence in a natural stream.

Isotropic Turbulence

In the cases of comparatively deep channels, it is possible that the turbulence may approach an isotropic condition. Inspection of Eq. 32 indicates that as the depth increases and the bottom shear decreases, the velocity gradient approaches zero, which is characteristic of isotropic turbulence. It follows that Eq. 12 cannot be used to define the ratio of the mixing length and the vertical-velocity fluctuation. Regardless of the fact that the velocity gradient may be zero, as in the case of the center of a pipe, the eddy viscosity has a finite value because heat or mass can be transferred across this plane. Because ϵ has a value greater than zero, the mixing length and the vertical-velocity fluctuation also have finite values. The approach described previously (13, 14) is appropriate in this case. However, the turbulence parameters, as defined by the theory, have not yet been related to the physical dimensions and the characteristics of a channel or a pipe. Additional research in this field is required before a determination can be made concerning the form of the velocity correlation curve. The only way to obtain values of the turbulence parameters is by a direct measurement. Values of the mixing length and the vertical-velocity fluctuation from actual measurements of the Mississippi River have been presented (25). Comparable values for estuaries have also been reported (26, 27). The mixing length and the vertical-velocity fluctuation were approximately 10% of the average depth and the mean-flow velocity, respectively. These values may be used as an approximation to determine the rate of the surface renewal, which is

$$r = \frac{0.10\, U}{0.10\, H} = \frac{U}{H} \quad \text{(37)}$$

Substitution of this value for r in Eq. 27 yields

$$k_2 = \frac{(D_L\, U)^{\frac{1}{2}}}{2.31\, H^{\frac{3}{2}}} \quad \text{(38)}$$

Eq. 38 approximates the reaeration coefficient for the case of isotropic turbulence in natural streams.

The rate of the surface renewal for nonisotropic turbulence has been defined, and a value has been approximated for isotropic turbulence. Therefore, it follows that, for an engineering application, it is necessary to determine the nature of the turbulence that prevails in any particular river. There is no sharp line of demarcation between isotropic turbulence and nonisotropic turbulence. The transition between the two types is defined by a range within which both may exist. When considering the case for which the effects of each are equal, it is observed that the rates of the surface renewal, as defined by Eq. 35 and Eq. 37, are equal. Therefore,

$$\frac{U}{H} = \frac{(H\, g\, S)^{\frac{1}{2}}}{\kappa\, H} \quad \text{(39a)}$$

Rearranging the terms yields

$$U = \frac{g^{\frac{1}{2}}}{\kappa} (H\, S)^{\frac{1}{2}} = C\, (H\, S)^{\frac{1}{2}} \quad \text{(39b)}$$

Eq. 39*b* is equivalent to the Chezy formula for flow in open channels. Assuming that κ ranges from 0.3 to 0.4, the Chezy coefficient, C, has an approximate range of from 14 to 20. Referring to Eq. 33, it can be seen that large values of the bottom shear and small values of depth produce marked velocity gradients. These conditions, which are associated with small values of C, indicate nonisotropic turbulence. Therefore, the turbulence can be assumed to be nonisotropic in cases for which the value of C is less than from 14 to 20, and isotropic for cases in which the value of C is greater than from 14 to 20. This range is also influenced by the ratio of the vertical-velocity fluctuation to the mean-flow velocity and by the ratio of the mixing length to the average depth, both of which were assumed to be 10%. In this study the distinction between the two types of turbulence was based on a value of C equal to 17. In cases for which it was not possible to estimate the value of C, the differentiation between the two types of turbulence was based on a criterion of depth. If the depth was less than 5 ft, turbulence was assumed to be nonisotropic, and if the depth was greater than 5 ft, it was assumed to be isotropic.

EFFECT OF TEMPERATURE ON THE REAERATION COEFFICIENT

The effect of the temperature on the coefficient of molecular diffusivity is given by the Stokes-Einstein equation (28):

$$D_L = \frac{B\, T_{\text{abs}}}{\mu} \quad \text{(40)}$$

in which T_{abs} is the absolute temperature and μ represents the coefficient of viscosity. If the coefficient of molecular diffusivity at any single temperature is known, the value of the constant, B, may be computed. A determination of the effect of the temperature on the reaeration coefficient may be made by substituting Eq. 40 in Eq. 27. The substitution results in

$$k_2\, \alpha\, (D_L)^{\frac{1}{2}}\, \alpha \left(\frac{T_{\text{abs}}}{\mu} \right)^{\frac{1}{2}} \quad \text{(41)}$$

The effect of temperature on the reaeration coefficient has been reported by others (29) as follows:

$$k_{2T} = (k_{2.20})\,1.016^{T-20} \qquad (42)$$

in which k_{2T} represents the value of the reaeration coefficient at temperature T; $k_{2.20}$ denotes the value of the reaeration coefficient at 20° C; and T designates temperature in degrees Centigrade. Eq. 42 has been used in the stream surveys that will be referred to subsequently. The ratios of the reaeration coefficient at 20° C to those at other temperatures, as computed by Eq. 41 and Eq. 42, are indicated in Table I. A comparison of these ratios indicates that the divergence is on the order of 2% between the two formulas.

TABLE I.—EFFECT OF TEMPERATURE ON THE REAERATION COEFFICIENT

Temperature, in degrees Centigrade	Coefficient of diffusion, in square feet per hour	Ratio of reaeration coefficient at 20°C to that at indicated temperature	
10	61	0.86[a]	0.85[b]
15	71	0.93[a]	0.92[b]
20	81	1.00[a]	1.00[b]
25	92	1.07[a]	1.08[b]
30	106	1.14[a]	1.17[b]

[a] Computed by Eq. 41. [b] Computed by Eq. 42.

The Stokes-Einstein equation has been used to determine the effect of the temperature on the coefficient of molecular diffusion.

DESCRIPTION OF EXPERIMENTAL APPARATUS AND METHOD

The critical point to be verified in the theoretical development is the relationship between the reaeration coefficient and the rate of surface renewal, as indicated by Eq. 27. The experimental apparatus consisted of a lattice work oscillating vertically in simple harmonic motion, which created a uniform degree of turbulence throughout the fluid (30). The lattice work was composed of three layers of screens, which were $\frac{3}{8}$ in. apart. Each layer was composed of two aluminum screenings, which were soldered together so that the openings of one were bisected in both directions by the wiring of the other. The net opening was $\frac{5}{32}$ in. square. The cylinder was approximately $5\frac{1}{2}$ in. in diameter and contained $6\frac{1}{4}$ in. of water, which was approximately 2,500 ml of water. The frequency of the oscillation of the lattice work was varied from 45 rpm to 120 rpm.

It was reasoned that the mixing length was proportional to the scale of the lattice work and to the amplitude of the oscillation (both of which were constant), and the vertical-velocity fluctuation was proportional to the frequency of oscillation, which was varied as described. The rate of the surface renewal would therefore be proportional to the speed of the oscillation. The amplitude of the oscillation of the lattice work was approximately equal to the depth of the water. This condition insured a uniform concentration of dissolved oxygen throughout the depth. In all tests the surface of the water was slightly above the highest position of the lattice work so that the surface renewal was caused only by the fluid turbulence and not by the lattice work breaking the surface.

Two sets of experiments were conducted. In the first, deaerated water was used and dissolved oxygen concentrations were determined by the polographic method of analysis (31). Samples were taken at from 5-min to 20-min intervals, depending on the speed of the oscillation. The temperature was measured at the time that each sample was taken. The reaeration coefficient, k_2, was determined graphically from a plot of the time periods of aeration versus the logarithm of the dissolved oxygen deficits. In the second set, a sodium sulfite solution was used. This method is based on the assumption that the rate of change of sulfite to sulfate is a direct measure of the rate of reaeration. The initial concentration of the sulfite was 8,000 ppm for each test, and its concentration was measured (32) at 2-hr or 3-hr intervals for a period of 12 hr. The average change in the concentration of the sulfite per hour was determined by the method of least squares. Although the sulfite method has been used frequently in the past as a means of measuring oxygen transfer, it has been criticized recently by some investigators. The pertinent test data and the computed reaeration coefficients for both experiments are shown in Table 2.

TABLE 2.—EXPERIMENTAL DATA

DEAERATED WATER		SULFITE SOLUTION			
Speed, in rpm	Temperature, in degrees Centigrade	Reaeration coefficient per hour	Speed, in rpm	Temperature, in degrees Centigrade	Reaeration coefficient per hour
45	19	0.19	47	27	5.06
47	19	0.23	57	27	5.42
55	17.5	0.31	68	27	5.55
55	17.5	0.37	94	27	6.30
70	19	0.71	...	...	...
90	20	1.1	...	...	...
94	19	1.3	...	...	...
118	20	1.6	...	...	...

VERIFICATION OF THE THEORY

In order to verify the relationship between the reaeration coefficient and the rate of surface renewal, the computed test coefficients were plotted against the square root of the speeds of oscillation, as shown in Fig. 1. As described in the previous section, it was reasoned that this speed is directly proportional to the rate of the surface renewal. The theory, according to Eq. 27, predicts that the reaeration coefficient varies as the square root of the rate of surface renewal; this proportionality is confirmed by the linear variation of the experimental data. The reaeration coefficients were converted to a common basis of 20° C by the proportionality indicated in Eq. 41. The data from both the deaerated water test and the sulfite test are plotted in Fig. 1. It is significant that the slopes of the lines defining the data are approximately equal and that both sets of data indicate good correlation. It may be concluded that the experimental work substantiates the theoretical development in this respect.

The following part of the verification consists of comparing values of the reaeration coefficient that have been previously reported by others with the values that were computed in accordance with the theoretical development presented herein. The reported values of the coefficient had been independently determined by others by substituting observed values of the biochemical oxygen demand, dissolved oxygen deficits, and time periods in Eq. 2. These values will be referred to as the observed coefficients, and those determined by the theory presented herein will be referred to as the computed coefficients. In selecting values for comparison, it was essential to use river sections in which the sludge deposits and the algae effects were at a minimum because such conditions would reflect false values of the reaeration coefficient. Fig. 2 shows a graphical comparison between the observed coefficients and the computed coefficients. The distinction between isotropic turbulence and nonisotropic turbulence was made on the basis of the Chezy coefficient in accordance with the criterion described previously. After the Chezy coefficient was defined for each river, the appropriate formula (Eq. 36 or Eq. 38) was selected to compute the reaeration coefficient. The range and the over-all correlation of the data are particularly significant and may be accepted as further evidence of the validity of the theoretical development.

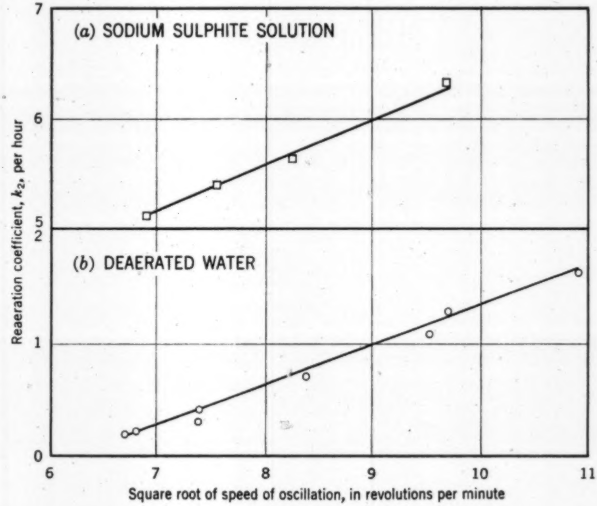


FIG. 1.—THE EFFECT OF AGITATION ON THE REAERATION COEFFICIENT

The Clarion River in Pennsylvania and its tributaries (33) provided excellent data on large values of the reaeration coefficient. The largest value shown represents a tributary of the Clarion River (Elk Creek), which is a shallow, turbulent stream with a steep slope, and, therefore, the biochemical oxygen demand removal by sedimentation was assumed to be negligible. It is probable that algae effects were also negligible. The smaller value was taken from downstream sections of the Clarion River where sludge deposits were at a minimum.

The Tennessee River tributary (34) is also a shallow, turbulent stream, and values of the reaeration coefficient for this stream are in the same range as those of the Clarion River. The upstream areas of this river were not included for comparison because data concerning the depths were lacking. The two values shown were averages that were weighted by time for periods of approximately one day and three days. F. W. Kittrell and Oscar W. Kochtitzky, Jr., A.M.ASCE, believe that a representative value for the entire stream is between 0.6 and 0.7, which may be compared with a value of approximately 0.8 as computed by the proposed formula (34).

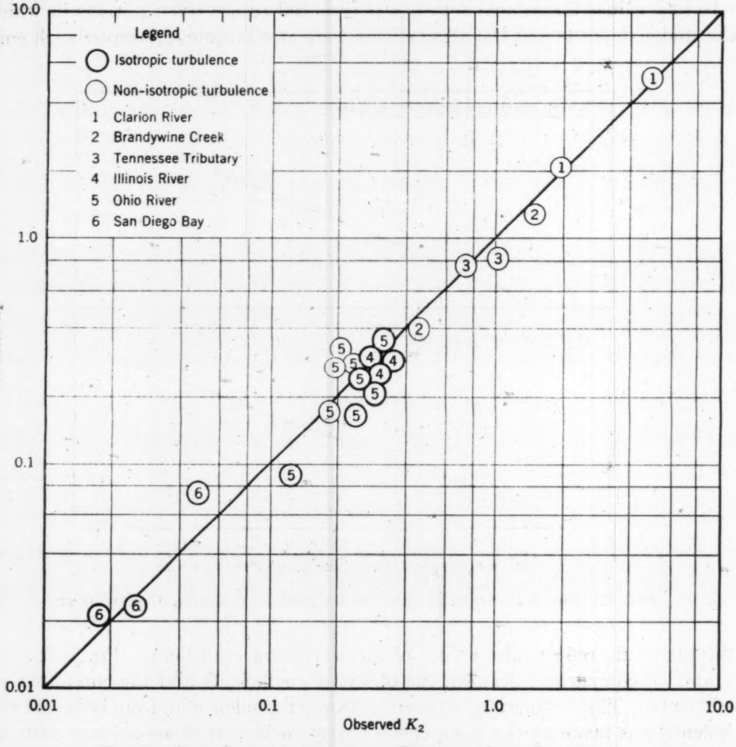


FIG. 2.—THE COMPARISON OF THE COMPUTED AND OBSERVED REAERATION COEFFICIENTS

Brandywine Creek (35) provided two additional points, which were in the category of large values. Regarding data on the depth, only the maximum center values were reported; the average depth was considered as two-thirds of the maximum depth. For these three cases the turbulence was considered nonisotropic.

The values of the reaeration coefficient of the Illinois River are for river sections downstream from the source of the pollution, in which sludge deposits were absent (36). That the algae influence was negligible was indicated by an hourly survey at one station, which showed a range of dissolved oxygen from 1.95 ppm to 2.04 ppm.

The smallest values of the reaeration coefficient were for San Diego Bay in California (8). For the Illinois River study and the San Diego Bay study, the turbulence was considered to be isotropic. Table 3 presents the observed values and the computed values of the reaereation coefficient for the rivers, with the pertinent hydraulic data.

TABLE 3.—COMPUTED AND OBSERVED REAERATION COEFFICIENTS

River	Depth, in feet	Velocity, in feet per second	Slope, in feet per 1,000 ft	Chezy coefficient	Temperature, in degrees Centigrade	Diffusivity, in square feet per hour $\times 10^6$	Time of renewal, in seconds	REAERATION COEFFICIENT, k_2, PER DAY	
								Computed	Observed
Elk	0.9	0.97	3.6	17	12	64	1.1[a]	5.3	4.8
Clarion	1.9	0.55	1.4	11	13	66	1.7	2.1	2.6 1.2 1.9[b]
Brandywine	2.5	...	1.56	...	20	81	2.8	1.3	1.5
	5.2	...	0.68	...	20	81	6.2	0.43	0.45
Tennessee	4.0	0.73	0.49	16	23	89	5.3[a]	0.64	1.31
	3.1	0.65	0.69	14	23	89	3.9[a]	0.97	0.99
								0.84[b]	1.12[b]
	2.0	0.98	0.75	25	23	89	1.6[a]	2.24	
	4.0	0.67	0.46	16	22	86	4.7[a]	0.66	
	3.9	0.72	0.47	17	22	86	5.2[a]	0.65	
								0.76[b]	0.72
Illinois	9.2	1.37	0.027	87	27	98	6.7	0.26	0.27
	9.0	1.57	0.027	101	27	98	5.7	0.29	0.32
	8.9	1.63	0.027	105	27	98	5.5	0.30	0.35
San Diego Bay	12	0.32	...	...	20	81	37.5	0.076	0.048
	32	0.50	...	...	20	81	64.0	0.022	0.018
	37	0.88	...	...	20	81	42.0	0.023	0.026

[a] Time of renewal reduced by the percentage of rapids in the stretch. [b] Average value—weighted by time of passage in river section.

The Ohio River survey (1) provided data for both high-water values and low-water values. It should be noted that empirical relationships were developed for this river, as indicated by Eq. 5. The depth in this formula refers to the depth above low water. In most sections the average depth from low water to the channel bottom was significant. Therefore, in computing the value of the reaeration coefficient in accordance with the theoretical formulas, one-half of this depth correction was added to the stage-height depth in order to secure a value that was most representative of the ratio of volume and

TABLE 4.—COMPUTED, OBSERVED, AND EMPIRICAL REAERATION COEFFICIENTS FOR THE OHIO RIVER

Station	Temperature, in degrees Centigrade	Velocity, in feet per second	Depth stage, in feet	Depth correction, in feet	Mean depth, in feet	Slope, in feet per 1,000 ft	Chezy coefficient	Diffusivity, in square feet per hour × 10^6	Time renewal, in seconds	Reaeration Coefficient, k_2, Per Day		
										Computed	Observed	Empirical
NONISOTROPIC TURBULENCE												
11-19	24.5	0.59	4.8	2.2	7.0	0.197	16	92	13	0.23	0.19	0.34
	24.1	0.32	3.6	2.2	5.8	0.197	9	91	12	0.29	0.29	0.24
	24.4	0.22	3.1	2.2	5.3	0.197	7	92	12	0.33	0.22	0.17
	20.5	0.19	2.6	2.2	4.8	0.197	6	82	11	0.36	0.14	0.13
										0.30[a]	0.21[a]	0.24[a]
23-65	20.0	0.43	3.8	2.6	6.4	0.123	15	81	16	0.22	0.19	0.13
	18.0	0.23	1.4	2.6	4.0	0.123	10	77	12	0.38	0.35	0.31
										0.30[a]	0.28[a]	0.22[a]
77-88	24.5	0.40	4.9	3.7	8.6	0.095	15	92	22	0.15	0.25	0.27
	20.4	0.40	3.9	3.7	7.6	0.095	15	82	21	0.18	0.72[b]	0.29
	18.4	0.22	3.4	3.7	7.1	0.095	9	78	19	0.18	0.14	0.15
										0.17[a]	0.19[a]	0.21[a]
104-349	19.2	0.51	1.1	4.4	5.5	0.167	17	79	13	0.28	0.20	0.20
ISOTROPIC TURBULENCE												
11-19	15.9	2.50	11.8	2.2	13.8	0.197	48	72	5.5	0.16	0.24	0.26
23-65	16.3	4.20	12.4	2.6	15.0	0.123	96	73	3.6	0.18	0.20	0.21
	22.9	1.36	6.0	2.6	8.6	0.123	42	88	6.3	0.27	0.33	0.34
	24.8	0.86	4.6	2.6	7.2	0.123	27	93	8.4	0.29	0.23	0.38
	24.4	0.53	4.0	2.6	6.6	0.123	19	92	12.5	0.25	0.26	0.25
										0.25[a]	0.26[a]	0.32[a]
77-88	16.8	3.68	10.6	3.7	14.3	0.095	100	74	3.9	0.19	0.80	0.08
	23.7	1.01	5.6	3.7	9.3	0.095	34	90	9.2	0.21	1.00	0.84
	25.0	0.62	5.3	3.7	9.0	0.095	21	93	14.5	0.18	0.51	0.56
										0.19[a]	0.77[a]	0.50[a]
104-349	16.7	3.72	14.3	4.4	18.7	0.167	66	74	5.0	0.13	0.18	0.16
	24.4	1.48	4.4	4.4	8.8	0.167	39	92	5.9	0.28	0.27	0.33
	25.8	1.18	3.8	4.4	8.2	0.167	32	95	7.0	0.28	0.20	0.27
	24.8	0.91	2.8	4.4	7.2	0.167	26	93	7.9	0.30	0.21	0.23
	21.3	0.96	2.9	4.4	7.3	0.167	27	84	7.6	0.28	0.17	0.19
										0.29[a]	0.21[a]	0.26[a]
349-461	17.3	3.90	19.4	4.8	24.2	0.068	96	75	6.2	0.09	0.13	0.12
	26.9	2.32	5.0	4.8	9.2	0.068	88	98	4.0	0.34	0.53	0.55
	26.0	1.98	4.0	4.8	8.8	0.068	81	96	4.4	0.33	0.26	0.41
	22.2	2.18	4.5	4.8	9.3	0.068	86	86	4.3	0.31	0.16	0.34
	20.2	1.41	1.7	4.8	6.5	0.068	66	82	4.6	0.37	0.34	0.33
										0.34[a]	0.32[a]	0.41[a]

[a] Arithmetic average. [b] Not included in average.

surface area. The stage height itself does not represent an average depth because this is dependent on the cross-sectional shape of the river. If the channel has steep sides, the stage height will represent an average depth, and this was generally true for the Ohio River. The theoretical values of the coefficient were based on the stage height plus the correction to the channel bottom. In Table 4 the computed values, the observed values, and the empirical values of the reaeration coefficient are presented for nonisotropic turbulence, and a similar comparison is made for isotropic turbulence. Data for certain sections of the Ohio River were not included in Table 4 because of localized conditions as follows:

Section	Condition
3 to 11	Regulation by dams during survey
65 to 77	Inconsistent hydraulic data
475 to 482	Data influenced by backwater from dam and depth correction greater than 6 ft
482 to 488 492 to 598 598 to 611	Low water slope extremely variable and depth correction greater than 6 ft

Reaeration values of the same order of magnitude for each section were averaged to compute the points shown in Fig. 2. The only case in which there was an extreme discrepancy between the observed and computed values (stations 77–88) was not shown. Slope data were available for low-water values only. The low-water slopes were used in the determination of the Chezy coefficient. The values of this coefficient are therefore inaccurate for the high-water conditions, but the order of magnitude of the coefficient is such that the turbulence can be assumed to be isotropic.

The data for the depths were given in the preceding references, except in the case of the Clarion River, for which the depths were determined from maps of the Corps of Engineers, United States Department of the Army. The data for the slopes were either given or determined from the United States Geological Survey Water Supply information. The values of temperature were reported in all the references cited.

CONCLUSIONS

A theoretical development of the mechanism of reaeration in natural rivers has been presented. Both isotropic turbulence and nonisotropic turbulence have been considered in the development. Laboratory experiments were conducted so that a verification of the fundamental relationship between the reaeration coefficient and the rate of the surface renewal could be obtained. The comparisons made between the coefficients observed from the river surveys and those computed by the proposed formulas are in good agreement.

APPENDIX I. BIBLIOGRAPHY

(1) **"A Study of the Pollution and Natural Purification of the Ohio River,"** by H. W. Streeter and E. B. Phelps, *Public Health Bulletin 146*, U. S. Public Health Service, Washington, D. C., 1925.

(2) **"Principles of Gas Absorption,"** by W. K. Lewis and W. C. Whitman, *Industrial and Engineering Chemistry*, Vol. 16, 1924.

(3) **"Absorption and Extraction,"** by T. K. Sherwood and R. D. Pigford, McGraw-Hill Book Co., Inc., New York, N. Y., 2d Ed., 1952.

(4) **"Theoretical Principles of Aeration,"** by P. D. Haney, *Journal, A.W.W.A.*, Vol. 46, April, 1954.

(5) **"Aerobic Biological Treatment of Organic Wastes,"** by W. W. Eckenfelder and D. J. O'Connor, *Proceedings*, 9th Industrial Waste Conference, Purdue Univ., Lafayette, Ind., May, 1954.

(6) **"Location of Sewer Outlets and the Discharge of Sewage into New York Harbor,"** by W. M. Black and E. B. Phelps, *Report to the Board of Estimate and Apportionment*, New York, N. Y., March, 1911.

(7) **"International Critical Tables,"** McGraw-Hill Book Co., Inc., New York, N. Y., Vol. 5, 1925.

(8) **"The Oxygen Resources of San Diego Bay,"** by I. Nusbaum and H. E. Miller, *Sewage and Industrial Wastes*, Vol. 24, December, 1952.

(9) **"Stream Sanitation,"** by E. B. Phelps, John Wiley & Sons, Inc., New York, N. Y., 1944.

(10) **"Deoxygenation and Reoxygenation,"** by C. J. Velz, *Transactions*, ASCE, Vol. 104, 1939.

(11) **"Significance of Liquid Film Coefficients in Gas Absorption,"** by P. V. Danckwerts, *Industrial and Engineering Chemistry*, Vol. 43, June, 1951.

(12) **"Über Flüssigkeiten bei sehr kleiner Reibung,"** by L. Prandtl, *Verhandlung*, 3d International Mathematics Cong., Heidelberg, 1904.

(13) **"Diffusion by Continuous Movements,"** by G. I. Taylor, *Proceedings*, London Math. Soc., Vol. 2, No. 20, 1920.

(14) **"Statistical Theory of Turbulence,"** by G. I. Taylor, *Proceedings*, Royal Soc. of London, Vol. 151, No. 1, 1935.

(15) **"Turbulence in Open Channel Flow,"** by A. A. Kalinske and J. H. Robertson, *Engineering News-Record*, Vol. 126, No. 15, April 10, 1941.

(16) **"Experiments on Eddy Diffusion and Suspended Material Transportation in Open Channels,"** by A. A. Kalinske and C. L. Pien, *Transactions*, Am. Geophysical Union, Vol. 24, No. 2, April, 1943.

(17) **"A Sanitary Survey of the Willamette River,"** by G. W. Gleason, *Bulletin*, Oregon State Agricultural College, Corvallis, Ore., Series No. 6, April, 1936.

(18) **"The Nature of the Oxygen Transfer Coefficient in Aeration Systems,"** by W. E. Dobbins, *Proceedings*, Manhattan College Conference on Biological Waste Treatment, New York, N. Y., April, 1955.

(19) **"Laws of Turbulent Flow in Open Channels,"** by G. H. Keulegan, *Research Paper 1151*, *Journal of Research*, National Bureau of Standards, U. S. Dept. of Commerce, Washington, D. C., Vol. 21, 1938.

(20) **"Velocity Distribution in Open Channels,"** by V. A. Vanoni, *Civil Engineering*, Vol. 14, June, 1941.

(21) **"The Present Status of Research on Sediment Transport,"** by Ning Chien, *Transactions*, ASCE, Vol. 121, 1956.

(22) **"Transportation of Suspended Sediment by Water,"** by V. A. Vanoni, *ibid.*, Vol. 111, 1946.

(23) **"Flow in a Channel of Definite Roughness,"** by R. W. Powell, *ibid.*

(24) **"The Layer of Frictional Influence in Wind and Ocean Currents,"** by C. G. Rossby and R. B. Montgomery, Woods-Hole Ocean Inst., Woods-Hole, Mass., Vol. 3, No. 3, 1935.

(25) **"The Role of Turbulence in River Hydraulics,"** by A. A. Kalinske, *Bulletin No. 27*, Proceedings of the 2d Hydraulics Conference, Univ. of Iowa Studies in Eng., Univ. of Iowa, Ames, 1943.

(26) **"Density Current Problems in an Estuary,"** by T. Hamada, *Proceedings*, Minnesota International Hydraulics Convention, September, 1953.

(27) **"Theoretical Considerations of the Motion of Salt and Fresh Water,"** by J. B. Schijf and J. T. Schonfeld, *Proceedings*, Minnesota International Hydraulics Convention, September, 1953.

(28) **"Chemical Engineers Handbook,"** by J. H. Perry, McGraw-Hill Book Co., Inc., New York, N. Y., 3d Ed., 1950, Section 8.

(29) **"Measures of Natural Oxidation in Polluted Streams, II: An Experimental Study of Atmospheric Reaeration under Stream-Flow Conditions,"** by H. W. Streeter, C. T. Wright, and R. W. Kehr, *Sewage Works Journal*, Vol. 8, No. 2, March, 1936.

(30) **"Experiments on Mechanics of Sediment Suspension,"** by H. Rouse, *Proceedings*, 5th International Cong. for Applied Mechanics, 1938.

(31) **"Oxygen Transfer in Submerged Fermentation,"** by A. W. Nixson and E. L. Gaden, *Industrial and Engineering Chemistry*, Vol. 43, No. 8, 1950.

(32) **"Standard Methods for the Examination of Water, Sewage and Industrial Wastes,"** Am. Public Health Assn., Inc., New York, N. Y., 10th Ed., 1955.

(33) **"Report on the Clarion River Pollution Abatement,"** Camp, Dresser & McKee, San. Water Board, Dept. of Health, Commonwealth of Pennsylvania, Harrisburg, Pa., March, 1949.

(34) **"Natural Purification Characteristics of a Shallow, Turbulent Stream,"** by F. W. Kittrell and O. W. Kochtitzky, *Sewage Works Journal*, Vol. 19, No. 6, November, 1947.

(35) **"A Comparison of the Pollution and Natural Purification of the Connecticut and Delaware Rivers and Brandywine Creek,"** by L. R. Setter, *Bulletin 545*, New Jersey Agri. Experiment Station, New Brunswick, N. J., June, 1932.

(36) **"The Pollution and Purification of the Illinois River Below Peoria,"** by W. H. Wisely and G. W. Klassen, *Sewage Works Journal*, Vol. 10, No. 3, May, 1938.

APPENDIX II. NOTATION

The following symbols, adopted for use in the paper and for the guidance of discussers, conform essentially with "American Standard Letter Symbols for Hydraulics" (ASA Z 10.2–1942) prepared by a committee of the American Standards Association with Society representation, and approved by the Association in 1942:

A = surface area;
B = constant in Eq. 40;
C = Chezy coefficient;
C_s = saturation value of oxygen in water (concentration at the gas-liquid interface);
c = dissolved oxygen concentration;
$\bar{c}$ = Laplace transform of c;
c_0 = initial dissolved oxygen concentration;

D = dissolved oxygen deficit:
  D_0 = initial dissolved oxygen deficit;
  D_t = dissolved oxygen deficit after time t;
D_G = gas-film diffusion coefficient;
D_L = liquid-film diffusion coefficient;
D_ϵ = eddy-diffusion coefficient;
E = empirical coefficient in Eq. 5;
e = naperian base;
g = gravity constant;
H = average depth of liquid;
H_s = depth above low water in feet;
h = height of column of water;
j = concentration of inherent characteristic of the fluid;
K_L = liquid-film coefficient;
K_1 = deoxygenation coefficient—base e;
K_2 = reaeration coefficient—base e;
k_1 = deoxygenation coefficient—base 10;
k_2 = reaeration coefficient—base 10;
L = biochemical oxygen demand;
L_0 = initial biochemical oxygen demand;
l = mixing length;
m = mass;
N_c = rate of transport of characteristic in y-direction;
N_m = rate of mass transfer per unit area;
n_1 = empirical exponent in Eq. 5;
P = partial pressure of oxygen;
Q = exponent in Eq. 10;
R_y = correlation coefficient;
r = rate of surface renewal;
S = slope of the river channel;
T = temperature;
T_{abs} = absolute temperature;
t = time;
U = velocity in the x-direction (forward-flow velocity);
U_m = mean velocity;
u = velocity fluctuation in the x-direction;
V = volume;
v = velocity fluctuation in the y-direction;
w = velocity fluctuation in the z-direction;
x,y,z = coordinate axes;
Y_L = liquid-film thickness;
y = depth;
ϵ = coefficient of eddy viscosity;
κ = von Kármán constant;
ρ = density;
τ = shearing stress; and
μ = coefficient of viscosity.

DISCUSSION

Thomas R. Camp,[4] **M. ASCE.**—The authors have made a valuable contribution to the field of sanitary engineering. Although the paper is directed toward the development of a rational theory for the determination of reaeration coefficients for polluted streams, the theory presented is applicable generally to all gas-transfer processes. Therefore, it has wide application in water and sewage treatment.

The authors examined the Lewis and Whitman two-film theory of the transfer of a gas to solution in a liquid and show by comparison of the diffusion coefficients that the resistance through the liquid film at the interface is so great as compared to the resistance through the gaseous film at the interface, through the main body of the gas, and the main body of the liquid that it controls the entire mechanism. It is also shown that, because of the high resistance through the liquid film at the interface, the concentration of the dissolved gas in the body of the liquid will be substantially uniform even with a very slight mixing of the liquid.

It has been shown previously[5] by Mr. Dobbins that the liquid-film coefficient, K_L, of Eq. 6 is substantially equal to D_L/b if the liquid film remains unbroken, in which D_L is the coefficient of molecular diffusion of the dissolved gas and b is the thickness of the liquid film. Mr. Dobbins has also shown[5] that, where the film is being renewed continuously by shearing forces in the liquid, the liquid-film coefficient is substantially equal to the square root of the molecular diffusion coefficient times the rate of renewal of the liquid film (Eq. 26). In this relationship the liquid-film coefficient is substantially independent of the thickness of the liquid film. The writer wishes to emphasize that only this second expression for the liquid-film coefficient has real meaning because any motion in the liquid creates shearing forces that will remove the liquid film. Therefore, in any real process, the liquid film is continuously removed and regenerated.

The authors' evaluation of the rate of surface renewal, r, in terms of the parameters of the modern turbulent-flow theory is a significant advance. This development indicates that the rate of renewal of the liquid film at the interface is substantially equal to the velocity gradient of the liquid at the interface for nonisotropic turbulence, and is approximately equal to the velocity of the liquid divided by the depth for conditions for which the turbulence is isotropic. The basic equation for the reaeration coefficient is Eq. 27. In order to use this equation, it is necessary to insert a value for the rate of the surface renewal. Eq. 36 is the resulting expression for the reaeration coefficient for conditions in which the turbulence is nonisotropic, which also includes a dimensional factor for changing the rate of the surface renewal from seconds^{-1} to days^{-1}. A companion expression for the reaeration coefficient for isotropic turbulence is given in Eq. 38, which is in terms of the liquid velocity, presumably the mean velocity in the channel or stream with relation to the bottom. If the velocity, as shown in Eq. 37, is in feet per second, a numerical constant similar to the value of 480 of Eq. 36 must be introduced in Eq. 38 if the reaeration coefficient is to be in days^{-1}.

---

[4] Cons. Engr., Camp, Dresser & McKee, Boston, Mass.

[5] "The Nature of the Oxygen Transfer Coefficient in Aeration Systems," by W. E. Dobbins, in "Biological Treatment of Sewage and Industrial Wastes," by Joseph McCabe and W. W. Eckenfelder, Reinhold Publishing Corp., New York, N. Y., 1956, p. 141.

Eq. 36 is expressed in terms of the slope of the hydraulic-grade line, and Eq. 38 is expressed in terms of the mean velocity. Since the slope of the hydraulic-grade line may be expressed in terms of the mean velocity and a friction factor or coefficient, it is possible to state Eq. 36 and Eq. 38 in the same terms. If this is done, it will be found that the reaeration coefficients are identical with a Chezy C of 14.2. As indicated previously, if the slope of the river, S, is expressed in terms of the mean velocity, the depth, and the Chezy coefficient, C, the reaeration coefficient in Eq. 36 is

$$k_2 = 480 \frac{D_L^{\frac{1}{2}} U^{\frac{1}{2}}}{C^{\frac{1}{2}} H^{\frac{3}{2}}} \dots\dots\dots\dots\dots\dots\dots (43)$$

The reaeration coefficient in Eq. 38 is in seconds^{-1} if the velocity is in feet per second. Hence, if Eqs. 38 and 36 are to have the same dimensions of days^{-1}, the velocity in Eq. 38 must be expressed in feet per day, or 86,400 times the value of the velocity in feet per second. The resulting equation is as follows:

$$k_2 = 127 \frac{D_L^{\frac{1}{2}} U^{\frac{1}{2}}}{H^{\frac{3}{2}}} \dots\dots\dots\dots\dots\dots\dots (44)$$

The factor $480/\sqrt{C}$ in Eq. 43 is equal to the factor 127 in Eq. 44. When the Chezy C has a value of 14.2, the reaeration coefficient determined by Eq. 43 has the same value as that given by Eq. 44.

Under the heading, "Verification of the Theory," a description was given of the measurements that were made of the reaeration coefficient of the water surface of a cylindrical vessel, which was approximately 5½ in. in diameter and which contained 6¼ in. of water. The vessel also contained a lattice work, which oscillated vertically in simple harmonic motion at speeds varied from 45 rpm to 120 rpm. The purpose of the tests was to confirm Eq. 27, which indicates that the reaeration coefficient varies as the square root of the rate of the surface renewal. Because the turbulent-flow theory states that the rate of the surface renewal should be proportional to the speed of oscillation, the reaeration coefficient should be in proportion to the square root of this speed. The tests verified this relationship, as indicated in Fig. 1.

Two sets of experiments were conducted, one with deaerated water and the other with a sodium sulfite solution. The results, as shown in Table 2, indicate that at the lesser speeds the reaeration coefficient for the sulfite solution was 22 times as great as that for the deaerated water, and at the greater speeds the ratio was decreased to a value of 4.8. This is a significant difference, which the authors have explained by the statement that "although the sulfite method has been used frequently in the past as a means of measuring oxygen transfer, it has been criticized recently by some investigators."

The sodium sulfite method is based on the assumption that the rate of change from sulfite to sulfate is a direct measure of the rate of reaeration. This is undoubtedly true, but the mechanism of reaeration with the sulfite solution is totally different from that which it is desired to measure. The liquid-film coefficient, K_L (that is, the rate at which oxygen passes through the liquid film to the main body of the liquid), should be measured in the absence of appreciable quantities of sulfite or other reducing agents within the liquid film itself. It is reasonable to assume that when a sulfite solution is used the concentration of sulfite within the liquid film must be about the same as that in the main body of the liquid. Hence, oxygen from the air will react immediately with the sulfite as the oxygen passes through the interface. More sulfite will diffuse through the liquid film to the interface to make up this deficit, and the product of the reaction—sulfate—will diffuse through the liquid film into the main body of the liquid. The rate of reaeration measured with the sulfite solution, therefore, depends not on the rate of diffusion of dissolved oxygen through the liquid film but on the rate of diffusion of sulfites outward and sulfates inward through the liquid film. With a concentration of approximately 8,000 ppm of sulfite in the main body of the liquid, as compared with from 8 ppm to 14 ppm for dissolved oxygen at saturation, it is obvious that high concentration gradients may develop for the diffusion of sulfate inward to the main body of the liquid.

If the sulfite method has been used frequently in the past as a means of measuring oxygen transfer as stated by the authors, it is curious that the great discrepancy between the rates obtained when using sodium sulfite and the rate obtained when using deaerated water has not been discovered previously. The sulfite method has been used widely for the comparison of efficiencies of different aerating devices. This is probably a valid use, but the method should not be used to determine the liquid-film coefficient, K_L, for gas transfer.

One of the prime functions of an activated-sludge aeration tank is to transfer sufficient oxygen from the air or from oxygen gas to the solution to support the biochemical activity in the tank. Similarly, the main purpose of a carbonation tank is to transfer gaseous carbon dioxide to solution in the liquid. The same fundamental law (Eq. 26) applies to both mechanisms.

Eq. 26 may be applied in the analysis of aeration in an activated-sludge aeration tank by a revolving brush at the surface of the mixed liquor. The brush must revolve at a rate to produce a velocity gradient at the surface of ample magnitude to result in the desired value of K_L. At normal temperatures, D is approximately equal to 0.09 sq cm per hr. If the velocity gradient that is produced is 10,000 ft per hr per ft or 167 ft per sec per ft, then K_L will have a value of 30 from Eq. 26. Such a velocity gradient may be produced by the brush at a peripheral speed of approximately 30 ft per sec if the velocity in the liquid from 2 in. to 3 in. away from the brush is not more than from 2 ft per sec to 3 ft per sec.

For the case of rising gas bubbles, the rate of renewal of the liquid film, r, is proportional to the rising velocity of the bubbles with respect to the surrounding liquid and is inversely proportional to the size of the bubbles as follows:

$$r = p\frac{\nu}{\lambda} \qquad (45)$$

in which ν is the bubble velocity, λ is the size, and p is a proportionality coefficient. In the viscous-flow region of bubble size and velocity, the maximum velocity gradient at the horizontal axis of the bubble is between $2\,\nu/\lambda$, as indicated by Hadamard-Rybczynski's law, and $3\,\nu/\lambda$, as indicated by Stokes' law. Therefore, it may be stated that the value of p in Eq. 45 must be considerably less than 2 because it represents an average velocity gradient around the liquid film.

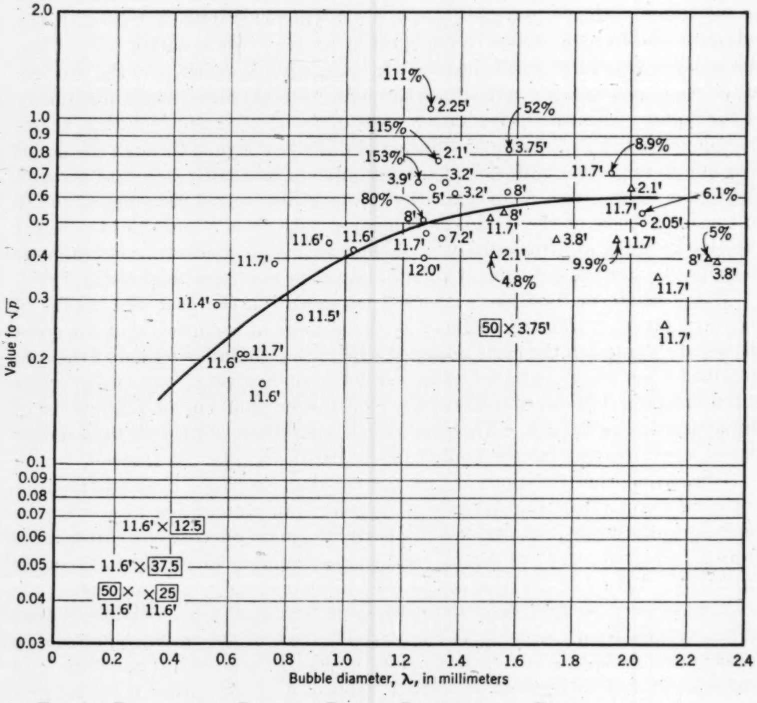


Fig. 3.—Relationship Between Bubble Diameter and Proportionality Coefficient

The experimental data of Arthur T. Ippen,[6,7] M. ASCE, have been analyzed by the writer to determine the actual values of p for rising bubbles as a function of the size of the bubbles. The results of this analysis are shown in Fig. 3.

If the value of r from Eq. 45 is inserted in Eq. 26, the following equation results:

$$K_L = \sqrt{p}\,\sqrt{D_L\frac{\nu}{\lambda}} \qquad (46)$$

---

[6] "The Determination of Oxygen Absorption in Aeration Processes," by A. T. Ippen, L. G. Campbell, and C. E. Carver, Jr., *Technical Report No. 7*, Hydrodynamics Lab., Massachusetts Inst. of Technology, Cambridge, May, 1952.

[7] "Oxygen Absorption and Turbulence Characteristics in Bubble Aeration," by A. T. Ippen and C. E. Carver, Jr., *Technical Report No. 14*, Hydrodynamics Lab., Massachusetts Inst. of Technology, Cambridge, April, 1955.

The experiments [6,7] included depths of aeration diffusers ranging from 2.1 ft to approximately 12 ft. The numerals at each experimental point plotted in Fig. 3 represent the depth of the tank in feet. Also shown in Fig. 3 by x's are experimental points in which the water contained a synthetic detergent. The concentration of the detergent in ppm is shown inside the block at each experimental point. It was found by Ippen and his coworkers that the value of the liquid-film coefficient was greater for shallow tanks than for deep tanks. When the data are plotted in the terms of $\sqrt{p}$, as shown in Fig. 3, the evidence does not conclusively show that tank depth has an effect. However, the presence of detergents has a marked effect in reducing the value of $\sqrt{p}$ and, hence, the value of the liquid-film coefficient, K_L. Fig. 3 also indicates the effect of the bubble size on the liquid-film coefficient. For example, the value of K_L for bubbles whose diameters were 0.6 mm is only approximately 40% of the value of K_L for 2-mm bubbles. This does not mean that the larger bubbles are more efficient. The smaller bubbles are more efficient despite the lower value of the liquid-film coefficient because they stay in the tank longer and present more interfacial area for gas transfer.

The results of experiments[6,7] with pure-oxygen bubbles are shown in Fig. 3 by triangles. The liquid-film coefficient, K_L, for pure oxygen seems to be somewhat less than the value for air, although the results are not conclusive. The experiments of Ippen and his colleagues were conducted in a lucite tube, which had an inside diameter of 5.5 in. From some preliminary experiments with still water in the column, it was found that the rate of oxygen absorption from the atmosphere through the water surface was very small. Therefore, the experimenters concluded that they could safely neglect this means of absorption of oxygen in reporting the results. In order to investigate this conclusion for turbulent water, the writer has shown the ratio of the water-surface area to the bubble-interfacial area for several of the experimental tests in Fig. 3. For the deep columns and the large gas flows, the ratio of the water-surface area to the bubble-interfacial area is from 5% to 10%, whereas for small gas-flow rates and shallow columns the ratio is as great as 153%. In Fig. 3 the largest values of $\sqrt{p}$ are for the largest ratios of water-surface area. However, it is not clear whether there was an appreciable rate of absorption of oxygen at the water surface, or whether the variation in the plotted points was due primarily to the experimental error.

It has been claimed that most of the absorption of dissolved oxygen in diffused-air aeration takes place from the atmosphere at the water surface and that the size of the air bubbles is not important. In order to investigate this claim, the writer has estimated the water-surface area per unit of volume of aeration tank for the case of diffused-air aeration at 1 cu ft per gal of sewage with a 6-hr aeration period. The aeration tank has been assumed to be 15 ft deep and 23 ft wide. The value of A/V in Eq. 6 for the water surface in this example is 0.0022 sq cm per cu cm. If the air bubbles are assumed to be 1.8 mm in diameter and to remain in the tank for 10 sec, the value of A/V for the air bubbles is 0.092 sq cm per cu cm. Therefore, the surface area per unit of volume for the air bubbles is about 42 times as great as for the water surface. If the rate of transfer from the water surface is to be of the same order of magnitude as the rate of transfer from the rising bubbles, the rate of replacement of the liquid film, r, for the water surface must be approximately 1,800 times that for the rising bubbles.

Actually, the rate of replacement of the liquid film for the water surface is probably much less than the rate for the rising air bubbles. The liquid-film replacement rate for the bubbles in the experiments cited previously has an average value of about 50 per sec. In the authors' studies of reaeration in natural streams, the highest rate of renewal in the most rapidly flowing stream was approximately 1 per sec. It is reasonable to suppose that the rate of renewal of the liquid film at the water surface of an aeration tank is much greater than 1 per sec because of the turbulence produced by the air bubbles breaking the surface. However, it is quite improbable that the rate of renewal of the liquid film at the water surface can be as great as that on the rising air bubbles. If the former is assumed, the conclusion is that the rate of oxygen transfer at the water surface is not more than about 1/42, or 2.4% of the rate of transfer from the air bubbles in an average diffused-air-aeration tank. Therefore, the bubbles are the principal means by which the gas transfer is accomplished, and the smaller the bubbles, the more effective the transfer.

ALEXANDER N. DIACHISHIN,[8] A.M. ASCE.—The authors' analysis of reaeration and their attempt to relate reaeration coefficients with the hydraulic characteristics of a stream are of considerable value. The following comments are of an elementary nature concerning their formulations, and present a detailed examination of some of the implications that are inherent in these formulations.

Theory.—Eq. 36, defining one of the two classes of reaeration turbulence, can be amplified by the use of Chezy's formula. Solving this formula for the fourth root of S yields

$$S^{\frac{1}{4}} = \frac{U^{\frac{1}{2}}}{C^{\frac{1}{2}} R^{\frac{1}{4}}} \qquad (47)$$

in which R is the hydraulic radius; the other notations remain the same as originally presented. For broad, shallow streams, the hydraulic radius is nearly equal to the depth of flow. Therefore, substituting the indicated relationship, as well as the relationship shown in Eq. 47, in Eq. 36 yields an alternate expression for nonisotropic turbulence,

$$k_2 = \frac{g^{\frac{1}{4}}}{2.31\,\kappa^{\frac{1}{2}}\,C^{\frac{1}{2}}} \, \frac{D_L{}^{\frac{1}{2}}\,U^{\frac{1}{2}}}{H^{\frac{3}{2}}} \qquad (48)$$

It is evident that this expression bears a striking resemblance to the authors' formulation (Eq. 38) for the case of isotropic turbulence. A comparison of Eqs. 38 and 48 indicates that the isotropic and nonisotropic expressions are equivalent when

$$g^{\frac{1}{4}} = \kappa^{\frac{1}{2}}\,C^{\frac{1}{2}} \qquad (49)$$

---

[8] Cons. Engr., Bergenfield, N. J.

Eq. 49 yields the same value for the critical Chezy coefficient as does the authors' method. However, it is interesting and important to note that whereas the "constant" for the nonisotropic case, $\frac{g^{1/4}}{2.31\,\kappa^{1/2}\,C^{1/2}}$, decreases with an increasing hydraulic radius, the isotropic constant, 1/2.31, remains constant regardless of the value of the hydraulic radius.

The relationship of k_2 and H with constant velocity and constant temperature is shown in Fig. 4(a). It is evident from this figure that k_2 decreases as the depth is increased for both the isotropic and the nonisotropic zones, the only difference being in the constants. Although Chezy's formula is not particularly applicable, because the depth of flow in the formula cannot change without a corresponding change in the velocity (slope being held constant), it can be surmised that because the Chezy coefficient is proportional to the depth, the constant for the nonisotropic case tends to decrease as the depth is increased. The decrease in the nonisotropic constant makes the k_2-values of the nonisotropic curve less than the corresponding isotropic values in the isotropic zone. However, regardless of the difference in the isotropic and nonisotropic curves, it is evident that with a constant velocity, k_2 will reach a maximum at small depths and will decrease continually with increasing depth.

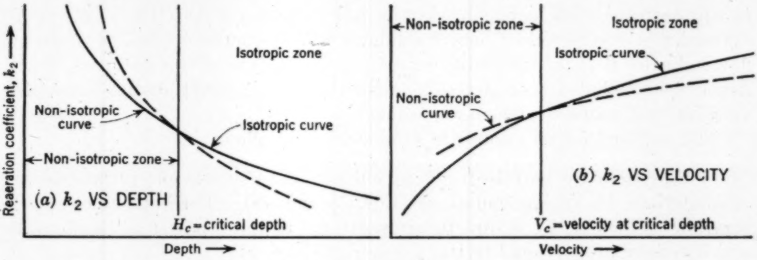


FIG. 4.—THE THEORETICAL EFFECT OF DEPTH AND VELOCITY ON REAERATION COEFFICIENT

The expected variation of k_2 with the velocity (depth and temperature being held constant) is shown in Fig. 4(b). From this figure it is evident that both the isotropic and the nonisotropic curves increase as the velocity is increased, and that the latter curve is above the former in the nonisotropic zone and below it in the isotropic zone. If only those sections of the curves that lie in the zones in which they are applicable are considered, an inflection point may occur at the velocity corresponding to the critical depth separating the two zones. Furthermore, the exponent of the velocity for the isotropic curve is $\frac{1}{2}$. Although this exponent is also $\frac{1}{2}$ for the nonisotropic curve, as shown by Eq. 48, it is possible that the value of the exponent would be hidden due to the changing value of the constant in the nonistropic case if it were necessary to determine the exponential value from experimental data. However, it is clear that this value for the velocity is equal to $\frac{1}{2}$ for velocities that are greater than the critical velocity—that is, for those velocities in the isotropic zone for the isotropic curve.

The allocation of isotropic zones and nonisotropic zones for both examples shown in Figs. 4(*a*) and 4(*b*) is somewhat arbitrary. Although the allocation of zones is easily understood for the constant-velocity case, as shown in Fig. 4(*a*), it is more complicated for the constant-depth case shown in Fig. 4(*b*), and there is some justification for reversing the allocation. If the two zones were transposed, and only those sections of the curves that lie in the zones in which they are applicable were used, it is apparent that the same inflection point would be maintained. However, instead of the curve taking an upward swing, it would tend to flatten at the inflection point.

The conditions of either constant depth and varying velocity, or constant velocity and varying depth, do not occur except in laboratory studies. For stream flow the velocity is directly proportional to some fraction of the depth. However, from an examination of Eqs. 36 and 38, some useful knowledge may be derived concerning the variation of k_2 in cases for which both the depth and the velocity change. Because a velocity term does not appear in Eq. 37 it is evident that k_2 decreases as the depth is increased and that a maximum k_2-value is expected at a minimum depth. In the isotropic zone in which Eq. 39 applies, k_2 will decrease with increasing depths and velocities if $\Delta(U^{\frac{1}{2}}) < \Delta(H^{\frac{3}{2}})$. An examination of either the Chezy formula or other formulas applicable to stream flow indicates that the inequality stated previously is usually valid, and, hence, it may be concluded that increasing river stages and corresponding increasing velocities will cause decreasing k_2-values.

The authors' formulations are summarized as follows:

1. Equations for both isotropic turbulence and nonisotropic turbulence are characterized by similar equations (Eqs. 38 and 48). For both cases the reaeration coefficient, k_2, is directly proportional to the $\frac{1}{2}$ power of the velocity and inversely proportional to the $\frac{3}{2}$ power of the depth.
2. For constant velocity and constant temperature, k_2 decreases as the depth is increased.
3. For constant depth and constant temperature, k_2 increases as the velocity is increased. The reaeration coefficient increases at an apparent rate that is no greater than the $\frac{1}{2}$ power of the velocity.
4. In streams maximum k_2-values will occur at minimum depths and correspondingly low velocities. Minimum k_2-values will occur at maximum depths and correspondingly high velocities.

Verifications of Theory by Field Data.—The use of Eq. 2 to determine the reaeration coefficients is common, as was noted by the authors. All other terms being known, except for k_2, the solution for k_2 can be accomplished by trial and error by substituting in Eq. 2 and finally obtaining a value satisfying the equation, or by any of a variety of approximation methods. However, although the solution for k_2 is mathematically straightforward, the dependence of this value on the other factors in Eq. 2 should not be minimized. These factors are approximated and, therefore, result in approximate k_2-values. Perhaps the greatest sources of inaccuracy for reported k_2-values are the dissolved oxygen deficits. The change in the dissolved oxygen deficit from one site on a stream to a downstream site reflects not only the depletions due to the oxygen requirements of pollutional materials and the additions due to reaeration, but also the oxygen requirements of life forms not involved in the pollutional-purification process. The writer has had occasion to compute reaeration coefficients utilizing the oxygen deficits for both an 8-hr period and a 24-hr period on successive days. There was a four-fold difference in the computed coefficients. In view of these considerations, it is surprising to note the almost exact agreement between the coefficients computed by the authors and the observed reaeration coefficients.

In order to test the authors' method further, data for a section of the Ohio River between station 598 and station 611 were analyzed (1). The data were from the same Ohio River survey used by the authors. The authors of the survey report classified the section between station 598 and station 611 as "relatively smooth," with an average low-water slope of 0.316 ft per 1,000 ft. Chezy coefficients were computed and the type of turbulence was assigned to each case in accordance with the procedure recommended by the authors, even to the extent of applying the recommended empirical depth correction to be added to the river stage, which is equal to one-half of the mean depth at extreme low water. Data for temperatures below 10° C were omitted in this study as they were in the paper. The computed results are shown in Table 5, and it is evident that there is a considerable discrepancy between the observed and the computed results.

TABLE 5.—REAERATION COEFFICIENTS—OHIO RIVER STATIONS 598–611

Mean depth, in feet	Reaeration coefficient, computed per day	Reaeration coefficient, observed per day	Remarks
27.2	0.067	0.170	isotropic
12.7	0.127	0.613	nonisotropic
11.8	0.144	0.925	nonisotropic
10.9	0.156	0.792	nonisotropic
11.3	0.143	1.140	nonisotropic
11.3	0.135	0.446	nonisotropic
Average	0.129	0.514	

Another example, chosen at random, is for the lower Little Tennessee River (Tennessee); the base data is given by Milo A. Churchill,[9] A.M. ASCE. The computed value indicates a k_2-value of 1.28 per day, whereas the observed value was approximately 10.0 per day. Another example in the writer's recent experience is for the Connecticut River (Connecticut), which evidenced a computed value of 0.28 per day and an observed value of 0.56 per day.

It is not clear from these examples whether the authors' method gives consistently low values, or whether the observed deviations of the computed values from the field-study values reflect variations from a mean value.

More important than the agreement or lack of agreement of the observed values with the computed values is the variation of the k_2-values at a given location with a change in depth and velocity. Sanitary engineers are often concerned with critical values of flow and reaeration rather than the average values, and the depth or flow at which minimum values of the reaeration coefficient occur may well be of critical importance.

---

[9] "Effects of Storage Impoundments on Water Quality," by Milo A. Churchill, *Transactions*, ASCE, Vol. 123, 1958.

It is evident that, according to the theory presented by Messrs. O'Connor and Dobbins, a minimum will occur at high river stages, which can be verified by an examination of the data in Table 4. Four out of five of the Ohio River sections that were analyzed have minimum computed k_2-values occurring at maximum river heights, whereas at the other station the computed minimum k_2-value occurs at an intermediate height. For one section the actual minimum k_2-value occurs at a maximum height, at two sections the minimum value occurs at intermediate heights, and at the remaining two sections it occurs at minimum heights. These results do not verify the theory.

Verifications of Theory by Experimental Data.—The authors state that

> "the critical point to be verified in the theoretical development is the relationship between the reaeration coefficient and the rate of surface renewal, as indicated by Eq. 27."

Eq. 27 can be written in the following equivalent form when the depth is kept constant, as was the case in the experimental apparatus of the authors:

$$k_2 = G\, r^{\frac{1}{2}} \qquad (50)$$

in which G is a constant. The test data presented presumably verify the theory because the speed of rotation of the experimental apparatus is directly proportional to the rate of the surface renewal. The theory, according to Eq. 50, predicts that the reaeration coefficient varies as the square root of the rate of the surface renewal, and their proportionality is seemingly confirmed by the linear variation of the experimental data for k_2 with the square root of the speed.

Utilizing the authors' criterion, their experimental data was fitted by the method of least squares for both a linear function and a square-root function of the speed. From the results of the statistical analyses shown in Table 6, no significant change in the correlation coefficient occurs when a square-root function is used instead of a linear function. It is unfortunate that the range of speeds used in the experimental tests was so limited. With a larger range extending several magnitudes, it would be possible to determine whether the effect of the speed is more correctly described by a linear or a square-root function. For the experimental range selected, if there is a square-root effect its influence in the range of speeds selected is minor.

TABLE 6.—CORRELATION COEFFICIENTS FOR EXPERIMENTAL DATA

	Deaerated water	Sulfite solution
Linear	0.994	0.993
Square root	0.995	0.991

Data from another experimental study (29) are also available to check the formulations. For a constant depth the results are shown in Table 7 for velocities that range from 1.6 ft per min to 4.6 ft per min. With regard to velocities greater than those for the range of from 1.6 ft per min to 4.6 ft per min, the report (29), presenting the results of the flume studies, states:

> "* * * the plotted points are shown to follow two distinct straight line trends, the slope breaking sharply upward at a velocity of flow approximat-

> ing 10 ft per minute. If the slope of each line based on the deaerated water tests be designated by (n) in the function (V^n), where (V) is the velocity of flow, the numerical value of (n) for the steeper line approximates 1.75 and the flatter line 0.50."

Thus, it appears that, at best, the theory is applicable to small velocities of less than 10 ft per min and certainly not for the entire range of velocities.

With reference to the effect of depth, the report states:

> "Little if any indication was evident from the foregoing data as to any consistent influence exerted by variations in the stream depth on the rate of surface reaeration, though plots of the data * * * have shown a tendency towards slightly increased surface rates of reaeration with increased depths. It is possible in the case at hand the differences in flow depth were too small to bring out clearly any well-marked effect of this variable, except to suggest that where the depth was increased, with a given velocity of flow, some hydraulic condition may have caused a slightly greater degree of turbulence than with more shallow depths."

TABLE 7.—EXPERIMENTAL REAERATION COEFFICIENTS[a]

Temperature, in degrees Centigrade	Reaeration coefficient per day
0 to 10	$0.32\ v^{0.48}$
10 to 20	$0.50\ v^{0.50}$
20 to 30	$0.75\ v^{0.51}$

[a] Bibliography reference (29).

Although the preceding statement referred primarily to shallow depths, the conclusion of the studies considering all depths was essentially the same—the variation of the depth of flow appeared to exert a slight influence. The foregoing does not support the theory, which predicts a considerable effect of height on the reaeration coefficient.

Conclusions.—Some of the results obtained by the authors' theory disagree substantially with those observed in other laboratory and field studies. Inasmuch as it is difficult to have prior knowledge of those instances in which the theory will apply, the actual measurements of reaeration coefficients still constitute good practice.

AALE PASVEER[10].—The work concerning the mechanism of reaeration in natural streams by Messrs. O'Connor and Dobbins proves that it is important to know that the turbulent flow of a river follows certain laws, resulting in a rate of surface renewal. This rate can be computed on a theoretical basis and is one of the factors that determine the rate of reoxygenation of the river water.

The study is not only of great value for an understanding of the reaeration process but also for its use in practice—that is, for the evaluation of the natural purification capacity of a stream.

The authors have succeeded in treating the mathematics of the reaeration process in a clear fashion, handling the problem with the aid of different mathematical models—that is, with and without a laminar film at the surface.

---

[10] Chemist-Bacteriologist, Research Inst. for Public Health Eng., National Health Research Council. The Hague, Netherlands.

They have shown that both methods lead to mathematically satisfying results. What is important is that they show that the eddy diffusivity in practical cases is much greater than the liquid-film diffusivity.

Because there is no necessity for assuming a stagnant liquid film, preference might be given to the model in which no stagnant laminar film at the surface is assumed but in which there is a rate of surface renewal.

In the case for the aeration with air bubbles, it can be shown that there is probably no "stagnant" liquid film at all.[11]

In the process of dissolving a gas into a liquid, the "resistance" in the boundary layer is due to the fact that the rate of a gas dissolving in a quiescent liquid is inversely proportional to the square root of the time. The rate of dissolving is at a maximum at the moment of creation of the surface and then rapidly decreases with time.

The writer's principal objection to the two-film or liquid-film concept is that the theory with its formula, as it is used in the aeration of water,

$$Q_d = A\, K_L\, (C_s - c_L)\, t \qquad (51)$$

is not always restricted to practical purposes in nonquiescent conditions. In Eq. 51, Q_d represents the quantity of dissolved oxygen. In addition, the theory obscures the fact that according to the theory of diffusion the quantity that is diffused is not proportional to the time but to the square root of the time. The writer therefore prefers, as a base, the use of the formula deduced by J. Stefan[12,13,14] from Fick's law of diffusion in the case of a poor soluble gas into quiescent water,

$$Q_d = 2\, A\, (C_s - c_L) \sqrt{\frac{D_L\, t}{\pi}} \qquad (52)$$

in which D_L is the coefficient of diffusion of oxygen in water. Eq. 52 also follows from Eq. 11. The authors have expressed some doubt as to the application of this formula to a liquid film of small thickness. Stefan made a detailed investigation of the conditions under which his formula is applicable. His conclusion was that the equation shows no greater divergence than 1% as long as $\pi^2 D_L\, t/h^2 < 3$, and no greater divergence than 2% as long as the value of this expression is less than 4.

From this it may be computed that for diffusion times of 1 sec and 100 sec the minimum column lengths, h, at which the divergence is less than 2% are 0.006 cm and 0.06 cm, respectively.[15]

The work of the authors shows that for those cases for which the surface renewal can be computed from the laws of turbulence it may be possible to determine the rate of reaeration.

---

[11] "Research on Activated Sludge. VI. Oxygenation of Water with Air Bubbles," by A. Pasveer, *Sewage and Industrial Wastes*, Vol. 27, October, 1955, p. 1130.

[12] "Ueber die Diffusion der Kohlensäure durch Wasser und Alkohol," by J. Stefan, *Wiener Sitz.*, Vol. 77, No. II, 1878, p. 371.

[13] "Ueber die Diffusion der Flüssigkeiten," by J. Stefan, *ibid.*, Vol. 79, No. II, 1879, p. 161.

[14] "Diffusion dans les liquides," by J. Duclaux, Hermann et Cie, Paris, 1936.

[15] "Research on Activated Sludge. 1. A Study of the Aeration of Water," by A. Pasveer, *Sewage and Industrial Wastes*, Vol. 25, November, 1953, p. 1253.

MYRTON C. RAND[16]. — The paper represents an important advance, both in a theoretical understanding of the mechanism of the reaeration process and in the field application of that understanding. Several points stand out as being of particular interest:

1. The careful and nearly rigorous derivation of the reaeration constant (k_2) as a function of the rate of surface renewal; although expressions of the same, or nearly the same, form have been presented previously, the present derivation sets forth the range of applicability of the final equation more clearly than most previous reports.
2. The expression of the rate of surface renewal, and, hence, of k_2, in terms of parameters susceptible to independent measurement; thus, for the first time, it is possible to estimate reaeration in a stream by methods that do not require prior observation of the reaeration rate.
3. The application of the modern theory of turbulence to reaeration in cases involving a flow of finite velocity and definite direction suggests a possible method of attack for the problem of reaeration in other bodies of water, such as lakes and reservoirs, in which turbulence is due primarily to wind action, thermal convection, and other characteristics.

In deriving the general equation for reaeration as a function of surface renewal, the authors use the assumption that the surface layer possessing infinitesimal thickness is saturated with dissolved oxygen instantaneously upon the exposure of a new element of surface. This assumption imposes a slight limitation on the generality of the resulting equation because the saturation of the surface layer must require a period of time that is finite, however small. The limitation does not become important as long as the average time of exposure of an element of surface is long compared with the time required for saturation of the surface layer. In other words, the assumption excludes only the state of extreme turbulence, which has been termed a "heaving surface."

The equations apply, and are intended to apply, only to bodies of water in which there exists a flow of finite velocity and measurable direction. The problem of reaeration in relatively quiet water exhibiting no net flow remains unsolved. However, this work may serve to extend the same reasoning to cases of the latter type.

DONALD J. O'CONNOR,[17] J.M. ASCE, AND **WILLIAM E. DOBBINS**,[18] M. ASCE. —The discussions have contributed considerably to the understanding of the problem of reaeration and its application.

Mr. Camp commented on the great difference between the sulfite tests and the aerated-water tests. He indicated further that the sulfite test should not be used to determine the liquid-film coefficient for gas transfer, although it has been used widely to compare the efficiencies of different aeration devices. The writers agree with this theory and appreciate Mr. Camp's clarifying study of the process in a sulfite solution. In stating that the method has been criticized recently, the writers had reference to a recent work in the fermentation field.[19] The preceding situation also has been treated mathematically by Danckwerts (2).

---

[16] Asst. Prof. of Sanitation, Rutgers Univ., New Brunswick, N. J.

[17] Asst. Prof., Dept. of Civ. Eng., Manhattan College, New York, N. Y.

[18] Associate Prof. of San. Eng., College of Eng., New York Univ., New York, N. Y.

Mr. Camp has contributed to the knowledge of reaeration by discussing bubble aeration and the application of the theory of this situation. His expression for the renewal rate for bubbles incorporates the velocity and diameter of the gas bubble. However, some measure of the scale of the fluid turbulence should be included.

The writers agree with Mr. Camp with respect to the relative effectiveness of bubble aeration and surface aeration in present practice.

Both Messrs. Camp and Diachishin have stated that Eq. 36 for the nonisotropic case may be expressed in a form in which the exponent of H is equal to $\frac{3}{2}$ as in the isotropic case. The writers have been aware of this fact, but they prefer the form of Eq. 36 because it eliminates the factor C which is difficult to estimate for any practical case. Furthermore, Eq. 36 expresses k_2 in terms that are more available as measurable parameters. The velocity term, which appears in Eq. 43 and Eq. 48, which are equivalent, is a function of the slope, depth, and some parameter characterizing the roughness of the channel. Hence, Eqs. 43 and 48 are not in their most fundamental form. For example, if Manning's expression for C is introduced into Eq. 43, the resulting expression for k_2 will be

$$k_2 = \frac{480\, n^{\frac{1}{2}}\, D_L{}^{\frac{1}{2}}\, U^{\frac{1}{2}}}{(1.486)^{\frac{1}{2}}\, H^{19/12}} \qquad (53)$$

Because the roughness coefficient, n, varies with H, the actual functional relationship between k_2 and H for a given river is not defined precisely by any of the equations presented thus far in the paper. Except for convenience or for comparison, there appears to be no justification in using the exponent of H as $\frac{3}{2}$ for the nonisotropic case simply because it agrees with the exponent for the isotropic case. The equation for the latter case was based on some simple assumptions that were derived from empirical measurements, such as the mixing length and the vertical velocity fluctuation being 10% of the average depth and flow velocity, respectively. The writers indicated that any theory that considers the roughness elements is the most appropriate one for natural rivers. However, due to the inadequacies of the existing theories for river flow, the expression for k_2 was stated in the form that eliminated the necessity for estimating the values of the bottom roughness parameters. Therefore, Mr. Diachishin is in error when he states that for both cases the reaeration coefficient is inversely proportional to the $\frac{3}{2}$ power of the depth. In addition, maximum k_2-values will not necessarily occur at minimum depths, as indicated by the computed data and observed data in Table 5.

The writers agree with Mr. Diachishin's comments concerning the inaccuracy in the reported values of the reaeration coefficient, particularly with regard to the oxygen factor of life forms not necessarily involved in the purification process. It is strongly emphasized that the theoretical development makes no attempt to incorporate this factor. Therefore, a comparison of the theoretical coefficient with the observed coefficient in which this factor is present is not valid, and such data neither prove nor disprove the theory. Mr. Diachishin, after appreciating this factor, ignored it in comparing values on the lower Little Tennessee River. He compared an observed value of 10 per day with a computed value of 1.28 per day. The former value was based on data collected during daylight hours. In this regard, Churchill states: "The dissolved oxygen varied between 5.85 ppm and 6.90 ppm, apparently due to oxygen added by algae during the daylight hours." In addition, he states "* * * (it brings out) the increase in (reaeration) rate during daylight hours due to the addition of oxygen by algae * * *." In using a value of 10.0, Mr. Diachishin measures the combined effects of reaeration and algae. In order to test the theoretical equations, the darkness values would be more representative. The data for the night condition were given, from which an observed reaeration coefficient was computed as 5.3 per day. In computing the theoretical reaeration coefficient for the Little Tennessee River, Churchill states:

---

[19] "Sulfite Oxidation as a Measure of Aeration Effectiveness," by J. S. Schultz and E. L. Gaden, *Industrial and Engineering Chemistry*, Vol. 48, December, 1956.

> "The width varies between 600 ft and 1,200 ft, and the mean depth at 7,000 cu ft per sec is on the order of 3.5 ft in the narrow sections and 1.5 ft in the wider reaches. Water velocities vary from 3.2 ft per sec in the deeper reaches to 4.2 ft per sec on the broad, shallow shoals."

Mr. Diachishin, in reporting a computed value of 1.28 per day, apparently ignored the shallow sections and based the computation on the deep section only. Such a computed value is not representative of the river stretch. When the reaeration coefficient is computed for each section and weighted according to the length of each section, the average value is 4.8 per day, as compared with an observed value of 5.3 per day.

Mr. Diachishin discussed the Ohio River survey data (1) and presented one example of the lack of agreement between the observed values and the computed values. The mean depth was determined by adding a depth correction to the stage height. This height referred to depth above low water, and, because the average depth from low water to channel bottom was significant, a depth correction of one-half this value was added to the stage height. This procedure was followed in order to obtain a value of depth that represented best the ratio of the volume to the surface area. The preceding procedure is justified if the depth correction is small. However, as this value increases, the possible error also increases. In the case examined by Mr. Diachishin, the depth from low water to the channel bottom was 11.7 ft. It is doubtful that the representative depth can be determined for stretches for which the correction was based on a value as significant as 11.7 ft. Consequently, the writers have not included stretches for cases in which the correction was of such magnitude, in spite of the fact that there was reasonable agreement between the computed and observed values in some cases. In addition, the slope of the sketch varied considerably, as was indicated in the paper, which indicates another possible source of inconsistency. There were other inaccuracies in these data, as indicated in Tables 4 and 5. Such data cannot be used either to confirm or to refute the theoretical development. These factors were presented in the paper, but, in view of Mr. Diachishin's comments, it was felt that they were worth repeating.

Mr. Diachishin states that the Ohio River data results "* * * do not verify the theory" because observed minimum reaeration coefficients occur at other than maximum depths and the theory indicates otherwise. The theory makes no such stipulation in this regard.

The writers appreciate Mr. Diachishin's analysis of the experimental data and agree with his conclusion that the range of speeds was too limited to indicate the relationship between the speed and the reareation coefficient.

With reference to another experimental study (29), it was believed that the data were not representative of the conditions that were used as a basis for the theoretical development. Flow was induced by a longitudinal propeller which was in one of the channels. Surface renewal was due not only to the turbulence of flow but also to rotary motion induced by the propeller. Furthermore, many tests were conducted under laminar flow conditions. Experiments were also conducted on depths of flow of 9 in., 12 in., and 15 in. Mr. Diachishin indicated that the conclusion showed no influence of depth on the reaeration coefficient. In the case of the 9-in. flow depth, the propeller broke the surface, and the authors themselves (29) are somewhat doubtful about the data. For the same velocity a greater speed of the propeller would be required for a greater depth, thus masking out the influence of depth in that experiment.

Mr. Pasveer objects to the two-film theory because "* * * it obscures the fact that according to the theory of diffusion the quantity that is diffused is not proportional to the time but to the square root of the time." The objection does not seem valid because the over-all transfer coefficient, K_L, as applied to a turbulent liquid, is actually a constant in the statistical sense, and the total quantity of gas absorbed increases directly with time as long as c_0 remains constant. For clarification it is necessary to distinguish between time in its general sense and the particular interval that is the average time of renewal of the surface. The latter is equal to $1/r$ and is a constant for any particular intensity of turbulence at the interface.

The equations presented by Mr. Pasveer result from the solution of the general differential equation for absorption of gas into a quiescent liquid (Eq. 9) under the boundary conditions:

$$c = c_0 \text{ when } y > 0 \text{ and } t = 0$$

$$c = C_s \text{ when } y = 0 \text{ and } t > 0$$

and

$$c = c_0 \text{ when } y = \infty \text{ and } t > 0$$

The concentration, c, at a distance, y, below the surface is given by

$$c - c_0 = (C_s - c_0) \operatorname{erfc}\left(\frac{y}{2\sqrt{D_L t}}\right) \qquad (54)$$

in which erfc is the error function, and the rate of transfer of gas across a unit area of interface is given by

$$N_m = (C_s - c_0)\sqrt{\frac{D_L}{\pi t}} \qquad (55)$$

Thus, the instantaneous rate of transfer decreases with time for any period during which the liquid remains quiescent.

The total gas carried across the interface during an interval of time, t', is given by $\int_0^{t'} N\,dt$, which is equal to $2\,(C_s - c_0)\sqrt{\frac{D_L\,t'}{\pi}}$, which is Mr. Pasveer's equation for Q_d. The average rate of transfer during the time, t', will be the total amount divided by t', which is given by

$$N = 2\,(C_s - c_0)\sqrt{\frac{D_L}{\pi\,t'}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (56)$$

If the assumed quiescent condition prevails for the time, t', and the whole mass of the liquid is then instantaneously mixed, this process being repeated indefinitely, the over-all average rate of transfer will be as given in Eq. 55. If the time of renewal, t', is replaced by the rate of renewal, $1 / t' = r$, the equation for the average rate of transfer is

$$N = \frac{2}{\sqrt{\pi}}\,(C_s - c_0)\sqrt{D_L\,r} \quad \ldots\ldots\ldots\ldots\ldots\ldots (57)$$

and

$$K_L = \frac{2}{\pi}\sqrt{D_L\,r} \quad \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

Eq. 58 is a constant, $2/\sqrt{\pi}$, or 1.13 times the value of K_L used by the writers in Eq. 26.

Eq. 26 was derived by neglecting minor factors in either Eq. 24 or Eq. 25. It is significant that the former equation was derived from boundary conditions based on a liquid film, whereas Eq. 25 was derived from boundary conditions similar to those advocated by Mr. Pasveer. Thus, it makes little or no difference whether or not the liquid film is assumed to exist. The factor, $2/\sqrt{\pi}$, which is the ratio of K_L obtained by Mr. Pasveer to the value of K_L that was obtained by the writers, is due to the fact that the writers used Danckwerts' age-distribution function (11) in their development, whereas Mr. Pasveer assumed that the surface replacement occurs at the same time over the entire surface. The first assumption is a closer representation of the actual physical conditions at the surface of a stream. It is remarkable not that the two different approaches result in different values for K_L, but that the expressions for K_L differ only by a constant factor equal to 1.13.

The writers agree with Mr. Pasveer's reference to Stefan's investigation of the conditions under which his formulas are applicable.

Mr. Rand discussed some factors concerning reaeration in natural bodies of water, such as wind action and thermal convection. The writers agree that the approach used may be applicable to these cases, particularly in the case of wind action. The discussion of the so-called heaving surface by Mr. Rand is particularly interesting. This condition may be represented by oceans and flow over rapids. He has stated that the assumption of instantaneous saturation upon exposure of a new element of surface may impose a limitation on the condition of a heaving surface. In addition, air bubbles may be entrained, and the transfer may be controlled by a different mechanism.

The effect of temperature on the reaeration coefficient, with a temperature coefficient of 1.016, was referenced incorrectly in the paper. This relationship is shown[20] by Gordon M. Fair and John C. Geyer, Members, ASCE, based on H. G. Becker's work. The coefficient indicated by the original reference (29) is 1.047. There is no apparent reason for this difference; work is being conducted at the present time which should clarify the issue.

---

[20] "Water Supply and Waste Water Disposal," by G. M. Fair and J. C. Geyer, John Wiley & Sons, Inc., New York, N. Y., 1954.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2935

OPEN-CHANNEL FLOW AT SMALL REYNOLDS NUMBERS

By Lorenz G. Straub,[1] M. ASCE, Edward Silberman,[2] and
Herbert C. Nelson,[3] Associate Members, ASCE

With Discussion by Messrs. Wallace M. Lansford and James M. Robertson; Chesley J. Posey; and Lorenz G. Straub, Edward Silberman, and Herbert C. Nelson

Synopsis

A summary and correlation are presented of the results of several theses studies on open-channel flow in a range of Reynolds numbers less than approximately 4×10^4. The hydraulic diameter is used as the length parameter in the Reynolds numbers. Smooth laminar flow, smooth turbulent flow, rough laminar flow, and rough turbulent flow are considered separately, as is transition from laminar flow to turbulent flow in smooth channels.

The results indicate that at these small Reynolds numbers, smooth channel flow, both laminar and turbulent, is quantitatively similar to smooth pipe flow. Rough channel flow is probably qualitatively similar to rough pipe flow and rough plate flow, but there is no adequate method available to correlate rough flows in the small Reynolds number range. Channel shape is important in laminar flow, but its entire effect may be determined theoretically. There is only a negligibly small channel-shape effect in smooth turbulent flow and a somewhat more pronounced effect in rough turbulent flow. Transition generally occurs at slightly higher Reynolds numbers in channels than in pipes, the exact effect depending on shape.

Note.—Published, essentially as printed here, in July, 1956, in the Journal of the Engineering Mechanics Division, as *Proceedings Paper 1031*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Director, St. Anthony Falls Hydr. Lab., and Prof. and Head, Dept. of Civ. Eng., Univ. of Minnesota, Minneapolis, Minn.

[2] Prof., St. Anthony Falls Hydr. Lab., Univ. of Minnesota, Minneapolis, Minn.

[3] Hydr. Engr., Research and Development, Kimberly-Clark Corp., Neenah, Wis.

Notation.—The letter symbols used in this paper are defined where they first appear, in the text or by illustration, and are assembled for convenience of reference in Appendix II.

INTRODUCTION

General.—Since 1930 a large number of more or less related experimental studies have been made at the University of Minnesota, at Minneapolis, on the hydraulics of flow in open and closed conduits. Much of the experimental work has been directed particularly toward establishing physical relationships of flow at small Reynolds numbers. Among these studies were several graduate theses concerned with steady, uniform flow in open channels in the laminar, transitional, and lower turbulent regions. A summary and a synthesis of the results of these studies, which were conducted at the St. Anthony Falls Hydraulic Laboratory of the university,[4,5,6,7,8,9,10] are presented herein.

Only flows with the Froude number less than unity (subcritical flow) and at high Weber numbers, or where capillary effects do not have significant influence on the flow pattern, are considered. Furthermore, the treatment is confined to that part of the channel length in which the boundary layer is fully developed—that is, in which the velocity profiles over the reach of channel considered are essentially constant.

The experimental studies naturally are grouped into the two following categories: Smooth channels and rough channels. These studies are further classified into laminar flow, transition from laminar to turbulent flow, and turbulent flow. Therefore, an analytical presentation is given for each type of flow, which is followed immediately by a review of the experimental results pertaining to that flow for various channel shapes. As of 1956 the most extensive experimental studies at the laboratory were with smooth channels, and, therefore, this category is given primary consideration. However, flow in rough channels at low Reynolds numbers has also received attention in the test program, and a review of the results is included although much additional research in this area remains to be done.

A Cartesian coordinate system is used in the algebraic treatment, with the positive z-axis oriented in the direction of flow, the (yz)-plane vertical, and the origin at the free surface. The velocity at any point in the channel is w, and the mean velocity is $\bar{w}$. The slope, S, is assumed as positive when the channel inclination is downward in the direction of flow.

---

[4] "Experimental Studies of Viscous Flow in Open Channels," by Edward Silberman, thesis presented to the University of Minnesota, at Minneapolis, in 1936, in partial fulfilment of the requirements for the degree of Master of Science.

[5] "Experimental Studies of Viscous Flow in a Triangular Open Channel," by S. H. Anderson, thesis presented to the University of Minnesota, at Minneapolis, in 1939, in partial fulfilment of the requirements for the degree of Master of Science.

[6] "Experimental Studies of Velocity Distribution at a Section in a 90° Triangular Channel," by Harry Purdy, thesis presented to the University of Minnesota, at Minneapolis, in 1947, in partial fulfilment of the requirements for the degree of Master of Science.

[7] "Flow in a Triangular Channel with Varied Boundary Roughness," by J. M. Forns, thesis presented to the University of Minnesota, at Minneapolis, in 1948, in partial fulfilment of the requirements for the degree of Master of Science.

[8] "A Study of Viscous Flow in an Open Channel Having a Circular Arc Cross-Section," by D. K. Donovan, thesis presented to the University of Minnesota, at Minneapolis, in 1949, in partial fulfilment of the requirements for the degree of Master of Science.

[9] "Experimental Studies of Viscous Flow in a Triangular Open Channel with Varied Central Vertex Angle," by Peter Chi-Te Feng, thesis presented to the University of Minnesota, at Minneapolis, in 1952, in partial fulfilment of the requirements for the degree of Master of Science.

[10] "Fluid Flow in Rough Triangular Channels," by H. C. Nelson, thesis presented to the University of Minnesota, at Minneapolis, in 1953, in partial fulfilment of the requirements for the degree of Master of Science.

Channel friction is evaluated by using the Darcy-Weisbach friction factor, f, defined by

$$S = \frac{f}{4\,R}\,\frac{\bar{w}^2}{2\,g} \qquad \text{(1)}$$

in which 4 R is the hydraulic diameter (R being the hydraulic radius or channel cross-sectional area divided by its wetted perimeter) and g is the acceleration of gravity. The friction factor is related to the Chezy C and Manning n by

$$C^2 = \frac{8\,g}{f} \qquad \text{(2)}$$

and

$$n^2 = \frac{(1.486)^2}{8\,g}\,f\,R^{1/3} \qquad \text{(3)}$$

FIG. 1.—GENERAL VIEW OF EXPERIMENTAL CHANNEL

EXPERIMENTAL SETUP

Several experimental setups were used in the studies. However, they were basically the same with respect to the scale of the tests, the system of circulation of the liquid mediums, and the instrumentation.

Fig. 1 is typical of the apparatus. Channels of rectangular cross section, fixed, 90° triangular cross section, variable triangular cross section, and circular-arc cross section were used. The test channels were formed from either a single rolled structural shape, or, for the triangular channel of adjustable central angle, from two structural channels back to back with a sheet of rubber cemented across the pair of flanges at the vertex. The rectangular channel was 15 ft long, and the other channels were either 20 ft long or 22 ft long. The channels were straight (probably within 0.01 in.), untwisted, and glossy smooth. The triangular channel of fixed vertex measured 90.7°. The central angle of the channel of controllable shape could be set closely by a screw adjustment arranged at four sections in the channel. In general an exceptionally high degree of precision in shape and control was possible.

A pumped recirculating system was used and included a low-level reservoir, as well as a smaller reservoir at the head of the channel. All the channels were provided with removable bell-mouthed entrances. The discharge was measured by intercepting the flow in a weighing tank for measured intervals normally of several minutes duration. Both water and kerosene were used in the experiments, and the temperature was controlled in order to maintain a constant viscosity for each test.

Each channel was provided with three piezometer wells, spaced at 5-ft intervals along its length. In the rectangular channel the piezometer wells were at distances of 2 ft, 7 ft, and 12 ft from the end of the bell-mouthed entrance. For the other channels they were either 7 ft, 12 ft, and 17 ft, or 8 ft, 13 ft, and 18 ft from the entrance. The fluid surface elevations in the wells could be measured to 0.001 in. by micrometer-fitted point gages. These gages were also used to set the channel slope and to measure the depth of flow. In establishing flow in the channel, the tail gate was adjusted so that the depths measured at the last two piezometer taps were identical within 0.001 in. As will be noted subsequently, even this procedure was not sufficient to insure uniform flow at the highest Reynolds numbers of laminar flow because of the developing boundary layer. However, for most experimental runs uniform flow was obtained.

Velocity data were obtained at the 12-ft piezometer well with a fine pitot tube, the velocity head being measured in an adjacent well and referred to the piezometer well for the reference pressure. The tube could be traversed vertically and laterally by a micrometer screw adjustment; the position was recorded to 0.01 in. Surface velocity observations were made on floating particles of sufficient thickness to avoid inhibition by surface tension phenomena.

SMOOTH CHANNELS—LAMINAR FLOW

Theory.—The Navier-Stokes equations[11] are applicable. With the stipulations stated previously, the pressure at the free surface is zero (atmospheric), the x-components and y-components of the velocity are zero everywhere, and the z-component, w, is not a function of z. Hence, the Navier-Stokes equations reduce to

$$p + \rho\, g\, y = 0 \qquad (4)$$

and

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -\frac{g\, S}{\nu} \qquad (5)$$

---

[11] "Hydro dynamics," by H. Lamb, Dover Publications, Inc., New York, N. Y., 1945, p. 577, Eq. 4.

in which p is the pressure, ρ is the density, and ν is the kinematic viscosity of the liquid. The external force is due to gravity and has the components per unit mass of $Y \approx - g$ in the y-direction and of $Z \approx g\,S$ in the z-direction. Eq. 4 shows that the pressure distribution is hydrostatic, leaving Eq. 5 to be solved for w when the appropriate boundary conditions are given. The solution of Eq. 5 is a two-dimensional problem in the (xy)-plane.

Eq. 5 is a form of Poisson's equation and is applicable to other problems, such as the determination of the streamlines for a fluid enclosed by a rotating cylindrical boundary and the stress due to torsion of a cylindrical shaft. Analytical solutions can be obtained whenever the given cross section can be mapped conformally on the interior of a circle. Approximate solutions can be obtained by numerical and other methods for any boundary configuration.[12]

The boundary conditions on Eq. 5 are:

$$w = 0 \qquad (6)$$

on all solid boundaries and, at the free surface,

$$\frac{\partial w}{\partial y} \approx 0 \text{ at } y = 0 \qquad (7)$$

The approximation sign is used in Eq. 7 because the presence of the atmosphere at the liquid surface results in a small drag and in a nonzero value of the derivative. In solving Eq. 5, the derivative is made equal to zero. The influence of the resulting approximation may be estimated by comparison with experimental data. Several useful solutions are given in Appendix I. (These solutions are the same as for closed conduits obtained by reflecting the solid boundary in the free surface.)

After Eq. 5 has been solved, the mean velocity, $\bar{w}$, is given by

$$\bar{w} = \frac{1}{A} \int\int w\,dx\,dy \qquad (8)$$

in which A is the cross-sectional area of the channel and the integral is taken over the entire cross section. Formulas for determining the values of $\bar{w}$ for several channel shapes are also given in Appendix I.

The equation defining the friction factor may be written

$$f = \frac{K}{\mathsf{R}} \qquad (9)$$

in which R is the Reynolds number given by

$$\mathsf{R} = \frac{4\,R\,\bar{w}}{\nu} \qquad (10)$$

and

$$K = \frac{32\,g\,S\,R^2}{\nu\,\bar{w}} \qquad (11)$$

---

[12] "Hydrodynamics," by H. L. Dryden, F. D. Murnaghan, and H. Bateman, *Bulletin No. 84*, National Research Council, Washington, D. C., 1932, Pt. II, Chapter II.

Because by Eq. 5 and Eq. 8, $\bar{w}$ is proportional to $g\, S/\nu$ and has a specific value for each channel shape, K is a purely numerical coefficient dependent only on the channel shape. Theoretical values of K for several channel shapes are also given in Appendix I. Eq. 9 plots as a straight line of slope minus 1 on full logarithmic paper.

Experiment.—Experimental data pertaining to Eq. 9 are plotted on four graphs in Fig. 2—Fig. 2(*a*) for rectangular channels, Fig. 2(*b*) for triangular channels with a 90° vertex angle, Fig. 2(*c*) for triangular channels with 60° and 120° vertex angles, and Fig. 2(*d*) for triangular channels with 30° and 150° vertex angles. Eq. 9, with applicable values of K taken from Appendix I, is also plotted on each of these figures. The value of K used for the 30° channel and 150° channel is the same as that used for 60°, 90°, and 120° because the value for 30° and 150° has not been computed. This value would seem to be approximately correct, extrapolating from the other solutions in Appendix I. Similar data for circular-arc and 45° vertex-angle, triangular channels are not shown. These data are not sufficiently different from those shown to warrant another figure. For the circular-arc channels, the data are grouped about the line $f = 64/\mathbf{R}$ and the data lie just below the line $f = 57/\mathbf{R}$ for the triangular channel.

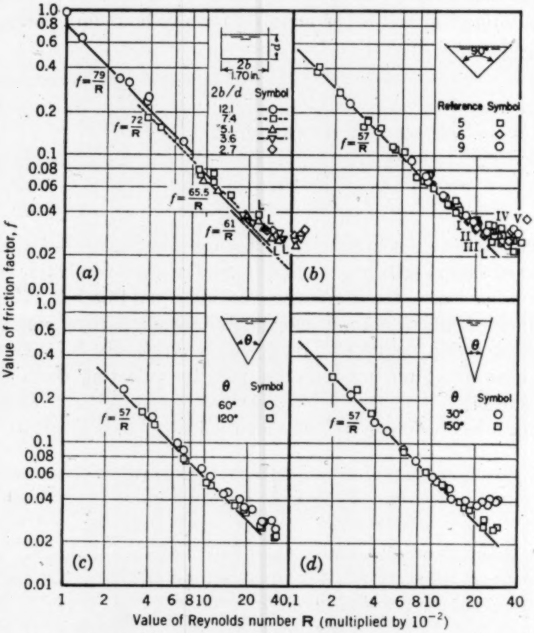


FIG. 2.—FRICTION FACTOR FOR SMOOTH LAMINAR FLOW

Experimental data pertaining to the solution of Eq. 5 for 90° triangular channels are given as velocity profiles in Fig. 3. The experimental points are compared with the theoretical results obtained from Appendix I. The experimental data correspond to points indicated by diamond-shaped symbols and marked I, II, and III in Fig. 2(*b*).

Although the goal of the present research was to investigate fully developed flow, the lengths of the channels made it necessary to accept some data where the boundary layer was clearly still developing. The exact length of the developing laminar boundary layer in a channel is not known. In a circular pipe the entrance length in hydraulic diameters is estimated to be on the order of 0.03 **R**.[13] If this criterion is applied to the present experiments, it is found that all the data obtained at Reynolds numbers of less than approximately 1,500 are in fully developed flow. However, as the Reynolds number increases, more and more of the data become influenced by the entrance region, depending on the hydraulic diameter for each experiment. Hence, the gradual departure of the data from the theoretical curves in Fig. 2 at the greater Reynolds numbers is not to be attributed entirely to transition, but it may be a result of including part of the entrance region in the channel reach for which data were obtained.

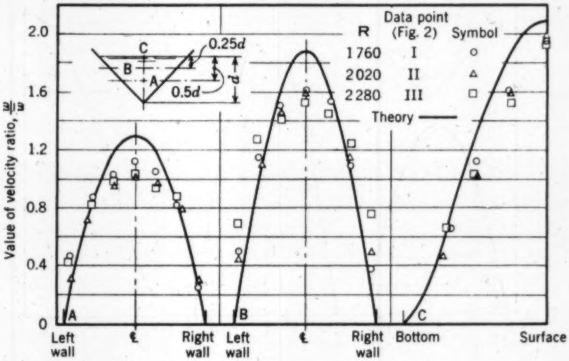


FIG. 3.—VELOCITY PROFILES FOR SMOOTH LAMINAR FLOW

It may be concluded that the theoretical results from Eqs. 5, 8, 9, and 11 give a reasonable estimate of the actual values in fully developed laminar flow in open channels. The entire effect of channel shape is given by these equations. The surface effect on the friction factor, which arises from assuming that the velocity derivative at the surface is equal to zero and from neglecting surface tension, is minor.

SMOOTH CHANNELS—TRANSITION TO TURBULENCE

For each of the data points shown in Figs. 2(*a*) and 2(*b*), the observer noted whether the flow appeared to be clearly laminar, clearly turbulent, or a combination of both. The observation for laminar flow was usually made by observing reflections of a long, straight object alined with the channel in the liquid surface. An unbroken reflection was regarded as an indication of pure laminar flow. The data point of greatest Reynolds number for which the flow appeared clearly laminar is indicated by a letter *L* on the symbol for that point.

---

[13] "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Clarendon Press, Oxford, England, 1938, pp. 301–304.

The solid symbol in Fig. 2(*a*) ($2\,b/d = 5.1$ at $\mathsf{R} = 2{,}550$) represents data obtained with the bell-mouthed entrance removed. Apparently the removal of the bell mouth, and thereby the disturbance of the flow at the entrance, did not influence the character of the flow. If the flow had been in a transitional rather than a laminar regime, a measurable effect would ordinarily have been expected.

Fig. 4 shows velocity profiles corresponding to data points IV and V in the transitional region of Fig. 2(*b*). These profiles are compared with those from laminar theory already shown in Fig. 3. Point IV closely approximates the previously shown laminar-flow data, with a slight tendency toward increased velocity nearest the wall. On the other hand, point V shows clearly the higher velocity near the wall characteristic of a turbulent boundary layer.

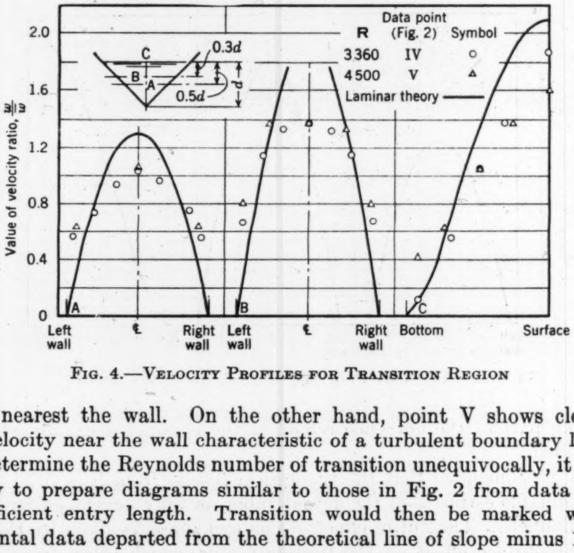


FIG. 4.—VELOCITY PROFILES FOR TRANSITION REGION

To determine the Reynolds number of transition unequivocally, it would be necessary to prepare diagrams similar to those in Fig. 2 from data obtained with sufficient entry length. Transition would then be marked where the experimental data departed from the theoretical line of slope minus 1. From the present observations it appears that there is a shape effect on the Reynolds number of transition. Values for rectangular, triangular, and circular-arc channels were estimated from Fig. 2 and similar curves as follows:

For rectangular channels—

Depth ratio $2\,b/d$	Transition R
7.4	2,350
5.1	3,100
3.6	3,600
2.7	2,650

For triangular channels—

Vertex angle	Transition R
30°	2,000
45°	2,300
60°	2,700
90°	2,350
120°	3,150
150°	2,800

For the triangular channels many more data were available at the 90° vertex angle than for the other angles. If equally representative data were available for all channels, it is possible that estimated transitional Reynolds numbers might be lower for the channels of vertex angle different from 90°. The estimated value of transition R for a semicircular-arc channel is 2,200.

Although the experimental channels were isolation mounted to prevent direct vibration by the pumping equipment, it is believed that there was still enough disturbance present to cause transition at the lowest possible Reynolds number. The foregoing values are therefore judged to be indicative of the lower Reynolds number of transition. This appraisal is supported by the solid data point in Fig. 2(*a*).

SMOOTH CHANNELS—TURBULENT FLOW

Theory.—In the absence of a complete theory of turbulent flow, the value of the Darcy-Weisbach friction factor for open channels can only be estimated on a semiempirical basis from earlier work on pipes. Such estimates have been made by G. H. Keulegan,[14] and the subject has been studied by Hunter Rouse, M. ASCE.[15] Keulegan made use of the logarithmic velocity profile, assuming this to be adequately descriptive at the velocity everywhere except in the laminar sublayer. For an average rectangular channel his results are equivalent to

$$\frac{1}{\sqrt{f}} = 2.03 \log \mathsf{R} \sqrt{f} - 0.99 \quad \text{(12a)}$$

and, for a 90° triangular channel, to

$$\frac{1}{\sqrt{f}} = 2.03 \log \mathsf{R} \sqrt{f} - 0.9 \quad \text{(12b)}$$

the shape factor being included in the last term. Eq. 12*b* is identical with the result for a smooth, circular pipe, which, to conform better with experimental results, is usually modified to

$$\frac{1}{\sqrt{f}} = 2 \log \mathsf{R} \sqrt{f} - 0.8 \quad \text{(12c)}$$

---

[14] "Laws of Turbulent Flow in Open Channels," by G. H. Keulegan, *Journal of Research*, National Bureau of Standards, U. S. Dept. of Commerce, Washington, D. C., Vol. 21, December, 1938, pp. 707–741.

[15] "Elementary Mechanics of Fluids," by Hunter Rouse, John Wiley & Sons, Inc., New York, N. Y., 1946, pp. 214–216.

For the present purposes, in which only small Reynolds numbers are involved, a power-law velocity profile is adequate and easier to handle than a logarithmic profile. Again the profile is assumed to be adequately representative of the velocity throughout the channel. The appropriate profile along a normal to a straight wall is

$$\frac{w}{w_*} = B\left(\frac{h\,w_*}{\nu}\right)^m \quad \text{......................} \quad (13)$$

in which $w_* = \sqrt{\tau_0/\rho}$ is the friction velocity; τ_0 is the wall friction per unit area where the normal joins the wall; h is the distance from the wall; and B and m are constants. For the smooth, circular pipe, H. Blasius gives the experimental result,

$$f = \frac{0.316}{\mathbf{R}^{1/4}} \quad \text{......................} \quad (14a)$$

in which the Reynolds number is based on the pipe diameter. Eq. 14*a* corresponds to $B = 8.6$ and $m = 1/7$ in Eq. 13. Eq. 14*a* will also apply to a semicircular channel if there are no secondary currents. For a very wide channel the profile, Eq. 13, with $B = 8.6$ and $m = 1/7$, will yield

$$f = \frac{0.333}{\mathbf{R}^{1/4}} \quad \text{......................} \quad (14b)$$

If the velocity profile is taken along a line other than a normal to a wall, Eq. 13 will no longer apply directly. For example, for the vertical profile along the line of symmetry of a 90° triangular channel, the velocity at any point, P_1, is given by Eq. 13 with $h = y/\sqrt{2}$. However, w_* varies from the vertex to the free surface. If $\bar{w}_*$ is the mean friction velocity defined by

$$\bar{w}_* = \sqrt{\frac{\bar{\tau}_0}{\rho}} = \sqrt{R\,g\,S} \quad \text{..................} \quad (15)$$

then

$$\frac{w_*}{\bar{w}_*} = \mathsf{F}\left(\frac{y}{d}\right) \approx \left(\frac{G\,y}{d}\right)^P \quad \text{..................} \quad (16)$$

in which G and P are empirical constants that measure the distribution of friction velocity along the wall. Using Eqs. 16 and 13, with $B = 8.6$ and $m = 1/7$, the profile along the line of symmetry of a 90° triangular channel is

$$\frac{w}{\bar{w}_*} = 8.2\left(\frac{G\,\nu}{d\,\bar{w}_*}\right)^{8P/7}\left(\frac{y\,\bar{w}_*}{\nu}\right)^{(1+8P)/7} \quad \text{.............} \quad (17)$$

Experiment.—In Fig. 5 experimental data for various rectangular and triangular channels in the range of lower turbulent Reynolds numbers are presented as graphs of friction factor versus Reynolds number. Eq. 14*a* is plotted for comparative purposes on all the graphs, and Eq. 14*b* is also plotted on one of them. Similar data obtained for circular-arc channels and for 45° triangular channels are not shown in the figures. For comparison Keulegan[14] gives the equivalent of

$$f = \frac{0.325}{\mathsf{R}^{1/4}} \quad \text{.........................} \quad (18)$$

as the mean line of all H. Bazins' data on smooth rectangular channels. The data in Fig. 5 could be represented equally well by Eq. 12*a* and Eq. 12*b* for the rectangular and triangular channels, respectively.

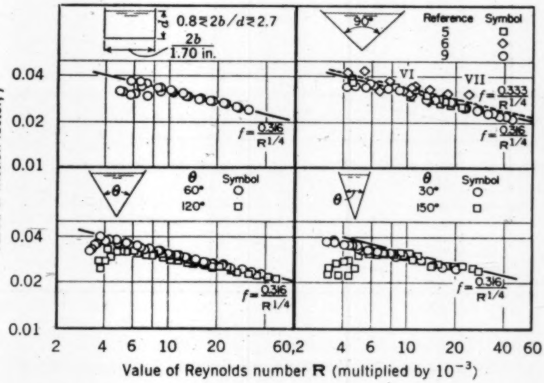


FIG. 5.—FRICTION FACTOR FOR SMOOTH TURBULENT FLOW

In Fig. 6 velocity profiles are shown for two of the data points for the 90° triangular channel. Profiles, both normal to the wall and along the line of symmetry, are plotted on full logarithmic paper to facilitate comparison with Eq. 13 and Eq. 17. The experimental data yield $m = 1/7$ and $P = 0.076$, and $B = 8.3$ and $B = 8.8$ for the two separate graphs. If the two sets of data were superimposed on one graph, however, $B = 8.6$ would adequately describe the average line through the data along the normals. For the two graphs $G = 1.14$ or $G = 1.36$. (It should be noted that the least accurate data in these experiments are shown in the measurement of the slope, which enters directly into the computation of $\bar{w}_*$, so that the coefficients, B and G, as determined in Fig. 6, are less accurate than the exponents, m and P.)

It is believed that the entrance-length problem in the turbulent-flow region was minor. In smooth pipes with disturbed entrance conditions at a Reynolds number on the order of 10^4, only twelve diameters are required theoretically for the wall friction to become equal to its value in fully developed flow.[16] Furthermore, the velocity profiles shown in Fig. 6 are characteristic of fully developed turbulent flow.

It is concluded from the data in Fig. 5 and Fig. 6 that at these small Reynolds numbers:

---

[16] "The Turbulent Boundary Layer in the Inlet Region of Smooth Pipes," by J. S. Holdhusen, thesis presented to the University of Minnesota, at Minneapolis, in 1951, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

1. Channel shape has relatively little influence on frictional loss.

2. The law given by Eq. 14a for the friction factor for smooth pipes is adequately representative of all smooth channels; Eq. 12c is also adequate.

3. The power-law velocity distribution given by Eq. 13, with $B \approx 8.6$ and $m = 1/7$, is acceptably representative of the true velocity distribution except near the free surface.

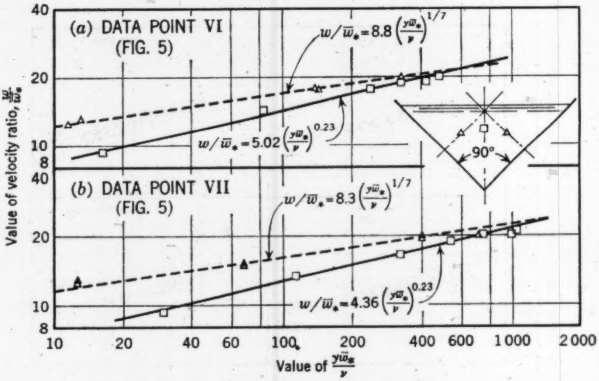


FIG. 6.—VELOCITY PROFILES FOR SMOOTH TURBULENT FLOW

ROUGH CHANNELS—LAMINAR FLOW

Theory.—Following a familiar line of reasoning for rough pipes,[17] the velocity in a 90° triangular channel, at a distance from the wall, equivalent to the roughness height, k, is approximately (from Appendix I)

$$w_k = 10\,\frac{k}{d}\,\overline{w} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

in which w_k is the velocity at distance k from the wall, d is the depth of liquid in the channel, and $k/d \ll 1$. The relative roughness is then given by the tip Reynolds number of the roughness and, assuming that the presence of the roughness does not affect the flow, is

$$\mathsf{R}_k = \frac{w_k\,k}{\nu} = 7.1\left(\frac{k}{d}\right)^2 \frac{4\,R\,\overline{w}}{\nu} = 7.1\left(\frac{k}{d}\right)^2 \mathsf{R} \quad\ldots\ldots\ldots\ldots (20)$$

Presumably there is some critical value of the tip-roughness Reynolds number below which the laminar flow is practically undisturbed by the wakes of the roughness elements. Terming this critical value R_{kc},

$$k < \left(\frac{\mathsf{R}_{kc}}{7.1\,\mathsf{R}}\right)^{1/2} d \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21a)$$

---

[17] "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Clarendon Press, Oxford, England, 1938, p. 311.

or

$$\frac{2\,R}{k} > \left(\frac{3.55\ \mathsf{R}}{\mathsf{R}_{kc}}\right)^{1/2} \qquad (21b)$$

if the laminar flow is to be uninfluenced by the roughness. The critical-roughness Reynolds number must depend on the shape and spacing of the roughness elements.

If the roughness height is greater than the value given by Eq. 21*a* and Eq. 21*b*, the friction factor can be expected to be larger than the computed value for laminar flow, and the velocity profile near the wall can be expected to be shifted away from the wall. No theoretical method is available for computing the magnitude of these effects. Not only the roughness height, but also its shape and spacing will enter into the result.

Experiments.—Experiments were conducted in the 90° triangular channel previously described, using rough surfaces. The surfaces were prepared by cementing abrasive cloth to the smooth channel walls. The abrasive material was composed of sharp-edged particles. Two grades of cloth were used, specifications of which are contained in Table I.

Results of the experiments at Reynolds numbers less than 5,000 are given in Fig. 7 as a plot of friction factor versus Reynolds number. The channel bottom is taken at the base of the roughness elements in computing the data. Velocity profiles were not measured. The theoretical result for laminar flow in smooth, 90° triangular channels is shown, for comparison, by the heavy line in Fig. 7. There is little doubt that the roughness of the type used has influenced the laminar flow.

TABLE I.—DESCRIPTIVE TABULATION OF ROUGH SURFACES

Grade of cloth	Range of grain diameter, in inches	Approximate mean diameter, in inches	Approximate mean spacing, in inches
Fine	0.0105 to 0.0132	0.0119	0.0144
Coarse	0.026 to 0.0307	0.0283	0.0386

If the friction factor and Reynolds number for the data in Fig. 7 are recomputed, taking the channel bottom at the tips of the roughness elements, the data are brought closer to the theoretical line, but the fine cloth data lie generally above the coarse cloth data.

It should be noted that part of the divergence of the experimental data in Fig. 7 might be attributable to a too short entry length, as was the case in Fig. 2. The entry length for these rough-channel data was 11 ft, which corresponds to a range of from 80 hydraulic diameters at some of the high Reynolds numbers to 400 hydraulic diameters at the smaller Reynolds numbers. Thus, there is some possible effect of entry length, particularly for Reynolds numbers more than 2,000.

It is concluded from Eq. 11 that the critical-roughness Reynolds number is less than 2 for the roughness used herein. This value is of smaller order than has been assumed previously for uniform sand roughness in pipes.[17]

ROUGH CHANNELS—TURBULENT FLOW

Theory.—No completely satisfactory theory for rough turbulent flow exists, even for pipes. It has become common practice to describe a rough surface by the height, k, or nominal height, k_s, of the roughness particles. Although this practice facilitates correlating friction-loss data in special cases, it must be used with caution in dealing with arbitrarily rough surfaces. Even when using surfaces roughened artificially in a systematic pattern, generally more than a single linear dimension is required to describe the roughness geometrically. Therefore, equations or formulas for friction loss that contain only the single roughness parameter, k or k_s, cannot be expected to be generally valid under all conditions. Although formulas involving two or more linear roughness measures have been proposed (by Henry M. Morris, Jr.,[18] M.ASCE, for example) and even appear to be verified by some of the experimental data presented subsequently herein, these formulas have not been tested sufficiently to warrant their publication as recognized theory.

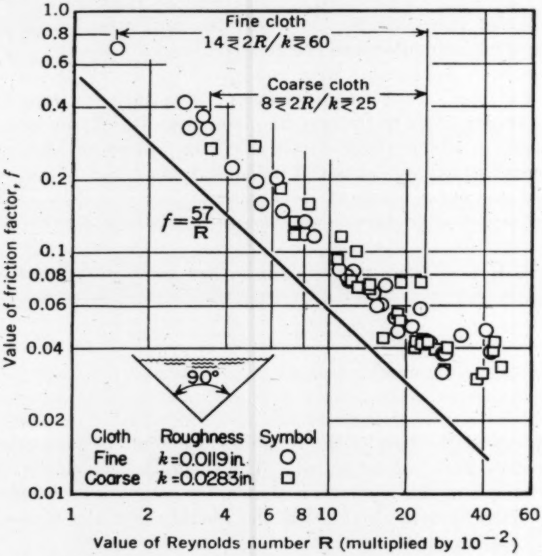


FIG. 7.—FRICTION FACTOR FOR ROUGH LAMINAR FLOW

The most satisfying investigation of rough surfaces to date has been that of J. Nikuradse for closely packed, uniform sand grains in pipes. The one linear dimension, k, suffices to measure the roughness because of the uniform shape and close packing of the grains. When the surface is sufficiently rough,

---

[18] "Flow in Rough Conduits," by Henry M. Morris, Jr., *Transactions*, ASCE, Vol. 120, 1955, pp. 373–398.

Nikuradse's data are represented[19] by

$$\frac{1}{\sqrt{f}} - 2 \log \frac{2\,R}{k} = 1.74 \qquad (22)$$

Eq. 22 represents the well-known flow regime, in which the friction factor has become independent of the Reynolds number and depends only on the relative roughness, $2\,R/k$. Based on Keulegan's analysis,[14] Eq. 22 may also be applied directly to a 90° triangular channel. Because this paper is primarily concerned with flow at small Reynolds numbers, Eq. 22 represents a limiting case only.

For surfaces other than those composed of uniform sand grains, it is common practice to replace k in Eq. 22 by k_s, the nominal roughness size, which is determined empirically for each surface from experimental data in order to make Eq. 22 valid. It is sometimes necessary to go to extremely high Reynolds numbers to accomplish this, and then the region of constant friction factor may be approached either from below or from above.

Between the region of applicability of Eq. 22 for rough flow and Eq. 12c for smooth flow, there exists a flow regime for which both viscosity and roughness influence the friction factor. It is this regime that is mainly involved herein. Except for Nikuradse's data on uniform sand-grain roughness in pipes, satisfactory correlation of data in this region has not been attained. It can be expected that the relationship between the friction factor and Reynolds number will depend on many factors—such as the form of the roughness, its average size, the average spacing of the separate elements and their relative orientation, and the viscosity and velocity. The relative roughness size can no longer be expressed as a function of the channel size only, but it must be measured with respect to the thickness of the laminar sublayer of the flow. Thus, for uniform sand grains the relative roughness can be expressed[19] by the parameter:

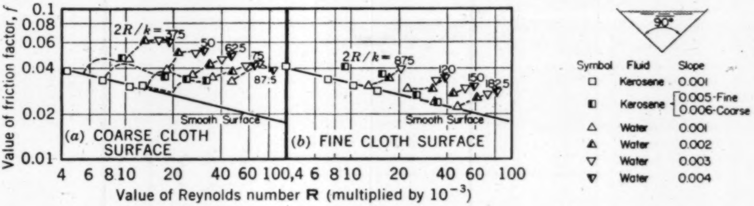


FIG. 8.—FRICTION FACTOR FOR ROUGH TURBULENT FLOW USING ABRASIVE CLOTH ROUGHNESS

$$\frac{k}{\dfrac{\nu}{\bar{w}_*}} = \frac{\mathsf{R}\sqrt{f}}{\dfrac{2\,R}{k}} \qquad (23$$

When plotted against the left side of Eq. 22, Eq. 23 correlates Nikuradse's uniform sand-grain data. The resulting experimental curve becomes asymptotic to Eq. 22 when the relative roughness given by Eq. 23 is equal to approximately 400, and to Eq. 12*c* when the relative roughness is approximately 16.

---

[19] "Elementary Mechanics of Fluids," by Hunter Rouse, John Wiley & Sons, Inc., New York, N. Y., 1946, pp. 205–206.

For other types of roughness the substitution of k_s for k in the definition of relative roughness, as is sometimes done, cannot be expected to produce the same correlation as for uniform sand grains, except by chance.

Experiments.—Two sets of experiments with rough surfaces in triangular channels have been completed. Both these involved measurements of bulk-flow quantities only and not of velocity profiles. One set was part of the study which has already been cited under the heading, "Rough Channels—Laminar Flow," using abrasive cloths as the surface roughness. Typical data from these experiments are shown in Fig. 8 as graphs of the friction factor versus the Reynolds numbers. Depths have been taken to the bottom of the roughness elements in these graphs. The lines having constant values of $2\,R/k$ have been drawn on the graphs, considering unplotted as well as plotted data points.

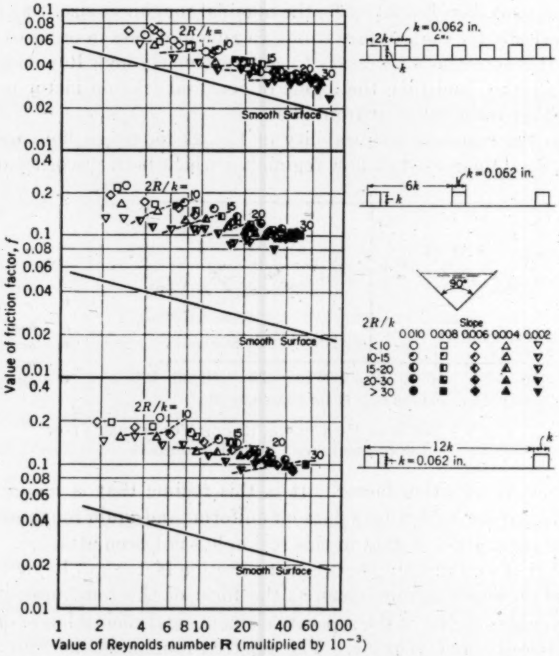


FIG. 9.—FRICTION FACTOR FOR ROUGH TURBULENT FLOW USING STRIP ROUGHNESS

The second set of experimental data was obtained in the triangular channel of variable cross section using strip-roughness elements. The strip-roughness elements of square cross section were laid completely across the channel walls, with their long dimension normal to the flow direction. Three spacings along the channel were used. Typical experimental data are shown in Fig. 9 as graphs of friction factor versus Reynolds number for the 90° channels at the three different spacings. Other typical data, showing the effect of channel shape, are presented in Fig. 10. Depths have been taken to the tops of the strips in both sets of curves.

In general the data presented in Figs. 8, 9, and 10 have not yet reached Reynolds numbers sufficiently high that Eq. 22 will apply. Even those few data which appear to indicate that a constant friction factor may have been reached should be regarded with caution because of the small quantity of data involved.

Any attempt to correlate these data by using the relative roughness defined by Eq. 23, together with an arbitrary value of k_s, fails except in the cases of the graphs of Fig. 9 and Fig. 10 for the spacing of 6 k. In these cases by choosing $k_s = 6\,k$, it is possible to obtain what appears to be some kind of correlation similar to that obtained for the Nikuradse sand-grain roughness, a different correlating curve applying to each channel shape. Other methods of correlation using additional length parameters are also largely futile. Hence, the problem of correlating friction-factor data for arbitrary rough surfaces at small Reynolds numbers is still unsolved.

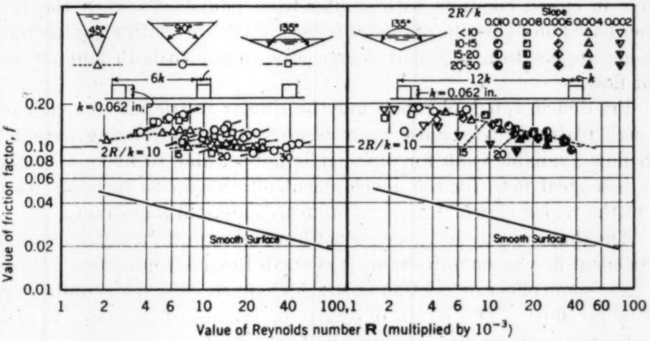


Fig. 10.—Friction Factor for Rough Turbulent Flow As Influenced by Channel Shape

The apparent effects of channel shape should be mentioned, particularly as shown for the largest strip spacing. In the case of the 135° channel with a strip spacing of 12 k, it appears that the effect of the strips eventually will become independent of $2\,R/k$, and that the friction factor will become a multiple of the smooth-surface friction factor. Similar results have been observed at higher Reynolds numbers for ship roughness[20] and for widely spaced roughness elements in pipes.[18] Although the 90° channel data, with a strip spacing of 12 *k* (Fig. 9), do not appear to follow the same law as the 135° channel data, they might have done so if the experiments could have been extended to a slightly higher Reynolds number at each depth. In that case the effect of shape would simply have changed the multiples to be applied to the smooth-surface curve. At the smaller strip spacings the shape effect is not very pronounced, and only one graph in Fig. 10 is given as typical.

---

[20] "Friction and Turbulence Stimulation," Pt. II, *Proceedings*, 7th International Cong. on Ship Hydrodynamics, Goteborg, Sweden, 1955, pp. 55–128.

These rough-channel data do not differ greatly in appearance from pipe data and from ship-model data at similar Reynolds numbers and for similar roughness, but no exact comparison is obtainable as yet.

CONCLUSIONS

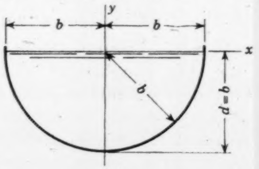
The following conclusions are drawn:

1. Friction and velocity distribution for flow in smooth, open channels at small Reynolds numbers are quantitatively equal to the corresponding values for flow in closed conduits with similar fixed boundaries when the Reynolds number based on hydraulic diameter is used as the correlating parameter. The foregoing applies to a sufficient degree of accuracy to both laminar and turbulent flow.
2. Friction in rough channels may be similar to friction in rough pipes or on rough plates, but, because there is no suitable correlating parameter for rough flow available even for pipes, it is not feasible to make a comparison. There is a great need for the development of a means to correlate rough-flow data in the region in which viscous influences are still important.
3. The shape of the channel does not have an important influence on friction for turbulent flow in smooth channels at small Reynolds numbers. However, it does have some effect on friction in rough channels. Shape is quite important for laminar flow. The effect of shape in laminar flow can be estimated theoretically; some results are given in Appendix I.
4. The lower critical Reynolds number for transition between laminar flow and turbulent flow in smooth channels depends to some extent on channel shape. The value ranges from 2,000 to more than 3,000, being generally larger than the value for closed-conduit flow.

APPENDIX I. FLOW PARAMETERS FOR SEVERAL OPEN CHANNEL SHAPES

ANALYTICALLY DERIVED SOLUTIONS

1. Semicircular Channel

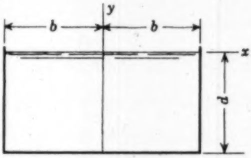


$$w = \frac{g\,S}{4\,\nu}\,b^2\left(1 - \frac{x^2}{b^2} - \frac{y^2}{b^2}\right)$$

$$\bar{w} = \frac{g\,S}{8\,\nu}\,b^2$$

$$K = 64$$

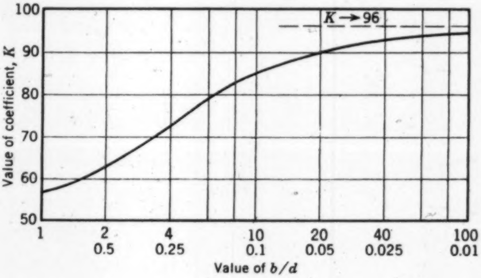
2. Rectangular Channel



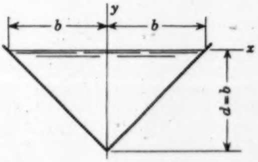
$$w = \frac{g\,S}{2\,\nu}\,d^2\left(1 - \frac{y^2}{d^2} - \frac{32}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2\,n+1)^3}\,\frac{\cosh\dfrac{2\,n+1}{2}\dfrac{\pi\,x}{d}}{\cosh\dfrac{2\,n+1}{2}\dfrac{\pi\,b}{d}}\cos\frac{2\,n+1}{2}\frac{\pi\,y}{d}\right)$$

$$\bar{w} = \frac{g\,S}{3\,\nu}\,d^2\left(1 - \frac{192}{\pi^5}\frac{d}{b}\sum_{n=0}^{\infty}\frac{1}{(2\,n+1)^5}\tanh\frac{2\,n+1}{2}\frac{\pi\,b}{d}\right)$$

$$K = \frac{96}{\left(1+\dfrac{d}{b}\right)^2}\,\frac{1}{1 - \dfrac{192}{\pi^5}\dfrac{d}{b}\displaystyle\sum_{n=0}^{\infty}\frac{2}{(2\,n+1)^5}\tanh\frac{2\,n+1}{2}\frac{\pi\,b}{d}}$$



3. 90-Degree Triangular Channel (Symmetrical)



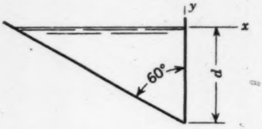
$$w = \frac{g\,S}{4\,\nu}\,d^2$$

$$\times\left(1 - \frac{(x+y)^2}{d^2} - \frac{32}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2\,n+1)^3}\,\frac{\cosh\dfrac{2\,n+1}{2}\dfrac{\pi\,(x-y)}{d}}{\cosh\dfrac{(2\,n+1)}{2}\,\pi}\cos\frac{2\,n+1}{2}\frac{\pi\,(x+y)}{d}\right)$$

$$\bar{w} = \frac{g\,S}{14.22\,\nu}\,d^2$$

$$K = 56.9$$

4. 60-Degree Triangular Channel (Vertical Leg)

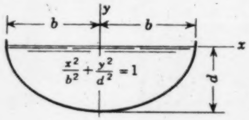


$$w = \frac{\sqrt{3}\, g\, S}{12\, \nu} \times d \left(\frac{x^2}{d^2} + 2\sqrt{3}\, \frac{x}{d} + 3 - 3\, \frac{y^2}{d^2} \right)$$

$$\bar{w} = \frac{g\, S}{20\, \nu}\, d^2$$

$$K = 53\tfrac{1}{3}$$

5. Elliptical Channel

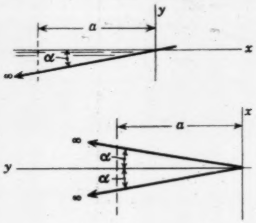


$$w = \frac{g\, S}{2\, \nu} \frac{b^2\, d^2}{b^2 + d^2} \left(1 - \frac{x^2}{b^2} - \frac{y^2}{d^2} \right)$$

$$\bar{w} = \frac{g\, S}{4\, \nu} \frac{b^2\, d^2}{b^2 + d^2}$$

$$K = \frac{128}{\left[1 + \frac{1}{4} \left(\frac{b-d}{b+d} \right)^2 + \frac{1}{64} \left(\frac{b-d}{b+d} \right)^4 + \frac{1}{256} \left(\frac{b-d}{b+d} \right)^6 + \cdots \right]}$$

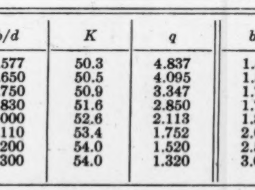
6. Edge or Bottom of Triangular Channel



$$\left.\begin{aligned} w &= \frac{g\, S}{2\, \nu} \frac{x^2 \tan^2 \alpha - y^2}{1 - \tan^2 \alpha} \\ \bar{w} &= \frac{g\, S}{6\, \nu} \frac{\sin^2 \alpha}{\cos 2\, \alpha}\, a^2 \\ K &= 48 \cos 2\, \alpha \end{aligned} \right\} \alpha < \frac{\pi}{4}$$

Numerical Solutions

7. Triangular Channel (Vertical Leg)



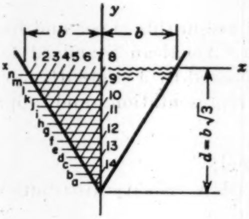
$$\bar{w} \approx \frac{q\, g\, S}{81\, \nu} \frac{b^3}{d}$$

Values of K and q[a]

b/d	K	q	b/d	K	q
0.577	50.3	4.837	1.500	53.50	1.027
0.650	50.5	4.095	1.580	53.20	0.935
0.750	50.9	3.347	1.732	53.33	0.780
0.830	51.6	2.850	1.760	53.50	0.750
1.000	52.6	2.113	1.880	53.10	0.662
1.110	53.4	1.752	2.000	53.60	0.497
1.200	54.0	1.520	2.500	54.20	0.285
1.300	54.0	1.320	3.000	55.40	0.225

[a] "Flow of a Viscous Incompressible Fluid: Expressions for a Uniform Triangular Duct," by H. Nuttall, *Engineering* (London), Vol. 178, September 3, 1954, pp. 299–300.

8. 60-Degree Triangular Channel (Symmetrical)



$$w = \frac{q\,S}{1{,}500\,\nu}\,b^2\,H$$

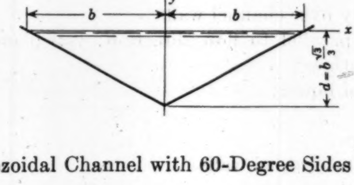
$$\bar{w} \approx \frac{g\,S}{9.4\,\nu}\,b^2$$

$$K \approx 56.6$$

VALUES OF H DETERMINED BY RELAXATION
($H = 0$ at all boundary points)

Pt	H	Pt	H	Pt	H
b—14	18.22	i—10	231.50	m—5	229.25
c—13	34.33	j—6	95.11	m—6	280.00
d—12	48.22	j—7	169.55	m—7	314.75
d—13	64.44	j—8	223 00	m—8	332.25
e—11	60.67	j—9	255.25	n—2	74.88
e—12	91.11	j—10	266.00	n—3	149.00
f—10	71.67	k—5	95.67	n—4	215.50
f—11	114.55	k—6	173.00	n—5	270.50
f—12	128.77	k—7	232.00	n—6	311.50
g—9	80.55	k—8	271.50	n—7	336.75
g—10	134.22	k—9	291.50	n—8	345.25
g—11	161.00	l—4	94.00	x—1	51.22
h—8	87.44	l—5	171.55	x—2	121.55
h—9	150.00	l—6	234.25	x—3	190.00
h—10	187.50	l—7	280.00	x—4	249.75
h—11	200.00	l—8	307.75	x—5	297.25
i—7	92.22	l—9	317.25	x—6	330.00
i—8	161.77	m—3	87.00	x—7	346.75
i—9	208.25	m—4	164.33		

9. 120-Degree Triangular Channel (Symmetrical)

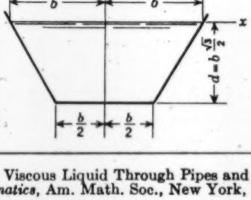


$$w = \frac{q\,S}{1{,}500\,\nu}\,d^2\,H$$

$$\bar{w} \approx \frac{g\,S}{9.4\,\nu}\,d^2$$

$$K \approx 56.6$$

10. Trapezoidal Channel with 60-Degree Sides



w: see footnote 21

$$\bar{w} \approx \frac{8\,g\,S}{5\,\nu}\,b^2$$

$$K = 60$$

---

[21] "Flow of Viscous Liquid Through Pipes and Channels," by J. L. Synge, *Proceedings, Symposium in Applied Mathematics*, Am. Math. Soc., New York, N. Y., Vol. IV, 1953, pp. 141–165.

APPENDIX II. NOTATION

The following symbols, adopted for use in this paper and for the guidance of discussers, conform essentially with "American Standard Letter Symbols for Hydraulics" (ASA Z 10.2–1942) prepared by a committee of the American Standards Association with Society representation, and approved by the Association in 1942.

A = cross-sectional area of channel;
B = constant coefficient in power-law velocity distribution;
b = half-width of channel;
C = Chezy coefficient;
d = depth of water in channel;
f = Darcy-Weisbach friction factor;
G = constant;
g = acceleration of gravity;
h = distance measured normally to a wall;
K = parameter in $f = K/\mathbf{R}$;
k = height of roughness element;
k_s = equivalent sand-roughness height;
m = exponent in power-law velocity distribution;
n = Manning n;
P = exponent;
p = pressure;
R = hydraulic radius;
$\mathbf{R}$ = Reynolds number $= 4\,R\,\bar{w}/\nu$;
S = channel and energy slope;
w = local velocity parallel to channel axis;
$\bar{w}$ = mean velocity parallel to channel axis;
w_k = velocity at distance k from wall (at tips of roughness elements);
w_* = friction velocity $= \sqrt{\tau_0/\rho}$;
$\bar{w}_*$ = mean friction velocity over channel walls;
x,y,z = coordinate axes—z, parallel to flow direction; (yz)-plane vertical with origin at free surface;
ν = kinematic viscosity of liquid;
ρ = density of liquid;
τ_0 = wall shear stress; and
$\bar{\tau}_0$ = mean shear stress over channel walls.

DISCUSSION

Wallace M. Lansford[22] **and James M. Robertson,**[23] **Members, ASCE.**—The theoretical relationships presented for viscous laminar flow in elliptical, rectangular, and 90° triangular open channels are contained in the theoretical results presented by J. Boussinesq in 1868[24] for viscous flow in rectangular and elliptical conduits. In applying these results to the appropriate open-channel cases, no effect of the free surface is assumed. In turbulent flow such an assumption is slightly in error, and the maximum velocity usually lies below, rather than on, the free surface.

It would be interesting to know if the authors' measurements of the velocity distribution in the case of laminar flow include a determination of the location of the filament of maximum velocity and if such a filament occurs on or below the free surface. For turbulent flows the occurrence of the maximum filament below the surface has been attributed to air resistance, surface tension, secondary currents, or some effect of turbulence. Observations of the occurrence or nonoccurrence of this phenomenon in laminar flow would at least represent the case of no turbulence. The possible importance of turbulence would be revealed, and some light could be shed on this intriguing but elusive question.

Studies of open-channel flow at the University of Illinois, at Urbana, although not as extensive as those reported by the authors, have been concerned with the turbulent-flow regime at mild slopes.[25,26] The subsequent presentation of the Illinois data supplements that of the authors.

The mean-flow relationships in a smooth, 90° triangular flume at Reynolds numbers of as much as 45,000 are shown in Fig. 2 and Fig. 5. The results obtained by W. M. Owen[25] on a similar smooth wooden channel at Reynolds numbers of from 20,000 to 275,000 are presented in Fig. 11 in comparison with the Minnesota data and smooth-pipe relationships. Barring any systematic errors, it would appear that at the higher Reynolds numbers the friction for triangular channels falls below the Blasius relationship for smooth pipes, as given by Eq. 14*a*.

The studies at Illinois have also been concerned with the nature of the turbulent flow in the triangular channel roughened with sand grains. This type of roughness was chosen because it is more nearly similar to Nikuradse's sand-grain roughness.[27] Fig. 11 also includes a comparison of the pipe and open-channel variation in friction factor with Reynolds number and relative roughness, $k/2\,R$. In contrast to Owen's[25] conclusion, from some of the same data these results show a transition region between the smooth and fully rough conditions for open-channel flow. Differences from the pipe results are apparent. Thus, the present sand grains appear to be effectively rougher than those of Nikuradse by approximately 1.4 times, and fully rough flow seems to occur at a lower Reynolds number. The first of these observations is not surprising because the sand grains used in the flume were not as closely packed as were Nikuradse's.

---

[22] Prof. of Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.

[23] Prof. of Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.

[24] "Memoire sur l'influence des frottements dans les mouvements reguliers des fluides," by J. Boussinesq, *Journal de Mathématiques Pures et Appliqués*, Series 2, Vol. 13, 1868, pp. 377–424.

[25] "Correlation Between Pipe Flow and Uniform Flow in a Triangular Open Channel," by W. M. Owen, *Transactions*, Am. Geophysical Union, Vol. 34, 1953, pp. 213–219.

[26] "Laminar to Turbulent Flow in a Wide Open Channel," by W. M. Owen, *Transactions*, ASCE, Vol. 119, 1954, pp. 1157–1164.

[27] "Laws of Flow in Rough Pipes," by J. Nikuradse, *Forschungsheft No. 361*, Verein Deutscher Ingenieure, 1933 (translation in *Technical Memorandum 1292*, National Advisory Committee for Aeronautics, Washington, D. C., 1952).

A better understanding of the nature of the flow is obtained through studies of the velocity distribution. Thus, in Fig. 6, velocity traverses obtained for turbulent flow in the smooth 90° flume have been presented. Similar measurements in the roughened triangular flume have been taken by D. B. Y. Chen.[28] These measurements were made in the same 55-ft flume used by Owen.[25] The sand-grain roughness was not identical, but grains of the same size were similarly attached. That the roughness was effectively the same is evident from the general agreement of the friction factors in Fig. 11. More baffling was added at the entrance of the flume to improve flow symmetry. The velocity measurements were taken at a station 45 ft from the head bay with a pitot-

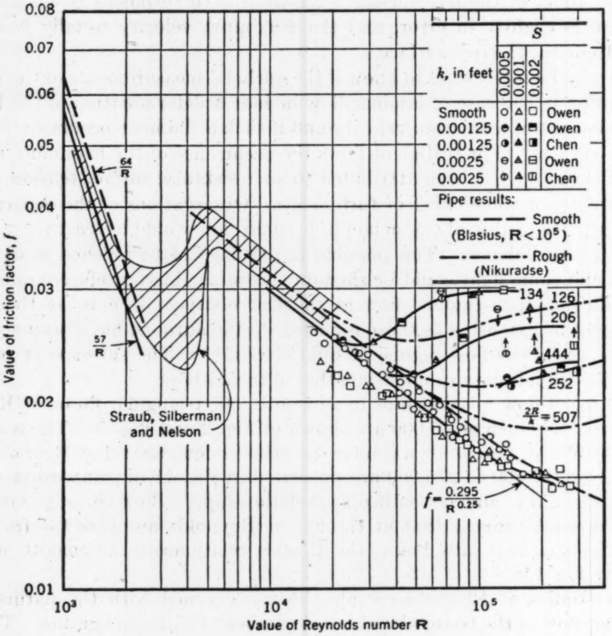


FIG. 11.—FRICTION FACTOR IN 90° FLUME COMPARED WITH PIPE RESULTS

static tube, which had a diameter of 0.106 in. For a flow depth of 0.518 ft, velocities were measured at thirty-two locations, and for a depth of 0.788 ft, forty-eight locations were used. The reliability of the velocity measurements was verified through comparison of integrated discharges with those obtained by weight. The integrated discharges were consistently smaller by approximately 2.75%.

---

[28] "A Study of the Velocity Distribution in an Open Channel Having a Triangular Cross Section," by D. B. Y. Chen, thesis presented to the University of Illinois, at Urbana, in 1955, in partial fulfilment of the requirements for the degree of Master of Science.

The results obtained for the smaller depth and roughness ($2\,R/K = 294$) are shown as contour plots in Fig. 12. The symbols in the upper plot represent the locations at which the velocities were actually obtained. The differences between the several plots are most apparent when one considers the maximum velocity in magnitude and location. As the Reynolds number increases, the ratio of the maximum velocity to the average velocity increases, and this maximum is found nearer the free surface. The thirty traverses made were analyzed by noting the ratio of maximum to average velocity, $w_m/\bar{w}$, and the ratio, ϵ, of the depth of the maximum-velocity location to depth of flow d. The results are shown in Table 2 and, in general, agree with the trend shown in Fig. 12. However, most of the measurements were taken in the region of rough conduit flow, in which the friction factor presumably depends only on the relative roughness and not on the Reynolds number. Variation of the velocity ratio with the relative roughness, as found by Nikuradse in rough pipes, is not evident from these data, due either to lack of precision or to the effect of the free surface.

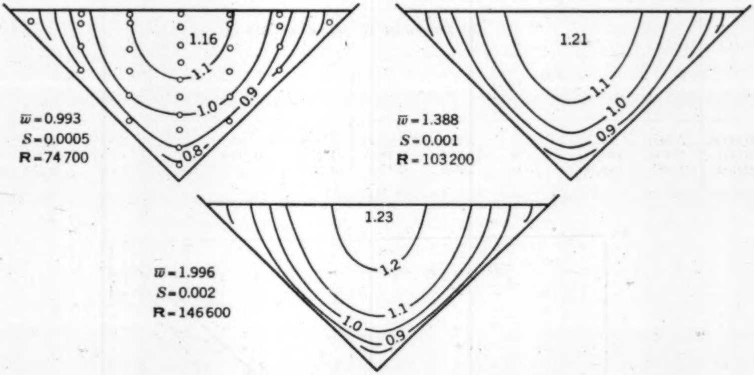


FIG. 12.—VELOCITY CONTOUR PLOTS FOR THREE REYNOLDS NUMBERS

Five of the velocity traverses taken by Chen were analyzed to determine velocity profiles in the rough flume in a manner similar to that presented by the authors in Fig. 6 for the smooth condition. Profiles have been obtained on the center line and on normals to the side walls passing through the point of maximum velocity. In the latter case, values of the velocity were determined from velocity contours sketched with the aid of the measured data. The resultant velocity profiles are presented in Fig. 13. Comparison of the profiles on the center line and the normal lines indicates effects similar to those

TABLE 2.—SUMMARY OF MAXIMUM VELOCITY DATA FROM A TRIANGULAR FLUME WITH ROUGH SIDES

Slope, S	(a) ROUGHNESS, $k = 0.00125$ FT					(b) ROUGHNESS, $k = 0.0025$ FT				
	$\bar{w}$, in feet per second	Reynolds number, $\mathbf{R}$	$\frac{k\, w_*}{\nu}$	$\frac{w_{max}}{\bar{w}}$	ϵ	$\bar{w}$, in feet per second	Reynolds number, $\mathbf{R}$	$\frac{k\, w_*}{\nu}$	$\frac{w_{max}}{\bar{w}}$	ϵ
	(A) DEPTH OF WATER, $d = 0.788$ FT									
	$\frac{2\,R}{k} = 444$					$\frac{2\,R}{k} = 206$				
0.0005	1.293	144,000	8.4	1.164	0.238	1.235	135,600	16.5	1.184	0.290
0.001	1.785	202,000	11.8	1.240	0.195	1.747	190,000	30.8	1.264	0.197
0.002	2.56	290.000	17.0	1.268	0.031	2.431	287,000	35.5	1.211	0.042
	(B) DEPTH OF WATER, $d = 0.518$ FT									
	$\frac{2\,R}{k} = 294$					$\frac{2\,R}{k} = 134$				
0.0005	0.989	74,360	6.9	1.175	0.212	0.908	65,200	13.5	1.231	0.254
0.001	1.398	104,000	9.8	1.204	0.169	1.275	94,000	19.1	1.181	0.212
0.002	1.988	146,000	13.6	1.232	0.135	1.863	134,000	26.6	1.240	0.171

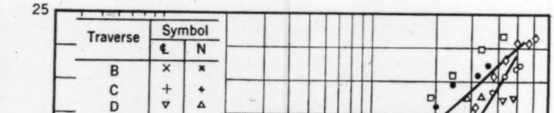


FIG. 13.—VELOCITY PROFILES FOR ROUGH TURBULENT FLOW IN A TRIANGULAR FLUME

shown in Fig. 6. The mean lines passing through the data have the following equations:

For the normal—

$$\frac{w}{\bar{w}} = 8.3 + 5.75 \log\left(\frac{y}{k}\right) \ldots\ldots\ldots\ldots\ldots\ldots (24a)$$

and, for the center line—

$$\frac{w}{\bar{w}} = -\,4.85 + 10.5 \log\left(\frac{y}{k}\right) \ldots\ldots\ldots\ldots\ldots\ldots (24b)$$

This form of equation, rather than an exponential form, is chosen to permit comparison with Nikuradse's pipe work. In making this comparison Eq. 24*a* is found to be close to Nikuradse's equation for the velocity profile on fully rough pipe flow. Such a similarity is coincidental because it has already been noted that the roughness heights of the sand grains used correspond to higher values of Nikuradse[28] sand-grain roughness. Another factor influencing these results is the variation in wall shear stress around the periphery of the flume. Thus, for the center-line traverses, the effective shear must be much less than the mean value used to compute $\bar{w}_*$.

An indication of the variation in the wall shear stress around the wetted perimeter of the flume has been obtained from the velocity measurements nearest the side walls. For each such measurement the Nikuradse rough pipe velocity profile equation, corrected for the different effective roughness height of the sand grains, was used to compute the value of the local shear-stress velocity, w_*. From a detailed analysis of the Nikuradse velocity measurements near the wall, this equation has been found to apply for values of $w_*\, k/\nu$ greater than 4.4. For the traverses analyzed, $\bar{w}_*\, k/\nu$ was not less than 8.4. This indicates an error when $w/\bar{w}^*$ is less than 0.52 $\left(\frac{\tau_0}{\gamma\, R\, S} \text{ less than } 0.23\right)$. The square of the ratio of w^* to the mean shear-stress velocity, $\bar{w}_*$, yields the ratio of wall shear stress to the average, or $\frac{\tau_0}{\gamma\, R\, S}$. The variation in wall shear stress up the side of the flume is presented in Fig. 14. Although there is considerable scatter and the traverse A appears out of line, a definite trend is apparent for the wall shear-stress variation.

For rectangular rough channels Keulegan[29] has analyzed Bazin's data in a similar fashion to obtain the peripheral variation in wall shear. He notes that the shear has a maximum value at the midpoint of the bottom and a minimum value at the corners. With this information he then verifies the agreement between Baxin's velocity traverses and the Nikuradse velocity equation. The writers believe that additional studies of shear and velocity distributions in turbulent open-channel flow will further a basic understanding of the phenomena.

---

[29] "Laws of Turbulent Flow in Open Channels," by G. H. Keulegan, *Journal of Research*, National Bureau of Standards, U. S. Dept. of Commerce, Washington, D. C., Vol. 21, December, 1938, pp. 733–735.

It is hoped that the material included herein will help to give a clearer insight into some phases of open-channel flow, thus supplementing the excellent coverage presented by the authors.

Traverse	A	B	C	D	E	F
Depth, d, in feet	0.788	0.518	0.518	0.788	0.788	0.788
Roughness, k, in feet	0.0025	0.0025	0.00125	0.00125	0.00125	0.00125
Slope, S	0.001	0.001	0.001	0.0005	0.001	0.002
Symbol	○	×	+	△	●	□

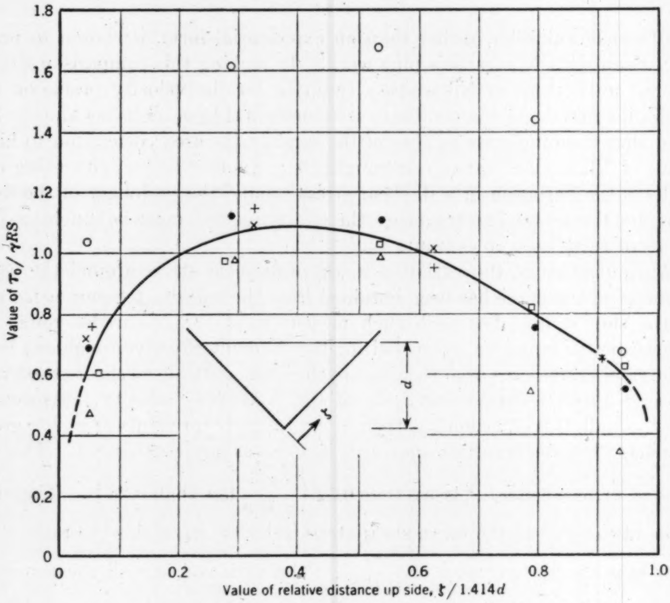


FIG. 14.—SHEAR-STRESS VARIATION UP THE SIDE OF A 90° FLUME

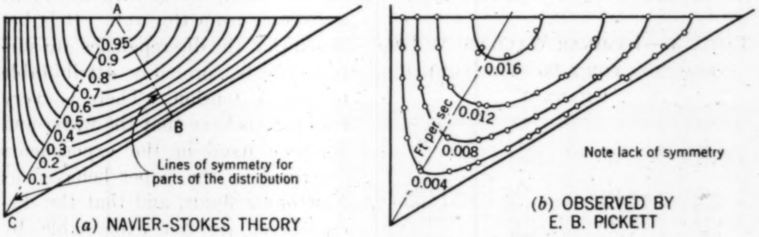
CHESLEY J. POSEY,[30] M. ASCE.—Part of one of the conclusions states that "* * * the entire effect of shape in laminar flow may be computed theoretically * * *." This seems to imply that the theory based on the Navier-Stokes equations is applicable to the extent that it will yield an accurate picture of the velocity distribution. There is good evidence to the contrary.

Fig. 3 shows a considerable deviation from theory, a deviation that is evidently systematic and not haphazard. Data obtained by Ellis B. Pickett, J.M. ASCE, show deviations of comparable magnitude (Fig. 15).[31] Theoretical studies based on differential equations that describe the behavior of actual fluids more precisely than the Navier-Stokes equations yield significantly different results.[32,33,34]

---

[30] Prof. and Head, Dept. of Civ. Eng., State Univ. of Iowa, Iowa City, Iowa.

[31] "A Direct Optical Method for Measuring Fluid Velocities in Laminar Flow," by Ellis B. Pickett, thesis presented to the State University of Iowa, at Iowa City, in 1950, in partial fulfilment of the requirements for the degree of Master of Science.



(*a*) NAVIER-STOKES THEORY

(*b*) OBSERVED BY E. B. PICKETT

FIG. 15.—VELOCITY DISTRIBUTION FOR LAMINAR FLOW

It is reasonable to expect that velocity distributions that differ appreciably from geometrical similarity might still have nearly the same over-all rate of friction loss. This could explain the agreement shown in Fig. 2.

LORENZ G. STRAUB,[35] M. ASCE, EDWARD SILBERMAN,[36] AND HERBERT C. NELSON,[37] ASSOCIATE MEMBERS, ASCE.—The writers appreciate the comments of Messrs. Lansford and Robertson and of Mr. Posey, which have served to augment the paper.

With respect to the filament of maximum velocity in laminar flow, the measurements described in the paper do not permit making a positive statement. Because approximately 15 min were required for each pitot-tube reading, it was not feasible to search for the filament of maximum velocity. Velocities were obtained by the tube at no closer than 0.2 in. below the surface (approximately one-quarter of the depth) and at 0.25-in. intervals below this depth. Within these limitations no velocity was found that exceeded the surface velocity in laminar flow. On the other hand, for most turbulent-velocity measurements with the same limitations, a maximum was found beneath the surface, even at Reynolds numbers as low as 6,000. After the paper was published, another thesis was completed in which laminar-flow velocities were measured by an electrical conductivity method.[38] The closest readings beneath the surface were about 0.2 depth, and, again, no maximum was found. Therefore, it appears that the maximum velocity in laminar flow may be at the surface. The isovels in Fig. 15(*b*), presented in Mr. Posey's discussion, seem to indicate maximum velocity at the surface also.

---

[32] "Experience, Theory, and Experiment," by C. Truesdell, *Bulletin 36*, State Univ. of Iowa, Iowa City, Iowa, 1956, p. 3.

[33] "Hydrodynamics of non-Newtonian Fluids," by R. S. Rivlin, *Nature* (London), Vol. 160, 1947, p. 611.

[34] "Overdetermination of the Speed in Rectilinear Motion of non-Newtonian Fluids," by J. L. Erickson, *Quarterly of Applied Mathematics*, Vol. 14, 1956, p. 318.

[35] Director, St. Anthony Falls Hydr. Lab., and Prof. and Head, Dept. of Civ. Eng., Univ. of Minnesota, Minneapolis, Minn.

[36] Prof., St. Anthony Falls Hydr. Lab., Univ. of Minnesota, Minneapolis, Minn.

[37] Hydr. Engr., Research and Development, Kimberly-Clark Corp., Neenah, Wis.

[38] "Laminar Flow Velocity Distribution in a 120° Triangular Channel," by H. Howe, thesis presented to the University of Minnesota, at Minneapolis, in 1956, in partial fulfilment of the requirements for the degree of Master of Science.

The conclusions regarding the applicability of laminar-flow theory to flow in open channels were intended to apply mainly to the friction factor and to wall shear stress (conclusions 1 and 3). Mr. Posey is correct in stating that the predicted velocity profiles are not accurately comparable with theory at a distance from the wall. It is believed that the discrepancies are surface effects, probably attributable to surface tension. There is every reason to believe that the water and kerosene used in the experiments described in the paper behaved as Newtonian fluids, and that the discrepancies are not explainable by use of differential equations based on non-Newtonian fluids. If Mr. Posey had included mean-flow data with Fig. 15(*b*) from Pickett's measurements, a better comparison could have been made with the theoretical results given graphically in Fig. 15(*a*) and analytically in Appendix I.

TABLE 3.—LAMINAR WALL SHEAR DISTRIBUTION FOR A 90° TRIANGULAR CHANNEL

$\frac{\zeta}{1.414\,d}$	$\frac{\tau_0}{\bar{\tau}_0}$	$\frac{\zeta}{1.414\,d}$	$\frac{\tau_0}{\bar{\tau}_0}$
0.05	0.424[a]	0.25	1.135[a]
0.10	0.700[a]	0.30	1.210[a]
0.15	0.868[a]	0.40	1.323[a]
0.20	1.014[a]	0.50	1.343[a]

[a] Symmetrical about $\frac{\zeta}{1.414\,d} = 0.5$.

In connection with Fig. 14 of Messrs. Lansford and Robertson, Table 3 shows the laminar wall shear distribution for the same channel taken from Appendix I.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2936

PILE TESTS, LOW-SILL STRUCTURE, OLD RIVER, LOUISIANA

By Charles I. Mansur,[1] M. ASCE, and Robert I. Kaufman,[2] J. M. ASCE

With Discussion by Messrs. Yoshichika Nishida; Stanley F. Gizienski; and Charles I. Mansur and Robert I. Kaufman

Synopsis

A comprehensive pile testing program was undertaken at the site for the low-sill structure for control of the Old River, Louisiana. The object of the program was to determine the required type, size, and length of piles necessary to carry the design compression and tension loading without any significant movement of the structure. Information regarding the driving of displacement and nondisplacement types of piles was also desired. Fourteen-inch **H**-piles and pipe piles ranging from 16 in. to 20 in. in diameter were driven and tested. The details of the loading arrangement and test procedures are described. Because from 50 ft to 60 ft of alternating strata of silty sands, sandy silts, and clay overlie the sand beneath the structure, both the total bearing capacity and the bearing capacity of only the section of the piles penetrating into the sand were measured. The load carried by that section was computed from strain measurements that were made on rods attached at different points along the pile. After the compression tests were completed, the piles were allowed to rest and tension tests were performed on all piles but one.

From the pile-load tests, either 20-in. steel-pipe piles, 20-in. precast-concrete piles, or 14-in., 73-lb-per-ft, steel **H**-beam piles, with respective penetrations of 15 ft, 12 ft, and 27 ft into sand, were considered satisfactory for carry-

---

Note.—Published, essentially as printed here, in October, 1956, in the Journal of the Soil Mechanics and Foundations Division, as *Proceedings Paper 1079*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Vice-Pres. and Chf. Engr., Independent Wellpoint Corp., Baton Rouge, La.; formerly Chief, Geology, Soils, and Materials Branch, Mississippi River Comm., Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss.

[2] Chief, Geology, Soils, and Materials Branch, Mississippi River Comm.; formerly Chief, Design and Analytical Section, Embankment and Foundation Branch, Soils Div., Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss.

ing the design load of 100 tons in compression and 40 tons in tension, with an ample factor of safety against both detrimental settlement and sudden plunging. The estimated average unit skin friction in the silts was 0.64 ton per sq ft for the piles tested in compression and 0.26 ton per sq ft for the piles tested in tension. The average angle of internal friction of the sand, which was computed from bearing-capacity formulas and the maximum load carried in the pile tip, was 33°. The strain-rod installation proved to be satisfactory and reliable in determining the distribution of applied load in the silt and sand strata.

INTRODUCTION

The soil conditions at the site for the low-sill structure necessitated the use of long piles, which were driven through alternating strata of silty sand, sandy silt, and clay into sand at a depth of from approximately 50 ft to 60 ft. Because the pile foundation for the structure would be required to carry great loads in compression and tension and because there could be little movement, a series of pile-loading tests was performed to determine the type, size, and length of piles required to carry the design loads without any significant movement. Information was also desired regarding the driving resistance of both the displacement and nondisplacement types of piles. It was considered that the pile-load tests should be conducted in a test excavation, which would relieve the overburden pressure and skin-friction effects that would exist if the piles were tested at the natural ground surface. The procedure that was used in making the tests, the types of piles tested, and the results and conclusions obtained are summarized herein.

DESCRIPTION OF STRUCTURE

The low-sill structure, which is a controlled spillway, is a principal feature of the Old River (Louisiana) control project, which was planned for the control of the flow of the Mississippi River into the Atchafalaya River. The structure is located on the west bank of the Mississippi River at a site that is approximately 35 miles south of Natchez, Miss. It is of reinforced concrete with vertical lift steel gates; its gross length is 566 ft between the faces of the abutment training walls; and it has eleven gate bays. Four bays on each end of the structure have a weir-crest elevation of 10 ft mean sea level (msl). The three central bays of the structure have a weir crest of −5 ft (msl).

The gated section of the structure and the abutment piers are founded on piles to insure the stability of the structure with respect to horizontal sliding and to prevent any detrimental settlement. For the design of the pile foundation, it was assumed that the piles would be capable of carrying design loads of 100 tons in compression and 40 tons in tension. Approximately 75% of the piles will be on a 2-on-1 batter; the remainder will be vertical.

FOUNDATION CONDITIONS

The low-sill structure and pile test site are located in an abandoned river channel of the Mississippi River. An aerial photograph of the pile test site in relation to the Mississippi River is shown in Fig. 1. The soils beneath the low-sill structure consist of alternating strata of silts, sandy silts, and silty sands, with some interspersed clay strata for a total thickness of from about 50 ft to 60 ft. Clean sands that are from 40 ft to 60 ft thick lie beneath the silty soils. The sands are in turn underlain by stiff Tertiary clays. On the basis of split-spoon borings made from the natural ground surface, penetration resistances obtained in the silty-sand strata below El. −5 vary from 25 blows per ft to more than 50 blows per ft. Below approximately El. −40, the split-spoon resistance varies from 50 blows per ft to more than 100 blows per ft. After 50 ft of overburden had been removed by excavation for the pile tests, the split-spoon resistances were only approximately one-half of those given in the foregoing.

PILE TEST EXCAVATION

FIG. 1.—PILE TEST SITE

It was not possible to conduct the tests along the axis of the structure because preload fills were to be constructed at the abutments. Such fills would have affected the stability of the test excavation slopes as well as the stresses at the bottom of the foundation. Therefore, a site was selected between the river and the structure, where the soil conditions were similar to those along the axis of the weir of the structure. Logs of split-spoon borings made at the test site are shown in Fig. 2. The numbers at the left of the borings in Fig. 2 are natural-water contents expressed in percentage of dry weight. The borings are classified in accordance with the unified soil classification system used by the Corps of Engineers, United States Department of the Army. Boring PT-1 was made in August, 1954, prior to excavation in order to determine whether the soils at the test site (El. 48) were similar to those at the site for the structure. Boring PT-1A was made in February, 1955, in the bottom of the pile test excavation (El. 0) to establish the effect of removing 50 ft of overburden on split-spoon driving resistances. The driving resistances (blows per foot) were determined with a standard split-spoon sampler (inside diameter equaling $1\frac{3}{8}$ in. and outside diameter equaling 2 in.) and a 140-lb hammer dropped 30 in.

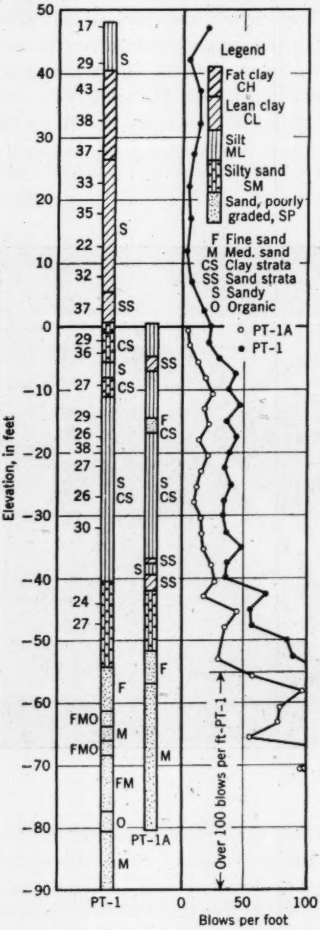


FIG. 2.—SPLIT-SPOON BORING DATA

LABORATORY TESTS

The shear strengths of the silts and sands that underlie the proposed structure were determined to estimate initially the bearing capacity of the piles. Laboratory tests on undisturbed samples taken from beneath the proposed structure indicated an average shear strength of $c = 0.1$ ton per sq ft for the silts and sandy silts, and a corresponding angle of friction, ϕ, equal to approximately 28°. The shear strength was selected from the results of consolidated-undrained triaxial tests and consolidated-drained direct shear tests. Consolidated-drained triaxial tests on both undisturbed and remolded samples indicated an average value of ϕ of approximately 36° for the sands.

Consolidation tests on representative samples of the silty stratum below the structure indicated that there would be significant settlement of the structure if the piles were founded in this stratum. Clay seams beneath the structure were even more compressible. This conclusion was substantiated by the 5-in. settlement of a plug installed in the silts below the proposed structure under the 30-ft-high preload fill for the south abutment. A rebound plug installed beneath the pile test excavation prior to excavation also showed that from 2 in. to 4 in. of rebound could be expected upon excavation.

PILE TEST PROGRAM

Test Program.—The seven piles tested are described in Table 1 and in Fig. 3. Although it would have been desirable to have driven and tested some piles on

TABLE I.—TEST PILE DATA

Pile no.	Type of test[a]	PILE DESCRIPTION			Length, in feet
		Size, in inches	Type	Weight in pounds per foot	
1	C	14	H-beam	73	81
2	CT	21	pipe	...	65
3	CT	14	H-beam with bottom plate	73	71
4	CT	17	pipe	...	66
5	CT	17	pipe	...	45
6	CT	19	pipe	...	65
7	C	18	pipe filled with concrete	...	65

[a] C indicates compression test; T indicates tension test.

a batter, loading difficulties and the extra cost of testing such piles outweighed the benefits. It was also believed that the load-carrying capacity of a batter pile would not be appreciably less than that of a vertical pile driven to the same depth. Experience at the Morganza (La.) Floodway control structure had also indicated that long, precast-concrete piles on the same batter (2 on 1) could be driven satisfactorily through a clay stratum that offered approximately the same resistance to driving as the silty soils at the low-sill structure site.[3]

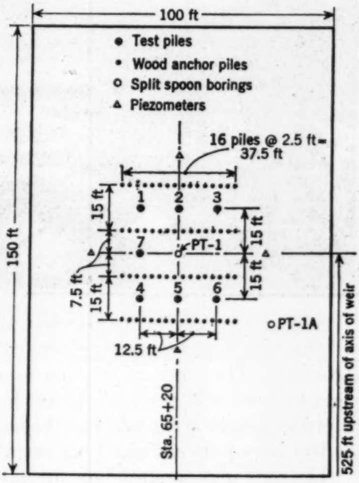


FIG. 3.—PLAN OF PILE TESTS

Excavation for Tests.—The pile-load tests were conducted in an excavation in order to relieve the overburden pressure and skin-friction effects that would have existed had the piles been tested at the ground surface. Fig. 4 shows some of the piles loaded in the test excavation. The bottom of the excavation was at El. 0 and had an inside clear area of 100 ft by 150 ft. The hydrostatic pressure in the deep sands below the pile test excavation was controlled by deep wellpoints, so that an uplift pressure was created at the top of the bearing sand that was approximately equivalent to that which will exist beneath the structure. During the tests the hydrostatic head in the deep sands was kept between El. 0 and El. 5. The effective surcharge at the top of sand during the driving of the piles and testing was approximately 2,500 lb per sq ft.

---

[3] "Review of Soils and Foundation Design and Field Observations, Morganza Floodway Control Structure," *Technical Memorandum No. 3-384*, Waterways Experiment Station, Vicksburg, Miss., August, 1954.

Types of Piles and Strain-Rod Installation.—The layout of the test piles and the anchor piles in the test excavation is shown in Fig. 3. Pile 1 and pile 3 were 14-in. H-beams; piles 2, 4, 5, and 6 were empty-pipe piles with a ¾-in-thick bottom plate. Pile 3 was also equipped with a square bottom plate that was ¾ in. thick. Pile 7 was a pipe pile and was filled with concrete.

Because the quantity of differential settlement that can be tolerated by the low-sill structure is small, it was essential that the pile foundations would not only provide an adequate factor of safety with respect to the failure of the piles by plunging, but also that it would insure that there would be no significant settlement of the structure over a long period of time. Considerable settlement could be expected if the carrying capacity of the pile tips in the sand was exceeded because of the compressibility of the silty soils beneath the structure. Such settlements along the structure would not be uniform because of the variable nature of the silty soils and the presence of random clay strata in the silty materials. Therefore, it was necessary to determine the load on a pile that could be carried in the underlying clean sand. In order to determine the distribution of load in the pile and the load carried in the sand during the tests, all the test piles were equipped with strain rods except pile 7.

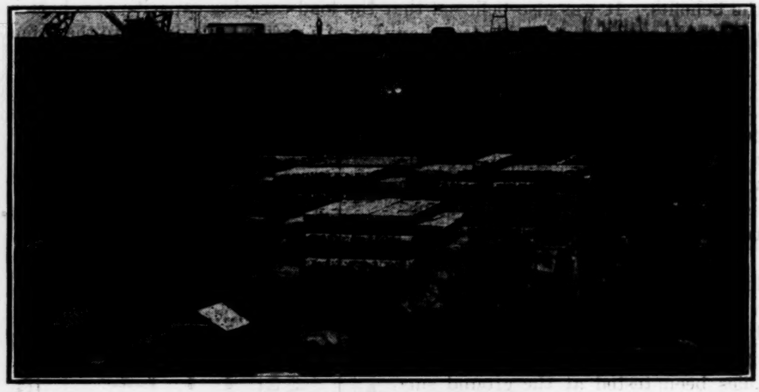


FIG. 4.—GENERAL VIEW OF LOADING TESTS

The rods were mounted on the outside of the pipe piles and were protected by 4-in. channels, which were welded continuously along opposite sides of the pile. Details of these rods are shown in Fig. 5. Six were installed on each pile except for pile 5 and pile 7. Only 5 rods were installed on the former pile because of its short length, and none were installed on pile 7, which was filled with concrete before driving. The channels resulted in an increase in the effective diameter of approximately 1 in. more than the actual outside diameter of the steel pipe. For that reason the pipe piles with actual outside diameters of 16 in., 18 in., and 20 in. had effective diameters of approximately 17 in., 19 in., and 21 in., respectively. Strain rods were attached to the web of H-beam piles

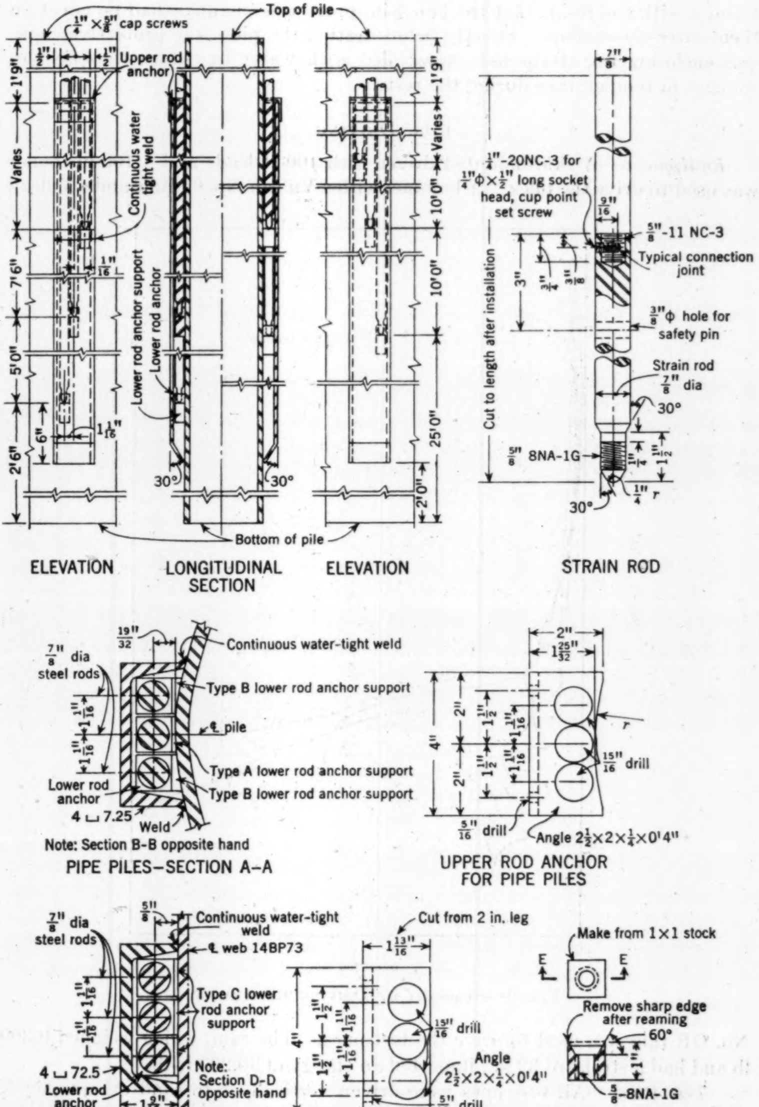

Top of pile
$\frac{1}{4}$" × $\frac{5}{8}$" cap screws
$\frac{1}{2}$"
$\frac{1}{2}$"
Upper rod anchor
1'9"
Varies
Continuous water tight weld
$\frac{1}{16}$"
7'6"
5'0"
Lower rod anchor support
Lower rod anchor
6"
$1\frac{1}{16}$"
2'6"
30°
30°
1'9"
Varies
10'0"
10'0"
25'0"
2'0"
Bottom of pile
ELEVATION
LONGITUDINAL SECTION
ELEVATION
$\frac{7}{8}$"
$\frac{1}{4}$"-20NC-3 for $\frac{1}{4}$"φ × $\frac{1}{2}$" long socket head, cup point set screw
$\frac{9}{16}$"
$\frac{5}{8}$"-11 NC-3
Typical connection joint
3"
$\frac{3}{4}$"
$\frac{3}{8}$"
$\frac{3}{8}$"φ hole for safety pin
Cut to length after installation
Strain rod $\frac{7}{8}$" dia
30°
$\frac{5}{8}$" 8NA-1G
1"
$1\frac{1}{2}$"
$\frac{1}{4}$" r
30°
STRAIN ROD
$\frac{19}{32}$"
Continuous water-tight weld
$\frac{7}{8}$" dia steel rods
Type B lower rod anchor support
$1\frac{1}{16}$"
℄ pile
$1\frac{1}{16}$"
$1\frac{1}{16}$"
Type A lower rod anchor support
Type B lower rod anchor support
Lower rod anchor
4 ⊔ 7.25
Weld
Note: Section B-B opposite hand
PIPE PILES–SECTION A–A
2"
$1\frac{25}{32}$"
2"
4"
2"
$1\frac{1}{2}$"
$1\frac{1}{16}$"
$1\frac{1}{2}$"
$1\frac{1}{16}$"
r
$\frac{15}{16}$" drill
$\frac{5}{16}$" drill
Angle $2\frac{1}{2}$×2×$\frac{1}{4}$×0'4"
UPPER ROD ANCHOR FOR PIPE PILES
$\frac{5}{8}$"
$\frac{7}{8}$" dia steel rods
Continuous water-tight weld
℄ web 14BP73
$1\frac{1}{16}$"
$1\frac{1}{16}$"
Type C lower rod anchor support
4 ⊔ 72.5
Lower rod anchor
$1\frac{9}{16}$"
Note: Section D-D opposite hand
H–PILES-SECTION C–C
Cut from 2 in. leg
$1\frac{13}{16}$"
4"
$1\frac{1}{2}$"
$1\frac{1}{16}$"
$1\frac{1}{16}$"
$1\frac{1}{2}$"
$\frac{15}{16}$" drill
Angle $2\frac{1}{2}$×2×$\frac{1}{4}$×0'4"
$\frac{5}{16}$" drill
UPPER ROD ANCHOR FOR H-PILES
Make from 1×1 stock
E
E
Remove sharp edge after reaming
60°
1"
$\frac{5}{8}$-8NA-1G
LOWER ROD ANCHOR SECTION E–E

FIG. 5.—DETAILS OF PILES AND STRAIN RODS

1 and 3 with the result that the two 4-in. protecting channels had no effect on their outer dimensions. Shortly before testing the piles, the protective channels enclosing the strain rods were filled with water to reduce the effect of changes in temperature during the tests.

PILE DRIVING

Equipment.—A skid-mounted driver with 100-ft leads and a 100-hp boiler was used to drive the piles. A hammer with a Vulcan No. 0 frame and a Vulcan No. OR ram was used to drive the test piles. The ram, which weighed 9,300 lb and had a stroke of 39 in., imparted an energy of 30,200 ft-lb.

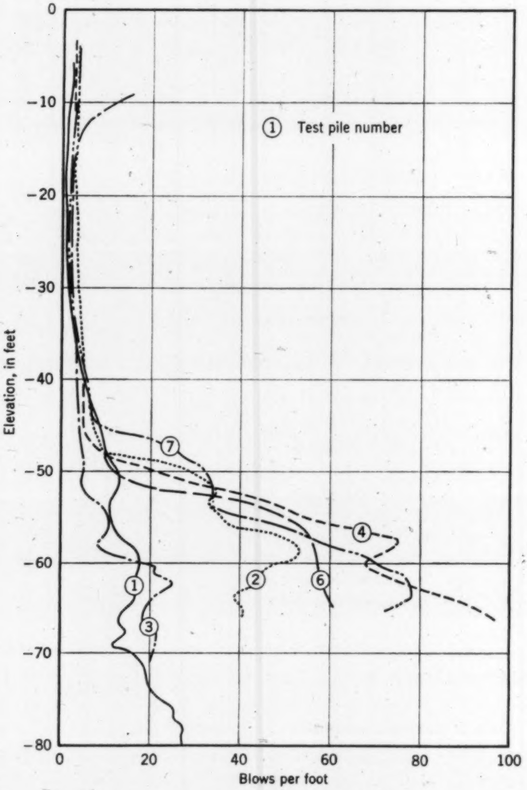


FIG. 6.—SUMMARY OF DRIVING RESISTANCE

Test Piles.—All test piles were driven vertically into sand except pile 5, which was stopped in the silty material overlying the clean sand. The average penetration of the piles in sand was from approximately 15 ft to 18 ft except for pile 1, which penetrated to a depth of 32 ft.

A summary of the driving resistances for each pile, except pile 5, is shown in Fig. 6. All the piles were driven with a Vulcan No. OR ram. The driving resistances were generally less than had been anticipated on the basis of the split-spoon driving resistances. The removal of 50 ft of overburden reduced the split-spoon driving resistance considerably and, presumably, reduced the pile-driving resistances. The driving resistances in the silty soils above approximately El. – 48 generally did not exceed from 5 blows per ft to 10 blows per ft. The top of the clean sand at approximately El. – 50 was marked by a sharp increase in the driving resistance of pipe piles, although only a minor increase in resistance occurred when the H-beam piles penetrated the clean sand. The final resistance for driving the piles the last 6 in. into sand ranged from 20 blows per ft for pile 3 to 96 blows per ft for pile 4. The final driving resistance of pile 5, which was founded in the overlying silt, was only 3 blows per ft.

LOADING AND MEASURING EQUIPMENT

Each loading installation consisted of a specially built jack pedestal, which was mounted on top of the test pile; either one or two hydraulic-jack rams mounted on the pedestal; and a platform to jack against, which was loaded with concrete weights supported on twelve timber anchor piles. A steel plate, which was 1 in. thick, was welded to the top of each test pile, and a thin layer of plaster of paris was used between these plates and the jack pedestals to level the tops of the pedestals. Pieces of plywood, which were ½ in. thick, were used at the top of the hydraulic jacks to provide uniform contact. The hydraulic jacks were of 200-ton capacity and 300-ton capacity. Hydraulic-jack pumps were located approximately 40 ft from the test piles for safety and were connected to their respective jacks with ¼-in. high-pressure steel pipe and copper tubing, which had an inside diameter of ¼ in. When two jacks were used on one pile, they were connected together with one pipe to the pump. Calibrated Bourdon pressure gages, which were connected to the high-pressure pipeline at the jack pump. were used to measure the pressure on the jacks.

The loading platform consisted of six 21-in., 62-lb I-beams, which were 16.5 ft long and were placed center to center with two 30-in., 132-lb I-beams which were 14 ft long. The latter were placed across, and welded to, the 21-in. I-beams symmetrically about the center of the platform and were placed 16 in. or 18 in. apart. For the compression tests the hydraulic jack or jacks operated against the two 30-in. I-beams beneath the platform. The anchor-pile caps consisted of 12-in.-by-12-in. timbers, which were approximately 14 ft long with 12-in.-square steel plates, which were 1½ in. thick, placed between the steel I-beams and the timber caps.

The platform was inverted as a unit for the tension test, and the hydraulic jacks were placed on top of the 30-in. I-beams.

Specially built frames, shown in Fig. 7, were mounted on each side of the piles to support the dials bearing on top of the strain rods. These frames were bolted to lugs welded to the piles. Long slots in the sides of the frames permitted adjustment of the dials over their respective rods. Dial gages reading direct to 0.001 in. were used on each rod. In addition to the dial frames, reference plates, as shown beneath the vertical deflection gage in Fig. 7, were bolted to the pile lugs. These plates served as contact surfaces for the stems of the dial gages used to measure the vertical movement of the piles. Two dial gages reading direct to 0.001 in. were used on each pile tested to measure the vertical movements of the tops. The gages were attached separately with adjustable supports to a reference beam so that their stems rested on the reference plates at diametrically opposed points. The dial supports were fastened to the reference beam with C-clamps, and the arms of the support were made equal in length and as short as practicable. The reference beams shown in Fig. 7 consisted of a 4-in. channel, which was 16.5 ft long and reinforced with a slightly shorter 6-in. channel, which was welded to the web of the 4-in. channel. The ends of the reference beam were supported by 4-in-.by-4-in. H-beams, which were driven approximately 11 ft into the ground. One end was fixed, and the other end rested on a smooth, rounded edge to permit contraction and expansion under varying temperature. The reference beam was kept shaded during the load tests. The beams were located as close as practicable to the test pile (approximately 1½ in. distant), and the anchors at the ends of the beams were placed as far as possible from the timber anchor piles (approximately 5 ft).

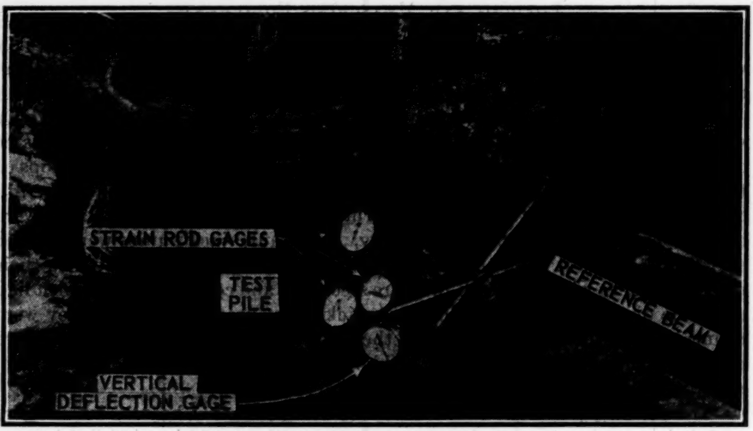


FIG. 7.—TYPICAL ARRANGEMENT OF REFERENCE BEAM, VERTICAL DEFLECTION GAGES, AND STRAIN-ROD GAGES

In order to check the settlement of the piles and to determine whether the reference beams moved during the tests, periodic readings were taken with an engineer's level on level rods, which were attached to each pile and at each end of the reference beam. A separate level was used for each pile being tested, and the level tripods were set in 1½-in. ID pipes, which were 3 ft long and had been driven so that they were flush with the ground. The elevations of the evels were checked periodically during the testing with readings taken on a benchmark, which was located in the excavation and consisted of a pipe that was driven into the sand stratum below El. − 55.

COMPRESSION TESTS

Loading Procedure and Observation of Pile Movement.—Concrete weights, from approximately 5 tons to 7.5 tons each, were used as ballast on the platform and were placed prior to loading the piles. The platform load was placed in two increments for pile 2 and pile 4 in order to prevent an excessive increase in effective surcharge at the test pile tip resulting from the weight of the loaded platform on the anchor piles. The first load increment was placed on the platform before the testing commenced, and the second increment was added during the test at a time when no load was on the pile.

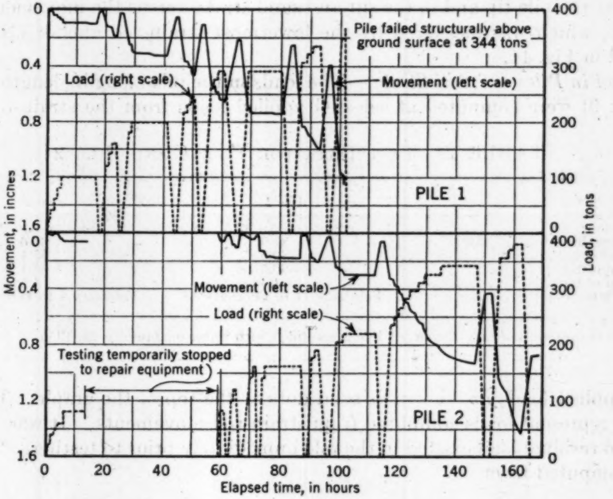


FIG. 8.—COMPRESSION LOADING DATA FOR PILE 1 AND PILE 2

The piles were loaded in compression from approximately two weeks to three weeks after they had been driven. Loads were applied in 20-ton increments during the first part of the test and in 15-ton increments during the last part. Each increment of load was intended to remain on the pile for 12 hr, or until the observed movement of the pile was less than 0.010 in. per hr, whichever occurred first. However, it was noted during the first loading period that from approximately 80% to 90% of the movement occurred as the load was being placed on the pile. Therefore, it was decided to have each subsequent load increment act on the pile for a period of only 1 hr. In order to establish the elastic and net load-movement curves for each pile, the load was removed at intermediate loads and the pile was allowed to rebound.

Continuous time-movement records of the pile butt, as determined from the average reading of two dial gages that were attached to the reference beam, were kept for each pile. Similar time-movement records were kept for all strain rods. At periodic intervals the settlement of the pile butt was also measured by an engineer's level.

Test Results.—Continuous records of pile load and movement versus elapsed time during compression tests on pile 1 and pile 2 are shown in Fig. 8. Time-movement curves indicate that for all test loads the movement was very rapid. Additional data concerning pile 1 and pile 2 are given in Table 2.

The compression test data for piles 1 through 6 are shown in Figs. 9, 10, and 11. Fig. 9 shows the location of the strain-rod anchors and the distribution of the applied butt load along the pile based on strain-rod observations. Plots of gross settlement and net settlement versus applied load are shown in Fig. 10. Loads at the pile tip and in the silt and sand strata versus the movement of the pile tip, which was measured by the lowermost strain-rod anchor (No. 1), is plotted in Fig. 11.

Load in Piles and Soil Strata.—The loads in the pile along its length (shown Fig. 9) were computed at selected applied loads from the strain-rod data.

TABLE 2.—Test Data for Pile 1 and Pile 2

	Pile 1	Pile 2[b]
Pile location[a]	Station 65 +07.5	Station 65 +20
Tested length, in feet	81.9	66.6
Imbedded length, in feet	80.5	65.1
Elevation of tip	−80.5	−65.1
Elevation of butt	1.4	1.5
Testing date (1955)	February 14 to February 18	February 4 to February 11

[a] 540 ft upstream of axis of weir. [b] Pile 2 was filled with water on January 26, 1955.

The applied loads are shown as solid dots at the top of the graph. The open circles represent loads computed from strain-rod movements. It was assumed that no residual load existed in the pile immediately prior to testing. The load was computed from

$$P = E\,A\,\frac{\Delta e}{\Delta L} \tag{1}$$

in which P denotes the average load in pounds between any two strain-rod anchors; E is the modulus of elasticity for steel (assumed to be 29×10^6 lb per sq in.); A represents the cross-sectional area of the steel pile, including channels, in square inches; Δe designates the difference in the observed strain, in inches, between adjacent strain-rod anchors; and ΔL is the distance between adjacent strain-rod anchors, in inches.

The accuracy of the loads computed from the strain-rod data is demonstrated by the favorable check of the computed loads near the top of the pile and the applied butt load. The load-distribution curves represent the load along the length of the pile for different applied loads at the top (solid dots). The slope of each curve at any depth is a measure of the rate at which the load

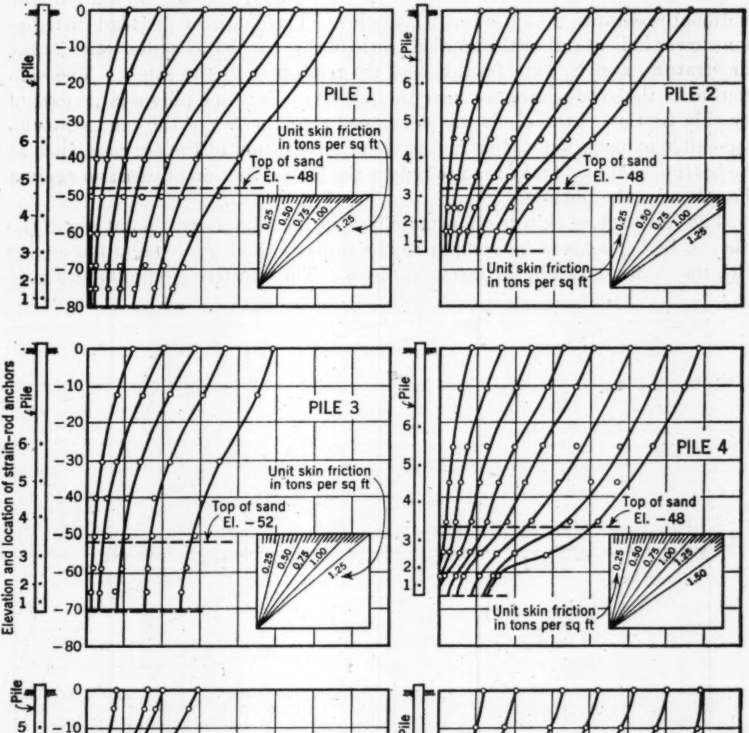


FIG. 9.—DISTRIBUTION OF LOAD IN PILES DURING COMPRESSION TESTS; INSERTS SHOW THE RELATIONSHIP BETWEEN THE SLOPES OF THE LOAD CURVES AND THE UNIT SKIN FRICTION IN THE SILT STRATUM

is transferred from the pile to the soil at that depth, and is a measure of the frictional resistance mobilized at that point. The shape of the load-distribution curves indicates that the applied pile loads are carried by skin friction in the silt stratum, and by skin friction and tip resistance in the sands. The skin friction in the sands decreases near the pile tip. For pipe piles a large part of the skin friction in the sands was developed near the top of the sand stratum. A possible explanation of this fact is that mobilization of the tip resistance at the bottom of the pile may tend to reduce the lateral friction of the sand against the pile near the bottom.

The quantity of skin friction mobilized in the silts at any point along the pile can be determined by comparing the slope of the load-distribution curves with the slopes shown in the insets in Fig. 9. The effective circumference used in determining the skin friction between the pile and the silty soils was taken as the minimum peripheral circumference of pile and soil for the H-beams and pipe piles with channels. It was considered that the shear strength of the silts would be less than the shear strength developed between the pile and the soil. The effective circumference used in determining the skin friction between the pile and the sands was also taken as the minimum peripheral circumference of the pile and soil for the H-beams. However, the actual circumference along the contact surface between the pile and the sand was used for the pipe piles with channels, because laboratory tests indicated that the coefficient of friction between sand and steel was less than the coefficient of friction between sand and sand. The effective circumference of the piles in the sands is slightly greater than that in the silts. Therefore, the skin friction in the sands is approximately 5% less than that indicated by the slope of the curves in the insets.

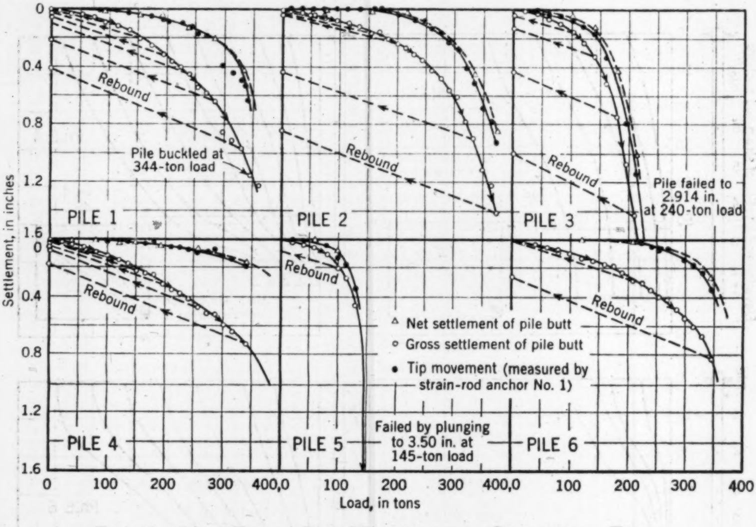


FIG. 10.—LOAD-VERSUS-PILE MOVEMENT FOR COMPRESSION TESTS

From the observed test data, a curve of gross movement of the pile butt versus the pile load was developed for each pile (Fig. 10). These curves were used in estimating the failure load of the piles. The net settlement of the pile is also shown on the same plots. This is the residual settlement that remained after the test load was removed from the pile, which was permitted to rebound. The difference between the gross and net settlements of the pile butt is the elastic compression of the pile, which, at any test load, was equal to the rebound that occurred upon release of the load. Also shown is the tip-movement curve of the pile, which was determined from observations on the bottom strain-rod anchor. Favorable agreement was obtained between the tip-movement curves and the net-settlement curves. The difference between the tip-movement curves and the net-settlement curves at a given butt load is believed to be the elastic compression in the pile resulting from locked-in stresses.

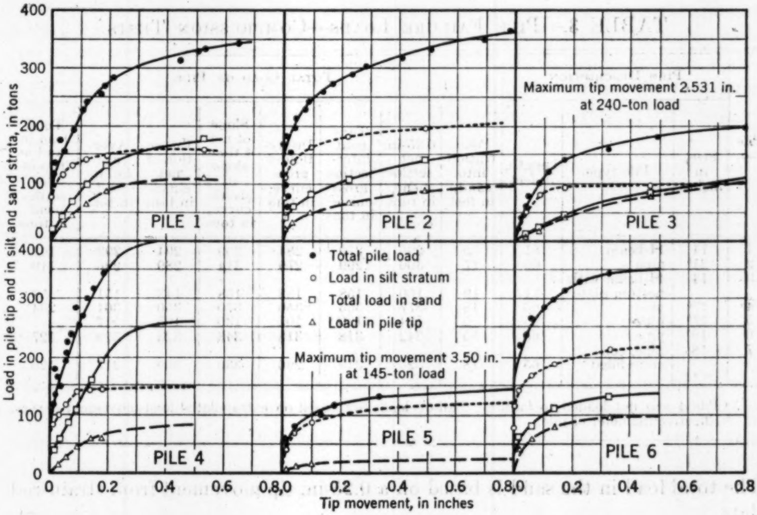


FIG. 11.—LOAD IN PILES AND SOIL STRATA VERSUS TIP MOVEMENT FOR COMPRESSION TESTS

From the curves in Fig. 9 and Fig. 10, load-versus-tip-movement curves were determined for the various soil strata and pile tip, as shown in Fig. 11. The loads carried by the pile tip and the sand stratum are closely related to the tip movement, as shown by the graphs in Fig. 11. The load-deformation characteristics of the silt stratum are less accurately represented by the relationship with the pile-tip movement because the tip movement is not representative of deformations in the silt stratum. However, the latter plot is also shown for comparison.

Pile Failure Loads.—Various procedures were used to obtain values for the pile failure loads, and an average was taken of the values for each pile. The criteria used to select failure loads were as follows:

a. The load that produced a plastic or net settlement of 0.25 in.;

b. The load indicated by the intersection of tangent lines drawn through the initial, flatter section of the gross-settlement curve into the steeper part of the same curve;

c. The load beyond which there was an increase in gross settlement disproportionate to the increase in load;

d. The load at which the slope of the plastic- or net-settlement curve was four times the slope of the elastic deformation curve; and

e. The load beyond which there was an increase in plastic or net settlement disproportionate to the increase in load.

The results of the analyses of each test are given in Table 3. Fig. 12 shows the failure loads plotted against the effective diameter of each test pile. The failure load of the pile is based on the average of five methods for computing this load.

TABLE 3.—PILE FAILURE LOADS—COMPRESSION TESTS

Pile no.	Pile Description			Total Load on Pile							
	Size, in inches	Pile type	Length, in feet	Penetration into sand, in feet	0.25-in. net settlement, in tons	Tangent intersection gross curve, in tons	Inspection of gross curve, in tons	Slope of net 4 times slope of elastic curve, in tons	Inspection of net curve, in tons	Average failure load, in tons	Total load in sand at 0.25-in. tip settlement, in tons
1	14	H-beam	81	32	296	279	284	309	294	292	142
2	21[b]	pipe	65	17	300	299	279	314	289	296	110
3	14	H-beam with bottom plate	71	19	160	158	138	158	143	151	58
4[a]	17[b]	pipe	66	18	377	365	350	370	350	361	234
5	17[b]	pipe	45	0	124	120	105	130	105	117	...
6	19[b]	pipe	65	15	342	318	318	343	323	329	127
7	18	pipe, concrete-filled	65	18	320	319	296	339	309	317	126

[a] Pile 4 was not loaded to failure. Failure loads are based on extrapolated load-movement curves.
[b] Effective diameter of pile.

The total load in the sand is based on a 0.25-in. tip movement from strain-rod data.

The load carried by the sand (point bearing and skin friction) at a pile-tip movement of 0.25 in. was selected as a maximum safe value for design purposes. From the load in sand versus tip-movement curves (Fig. 11), this load is approximately 80% of the maximum load. The maximum safe loads carried by the sand are shown in Table 3. The load in sand versus effective diameter of the pile is also plotted in Fig. 12.

Required Sizes and Lengths of Piles.—In order to establish the required size and length of pile to carry the design load of 100 tons in compression, curves of best fit were drawn through the plotted points for total failure load versus effective pile diameter, and total load in the sand versus effective pile diameter in Fig. 12. Except for pile 1 (14-in. H-beam), the piles penetrated from approximately 15 ft to 19 ft into bearing sand. On the basis of this penetration and the data plotted in Fig. 12, a 20-in. pipe pile penetrating 15 ft into bearing sand could carry the design load of 100 tons in sand with a safety factor of 1.35. Test pile 1 carried a total load of 142 tons in sand and penetrated 32 ft into the sand. Therefore, it was estimated that a 14-in. H-beam could carry the design load in sand with a 27-ft penetration. A 20-in. precast-concrete pile with a 12-ft penetration would have the same load-carrying capacity in sand as a 20-in. pipe pile with a 15-ft penetration. The shorter length for precast-concrete piles, as compared with the steel-pipe piles, is based on an estimated 40% greater friction between concrete and sand, compared with steel and sand, as obtained from laboratory shear tests.

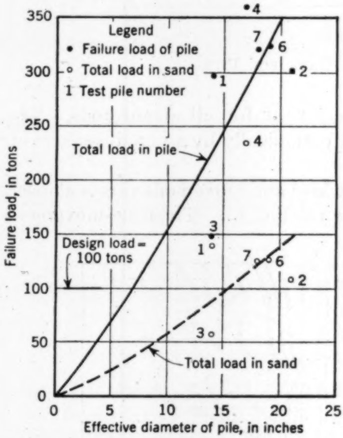


FIG. 12.—PILE FAILURE LOADS VERSUS PILE DIAMETER FOR COMPRESSION TESTS

Approximate analyses utilizing laboratory consolidation data for the silts and settlement observations within the silt stratum beneath one of the preload fills indicate that the silt stratum beneath the structure can carry approximately 20% of the design pile load without appreciable settlement over a long period of time. Therefore, the piles are considered to have a factor of safety of from 1.5 to 1.6 against detrimental settlement. Such a pile will have a safety factor of approximately 3.5 with respect to failure by plunging, based on mobilizing the full strength of the silts and sands.

On the basis of the load-versus-movement curves, no movement of the pile tip will occur as the load is applied. However, there will be minor elastic compression of the pile tip. As the 100-ton design compression load is eventually transferred from the silts to the sands as a result of consolidation of the silt, the tip of the pile will settle approximately 0.20 in. The latter value is significant because it indicates the magnitude of the probable settlement of the whole structure.

TENSION TESTS

Loading Procedure and Observation of Pile Movement.—Tension tests were performed on piles 2 through 6 from approximately two to three weeks after the piles were tested in compression. Loads were applied generally in 20-ton increments. The piles were tested either to failure or to a maximum tension load of 200 tons. Each load increment, except as otherwise indicated subsequently, was to remain on the pile until the rate of movement was less than 0.01 in. per hr, or for 12 hr, whichever period was of less duration; each increment was held for a minimum of 1 hr. At intermediate loads the load was removed from the pile, which was allowed to reach equilibrium. The rate of load application and removal was 1.0 ton per min.

Continuous time-movement records of the pile butt, which were determined from the average reading of two dial gages attached to the reference beam, were

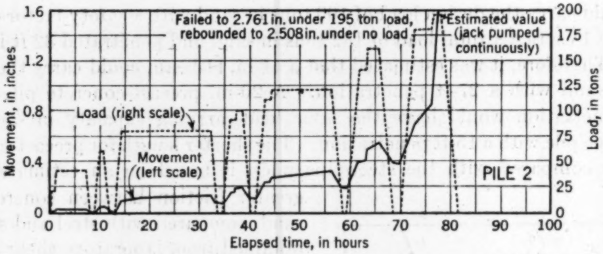


Fig. 13.—Tension Loading Data for Pile 2

kept during each test. Similar records were kept for all strain rods. The movement of the pile butt was also checked periodically by an engineer's level, as described for the compression tests.

Test Results.—A continuous record of pile load and movement versus elapsed time during the tension tests on pile 2 is shown in Fig. 13. The time-movement

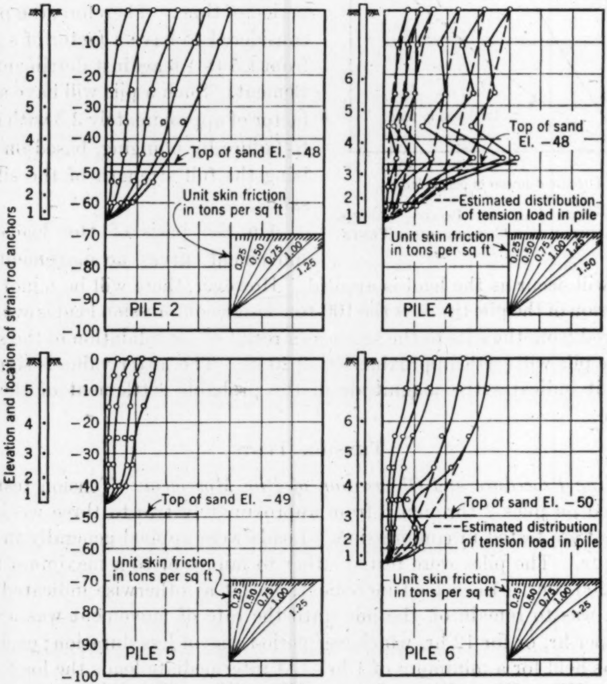


Fig. 14.—Distribution of Load in Piles During Tension Tests; Inserts Show the Relationship Between the Slopes of the Load Curves and the Unit Skin Friction in the Silt Stratum

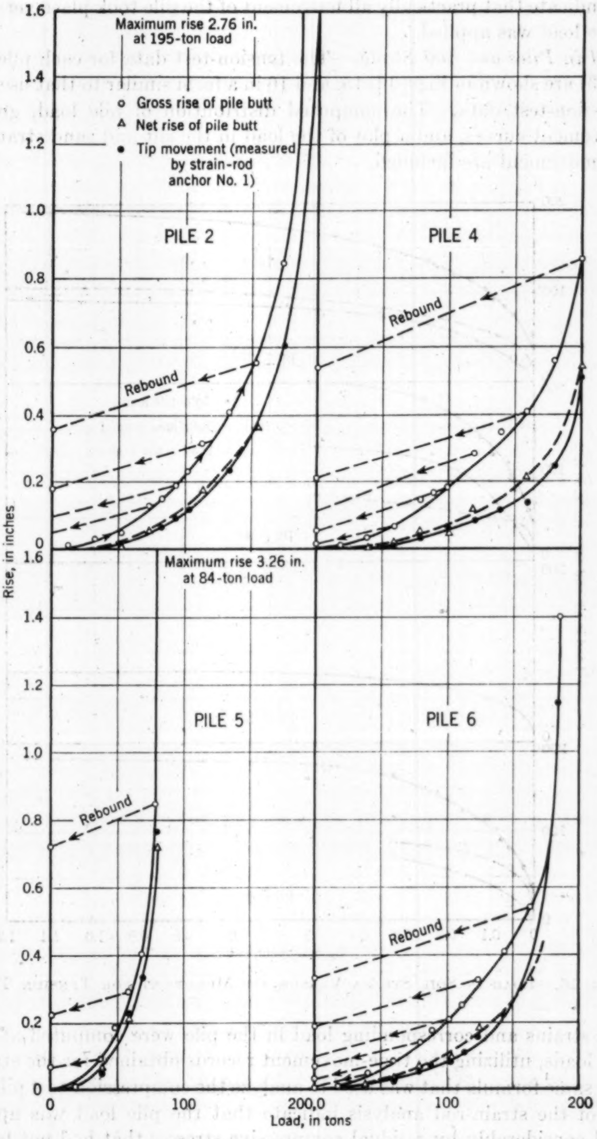


FIG. 15.—LOAD-VERSUS-PILE MOVEMENT FOR TENSION TESTS

curves indicate that practically all movement of the pile took place very rapidly after the load was applied.

Load in Piles and Soil Strata.—The tension-test data for each pile (except for pile 3) are shown in Figs. 14, 15, and 16 in a form similar to that used for the compression-test data. The computed distribution of pile load, gross- and net-movement curves, and a plot of the load in the silt and sand strata versus the tip movement are included.

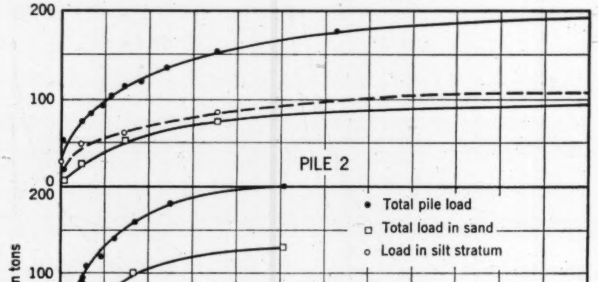


FIG. 16.—LOAD IN SOIL STRATA VERSUS TIP MOVEMENT FOR TENSION TESTS

The strains and corresponding load in the pile were computed at selected applied loads, utilizing the time-movement records obtained for the strain rods and the same formula that was used to analyze the compression-test piles. The results of the strain-rod analysis indicate that the pile load was apparently affected considerably by residual compressive stresses that had not been fully dissipated after the compression tests. Unusually large and often erratic tensile strains were manifested within the pile during the application of tension loads. Based on observed data, the computed pile loads are shown for various pile loads in Fig. 14. In general, scatter in the pile loads usually occurred near the top of the sand stratum, where the intensity of the load carried by the soil had been at a maximum during the compression tests. This scatter may result from residual compressive stresses in the pile and soil. In addition, it appears that the residual compressive stresses in the pile are less for those piles that failed by plunging than for those that were not tested to failure, because little or no scatter was present in the data for the former. However, the estimated distribution of load in the piles shown in Fig. 14 is believed to be reasonably accurate. The load-distribution curves indicate that the applied tension loads are carried by skin friction in both the silt and sand strata. The skin friction at any depth in the soil strata can be determined by comparing the slope of the load-distribution curves with the slopes shown in the inset. As in the compression tests, the skin friction in the sands is approximately 5% less than that indicated by the slope of the curves in the inset.

Gross-movement curves of the pile butt versus pile load are shown in Fig. 15 for the various piles. These curves were used to estimate the failure load of the piles in tension. The net rise of the pile butt is also shown on these graphs and is equal to the gross rise of the pile butt minus the elastic movement that resulted when the pile load was removed. Also shown for comparison on the same plot is the tip-movement curve, which was plotted from information that was obtained from observations on the bottom strain-rod anchor. The tip-movement curves and the net-rise curves are in agreement. As stated for the compression tests, the difference between the two curves at a given butt load is attributed to the elastic strain in the pile that results from locked-in stresses.

TABLE 4.—PILE FAILURE LOADS—TENSION TESTS

Pile no.	PILE DESCRIPTION			TOTAL LOAD ON PILE			Total load in sand at 0.25-in. tip movement, in tons
					Inspection of Curve		
	Size, in inches	Pile type	Length, in feet	0.25-in. gross rise, in tons	Gross curve, in tons	Net curve, in tons	
2	21[a]	pipe	65	106	135	135	75
3	14	H-beam with bottom plate	71	53	50	50	28
4	17[a]	pipe	66	116	160	162	115
5	17[a]	pipe	45	56	55	55	...
6	19[a]	pipe	65	112	135	140	65

[a] Effective diameter of pile.

Fig. 16 shows a plot of the loads in the silt and sand strata versus movement of the pile tip as determined from movement of the bottom strain-rod anchor. The curves represent approximately the load-deformation characteristics of the two strata. The tip movement is not representative of the deformation in the silt stratum.

Pile Failure Loads.—The failure loads of the piles in tension, which are shown in Table 4, were based on an inspection of the gross- and net-rise curves for the load beyond which there was an increase in movement that was disproportionate to the increase in load. A second condition of failure was based on limiting the gross rise to tolerable amounts. A gross rise of 0.25 in. was taken to be the limiting criterion for this condition. As shown in Table 4, the loads determined for the latter condition tend to be approximately equal to or less than the failure loads based on the net- and gross-rise curves. The failure loads versus effective diameter of each test pile are plotted in Fig. 17. The 20-in.-diameter pipe pile that was selected for the structure from the compression test had a safety factor of approximately 3 with respect to the ultimate capacity of the pile in tension. The other piles selected for the structure on the basis of compression-test data also have high factors of safety with respect to the design tension load, which was 40 tons. The total load in the sand is based on a 0.25-in. tip movement determined from strain-rod data.

The maximum tension loads for each pile that can be carried by the sands alone are also shown in Table 4 and were based on a tip movement of 0.25 in. as

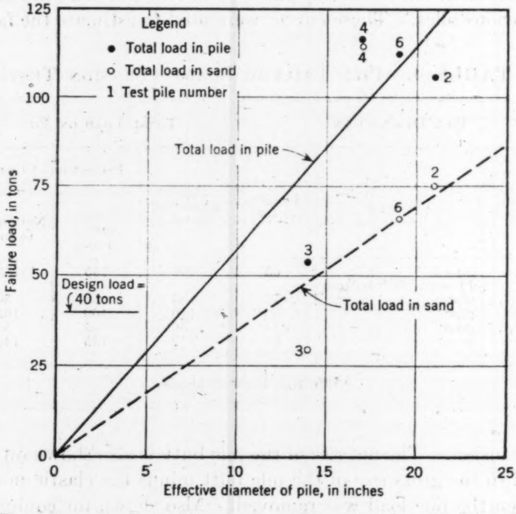


FIG. 17.—PILE FAILURE LOAD VERSUS PILE DIAMETER FOR TENSION TESTS

determined by the strain rods. The maximum tension loads versus the effective diameter of the pile are also plotted in Fig. 17. The sand alone is capable of carrying the design tension load with a safety factor equal to 1.75 for the proposed 20-in.-diameter pile. The factors of safety for the other proposed piles are considered adequate.

REQUIRED PILE–DRIVING RESISTANCE AND LENGTH

The capacity of the piles in compression was computed from dynamic pile-driving formulas (Eqs. 2, 3, and 4). The equations are for a single-acting hammer. Eq. 2 and Eq. 3 include a factor of safety of 6, and Eq. 4 includes a factor of safety of 1—

$$\text{(Engineering-News)} \quad R = \frac{2\,W\,h}{s + 0.1} \cdots\cdots\cdots\cdots\cdots (2)$$

$$\text{(Engineering-News Modification 1)} \quad R = \frac{2\,W\,h}{s + 0.1\,\dfrac{P}{W}} \cdots\cdots (3)$$

and

$$\text{(Pacific Coast Uniform Building Code)} \quad R_u = \frac{12\,W\,h\,\dfrac{W + K\,P}{W + P}}{s + \dfrac{12\,R_u\,L}{A\,E}} \ldots (4)$$

in which R is the allowable carrying capacity of the pile, in pounds; R_u is the ultimate carrying capacity of the pile, in pounds; W designates the weight of the falling mass, in pounds; h denotes the height of free fall of the ram, in feet; s represents the set of blow, in inches; P equals the weight of the driven pile, in pounds; L is the length of the pile, in feet; A represents the average cross-sectional area of the pile, in square inches; E denotes the modulus of elasticity of the pile material, in pounds per square inch; and K is a coefficient that equals 0.25 for steel piles and 0.10 for all other piles.

TABLE 5.—RELATIONSHIP BETWEEN PILE-DRIVING FORMULAS AND FAILURE LOADS

Pile no.	PILE DESCRIPTION			Failure load in sand, in tons[b]	Blows per foot (last 6 in.)	Failure set, in inches (last 6 in.)	ENGINEERING-NEWS		ENGINEERING-NEWS (MODIFICATION 1)		PACIFIC COAST UNIFORM BUILDING CODE	
	Size, in inches	Pile type	Length, in feet				R, in tons[c]	Ratio[d]	R, in tons[c]	Ratio[d]	R, in tons[e]	Ratio
1	14	H-beam	81	142	32	0.375	64	2.22	69	2.06	148	0.96
2	21[a]	pipe	65	110	40	0.30	76	1.45	83	1.33	170	0.65
3	14	H-beam with bottom plate	71	58	20	0.60	43	1.35	46	1.26	132	0.44
4	17[a]	pipe	66	234	96	0.125	134	1.75	172	1.36	183	1.28
5	17[a]	pipe	45	...	3	4.0	7	...	8	...	36	...
6	19[a]	pipe	65	127	64	0.187	105	1.21	123	1.03	181	0.70
7	18	pipe, concrete-filled	65	126	80	0.15	121	1.04	79	1.59	160	0.70
Average, excluding pile 3 and pile 5							...	1.51	...	1.44	...	0.86

[a] Effective diameter of pile. [b] Failure load based on 0.25-in. tip movement. [c] Formula capacities are allowable loads. [d] Ratio is the test failure load in sand divided by the formula capacity. [e] Formula capacities are ultimate loads for which a safety factor of 4 is recommended.

The pile-driving records indicate that the driving resistance in the silts was only a small fraction of that developed in the sands. Therefore, if any relationship existed between the test capacities and the capacities that were computed from dynamic pile-driving formulas, this relationship should be based on the failure load in sand. The driving resistance of the last full 6 in. of driving was used in each formula. These values are given in Table 5 with the computed capacity, the failure load in sand, and the ratio of the failure load in sand to the indicated formula capacity for each pile-driving formula. The failure load in sand is that which was carried by the sand stratum at the time the tip movement was 0.25 in.

As shown in Table 5, the failure loads in sand are approximately 50% greater than those computed from either of the *Engineering-News* formulas. The Pacific Coast Uniform Building Code formula, which correlated well with observed tip capacities for the Morganza Floodway control structure, indicated computed capacities for the compression test piles to be from 4% to 35% greater than the observed capacity in sand, and averaged approximately 15% greater. On the basis of these data, the Pacific Coast formula was used to estimate the required driving resistances for the proposed types and lengths of piles to be used.

Required Driving Resistances.—Required driving resistances were established for 20-in. pipe piles, 20-in. precast-concrete piles, and 14-in. H-beam piles from a consideration of (*a*) the pile capacities, which were computed by using the Pacific Coast formula (Eq. 4); (*b*) the relationship between the capacities computed from Eq. 4 and test failure loads (Table 5); and (*c*) the driving records of the test piles. The reduction in hammer efficiency due to inclination in driving the piles on a batter was also considered. Pile capacities were computed for a Vulcan OR hammer and were based on estimated average lengths of each type of pile.

TABLE 6.—UNIT SKIN FRICTION IN SILT FOR COMPRESSION TESTS

Pile no.	MINIMUM STRAIN IN SILTS AT 0.20 IN.		UNIT SKIN FRICTION		Average K-value for $\phi = 28°$
	Pile load, in tons	Silt stratum load, in tons	Average, in tons per square foot	Maximum, in tons per square foot	
1	240	140	0.62	0.70	1.56
2	260	172	0.65	0.73	1.58
3	140	93	0.38	0.52	0.90
4	320	143	0.66	1.08	1.60
5	118	118	0.49	0.65	1.29
6	302	190	0.76	0.94	1.78
Average, excluding pile 3	...	...	0.64	0.82	1.56

SHEAR STRENGTH OF SILT STRATA

Compression Tests.—The observed distribution of shear stress in the soil adjacent to the piles was not uniform. It decreased generally from a maximum at the top of the pile to a minimum at the tip. Furthermore, as the load on the pile was increased, the maximum shearing resistance of the silty soils adjacent to the pile moved progressively downward (Fig. 9).

The average unit skin friction of the silts was computed for each pile from the part of total load carried by the silt stratum, and the values are shown in Table 6. Also shown are the maximum values of unit skin friction, which were determined from the maximum slope of the load-distribution curves for the same pile load that was used to determine the average unit skin friction. The average and maximum unit skin frictions are appreciably lower for pile 3 than for the other piles. The skin friction in the silts was probably reduced for pile 3 (14-in. **H**-beam with bottom plate) because of a zone of loose material that is believed to exist between the flanges of the **H**-beam as a result of the plate on the bottom of the pile.

The observed distribution of unit skin friction in the silty soils at piles, 1, 2, 4, and 6 is shown in Fig. 18. The average unit skin friction and the unit skin friction computed from theoretical concepts and the design shearing strength

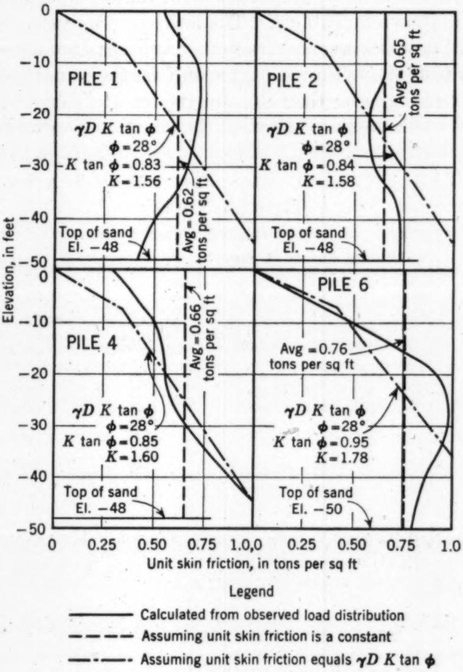


FIG. 18.—DISTRIBUTION OF SKIN FRICTION IN SILTS FOR COMPRESSION TESTS (NOTE: UNIT SKIN FRICTION COMPUTED FOR PILE LOAD CORRESPONDING TO 0.20-IN. MOVEMENT AT TOP OF SAND)

are also indicated. The unit skin friction was computed for a pile load corresponding to a movement at the top of the sand equal to 0.20 in.

The coefficient of earth pressure, *K*, at any depth in the silt stratum was computed from

$$s = \gamma\, D\, K \tan\phi \dots\dots\dots\dots\dots\dots \quad (5a)$$

or

$$K = \frac{s}{\gamma\, D \tan\phi} \dots\dots\dots\dots\dots\dots \quad (5b)$$

in which s represents the unit skin friction of the silts at depth D; γ is the effective unit weight of the silts based on piezometric data; K denotes the coefficient of earth pressure; and ϕ is the angle of internal friction and is equal to 28°. The average values of K that are shown in Fig. 18 and in Table 6 were computed for each pile from the corresponding average unit skin friction, using the average value of γD for the section of the pile embedded in the silty soils and a value of ϕ equal to 28°. The average K-value was approximately 1.6 for the compression tests.

Tension Tests.—Unit skin friction along the portion of the piles in the silt stratum and values of K were computed for the tension tests as for the compression tests and are shown in Table 7. The maximum unit skin friction averages approximately $2\frac{1}{2}$ times the average friction. A comparison with similar analyses for compression tests shows that the value of K for the compression tests was approximately $2\frac{1}{2}$ times greater than that for the tension tests. For compression loads the lateral earth pressure exceeded the effective overburden pressure and tended to approach a passive value. For tensile loads the lateral earth pressure was less than the effective overburden pressure and was approximately equal to an at-rest value.

TABLE 7.—UNIT SKIN FRICTION IN SILT FOR TENSION TESTS

Pile no.	MINIMUM STRAIN IN SILTS AT 0.20 IN.		UNIT SKIN FRICTION		Average K-value for $\phi = 28°$
	Pile load, in tons	Load in silt, in tons	Average, in tons per square foot	Maximum, in tons per square foot	
2	118	63	0.24	0.35	0.58
3	54	28	0.12	0.23	0.26
4	152	52	0.24	0.52	0.58
5	50	50	0.25	1.25	0.62
6	128	79	0.31	0.42	0.75
Average, excluding pile 3	...	...	0.26	0.62	0.63

SHEAR STRENGTH OF SAND STRATA

Compression Tests.—The load carried by the sand was divided into two parts: That carried by skin friction along the sides of the pile and that developed by compressive forces at the pile tip. As previously noted, the maximum skin friction in the sands appeared to be in the sands above the pile tip, with a lesser skin friction near the tip. For that reason a great difference may be expected between the average unit skin friction and the maximum unit skin friction that is developed by the sand. The average skin friction and the maximum skin friction corresponding to a tip movement of 0.25 in. are shown in Table 8 for the piles penetrating into sand. The average friction in the sands was determined from the relationship (Fig. 11) between the pile load that was carried by skin friction in the sand and the pile-tip movement. The maximum skin friction in the sands was determined from the maximum slope of the "distribution of load in pile" curves shown in Fig. 9. The computed values of K for the sand are also indicated in Table 8.

Both the average unit skin friction and the maximum unit skin friction for pile 3 (14-in. H-beam with bottom plate) were much less than the average of the group. A zone of loose material may have formed above the bottom plate, thereby reducing the shear strength of the sands. On the other hand, the skin friction in sand for pile 4 was considerably greater than the average skin friction and accounts for the large total failure load for this pile. Pile 4 also exhibited the greatest driving resistance, and it is probable that the large values of skin friction in the sand for this pile may reflect the presence of a dense pocket of sand, or possibly a gravel pocket with a large shearing resistance. Excluding the results from piles 3 and 4, the average value and the maximum value for the unit skin friction are 0.39 ton per sq ft and 0.70 ton per sq ft, respectively. The average K-value is 0.31, based on an average laboratory shear strength of $\phi = 36°$ for the sand. The maximum unit skin friction is approximately twice the average unit skin friction. A comparison between the average values of the friction in sand and the corresponding values for silt indicates that the average friction for silt was approximately 1.7 times the corresponding value for sand. The lesser friction in the sands may reflect the pattern of shearing failure beneath a pile tip, as suggested by Karl Terzaghi,[4] Hon. M. ASCE. Compressive forces at the pile tip result in a zone of radial shear beneath the pile tip, which causes a radial movement of the soil that will tend to reduce the lateral earth pressure and skin friction of the sand on the surface of the pile immediately above the tip. From the load-distribution curves, the skin friction appears to be reduced significantly for a distance of from 6 ft to 12 ft above the pile tip.

TABLE 8.—UNIT SKIN FRICTION IN SAND FOR COMPRESSION TESTS

Pile no.	UNIT SKIN FRICTION IN SAND		Average K-value for $\phi = 36°$
	Average, in tons per square foot	Maximum, in tons per square foot	
1	0.31	0.63	0.25
2	0.42	0.60	0.32
3	0.10	0.24	0.08
4	1.84	3.80	1.38
6	0.43	0.87	0.37
Average, excluding pile 3	0.75	1.47	0.58
Average, excluding piles 3 and 4	0.39	0.70	0.31

The tip-load-versus-tip-movement curves are shown in Fig. 11. The tip load corresponding to a 0.25-in. tip movement averages approximately 65% of the total load carried by the sand. The theoretical tip load of a pile penetrating into sand is given by Terzaghi's formulas:

$$\text{(circular piles)} \quad Q = \pi R^2 (\gamma D N_q + 0.6 \gamma R N_\gamma) \ldots\ldots\ldots\ldots (6)$$

---

[4] "Soil Mechanics in Engineering Practice," by Karl Terzaghi and Ralph B. Peck, John Wiley & Sons, Inc., New York, N. Y., 1948.

and

$$\text{(square piles)} \quad Q = 4\,R^2\,(\gamma\,D\,N_q + 0.8\,\gamma\,R\,N_\gamma) \ldots\ldots\ldots (7)$$

in which N_q and N_γ are bearing-capacity factors for a given angle of internal friction; $\gamma\,D$ is the effective surcharge at the pile tip; and R is the radius of a circular pile or one-half the width of the square pile. The terms $0.6\,\gamma\,R\,N_\gamma$ and $0.8\,\gamma\,R\,N_\gamma$ in Eq. 6 and Eq. 7 have little effect on the computed bearing capacity of the piles and were neglected. An attempt was made to evaluate the shearing strength of the sand by using Eq. 6 and Eq. 7. Using the effective surcharge at the pile tip and the tip capacity at a tip movement of 0.25 in., the values of the shear strength in terms of an angle of internal friction were computed for the sand and are listed in Table 9. The average ϕ-value is less than that of 36° obtained from the laboratory tests on the sands. However, the maximum tip resistance was developed at movements that were considerably greater than 0.25 in. The average computed angle of internal friction, based on the maximum tip resistance, was approximately 33°, which agrees closely with the laboratory value. The unit tip load for the H-beam piles was computed by using the combined areas of steel and soil enclosed by the pile flanges. The unit tip load of the pipe piles was computed by using the area of the pile tip plus the projected area of the channels enclosing the strain rods.

TABLE 9.—VALUES OF ANGLES OF INTERNAL FRICTION OF SAND

Pile no.	Tip capacity, in tons	Unit tip capacity, in tons per square foot	Effective surcharge, $\gamma\,D$, in pounds per square foot	Angle of internal friction, ϕ°
1	95	68	4,395	33
2	69	31	3,600	28
3	52	37	3,800	29
4	72	48	3,660	31
6	92	50	3,590	32
Average	...	...	...	31

TABLE 10.—UNIT SKIN FRICTION IN SAND FOR TENSION TESTS

Pile no.	UNIT SKIN FRICTION		Average K-value for $\phi = 36°$
	Average, in tons per square foot	Maximum, in tons per square foot	
2	0.63	1.25	0.56
3	0.31	0.75	0.26
4	1.32	2.20	1.08
6	0.83	1.70	0.74
Average, excluding pile 3	0.93	1.72	0.79
Average, excluding piles 3 and 4	0.73	1.48	0.65

Tension Tests.—In the tension tests all the pile load in the sand was carried by skin friction. The average skin friction in sand was computed from the load in the sand at a tip movement of 0.25 in., as shown by the curves in Fig. 16 and as listed in Table 10 for each pile. The maximum skin friction in the sand was determined from the slope of the "distribution of load in pile" curves shown in Fig. 14. The computed values of K for $\phi = 36°$ are shown in Table 10. Thus, the average skin friction of the sands in probably approximately 0.75 ton per sq ft, with a maximum value of about 1.5 tons per sq ft. Based on the average laboratory shear strength of $\phi = 36°$ for the sands, the coefficient of earth pressure, K, for piles in tension was about 0.7, or twice the earth-pressure coefficient computed on the basis of the compression tests. The lateral earth pressure in the sands during the compression tests was reduced as a result of the failure pattern below the pile tip. Favorable agreement for the tension tests exists between the earth-pressure coefficient obtained for the silts ($K = 0.63$) and that for the sands ($K = 0.7$).

CONCLUSIONS

On the basis of the tests, the following piles were considered to be satisfactory for carrying design loads of 100 tons in compression and 40 tons in tension:

Type	Penetration in bearing sand, in feet
20-in. steel pipe	15
20-in. precast concrete	12
14-in., 73-lb-per-ft, steel H-beam	27

The settlement of the structure on piles should be relatively small, consisting of an elastic compression of the piles during construction and an eventual settlement of the pile tips of approximately 0.20 in. The settlement will occur as the load is gradually transferred from the silts to the sand as a result of consolidation of the silts, and most of this settlement should occur during construction.

The estimated average unit skin friction in the silts is 0.64 ton per sq ft for the piles tested in compression and 0.26 ton per sq ft for those tested in tension. The estimated average unit skin friction in the sands was about 0.4 ton per sq ft for the piles tested in compression and 0.75 ton per sq ft for those tested in tension.

The average angle of internal friction of the sand computed from bearing-capacity formulas and the maximum load carried by the pile tip was 33°, which agrees favorably with the results that were obtained by laboratory shear tests.

The strain-rod installation proved to be a satisfactory and reliable method for determining the distribution of applied load in the silt and sand strata.

ACKNOWLEDGMENTS

The pile tests were planned by the Mississippi River Commission and Waterways Experiment Station at Vicksburg, Miss. The tests were conducted by the New Orleans District; the Waterways Experiment Station was responsible for analyzing the data. Walter C. Sherman, Jr., J.M. ASCE, assisted the writers in the analysis of the test results.

DISCUSSION

Yoshichika Nishida[5].—By computing the coefficient of earth pressure, K, from the observed values, the authors have shown that the value of K in the pile compression was different from the corresponding value in the tension tests. For this reason the distribution of the skin friction is different for compression than for tension because of the effect of the point resistance of the pile tip. Furthermore, it signifies that the skin-friction term in the bearing capacity of the pile should not be obtained from the pulling tension test of the pile. The skin-friction term and the toe term cannot therefore be independent of each other, although many computations by pile formulas are obtained with no relationship between the two terms. The writer believes that this fact is of great value.

The authors have stated that if K is constant, the skin friction, s, is proportional to the depth in a stratum, in order to simplify the situation. The writer believes that s is not simply proportional to the depth, as shown in Fig. 15, although the rigorous relationship between the skin friction and the lateral earth pressure is as yet unknown, except for some attempts by the writer. That the average value of unit skin friction in sand for the compression tests is approximately 1/1.7 times the corresponding value in silt is perhaps due to the effect of the pile tip in sand. The effect of the pile tip also results in the difference in the values of K between compression and tension for sand. However, the writer wishes to know why the difference exists in the unit skin friction obtained for the compression test and for the tension test in silt in spite of the distance from the pile tip. Is there no cohesion in silt? A little cohesion or moisture in sand or silt seems to have a considerable effect on the strength of soils. The pile in a silt stratum swells laterally due to the great deformation under compressive loads, as seen from the difference between the gross settlement and the net settlement in Fig. 7, because the pile stands on the sand stratum, which is firmer than the silt stratum. However, there is no swelling of the pile in the case of the tension tests. A greater swelling in the lateral direction of the pile causes a higher pressure in that direction, which explains the difference of K in silt between compression and tension. In sand the point resistance causes the tension stress near the tip, and it reduces the earth pressure, as has been explained. Therefore, the swelling of the pile, even if considered, has a much lesser effect.

Stanley F. Gizienski,[6] M. ASCE.—Although the pile tests described by the authors were made specifically to determine the type, size, and length of piles required to support a proposed low-sill structure, the results confirm several theoretical concepts regarding the distribution of loads along the length of a pile. The excavation of 50 ft of overburden at the test site, the driving of seven test piles and sixty-four wood anchor piles, the elaborate instrumentation, and the installation of well points all involved considerable expense. This was justified undoubtedly by the size and value of the project. The completeness with which these tests were made and analyzed and the fact that they are made available to engineers provide additional justification for such an expense.

---

[5] Instr. of Civ. Eng., Univ. of Kanazawa, Kanazawa-shi, Japan.

[6] Civ. Engr., Woodward, Clyde, Sherard & Associates, Oakland, Calif.

The authors have based their conclusions on the measurements of pile deformations at various depths for different load conditions. The soil conditions around and below the test piles as installed are assumed to be the same as they will be for the actual structure. This assumption will not be true if the closely spaced wood anchor piles that were used during the test have densified the surrounding soil. The depths to which the anchor piles were driven have not been indicated. The writer wonders if there was any densification of the surrounding soil by the driving of either the anchor piles or the test piles. An examination of Table 5 shows that the resistance to the driving of the pile for the last 6 in. was approximately 60% higher for pile 6 (19-in. diameter) than for pile 2 (21-in. diameter). Both piles were driven to the same depth, although pile 2 was embedded 2 ft more in the sand strata than pile 6. The supporting capacity of the latter pile was greater for friction in the silt layer than for friction and end bearing in the sand. Was pile 6 one of the last piles to be driven?

TABLE 11.—PERCENTAGE OF TEST LOAD CARRIED BY FRICTION

Loading test number	PILE DESCRIPTION			Length unsupported and in soft soils, in feet	Test load, in tons	Percentage carried by friction
	Size, in inches	Weight, in pounds	Length,[a] in feet			
1	14	102	215	172	200	41
3	14	89	137	54	121	41
15	14	73	81.3	22	150	55
16	12	53	71.1	39	150	21
17	14	89	110.3	80	150	30

[a] All piles driven to point bearing on rocks.

The curves for load distribution in piles (Fig. 6 and Fig. 11) and the curves for load versus movement (Fig. 7 and Fig. 12) furnished the basic data from which additional curves and conclusions were extrapolated. Fig. 6 represents the distribution of load in piles for various depths and is based on values measured at six or seven points along the length of the pile. When fitting a curve to these points, the lower part of the curve should have a vertical tangent at the bottom of the pile where end-bearing is developed. The authors' statement that "the skin friction in the sands decreases near the pile tip" is true of all soils in which some end support is developed. According to the load-distribution curves (Fig. 6), a vertical tangent indicates that this is the end of frictional resistance and whatever remains unsupported is carried by point resistance. In an analogous fashion, all curves for load distribution in piles in tension (Fig. 11) should have a vertical tangent at the ground surface.

The authors' statement (under the heading, "Compression Tests: Load in Piles and Soil Strata") that " * * * applied pile loads are carried by skin friction in the silt stratum, and by skin friction and tip resistance in the sands" is in accordance with theory and should be emphasized. Even in the case of piles supposedly driven to "point bearing" on rock, there is evidence that a part of the load is carried by friction. An examination of the results of several loading tests on **H**-piles laterally supported by soft and stiff soils shows this to be true. The values in Table 11 have been computed from a summary of these loading tests.[7]

The percentage of the test load carried by friction has been computed from a comparison of the actual settlements[7] with the theoretical decrease in the length of the pile under the test load, assuming uniform compression across the cross section of the pile. The value of the modulus of elasticity of steel is taken as 29 × 10^6 lb per sq in.

The gross and net settlement of the pile butt were measured by the authors. The net settlement is the residual settlement after the load and rebound of the pile have been removed. In Fig. 7 and Fig. 12, the curve of tip movement is also presented. Although there is a close agreement between this curve and the curve of the net settlement of the butt, the latter is located above the former in all cases in Fig. 7, except for pile 5 where both curves coincide. In tension tests (Fig. 12) the relative position of these curves is reversed. The hypothesis of locked-in or residual stresses cannot be discussed unless the authors state what they believe to be the origin of such stresses. It is necessary to know how the tip movement was measured before or after the load was removed from the pile—that is, determining if the elastic rebound of the sand under the pile tips was considered.

CHARLES I. MANSUR,[8] M. ASCE, AND ROBERT I. KAUFMAN,[9] J. M. ASCE.—The writers appreciate the interest shown by the discussers in the pile loading tests.

It is possible that the coefficient of earth pressure, K, and the unit skin friction in silt was greater in the compression tests than in the tension tests because of the "swelling" of the pile in compression, as proposed by Mr. Nishida. In the tension tests the average value of K in silt was approximately 0.6, which is about equal to an at-rest value, whereas in the compression tests the average value of K in silt was approximately 1.6, approaching a value of passive earth pressure.

Mr. Nishida has asked whether the shearing strength of the silty soils is composed partly of cohesion. A shear strength of approximately $\phi = 28°$, $c = 0.1$ ton per sq ft, was obtained from consolidated-undrained triaxial compression and consolidated-drained direct-shear tests on undisturbed silt samples. A shear strength of about $\phi = 33°$, $c = 0$, was obtained from consolidated-drained direct-shear tests on undisturbed specimens of silty sand. Because the cohesion was relatively small, it was neglected in the design of the pile foundation and analyses of the pile-load test data.

---

[7] "Bethlehem H-Piles," *Catalog 223*, Bethlehem Steel Co., Bethlehem, Pa., 1949, p. 13, Table 2.

[8] Vice-Pres. and Chf. Engr., Independent Wellpoint Corp., Baton Rouge, La.; formerly Chief, Geology, Soils, and Materials Branch, Mississippi River Comm., Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss.

[9] Chief, Geology, Soils, and Materials Branch, Mississippi River Comm.; formerly Chief, Design and Analytical Section, Embankment and Foundation Branch, Soils Div., Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss.

Mr. Gizienski has questioned the effect of driving closely spaced wood anchor piles (on 2.5-ft centers) on the density of the foundation soils, and whether differences existed between the density of the foundation in the pile test excavation and that of the foundation for the structure in which no anchor piles are to be driven. The timber anchor piles had an embedded length of approximately 44 ft and, therefore, penetrated only into the silty part of the foundation. Although driving these piles probably caused the silty soils to densify, no measurements were made to determine this effect. Those that were made when driving the piling for the structure indicate that the bottom of the excavation settled from approximately 0.5 ft to 1.0 ft as a result of driving the structure piling (14-in. H-beams), which is spaced on approximately 5-ft centers. It is believed that the settlement results primarily from the densification of the silty portion of the foundation. In general, conditions at the test piles in the pile test excavation after the anchor piles and test piles had been driven would not differ significantly from those at the structure after all the structure piling had been driven. Furthermore, because the piles were designed to support the design load by point bearing and skin friction in the deep sand stratum with an adequate safety factor, any differences in density in the upper silt stratum beneath the test excavation, as compared with those beneath the structure, were minor with respect to the load-supporting capacity of the section of pile embedded in sand.

The sequence of driving the test piles did not significantly affect their penetration resistance or static capacity, and variations in these resistances and capacities reflect the variability of in-place soil conditions. Mr. Gizienski noted that the final penetration resistance of pile 6 (19-in. diameter) was approximately 60% greater than for pile 2 (21-in. diameter). Pile 2 was at the center of the row of test piles which comprised piles 1, 2, and 3 (Fig. 3). Pile 2 was driven after pile 1 and pile 3. Pile 6 was at the end of the row which consisted of piles 4, 5, and 6 and was the first to be driven in this row. Pile 6 was driven several days prior to the driving of pile 2. If the sequence of driving had had a significant effect on the driving resistance, it would have been reasonable to expect that pile 2 would have had a greater driving resistance than pile 6. However, the final penetration resistance of pile 6 was greater than that of the former. Furthermore, pile 4 (17-in. diameter) offered the greatest resistance to driving of all piles and had the greatest failure load in sand. This pile was at the end of a row (Fig. 3) and was driven after pile 6, but before pile 5. Pile 7 was the last to be driven. The anchor poles were driven after the test piles. Thus, when pile 4 was being driven, the closest one, which had already been driven into sand, was 25 ft away. Based on the preceding, the sequence of pile driving had no noticeable effect on the pile-driving resistance.

Mr. Gizienski has inquired also about the procedures used to determine the tip movement of the pile and "residual" stresses in the piles in the tension tests. The tip movement of the loaded pile was determined from the movement of the bottom strain-rod anchor. The net movement of the pile was determined as the residual settlement or rise remaining after the test load was removed from the pile, which was permitted to rebound. In general, the net movement agreed closely with that of the tip at a given load, although some difference was noted, as shown in Fig. 10 and Fig. 15. The "locked-in" or residual stresses and their effect on tension-test data are believed to have developed in the soil and pile in the compression tests, which were performed prior to the tension tests. It is believed that piles loaded in compression to the ultimate capacity of the foundation developed high compressive stresses and strains that were not relieved completely upon subsequent unloading of the pile. Loading these piles in tension would tend to relieve the compressive stresses further, although, based on tension-test data for pile 4 and pile 6 in Fig. 14, it appears that sections of these piles in the vicinity of, and immediately above, the top of the sand may still have been in compression during the tests. If the residual compressive stresses had been relieved completely in the tension tests, or if the tension tests had been performed prior to the compression tests, the "load-in-pile" curves in Fig. 14 probably would have been less erratic.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

TRANSACTIONS

Paper No. 2937

DESIGN OF VENTURI FLUMES IN CIRCULAR CONDUITS

BY EDWIN A. WELLS, JR.,[1] A. M. ASCE, AND HAROLD B. GOTAAS,[2] M. ASCE

WITH DISCUSSION BY MESSRS. KEENO FRASCHINA, AND EDWIN A. WELLS, JR., AND HAROLD B. GOTAAS

SYNOPSIS

The results of extensive experimental studies to determine the accuracy and design criteria for Venturi flumes of the type first recommended by Harold K. Palmer and Fred D. Bowlus, Members, ASCE, are presented. The coefficient of discharge for the flumes and the hydraulic aspects of Venturi flow, such as energy criteria, channel slope, submergence, and velocity of approach, are examined. The studies and results show the influence of various flume dimensions, such as throat length, side slope, base height, transitions, and point of depth measurement, on the accuracy of the flume.

INTRODUCTION

Since 1915, many Venturi flumes have been installed for the measurement of flow of sewage and irrigation waters in open channels. The method of design for most of these flumes can be traced to one of three sources: (*a*) English flumes, which were rectangular in shape and were based on early work in India (1908–1914) and on the writings of F. V. A. E. Engel[3, 4, 5] in England (1933); (*b*) the Parshall flume originated (1915) by V. M. Cone[6, 7] and modified

---

NOTE.—Published, essentially as printed here, in April, 1956, in the Journal of the Sanitary Engineering Division, as *Proceedings Paper 928*. Positions and titles given are those in effect when the paper or discussion was approved for publication in Transactions.

[1] Municipal Financing Consultant, Stone & Youngberg, San Francisco, Calif.

[2] Dean, Technological Inst., Northwestern Univ., Evanston, Ill.

[3] "Non-Uniform Flow of Water: Problems and Phenomena in Open Channels with Side-Contractions," by F. V. A. E. Engel, *The Engineer*, London, Vol. 155, 1933, pp. 392–394.

[4] *Ibid.*, pp. 429–430.

[5] *Ibid.*, pp. 456–457.

[6] "Irrigation Investigations," by V. M. Cone, *28th Annual Report*, Agri. Experiment Station, State Agri. College of Colorado., Fort Collins, Colo., 1915, p. 16.

[7] "The Venturi Flume," by V. M. Cone, *Journal of Agricultural Research*, U. S. Dept. of Agriculture, Washington, D. C., Vol. 9, 1917, pp. 115–129.

and tested extensively (1917–1927) by Ralph L. Parshall,[8,9,10] A.M. ASCE; and (*c*) flumes of the type suggested and developed (1936) by Harold K. Palmer and Fred D. Bowlus,[11] Members, ASCE.

The results of laboratory experiments, which were conducted to provide data on the accuracy of the Palmer-Bowlus flume and on the importance of various design criteria, are reported herein. This type of Venturi flume is characterized by a throat of uniform cross section and a length that is approximately equal to one diameter of the pipe in which it is to be installed. Its shape, as suggested by Palmer and Bowlus, makes it inexpensive to construct, simple to operate, easy to install and maintain, accurate in its measurement of flow, and low in energy loss. Although it is not as widely known as the Parshall flume, it has been installed extensively in the western United States since 1937.

Seven flumes conforming to the Palmer-Bowlus flume shape were built and tested to determine the accuracy of the flume under normal operating conditions. One flume was tested under varying conditions of channel slope. A second series of flumes was built with interchangeable parts to study the importance of flume dimensions, such as throat length, base height, and entrance transition. Each design variable is examined separately herein.

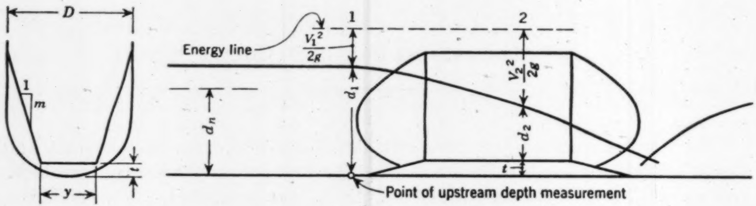


FIG. 1.—ANALYSIS OF FLOW IN THE VENTURI FLUME

DERIVATION OF LAWS

Rating Curve.—Using Fig. 1 as a basis for analysis of the flow in the Venturi flume, the Bernoulli equations for the energy at the two sections may be written as

$$E_1 = d_1 + \frac{V_1^2}{2\,g} \qquad (1)$$

and

$$E_2 = d_2 + \frac{V_2^2}{2\,g} + t \qquad (2)$$

---

[8] "Report of the Irrigation Investigations Engineer," by R. L. Parshall, *33d Annual Report*, Agri. Experiment Station, State Agri., College of Colorado, Fort Collins, Colo., 1920, pp. 31–36.

[9] "The Venturi Flume," by Ralph L. Parshall and Carl Rohwer, *Bulletin No. 265*, Agri. Experiment Station, State Agri. College of Colorado, Fort Collins, Colo., 1921.

[10] "The Improved Venturi Flume," by Ralph L. Parshall, *Transactions*, ASCE, Vol. 89, 1926, pp. 841–851.

[11] "Adaptation of Venturi Flumes to Flow Measurements in Conduits," by Harold K. Palmer and Fred D. Bowlus, *ibid.*, Vol. 101, 1936, pp. 1195–1216.

Equating the foregoing and solving for the depth of flow at section 1,

$$d_1 = d_2 + t + \frac{V_2^2}{2\,g} - \frac{V_1^2}{2\,g} \qquad (3)$$

Because the flume should operate under the free discharge condition, the flow must pass through critical depth in the throat in which

$$d_2 = d_c \qquad (4)$$

and

$$\frac{V_2^2}{2\,g} = \frac{V_c^2}{2\,g} = \frac{A_c}{2\,B_c} \qquad (5)$$

Eq. 3 may be rewritten as

$$d_1 = d_c + t + \frac{A_c}{2\,B_c} - \frac{V_1^2}{2\,g} \qquad (6)$$

For a given flow rate, Q, the variable quantities, $\frac{V_1^2}{2\,g}$, d_c, and $\frac{A_c}{2\,B_c}$, have fixed values and, hence, Eq. 6 may be written as

$$Q = f\,d_1 \qquad (7)$$

Several methods have been suggested for solving Eq. 7. The method proposed by Filippo Arredi[12] in 1936 and clarified by Harvey F. Ludwig,[13] M. ASCE, and the Los Angeles (Calif.) County Sanitation Districts was used in this investigation because it is considered to be more easily adapted to irregularly shaped sections than other procedures. Another convenient method has been proposed by John H. Ludwig and Russell G. Ludwig,[14] Associate Members, ASCE. An Arredi diagram and computations for rating a flume are presented in the Appendix.

A computed "rating" curve of Eq. 7 for a given Venturi flume installation will be referred to hereafter as the theoretical curve of that flume. The theoretical curve is based on a U-shaped channel rather than on a circular channel. If the sides of the throat consist of planes that extend unbroken to the top of the channel, Eq. 7 has the logarithmic form, $Q = k\,d_1^n$.

In deriving Eq. 6 and Eq. 7 for rating the flume, several assumptions have been made, the most important of which are: (*a*) There are no energy losses between sections 1 and 2; and (*b*) the streamlines are parallel and are perpendicular to both sections. Actual flow conditions, however, vary slightly from these assumptions because side-wall friction and other energy losses influence the flow. The character of the flow at section 2 is at variance with the original assumption because, unless correction factors are used, the Bernoulli equation is valid only where there is no curvature of the streamlines. It is apparent that the use of the "average velocity" and of the "velocity head" when the actual velocity distribution has not been considered is a simplification of the conditions that exist. The drawdown through section 1, although small, also affects the upstream depth, d_1. The magnitude of the effect of these assumptions on the accurate measurement of the quantity of water passing through the flume is given quantitatively by the discharge coefficient which, as shown later, does not vary more than 3% from the theoretical value for most standard flumes. For this investigation performance was considered to

---

[12] Discussion by Filippo Arredi of "Adaptation of Venturi Flumes to Flow Measurements in Conduits," by Harold K. Palmer and Fred D. Bowlus, *Transactions*, ASCE, Vol. 101, 1936, pp. 1231–1235.

[13] "Palmer-Bowlus Meters for Measurement of Flow in Sewers," by Harvey F. Ludwig, *Syllabus 01.01*, Dept. of Civ. Eng., Univ. of California, Berkeley, Calif., 1950.

[14] "Design of Palmer-Bowlus Flumes," by John H. Ludwig and Russell G. Ludwig, *Sewage and Industrial Wastes*, Vol. 23, 1951, pp. 1096–1107.

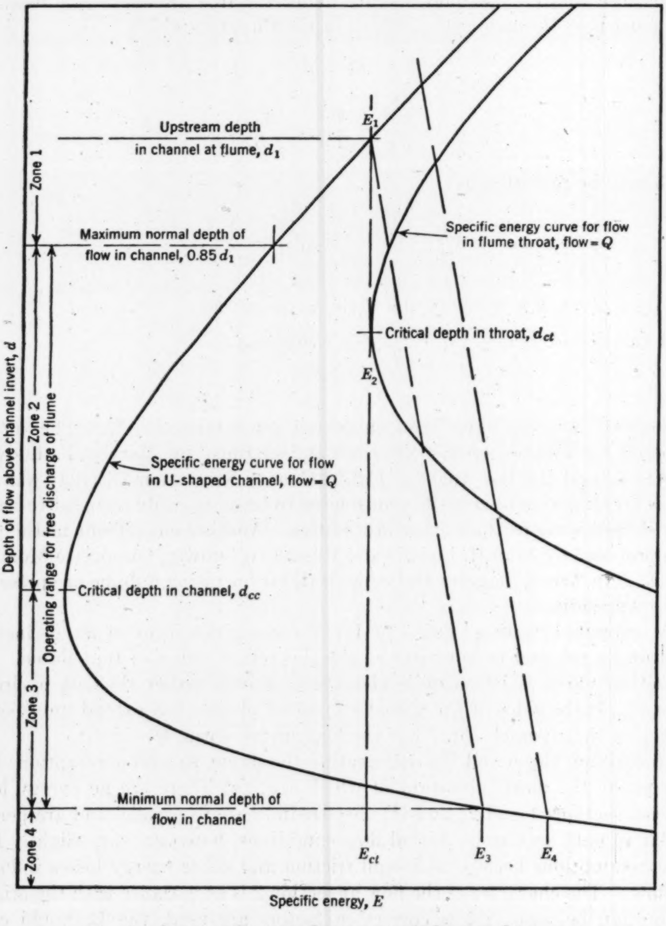


FIG. 2.—SPECIFIC-ENERGY CURVES FOR U-SHAPED CHANNEL AND VENTURI FLUME THROAT

be satisfactory when the coefficient of discharge did not exceed a range of from 0.95 to 1.05.

Energy Criteria.—The rate of flow, Q, becomes a function of the upstream depth, d_1, when the specific energy, E_1, of the flow in the pipe is equal to the specific energy, E_2, of the same flow at critical depth in the throat of the flume. On the basis of this criterion, the theoretical curve of flow as opposed to that of depth may be computed without regard to the conditions of actual usage. However, in order to install a flume in a given channel so that it will operate properly, additional energy criteria must be satisfied.

The principle of the Venturi flume, as well as the energy criteria essential for its proper design and operation, may be described by the use of a specific-energy diagram. Fig. 2 shows the specific-energy curves for a given rate of flow in the U-shaped channel and in the throat of a Venturi flume. The possible conditions of normal flow in the channel prior to the installation of a flume are also shown, and the normal-flow conditions are separated into four zones based on depth of normal flow. In all cases, the dimensions of the channel and the flume, the rate of flow, and, therefore, the theoretical upstream depth, d_1, are held constant.

Zone 1. Tranquil Channel Flow—Submerged Discharge.—If the normal depth, d_n, of flow in the channel without the flume is greater than d_1, the constriction produced by the flume will merely create a depression in the water surface that is equal to the increase in velocity head. Thus, the flow would not pass through critical depth, a condition that is referred to as submerged discharge.

Zone 2. Tranquil Channel Flow—Free Discharge.—The normal depth of flow, d_n, in the channel usually is less than the upstream depth, d_1, required for operation of the flume and is greater than the critical depth, d_{cc}. Installation of the flume in the channel will cause water to back up until it reaches depth d_1. From this depth the water flows into the flume and passes through critical depth within its throat. To assure that the flume is not submerged, the normal depth of flow in the channel should be somewhat less than the upstream depth, approximately $0.85\, d_1$ or less.

Zone 3. Rapid Channel Flow—Free Discharge.—If the normal depth of flow in the channel is less than the critical depth, d_{cc}, placing the flume in the channel will cause the water to jump to its conjugate depth with a subsequent loss of energy. The minimum normal depth, $(d_n)_{\min}$, as shown on the curve, represents the normal flow for which the flume will continue to function properly. In this case the difference in specific energies E_3 and E_{ct} represents the loss of energy in the jump as the water surface passes from $(d_n)_{\min}$ to d_1. This would be typical of the case in which a jump occurred immediately upstream from the flume. For other instances in which d_n is less than d_{cc} and greater than $(d_n)_{\min}$, the hydraulic jump would occur farther upstream, and the water surface would back up to depth d_1.

Zone 4. Rapid Channel Flow—Unstable Discharge.—Where the normal depth of flow, d_n, is less than $(d_n)_{\min}$ and where the energy of the flow is correspondingly greater than E_3 and is E_4, energy $E_4 - E_3$ is dissipated by turbulence in the upstream channel after the jump has occurred. If the energy of the flow exceeds E_3, the conditions of discharge would be unstable and would not conform to Eq. 7. Conditions may be such that no pond forms upstream and there is shooting flow through the upstream transition of the flume.

Satisfactory operation of the flume can be obtained only in zones 2 and 3. A principal objective of the experimental investigation was to determine the practical limits of these zones. The upper limit of zone 2, $(d_n)_{max}$, is described in the section entitled, "Experimental Results and Analysis: Submergence." The lower limit of zone 3, $(d_n)_{min}$, is described under the heading, "Experimental Results and Analysis: Channel Slope."

LABORATORY EQUIPMENT AND EXPERIMENTAL PROCEDURES

Experimental Channel.—The 36.67-ft-long channel consisted of three sections, which made it possible to keep the bottom of the center section (test section) level while varying the slope of the upstream and downstream channels as desired. The circular parts of the channel were 12-in.-inside-diameter, asbestos-cement pipe with special butt joints to reduce the surface waves. The pipe was supported on wide-flange steel girders and held between the two flanges. The test section of the channel consisted of a 4-ft plywood box set on top of the 4-ft 6-in. center section to form a U-shaped test section. A tail gate was placed at the end of the channel so that water could be backed up to the test section for submergence experiments.

The quantity of water, Q, passing through the channel was determined by the use of calibrated nozzle lines, which were calibrated volumetrically immediately prior to the study.

Flumes.—The flumes were made of sheet metal and had wooden ribs to hold them rigid. They were of two different types: (*a*) Solid flumes and (*b*) erector flumes, some of which are shown in Fig. 3. Seven different solid flumes of one-piece construction were made and tested. Those of the erector series were of three-piece construction—that is, they consisted of entrance and exit transitions and the throat—and ten of this type were tested. Table 1 lists the dimensions of all the flumes used in the study.

Free Discharge Experiments.—Experimental tests were made to calibrate the different flumes and to accumulate sufficient data for analysis of their behavior under varying conditions. Each flume was installed for a series of from 15 tests to 25 tests, beginning with full pipe flow and decreasing to minimal flows. A single test consisted of recording rate of flow, manometer, and surface profile readings after steady-state conditions had been obtained. The upstream depth, d_1, was measured by a point gage in a stilling well opposite the upstream edge of the entrance transition to the flume.

The values of Q and d_1, plotted to show the actual flow as a function of measured depth, are the "calibration" or "actual test" curve.

All flumes were checked by two or more series of tests. In many of the test curves shown, the flume was removed after the first test, and an additional test series run under similar conditions at a later date. In no case did a flume fail to reproduce the calibration curve obtained from the earlier runs.

Table 2 presents a summary of the conditions under which the various flumes were tested.

Submergence Tests.—A standard test run was made with the flume in a normal position. The point-gage reading for depth d_1 was carefully checked to find the mean reading. The tip of the point gage was then raised 0.005 ft and checked to see that normal surface undulations in the stilling well did not touch it. The tail gate at the end of the channel was 15 ft from the flume and was raised slowly until the submergence of the flume caused the water in and above it to rise and touch the gage at d_1. The depth of water below the flume was then determined with a second point gage, and the submergence ratio was

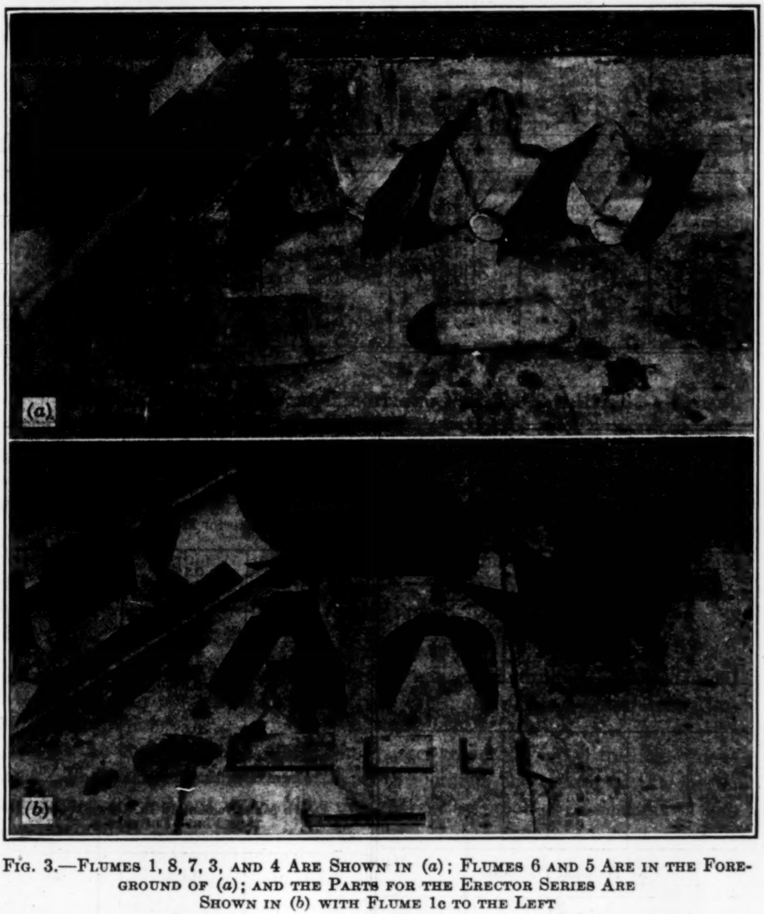


FIG. 3.—FLUMES 1, 8, 7, 3, AND 4 ARE SHOWN IN (*a*); FLUMES 6 AND 5 ARE IN THE FOREGROUND OF (*a*); AND THE PARTS FOR THE ERECTOR SERIES ARE SHOWN IN (*b*) WITH FLUME 1c TO THE LEFT

TABLE 1.—FLUME DIMENSIONS

No.	Throat Length, in Feet	Side Slopes	Base Height, in Feet	Base Width, in Feet	Entrance Transition Slope
Solid Flumes					
1	1.00	1:3.07	0.089	0.333	1:3
3	1.00	1:2.05	0.083	0.333	1:3
4	1.00	1:1.00	0.096	0.321	1:3
5	1.00	...	0.089	...	1:3
6	1.00	...	0.179	...	1:3
7	1.00	1:3.04	0.035	0.333	1:3
8	1.00	1:3.02	0.184	0.343	1:3
Erector Flumes					
1a	2.00	1:3	0.095	0.354	1:3
1b	1.50	1:3.08	0.093	0.340	1:3
1c	1.00	1:3.20	0.090	0.337	1:3
1d	0.67	1:3.20	0.090	0.337	1:3
1e	0.33	1:3.20	0.090	0.337	1:3
1f	0.12	1:3.20	0.090	0.337	1:3
1g	1.00	1:3.20	0.090	0.337	1:2
2c	1.00	1:3.20	0.187	0.401	1:3
2d	0.67	1:3.20	0.187	0.401	1:3
2f	0.12	1:3.20	0.187	0.401	1:3

TABLE 2.—TEST CONDITIONS FOR VARIOUS FLUMES

No.	Indicated Channel Slope	Number of Test Runs
Solid-Flume Test		
1	0.1%; 0.5%	97
3	0.1%; 0.5%	112
4	0.1%; 0.5%	89
5	0.1%; 0.5%	97
6	0.1%	42
7	0.1%	58
8	0.1%	53
Submergence Test		
1	0.1%	15
3	0.1%	18
6	0.1%	19
7	0.1%	14
8	0.1%	12
1c	0.5%	35
No Terminal Transition		
1c	0.5%	17
1:2 Entrance Transition		
1g	0.1%	22

No.	Indicated Channel Slope	Number of Test Runs
Erector Model Test		
1a	0.1%	62
1b	0.1%	33
1c	0.5%	128
1c	2.0%	107
1c	3.0%	67
1c	4.0%	41
1c	4.5%	17
1c	5.0%	10
1d	0.5%	39
1e	0.5%	21
1f	0.5%	36
2c	0.5%	40
2d	0.5%	41
2f	0.1%	22
Depth Measurement 12 In. Downstream from Flume Entrance		
1	0.1%	23
Depth Measurement 24 In. Downstream from Flume Entrance		
1	0.1%	21

computed. The channel was level from d_1 to the tail gate. The distance between the point gages was approximately 10 ft.

EXPERIMENTAL RESULTS AND ANALYSIS

Free Discharge Experiments.—Nine different standard shapes were tested throughout a wide range of flows and energy conditions. A standard flume is defined as one having a throat length equal to the diameter of the pipe. The investigation of its behavior under normal, free discharge conditions was a principal objective. Table 3 shows the flow range within which measurements were made for each flume. The upstream depth, d_1, and rate of flow, Q, which determine the actual test curve, are compared with the theoretical discharge curve for each of these flumes in Figs. 4 and 5. The correlation between the actual and theoretical curves is examined under the heading, "Experimental Results and Analysis: Coefficient of Discharge."

TABLE 3.—FLOW RANGES FOR STANDARD FLUMES

No.	Minimum flow, in cubic feet per second	Maximum flow,[a] in cubic feet per second	Ratio, maximum to minimum flow
8	0.18	1.16	6.5
2c	0.18	1.25	7.0
1c	0.18	1.49	8.3
1	0.18	1.55	8.8
7	0.18	1.78	9.9
3	0.17	2.10	11.7
4	0.17	2.70	15.0
6	0.18	2.70	15.0
5	0.18	2.80	15.5

[a] Maximum flow read from test curves at 0.95-ft depth.

The flumes were tested under the most desirable operating conditions (an upstream energy line slope of less than 0.5%), and the data shown in Figs. 4 and 5 were used as a basis of comparison of further tests on different aspects of design and operation. The actual test curve for flume 1c was chosen as the standard curve for comparison purposes in the experiments involving changes in dimensions. Because flumes would be most widely used under conditions similar to the free discharge experiments, when they are functioning properly their characteristics are as follows:

1. In the upstream channel the flow is smooth and tranquil. There should be no aeration or prominent surface waves, especially at the point of upstream-depth measurement.
2. At the flume the water enters and passes smoothly and with little turbulence. The surface profile should drop throughout the length of the flume. Streamlines are evident in the flow even after the point of critical depth has been reached.
3. In the downstream channel a shooting flow is evident on the downstream side of the flume, indicating that the free discharge necessary for proper functioning prevails. In no case should the flow merely "neck down"—that is, show a smooth surface depression. In some instances a hydraulic jump may form immediately downstream. Operation will be satisfactory if the upstream edge of the jump remains below the throat of the flume.

Flume 5, a broadcrested weir with a height of about 1 in., did not perform satisfactorily because it did not provide sufficient channel constriction to develop critical-flow conditions.

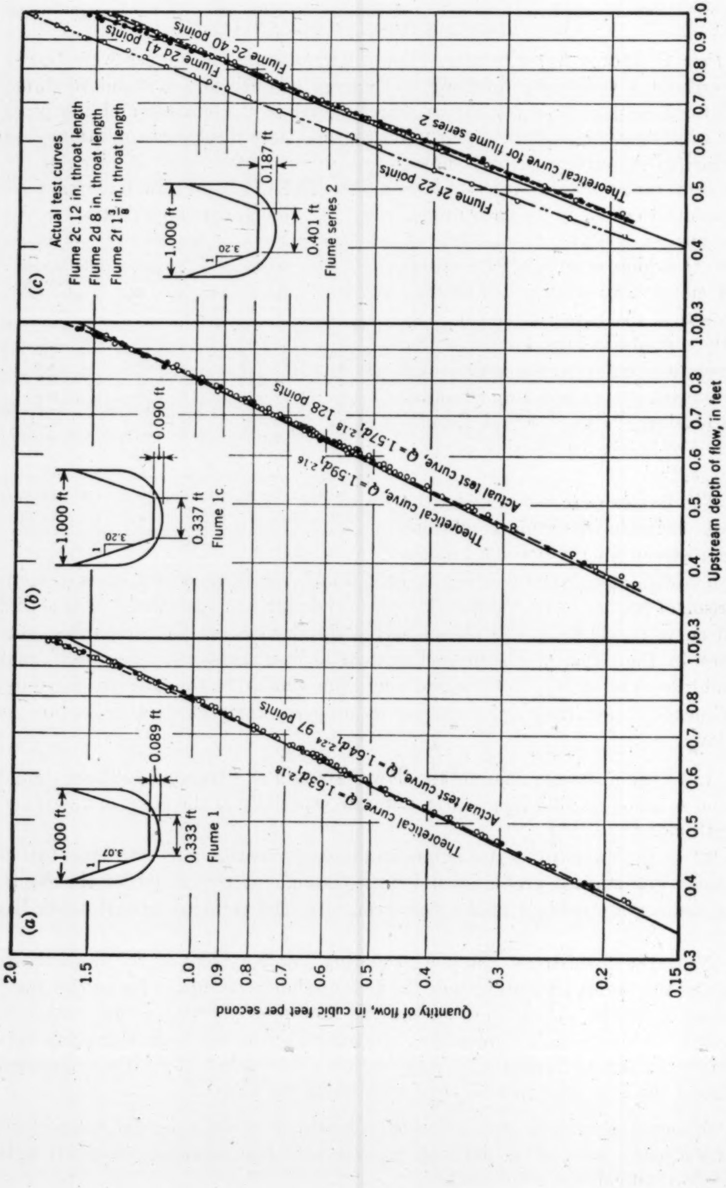


FIG. 4.—RATING CURVES FOR FLUMES 1, 1c, AND 2

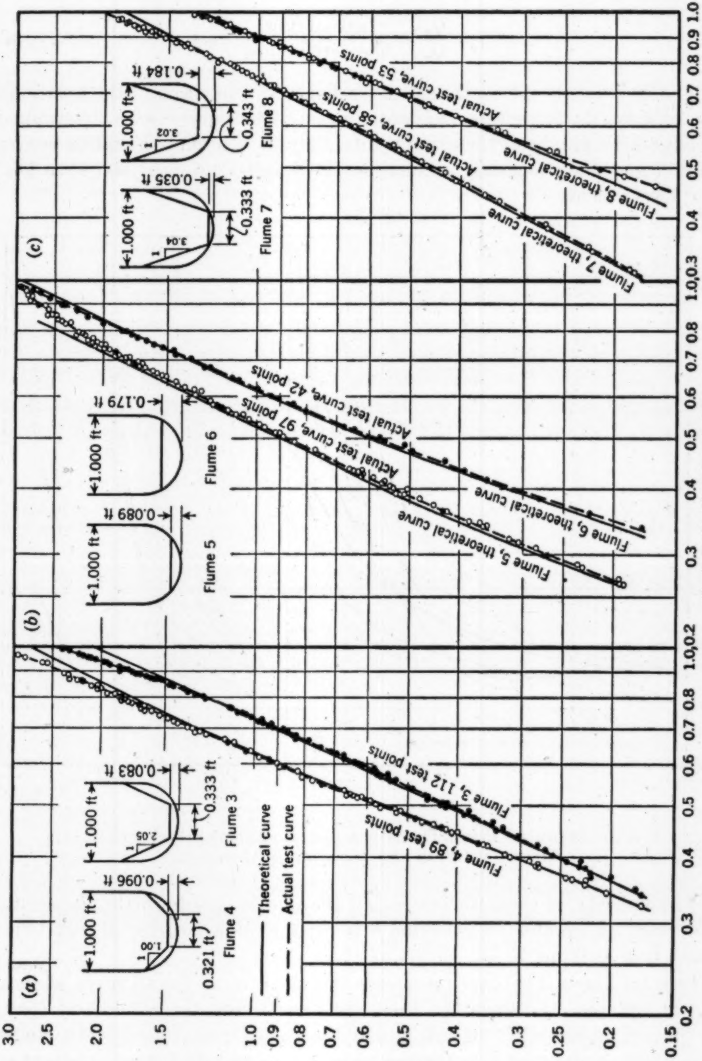


FIG. 5.—RATING CURVES FOR FLUMES 3 THROUGH 8

Coefficient of Discharge.—The coefficient of discharge, C_d, of a flow-measuring device is given by

$$C_d = \frac{Q_a}{Q_t} = \frac{\text{actual flow}}{\text{theoretical flow}} \qquad (8)$$

Fig. 6(*a*) presents the coefficient of discharge as a function of the upstream depth of flow, d_1, for seven standard flumes. The curves were determined from large-scale graphs of the experimental data used to plot the rating curves presented under the heading, "Experimental Results and Analysis: Free Discharge Experiments."

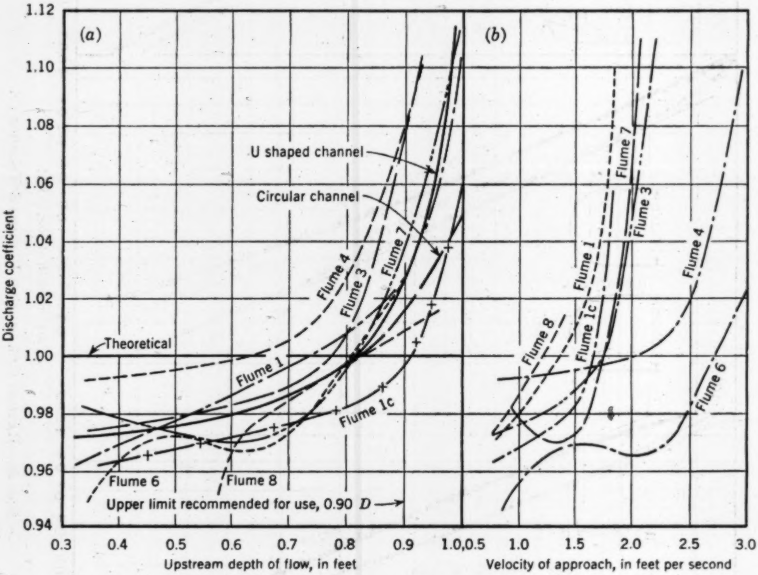


FIG. 6.—STANDARD FLUME DISCHARGE COEFFICIENT AS A FUNCTION OF UPSTREAM DEPTH AND VELOCITY OF APPROACH

The similarity of these curves for all the flumes is noteworthy, especially considering, for example, that flume 6 is capable of measuring almost twice as much flow as flume 1.

The solid line in Fig. 6(*a*) represents the mean trend for all seven shapes. That the difference between the theoretical curve and the actual test curve for a properly designed flume will be less than 3% for upstream-depth ranges between 0.40 ft and 0.90 ft is thereby indicated. At 0.82 ft the coefficient of discharge is 1.00 and does not remain constant throughout the range of flows measured by the flume. The variation shown here for trapezoidal flumes is similar to that described by Engel[3] for those that are rectangular.

In the laboratory model, when the flow from the circular upstream channel entered the **U**-shaped test channel, flow conditions were not uniform in the top surface of the test channel at upstream depths greater than 0.5 diameter. The curves shown in Fig. 6(*a*) are based on a comparison of the actual test curve of the flume with a computed rating curve, which, in turn, was based on the assumption that the upstream channel was U-shaped and of sufficient length for uniform flow conditions to exist. If it were assumed that the upstream channel had been circular, the trend of these curves would be as shown by the heavy dashed line. The difference between these two curves appears to be insignificant for practical purposes. Generally, the computation of the theoretical rating curve would be based on a U-shaped upstream channel.

The results demonstrate conclusively that the standard flume shapes are capable of accurate flow measurement when they are installed to satisfy the basic energy criteria. A reasonably accurate measurement of flow can be obtained with upstream depths as great as $0.95\ D$, and accuracies within 3% of the theoretical rating curve can be obtained readily at depths of as much as $0.90\ D$.

Velocity of Approach.—From the standpoint of operation, these experiments indicate that velocity of approach is not relatively important as a design limitation. Palmer and Bowlus[11] have stated that the velocity of approach should be, ideally, between 2.0 ft per sec and 2.5 ft per sec. This concept has been amplified by other engineers. It should be at least from 1.5 ft per sec to 2.0 ft per sec at low flows where it is necessary to prevent deposition of solids upstream from the flume. Before any arbitrary range is established, however, the following experimentally based facts should be considered:

1. The results of the 12-in. pipe experiments indicated that properly proportioned flumes would operate satisfactorily, with velocities of approach ranging from 0.8 ft per sec to 3.0 ft per sec as demonstrated in Fig. 6(*b*). For kinematic similarity in large pipe sizes, the approach velocity would increase as the square root of the diameter ratio.
2. The assumption of reasonable approach velocities does not preclude satisfactory functioning of the flume. This has been demonstrated by the results of experiments with supercritical channel velocities and variable throat length.

Submergence.—Submergence of a Venturi flume is expressed as a percentage ratio of tailwater depth to the upstream depth of flow, in which tailwater depth is referred to channel invert at the point of upstream depth measurement. The maximum submergence ratio is a measure of the upper limit of tailwater elevation for which free discharge can be maintained in the flume. Where steady flow conditions exist, the tailwater depth may be assumed to equal the normal depth of flow in the channel. Thus, the maximum submergence factor becomes a measure of the upper limit of zone 2 of Fig. 2.

Experiments were made to determine the maximum submergence ratio at different rates of flow for six standard flumes. Fig. 7 shows the maximum submergence, S_m, as a function of the upstream depth of flow. This submergence is the upper limit of tailwater elevation for a free discharge condition to exist at the flume. It is expressed as a ratio of tailwater depth to upstream depth of flow. The results indicate that this is approximately a linear function through the range of depths shown (from 0.4 ft to 0.95 ft). Fig. 7 shows that at a depth of 0.90 ft the ratio is about 85% for the flumes tested.

When a flume is placed in a channel having a continuous slope through the measuring station, the fact that the tailwater may drown the flume—that is, may exceed the limit of maximum submergence—presents a real hazard. For this reason it is necessary to limit the normal depth of the maximum flow to be measured. Thus, if it is assumed that the maximum upstream depth, d_1, is not to exceed 0.90 D and that the maximum recommended submergence is approximately 85% of this depth, the normal depth of the flow in the channel should not exceed 0.75 D.

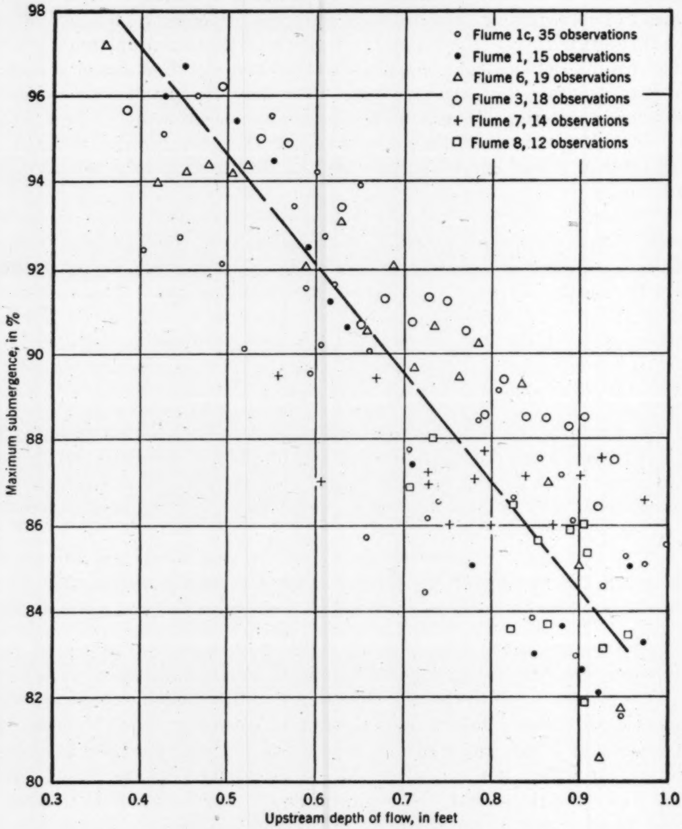


FIG. 7.—MAXIMUM SUBMERGENCE FOR STANDARD FLUMES

Channel Slope.—Water-surface measurements with flume 1c both in and out of the channel were made through a wide range of upstream channel slopes in order to ascertain conformance of the flume to the energy criteria previously described. Fig. 8 presents the calibration curves for the flume 1c experiments. Different upstream channel slopes were expressed in terms of channel slope because uniform flow conditions could not be obtained, and, therefore, the slope of the energy gradient was difficult to determine. At 2% slope the calibration curve remained unchanged from that developed during the free

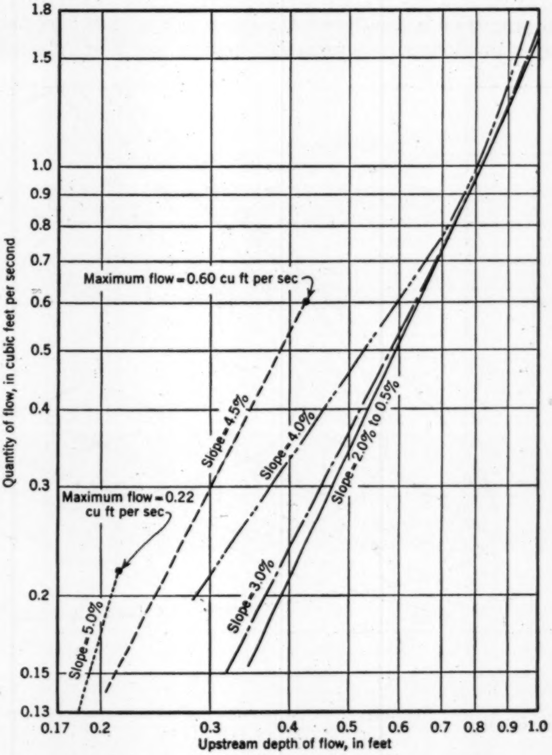


FIG. 8.—EFFECT OF SLOPING APPROACH CHANNEL FOR FLUME 1c

discharge tests made under ideal conditions. When the channel slopes were greater than 3%, there was only a vague resemblance to the usual curve. At slopes greater than 3.0%, the water became extremely turbulent. At 4.0%, 4.5%, and 5.0%, no backup would occur, and one had to be formed artificially. Sometimes, shooting flow would wash the backup pond through the flume. No data were taken under the latter condition. Hence, the 4.5% and 5.0% curves are discontinuous.

This phenomenon may be shown to conform with the theory developed for the flumes by the use of Fig. 9. The three sets of curves shown are: (*a*) Specific energy curves for the flows of 0.4, 0.8, 1.2, 1.6, and 2.0 cu ft per sec in the U-shaped upstream channel; (*b*) experimental "normal" depth curves for the channel slopes shown, determined on the basis of actual Q and channel depth at d_1; and (c) curves of $\frac{V^2}{2\,g\,d_n}$—that is, lines of proportionality between velocity head and normal depth of flow in the channel.

When d_1 of flume 1c for an indicated flow is plotted on its corresponding specific-energy curve and a vertical line is projected downward from that point, the vertical projection will intersect the lower half of that curve at the point indicated by a small circle. Several such points, which are plotted in Fig. 9, lie approximately on the 2.0%-slope line, indicating that the 2.0% channel slope is approximately the maximum at which flume 1c will function properly throughout the range of flows represented; Fig. 8 shows this to be true.

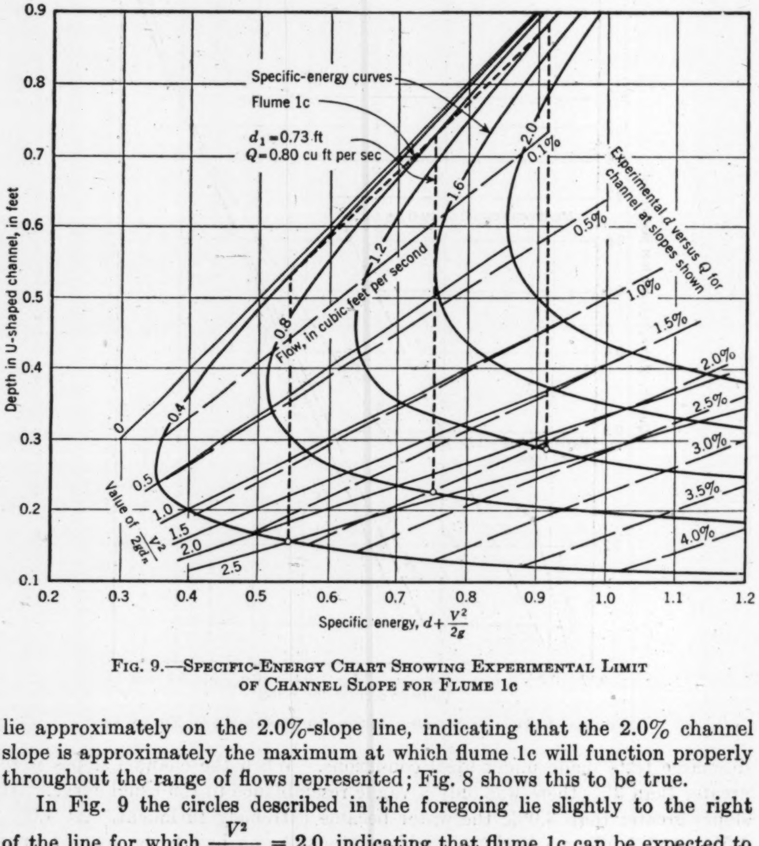


FIG. 9.—SPECIFIC-ENERGY CHART SHOWING EXPERIMENTAL LIMIT OF CHANNEL SLOPE FOR FLUME 1c

In Fig. 9 the circles described in the foregoing lie slightly to the right of the line for which $\frac{V^2}{2\,g\,d_n} = 2.0$, indicating that flume 1c can be expected to function properly in a channel in which the velocity head does not exceed twice the normal depth of flow. The $\left(\frac{V^2}{2\,g\,d_n} = 2.5\right)$-line very nearly defines the upper limit of energy in the flow for which the flume may be expected to function properly because the energy content is only slightly in excess of that needed to operate the flume.

Further investigations with other standard flumes indicated that, if $\frac{V^2}{2\,g\,d_n}$ is less than 1.5, operation of the flume will be satisfactory. Where the proportionality number exceeds 1.5 for channel flow at a proposed installation, the limiting number should be determined by plotting an upstream depth, d_1, for the given flume in Fig. 9. Thus, for the lower limit of zone 3 in Fig. 2, $d_n = \frac{V^2}{3\,g}$ is arbitrarily assumed as the minimum value for design purposes. It is suggested that this check of energy content of normal channel flow be made on the maximum flow to be measured by the flume.

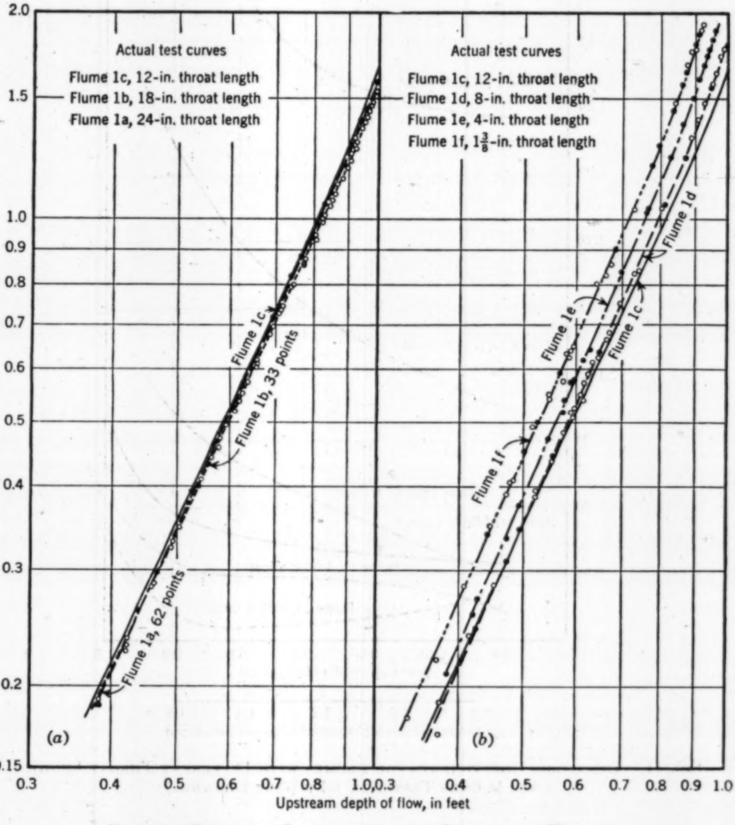


FIG. 10.—EFFECT OF LENGTHENING AND SHORTENING THROAT

Throat Length.—Six similar erection flumes (No. 1 series in which the throat length varied from $D / 8$ (0.12 ft) to $2\,D$ (2.0 ft)) were investigated to determine the effect of the throat length on the accuracy of flow measurement.

Fig. 10 shows the effect of such length on the calibration curve of flume 1c. Fig. 11 demonstrates the influence of throat length on the discharge coefficient for different depths of flow. For example, if two flumes with throat lengths of $D/8$ and D, respectively, were used with an assumed discharge coefficient of 1.0, the flow rate for the short flume might be as much as 30% less than that of the longer one. The results indicate that the throat length should be approximately equal to the pipe diameter. It should in no case be less than approximately 0.60 D.

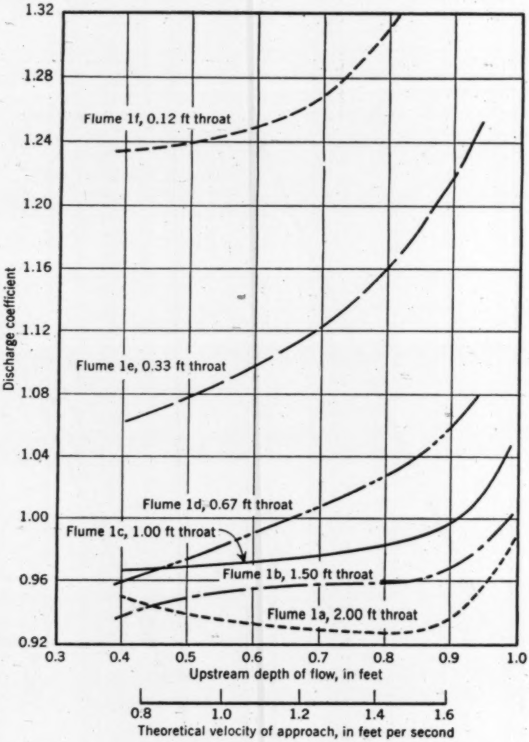


FIG. 11.—DISCHARGE COEFFICIENTS FOR FLUMES WITH DIFFERENT THROAT LENGTHS (ALL FLUMES TESTED IN 1-FT-WIDE CHANNEL)

Side Slope.—Flumes 1, 3, and 4 (Figs. 4(*a*) and 5(*a*)) had side slopes of 1 on 3, 1 on 2, and 1 on 1, respectively. Flume 6 (Fig. 5(*b*)) was a broad-crested weir and had no sides as such but used the channel walls as part of the throat section. Observation of the flow characteristics and the similarity of the calibration curves indicates that the shape of the throat is relatively unimportant providing that it satisfies the necessary energy relationships.

The rating curves for flumes 3, 4, and 6 were not continuous functions because the sides of the throat intersected the channel considerably below the top of the pipe. The fact that the throat is partly trapezoidal and partly rectangular or circular merely complicates the computations but otherwise does not impair the accuracy of the flume.

Base Height.—Palmer and Bowlus[11] use $D / 8$ and $D / 11$ for the height of the base above the pipe invert, whereas Ludwig and Ludwig[14] use $D / 12$. Some of the English flumes[15] have no base, and they rely on the side constriction to force the flow through critical velocity. The base of a trapezoidal flume serves two functions: (*a*) It holds the side walls in place with respect to each other; and (*b*) it aids in forming a plane-shaped throat, which is convenient, though not essential, in rating the flume. The base does not add any safeguard against submergence that would not be accomplished by increasing the side-wall constriction.

The base height, *t*, is defined as the difference in elevation between the channel invert at the point of measurement of the upstream depth and the top of the flume base at the point of occurrence of the critical depth. The flow in the throat of a Venturi flume is curvilinear so that any determination of the point of critical depth is hypothetical. However, for practical purposes it may be assumed that the critical depth occurs at a point near the center of the throat. For example, in flume 1c, the critical depth (vertical d_c) moves from 6 in. to 9 in. below the upstream edge of the throat as the flow increases.

The different base heights shown in Table 4 were studied to determine their relative importance on flume operation. Fig. 6, which presents discharge coefficients for the various flumes, also shows the effect of base height on flow measurement. Flumes 8, 6, and also flume 2c (which is not shown) are high based and did not function satisfactorily at very low flows. In the medium-height range, flumes 1 and 1c performed satisfactorily within the flow limits of the experiment. Flume 7, constructed so that the trapezoidal section rested on the channel bottom, maintained reasonable accuracy throughout all flows. Observation of the calibration curves of these flumes and a comparison of their discharge coefficients shows that the height of the base is of minor importance, although it appears desirable to use a low-flume base.

TABLE 4.—VARIATION OF HEIGHT OF BASE ABOVE INVERT

Base	ERECTOR SERIES		BROAD WEIR		SOLID FLUMES	
	Flume	*t*, in feet	Flume	*t*, in feet	Flume	*t*, in feet
High	2c	0.167	6	0.179	8	0.184
Medium	1c	0.090	...	...	1	0.089
Low	...	...	...	...	7	0.035

Terminal Transition.—Experiments were conducted on flume 1c with the terminal transition removed entirely. In this case, the flume consisted of an entrance transition and a throat so that the water shot, weir-like, back into the channel. Fig. 12(*a*) shows the observations plotted on the calibration curve of flume 1c with a terminal transition. The absence of terminal transition has no effect on the flow upstream nor on the accuracy of the flume. However, its absence would cause additional energy losses downstream from the flume.

---

[15] "Venturi Flume Flow Meter," by A. Linford, *Civil Engineering*, London, Vol. 36, 1941, pp. 582–588. (See Abstract, *Journal*, A.W.W.A., Vol. 34, 1942, pp. 1473–1475.)

Entrance Transition.—Palmer and Bowlus[11] state that the entrance transition (sides and base) for standard flumes should be approximately 1 on 3. The length may be restricted by difficulties of installation, and shorter transitions might cause excessive turbulence and loss of energy.

Experiments were made on flume 1c with an entrance transition having a ratio of 1 to 2. Fig. 12(*b*) presents these observations plotted on the calibration curve of flume 1c, with a 1-on-3 entrance transition. Because the difference between these two curves is negligible, it may be assumed that minor changes in the slope of the entrance transition will have no effect on the accuracy of the flume.

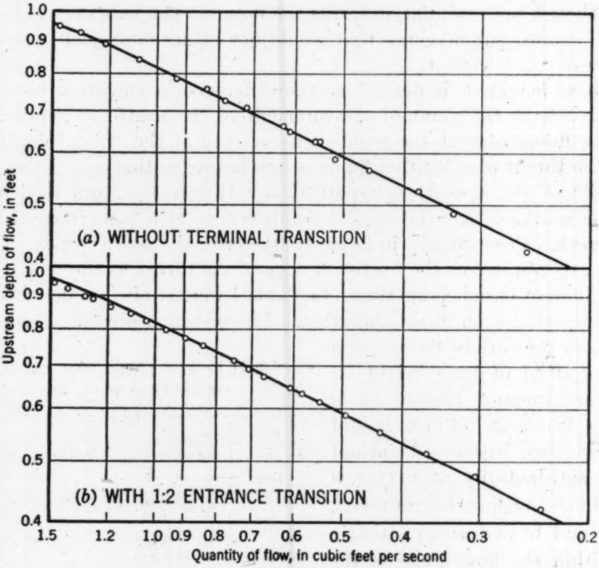


FIG. 12.—EFFECT OF TRANSITION IN COMPARISON WITH ACTUAL TEST CURVE FOR FLUME 1c

Point of Upstream Depth Measurement.—The upstream depth of flow, d_1, is defined as the flow depth in the channel at the point where water enters the transition section and was measured by a point gage in a stilling well. The flumes were placed in the test section so that the port of the stilling well was adjacent to the leading edge of the side transitions.

When flume 1c was moved downstream below the stilling-well port, a small drawdown curve of increasing curvature with increasing flow was effected. Thus, the point of measurement of the upstream depth should be at the entrance to the flume, or additional loss of energy in the channel will be included in the measurement.

CONCLUSIONS

The following conclusions concerning Venturi flume design and behavior can be made:

1. The Arredi rating method will give satisfactory results with a high degree of accuracy throughout a wide range of flows and channel conditions.
2. The discharge coefficient varies from 0.97 to 1.03 as a function of depth of upstream flow for normal operation when the depth of upstream flow does not exceed 0.90 of the pipe diameter (0.90 *D*).
3. Standard flumes will function properly if the velocity head at depth for the maximum flow is no greater than 1.5 times the normal depth. For special cases this ratio may be exceeded if proper investigation is made of the result.
4. The velocity of approach can be greater than 2.0 ft per sec and is not relatively important as a design criterion.
5. The maximum submergence should not exceed 85% of the upstream depth at the maximum flow.
6. These flumes will measure flows at depths of as much as 0.75 *D* under normal flow conditions—that is, at 90% of the maximum pipe capacity—but the maximum depth may vary with the slope of the pipe and its roughness.
7. The throat length of 1.0 *D* is preferable, but it may vary from 0.6 *D* to 1.5 *D* without greatly affecting accuracy.
8. The sides of the flume may slope as desired providing it is properly rated. Side slopes of 1 on 2 with a bottom width of *D* / 3 permit the measurement of a wide range of flow (of from 0.2 cu ft per sec to 1.75 cu ft per sec for a 12-in.-diameter pipe).
9. The height of the base for trapezoidal flumes should be between *D* / 10 and *D* / 20.
10. An entrance transition of 1 on 3 is desirable, but small changes in the slope do not affect the accuracy of the flume.
11. The slope of the exit transition has no effect on the accuracy of measurement. Such a transition is desirable only to conserve energy.
12. The point of upstream-depth measurement should be no more than 0.5 *D* upstream from the entrance to the flume.

ACKNOWLEDGMENTS

The research on the Palmer-Bowlus flumes was sponsored jointly by the Civil Engineering Division and the Sanitary Engineering Research Laboratory of the University of California, at Berkeley.

Joe W. Johnson, M. ASCE, and Erman A. Pearson, A.M. ASCE, assisted the writers in reviewing the analysis of the basic research. Special acknowledgment is made to Jayantilal Parikh for his help in conducting the laboratory experiments.

Norman H. Anderson of the Los Angeles County Sanitation Districts encouraged the testing program and made many valuable suggestions regarding the practical installation problems of Venturi flumes.

APPENDIX. RATING-CURVE COMPUTATIONS FOR FLUME 1c

Several graphical or semigraphical methods may be used to compute the rating curve. The Arredi method, as clarified by Harvey F. Ludwig, was used to compute the rating curves of the flumes used in this research because it is easily adaptable to differently shaped flumes and channels.

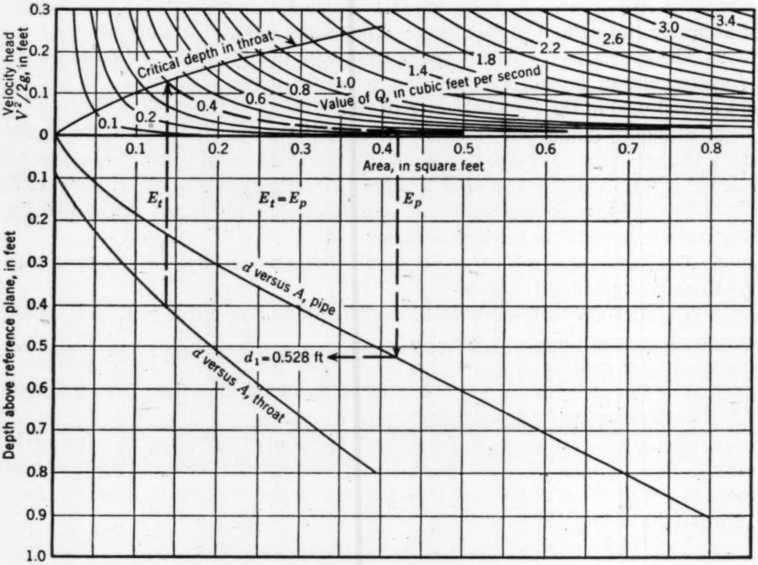


FIG. 13.—ARREDI DIAGRAM FOR FLUME 1c

TABLE 5.—COMPUTATIONS FOR RATING CURVE FOR FLUME 1c

d_c (1)	$d_c + t$ (2)	$y + (d_c/m)$ (3)	$A = d_c\left(y + \frac{d_c}{m}\right)$ (4)	$2B = 2y + (4 d_c/m)$ (5)	$\frac{V^2}{2g} = \frac{A}{2B}$ (6)	d_p (7)	A_p (8)	Q (9)	d_1 (10)
0.10	0.190	0.368	0.0368	0.799	0.0460	0.10	0.0408	0.2	0.378
0.20	0.290	0.400	0.0800	0.924	0.0866	0.20	0.1117	0.4	0.528
0.30	0.390	0.431	0.1293	1.049	0.1232	0.30	0.1980	0.6	0.636
0.40	0.490	0.462	0.185	1.174	0.1577	0.40	0.2930	0.8	0.727
0.50	0.590	0.495	0.2475	1.299	0.1905	0.50	0.3925	1.0	0.807
0.60	0.690	0.524	0.3143	1.424	0.221	0.70	0.5925	1.2	0.877
0.70	0.790	0.556	0.389	1.549	0.251	0.90	0.7925	1.4	0.947

Using the Arredi diagram,

$$d_1 = d_c + t + \frac{A_c}{2\,B_c} - \frac{V_1^2}{2\,g} \quad \text{..................} (9)$$

is solved graphically for values of d_1 corresponding to assumed values of Q. Three curves are plotted as shown in Fig. 13. Two show the area as a function of distance above the channel invert for the channel section and the throat section of the flume. A third curve shows the area as a function of velocity head for critical flow in the throat. The computations for these curves are shown in Table 5.

The following explains the computations made:

1. Assume values of critical depth in throat (Col. 1);

2. Add height of base, t, to obtain height of surface above pipe invert (Col. 2);

3–4. Find area as $d_c\,(y + d_c/m)$, and plot on Arredi diagram as d_t versus A (Cols. 3 and 4);

5. Find $2\,B$ as $2\,y + 4\,d_c/m$ (Col. 5);

6. Find $\frac{V^2}{2\,g} = \frac{A}{2\,B}$ for throat, and plot $\frac{A}{2\,B}$ versus A on Arredi diagram (Col. 6);

7. Assume depth in U-shaped pipe (Col. 7);

8. Find corresponding pipe area (from standard handbooks), the section above 0.5′ being rectangular; plot d_p versus A on Arredi diagram (Col. 8);

9. Assume value of rate of flow (Col. 9); and

10. From diagram, scale value $\left(d_c + t + \frac{V^2}{2\,g}\right)_t$ for a given flow rate (an example is shown for 0.4 cu ft per sec). With this value, find the depth in the pipe, d_1, for which $\left(d_1 + \frac{V^2}{2\,g}\right)_p = \left(d_c + t + \frac{V^2}{2\,g}\right)_t$ (Col. 10).

DISCUSSION

Keeno Fraschina[16].—The North Point Sewage Treatment Plant of San Francisco (Calif.), which was placed in operation in December, 1951, is a primary treatment plant designed for an average dry-weather flow of 65×10^6 gal per day and a maximum wet-weather flow rate of 150×10^6 gal per day.

Plant inflow is measured as the summation of flow in four Parshall flumes that are used to regulate velocity of flow in four grit tanks. Plant outflow is measured by a Palmer-Bowlus flume in a covered channel between the sedimentation tanks and the postchlorination contact tank. The Parshall flumes are followed by a sewage-lift station. Consequently, the plant outflow rate depends on sewage and sludge pumping rates and varies from the plant inflow rate accordingly.

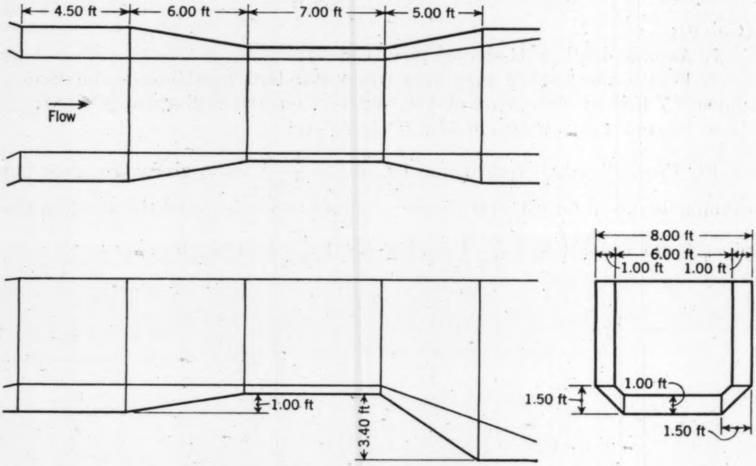


Fig. 14.—The Palmer-Bowlus Flume

The Palmer-Bowlus flume is designed for a flow range of between 15×10^6 gal per day and 150×10^6 gal per day. The flume is accurately constructed of smooth concrete to the dimensions shown in Fig. 14. The upstream channel is trapezoidal rather than circular. The theoretical rating curve was derived by a trial-and-error solution of Eq. 2. Dimensions and flow characteristics listed as essential in the paper are given in Table 6. The value of D is taken as the width of the upstream channel, or 8.00 ft.

The flume differs from the design recommendations (under the heading, "Conclusions") as follows:

1. The base height, $D/8$, slightly exceeds the recommended range of from $D/10$ to $D/20$ given for trapezoidal flumes.

---

[16] Supt., North Point Sewage Treatment Plant, San Francisco, Calif.

2. The slope of the entrance transition of 1 on 6 is less than the recommended slope of 1 on 3.

The foregoing differences seem too inconsequential to affect flume accuracy.

TABLE 6.—DIMENSIONS AND FLOW CHARACTERISTICS FOR PALMER–BOWLUS FLUME

Item	Value
Throat length	0.88 *D*
Side slope, all sections	1:1
Base height	*D*/8
Entrance transition slopes:	
Sides	1:6
Base	1:6
Terminal transition slopes:	
Sides	1:5
Base	1:1½
Distance from flume entrance to point of upstream-depth measurement	0.09 *D*
Upstream channel slope:	
First 2 ft upstream of flume entrance	level
Balance	<0.1%
Velocity of approach, in feet per second[a]:	
15×10^6 gal per day	1.5
150×10^6 gal per day	5.0
Velocity head, maximum flow[a]	0.07 *D*
Submergence at 150×10^6 gal per day[b]	85%

[a] From theoretical rating curve; flume not accessible for measurement. [b] Ratio, observed tailwater depth to d_n from the theoretical rating curve.

Flume Accuracy.—The only method of checking the accuracy of the flume is by making a comparison with plant inflow measured in the Parshall flumes. Table 7 shows such a comparison as measured by daily average flow rates on twenty-one weekdays during July, 1956. A statistical test of the differences

TABLE 7.—DETERMINATION OF FLUME ACCURACY (FLOWS IN 10^6 GAL PER DAY)

Date in July, 1956	Inflow from Parshall flumes (1)	Sludge from sludge Venturi meter (2)	Sewage outflow from Palmer-Bowlus flume (3)	Sewage outflow, Col. 1–Col. 2 (4)	Col. 3-Col. 4 (5)
2	42.8	0.9	42.1	41.9	0.2
3	42.2	0.9	42.0	41.3	0.7
5	42.7	0.8	41.9	41.9	0.0
6	43.1	0.8	42.7	42.3	0.4
9	43.7	0.8	43.2	42.9	0.3
10	42.9	0.8	42.8	42.1	0.7
11	43.1	0.8	42.9	42.3	0.6
12	43.0	0.8	41.8	42.2	−0.4
13	43.4	0.9	42.4	42.5	−0.1
16	43.4	0.9	41.7	42.5	−0.8
17	43.1	0.9	42.0	42.2	−0.2
18	43.8	0.9	42.2	42.9	−0.7
19	42.7	0.9	41.7	41.8	−0.1
20	43.6	0.9	41.5	42.7	−1.2
23	43.2	0.9	41.9	42.3	−0.4
24	43.5	0.9	42.4	42.6	−0.2
25	43.7	0.9	42.4	42.8	−0.4
26	42.6	0.9	41.7	41.7	0.0
27	42.4	0.9	41.8	41.5	0.3
30	42.9	0.8	41.9	42.1	−0.2
31	43.0	0.9	42.5	42.1	0.4
Average	43.1	0.9	42.1	42.2	−0.1

in Col. 5 by the "Students *t* Test" gives a *t*-value[17] of 0.47. The probability of obtaining an equal or greater value of *t* is more than 60%, from which it can be concluded that there is no statistically significant difference between the two methods of measurement. This is also shown diagrammatically in Fig. 15, in which the data in Cols. 3 and 4 are plotted on arithmetic probability paper. All points fall on a single straight line, and both sets of measurements can be considered to be normally distributed with the same inherent variability. Errors of measurement by the primary measuring elements and their flow integrating devices are included in this variability.

It would be desirable to supplement the average daily flow readings given in Table 7 with instantaneous readings taken over the entire range of flow. Unfortunately, steady flow conditions in the lift-pump sump cannot be maintained long enough to make accurate measurements. Nevertheless, influent and effluent flow charts (each readable to 10^6 gal per day) seldom differ by more than 10^6 gal per day over the range of from 15×10^6 gal per day to 100×10^6 gal per day, when the sump level remains relatively constant and sludge withdrawal from sedimentation is at a steady rate of 0.9×10^6 gal per day.

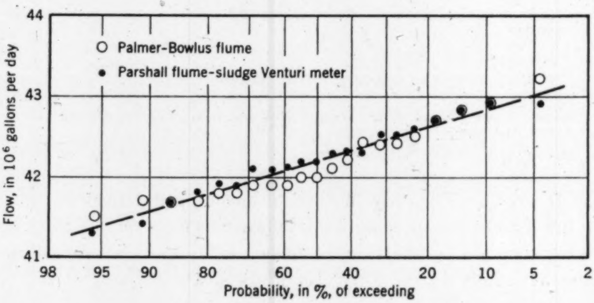


FIG. 15.—THE PROBABILITY OF FLOWS AT NORTH POINT PLANT

Accuracy of this Palmer-Bowlus flume can be considered to be equal to that of the Parshall flumes and probably well within the limit of 3% found by the authors.

The flume responds promptly to changes in flow. A hydraulic jump is observable within or below the exit transition section under all flow conditions. There is no evidence of solids deposition in the channel ahead of the flume at flows of as low as 15×10^6 gal per day. No maintenance has been required in the five years the flume has been in service.

EDWIN A. WELLS, JR.,[18] A. M. ASCE, AND **HAROLD B. GOTAAS,**[19] M. ASCE.—The Venturi flume at the North Point Sewage Treatment Plant in San Francisco described by Mr. Fraschina is one of the largest Palmer-Bowlus flumes. The writers visited this installation while it was under construction. The methods used in the construction were most precise, and the flume design conformed, in general, with the findings of laboratory studies conducted at that time.

---

[17] "Statistical Methods for Chemists," by W. J. Youden, John Wiley & Sons, Inc., New York, N. Y., 1951, p. 28.

[18] Municipal Financing Consultant, Stone & Youngberg, San Francisco, Calif.

[19] Dean, Technological Inst., Northwestern Univ., Evanston, Ill.

Mr. Fraschina's study of the behavior and accuracy of this flume has indicated that the performance of a flume of this size conforms closely with the results of the laboratory tests. As he has noted, the fact that the upstream channel is not circular has no relative bearing on the desirability or usefulness of the Palmer-Bowlus type of Venturi flume for measuring flow.

Except for minor variations, the North Point Palmer-Bowlus flume meets the general criteria that have been found applicable. It should be emphasized that the energy criterion developed in the laboratory studies is satisfied by the nature of the installation and by its relative position to the hydraulic grade line through the sewage treatment plant.

Mr. Fraschina has performed the first large-scale correlative tests to come to the attention of the writers. They provide a valuable background for the future utility of the Palmer-Bowlus flume. The dimensions of each such flume can be measured after it has been built, and the rating curves can be modified to reflect even minor variations in the final construction. Because of the 'standard" nature of the Parshall flume, rating curves cannot be modified where the final flume as built differs from its original design.

The writers have been advised that several Palmer-Bowlus flume installations (in sewers) do not perform the metering functions satisfactorily. This can probably be traced to the fact that prior to the availability of the energy criterion presented from laboratory tests the installation was designed principally on the basis of dimensional variables and approach velocity.

The most valuable product of the laboratory research is the development of a working criterion for the design of the Palmer-Bowlus flume based on its energy relationships. Because Venturi-type measuring flumes must be installed where rapid flow normally occurs, in sewers or in treatment plant channels, an understanding of the energy limits within which such a flume can be used should increase its utility. For conservative design these flumes have heretofore been largely restricted to uses where tranquil flow can be assured. The Parshall flume, which was originally developed solely for use in earthen irrigation ditches in which there is tranquil flow, has been used unwittingly for a variety of installations in which there is rapid flow. To the writers' knowledge, no energy criterion similar to that available for Palmer-Bowlus flumes has ever been developed for the Parshall flume.

The Palmer-Bowlus flume is becoming more widely used in the western United States. For normal installations the design has not proved difficult or cumbersome. However, because standard Palmer-Bowlus flumes have not been developed to the point at which they are readily available in engineering handbooks, this type is not as popular and as well understood as the Parshall flume. Standard dimensions and rating curves for Palmer-Bowlus flumes need further development in order that the normal metering problems, such as those suggested by Ludwig and Ludwig,[14] can be handled.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2938

IMPORTANCE OF GROUND WATER IN THE NATIONAL ECONOMY

BY ALBERT G. FIEDLER,[1] M. ASCE

SYNOPSIS

The use of water derived from underground sources by wells has increased greatly. Although ground water meets only approximately 15% of the United States' total water requirements, it is an important resource on which a large part of the population depends. There is no national ground-water problem as such, but there are numerous widely scattered problem areas, each differing more or less markedly from neighboring areas because of distinctive geologic and hydrologic environment and other local factors. Typical ground-water problems in six areas are reviewed, and the solution of each must rest on adequate knowledge of the character and capacity of the ground-water reservoirs involved, and on comprehensive planning for the best use of the available supply. It will be complex and expensive to solve many of the major problems, and practically all problems will require considerable public education and support to achieve adequate solution.

INTRODUCTION

As has been clearly stated in the report[2] on water-resources policy, there is no "national" water problem. Instead, there are individual problems distributed over the United States, relating to the use and development of the water resources, which differ in character and severity from place to place. It is appropriate to devote attention to the importance of ground water and ground-water problems in the economy of the United States.

The great attention that water resources have been receiving is substantial evidence that both public and private interests have become increasingly aware that continued economic expansion is limited by the availability of an adequate water supply of good quality. Inexhaustible supplies of potable

---

NOTE.—Published, essentially as printed here, in June, 1957, in the Journal of the Sanitary Engineering Division, as *Proceedings Paper 1271*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Asst. Chief, Water Resources Div., Geological Survey, U. S. Dept. of the Interior, Washington, D. C.

[2] *Water Resources Law*, Report of the President's Water Resources Policy Comm., Washington, D. C., 1950.

water do not exist, and it is well recognized that stock must be taken of the water resources that are available in order that their use and conservation may be planned wisely in order to meet the increasing demands of modern industrial civilization.

It is obvious that there is no single solution that can be applied to all water problems. A wide variety of solutions are required, each adapted to the particular conditions and demands of the area under consideration. There are water-supply shortages in many places. In others, at present rates of withdrawal or at prospective rates, shortages will occur in the near future. In still other areas in which ground water is a major source of supply, it is exhausted by being withdrawn from the underground reservoirs at a rate far greater than the rate of replenishment. In such areas the water is, in effect, mined. Although there are many water problems throughout the United States in both humid and arid regions, their multiplicity does not indicate that the nation is "running out of water."

Despite pessimistic predictions that the United States will not be able to meet the large prospective demand for water, the situation, as a whole, is not dark. The United States is one of the best watered nations in the world, and although population and industry are expanding, technical knowledge of water resources also is growing, even though not in proportion to demands for information. However, problems are being defined more clearly, and new solutions are being developed constantly. In many cases the solutions will involve large expenditures. However, as in many problems, such solutions concern economical rather than physical feasibility, and necessary economic adjustments will be made wherever failure to make them would result in a still greater cost to the community and the country as a whole.

IMPORTANCE OF GROUND WATER INDICATED BY SIZE OF INDUSTRY

The importance in the national economy of water developed from the ground by wells can be illustrated by examining the well-drilling industry. The Water and Sewerage Industry and Utilities Division of the Business and Defense Services Administration (United States Department of Commerce) provides the best estimate available of the size of this industry.[3] Although no complete census has been taken, it is estimated that there are more than 9,000 well-drilling contractors in the United States. This figure has been adjusted to allow for a relatively large percentage of part-time operators who construct a few wells each year but have other activities as their major occupation.

It is estimated that the 9,000 well-drilling contractors operate approximately 22,000 drilling machines and that they employ about 50,000 workers. Estimates of the number of wells constructed each year range from 460,000 to 600,000. The latter figure seems somewhat high in relation to estimates of materials used to construct the wells and equip them with pumps. Therefore, it seems reasonable to estimate the total number of wells (as of 1957) constructed each year as approximately 500,000. The new construction involves expenditures estimated at $475,000,000 in 1953. This sum does not include dug wells and small driven and bored wells installed by hand, many of which are still being used in some areas.

---

[3] "Water Well Drilling—a Big Industry," by W. L. Picton, *Water Well Journal*, Champaign, Ill., November–December, 1954.

Walter L. Picton, A. M. ASCE, has estimated[3] that the total number of wells in operation has grown from an estimated 7,000,000 in 1900 to approximately 13,500,000 in 1957, and that by 1975 the total may reach 16,000,000. In reviewing these figures it should be realized that a substantial part of the annual construction—perhaps approximately 300,000 of the 500,000 wells—represents the replacement of wells that have reached the end of their economic life, so that approximately 200,000 wells are added to the total inventory each year.

The total water requirements of the United States have grown tremendously. It is estimated that in 1900 the aggregate use of water was approximately 40×10^9 gal per day, of which 7×10^9 gal per day were supplied by wells. The total use in 1955 is estimated at about 260×10^9 gal per day, of which almost 40×10^9 gal per day were obtained from underground sources Assuming that the economy will expand and that present trends will continue, it is reasonable to predict that normal growth in population, nonfarm homebuilding, movement of population to suburban areas, modernization of farms, industrial growth, and the demand for water for irrigation will cause a large increase in the use of water from wells. By 1975 the quantity developed from the ground may reach 69×10^9 gal per day, according to Picton.

The well-drilling industry is not large if compared with other segments of the economy, but it has great importance when viewed in relation to the present standard of living. It has been a significant factor in the decentralization of many industrial operations, in the movement of population from cities to suburban and rural areas, and in the expansion of irrigation in both humid and semiarid regions.

PROBLEM AREAS

As a "penalty" for the many benefits accruing from ground-water use, its development in many sections of the United States has created certain problems. Perhaps a good way to show the varied and individual nature of the problems would be to examine a few briefly. They all have one factor in common: There is danger that there will not be enough water of the necessary quality or temperature, at the specific points where it is desired, and at a convenient or feasible cost. On other major points, such as local geology and hydrology, local economic situation and pattern of water needs, and the local background of water law as it influences the ways in which the right to use water can be acquired, each problem area differs more or less markedly from its neighboring areas.

The western United States has its full share of major water problems. One might say more than its share if it were not that the situation in the west is the inevitable result of regional hydrology and national history, and therefore is hardly surprising or unexpected. In 1874, in spite of the most optimistic claims of promoters as to the west's fertility and productiveness, J. W. Powell made an astute observation and prediction: There would never be enough water for all the irrigable acreage. In fact, there would be only enough water for from 1% to 3% of the west's total area. In making this prediction he neatly bracketed the existing (1958) irrigated acreage in the west—roughly 2% of the total area. It is evident that the limit of practical irrigation in the west has been reached. New irrigation projects are already balanced by the retirement of farmland as the result of water depletion, soil deterioration, suburban growth, and highway and industrial construction. According to Fred A. Seaton,[4] the foregoing and other causes have removed more than 5,500,000 acres of land from farm use, a substantial part of it being irrigated land, in the period from 1954 to 1956.

Roswell Basin.—The Roswell artesian basin[5] in New Mexico is one problem area. Although it differs from the typical western valley in geology and hydrology, the Roswell Basin shares with many others the characteristic of a perennial natural water supply which, although it is large, can be so overdeveloped that its usefulness in supporting the economy of the region can be impaired.

The basin lies on the easterly dip slope of the Sacramento Mountains. Its history dates from the time when the mountains themselves were formed by the uplift and eastward tilting of a huge block of the earth's crust. The broken western edge of the block forms the steep west face of the mountains, overlooking the Tularosa Basin into which the adjacent block foundered. As the Sacramento block rose, streams flowing from its crest carried erosional debris eastward, forming a part of the vast alluvial apron whose remnants are known as the High Plains. Subsequent uplift and erosional and solutional activity resulted in the trenching of the alluvial apron and isolation of the High Plains from the mountains. In the Roswell area the separation of mountains and plains was achieved by the southward-flowing Pecos River. The river probably once flowed somewhat west of its present course, at a higher level, and reached its present position by gradual retreat down the dip slope of the beds along the flanks of the Sacramento Mountains. As the Pecos River shifted eastward, it cut down and, together with its westward tributaries, stripped away still more of the younger sedimentary rocks of the mountain block, eventually exposing the soluble limestone that was to become the aquifer of the Roswell Basin. The river retreated eastward down the dip slope faster than it could remove all the rocks above the limestone. The result was a belt of limestone exposed on the east flank of the mountains. East of this belt the limestone strata dip beneath the still unremoved shale, gypsum, and sandstone of the overlying beds.

As the mountains rose, greater precipitation was induced along their crest, and as larger areas of the limestone strata were exposed by erosion, progressively larger quantities of water entered through cracks and moved down the dip in the limestone. Much of the water returned to the surface in springs at the retreating featheredge of the younger rocks, but some of it passed beneath those rocks, gradually enlarging the crevices as it moved eastward. Farther down the dip the openings in the rocks became so few and other outlets so remote that it was easier for the water to seep upward through the overlying rocks and into the alluvium along the Pecos River than to move farther east.

---

[4] "Secretary Seaton Says We Must Bring Water to Every Thirsty Acre," by Fred A. Seaton, *Relamation News*, National Reclamation Assn., Washington, D. C., November, 1956.

[5] "Geology and Ground Water Resources of the Roswell Artesian Basin, N. Mex.," by A. G. Fiedler and S. S. Nye, *Water-Supply Paper 639*, Geological Survey, U. S. Dept. of the Interior, Washington, D. C., 1933.

In time the ground-water system developed to the stage that existed when man discovered the artesian basin: Water entered the outcrop of the limestone on the mountain slope, passed beneath the confining beds of younger rocks in openings that the water had created or enlarged by solution, seeped upward under the artesian head provided by the altitude of the recharge area, entered the alluvium deposited along the river, and eventually discharged into the river or was captured by the thirsty flood-plain vegetation.

The hydrologic make-up of the Roswell Basin has been described in detail because the description emphasizes an important point: The availability of ground water in any area is governed by the hydrology of that area, which is different in one or more major respects from the hydrology of all other areas, in spite of other similarities.

It was soon discovered that the Roswell Basin provided a bountiful supply of water by natural flow by drilling into the limestone that was made cavernous and permeable by circulating water. Some wells flowed thousands of gallons a minute, and it is not surprising that early water users considered the source inexhaustible. However, as development mushroomed during the period from 1905 to 1915, it soon became apparent that there was a limit to the water supply. As well after well was drilled, the artesian head declined. The yields of all the wells diminished, and the wells on higher ground stopped flowing altogether. Poorly constructed wells or rusted casings in the older ones allowed water to leak directly into the alluvium at a rate greater than was permitted by the relatively tight rocks through which it was necessary for the water to discharge under natural conditions. It was necessary to install and operate expensive pumping equipment to continue the irrigation of lands previously supplied with flowing artesian water. Ultimately, the decline in head became so great that many farms along the western side of the basin at higher elevations were abandoned because crop returns were inadequate to meet the high cost of pumping the artesian water.

Thus, early in the 1920's the economy of the area began to change. The Geological Survey (United States Department of the Interior) was requested to investigate the basin in cooperation with the state engineer in order to determine how much water could be developed perennially so that the state would have a basis for the inevitable regulatory action. The Survey's investigation[5] formed the basis of a state law,[6] which was enacted in 1927, declaring that waters "in underground streams, channels, artesian basins, reservoirs, or lakes, the boundaries of which may be reasonably ascertained by scientific investigations or surface indications" were public waters and subject to appropriation for beneficial uses under the existing state laws relating to the appropriation of waters from surface streams. In order to test the constitutionality of this legislation, two suits were brought into the district court, and in 1929 the State Supreme Court of New Mexico rendered a far-reaching opinion upholding the law in principle but declaring it unconstitutional. The objection was strictly a technical one: The law incorporated an existing statute, but referred to it only by title instead of quoting it in full, as required by the state's constitution. A new act,[7] drawn in order to overcome the court's objections, was passed in 1931. This law has withstood all legal attacks and has served as a model for similar laws in several other states. It provides a basis on which the use of water can be controlled so that its use can be held within safe limits.

---

[6] *Laws*, approved March 16, 1927, New Mexico State Legislature, 8th Session, Chapter 182.

In the atmosphere provided by the knowledge of the basin's water resources and by the control of development under the state law, the local economy was strengthened. Leaky wells were repaired or sealed to reduce the loss of water through them. Pumping of ground water from the alluvium, which was recommended by the Survey as a means of providing water for additional acreage without greatly increasing the draft on the artesian reservoir, was expanded. Other recommendations of the investigation were also implemented, and the agricultural economy of the basin again was placed on a sound basis.

It would be gratifying to report that this area has no water problems, but this is not correct. As a result of high prices for farm products during and since World War II, the withdrawal of ground water from the basin as a whole has increased persistently, despite the control afforded by the state law. This increased pumping has created a situation in which ground-water levels have declined to new lows, saline water has encroached from the east, and the agricultural economy of the basin again is being threatened.

An adjudication of the water rights in the artesian basin is currently (1958) in progress. When completed, this procedure will establish the priority of rights and undoubtedly will eliminate from the irrigated area some acreage for which there are no valid water rights. However, the increase in the use of ground water for irrigation has been so great that the reduction in draft resulting from the elimination of acreage having no valid water right probably will not be sufficient to bring the remaining draft within the safe yield. Further reduction in draft will undoubtedly be necessary, either through limiting the use of water on acreage of low water-right priority, through a reduction in the use of water on water-right land, or through the relentless action of economic and hydrologic law.

In time a part of the water will undoubtedly be diverted to certain industrial or other uses that give a greater economic return than irrigation. Nevertheless, the goal of economic security will be reached only through the recognition of the nature of the problem by both water users and the public, and by hard work and cooperation on the part of all for many years. No permanent solution of the ground-water problem will be successful unless it is based on the widespread acceptance of the fact that, in the long run, water use cannot be continued at a rate that is substantially more than the perennial yield of the ground-water reservoirs.

The Roswell Basin is an area which, although it has a large natural ground-water supply, has a hydrologic setting favorable to a permanent water supply, and is in a state having a model water law, requires constant vigilance and persistent effort. Many other areas have less assets and more liabilities, and yet must achieve a comparable solution.

---

[7] *Laws*, approved 1931, New Mexico State Legislature, 10th Session, Chapter 131.

Southern High Plains.—Not far from the Roswell Basin is an area in which the water problem is even more difficult—the High Plains. The High Plains area has extensive natural ground-water storage but negligible natural recharge; that is, the ground water resembles coal or oil in that to use it on a large scale is to mine it out.

The High Plains are a vast apron of alluvium spread by the eastward-flowing streams that carried water and erosional debris from the Rocky Mountains. After the apron was laid down, it was isolated from its mountain source by erosion, and it was divided into several large segments by the major streams crossing it. Thus, the High Plains consist of several large areas, each of which depends on local precipitation for its ground water. Because precipitation is light (20 in. or less in much of the area), evaporation is high, and there is a tight subsoil in a large part of the plains, nearly all the precipitation is discharged by evaporation and transpiration. Little runs off, and an average of only a fraction of 1 in. per yr can penetrate to the zone of saturation in the lower part of the alluvial apron, which rests on the tight rocks below.

The southern High Plains[8] of eastern New Mexico and western Texas, south of the Canadian River, are a vast underground reservoir, which stores approximately 200×10^6 acre-ft of water but is replenished at a rate of probably less than 100,000 acre-ft and perhaps less than 75,000 acre-ft per yr.

Domestic, stock, municipal, and industrial water supplies in the High Plains are obtained almost completely from wells because surface-water supplies are almost nonexistent. However, until the 1930's there were few irrigation wells. Then the droughts and crop failures clearly demonstrated that precipitation could not be depended on for successful crop growth year after year. Since about 1935 the development of ground water for irrigation in the southern High Plains, particularly in Texas, accelerated at a rate unmatched elsewhere in the United States. Each year a new record is set in the number of wells in use and quantity of water pumped. Pumping in the High Plains has been the largest single factor in the tenfold increase in ground-water pumpage in Texas in the last decade, to a total for the state of more than 7×10^6 acre-ft per yr, second only to California. This water is being mined in the High Plains because the natural discharge at the edges of the plains continues unabated and will not be affected substantially by the pumping for many years in the future. The increased pumping in the New Mexico section of the southern plains has been less spectacular, but it is still impressive.

What type of situation faces the users of ground water in the southern High Plains? At first many refused to believe that the water supply was not inexhaustible. Their contention was strengthened by the heavy rains of 1941, which raised the water table to record heights and gave every impression of abundance. However, the rains of 1941 were the heaviest, or almost the heaviest, on record in much of the southwestern United States, and they probably will not be repeated, on the average, for 50 yr or 100 yr. As evidenced by the fact that most recharge on the High Plains occurs in such years and that little or no recharge occurs in average or dry years, it is easy to realize that the

---

[8] "Geology and Ground Water in the Irrigation Region of the Southern High Plains of Texas," *Progress Report* 7, Texas Board of Water Engrs., 1949.

1941 recharge was only a "drop in the bucket" compared with the pumpage since that year.

Thus, the well owners of the High Plains have gradually become aware that they are pumping from storage. It is too early to state that they have have become adjusted to the idea because it is not known exactly how the region will meet the problem of depletion of stored ground water. If events are allowed to run their course without interference, it will be necessary to abandon irrigation from wells in certain areas, beginning at centers of heavy pumping and growing outward from those centers as the water is depleted in the outlying areas. Landowners in the affected areas will have to turn to dry farming, grazing, or other activities, or sell out and move. Areas in which the saturated thickness of the water-bearing formation is small will be the first to experience difficulty in obtaining water in quantities sufficient for irrigation, whereas those areas in which the saturated thickness of the formation is large may have adequate water for heavy irrigation for a relatively much longer period.

What are the possible solutions? Storage and distribution of the available flow of the major streams might help locally but could not replace the water from wells—there is not enough surface water. Importing large quantities of water from the more humid regions to the east is a dream of the future, as is conversion of whatever saline water may be present. Artificial recharge of some of the water that now ponds and evaporates is somewhat promising, but how far it can go toward meeting future water demands that are as large as the present demands is still an unanswered question. Economy in the use of ground water in irrigation can help to some extent at least, and efforts in that direction are widespread.

The problem is complicated by the fact that a single aquifer is divided by a political boundary into two parts. In each part the approach to conservation and allocation of ground water differs widely from that in the other because of basic differences in legal philosophy. In New Mexico the appropriation principle is followed. The doctrine of prior appropriation is based on the assumption that the water supply is perennial, whereas the water supply in the High Plains is exhaustible, in so far as large demands are concerned, unless artificial recharge can be practiced on a scale far greater than seems possible under present conditions. Thus, it would be impossible to acquire rights to the use of water at a total rate greater than the average recharge, which would mean that only domestic and limited municipal and other uses could be sustained perennially. The state engineer allows additional water to be appropriated until, in his judgment, a point is reached at which existing developments can continue successfully for a period of 40 yr before the water is so depleted that the developments become uneconomical. Further applications are denied because they would shorten the useful economic life to less than the 40-yr period selected as the minimum necessary to compensate the individual, the community, and the state as a whole for the exhaustion of this natural resource.

Texas law recognizes the principle of prior appropriation to some extent for surface streams and so-called underground streams, but not for so-called percolating water, which includes the water beneath the High Plains. A specific statute[9] provides for the recognition of underground reservoirs of percolating water and for the formation of water-conservation districts having the power to establish regulations governing the conservation of ground water. One such district, covering essentially all the southern plains in Texas, has been formed. It actively promotes the conservation of water by eliminating wasteful use and by research on artificial recharge of storm water.

Here, too, is an area whose problem will be solved only by strenuous effort over the years, using means that have never been utilized before because a problem of this kind and on this scale has never been encountered before. If the problem is solved successfully, as it must be, a significant addition will have been made to man's "bag of tools" for handling analogous problems elsewhere.

Miami, Fla.—The humid eastern United States also has water problems. For example, an area that is about as humid as any in the east is the Miami area of Florida, where the average annual rainfall is approximately 60 in. Although the climate is warm and the loss of water by evaporation and transpiration is high, the area is underlain by soil and rocks that are so permeable that a large quantity of rainfall escapes evapotranspiration and recharges the ground-water body. The permeability and transmissibility of the sand-and-limestone aquifer are among the highest known in the United States and the world. For example, pumping at a rate of more than 40×10^6 gal per day from the city's Miami Springs-Hialeah well field produces a cone of depression in the water table that is less than 4 ft deep. The maximum yield of the more productive wells throughout the area is thousands of gallons per minute, and is determined more by practical considerations of pump and casing size than by the yield of the aquifer.

Where, then, is there a problem in this veritable paradise? The answer lies in salt water and in drainage canals. Much of the metropolitan Miami area lies on a low coastal ridge and is not subject to widespread flooding. However, in certain coastal areas and in the outlying areas, flooding occurs during certain parts of most years so that the land either must be drained or filled to make it usable for ordinary development. The drainage canals have lowered the water table and with it the head that formerly kept the aquifer so full of fresh water that fresh springs discharged along the shore. With the lowering in head, salt water has moved inland on a broad front along the shore and in the form of underground tongues along the canals. The salting of nearshore wells, including those of the Coconut Grove well field, brought the problem to public attention in the early 1930's. However, the problem did not become acute until 1939, when drought conditions and a resultant low canal flow permitted salt water from Biscayne Bay to advance upstream in the Miami Canal to a point beyond the Miami Springs-Hialeah well field. As a result of the lowering of the water table caused by pumping, the salt water seeped out of the canal through its permeable bottom and sides and entered some of the public-supply wells. Meanwhile, the landward advance of the underground salt-water tongues along the Miami Canal and other tidal canals was accelerated. It was at this time that the Geological Survey's ground-water studies, made in cooperation with the Florida Geological Survey, were broadened to include cooperation with Dade County, the cities of Miami and Miami Beach, and other agencies, which resulted in an intensive study[10] of the hydrology of the Miami area and the rest of southeastern Florida. This study showed that the Miami area can rely on a substantial ground-water supply as long as salt-water encroachment in and along the tidal canals can be controlled. The principal methods of control are adjustable dams, which are placed in the canals and opened during normal and high water levels to allow the canals to control the rise of the water table, but are closed during low water levels to halt the inland flow of salt water in the canals and to maintain a higher water table.

---

[9] *House Bill 162*, approved June 2, 1949, Texas State Legislature, 51st Session.

At present (1958) the salt-water problem is more or less under control, but a few troublesome spots remain. Certain tidal canals are still uncontrolled, and the control structures in others are too far upstream to restrain salt-water encroachment nearer the coast. Also, in recent years many new tidal canals have been proposed for low-lying coastal areas in order to obtain fill for reclaiming swampy land for homebuilding and to provide access to the bay for pleasure boats. In effect, each canal would be an extension of Biscayne Bay, bringing salt water inland as far as the canal extends and with it the threat of additional salt-water encroachment. Wherever feasible, appropriate control measures should be taken to safeguard the precious ground-water supply.

Long Island, New York.——The greatest metropolis in the United States—New York (N. Y.)—has had a ground-water problem that is similar in some respects to that of Miami. The water supply for New York City is obtained primarily from upstate surface reservoirs, but ever since the early days a large share of the supplies of both domestic and industrial water on Long Island has been obtained from wells. Long Island accumulated much of the debris scraped up and carried along by the southward-moving glaciers of the Pleistocene epoch, several thousands of years ago. Advancing from rocky New England, the ice sheets amassed vast quantities of comminuted rock and dumped it, blanketing a ridge of eroded Cretaceous coastal-plain sediments. The glaciers built up a compound ridge of largely permeable glacial drift, which forms most of what is now known as Long Island. This sandy drift, recharged from precipitation at a rate of approximately 1,000,000 gal per day per sq mile, makes of Long Island's 1,400 sq miles an aquifer having a potential yield of millions of gallons per day. Beneath the drift are Coastal Plain strata which include many beds of sand that form productive aquifers. These aquifers do not have their own source of recharge because they depend on rainfall passing through the overlying glacial drift. For this reason they do not actually add to the ground-water potential. However, they do provide a means of obtaining water in areas in which the water in the drift is scanty or of poor quality. In fact, because of the possibility of contamination, there is a tendency to develop water supplies in the deeper aquifers, and nearly half the total pumpage is from these older deposits.

---

[10] "Water Resources of Southeast Florida," by G. G. Barker et al, *Water Supply Paper 1255*, Geological Survey, U. S. Dept. of the Interior, Washington, D. C., 1956.

Long Island's abundant ground water has long been recognized and utilized. Because the population is greatest at the western end, the ground-water withdrawal has been concentrated there also. In fact, the withdrawal became so great that by the late 1920's the water table had been drawn below sea level in a large part of Brooklyn (N. Y.), and the lowering extended to the shore in broad stretches and induced salt-water encroachment. The recognition of this danger led in the early 1930's to the enactment of a state law[11] empowering the State Water Power and Control Commission to control ground-water withdrawal on Long Island, except for agricultural purposes. As a result of that agency's regulations, there has been a considerable reduction in ground-water withdrawal in the areas of heaviest pumping. Ground water used for air conditioning, but not contaminated by that use, is returned to the ground at the rate of tens of millions of gallons per day in what is the most advanced development of artificial recharge through wells yet undertaken. Little by little, public authorities have closed down or reduced the withdrawal from the well fields that formerly supplied public water in Brooklyn and Queens (N. Y.), substituting water from the upstate sources. The water table in Brooklyn has now (1958) recovered so that salt water no longer actively encroaches, or at least it moves at slow rates only. Nevertheless, the demand for relatively inexpensive ground water for numerous industrial and commercial uses in this congested area is likely to continue. Therefore, it will be necessary to keep a close watch on the area to make sure that the ground-water situation does not retrogress. Meanwhile, ground-water pumping for public and industrial supply and for irrigation has increased markedly in Nassau and Suffolk Counties to the east, and great care will have to be exercised to prevent problems in those counties similar to those that have plagued western Long Island. The maximum development of the ground-water resources of this productive island on a perennial basis will require perpetual vigilance because the island is surrounded by salt water that is always ready to move inland in response to the lowering in head accompanying ground-water pumping.

Midwestern States.—Different ground-water problems exist in parts of a large midwestern area of the United States that is underlain by artesian sandstones of the Cambrian and Ordovician epochs. The area in which the sandstones yield usable water includes eastern and southeastern Minnesota; western, southern, and eastern Wisconsin; northern Illinois; and northeastern Iowa. The strata cover the flanks of highlands of Precambrian rock in central Minnesota and Wisconsin, dipping to the south and west into Iowa and Illinois, and to the east into the Michigan basin. From the areas in which they crop out at the surface or beneath glacial drift, they pass beneath younger Paleozoic rocks, and the water in them becomes confined under artesian pressure. Although the permeability of the sandstones cannot compare with that of the aquifers of the areas described previously, they yield from a few hundred gallons per minute to a few thousand gallons per minute to wells penetrating their combined thickness, which in places is more than 1,000 ft. In addition, they are accessible in many places in which aquifers in the overlying younger Paleozoic rocks and in the glacial drift yield little water or water of poor quality.

---

[11] *Laws*, enacted April 18, 1933, New York State Legislature, 156th Session, Chapter 206.

The principal problem in the utilization of the sandstone water supply is an economic one. Because of their accessibility, the sandstones have been widely developed as sources of municipal and industrial water supply. Even though sandstones have been replaced as the prime sources of the principal public water supplies in the larger metropolitan areas, including Chicago (Ill.), Milwaukee (Wis.), and St. Paul-Minneapolis (Minn.), they are pumped heavily for industrial use and for the public water supply of numerous suburban communities. Because they are not highly permeable compared with better aquifers, because they have the small storage coefficient characteristic of artesian aquifers, and because the heavy-pumping areas are far from the principal recharge areas, the water levels in the sandstones have declined greatly since early days—by hundreds of feet in many places. Still, the sandstones have not been unwatered because they lie deeper than even the depths to which the water levels have been drawn, and water can still be pumped as long as the user is willing to lift the water hundreds of feet. Eventually, the pumping rates in the areas of heavy development will stop increasing, or may even decrease, as the cones of depression extend and merge and the hydraulic gradients toward the stabilized bottoms of the cones become more gentle. The pumpage, too, will decrease as lifting the water becomes as expensive as its purchase from the public mains. There have been some inevitable hardships as users who became accustomed to low-cost water pumped from moderate depths were forced to deepen wells, lower pumps, or even stop pumping altogether. On the other hand, the water has contributed substantially to the prosperity of this important industrial and farming region, and the only means by which it can be obtained in the quantities that have been developed has been to tolerate the declines in water level. In large areas between the centers of heaviest pumping, the water levels have not been drawn down far, and water is still available with moderate lifts. For the maximum development of the available supply in these sandstones, it is obvious that, wherever economically feasible, some of the water demand in heavy-pumping centers might be met profitably by pumping from these moderate-lift areas.

Throughout the area in which the artesian sandstones provide relatively large quantities of water, there are less extensive aquifers, such as Paleozoic limestones, glacial outwash, and valley fill along recent and preglacial valleys. These are undeveloped for the most part, and few quantitative data on their capacity are available. However, these aquifers yield substantial supplies in some places, and their availability for further development should not be overlooked.

Yazoo Delta, Mississippi.—In reviewing examples of the variety and range of ground-water problems in the United States, one might select the Yazoo Delta of northwestern Mississippi. The delta is an area of rapidly expanding development of ground water for irrigation. It does not presently (1958) have serious problems, but some may develop unless steps are taken reasonably soon to forestall them.

The Yazoo Delta is a part of the prehistoric valley of the Mississippi River, which, in this its area of widest meandering, has wandered as far east as the present course of the Yazoo River in Mississippi and as far west as the valleys of the Arkansas River and Ouachita River in Arkansas. During the Pleistocene epoch, when the climate became warm and the glaciers of the north melted back, the Mississippi River carried vast floods of melt water, which scoured out a huge, nearly flat-bottomed trench many miles wide and several hundred feet deep. The trench was partly filled late in the ice age, or at the beginning of the Recent epoch, with glacial debris, much of it coarse and well sorted. Later, as the Recent epoch continued, the flow of the streams slackened, and the coarse glacial outwash was covered by deposits of silt and clay.

Irrigation in the Yazoo Delta was first practiced using water from streams, but around 1950 a rapid development of irrigation from wells began when farmers realized that a huge reservoir of water was available wherever one chose to drill and which was more reliable than the surface-water supplies that diminished in periods of drought, just when they were needed the most. By the end of 1955 approximately 1,000 irrigation wells had been put down, and by the end of 1956 the total probably neared 1,200; the pumpage in 1956 was 500,000 acre-ft. Bolivar County in the west-central part of the delta became the center of extensive development. Comparative measurements made in 145 observation wells in the county show there was a mean lowering of the water level of 1.1 ft in the 2-yr period from August, 1954, to August, 1956.

In 1956 a third of the irrigation wells were used for the cultivation of rice. The silty and clayey subsoil makes flooding of rice practicable, but it also impedes recharge from precipitation and irrigation water. For the possible significance of this situation, one has only to examine the Grand Prairie of Arkansas across the Mississippi River, where conditions appear to be somewhat similar. There the pumping of water for rice irrigation and the impeding of recharge by the tight subsoil have caused the depletion of ground water to such an extent that it has been necessary to find means to replenish the ground water by artificial recharge through wells or other media. In comparing any two areas, it is not possible to draw an exact parallel between the Grand Prairie and the Yazoo Delta because the geologic and hydrologic conditions in the two areas are not identical. Nevertheless, there are enough points of similarity to emphasize the need for anticipating similar problems in the delta, and for the state or local agencies to take appropriate action in spacing wells, regulating withdrawals, if necessary, and practicing artificial recharge. Comprehensive ground-water studies were begun in the area late in 1953 as a cooperative state-federal project, and are still being conducted with the Mississippi Board of Water Commissioners as the cooperating state agency. These studies will provide a sound basis of hydrologic facts for the state and its administrative subdivisions to promote a full yet safe development of the great ground-water resources of the Yazoo Delta.

CONCLUSIONS

Lessons to be Learned.—What are some of the lessons that might be gained from the case histories presented and from the results of other hydrologic studies throughout the United States? Certainly, a primary need is to recognize that although emphasis herein has been placed on ground water, its problems and those of surface water are bound together. It is impossible to consider the use of one resource without recognizing the extent to which the other may be affected. The effect may be adverse, as, for instance, when the pumping of grond water depletes the flow of a stream whose water is already fully utilized, or when new diversions from a stream reduce the recharge to a downstream aquifer. On the other hand, it may be possible to use surface water to remedy a ground-water shortage, or conversely.

In all six problem areas described, surface water is significant in one respect or another. In the Roswell Basin, surface water is involved because the water of the Pecos River is fully appropriated, and the river cannot be relied on as a source of water to compensate for a ground-water shortage. In addition, pumping and consumptive use of ground water deplete the river, whose base flow depends to a large extent on the upward seepage of water from the artesian basin.

In the High Plains, also, there is insufficient surface water to replace the diminishing ground-water resources. The proposed projects would make water available from the Canadian River to meet a substantial part of the municipal and industrial needs of the area but only a fraction of the irrigation requirement. Some water running off into the many shallow ponds of the plains will be salvaged also for direct use or for recharge to the aquifer through wells. Although the foregoing does not answer the problem completely, any available surface water will contribute greatly to the economy of the High Plains.

In southeastern Florida the control of surface water is perhaps the most essential ingredient in the successful use of the abundant ground water. In this area a delicate balance must be preserved between replenishment and disposal of water. The replenishment from rainfall is great and often occurs at nearly unmanageable rates, as whenever a hurricane dumps many inches of water over the whole area in a day or two. On the other hand, the combination of a highly permeable aquifer and a system of drainage canals results in the rapid draining of water during dry weather unless steps are taken to halt it.

It would be possible to stress the significance of surface water in the other three ground-water problem areas that have been cited. However, to do so would only emphasize further the fact that ground water and surface water must be considered together, both as sources of water and as interrelated resources, neither of which can be developed safely without considering the effect on the other.

It is important to note that the national water supply is not diminishing, in so far as can be ascertained, but remains approximately the same over the long term. There are wet and dry cycles, ranging in duration from a few years to a few decades, that must be considered, and less definite wet and dry trends that may have periods of from 50 yr to 100 yr, or more. However, there is no reason to fear that the United States is gradually drying up.

Thus, the same total water supply can be relied on in the long run. Nevertheless, it is still necessary to face the problems caused by the droughts that recur periodically, such as the one that has persisted in much of the southwestern United States ever since the record-breaking rains of 1941, and the one that has affected a sizable part of the midwestern and eastern states since 1955. Each new drought brings problems more severe than before because each year more water is used and the limits of supply are approached more closely in increasing localities. When normal or wet spells are used to expand water use to the limit, the problems presented by the inevitable drought are compounded. The water shortage in New York City in 1949 and 1950 is a dramatic example. Not even much of a drought was necessary to reveal the impossibility of continuing, year after year, to take advantage of the extra water provided by the wet years. Fortunately, the water shortage caused relatively little real hardship, and the city already had planned for additional sources that would have been used if World War II had not occurred. Nevertheless, the situation is a clear example of what may happen in a less spectacular but perhaps more serious fashion in other areas if the development and conservation of water do not keep up with demand.

With a relatively uniform water supply, yet one that is reduced significantly by drought for periods of from one year to several years, and with water demands rising every year, what can be done? More water will be bought, waste will be reduced, or demand will be lowered.

Basic Attack on Problems.—In order to get the water needed at a reasonable price, several conditions must be met:

First and foremost, one must determine how much water is used and how much can be used. This involves the considerable task of maintaining an up-to-date inventory of water uses, and the much greater and almost overwhelming task of measuring the national water supply. The latter task will require the filling in of many gaps in the present knowledge of the basic physical principles that control the occurrence of water in different environments, as well as the forces involved in the movement of water from one environment to another. As both an essential part and an extension of this research, it is necessary to utilize the knowledge of basic principles to locate and characterize each surface and subsurface source of water. In addition, an attempt should be made to estimate closely the quantity of water available from each source under varying weather conditions and water-use pattern.

With this knowledge, the individual problems must be handled as they arise, anticipating them as much as possible in order that they may be solved in their earliest stages when it is still easy and inexpensive to do so. It must be recognized that each problem is different; each is a product of the local geologic and climatic environment and history of water development. For most of the major problems, the solution will be difficult, complicated, and expensive, and will require considerable public education to secure adequate public support. A solution that might work in one area may be impracticable in another hydrologically similar area because of traditions of water use that have become expressed in law.

The most efficient utilization of water resources require long-term planning and the control of development. In the United States legal control of the use of water lies in the states themselves, and interstate problems are handled by compacts agreed to by the United States Congress, or by interstate lawsuits decided in the United States Supreme Court. Because interstate difficulties are only a small part of the water problems as a whole, it is within the framework of state law that the planning and control of water use must be accomplished. Although existing law may handicap efforts occasionally to achieve long-range planning and adequate control of water development, and even though public sentiment may not favor the changes in the law necessary for effective water use, in most states much more can be accomplished along these lines than has been done in the past.

Acknowledgments

The ground-water reports of investigations, chiefly by the Geological Survey and its cooperating agencies, provide the basis for the geologic and hydrologic material presented. Special acknowledgment is made to Charles L. McGuinness for his assistance in the preparation of the paper, and to the Geological Survey Ground Water Branch district supervisors for reviewing the material on the ground-water problem areas in their districts.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2939

EARTHQUAKE RESISTANCE OF ROCK-FILL DAMS

BY RAY W. CLOUGH[1] AND DAVID PIRTZ,[2] ASSOCIATE MEMBERS, ASCE

WITH DISCUSSION BY MESSRS. NICOLS N. AMBRASEYS; JOHN V. SPIELMAN; AND RAY W. CLOUGH AND DAVID PIRTZ

SYNOPSIS

An experimental investigation of the effects of earthquakes on rock-fill dams with earthen cores is described. The tests were performed on (1/150)-scale models of two types of dams—one having an inclined core near the upstream face, and the other having a central core. The models were subjected simultaneously to water loadings and to simulated earthquakes that were generated by the shaking table on which the models were constructed.

The test results show that rock-fill dams inherently are resistant to earthquakes because of their flexible structure. No catastrophic slippages occurred in the models when they were subjected to ground accelerations exceeding the acceleration of gravity.

INTRODUCTION

A rock-fill dam may be defined as one in which the thrust of the impounded water is carried by an embankment of rock or gravel.[3] The structure may be made watertight either by facing it with steel or concrete slabs, or by an impervious core of compacted earthen materials. This paper will be limited to a presentation of rock-fill dams of the latter type. In general, such dams may be divided into two broad classifications: Those having a sloping impervious core near the front face, and those having a vertical core which is centrally located.

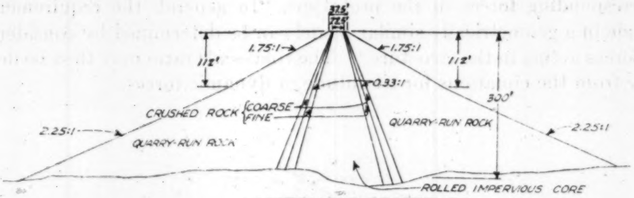
NOTE.—Published, essentially as printed here, in April, 1956, in the Journal of the Soil Mechanics and Foundations Division, as *Proceedings Paper 941*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Associate Prof. of Civ. Eng., Univ. of California, Berkeley, Calif.

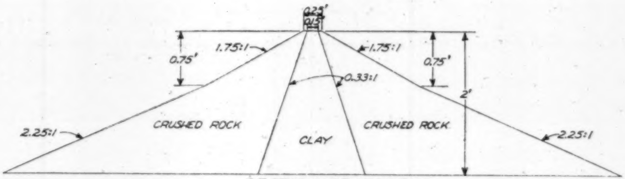
[2] Associate Prof. of Civ. Eng., Univ. of California, Berkeley, Calif.

[3] "Rock-Fill Dam Design and Construction Problems," by D. J. Bleifuss and J. P. Hawke, *Proceedings-Separate No. 514*, ASCE, October, 1954.

Dams with sloping cores are more easily constructed and are more economical than those having vertical cores. However, because of uncertainties as to their stability when subjected to earthquakes, some doubt has been expressed as to the suitability of sloping-core dams for use in the western regions of the United States.



SECTION OF PROTOTYPE



SECTION OF MODEL

FIG. 1.—SECTION VIEWS OF SLOPING-CORE ROCK-FILL DAM

The purpose of the investigation was to determine qualitatively, by model tests, the effects of earthquakes on the two types of impervious-core rock-fill dams, and to compare their earthquake resistance.

The prototype of the sloping-core dam was a preliminary design of the Kenney Dam, a structure approximately 300 ft high that was recently constructed on the Nechako River in British Columbia. The prototype of the central-core dam was a hypothetical structure similar to Mud Mountain Dam on the White River in Washington, but it was scaled to a height of 300 ft so that its dimensions would be comparable with those of the sloping-core dam. Cross-sectional dimensions of the prototypes and models are shown in Figs. 1 and 2.

During the test program, models of the sloping-core dam were subjected to simulated earthquakes of various intensities with the water in the reservoir at three different levels—empty, 4/10 full, and full. The model of the central-core dam was tested only in the full-reservoir condition. The simulated earthquakes ranged in intensity from maximum ground accelerations of less than 0.10 g (1/10 of the acceleration of gravity) to about 1.25 g.

DESIGN OF THE MODELS

Similitude Requirements.—In order that results of the model tests might be representative of the earthquake response of the prototypes, it was necessary to consider the requirements of model similitude. In a structural model intended for dynamic loading, complete similitude is obtained if there is similarity between the model and prototype with respect to forces, lengths, and periods of time. Length similitude is obtained by making the model geometrically similar to the prototype. Similitude of time is obtained if every event in the model is made proportional in duration to the corresponding event in the prototype. Similarity of forces requires that all forces in the model have a constant ratio to the corresponding forces in the prototype. In general, the requirements of similitude in a geometrically similar model can be determined by consideration of the forces acting in the structure.[4] The time-scale ratio may then be derived directly from the equations for similitude of dynamic forces.

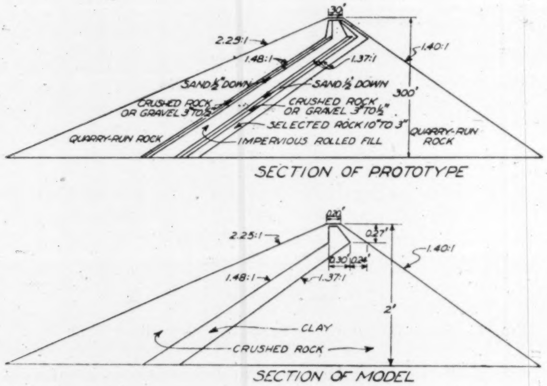

30'
2.25:1
1.48:1
1.37:1
1.40:1
SAND ½" DOWN
SAND ½" DOWN
300'
CRUSHED ROCK OR GRAVEL 3" TO ½"
CRUSHED ROCK OR GRAVEL 3" TO ½"
SELECTED ROCK 10" TO 3"
QUARRY-RUN ROCK
IMPERVIOUS ROLLED FILL
QUARRY-RUN ROCK
SECTION OF PROTOTYPE
0.20'
0.27'
2.25:1
1.40:1
0.30' 0.24'
1.48:1
1.37:1
2'
CLAY
CRUSHED ROCK
SECTION OF MODEL

FIG. 2.—SECTION VIEWS OF CENTER-CORE ROCK-FILL DAM

In the following presentation, the various symbols will be defined as they are introduced. The subscripts, m and p, will be used to refer to the model and prototype, respectively.

The first step in the design of the models that were used in the investigation was the selection of a suitable length scale. The space that was available for testing limited the end-to-end length of the model to approximately 7 ft. Accordingly, the height of the model was established at 2 ft because a length-to-height ratio of at least 3 was considered desirable to minimize the effects of end restraint. The ratio of model height to prototype height determined the length scale, λ. Thus, $\lambda = \frac{L_m}{L_p} = \frac{2}{300} = \frac{1}{150}$, in which L represents any linear dimension; all dimensions of the model were maintained in this scale.

The next step was the determination of the model ratio of forces. Five types of forces were of importance in the analysis: (*a*) Dead weight of rock-fill and core material; (*b*) applied water load; (*c*) inertia forces due to earthquake accelerations; (*d*) forces associated with elastic deformations in the structure; and (*e*) forces connected with the ultimate strength of the structure. For similitude to exist, the ratio of each model force to the corresponding prototype force must be the same for all types of forces. The force ratio will be designated by the symbol, α, with appropriate subscripts to indicate the type being considered.

---

[4] "General Structural Similtude," by R. W. Carlson in "Tests on Structural Models of Proposed San Francisco-Oakland Suspension Bridge," by G. E. Beggs, R. E. Davis, and H. E. Davis, Univ. of California Publications in Eng., Berkeley, Calif., Vol. 3, No. 2, 1933, Appendix A.

In the analysis the ratio of the dead-weight forces in the model to those of the prototype determined the value of the force ratio. The dead-weight force is given by the product of the unit weight, γ, and the volume, V, but the volume is proportional to the cube of the length. Thus,

$$\alpha_{dw} = \frac{\gamma_m\, V_m}{\gamma_p\, V_p} = \frac{\gamma_m}{\gamma_p}\left(\frac{L_m}{L_p}\right)^3 = \alpha_{dw} = \frac{\gamma_m}{\gamma_p}\lambda^3 \qquad \text{(1)}$$

In a similar analysis the ratio of applied fluid forces will have this same value only if the model fluid has a specific gravity equal to the ratio of unit weights of the model materials, γ_m/γ_p.

The ratio of inertia forces, α_i, is given by the ratio of the products of masses, M, and accelerations, a. Thus, because the ratio of masses is the same as the ratio of weights, $(\gamma_m/\gamma_p)\,\lambda^3$,

$$\alpha_i = \frac{M_m\, a_m}{M_p\, a_p} = \left(\frac{\gamma_m}{\gamma_p}\right)\lambda^3 \frac{a_m}{a_p} \qquad \text{(2)}$$

In order for this force ratio to equal the ratio of dead-weight forces, the model accelerations must equal the prototype accelerations,

$$\frac{a_m}{a_p} = 1 = \frac{\dfrac{L_m}{T_m^2}}{\dfrac{L_p}{T_p^2}} = \lambda\left(\frac{T_p}{T_m}\right)^2 \qquad \text{(3)}$$

from which

$$\left(\frac{T_m}{T_p}\right)^2 = \lambda \qquad \text{(4)}$$

in which T represents time. Finally, designating the model ratio of time T_m/T_p by the symbol, τ, the time scale becomes

$$\tau = \sqrt{\lambda} = \sqrt{\frac{1}{150}} \qquad \text{(5)}$$

Therefore, in order to obtain similitude of inertia forces, events on the model, such as the periods of vibration, must take only 8.2% as long as the corresponding events in the prototype.

Forces that are associated with elastic deformations in the structure depend primarily on the effective modulus of rigidity of the materials because in rock-fill dams the only important deformations that are developed by earthquakes are shearing distortions. The force that is required to produce a given elastic strain is the product of the unit strain, e, the modulus of rigidity, G, and the cross-sectional area, A. Thus, the ratio of forces due to elastic deformations,

α_e, is given by

$$\alpha_e = \frac{e_m\, G_m\, A_m}{e_p\, G_p\, A_p} \dotfill (6)$$

In order to maintain geometric similitude, the strain in the model must equal the strain in the prototype. The ratio of areas is equal to the square of the length scale. Thus,

$$\alpha_e = 1\left(\frac{G_m}{G_p}\right)\lambda^2 \dotfill (7)$$

Equating Eq. 7 with the previously established force ratio yields the relationship,

$$\frac{G_m}{G_p}\lambda^2 = \left(\frac{\gamma_m}{\gamma_p}\right)\lambda^3 \dotfill (8)$$

from which the ratio of moduli of rigidity required for similitude is

$$\frac{G_m}{G_p} = \left(\frac{\gamma_m}{\gamma_p}\right)\lambda \dotfill (9)$$

Finally, in order for the model to simulate failures in the prototype, not only the force, but also the strength properties of the materials must be simulated. The model should fail at a force corresponding to the force that would produce failure in the prototype. The strength of the materials is assumed to depend on two factors—the angle of internal friction and the cohesion. Similitude requires that the ultimate model force, which is associated with each of these factors separately, must be in the proper ratio to the corresponding prototype force.

Because the angle of internal friction is a dimensionless quantity, its magnitude is not affected by change of scale. Consequently, internal friction forces can be simulated only if the angle of internal friction in the model is equal to that in the prototype. The ultimate cohesive force is given by the product of the cohesion, C, and the area, A. Therefore, the ratio of cohesive forces is given by

$$\alpha_c = \frac{C_m}{C_p}\left(\frac{A_m}{A_p}\right) = \frac{C_m}{C_p}\lambda^2 \dotfill (10)$$

Equating Eq. 10*b* with the established force ratio yields the required ratio of model cohesion to prototype cohesion,

$$\frac{C_m}{C_p} = \left(\frac{\gamma_m}{\gamma_p}\right)\lambda \dotfill (11)$$

Selection of Model Materials.—After the relationships between the model and the prototype that are required by similitude had been determined, the next step in the design of the models was to select materials that would satisfy these requirements as completely as possible.

The dumped rock fill of the prototype was assumed to consist of essentially granular material ranging in size from sand to massive boulders and having an average unit dry weight of 110 lb per cu ft. The cohesion of this material was assumed to be negligible, and the angle of internal friction was taken at 45°. (This value is somewhat greater than that normally used in the design of rock-fill dams, but its application is justified by available test data.[5]) For the rock fill of the model, a crushed quartzite was selected which would pass a $\frac{3}{8}$-in. sieve and be retained on a No. 30 sieve. The unit dry weight of this material was 96 lb per cu ft. Thus, the unit weight ratio of the materials was

$$\frac{\gamma_m}{\gamma_p} = \frac{96}{110} = 0.87$$

and the force ratio, α, was

$$\alpha = \frac{\gamma_m}{\gamma_p} \lambda^3 = 0.87 \left(\frac{1}{150}\right)^3$$

The value of the angle of internal friction of the model material, as determined by direct-shear tests using relatively low normal loads, was 48° and the value of the same angle as determined by the vacuum triaxial test was 42°. Both these values sufficiently approximated the corresponding value of the prototype.

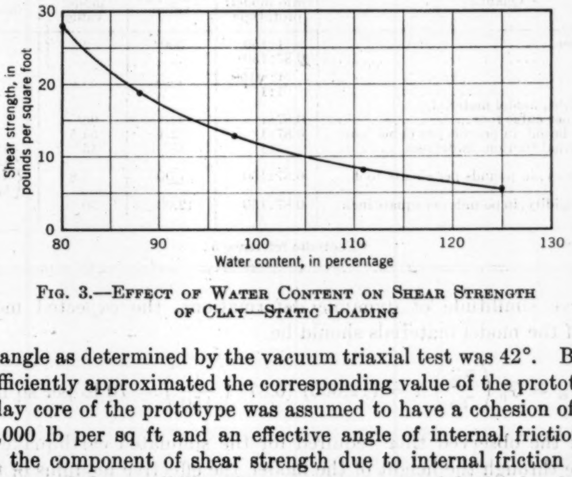


FIG. 3.—EFFECT OF WATER CONTENT ON SHEAR STRENGTH OF CLAY—STATIC LOADING

The clay core of the prototype was assumed to have a cohesion of approximately 1,000 lb per sq ft and an effective angle of internal friction of 20°. However, the component of shear strength due to internal friction was neglected because of the possibility that pore pressures might be developed with sufficient magnitude to eliminate it. Thus, for similitude a model material was required having a cohesion of

$$C_m = C_p \frac{\gamma_m}{\gamma_p} \lambda = 1{,}000\ (0.87) \left(\frac{1}{150}\right) = 5.8 \text{ lb per sq ft}$$

In addition, it was necessary that the material provide an impervious core in the model and be stiff enough to be placed without difficulty during construction of the model. A commercially available kaolin clay was suitable for the purpose. Because of its low plasticity it was easy to mix and handle. At a water content of 125% it had the required shear strength of 5.8 lb per sq ft.

---

[5] "A Review of Special Problems on Soil Shear Strengths," by G. Fuquay, paper presented at the meeting of the Soil Mechanics and Foundations Div., ASCE, San Francisco, Calif., January, 1954.

The static shear strength of the clay, as determined by a quick, undrained, direct-shear test, is plotted as a function of water content in Fig. 3. The direct-shear test load was applied slowly for a period of 5 min.

It was also advisable to investigate the strength of this material when it was subjected to rapid loadings because previous investigators had observed a considerable increase (from 40% to 200%) in the strength of saturated clays under such conditions.[6] Results of this study showed that when the kaolin mixed to a water content of 125% under rapid loading (0.03 sec) it had a strength 70% greater than when subjected to a static loading (failure time of 5 min). Therefore, the model material is similar to the prototype materials in this respect.

TABLE 1.—SIMILITUDE ANALYSIS DATA

Quantity	Required ratio model: prototype	Prototype value	Required model value	Actual model value
Length, in feet	1:150	300	2	2
Force	0.87:150[a]	...	...	...
Time	$1:\sqrt{150}$	...	...	...
Acceleration	1:1	...	...	...
Unit dry weight, model material, in pounds per cubic foot	0.87:1	110	96	96
Unit weight, liquid, in pounds per cubic foot	0.87:1	62.4	54.5	62.4
Angle of internal friction, in degrees	1:1	45°	45°	42° to 48°
Cohesion of clay, in pounds per square foot	0.87:150	1,000	5.8	Model E 7.2 Model F 7.7 Model G 5.0
Modulus of rigidity, in pounds per square inch	0.87:150	12,000	70	120

[a] Footnote reference 5.

The effective modulus of rigidity of the materials that were used in the construction of rock-fill dams was on the order of 12,000 lb per sq in.[7] In order to preserve similitude of dynamic deformations, the expected modulus of rigidity of the model materials should be

$$G_m = G_p \left(\frac{\gamma_m}{\gamma_p} \right) \lambda = (12{,}000)\ (0.87) \left(\frac{1}{150} \right) = 70 \text{ lb per sq in.}$$

Based on the observed time required for the simulated earthquake shock to propagate through the height of the model, the effective modulus of the model dam materials was computed as 120 lb per sq in., or approximately twice as great as it should have been. The principal effect of this discrepancy in rigidity was to reduce the natural period of vibrations of the model from approximately 0.09 sec as required by similitude to about 0.07 sec. It will be demonstrated subsequently that this reduction of period in the model would tend to increase the model stresses, and, thus, the results obtained with the model should be conservative.

It was noted in the previous section that in order to maintain similitude of the applied loads the specific gravity of the liquid loading of the model should equal the ratio of the unit weights, $\gamma_m/\gamma_p = 0.87$. Although it might have been possible to obtain such a liquid, it was satisfactory to use water for loading the model. The effect of this discrepancy was to increase the loads on the model beyond the required value, and again the model results should be conservative.

---

[6] "Research on Stress-Deformation and Strength Characteristics of Soils and Soft Rocks under Transient Loading," by A. Casagrande and W. L. Shannon, *Soil Mechanics Series 31*, Harvard Graduate School of Eng., Cambridge, Mass., No. 447, June, 1948.

[7] "Seismic Investigations of Hansen Dam and Site," by C. A. Heiland, report to the Office of the U. S. Engr., South Pacific Div., San Francisco, Calif., 1938.

Summary.—The results of the similitude analysis and model design are summarized in Table 1.

From this table it is seen that all the required model values have been achieved with the exception of the density of liquid and the modulus of rigidity of the model materials. As was stated previously, the discrepancy in both the items is on the conservative side—that is, the tendencies toward failure should be greater in the model than in the prototype.

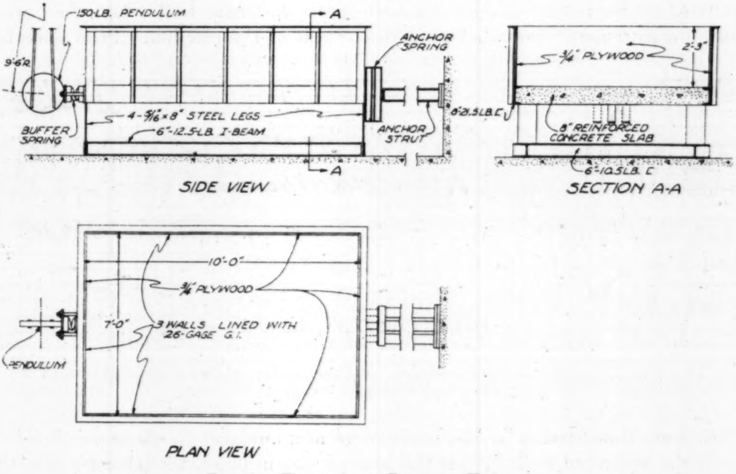

150-LB. PENDULUM
A
ANCHOR SPRING
9'-6" R.
4-5/16" × 8" STEEL LEGS
6"-12.5-LB. I-BEAM
BUFFER SPRING
ANCHOR STRUT
A
SIDE VIEW
2'-3"
3/4" PLYWOOD
8"-21.5 LB. C
8" REINFORCED CONCRETE SLAB
6"-10.5 LB. C
SECTION A-A
10'-0"
3/4" PLYWOOD
7'-0"
3 WALLS LINED WITH 26-GAGE G.I.
PENDULUM
PLAN VIEW

FIG. 4.—DETAILS OF SHAKING TABLE

TEST EQUIPMENT AND INSTRUMENTATION

Shaking Table.—The simulated earthquake was applied to the model by a shaking table. The top of the table was a reinforced concrete slab, which was 8 in. thick and was cast within a frame of 8-in. structural steel channels. The slab was supported at each corner by a flexible steel leg, which permitted the slab to move freely in the longitudinal (upstream and downstream) direction, and the flexible steel provided high restraint to lateral and torsional movement. Walls, which were 27 in. high and cross braced to reduce local vibrations, were provided in order to retain the model dam and reservoir. A sketch showing the general construction features of the shaking table is shown in Fig. 4. Motion was imparted to the table by a 150-lb pendulum striking against a buffer spring at the upstream end of the table. The other end of the table was anchored by a heavy spring, which controlled its frequency of vibration.

The motion produced by this type of system may be divided into two phases.[8] In the first phase, while the pendulum is in contact with the buffer spring, it acts as a system with two degrees of freedom. After the pendulum has rebounded, the ensuing table motion is a damped, simple harmonic motion, with the degree of damping depending on the quantity and character of the materials on the table. Typical records of the table displacements and accelerations are shown in Fig. 5.

The proximity of the vertical upstream wall of the shaking table to the face of the dam constituted a difference between model and prototype conditions, which, it was felt, might cause some discrepancies in the response of the model. Specifically, it seemed that a pressure wave might develop at this face during a downstream acceleration of the table and that the wave might affect the model. In order to prevent this occurrence, an air cushion in the form of a blanket of inflated rubber tubes was mounted against the upstream wall. Dynamic pressure measurements indicated the effectiveness of the cushion. Prior to its installation, downstream accelerations were accompanied by pressure increases near the upstream wall. After the cushion was in place, the pressure near the upstream face decreased when the table was accelerated downstream, as would be expected in the prototype when the dam tended to move away from the reservoir.

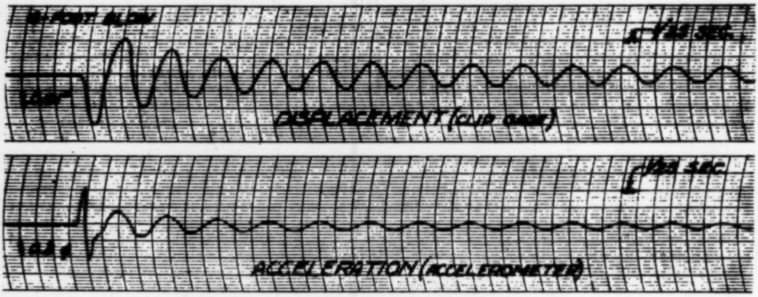


FIG. 5.—TYPICAL OSCILLOGRAMS OF DISPLACEMENT AND ACCELERATION OF SHAKING TABLE

Instrumentation.—The primary instrumentation for the test program consisted of a clip gage and accelerometer, which were mounted on the shaking table to record its displacements and accelerations—that is, to measure the intensity of the earthquake applied to the model. In addition, a linear, variable, differential-transformer type of displacement gage was used for measuring the displacements of the top of the dam with respect to the base. The core of this gage, which was attached to the dam, weighed only a few ounces, so that its mass had a negligible effect on the forces developed in the dam. However, the gage could be used only with earthquakes of rather light intensity because strong earthquakes were accompanied by settlement of the dam, which rendered the gage inoperative. In a few tests a second accelerometer was buried near the top of the dam in order to compare the accelerations that developed in this area with those that developed at the base of the model.

---

[8] "Vibration Research at Stanford University," by L. S. Jacobsen, *Bulletin*, Seismological Soc. of America, Vol. 19, No. 1, March, 1929.

The output of the various gages was recorded on two-channel, direct writing oscillographs. The maximum frequency for which this equipment was designed is about 100 cycles per sec. A few check tests were made using a high-frequency, photographic recording oscillograph in order to determine whether any significant part of the response was missed by the low-frequency recorders. However, there was no appreciable difference between the record obtained with the two types of oscillographs. Therefore, the direct writing recorder was used on all subsequent tests.

For the purpose of the investigation, the effect of the simulated earthquakes on the models was indicated by the change of shape or deformation of the models. The shape was measured with a profileometer, which consisted of a depth gage mounted on rails above the dam. By measuring the distance down to the surface of the dam at intervals across the midsection, an accurate measure of the profile was obtained.

CONSTRUCTION OF MODELS

In constructing the models the part of the structure that was downstream from the sloping core was placed and screeded to shape. The clay blanket was placed on the upstream surface of this main structure. Because of the low shear strength of the core material, the upstream rock had to be built up together with the clay, in order to hold it in position. Screed boards, sliding on wooden strips that were clamped to the walls of the table, aided in obtaining the proper shape for the cross section.

The prototype core consisted of a central clay core proper, above and below which was a filter layer consisting of sand and relatively fine crushed rock (Figs. 1 and 2). To approximate a corresponding condition in the models, which had clay cores only, the thickness of the model core was made to correspond to the thickness of the prototype core proper plus one-half of the thickness of the adjacent filter layers.

All the rock (upstream and downstream) was moistened with a fine spray before the core of the dam was constructed so that it would not absorb water from the clay. The original water content of the clay was purposely made greater than that desired at the time of the test because it was known that some water would be lost as a result of consolidation. The desired shear strength of 5.8 lb per sq ft corresponded to a water content of 125%, in accordance with the relationship shown in Fig. 3.

GENERAL PROCEDURE OF TESTING

In order to minimize the loss of water from the clay blanket due to consolidation, the model was tested as soon as possible after its construction was completed. Each model was subjected to a series of earthquakes of increasing magnitude. Each earthquake consisted of an initial impact of the pendulum plus two rebound blows, after which the shaking table was permitted to oscillate freely until the motion was finally damped out. During each motion records were taken of acceleration and displacement of the shaking table (base of dam) and the top of the dam. Profile measurements were also made prior to filling the reservoir, after filling the reservoir, and after each test earthquake.

The magnitude of the accelerations developed by the earthquakes was controlled by specifying the chord of the arc through which the pendulum swung. For the weaker motions the magnitude of the maximum acceleration was nearly proportional to the chord distance—for the stronger ones the maximum accelerations increased more rapidly than the chord distances.

Immediately after the model was tested, the reservoir was drained, and six representative samples of the clay blanket were taken so that the shear strength of the clay could be determined.

Results of Tests

General.—Eight different models were tested. Two consisted of the downstream rock structure for a sloping-core dam (without any clay core), five contained sloping cores and water in the reservoir at various levels, and one was a model of a dam with a central clay core.

Downstream Rock Structure Only.—Tests on the two models of the downstream rock structure for a sloping-core dam (downstream and upstream slopes of 1.40:1) were conducted to study the change in slope and crest settlement of a rock fill that had been placed approximately at its angle of response (1.35:1). At accelerations as great as 0.4 *g*, the profiles for both models showed that little change occurred and that the settlement at the crest was 0.6% of the height of the model. This settlement is no more than would be expected in the prototype for static conditions during a period of several years.

Model with Sloping Clay Core.—Of the five models with sloping cores that were tested, only the results of the third and fourth (models E and F) will be cited. The first two models were used for preliminary studies to develop methods of construction and instrumentation and the last model was used for checking.

Model E had a sloping clay core, a downstream slope of 1.40:1, and an upstream slope of 2.25:1, as shown in Fig. 1. The model (with water at the full-reservoir level) was tested 3 hr after construction. Immediately after the test the average water content of the core was 115%, and the corresponding shear strength was 7.2 lb per sq ft (Fig. 3). The observed accelerations and displacements are given in Table 1, and the profiles are shown in Fig. 6. These profiles indicate accumulated displacements, and the effect of any single earthquake is indicated by the change in profile from one curve to the next.

The first profile in Fig. 6 shows a considerable change in section as a result of filling the reservoir to its high-water elevation (1.82 ft). The crest settled a distance equal to 0.2% of the height of the model and moved downstream approximately 2.5% of the height of the model. Thereafter, little change occurred until an acceleration of 0.4 *g* was reached. At that stage the crest settled noticeably and rounded off, the upstream face moved downstream, and the downstream face bulged. At greater accelerations these movements continued.

In the upstream face the principal changes in section occurred in the upper half of the section, even at the higher accelerations. From the midheight of the section to the three-quarter height, the slope flattened considerably. From the three-quarter height to the rounded crest, the slope remained constant, but this section of the face shifted downstream.

The bulging of the downstream face progressed as follows: At an acceleration of 0.4 *g*, the slope of the upper half of the section flattened slightly. At an acceleration of 1.1 *g*, the slope of the upper half became even flatter, and the slope near the base became steeper.

Model F was identical to model E but was tested at different conditions of water load. The model with the reservoir empty was tested 2 hr after construction at accelerations as great as 0.36 *g* and was tested again with the reservoir 0.4 full. The average water content of the core, determined immediately after the test, was 113%, and the corresponding shear strength was 7.7 lb per sq ft. The accelerations and displacements are shown in Table 2, and the profiles are shown in Fig. 7.

QUAKES		ACCELERATION, g
EACH TEST	TOTAL	
0	0	0
RESERVOIR FULL		
1	1	0.08
1	2	0.14
1	3	0.26
2	5	0.43
1	6	0.50
1	7	0.60
1	8	0.72
1	9	0.81
1	10	1.11

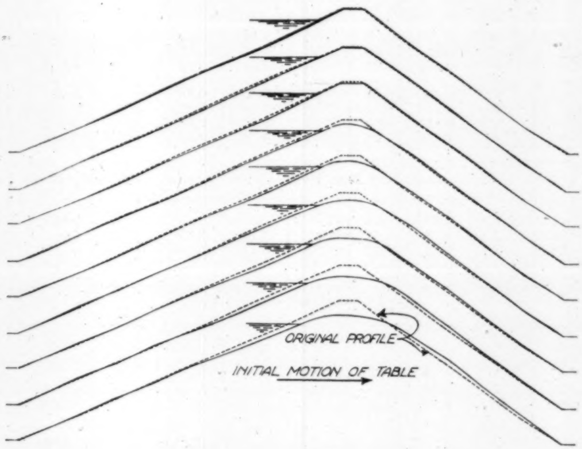


FIG. 6.—PROFILES OF MODEL E—SLOPING-CORE ROCK DAM

When the reservoir was empty and the model was subjected to accelerations to 0.36 *g*, the top half of the model settled without appreciable change in slopes, and the lower half of the upstream face bulged slightly. These movements indicated that a slight slippage had occurred in the clay core. A stability analysis (not including earthquake effects) indicated that the clay core and upstream rock were less stable when the reservoir was empty than when it was full.

The critical depth of the reservoir producing the least stable condition occurred when the reservoir was filled to a depth of 0.80 ft, which had been determined by stability analysis (not including earthquake effects). A new series of tests was made, the results of which are also shown in Table 2 and Fig. 7.

At an acceleration of 0.3 g, the crest settled an additional 0.8% of the height of the model, the upper half of the upstream face settled slightly, and the upstream face bulged slightly near the base. There was no change in the downstream face at this acceleration. At greater accelerations, the motions of the upstream face continued and indicated a slippage in the clay core.

TABLE 2.—RESULTS OF TESTS ON MODELS E, F, AND G

Test no.	Chord length of blow, in feet	TABLE ACCELERATION, g					MAXIMUM TABLE DISPLACEMENT, IN INCHES		TOP DISPLACEMENT, IN INCHES		
		Maximum		Average peak[a]	Downstream Maximum				Maximum		Permanent
		Downstream	Upstream		2nd blow	3rd blow	Downstream	Upstream	Downstream	Upstream	Downstream
MODEL E,[b] RESERVOIR FULL, AGE AT TEST, 3 HR											
E-1a	1	0.08	0.08	0.05	...	...	0.007	0.006	0.006	0.004	0.001
E-1b	2	0.14	0.12	0.06	0.11	0.08	0.013	0.012	0.022	0.010	0.003
E-2	3	0.26	0.20	0.09	0.21	0.13	0.020	0.018	0.055	0.020	−0.050
E-3	4	0.43	0.41	0.13	0.32	0.24	0.029	0.023	...	...	...
E-4	5	...	...	...	...	...	0.033	0.029	...	...	...
E-5	6	0.60	0.54	0.18	0.40	0.30	0.043	0.032	...	...	...
E-6	7	0.72	0.72	0.20	0.49	0.32	0.051	0.038	...	...	...
E-7	8	0.81	0.84	0.24	0.50	0.36	0.056	0.041	...	...	...
E-8	10	1.11	1.28	0.33	0.78	0.53	0.075	0.052	...	...	...
MODEL F,[b] RESERVOIR EMPTY, AGE AT TEST, 1HR TO 1½ HR											
F-1a	1	0.10	0.06	...	...	...	0.007	0.007	0.001	0.001	0.000
F-1b	2	0.18	0.12	0.09	0.10	0.09	0.015	0.015	0.024	0.004	0.013
F-1c	3	0.28	0.20	0.12	0.18	0.15	0.022	0.020	0.050	0.032	−0.011
F-2	4	0.36	0.32	0.13	0.20	...	0.030	0.025	...	...	...
MODEL F,[b] RESERVOIR 0.4 FULL, AGE AT TEST, 2 HR											
F-3a	1	0.08	0.07	0.04	0.02	0.02	0.007	0.007	...	...	...
F-3b	3	0.31	0.28	0.12	0.21	0.14	0.020	0.017	...	...	...
F-4	5	0.56	0.50	0.17	0.42	0.24	0.040	0.027	...	...	...
F-5	7	0.75	0.70	0.24	0.60	0.35	0.055	0.035	...	...	...
F-6	10	1.23	1.20	0.36	0.81	0.61	0.075	0.055	...	...	...
MODEL G,[c] RESERVOIR FULL, AGE AT TEST, 3 HR											
G-1a	1	0.05	0.05	...	...	...	0.015	0.010	0.011	0.002	0.005
G-1b	1	0.05	0.05	...	...	...	0.015	0.010	0.012	0.003	0.005
G-2	2	0.12	0.12	0.05	0.11	0.06	0.015	0.015	0.015	0.007	0.000
G-3	3	0.26	0.23	0.08	0.18	0.12	0.022	0.015	...	...	...
G-4	4	0.37	0.36	0.11	0.30	0.21	0.030	0.020	...	...	...
G-5	5	0.51	0.51	0.14	0.40	0.26	0.037	0.025	...	...	...
G-6a	6	0.63	...	...	...	...	0.042	0.032	...	...	...
G-6b	6	0.65	0.66	0.18	0.47	0.34	0.042	0.032	...	...	...
G-7	7	...	...	...	...	...	0.052	0.037	...	...	...
G-8	8	0.95	0.95	0.26	0.70	0.50	0.060	0.042	...	...	...

[a] Average of maximum second and third downstream acceleration and second upstream acceleration, as an indication of harmonic accelerations. [b] Sloping core; downstream slope of face, 1.40:1; upstream slope of face, 2.25:1. [c] Center core; lower section of downstream and upstream slopes, 2.23:1, and upper section, 1.75:1.

Model with Center Clay Core.—Model G was built with a vertical clay core at the center of the section, as shown in Fig. 2. The shape of the prototype was similar to that of Mud Mountain Dam, but the rock and clay that were used in model G were the same as those that were used in the sloping-core models. Model G had an upstream slope and a downstream slope of 1.75:1 over the upper 0.37 of its height, and slopes of 2.25:1 over the lower 0.63 of its height. It was tested 3 hr after construction, with the reservoir filled to the high-water mark (1.83 ft), which a stability analysis had indicated as the critical condition. The average water content of the core immediately after the test was 132%, which by extrapolating from Fig. 3 indicated a shear strength of approximately 5.0 lb per sq ft. The accelerations and displacements are given in Table 2, and the profiles are shown in Fig. 8.

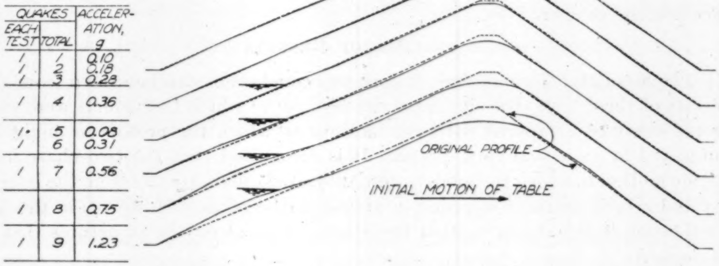


FIG. 7.—PROFILES OF MODEL F—SLOPING-CORE ROCK DAM

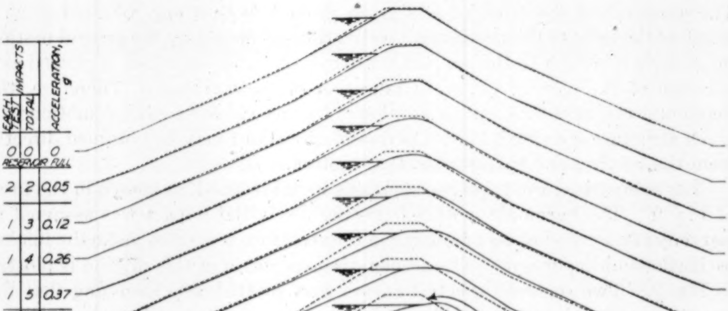


FIG. 8.—PROFILES OF MODEL G—CENTER-CORE ROCK DAM

During the filling of the reservoir, the top part of the upstream face of the model (the part with the 1.75:1 slope) shifted appreciably downstream. At an acceleration of 0.05 g, settlement occurred all along the upstream slopes—the settlement was greater near the top. At this acceleration the crest began to round off and dip upstream. At accelerations as great as 0.26 *g*, these movements of the upstream face and the crest continued, and at an acceleration equal to 0.26 *g*, the top part of the downstream face bulged. At greater accelerations the settlement of the entire upstream face continued, and the bulging of the downstream face increased and progressed downward near the base, but the largest changes in section were still in the top part of the model. At the greatest acceleration (0.95 *g*) that was imposed on this model, the upstream face settled until it approximated a uniform slope that was flatter than the original slope of 2.25:1 of the lower part of the model. Both slopes of the downstream face became steeper.

ANALYSIS OF RESULTS

The Simulated Earthquake.—Before any conclusions can be drawn from the results of these tests, the simulated earthquake to which the models were subjected must be compared with the motions to which the prototype might be subjected in an actual earthquake. It is recognized that the two-phase harmonic motion to which the models were subjected differs greatly from the irregular and erratic motions recorded in actual earthquakes. However, in this investigation it is not the motion itself but its effect on the structures that is important.

A comparison of the effects of the simulated earthquake with an actual one is made best by comparing the acceleration spectra of the two types of motion. The acceleration spectrum of any given ground motion may be defined as a graph of the ratio of the maximum acceleration developed by the ground motion in a simple oscillator to the maximum ground acceleration, which is plotted as a function of the natural period of vibration of the oscillator. Therefore, the maximum response that will be developed by any specified ground motion in a given structure (having a known period of vibration) may be obtained directly from the acceleration spectrum of that motion.

For comparison the table accelerations were idealized, as shown in the inset of Fig. 9. The harmonic motion following the initial impact was assumed to last only two cycles because damping in the structure tended to make the further oscillations unimportant. The acceleration spectrum of this motion is plotted in Fig. 9. Two separate spectral curves were plotted, one assuming that the structure was undamped, and the second assuming a viscous damping factor of 20% of critical damping. For comparison two spectral curves[9] for the north-south component of the El Centro, Calif., earthquake of May 18, 1940, have been smoothed out slightly and plotted on the same graphs. The time scale of the El Centro earthquake spectrum has been reduced to the model scale in order to make the comparison.

The agreement between the two sets of curves is surprisingly good. Comparing the undamped table motion spectrum with the earthquake spectrum for 2% critical damping, it is noted that the two curves are nearly identical for structures having periods between 0.03 sec and 0.07 sec (model scale). However, structures with longer periods tend to develop resonance with the low-frequency harmonic motion of the table, and, consequently, the table spectrum increases in amplitude again for these longer periods in contrast with the earthquake spectrum, which shows a continual decrease toward the longer periods.

---

[9] "Lateral Forces of Earthquakes and Wind," by the Joint Committee of the San Francisco, Calif., Section, ASCE, and the Structural Engrs. Assn. of Northern California, *Transactions*, ASCE, Vol. 117, 1952, p. 716.

A comparison of the spectral response curves for structures having 20% of critical damping is of greater significance because the rock-fill dam has a great capacity for internal damping. The curves are almost identical throughout the entire range of periods considered, leading to the conclusion that the maximum response developed in the dam by the simulated earthquake is nearly the same as the maximum response that would be developed by an actual earthquake.

The effect of the lack of similitude in the shear modulus of the model material is also indicated by the spectral response curves. The principal effect of this discrepancy was to reduce the natural period of vibration of the models from 0.091 sec to 0.069 sec. The acceleration spectra show that although the spectral response ratio would be only 0.80 if the model had the correct rigidity, the more rigid material that was actually used had the effect of increasing this ratio to about 1.20. Thus, the lack of similitude in this respect increases the forces developed in the model.

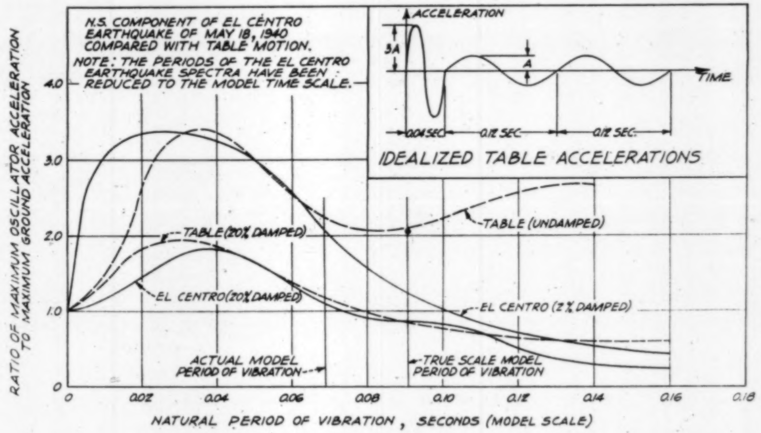


FIG. 9.—COMPARISON OF ACCELERATION SPECTRA FOR EL CENTRO, CALIF., EARTHQUAKE AND SHAKING TABLE

Dynamic Response of the Models.—The response of the model to the simulated earthquake was evaluated from the records of the displacement of the top of the dam relative to the base. These displacements, together with the records of the table displacement, made it possible to deduce the motion of the structure as a whole. The typical motion that was observed is shown in Fig. 10. The upper curve shows the total movement at the gage position near the top of the dam, and the lower curve shows the table motion or the movement of the base of the dam. The position of the axis of the dam at intervals during the motion is shown in the diagrams between the two curves.

The curves and diagrams in Fig. 10 show that the base of the dam moved downstream, leaving the top almost stationary. Then, after a time lag of from 0.02 sec to 0.03 sec, the top of the dam moved over beyond the base. In the following cycle or two the top quickly came into phase with the base. During the ensuing harmonic motion the whole model moved as a unit.

In general, as a result of distortions within the dam, the top moved farther than the base during harmonic oscillation and, therefore, was subjected to greater accelerations. This amplification was from 25% to 35% in the tests that were recorded. However, it had little significance because the maximum accelerations were associated with the initial blow rather than with the harmonic motion. The initial shock was felt more strongly at the base than at the top—the flexibility of the model serving as a sort of shock absorber—which prevented the transmission of the maximum accelerations directly to the top.

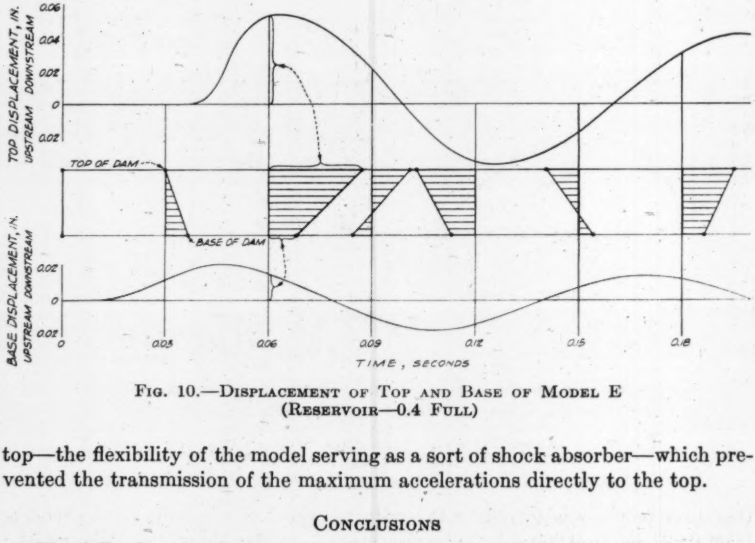


FIG. 10.—DISPLACEMENT OF TOP AND BASE OF MODEL E (RESERVOIR—0.4 FULL)

CONCLUSIONS

The following conclusions regarding the results of this investigation seem to be justified:

1. The strength of the model was designed in proper scale to the prototype structure, and the acceleration spectrum indicates that the model earthquake was approximately equivalent to a typical prototype earthquake. Therefore, it may be concluded that the results observed in the model can be extrapolated to the prototype. The effect of a test earthquake on the model should be equivalent to the effect of an actual earthquake (having the same maximum acceleration) on the prototype.

2. The applied earthquakes produced no significant effects (appreciable changes of section) on the models until the progressively increasing accelera-

tions exceeded 0.4 *g*. The greatest accelerations that have been recorded with reliable strong-motion seismographs during actual earthquakes have been less than 0.4 *g*. Therefore, it can be concluded that rock-fill dams similar to the models that were tested would not be damaged seriously by earthquakes of the magnitudes observed in the western states during the last 30 yr.

3. When the accelerations of the test earthquakes were increased to more than 1 *g*, the models suffered only minor changes of shape, indicating that with the strongest conceivable earthquakes the prototype structures would show damage only in the attached rigid structures and appurtenances. It is unlikely that settlements large enough to cause overtopping of the structure would result.

4. The high degree of earthquake resistance exhibited by the models may be attributed to two principal factors: (*a*) The materials of which the dam is constructed, both clay and granular, show increased strength when subjected to dynamic loading,[6,10] and, therefore, the capacity to resist earthquake forces is greater than the capacity to resist static forces; and (*b*) the rock-fill dam is a naturally flexible structure and can undergo large distortions without appreciable damage.

In connection with the latter point, it may be noted that the customary method of designing structures to resist earthquakes is not applicable to rock-fill dams. The usual method of taking account of earthquake effects in design is to assume a reasonable value for the maximum ground accelerations, approximately 0.1 *g*, and then to apply a static horizontal force to the structure equal to the mass of the structure multiplied by the acceleration. The equivalent static force approximates the inertia forces to which the structure would be subjected if it were completely rigid. However, in a rock-fill dam this force has no significance because if it should exceed the shearing resistance of the dam the only effect would be to initiate shearing displacements. Before much displacement could occur, the direction of the ground acceleration would be reversed as the ground oscillated back, and the shearing displacements would cease. Because of the flexible nature of the dam structure, these shearing displacements would not be important. The action would be similar to the response of a deck of playing cards to a small horizontal oscillation of the table on which it lay. Provided that the displacement of the base was only a small percentage of the width of the deck, no harm would result. As previously stated, settlements and cracking of the attached rigid structures would occur, but the basic dam structure would be undamaged.

5. These conclusions apply to both types of rock-fill dams with impervious cores. However, the results of the investigation indicate that the sloping-core type is somewhat more earthquake resistant than the center-core type because its structure is more closely bound together. In the center-core dam the vertical core breaks the continuity between the upstream and downstream segments of the rock fill and constitutes a zone of weakness, whereas the entire structure of the sloping-core dam acts as a unit. Either type of dam can be

---

[10] "Investigation of the Effect of Transient Loading on the Strength and Deformation Characteristics of Saturated Sands," by H. B. Seed and R. Lundgren, paper presented at the Annual Meeting of the A.S.T.M., Chicago, Ill., June 16, 1954.

made stable and will exhibit high resistance to earthquake action, but because of its greater rigidity the sloping-core dam should show less of the settlement that accompanies the shearing distortions.

ACKNOWLEDGMENTS

The work described herein was sponsored by the International Engineering Company of San Francisco, Calif. Donald J. Bleifuss and James P. Hawke, Members, ASCE, gave much valuable advice and assistance. The investigation was performed under the immediate supervision of Raymond E. Davis, M. ASCE, of the University of California, at Berkeley. George Wendell worked with the writers on all phases of the project.

DISCUSSION

NICOLS N. AMBRASEYS,[11] J. M. ASCE.—The authors' exposition of a difficult and almost neglected subject is extremely valuable. However, the writer believes that the deductions as to the behavior of soils during seismic vibrations are not correct.

The paper states (under the heading, "Conclusions") that "the materials of which the dam is constructed, both clay and granular, show increased strength when subjected to dynamic loading * * *."

The foregoing statement implies two conditions—first, that the simulated earthquake vibrations were equivalent to the transient loading tests previously cited,[6,10] and secondly, that the behavior of soils during seismic vibrations occurring in nature is also equivalent to that of samples that were tested in transient loading tests.

Based on existing evidence,[12,13,14,15,16] it seems justified to assume that there is an increase in the strength of soils that are subjected to dynamic loading. The writer did not understand the authors' reasoning in connecting seismic effects with dynamic effects on the strength of soils. The writer believes that results obtained from dynamic loadings have been used unjustifiably for determining the strength that might be expected in soils subjected to seismic vibrations.

Some examples of the damage to structures by the change of the stress distribution in the ground, or the change of mechanical properties of the soil during earthquakes, are the settlement of buildings, dams, landslides, and the spouting of liquified soil. These effects indicate that during earthquakes the strength of the soil decreases (which is inconsistent with the effect of dynamic loading on soils), and this decrease resembles the condition when a mass of soil is vibrated.[17,18,19,20,21]

It has been proved[22] that earth dams of approximately isosceles cross section and, to a certain extent, rock-fill dams subjected to horizontal vibrations suffer only from shear. By neglecting the vertical seismic component, the situation appears to bear no relation to a single-half phase of transient loading.

---

[11] Researcher, Imperial College of Science and Technology, London, England; formerly Teaching Asst., National Technical Univ., Athens, Greece.

[12] "The Behavior of Soils Under Dynamic Loadings," *Contract DA-49-129-ENG-227*, Massachusetts Inst. of Technology, Cambridge, Mass., August, 1954.

[13] "Investigation of the Effect of Transient Loading on the Strength and Deformation Characteristics of Saturated Sands," by H. Seed and R. Lundgren, *Proceedings*, A.S.T.M., Vol. 54, 1954.

[14] "Characteristics of Cohesionless Soils Affecting the Stability of Slopes and Earth Fills," by A. Casagrande, *Contributions to Soil Mechanics*, Boston Soc. of Civ. Engrs., Vol. 2, 1940, p. 257.

[15] "Research on Stress-Deformation and Strength Characteristics of Soils and Soft Rocks Under Transient Loading," by A. Casagrande and W. Shannon, *Soil Mechanics, Series No. 31*, Harvard Univ., Cambridge, Mass., 1948.

[16] "Strength of Soils Under Dynamic Loads," by A. Casagrande and W. Shannon, in "Panama Canal—The Sea-Level Project: A Symposium," *Transactions*, ASCE, Vol. 114, 1949.

[17] "The Behaviour of Soils During Vibrations," by T. Mogami, *Proceedings*, 3d International Conference on Soil Mechanics, Zürich, Vol. 1, 1953, pp. 152–155.

[18] "The Dynamical Properties of Soils," by T. Mogami, *1st Report*, Inst. of Science, Univ. of Tokyo, Japan, Vol. 8, No. 1, 1954, pp. 31–38.

[19] "The Dynamical Properties of Soils," by T. Mogami, *2d Report*, Inst. of Science, Univ. of Tokyo, Japan, Vol. 9, No. 1, 1955, pp. 41–52.

[20] "Experiments with a Shaking Machine," by F. Rogers, *Bulletin*, Seismological Soc. of America, Vol. 20, No. 3, 1930, pp. 147–159.

[21] "Motion of a Soil Subjected to a Simple Harmonic Ground Vibration," by L. Jacobsen, *ibid.*, pp. 160–195.

[22] "Fundamental Considerations on the Earthquake Resistant Properties of Earth Dams," by M. Hatanaka, *Bulletin No. 11*, Disaster Prevention Research Inst., Kyoto, Japan, 1955.

Compressive stresses actually increase because of the combined effect of the horizontal component and the additional vertical component of the earthquake force, as do the shearing stresses that are related to the compression. The bearing capacity of a soil depends on its shearing resistance, which, in turn, is controlled by the internal friction, the cohesion of the particles, and the pore water pressures. During an earthquake the shearing strength of normally consolidated soils is probably partly or even wholly destroyed by the combined effects of thixotropic and remolding losses. In addition, an increase in pore pressure, which results from the rearrangement of the particles of a semisaturated soil, contributes to the reduction in the soil shearing strength.

During the first disturbance the behavior of a soil is similar to that occurring in rapid shearing, but usually the shear stress does not exceed the strength of the material. According to the relative rate of strain and drainage conditions, pore pressure may decrease rapidly for overconsolidated soils with a consequent increase in the strength of the material. The negative phase follows and the motion is reversed. In the case of saturated soils the rearrangement of the grains of the soil affects the interlocking ratio as well as the void ratio during this phase, with a consequent increase of the pore pressures, which, with local remolding phenomena, decreases the strength that was developed at the end of the first phase. After sufficient cycles, a considerable reduction in strength may take place.

TABLE 3.—SUMMARY OF TEST RESULTS FOR EXPERIMENTS E, F, AND G

	Model E[a]	Model F[b]	Model G[a]
Water content, in percentage			
Initial	125	125	125
Final	115	113	132
Change, in percentage	−8	−9.6	5.6
Shear strength, in pounds per square foot			
Initial	5.8	5.8	5.8
Final	7.2	7.7	5.0
Change, in percentage	24	33	−14
Elapsed time, in hours	3	2	3
Approximate consolidating pressure, in pounds per square foot	25	50	2 to 3

[a] Water in dam. [b] No water in dam.

The foregoing hypothesis can be checked by analyzing the observed variation of the strength of the core material during experiments E, F, and G. Because of the geometry of the construction of models E and F, the consolidating pressures for the core material were greater than for model G. Furthermore, consolidation pressures for model F were greater than those for model E because of the difference in the unit weight of the overlying granular burden due to uplift. Model F gave a 33% net increase in strength after shaking (increase in strength due to 3-hr consolidation minus a decrease due to vibration). A corresponding value for model E was 24%, and in the case of model G there was a decrease in net strength of 14% (with the vibration effects predominating). A summary of the results is shown in Table 3.

A mean value of 15% is assumed[23] for the reduction of shear strength due to vibration for accelerations of as much as 1 g, which results in the following conditions after consolidation but before shaking:

	Model E	Model F	Model G
Approximate consolidation pressure, in pounds per square foot	25	60	3
Shear strength, in pounds per square foot	8.0	8.6	5.9

The preceding data could be checked if there were records of observations of the consolidation characteristics of the material used. By using such records the mean value of the reduction of shear strength could be determined indirectly.

The writer believes that the high degree of earthquake resistance exhibited by the models can be attributed to the fact that rock-fill dams are capable of withstanding large distortions without appreciable damage, especially when founded on firm ground. The increase in strength exhibited by the cores of models E and F is attributed mainly to the consolidation and thixotropic effects that occurred during the 2-hr-to-3-hr period of loading of the cores by the overlying material.

The writer believes that the foundation is a more important factor governing the stability of an earth-fill dam or a rock-fill dam. These types of dams can frequently be constructed on comparatively weak ground. In such a situation two phenomena would occur in the foundation, namely settlement and the creation of discontinuities within the foundation owing to the large stresses. The foregoing result in failure of the slopes because of the decrease of the bearing capacity of the foundation.

The hydrodynamic pressures on the model during the simulated earthquake vibrations, although small, were 40% less than the pressures corresponding to an infinitely long reservoir, or even to a reservoir whose length is three times the depth of the water.

JOHN V. SPIELMAN,[24] A. M. ASCE.—The paper describes an interesting model study, and it is concluded that the sloping-core model that was tested was equally stable or more stable than the central-core model. However, the writer objects to some of the other conclusions.

The statement is made (under the heading, "Design of the Models: Similitude Requirements") that "the strength of the materials is assumed to depend on two factors—the angle of internal friction and the cohesion." This statement entirely neglects the internal stresses (compressive, tensile, and shear) in the rock-fill material. How these stresses affect the problem will be discussed herein.

In a rock-fill dam many of the stones have point bearing on adjacent stones. If readjustments or increased loadings occur as a result of settlement, earthquake movement, water load, or other causes, there is usually some crushing and shearing at these points of bearing. This would contribute to the over-all settlement of the fill.

---

[23] "On the Bearing Capacity of Soils During Earthquakes," by S. Okamoto, Univ. of Tokyo Japan, June, 1956.

[24] Superv. Engr., State Dept. of Water Resources, Los Angeles, Calif.

In the model tests it is believed that the crushing strength of the model material representing the rock fill should have had the scale ratio of

$$\frac{\gamma_m}{\gamma_p}\lambda = 0.87\left(\frac{1}{150}\right) \qquad (12)$$

For example, if the rock used in the prototype dam had a crushing strength of 10,000 lb per sq in., the strength of the model material should have been 58 lb per sq in. It is believed that the effect would have been to increase the settlement appreciably.

If the rock-fill material in the model were damp, it would have a slight cohesion. In the model this might be an appreciable proportion of the total shear strength, whereas in the prototype dam it would be negligible compared with the friction.

The settlement is the principal criterion noted in the testing, and the writer questions whether there is sufficient similitude to extrapolate the results to the prototype safely.

RAY W. CLOUGH[25] AND DAVID PIRTZ,[26] ASSOCIATE MEMBERS, ASCE.—The writers appreciate the interest exhibited in the discussions and wish to amplify the paper with respect to the material presented by the discussers.

Mr. Spielman has recognized correctly the lack of similitude in the model with respect to the crushing strength of the rock. Ideally, the ratio of model rock strength to prototype rock strength should have been the same as the ratio of the moduli of rigidity and the ratio of the shear strengths. To achieve this would have required a crushing strength in the model rock equal to from approximately 50 lb per sq in. to 100 lb per sq in. (as stated by Mr. Spielman). The writers were aware of the discrepancy at the time the model was designed, but disregarded it as relatively unimportant to the primary action that was being studied. It is true that local crushing of the rock could have had a noticeable effect on the settlement of the rock structure, but, contrary to Mr. Spielman's statement, the magnitude of settlement was not the principal factor investigated. The primary objective was to learn whether the structure would exhibit tendencies toward shear failure through the clay core; local crushing would have had little effect on the shear failure. The actual magnitude of settlement would depend on many factors that are more important than local crushing, such as the intensity and duration of the earthquake as well as the degree of compaction of the rock obtained during its original placing. Consequently, only the relative magnitudes of settlement that were observed with different models have any significance in the tests, and the lack of similitude with respect to crushing strength would have had no effect on the relative settlements.

---

[25] Associate Prof. of Civ. Eng., Univ. of California, Berkeley, Calif.

[26] Associate Prof. of Civ. Eng., Univ. of California, Berkeley, Calif.

In answer to Mr. Spielman's question concerning the cohesion in the wet rock structure due to surface-tension effects, it is believed that the magnitude of the cohesive forces is completely negligible for the size of rock particles that were used in the investigation (between $\frac{3}{8}$ in. and No. 30 sieve).

Mr. Ambraseys introduced several interesting ideas with regard to the action of the model materials during an earthquake. The principal contention seems to be that the increase of soil strength that was demonstrated in the dynamic tests would not be developed during earthquakes because of the alternating nature of the earthquake motion as compared with the one-directional dynamic tests. Mr. Ambraseys referred to observed earthquake damage as well as laboratory experiments in support of this contention. However, the writers believe that the available evidence supports the original claims. It must be recognized that the structure that is being considered does not consist of natural soils but of carefully placed rock with a highly compacted clay core. For this reason observations of damage due to the failure of natural soils during earthquakes would appear to have little bearing on the expected response of a rock-fill dam. Similarly, the results of laboratory tests on sandy soils that are subjected to vertical vibrations[27] do not appear to be pertinent to the present investigations.

With regard to the rock structure of both the model and the prototype, it is likely that the dynamic-strength increases are small. However, there is little reason to expect differences between one-directional-load effects and alternating-load effects in the material because pore pressures, thixotropy, and remolding effects are not involved. With reference to the model clay core, the increase of dynamic strength as compared with the static strength (which was measured in the one-directional tests) should apply equally as well to alternating loadings, because for this supersaturated, semifluid kaolin mixture there should be no difference in pore-pressure effects, and remolding or thixotropic effects are neglible.[28] The analysis presented by Mr. Ambraseys to support his hypothesis regarding the loss of shear strength of the core during earthquakes has no significance because it is based on the erroneous assumption that the water content of all samples before testing was 125%. Actually, as was stated in the paper, the initial water content was made greater than 125% in all cases in an effort to provide the desired shear strength at the time of the test. The shear strength would increase with time because of consolidation.

The situation is different with respect to the prototype clay core because it is believed that thixotropic effects might be present to some degree, depending on the material involved. However, the writers are not aware of any evidence that remolding losses will be significant in compacted clays, nor is it likely that earthquake distortions could cause any significant pore pressures in the carefully compacted material. Thus, the loss in strength that might be expected in alternating loadings would be due only to thixotropy, and the gain in strength associated with dynamic loadings (as measured in one-directional tests) would probably more than offset the thixotropic loss.

---

[27] "The Behaviour of Soils During Vibrations," by T. Mogami, *Proceedings*, 3d International Conference on Soil Mechanics and Foundation Eng., Zürich, Vol. 1, 1953, pp. 152–155.

[28] "Thixotropic Characteristics of Compacted Clays," by H. B. Seed and C. K. Chan, *Proceedings Paper 1427*, ASCE, November, 1957.

The writers agree with Mr. Ambraseys that the increase of strength that was exhibited by the cores of models E and F, as compared with the intended value, is due to consolidation. It was not intended to imply that the increases were a result of dynamic effects. The increase in strength represents a lack of similitude in the model which was entirely unintentional but unavoidable.

The writers also concur with Mr. Ambraseys with regard to the importance of foundation conditions in the over-all strength of the dam. However, the models were designed to evaluate the stability of a particular prototype structure that happened to be found on solid rock and, consequently, the models were constructed on a solid base. Therefore, it is apparent that the observed results may be applied to other foundation conditions only with caution.

With regard to the length of the reservoir behind the dam, the tests were conducted so that the initial impact caused the dam to move away from the reservoir. The tests were deliberately designed in this fashion because the principal objective of the study was to observe tendencies toward slip failures in the clay core, and these tendencies were accentuated by downstream accelerations. Therefore, the length of the reservoir would have little effect on failures in the clay core, and the lack of similitude in this respect was considered unimportant (although it is granted that downstream settlement effects might have been slightly more pronounced if the reservoir had been longer).

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2940

HARRISON COUNTY (MISSISSIPPI) ARTIFICIAL BEACH

By Francis F. Escoffier,[1] A. M. ASCE

Synopsis

The natural beach that existed along the Harrison County (Mississippi) shore disappeared when a sea wall was completed in 1928. This was followed by the loss of backfill through the joints and drains in the sea wall and later by some damage to the wall during a hurricane in 1947. Studies indicated that an artificial beach would provide needed protection to the sea wall and would also provide a desirable recreational facility for the public.

Introduction

The shore line under consideration extends from St. Louis Bay to Biloxi Bay, which is a distance of approximately 27 miles, and lies entirely within Harrison County, Mississippi (Fig. 1). A natural beach, which existed originally along this shore, depended on the erosion of the bluff at its landward limit for a natural supply of sand. The construction of a bulkhead or sea wall to protect the bluff could therefore be expected to result eventually in the loss of the beach.

Construction of the Sea wall

In 1915 a highly destructive hurricane destroyed more than half of U. S. 90 between Pass Christian and Biloxi and caused about $13,000,000 worth of damage to beach-front property in Mississippi and Louisiana. Mississippi then passed a law appointing a commission to study the subject and authorizing the coastal counties to issue bonds for the construction of sea walls. During the period from 1925 to 1928, Harrison County constructed approximately twenty-four miles of sea wall at a cost of $3,400,000. This wall extended from Henderson Point to the Biloxi lighthouse. It consisted of an inclined concrete slab supported on piling, and a series of steps formed its upper surface. The top

---

Note.—Published, essentially as printed here, in September, 1956, in the Journal of the Waterways and Harbors Division, as *Proceedings Paper 1060*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Hydr. Engr., Mobile Dist., Corps of Engrs., U. S. Dept. of the Army, Mobile, Ala.

elevation varied from 8 ft to 11 ft above mean sea level, the height being governed by the elevation of the back-shore area. The slab was supported at its toe by a continuous concrete sheet-pile curtain wall and at the rear by square concrete bearing piles. Storm drainage was provided through the wall by concrete pipes, which were encased in concrete collars on the seaward side of the wall.

The beach in front of the sea wall disappeared a short time after the wall was constructed, and much of the sand backfill was lost through defective joints and drains in the wall. The sidewalk then settled and disintegrated, and the adjoining highway was threatened.

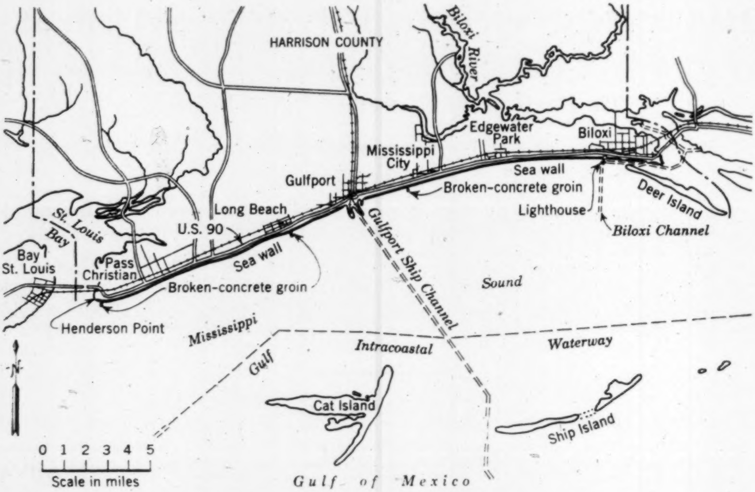

HARRISON COUNTY
Biloxi River
Edgewater Park
Biloxi
Mississippi City
Sea wall
Gulfport
Broken-concrete groin
Deer Island
Lighthouse
Biloxi Channel
Long Beach
St. Louis Bay
U.S. 90
Sea wall
Bay St. Louis
Pass Christian
Broken-concrete groin
Gulfport Ship Channel
Sound
Henderson Point
Mississippi
Intracoastal
Waterway
Gulf
Cat Island
Ship Island
N
0 1 2 3 4 5
Scale in miles
Gulf of Mexico

FIG. 1.—HARRISON COUNTY (MISSISSIPPI) BEACH

THE COOPERATIVE STUDIES

In March, 1942, the Harrison County Board of Supervisors made a formal application to the Corps of Engineers (United States Department of the Army) for a cooperative study to determine the best method for repairing and protecting the existing sea wall against further damage, and to furnish plans for establishing and protecting a beach along the shore line.

After making a study of the problem, the Chief of Engineers, acting on the recommendations of the Beach Erosion Board, recommended the immediate initiation of a program of sea-wall repair and beach construction at an estimated cost of $620,000, and the institution of an adequate program of periodic inspection and maintenance of both the beach and the sea wall. It was further recommended that no share of the expense of the improvement be borne by the United States.

A policy of federal assistance in the construction of works for the improvement and protection of publicly owned shores against erosion by waves and currents was set forth in 1946.[2] Following the enactment of this law, Harrison County initiated a second cooperative study to supplement the first and to reconsider the questions of federal participation.

Before this second study could be completed, the hurricane of September, 1947, occurred. This hurricane moved inland over the Mississippi River delta in a northwesterly direction and passed through New Orleans (La.). The Harrison County shore was in the right half of the hurricane and was exposed to winds with velocities as high as 100 miles per hr. The wind came generally from an easterly quadrant and drove the water before it into Mississippi Sound and the neighboring bodies of water. The result was the creation of a storm tide that increased in height progressively from east to west and reached a maximum known stage of El. 15.2 at Bay St. Louis.

No observations are available with respect to the height of waves in the sound during the hurricane. However, formulas for the height of waves in shallow bodies of water, developed from observations made in Lake Okeechobee (Florida), indicate that the height of waves at the sea wall probably did not exceed 10 ft.

The sea wall was completely overtopped, and much of the land behind it was inundated. The waves passed over the wall to attack the highway and those buildings that were not on high ground. More than nine hundred buildings were destroyed, and more than eight thousand five hundred were damaged. The cost of repair to roads and bridges in Mississippi was $544,000. Most of the fill behind the sea wall was lost, but the wall itself failed at only a few points. Failure, where it did occur, was caused by hydrostatic pressure, which lifted the stepped slab from its supporting piling.

The second cooperative study, completed in 1948, took into account the changes caused by the 1947 hurricane and went into greater detail than the first study, particularly with respect to repairs to the drainage system. This study, as did the previous one, indicated that a hydraulic fill placed seaward of the sea wall, if properly maintained, would be the most suitable method of stopping the leakage of sand through the wall and of prolonging the life of the wall. In addition, such a beach would provide a recreational facility for the public. In conformance with the policy set forth in the new law, the Chief of Engineers, again acting on the recommendations of the Beach Erosion Board, recommended federal participation to the extent of $1,133,000 toward the repair of the sea wall and its protection by the construction of a beach. This sum was one-third of the original cost of the wall, which was $3,400,000. The amount of federal aid was based on the sea-wall proviso[2] of Public Law 727, which states:

> "* * * that where a political subdivision has heretofore erected a seawall to prevent erosion, by waves and currents, to a public highway considered by the Chief of Engineers sufficiently important to justify protection, Federal contribution toward the repair of such wall and the protection

---

[2] *Public Law 727*, 79th Cong., approved August 13, 1946.

thereof by the building of an artificial beach is authorized at not to exceed one-third of the original cost of such wall * * *."

The plan of improvement formulated in the studies was adopted in the 1948 River and Harbor Act.[3] This plan included repairing the stepped concrete slab by the pressure concrete or "gunite" method, replacing backfill, pumping the hydraulic fill for the beach, and reconstructing the drainage system. The total volume of sand required for the beach was 5,985,000 cu yd. The drainage plan included a collecting sewer back of the sea wall, discharging through relatively few laterals across the beach. All drainage lines were designed to minimize differential settlement and to assure tight joints in order to prevent the infiltration of sand. Sewer pipes laid across the beach were anchored at their seaward ends by creosoted timber piling structures. At the larger outfalls the plan specified that drainage be carried across the beach between two parallel rows of interlocking concrete sheet piles.

Construction of the Beach

Before the dredging operation was begun, the county built three long groins of broken concrete. These groins, together with the Gulfport and Pass Christian harbor peninsulas and a few previously existing concrete groins at the Biloxi lighthouse, divide the beach into five separate compartments. The most important groin is at Henderson Point, which confines the beach at its western extremity.

The dredging operation was performed by two hydraulic dredges working toward each other, one beginning at Henderson Point and the other at the Biloxi lighthouse. Work was commenced early in January, 1951, and completed in December of that year.

The sand was obtained from borrow pits that were at least 1,500 ft from the sea wall and from either side of the Gulfport harbor. First, a ridge about 1,000 ft long was constructed at a distance of 300 ft from the sea wall. The intervening space between the ridge and the wall was then filled with a volume of material that was somewhat more than that required to bring the beach to the specified cross section. The excess was provided to forestall any need for the dredges to return for repumping and also to provide material for refilling behind the sea wall. This process was repeated in steps of approximately 1,000 ft. It was found subsequently that the retaining dike in beach construction was unnecessary. Actually, the dike prevented the runoff of the undesirable fine material.

In order to provide continuous drainage from the land side of the sea wall, depressions were left across the beach at the outfalls, the placing of which followed the hydraulic fill as closely as possible. The flow line of the discharge end of the pipes was established at El. − 0.5 at distances from the sea wall varying from 248 ft for the 42-in. pipes to 300 ft for the 18-in. pipes. The upper half of the outer end of the pipes was left exposed, but most of the remainder of the pipes were entirely covered with sand.

The specified beach cross section included a 50-ft level crown at El. 5 adjacent to the sea wall and a seaward slope of 1 on 50 down to natural ground surface. Every effort was made to place the material within the prescribed slope limits, but, because the underwater slope assumed by the sand was much steeper than that specified, it was necessary to overpump the outer slope and later to dress the excess material landward with bulldozers. Present (1958) practice is to place the estimated quantity with the berm at the required elevation, and to allow the slope seaward of the berm to assume its natural slope under wave action.

---

[3] *Public Law 858,* 80th Cong., 2d Session, Chapter 771.

As soon as the beach was in place, it was observed that the waves produced a weak littoral drift toward the west. That the drift was westward could be inferred from the accumulation of sand on the east side of the groins and outfalls, and from the erosion that appeared on the west of these structures. Sand has escaped around the outer end of the Henderson Point groin and has formed a small pocket beach on the west side of it.

A ridge or berm with a crest at El. 3 was formed. This change in the shape of the beach left many outfall lines protruding into the water. It also left pools of water impounded landward of the ridge. In order to eliminate the pools and to obtain a more uniform beach slope, the sand was regraded with bulldozers to a new cross section consisting of a 1-on-100 slope extending from the sea wall to the ridge crest and a 1-on-10 slope from the crest to the low-water line. Seaward of this line the sand assumed a rather flat natural slope.

In some cases the erosion on the west side of the pipes was severe enough to be objectionable. The county engineers developed a type of spur groin that is proving effective in remedying this situation. These spur groins are built of broken concrete and extend westward from the outer end of the pipes and parallel to the sea wall. They are 50 ft long and are placed to the same elevations as the top of the pipes. The spur groins function by intercepting the waves and thereby reducing the attack on the beach behind them.

The outfall pipes and flumes tend to fill with sand at their outer ends. However, in most cases the storm waters flowing through the pipes are capable of flushing out the sand. This is not true of the flumes, and these must be cleaned out from time to time. The large outfall flume at the Veterans Hospital at Gulfport has been a particular problem. Extending the east wall approximately 50 ft with a broken-concrete groin to impound the littoral drift has partly solved the problem. The results have been good, but it is believed that the groin should be extended somewhat farther and raised to a height of approximately 3 ft. A spur groin has been constructed at the outer end of the west wall to reduce erosion west of the flume.

The observed slope adjustments, the effect of the sand ridge on the outfalls, and other experiences with the finished beach were used as a guide when the county extended the beach eastward along the waterfront in Biloxi. This extension, which was completed in 1954, was designed for El. 5 at the sea wall, a 1-on-100 slope for 220 ft to a crest at El. 2.8, and then a slope of 1 on 10 to the natural floor in the sound. The beach was pumped successfully without using a retaining dike at the seaward edge of the beach, and the use of such dikes in the future construction of artificial beaches is considered inadvisable. No appreciable slope adjustment due to the action of the tides and waves has been observed.

A comparison of surveys made over a long period of time is the only accurate method of determining the rate at which sand is lost from a beach. An accurate estimate of this rate for the Harrison County Beach cannot be given at yet (1958). In an attempt to determine the changes that had occurred between 1951 and 1953, groups of five cross sections each were taken at 1-mile intervals. They were located so as not to experience the groin effects of the pipe outfalls. A typical section is shown in Fig. 2. The readings at the toe of the 1951 slopes are indefinite because of the mud found there. It was necessary to make allowance in the computations for the sand that was removed from the beach to backfill the sea walls. It was finally estimated that the annual loss from the beach extending from Henderson Point to the Biloxi lighthouse was approximately 32,500 cu yd for the period from 1951 to 1953.

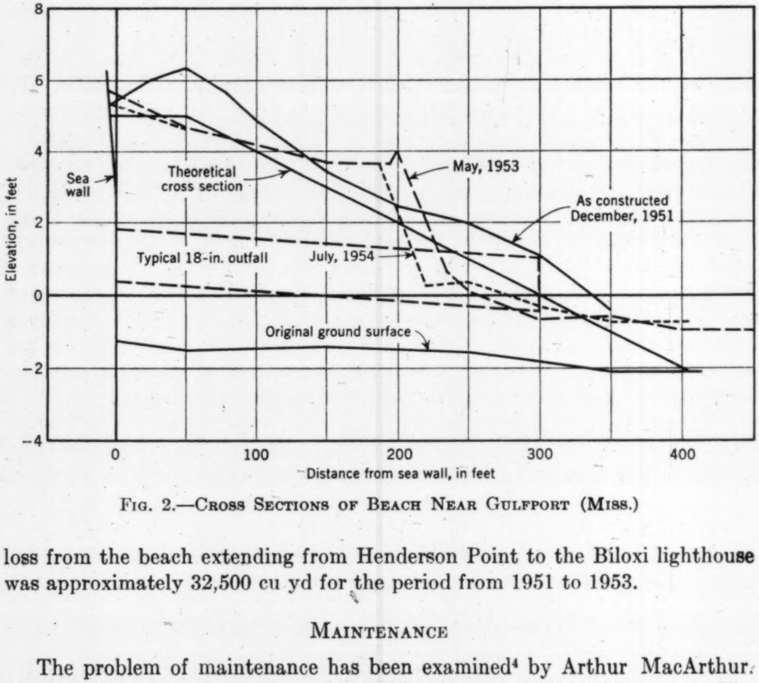


Fig. 2.—Cross Sections of Beach Near Gulfport (Miss.)

Maintenance

The problem of maintenance has been examined[4] by Arthur MacArthur. The maintenance program is performed by a crew of about fifty men. The work includes reshaping the beach; filling low areas; removing wind-blown sand from the sea wall and the adjacent roadway; removing grass, debris, and trash from the beach; cleaning outfall pipes and flumes; and constructing broken-concrete groins as needed at the seaward ends of these pipes and flumes. The equipment used includes six trucks, eight bulldozers, two seaman mixers, one small dragline, and one sanitizer. The mixers are used to remove grass and other vegetation from the beach by completely uprooting it and leaving it on the beach to be dried by the sun. Debris left by the tide is gathered by hand and burned. The sanitizer is a tractor-drawn machine that picks up the sand to a depth of approximately 4 in. and removes all sharp-edged shells, broken glass, and other objects that would otherwise be a menace to the users of the beach. The cost of maintaining the beach averages approximately $10,000 per month, according to the engineers employed by the county. No beach replenishment by dredging has been necessary since the completion of the fill, and no major repairs to the drainage system or to the sea wall have been required (as of 1958).

---

[4] "Maintenance of the Harrison County, Mississippi, Sloping Beach," by Arthur MacArthur, *Shore and Beach*, Am. Shore and Beach Preservation Assn., Washington, D. C., Vol. 24, No. 1, 1956.

Conclusions

The artificial beach along the Harrison County sea wall has proved effective in preventing the escape of backfill. Too short a time has elapsed since its completion to determine its resistance to erosion, particularly in view of the fact that no hurricane or severe tropical disturbance, with accompanying high tide and severe wave action, has directly attacked the project area. Observations indicate that the rate of erosion has been small and that replenishment will be required only at rare intervals.

The beach is used extensively as a recreational facility. Although no actual count has been taken of the daily visitors to the beach, its extensive use has been noted by county officials and by representatives of the Corps of Engineers. There has also been a considerable expansion in hotels, tourist cottages, and other tourist accommodations.

Acknowledgments

The writer wishes to express his appreciation to Mr. MacArthur and to J. K. Muether for furnishing much of the data presented herein.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2941

MOMENTS IN FLAT SLABS

By Mark W. Huggins,[1] M. ASCE, and Watone L. Lin[2]

With Discussion by Messrs. James Chinn, and Mark W. Huggins and Watone L. Lin

Synopsis

The results of a study of moments in a cast-aluminum model of a flat-slab floor are summarized. All panels that were used in the study were square. The columns and capitals were round, and there were no drop panels. The model was two bays by three bays and was tested under air pressure with a pattern-type loading.

Moments obtained from SR-4 strain-gage readings are compared with those that were obtained by a continuous-frame analysis following the American Concrete Institute code.

Introduction

Frame analysis of a flat-slab floor, according to the American Concrete Institute (ACI) code (ACI-318-51),[3] has been widely adopted because of its simplicity and flexibility. By the recommended procedure, flat slabs are analyzed as

> "* * * a number of bents, each consisting of a row of columns and strips of supported slab, each strip bounded laterally by the center line of the panel on either side of the row of columns. The bents shall be taken longitudinally and transversely of the building."[3]

It is usual to make the analysis using two-cycle moment distribution for a pattern-type live load. The critical sections for negative moment may be assumed to be not more than a distance equal to $X L$ from the column center lines, in which

$$X = 0.073 + 0.57 \frac{A}{L} \qquad (1)$$

Note.—Published, essentially as printed here, in July, 1956, in the Journal of the Structural Division, as *Proceedings Paper 1020.* Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions.*

[1] Associate Prof. of Civ. Eng., Univ. of Toronto, Toronto, Ontario, Canada.

[2] Asst. Bridge Design Engr., Dept. of Highways, Toronto, Ontario, Canada.

[3] "Building Code Requirements for Reinforced Concrete," *ACI-318-51*, A.C.I., April 5, 1951.

in which A represents the radius of the column capital and L denotes the span length of the slab center to center of the columns in the direction in which bending is considered.

Finally, the moments are allotted to column strips and middle strips in the same ratio as in design by the ACI moment coefficients.

At various times, the reliability of this procedure has been questioned. The results of an experimental investigation of a single-story, cast-aluminum flat-slab model (Fig. 1) are compared herein with moments that were found by the ACI recommended method of analysis.

These results indicate that the positive moments should be increased and that the critical negative moments could be reduced slightly. However, it is evident that a more extensive investigation is desirable.

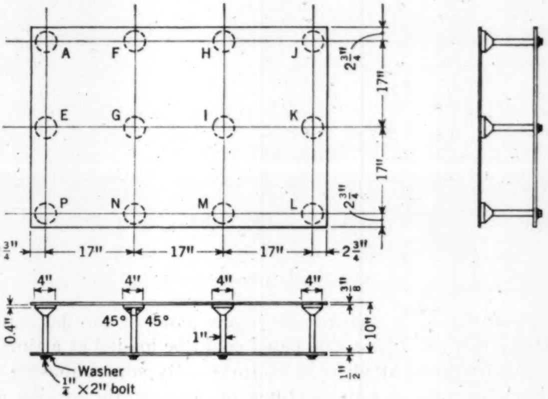


FIG. 1.—FLAT-SLAB MODEL

MODEL

The model (Fig. 1) was made in two parts—a top plate with columns and a bottom plate. The upper part was made of cast aluminum and was intended to have a slab thickness of ½ in., but due to faulty casting it became necessary to machine it to ⅜ in. in order to obtain satisfactory surfaces for the application of the SR-4 strain gages. The bottom plate was made of rolled aluminum. The connections between the columns and the bottom plate were made by 2-in. bolts, which had diameters of ¼ in.

Aluminum was chosen as the model material because

1. It is easily cast and is light.
2. It is nearly homogeneous, isotropic, and perfectly elastic.
3. It has a low value of Young's modulus of elasticity, which results in greater strains under small loads and, as a consequence, permits the use of smaller loads.

LOADING SYSTEM

All loads were applied by pneumatic pressure using the loading frame shown in Fig. 2 and the pressure cell shown in Fig. 3. The frame was made of structural-steel angles and plates. The pressure, which was applied by an automobile pump, was measured by a mercury manometer. The pressure cell or loading box had the same dimensions as one panel of the flat slab (17 in. by 17 in.). Using this procedure, one panel could be loaded at a time, and strain readings could be taken at all gage stations. By superimposing results that were obtained for various combinations of loading, the strains and stresses could be obtained for any desired loading pattern.

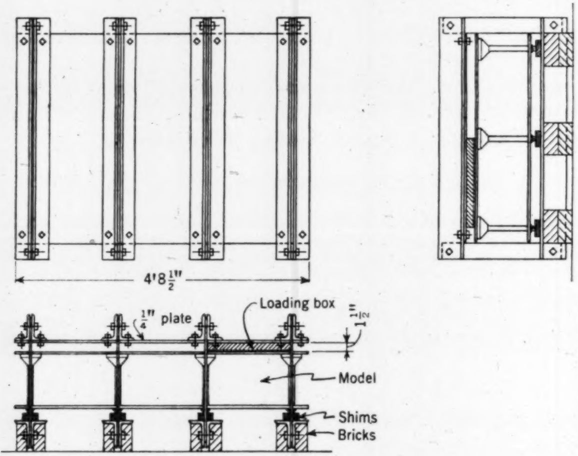

4'8½"
¼" plate
Loading box
1½"
Model
Shims
Bricks

FIG. 2.—LOADING FRAME

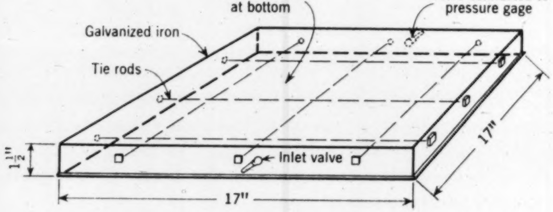

Rubber sheet at bottom
Valve connected to pressure gage
Galvanized iron
Tie rods
1½"
Inlet valve
17"
17"

FIG. 3.—LOADING BOX

The loading box had tops and sides of galvanized iron, and the bottom consisted of a sheet of rubber, which was clamped by the galvanized iron to form an airtight unit. To reduce distortion the box was reinforced by tie rods, as shown in Fig. 3.

With the loading box in place, the model was jacked up so that the top of the box was in close contact with the top steel plate of the loading frame. Steel

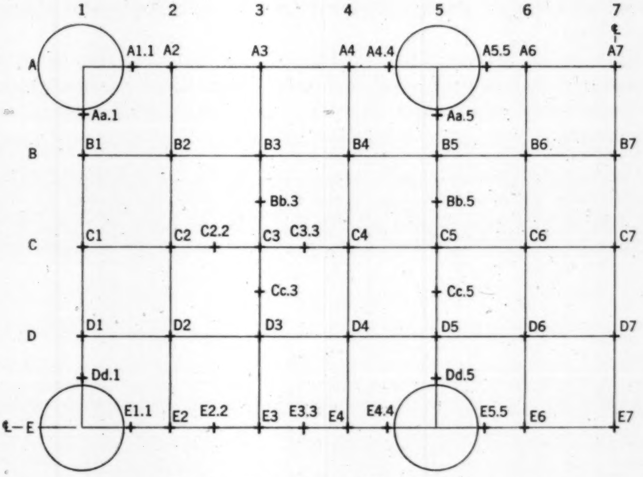

1
2
3
4
5
6
7
℄
A
B
C
D
℄–E
A1.1
A2
A3
A4
A4.4
A5.5
A6
A7
Aa.1
Aa.5
B1
B2
B3
B4
B5
B6
B7
Bb.3
Bb.5
C1
C2
C2.2
C3
C3.3
C4
C5
C6
C7
Cc.3
Cc.5
D1
D2
D3
D4
D5
D6
D7
Dd.1
Dd.5
E1.1
E2
E2.2
E3
E3.3
E4
E4.4
E5.5
E6
E7

Fig. 4.—Location of SR-4 Gages

shims were then placed under each column to hold the box in this position, and the jacks were removed. When air was pumped into the box the pressure was transmitted to the model as a uniformly distributed load.

Strain Gages and Strain Measurements

All strains were measured by SR-4 gages, which were located as shown in Fig. 4. In order to reduce the labor involved in reading the gages and also to reduce costs, only one-quarter of the model was covered with gages.

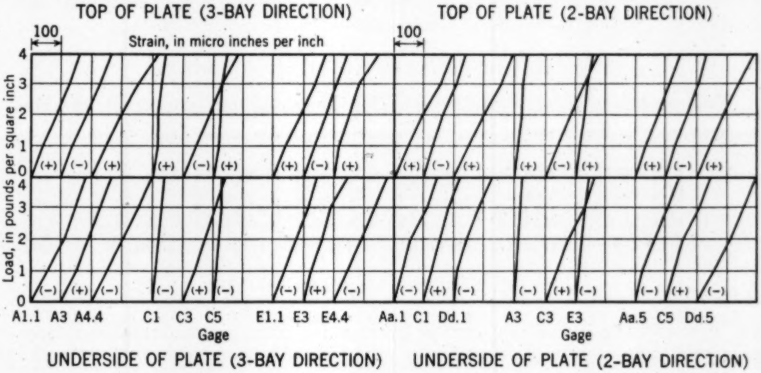


Fig. 5.—Load–Strain Curves for Load on Panel AFGE

No attempt was made to obtain principal stresses. Instead, the strains that were parallel to the column center lines were determined. The *AX*-5 gage was used to measure the strain in two perpendicular directions at each station.

Gages were identified by numbers determined by their position on the grid, as shown in Fig. 4.

Because it was necessary to record the strains of many gages for each load increment, a switch control board was built. Variations in contact resistance at the board caused some early difficulty, but eventually the system operated satisfactorily.

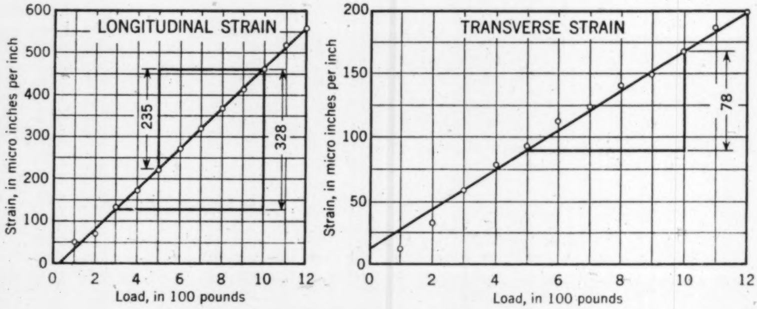


FIG. 6.—LOAD-STRAIN CURVES FROM TENSION-TEST DATA

Readings were taken at loadings of 1 lb per sq in., 2 lb per sq in., 3 lb per sq in., and 4 lb per sq in. of surface. A load-strain diagram (of which Fig. 5 is a typical example) was prepared from the readings for each gage in both directions. From these diagrams it was possible to detect faulty readings and to make new ones when necessary. The deviations from straight lines are due to the variations in contact resistance in the switching panel.

To allow for slack take-up the zero loading was taken as 1 lb per sq in. Because the maximum loading was 4 lb per sq in., all values given in the following tables and diagrams are for a load increment of 3 lb per sq in.

PHYSICAL PROPERTIES OF THE CAST ALUMINUM

Young's modulus and Poisson's ratio for the cast aluminum were determined by tests on a ½-in.-diameter bar, which was cast at the same time as the model.

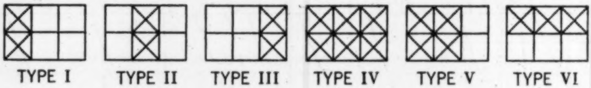


FIG. 7.—FUNDAMENTAL LOADING PATTERNS

Fig. 6 shows the load-strain curves from which Young's modulus was determined as 10,900,000 lb per sq in. and Poisson's ratio as 0.33.

LOADING PATTERN

Because only one panel was loaded at a time, it was necessary to obtain strains for the required loading patterns by summing the strain measurements from single-panel loadings. The fundamental loading patterns are shown in Fig. 7.

The following combinations for maximum-positive moments and maximum-negative moments for the model for the three-bay direction were obtained from the various types of loading, as shown in Fig. 7.

Moment	Location	Loading
Maximum positive.....	exterior panel	Type I
Maximum positive.....	interior panel	Type II
Maximum negative.....	interior support	Type V

For the two-bay direction the maximum-positive moment occurred for a Type VI loading. A Type IV loading resulted in the maximum-negative moment.

COMPUTATION OF STRESSES AND MOMENTS

In an elastic plate subjected to stresses σ_x and σ_y, the strains in the x-direction and y-direction are

$$\epsilon_x = \frac{1}{E}(\sigma_x - \mu\,\sigma_y) \quad \text{(2)}$$

and

$$\epsilon_y = \frac{1}{E}(\sigma_y - \mu\,\sigma_x) \quad \text{(3)}$$

which yields

$$\sigma_x = \frac{E}{1-\mu^2}(\epsilon_x + \mu\,\epsilon_y) \quad \text{(4)}$$

and

$$\sigma_y = \frac{E}{1-\mu^2}(\epsilon_y + \mu\,\epsilon_x) \quad \text{(5)}$$

In general, the top fiber stresses and the bottom fiber stresses at any point were unequal. This inequality was due to a certain amount of shear in the columns, thus producing thrust in the slab. To eliminate this complication from the bending-moment computations, an average was taken between the top stresses and the bottom stresses.

From the stresses computed by Eq. 4 and Eq. 5, the bending moments per inch of slab width were also computed. These moments are summarized in Table 1.

MOMENT DIAGRAMS

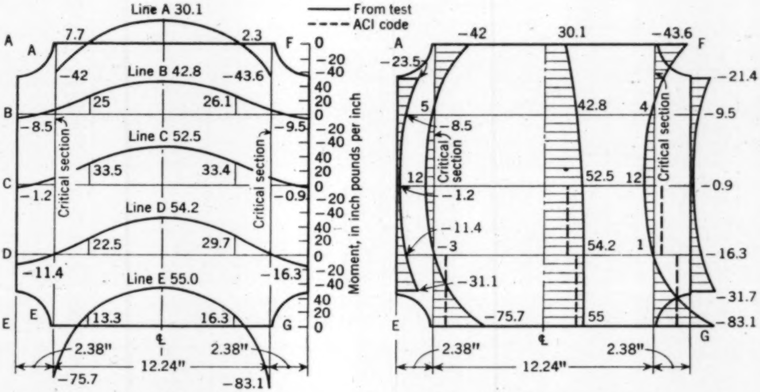
The moments shown in Table 1 occur at the gage points. Moment diagrams (Fig. 8 through Fig. 16) were plotted from these values to show the distribution of moments along and across the panels under the various loading conditions. In order to compare the test results with the ACI code analysis, it was necessary to find the average moments across the recommended design sections. The quantities of moment at the design sections in the column strips and in the middle strips were obtained from the moment diagrams by determining the areas under the curves for each strip. For many of the sections, curves have been plotted from only five points, whereas at least seven would have been preferable. This has resulted in considerable uncertainty in the negative moments taken at points other than the gage stations. For this reason it would be advisable to assume possible errors as great as 10% in the average moments at the critical sections for negative moment. The mid-span moments are much more accurate, except in cases in which they are small. It would have been preferable to use more gages nearer the supports and less gages at the center of the panel.

TABLE 1.—MOMENTS AT GAGE POINTS IN INCH-POUNDS PER INCH FOR 3-LB.-PER-SQ.-IN. LOAD

LOAD PATTERN	TYPE I $\leftarrow\!\updownarrow^{y}\!\rightarrow x$		TYPE II		TYPE III		TYPE IV		TYPE V	TYPE VI
Gage	M_x	M_y	M_x	M_y	M_x	M_y	M_x	M_y	M_x	M_y
A1.1	−42	−23	0	0.4	0	0	−42	−23	−42	−22
A2	7.7	−11	0	0	0	0	7.7	−11	7.7	−14
A3	30	−1.1	−3.5	−0.3	0	0	27	−1.4	27	3
A4	2.3	−12	−8.2	−2.6	0	0	−5.9	−14	−5.9	−14
A4.4	−44	−25	−18	−5.9	0	0	−61	−31	−61	−32
A5.5	−12	−4	−39	−23	9.1	−3.0	−42	−30	−51	−24
A6	−5.9	−2.1	8.0	−9.6	0	0	2.1	−12	2.1	−12
A7	−3.8	0	25	−3.9	−3.7	0	17.8	−3.9	21	−1.5
Aa1	−23	−40	−0.5	−1.7	0	0	−24	−42	−24	−42
Aa5	−21	−37	−26	−43	0	0	−47	−80	−47	−72
B1	−8.5	2	−1.4	−0.4	0	0	−9.9	−1.6	−9.9	3.1
B2	25	23	−4.6	1.4	0	0	20	25	20	29
B3	43	30	−4.4	−4.2	0	0	38	26	38	38
B4	26	16	−8.0	3.6	0	0	18	20	18	24
B5	−9.5	5.4	−16	−0.9	0	0	−26	4.5	−26	7.5
B6	−9.6	1.7	21	17	−2.0	−0.7	9.6	18	12	22
B7	−5.8	1.1	46	29	−4.7	1.5	35	32	40	35
Bb.3	46	36	−5.4	3.7	0	0	40	40	40	46
Bb.5	−4.3	21	−7.1	17	0	0	−11	38	−11	41
C1	−1.2	20	0.4	1.4	0	0	−0.8	21	−0.8	27
C2	33	38	−4.0	1.3	0	0	29	40	29	46
C2.2	44	41	−7.2	0	0	0	36	41	36	45
C3	52	40	−9.0	0	0	0	43	40	43	46
C3.3	48	45	−11	−3.5	0	0	37	41	37	49
C4	33	39	−17	5.3	0	0	16	44	16	51
C5	−0.9	22	−1.9	24	0	0	−2.8	44	−2.8	52
C6	−8	11	33	36	−2.9	−0.9	22	46	25	52
C7	−10	4.0	51	42	−6.5	1.3	35	47	41	52
Cc.3	49	36	−9.6	3.9	0	0	40	40	40	43
Cc.5	−7.1	13	−7.1	14	0	0	−14	28	−14	40
D1	−11	1.4	0	−0.7	0	0	−11	0.7	−11	7.6
D2	22	9.3	−1.7	−0.7	0	0	21	8.6	21	18
D3	54	23	−8.6	−2.8	0	0	46	21	46	27
D4	30	21	−9.4	−0.7	0	0	20	20	20	22
D5	−16	−2.6	−19	−4.4	0	0	−35	−7.0	−35	9.1
D6	−10	1.5	29	13	−2.9	−0.9	15	13	18	20
D7	−9.2	−1.0	58	26	−7.1	0	42	25	49	35
Dd.1	−31	−55	0	0	0	0	−31	−55	−31	−41
Dd.5	−32	−61	−23	−44	0	0	−55	−105	−55	−81
E1.1	−76	−47	−1.7	−0.5	0	0	−77	−48	−77	−22
E2	13	−27	0	0	0	0	13	−27	13	−13
E2.2	45	−7.6	0	0	0	0	45	−7.6	45	−5.1
E3	55	−1.0	−8.6	−2.9	0	0	46	−3.9	46	−2.9
E3.3	46	1.0	−12	−4.5	0	0	34	−3.5	34	−4.9
E4	16	−16	−20	−11	0	0	−3.6	−27	−3.6	−18
E4.4	−83	−43	−30	−12	0	0	−113	−55	−113	−33
E5.5	−47	−19	−69	−42	3.6	1.2	−112	−60	−116	−31
E6	−24	−13	15	−20	−2.3	−0.7	−12	−34	−9.4	−20
E7	−11	−2.8	54	−2	−10	−5.5	32	−10	42	−7.9

As a check on the accuracy of the results, the sum of the average negative moments at critical sections and the positive moment at the panel center line was computed for the full width of the model between two critical sections in each bay. The result was compared with values computed from $wl^2/8$, in which w and l, for the three-bay direction, are defined as follows:

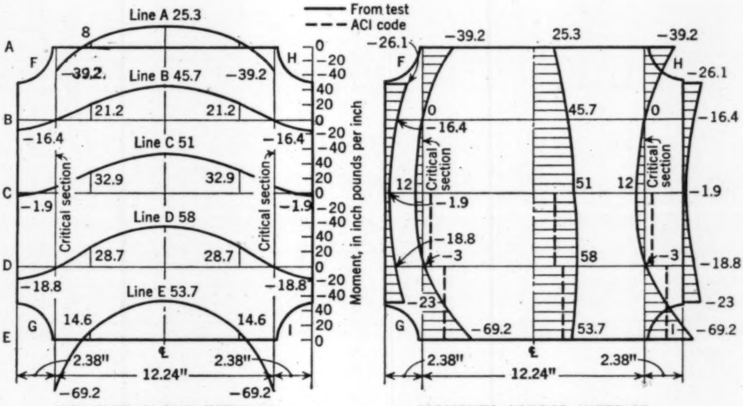
$$w = 3 \times 17 \times 2 = 102 \text{ lb per linear in.}$$



MOMENTS ALONG EXTERIOR PANEL AFGE

MOMENTS ACROSS EXTERIOR PANEL AFGE

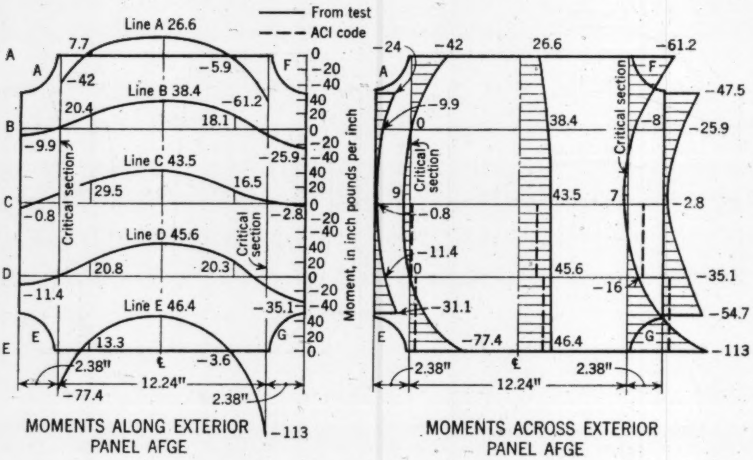
FIG. 8.—MOMENT DIAGRAMS FOR PANEL AFGE (3-BAY DIRECTION) FOR TYPE I LOADING



MOMENTS ALONG INTERIOR PANEL FHIG

MOMENTS ACROSS INTERIOR PANEL FHIG

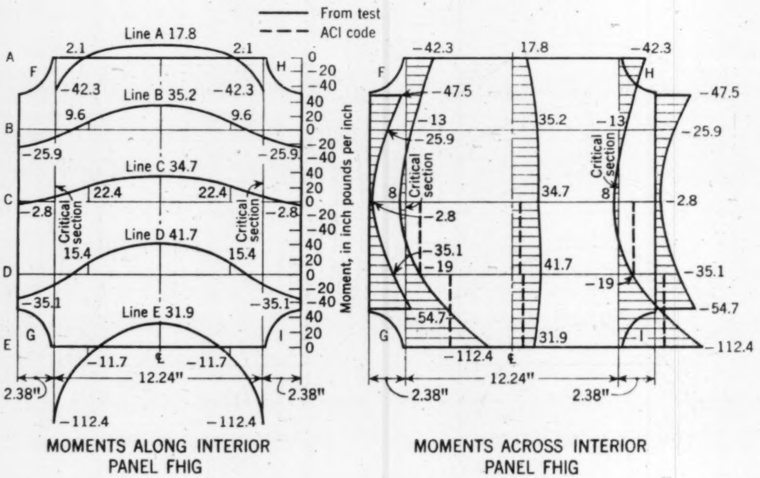
FIG. 9.—MOMENT DIAGRAMS FOR PANEL FHIG (3-BAY DIRECTION) FOR TYPE II LOADING


From test
ACI code
Line A 26.6
7.7
-42
-5.9
Line B 38.4
-61.2
20.4
18.1
-9.9
Line C 43.5
-25.9
29.5
16.5
-0.8
-2.8
Line D 45.6
20.8
20.3
-11.4
Line E 46.4
-35.1
13.3
-3.6
2.38"
12.24"
-77.4
2.38"
-113
Critical section
Moment, in inch pounds per inch
A
B
C
D
E
F
G
℄
-24
-42
26.6
-61.2
-47.5
-9.9
0
38.4
-8
-25.9
9
43.5
7
-2.8
-0.8
-11.4
0
45.6
-35.1
-16
-31.1
-54.7
-77.4
46.4
-113

MOMENTS ALONG EXTERIOR PANEL AFGE

MOMENTS ACROSS EXTERIOR PANEL AFGE

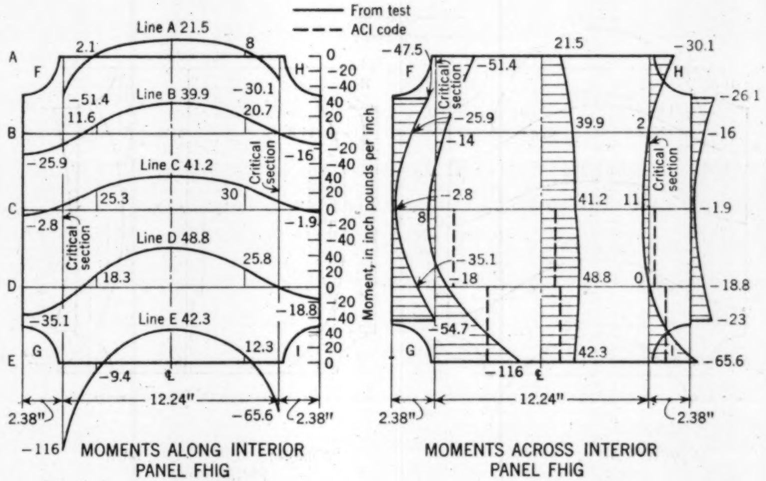
Fig. 10.—Moment Diagrams for Panel AFGE (3-Bay Direction) for Type IV Loading


From test
ACI code
Line A 17.8
2.1
2.1
-42.3
-42.3
Line B 35.2
9.6
9.6
-25.9
-25.9
Line C 34.7
22.4
22.4
-2.8
-2.8
Line D 41.7
15.4
15.4
-35.1
-35.1
Line E 31.9
-11.7
-11.7
12.24"
2.38"
2.38"
-112.4
-112.4
Critical section
Moment, in inch pounds per inch
A
B
C
D
E
F
G
H
I
℄
-42.3
17.8
-42.3
-47.5
-47.5
-13
35.2
-13
-25.9
-25.9
8
34.7
8
-2.8
-2.8
-35.1
-19
41.7
-19
-35.1
-54.7
-54.7
31.9
-112.4
-112.4

MOMENTS ALONG INTERIOR PANEL FHIG

MOMENTS ACROSS INTERIOR PANEL FHIG

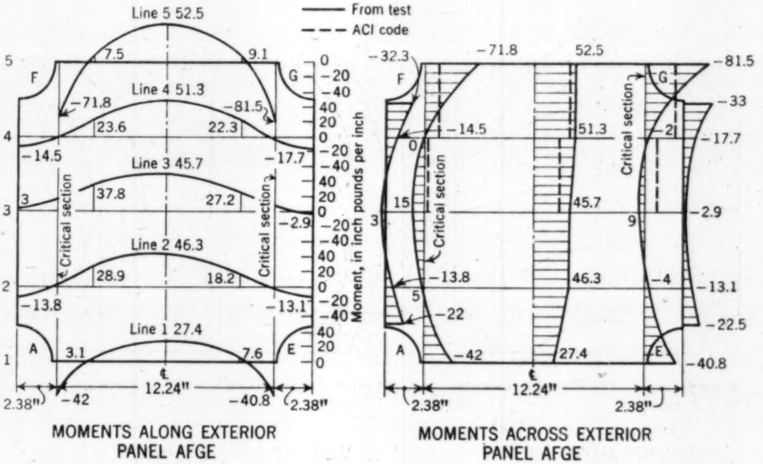
Fig. 11.—Moment Diagrams for Panel FHIG (3-Bay Direction) for Type IV Loading



MOMENTS ALONG INTERIOR PANEL FHIG

MOMENTS ACROSS INTERIOR PANEL FHIG

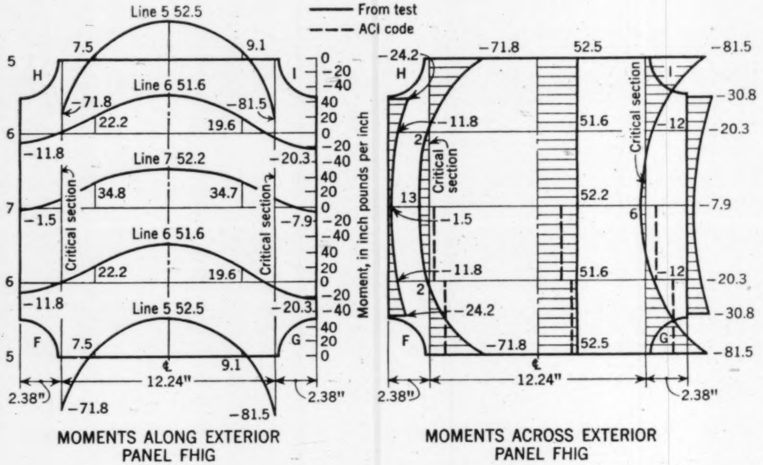
Fig. 12.—Moment Diagrams for Panel FHIG (3-Bay Direction) for Type V Loading



MOMENTS ALONG EXTERIOR PANEL AFGE

MOMENTS ACROSS EXTERIOR PANEL AFGE

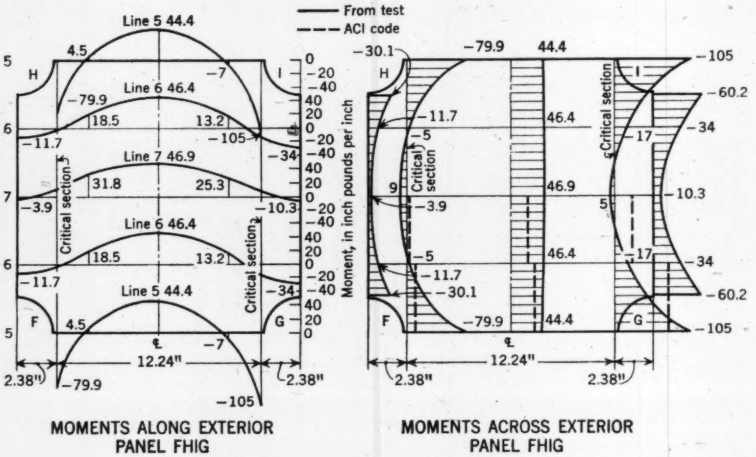
Fig. 13.—Moment Diagrams for Panel AFGE (2-Bay Direction) for Type VI Loading



MOMENTS ALONG EXTERIOR PANEL FHIG

MOMENTS ACROSS EXTERIOR PANEL FHIG

Fig. 14.—Moment Diagrams for Panel FHIG (2-Bay Direction) for Type VI Loading



MOMENTS ALONG EXTERIOR PANEL FHIG

MOMENTS ACROSS EXTERIOR PANEL FHIG

Fig. 15.—Moment Diagrams for Panel FHIG (2-Bay Direction) for Type IV Loading

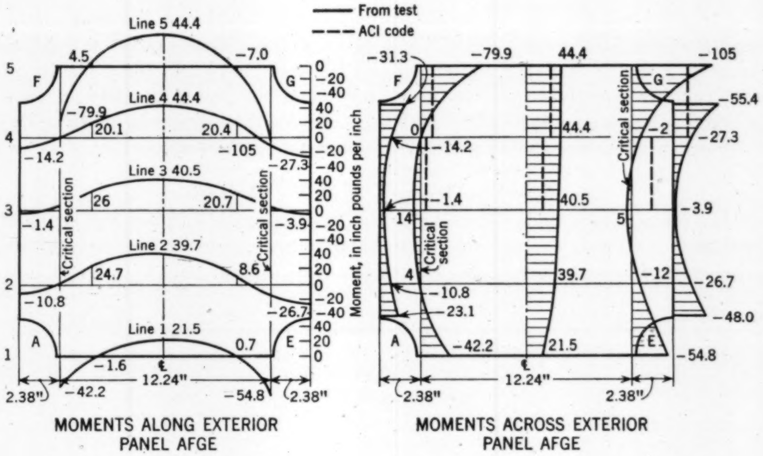


Fig. 16.—Moment Diagrams for Panel AFGE (2-Bay Direction) for Type IV Loading

and

$$l = L - 2 \times L = L\,(1 - 2\,X) = 17\left[1 - 2\left(0.073 + 0.57\,\frac{2}{17}\right)\right] = 12.24 \text{ in.}$$

For the two-bay direction, w and l are defined as

$$w = 3 \times 17 \times 3 = 153 \text{ lb per linear in.}$$

and

$$l = 12.24 \text{ in.}$$

The results are shown in Table 2.

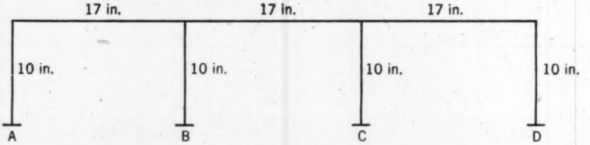
TABLE 2.—Comparison of Moments Determined by Tests with Moments Determined by $\dfrac{w\,l^2}{8}$

Load pattern	Three-Bay Direction				Two-Bay Direction		
	Panel	Moment, in Inch-Pounds		Error, in percentage	Moment, in Inch-Pounds		Error, in percentage
		Test	$\frac{w\,l^2}{8}$		Test	$\frac{w\,l^2}{8}$	
Type I	Exterior	2,010	1,910	5.0	...	...	...
Type II	Interior	2,070	1,910	7.5	...	...	...
Type V	Exterior	2,030	1,910	5.8	...	...	...
Type V	Interior	2,020	1,910	5.3	...	...	...
Type IV	Exterior	2,030	1,910	5.8	3,110	2,870	7.7
Type IV	Interior	2,040	1,910	6.3	...	...	...
Type VI	...	...	...	...	3,130	2,870	8.0

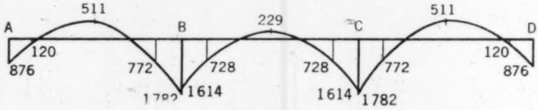
In every case the test moments are greater than those computed from $w l^2/8$, and the maximum error is 8%. The errors are probably the result of a difference in E and μ (between the model and the test piece) and the difficulty in determining negative moments accurately from the small quantity of gage points. Because no correction has been made for these errors in plotting the curves, an allowance should be made in interpreting the results.

Analysis by ACI Code Procedure

In the analyses made according to the ACI code, the columns and strips of the supported slab were considered to have infinite moments of inertia within the limits of the column capital (90° capitals used). Moments were determined at all supports and at the mid-span using two-cycle moment distribution. Negative moments were then determined at the prescribed critical sections at distances equal to $X L$ from the column center lines. Finally, the moments were apportioned to the column strips and middle strips as prescribed in the code.



(a) FRAME



(b) MOMENTS AT CRITICAL SECTIONS

FIG. 17.—CRITICAL-SECTION MOMENTS FOR FRAME

TABLE 3.—FRAME CONSTANTS FOR FIG. 17

Frame constant	Slab	Column
Fixed-end moments	$0.101\ w l^2$	...
Stiffness factor	$7.98 \frac{E I}{L}$	$10.2 \frac{E I}{L}$
Carry-over factor	0.673	...

Fig. 17 and Tables 3 and 4 show a typical ACI analysis for a frame that has all spans uniformly loaded. Tables 5 and 6 give the moments at the critical sections and at mid-span before their apportionment to column strips and middle strips. In Table 7 the apportioned moments, which are expressed in inch-pounds per inch, are tabulated for column strips and middle strips, as obtained by ACI analyses and by test. The test moments given are average moments in the design strips. Table 8 shows a typical method of apportioning the analysis moments to column strips and middle strips in accordance with ACI-318-51.

TABLE 4.—Two-Cycle Moment Distribution[a]

Support	A		B			C			D
	$\frac{wl^2}{8} = 1840$								
	Carry-Over Factor = 0.673								
Distribution factor	0.412		0.292	0.292		0.292	0.292		0.918
Fixed-end moment	−1,490		−1,490	−1,490		−1,490	−1,490		−1,490
Carry over	0		−413	0		0	−413		0
Summation	−1490		−1,903	−1,490		−1,490	−1903		−1,490
Distribution	+614		+121	−121		−121	+121		+614
Total	−876	+511	−1,782	−1,611	+229	−1,611	−1,782	+511	−876

[a] All spans loaded with a uniformly distributed load.

TABLE 5.—Moments by ACI Frame Analysis—Three-Bay Direction[a]

Support	1		2			3			4
Load pattern	Section								
	C.S.[b]	Center	C.S.	C.S.	Center	C.S.	C.S.	Center	C.S.
Type I	−208	642	−422	...	...	...	...	...	...
Type II	...	...	...	−380	577	−380	...	...	...
Type V	−132	468	−846	−827	404	−272	...	...	...
Type IV	−120	511	−772	−728	229	−728	−772	511	−120

[a] All moments are in inches and pounds per 17-in.-wide strip. [b] C.S. refers to the critical sections specified in the ACI code.

TABLE 6.—Moments by ACI Frame Analysis—Two-Bay Direction[a]

Support	1		2			3
Load pattern	Section					
	C.S.[b]	Center	C.S.	C.S.	Center	C.S.
Type VI	−207	642	−423	...	...	...
Type IV	−137	451	−877	−877	451	−137

[a] All moments are in inches and pounds per 17-in.-wide strip. [b] C.S. refers to the critical sections specified in the ACI code.

Comparison of Test Results and Analyses by ACI Code

The most important differences between test moments and analysis moments occur at mid-span, in which, in all cases, the test moments are greater (Fig. 18). Although the average support moments are less by test than by analysis, the variation in intensity of the negative moment across a column

TABLE 7.—COMPARISON OF TEST AND ACI MOMENTS ON COLUMN STRIPS AND MIDDLE STRIPS[a]

Load pattern and direction	Source of values	EXTERIOR PANEL						INTERIOR PANEL					
		COLUMN STRIP			MIDDLE STRIP			COLUMN STRIP			MIDDLE STRIP		
		Mid-span	Critical Section		Mid-span	Critical Section		Mid-span	Critical Section		Mid-span	Critical Section	
			Exterior	Interior		Exterior	Interior						
Type I	Test....	55.0	−30.0	−30.0	53.0	8.0	9.0	...	...		...	...	
	Analysis.	44.0	−19.7	−35.7	31.5	− 4.9	−12.9	...	...		...	...	
Type II	Test....	...	...	...	...	...	...	56.0	−28.0		55.0	7.0	
	Analysis.	...	...	...	...	...	...	39.3	−33.2		28.6	−11.6	
Type V	Test....	46.0	−28.0	−53.0	45.0	6.0	0	45.0	−56.0		45.0	0	
	Analysis.	32.2	−11.7	−73.5	23.0	− 2.8	−26.0	27.9	−72.2		20.2	−25.1	
Type IV	Test....	46.0	−28.0	−53.0	45.0	6.0	0	37.0	−54.0		38.0	0	
	Analysis.	35.3	−11.4	−67.5	25.2	− 2.8	−23.6	15.6	−63.9		11.2	−22.2	
									Exterior	Interior		Exterior	Interior
Type VI	Test....	52.0	−27.0	−33.0	48.0	11.0	6.0	52.0	−27.0	−36.0	52.0	10.0	0
	Analysis.	44.0	−19.6	−36.8	31.5	− 4.8	−13.0	44.0	−19.6	−36.8	31.5	− 4.8	−13.0
Type IV	Test....	44.2	−29.0	−50.0	42.0	10.0	− 1.0	45.0	−29.0	−55.0	46.0	5.0	− 2.0
	Analysis.	31.0	−13.0	−76.3	22.2	− 3.2	−26.8	31.0	−13.0	−76.3	22.2	− 3.2	−26.8

[a] Moments given are average values in inch-pounds per inch. To locate areas to which comparative moments apply, see dotted moments in Fig. 8 through Fig. 16.

TABLE 8.—TYPICAL APPORTIONMENT OF ANALYSIS MOMENTS—ACI CODE METHOD (1951)

Panel load	Strip	Moment	Apportionment of moment to strips[a]	Moment, in inch-pounds per inch
Exterior	Column	positive	$\frac{0.28}{0.20+0.28}\times 511$	35.3
		exterior negative	$\frac{0.41}{0.41+0.10}\times 120$	11.4
		interior negative	$\frac{0.50}{0.50+0.176}\times 772$	67.5
	Middle	positive	$\frac{0.20}{0.20+0.28}\times 511$	25.2
		exterior negative	$\frac{0.10}{0.41+0.10}\times 120$	2.8
		interior negative	$\frac{0.176}{0.50+0.176}\times 772$	23.6
Interior	Column	positive	$\frac{0.22}{0.22+0.16}\times 229$	15.6
		negative	$\frac{0.46}{0.46+0.16}\times 728$	63.9
	Middle	positive	$\frac{0.16}{0.22+0.16}\times 229$	11.2
		negative	$\frac{0.16}{0.46+0.16}\times 728$	22.2

[a] Table 1004—ACI-318-51; see Fig. 17.

strip is considerable. The maximum-negative test moment per unit width is always substantially greater than the corresponding average ACI-value.

When the large differences between test values and analysis values were first noted, it was thought that some settlement of the supports might be responsible. The shims were removed from beneath the interior columns, and tests were made. The changes in positive moment were found to be negligible.

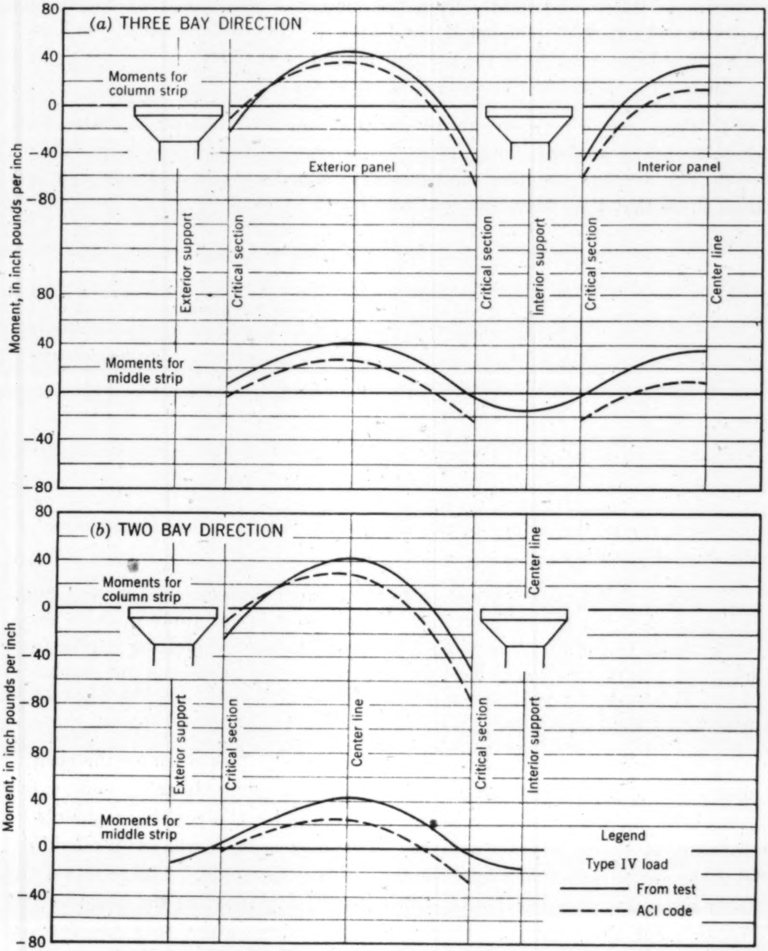


FIG. 18.—COMPARISON OF MOMENT DIAGRAMS FROM TEST RESULTS AND FROM ACI CODE ANALYSIS

Fig. 18 shows that the maximum-negative moments in the middle strips, which occurred on the column center lines rather than at the specified critical sections, were approximately equal to the moments found by the ACI code analyses.

CONCLUSIONS

The tabulated results show that the moments in the flat slab used in the experiment differ consistently from the moments determined by the frame-analysis method recommended by ACI-318-51. The differences are sufficient to warrant some changes in the present method of analysis.

In general terms the two main differences are:

1. The positive moments that were determined by test were greater than those found by analysis.
2. The average negative moments determined by test methods were less than those that were found by analysis.

TABLE 9.—CORRECTION FACTORS FOR POSITIVE MOMENTS

Panel	Strip	Factors for Moments at Mid-Span	
		Dead-load moments	Live-load moments
Exterior	Column	1.40	1.25
Exterior	Middle	1.80	1.65
Interior	Column	2.35	1.40
Interior	Middle	3.40	1.90

However, a careful examination of the test results shows that there is some inconsistency in the values of the negative moments. This is attributed to three possible causes:

a. Some of the gages were close to the column capitals, and, consequently, it is probable that the ordinary flexure formula does not apply in these areas.

b. Some of the negative moments were obtained by plotting curves from only five points, whereas at least seven points would have been preferable.

c. The contact resistances of the switch control panel for the SR-4 gages were larger than desirable.

However, no such inconsistency appears in the values given for the positive moments. Also, the test results for the negative moments are in close agreement with the analysis, whereas the test results for the positive moments do not agree favorably with the analysis. For this reason, as well as for reliability, changes are recommended only for the positive moments. The results found from the frame analysis should still be used for computing the negative moments. For the positive moments the recommendations take the form of factors by which the analysis moments should be multiplied to determine those of design. A list of these factors is given in Table 9.

It is hoped that further tests will be conducted to provide similar factors for use with the negative moments. In such tests the following precautions could be taken:

i. More gages should be applied in the negative-moment areas.

ii. The model should be made in such a form that the outside edges of the slab are tangent to the outside edges of the columns.

iii. A switch control panel that has contacts of low, constant resistance should be used for the SR-4 gages.

ACKNOWLEDGMENTS

The writers wish to acknowledge the early work performed on this project by Keith R. Ebern and Kenneth W. Moore at the University of Toronto (Ontario, Canada). Their work on an earlier, smaller scale model led to many of the improvements that were incorporated in the test described herein.

In addition, appreciation is extended to William H. Carr who did much of the preliminary work in the planning and preparation of the model.

DISCUSSION

JAMES CHINN,[4] J.M. ASCE.—One provision of the 1951 ACI Building Code that the authors did not apply to their analytical moments is the one (Sec. 1002 (a.) 8) that would allow the numerical sum of the maximum-positive and the average of the maximum-negative bending moments to be as low as $\frac{1}{10}\, W_{av}\, L \left(1 - \frac{4\, A_{av}}{3\, L}\right)^2$. In the foregoing, W_{av} is the average of the total load on two adjacent panels, L is the span length of a flat slab panel center to center of supports, and A_{av} is the average distance in the direction of span from the center of support to the intersection of the center line of the slab thickness with the extreme 45° diagonal line. The 1956 ACI Building Code (Sec. 1002 (a)) allows this sum to be as low as $0.09\, W\, L\, F \left(1 - \frac{2\, c}{3\, L}\right)^2$, in which $F = 1.15 - c/L \geqq 1$.

The experimental results (Table 2) indicate that the sum is more nearly $\frac{1}{8}\, W\, L$. This result shows that the moments from statical considerations are not reduced by two-way action as is commonly assumed. Frank E. Richart, Jr., A. M. ASCE, has verified[5] this in the case of reinforced-concrete column footings. It is hoped that additional tests will be performed on concrete to determine if the 0.09 coefficient is justified.

Although the code states that bents should be bounded laterally by the panel center lines, it seems more correct that they be bounded by lines of zero shear. In the case of the bent containing the first interior columns, the line of zero shear lies between the mid-panel and the five-eighths point.

The assumption that the moment of inertia of the slab becomes infinite at the edge of the column capital does not seem reasonable when there is no drop panel. If the moment of inertia does not become infinite until a point within the capital, the fixed-end slab moments will be reduced and the analytical moments will agree more favorably with the experimental ones.

The differential equation that governs the stresses and deflections of a slab is

$$\frac{\partial^4 w}{\partial x^4} + 2\,\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q\,(1 - \mu^2)}{E\,I} \qquad (6)$$

in which w is the vertical deflection of the slab, q is the load intensity, and the moments are given by

$$M_x = -\,\frac{E\,I}{1 - \mu^2}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\mu\,\partial^2 w}{\partial y^2}\right) \qquad (7a)$$

and

$$M_y = -\,\frac{E\,I}{1 - \mu^2}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2}\right) \qquad (7b)$$

---

[4] Associate Prof. of Civ. Eng., Univ. of Colorado, Boulder, Colo.

[5] "Reinforced Concrete Wall and Column Footings," by Frank E. Richart, *Proceedings*, A. C. I., Vol. 45, October, 1948, pp. 97-128 and pp. 273-260.

It is evident that the moments in a slab are dependent on the value of Poisson's ratio. An analysis such as that of a frame, which does not include the effect of Poisson's ratio, cannot therefore yield exact results.

The effect of Poisson's ratio also makes it doubtful how correctly experimental moments, which were determined from an aluminum model for which $\mu = 0.33$, can be applied to concrete that has values of from $\mu = 0$ to $\mu = 0.15$. A uniformly loaded, simply supported, square plate will be used to illustrate how much of an effect μ can have on moments.[6] The deflections are given by

$$w = \frac{16\, q\, (1 - \mu^2)}{\pi^6\, E\, I} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{\sin \frac{m\, \pi\, x}{a} \sin \frac{n\, \pi\, y}{a}}{m\, n \left(\frac{m^2 + n^2}{a^2} \right)^2} \qquad (8)$$

and

$$w = w_0\, (1 - \mu^2) \qquad (9)$$

in which w_0 is the deflection for $\mu = 0$. Moments are given by

$$M_x = - \frac{E\, I}{1 - \mu^2} \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right) \qquad (10a)$$

and

$$M_{x0} = - E\, I \frac{\partial^2 w_0}{\partial x^2} \qquad (10b)$$

together with

$$M_y = - \frac{E\, I}{1 - \mu^2} \left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} \right) \qquad (11a)$$

and

$$M_{y0} = - E\, I \frac{\partial^2 w_0}{\partial y^2} \qquad (11b)$$

Eqs. 10 and 11 yield

$$M_x = - \frac{E\, I}{1 - \mu^2} (1 - \mu^2) \left(\frac{\partial^2 w_0}{\partial x^2} + \mu \frac{\partial^2 w_0}{\partial y^2} \right) = M_{x0} + \mu\, M_{y0} \qquad (12a)$$

and

$$M_y = - \frac{E\, I}{1 - \mu^2} (1 - \mu^2) \left(\frac{\partial^2 w_0}{\partial y^2} + \mu \frac{\partial^2 w_0}{\partial x^2} \right) = M_{y0} + \mu\, M_{x0} \qquad (12b)$$

The relationships indicated in Fig. 19 for the moments at the corner and at the center of the slab can be shown easily. The magnitudes of these moments in terms of the moments for $\mu = 0$ are

At the corner	At the center
$M_x = (1 - \mu)\, M_{x0}$	$M_x = (1 + \mu)\, M_{x0}$
$M_y = (1 - \mu)\, M_{y0}$	$M_y = (1 + \mu)\, M_{y0}$

---

[6] "Afleiding van Eenvoudige Formules voor de Berekening van Rechthoe Kige op Vier Zijden Opgelegde Platen in Gewapend Beton voor Gelijk Matige Volbelasting," by P. P. Bijlaard, *De Ingenieur*, No. 23, 1935.

The moments (M_x and M_y) for $\mu = 0$ and $\mu = 0.15$ are, in terms of the moments for $\mu = 0.33$,

	$\mu = 0$	$\mu = 0.15$
Corner	1.5	1.275
Center	0.75	0.8625

Unfortunately, Eqs. 9, 12*a*, and 12*b* cannot be applied to the model tested because they are not general equations, but special ones applying to rectangular plates with nondeflecting side supports. For the case of a simply supported round plate, for example, the effect of μ becomes more complicated.

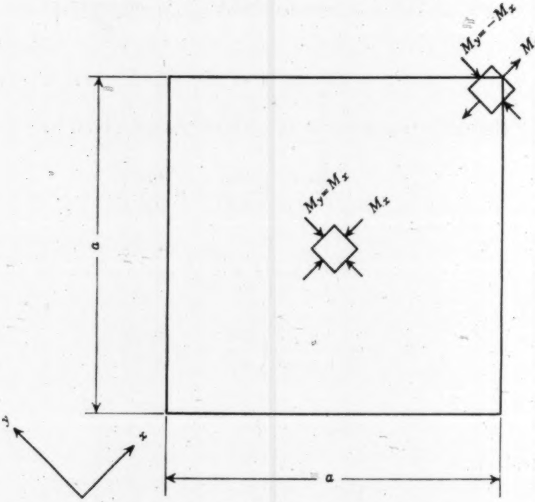


FIG. 19.—UNIFORMLY LOADED, SIMPLY SUPPORTED SQUARE PLATE

The writer does not know of any way of applying a correction for the effect of μ to the experimental results from the aluminum model. Adding to the difficulty is the fact that the stiffness of the slab is affected by μ, but the stiffness of the column is not. Therefore, the column moments will differ for different values of μ.

MARK W. HUGGINS,[7] M. ASCE, AND WATONE L. LIN.[8]—The careful and critical discussion by Mr. Chinn of the effect of Poisson's ratio is particularly valuable. He has indicated that the actual positive moments in a concrete slab would be less than those found in the test slab, and the negative moments in a concrete slab would be greater than the moments present in the test slab.

---

[7] Associate Prof. of Civ. Eng., Univ. of Toronto, Toronto, Ontario, Canada.

[8] Asst. Bridge Design Engr., Dept. of Highways, Toronto, Ontario, Canada.

However, it is apparent that the actual positive moments should be considered as being greater than those obtained by the ACI recommended method of analysis.

The 1951 ACI code states:

> "The numerical sum of the maximum positive and the average maximum negative bending moments for which provision is made in the design in the direction of either side of a rectangular panel shall be assumed as not less than $\frac{1}{10} W_{av} \cdot L \left(1 - \frac{4\, A_{av}}{3\, L}\right)^2$."

Thus, it makes no provision for downward adjustment of moments.

The somewhat similar section 1002(a) of the 1956 code is intended for "flat slabs within the limitations of section 1004." The model tested does not satisfy those limitations.

Mr. Chinn's proposal that the bents should be bounded laterally by lines of zero shear is excellent.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2942

RELATIONSHIP OF IRRIGATION TO PUBLIC HEALTH

By Lloyd E. Myers, Jr.,[1] A. M. ASCE

Synopsis

In planning and operating irrigation projects, the engineer should be aware of the health problems that arise. Serious mosquito-caused public problems exist in many irrigated areas of the United States. These problems are primarily the result of the same faulty irrigation structures and practices that cause water waste, damaged soils, and reduced crop production.

Introduction

The benefits of irrigation are so great, so obvious, and so well known that it is sometimes difficult to realize that there may also be some adverse effects. The favorable results irrigation may have on public health, through improved living standards made possible by the increased production of food and fiber, are recognized. The unfavorable results irrigation may have on public health are not as well known and are not commonly recognized. This presentation is devoted to those unfavorable and unpopular aspects.

Some diseases associated with irrigation in other countries are not of great concern in the United States. For example, schistosomiasis, a serious disease caused by blood flukes, is not prevalent, although it is estimated that approximately 70% of Egypt's population is infected.[2] Filth-borne diseases, such as typhoid and dysentery, which are associated with the use of polluted irrigation water for drinking or for irrigation of crops consumed raw, have been greatly reduced in the United States. However, problems still exist that are associated with irrigation-caused biting insects, such as black flies and mosquitoes. Although mosquito-borne yellow fever and malaria have been practically eliminated in the United States, the problem of mosquito-borne encephalitis remains to be solved.

Note.—Published, essentially as printed here, in May, 1957, in the Journal of the Irrigation and Drainage Division, as *Proceedings Paper 1235*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Technical Staff Specialist, Western Soil and Water Management Section, Soil and Water Conservation Research Branch, Agri. Research Service, U. S. Dept. of Agriculture, Beltsville, Md.

[2] "Mosquito Problems in Irrigation Areas and Their Prevention," Communicable Disease Center, U. S. Public Health Service, Atlanta, Ga., December, 1951.

Encephalitis means "inflammation of the brain" and is commonly termed "sleeping sickness." It results from infection by viruses and can cause serious brain damage or death. There is no curative medical treatment known, and immunizing vaccines are not available for humans (as of 1958). Although other mosquitos are capable of transmitting the encephalitis viruses, Culex tarsalis is the major vector and is frequently termed the "encephalitis mosquito." In many areas of the United States, the disease is common in horses, and the viruses that affect horses are, with the possible exception of the St. Louis (Mo.) strain, the same which affect humans. Severe human epidemics have occurred in the United States, and some are worth mentioning. An epidemic of 1,000 cases with 200 deaths struck St. Louis in 1933. The Dakotas and neighboring states had 2,800 cases in 1941. There were 813 cases with 58 deaths reported in California in 1952, and other outbreaks of the disease have occurred in various parts of the United States since that time. It is reliably estimated that more than 1,000 cases occur annually in the United States although a majority are not recognized. It is true that not all encephalitis epidemics are associated with irrigation, but many of them are, such as the 1952 epidemic in California. The potential for epidemics in many other irrigated areas is high and is increasing with the development of irrigation. The constant threat of irrigation-caused encephalitis epidemics and the constant occurrence of smaller nonepidemic outbreaks are serious matters and cannot be ignored.

Mosquitoes

Mosquitoes not only carry human disease but are of sufficient economic importance to warrant consideration of that factor.[3] Some of the ways in which mosquitoes cause economic loss are (*a*) animal diseases, such as encephalitis of horses, fowl pox, heartworm of dogs, and myxomatosis or "big head" of rabbits, are often spread by mosquitoes; (*b*) dairy, beef, and poultry production is often seriously reduced; (*c*) real-estate values are depressed; (*d*) labor becomes difficult to hire; (*e*) industrial development is retarded or prevented; (*f*) tourist trade is reduced or eliminated; (*g*) outdoor living and recreation are restricted or prevented; and (*h*) mosquito control is costly. Approximately $4,000,000 are spent annually on mosquito control in California, primarily on problems directly associated with irrigation. Hundreds of thousands of dollars more are spent for mosquito repellents, household sprays, and medications for bites. The total of similar expenditures in all irrigated areas must be staggering.

Mosquitoes have certain habits and abilities (other than their ability to carry disease) that are responsible for their adaptation to, and their importance in, irrigated areas. There are approximately 2,000 separate species and subspecies that have been identified in the world, and approximately 120 species have been identified in the western United States. The large number of mosquito species is worth noting because each species has slightly different habits and abilities. Some of these habits are of only incidental interest to irrigation engineers, such as the fact that the female Aedes varipalpus prefers to lay her eggs in tree holes, or that Mansonia perturbans larvae have spikes on their tails, which they sink into hollow cat-tail or tule roots to obtain air for breathing. However, some habits and abilities are of primary importance.

---

[3] "The Economic Significance of Mosquitoes to Agriculture in California," by Deane P. Furman, *Proceedings*, 21st Annual Conference, California Mosquito Control Assn., Turlock, Calif., February, 1953.

The flight range of mosquitoes is highly variable. The Anopheles species usually stay within a mile or two of their source. Some species, such as Anopheles freeborni, make dispersal flights in the fall, at which time they seek favorable sites for hibernation, and in the spring, when they emerge from winter hibernation. During fall dispersal flights they may fly for many miles. The Culex species normally fly only from 2 miles to 4 miles, but observations during the 1952 encephalitis epidemic in California indicated that Culex tarsalis probably flew much farther than this. Some Aedes species have been shown to fly for more than 20 miles. These long flight ranges mean that mosquitoes will spread rapidly throughout an irrigated area and can affect humans and animals at some distance from the area.

Egg-laying habits of mosquitoes are of considerable importance because they relate to irrigation practice. From the standpoint of irrigation, mosquitoes can be divided into two basic types—those laying eggs on the water surface and those laying eggs on moist soil. Eggs of Culex, Anopheles, and other genera are laid on the water surface and after several days hatch into larvae. The larvae, or "wigglers," progress through four stages, or instars, and develop into pupae, or "tumblers," from which the adult mosquitoes emerge. Because this process takes a minimum time of approximately from five days to six days, even in the hottest California weather, these mosquitoes propagate primarily in permanent or semipermanent water collections, such as those found in borrow pits, poorly maintained drains or ditches, or structures that hold water when not in use. Despite the relatively long period required for completion of the aquatic cycle, these mosquitoes are commonly produced within irrigated fields.[4]

Eggs of Aedes, Psorophora, and other genera are laid on moist soil, which will probably be reflooded at some future date. The eggs mature, become hatchable, or are conditioned in the dry state and usually hatch within a few minutes after being covered by water. The aquatic cycle is the same as for other mosquito genera. It should be emphasized that the aquatic cycle of all mosquitoes must be completed in water; mosquitoes cannot propagate in damp grass. Because of the immediate hatching, the egg-to-adult cycle can be completed in four days when mean temperatures of approximately 85°F prevail.[5] Prior to the development of irrigation, these were floodwater mosquitoes whose eggs were laid where they would be inundated by high water in spring or fall floods, and ordinarily only one or two broods were hatched each year. The practice of intermittent irrigation is ideally suited to the needs of these mosquitoes, and in a majority of cases every irrigation produces a new brood. Even though water is not ordinarily supposed to stand for four days within intermittently irrigated fields, the mosquitoes are produced within a majority of the irrigated fields in most, if not all, irrigated areas.

---

[4] "Preliminary Field Observations on the Aquatic Stages of Irrigated Pasture Mosquitoes in Western Nebraska with Particular Emphasis on Aedes dorsalis," Communicable Disease Center, U. S. Public Health Service, Atlanta, Ga., June, 1953.

[5] "A Cooperative Ecological Study of Mosquitoes of Irrigated Pastures," by Richard C. Husbands and Bettina Rosay, *Proceedings*, 20th Annual Conference, California Mosquito Control Assn., Berkeley, Calif., February, 1952.

The common idea that sloughs and swamps are responsible for most mosquito problems is incorrect. Mosquito problems in irrigated areas are primarily the result of mosquito production caused by faulty irrigation structures and practices. For example, for many years people living in the Milk River valley in Montana had blamed their mosquito problem on the numerous sloughs in the valley. An intensive study made by the United States Public Health Service in 1952 showed that 90% of the mosquito production was caused by irrigation, and more than 70% occurred within the boundaries of irrigated fields.[6]

Water which stands for four days on the soil surface in an irrigated field will, with very few exceptions, damage both the crop and the soil.[7] Because water must ordinarily stand for four days or more before mosquito production can occur, such production within an intermittently irrigated field indicates an undesirable situation from an agricultural standpoint.

The desirability and the possibility of greatly reducing or eliminating the mosquito problem by adopting good irrigation practices have been shown by several studies. A cooperative investigation in the San Joaquin Valley of California by the California State Health Department and the Soil and Water Conservation Research Branch, Agricultural Research Service (United States Department of Agriculture), has shown that mosquito production, as might be expected, is associated with irrigation efficiency.[8] Three border-strip irrigated pastures on similar soils were studied with the following results:

Irrigation efficiency, in percentage	Mosquito production
25	Abundant
35	Moderate
66	None

A cooperative investigation by the Public Health Service and the Agricultural Research Service in the Milk River valley produced the following results: By using fertilizers and timing irrigations to allow 60% of the available soil moisture to deplete between irrigations, formerly abundant mosquito production in a hay field was completely eliminated and hay production was increased fivefold.

The foregoing information has been presented to show that mosquito production within intermittently irrigated fields is unnecessary and undesirable from both agricultural and public health standpoints. Mosquito production in ditches, borrow pits, equalizing reservoirs, and other irrigation and drainage structures is undesirable from the standpoints of water conservation, weed control, maintenance, and operating efficiency. Space does not permit a complete analysis of each situation, but a brief listing of various causes for mosquito production associated with irrigation can be made as follows.

---

[6] "Mosquito Investigations," *First Progress Report*, Milk River Irrig. Project, Montana, Communicable Disease Center, U. S. Public Health Service, Atlanta, Ga., April, 1953.

[7] "Soil and Water Management in Relation to Mosquito Control," by Lloyd E. Myers, Jr., *Proceedings*, 24th Annual Conference, California Mosquito Control Assn., Turlock, Calif., January, 1956, pp. 11-13.

[8] "Irrigation Efficiency and Mosquito Production," by Lloyd E. Myers, Jr., *Mosquito News 16*, June, 1956, pp. 133-136.

CAUSES FOR MOSQUITO PRODUCTION

Irrigation Storage Reservoirs.—Ordinarily, irrigation storage reservoirs do not present problems because of falling water levels throughout the summer. Occasional problems are caused because of uncleared marginal vegetation and undrained marginal pools.

Distribution Systems.—These systems cause mosquitoes when (*a*) equalizing reservoirs have shallow, weedy margins; (*b*) seepage from canals creates swampy areas on lower-lying lands; (*c*) ditch bottoms are not on grade, causing ponding during nonuse periods; (*d*) structures are not on grade, thereby ponding water upstream; (*e*) leaky structures create downstream ponds; (*f*) undrained borrow pits hold water from seepage or runoff; (*g*) structures hold residual water when not in use; and (*h*) maintenance is inadequate, thus resulting in retarded channel flow, weed growth, and other situations favorable for mosquito propagation.

Irrigated Lands.—Inadequate drainage, both surface and subsurface, and overapplication of irrigation water contribute to mosquito production. Overapplication is caused by (*a*) poor measurement of water by the farmer or water-supply agency; (*b*) inefficient irrigation methods; (*c*) inadequate land preparation, causing ponding within fields; (*d*) lack of information to establish proper irrigation schedules; and (*e*) delivery schedules not corresponding to proper irrigation schedules. Crops and soils are often damaged by overirrigation.

Mosquitoes commonly result if the irrigated land is composed of problem soils with low infiltration rates due to compaction, alkali damage, or poor soil structure. Problem soils are frequently caused by poor soil management practices.

CONCLUSIONS

All irrigation-mosquito causes given relate directly to the activities of irrigation and drainage engineers, who have primary responsibility for planning, designing, and constructing irrigation and drainage works. The engineers must also obtain the information farmers need for selecting, planning, and implementing suitable irrigation practices. Therefore, they are also responsible for mosquito problems associated with irrigation.

The relationship between public health, mosquito problems, and irrigation and drainage is not an unfortunate situation to be looked on with regret and misgivings. Instead, it is a powerful tool for improving irrigation and drainage works and practices. Irrigation and drainage engineers have the ability and the responsibility to play a leading role in developing the methods for using this tool. They have an opportunity and an obligation to convert an existing debit into a future credit. The future prosperity, comfort, health, and even the lives of many people depend on the proper recognition of that obligation and the intelligent use of that opportunity.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2943

SHORESIDE FACILITIES FOR SPECIAL-PURPOSE SHIPS

BY HOWARD J. MARSDEN[1]

SYNOPSIS

A brief history and an analysis of the current (1958) situation regarding the operations of special-purpose vessels are presented. Also examined are the layout, general design criteria, and technical analyses of roll-on roll-off, lift-on lift-off, container, and conveyer terminals for trailerships, trainships, and containerships. Consideration is also given to berth and pier facilities, transfer bridges, marshalling areas, receiving and delivery areas, and subsidiary facilities areas.

INTRODUCTION

The rising cost of conventional cargo-handling methods at ports in the United States has created great interest in roll-on roll-off, lift-on lift-off, and similar types of shipping. All these operations use the principle of the transfer between land and sea carriers of loaded highway trailers, rail cars, and containers. This principle is a version of the unit-load principle as contrasted with the handling of many individual pieces of cargo. The savings in vessel port time and cargo-handling costs are potentially enormous.

Historically, ferries were the first roll-on roll-off vessels. They still perform their valuable service daily, plying the waters of many harbors. In 1860 trainship service was inaugurated in Scotland to convey rail cars across the Firth of Forth and the Tay River. Subsequently, trainship operations were established in other countries, and in 1892 the first train ferry in the United States began operations on Lake Michigan. With the development and accelerated use of automobiles and trucks, ferries were established to transport these vehicles. Although ferries have been largely replaced by bridges and tunnels on the shorter water crossings, they are used increasingly on long runs. Containers have also been utilized for many years to transport goods in general-

---

NOTE.—Published, essentially as printed here, in May, 1957, in the Journal of the Waterways and Harbors Division, as *Proceedings Paper 1248*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Chief, Div. of Port Development, Maritime Administration, Federal Maritime Board, U. S. Dept. of Commerce, Washington, D. C.

cargo ships, and consideration has been given to the use of ships designed or modified specifically for the efficient handling and transportation of containers. The use of roll-on roll-off and lift-on lift-off methods of handling was accelerated during World War I and World War II.

Although several commercial roll-on roll-off and lift-on lift-off deep-water-vessel operations have been conducted from United States ports for some time, the first specially designed ocean vessel for highway-vehicle transportation was completed in January, 1958. This vessel, the United States Navy ship, *Comet*, was constructed for the Military Sea Transportation Service (United States Department of Defense) and can transport one-sixth of the vehicles of an armored division, or approximately 703 vehicles. The only current deep-water, roll-on roll-off trailership operation is a service between Jacksonville (Fla.), San Juan (Puerto Rico), and the Virgin Islands. This service uses vessels that can carry fifty-two trailers on two decks. However, the propelling machinery has been removed from the ships, which are towed. A self-propelled vessel (landing ship-dock) was converted for this service in 1957, but, it has had propulsion-machinery problems to date, making it difficult to evaluate its suitability and efficiency.

Caution has supplanted the enthusiasm prevalent in 1956 among the prospective builders of such special-purpose ocean-going vessels. In 1955 applications for Federal Ship Mortgage and Loan Insurance for these ships were approved, in principle, by the Maritime Administration (United States Department of Commerce). Since that time (as of 1958), applications for twenty-four trailerships, eleven trainships, and two train-trailerships have been received. Many have been approved, in principle, for government mortgage insurance purposes. However, construction contracts have not as yet been awarded for any of these vessels. Ship operators have apparently made a second and more technical appraisal of the over-all situation, particularly with respect to roll-on roll-off operations versus lift-on lift-off methods, as well as loading the complete highway trailer versus loading the trailer body only—that is, loading mobile transportation equipment aboard as compared with a purely container type of operation. One operator abandoned the concept of a roll-on roll-off trailership in favor of the placement on board of only the trailer body in the form of a cargo container. This operator has converted several C-2 type vessels to lift-on lift-off trailerships capable of loading on board 226 thirty-five-foot truck-trailer bodies with two self-contained traveling cranes.

The vessels considered to date (as of 1958) vary widely in their loading media. Some are end loaders, some side-port loaders, and some combine both features. One lift-on lift-off type of vessel loads and discharges rail cars through a "well" opening in the ship. Except for two such self-loading trainships, only three of the foregoing vessels would have ship-furnished ramps, and even these ships would need terminal ramps or transfer bridges at certain ports and under certain tidal conditions.

TERMINAL LAYOUT

Because of the widespread interest in special-purpose vessels, the Maritime Administration conducted a study of the design criteria for the shoreside facilities necessary for the various types of vessels and the loading and unloading media being considered.[3] Although the uncertain direction of the trend in this field may make this presentation of terminal design criteria somewhat premature, the preliminary data may be of interest to the engineer as well as the potential terminal and ship operators.

Generally, the layout of a terminal for such vessels is influenced greatly by the variation of the water level in the berth. The dimensions of the vessel, the frequency of arrivals at a terminal, and the number and characteristics of the cargo units discharged and loaded per berthing determine the type and size of the facilities to be provided.

Rolling Containers.—The size and weight of highway trailers are controlled by state motor vehicle laws. Because of the low density of most general-cargo commodities, the cargo carried by a highway semitrailer rarely exceeds 20 tons. A standard highway tractor has sufficient tractive effort to haul a trailer carrying a maximum load of 25 tons on a 12% grade. With a minimum underclearance of 10 in. for the trailer landing gear in the fully retracted position, the 50-ft tractor-trailer combination recommended for trailership operation can negotiate a change in grade of 16% without a vertical curve. With a minimum underclearance of 9½ in., railroad cars up to 60 ft long can attempt a change in grade of 6.5% without a vertical curve.

Both the trailership and the trainship terminal should have the following principal facility units:

1. A ship berth is necessary.
2. A marshalling area should be included. This area provides space for the accumulation of rail cars or trailers arranged in the order in which they should be loaded on the ship and for the immediate storage of units unloaded from the vessel. It should be as near as possible to the ship berth in order to minimize the quantity of hauling units needed to unload or load the vessel.
3. A receiving and delivery area is necessary. In the case of a trainship terminal, such an area is a railroad yard with a capacity of one shipload. In the case of a trailership terminal, this area is a gate house where trucks are checked. Space is included for trucks awaiting gate clearance, as well as rail tracks and unloading ramps for handling trailers on flat cars ("piggy-back").
4. A subsidiary facilities area, which need not be at the terminal, would include (*a*) a consolidation and transfer area for outbound less-than-truckload cargoes or less-than-carload cargoes and the breakdown of inbound full loads; (*b*) a holding area for short-term storage of cargo units; and (*c*) a repair and maintenance area for minor repairs to trailers.

The ship berths should be readily accessible to deep water and, if possible, should be oriented in such a fashion that docking and undocking ships can be performed without using tugs. The choice of the type of berth to be used at a particular location is influenced by conditions of tide, wind, current, and accessibility.

ROLL-ON ROLL-OFF TERMINALS

Ship-berthing facilities can be grouped into five general types: Marginal wharves, finger piers, slip wharves, ferry slips, and offshore berths. Marginal wharves can be used not only for side-loading vessels but for stern-and bow-loaders if offshore structures or pontoons are added to support a transfer bridge; finger piers and slip wharves are suitable for either stern-, bow-, or side-loading vessels; ferry slips are useful for bow- or stern-loaders; and an offshore berth, consisting of breasting and mooring dolphins, may be constructed in deep water in which dredging would not be necessary. However, the latter facility usually requires the construction of a long over-water approach. The berth should have sufficient length, width, and depth under all conditions of tide and current for the safe docking and undocking of the largest ships that are expected to use the terminal. The following minimum single ship-berth dimensions are desirable to provide adequate space for berthing and mooring vessels:

---

[3] "Shoreside Facilities For Trailership, Trainship and Containership Services," Tippetts-Abbett-McCarthy-Stratton, New York, N. Y., November, 1956.

1. Length—length of vessel plus beam (minimum);
2. Width—beam of vessel, plus allowance of 100 ft for tug maneuvering; and
3. Depth—loaded draft of vessel, plus allowance of approximately 2 ft for lower low tides and uneven vessel trim, plus a 2-ft bottom clearance.

The transfer bridge for connecting the ship and the shore at a roll-on roll-off terminal may be a single-span or multiple-span bridge with intermediate supports that are hinged at the inshore end and are movable vertically at the outshore end to meet the loading deck of the vessel. Outshore-end and intermediate supports, if any, can be held, and the elevation can be adjusted by several methods, including cables attached to overhead winches, hydraulic pistons, or pontoons with adjustable ballast tanks. The length of the bridge is determined by the tide range of the port, the elevations at light and loaded draft of the loading vessel to be served, and the allowable grade on the bridge. A transfer bridge should be long enough to serve an empty vessel at the average yearly highest tide and a loaded vessel at the average yearly lowest tide. Many existing and proposed roll-on roll-off vessels are loaded on a single deck, with the transfer of units to other decks within the vessel by internal ramps or elevators. For these ships a single-level transfer bridge can be used, although at ports having tidal ranges of more than 15 ft, one that is multilevel might be considerably shorter and less costly. The upper level of such a bridge could be used for the higher-tide elevations and the lower level, for the lower-tide range. For vessels designed for direct loading to several decks, a multilevel bridge is necessary.

The transfer bridge at a trailership terminal is not fixed to the vessel, and a sliding plate is used to span the gap between the bridge deck and the vessel deck. The bridge may be a girder or truss construction and have a solid or open grid-type deck. Its grade should be limited to 10% in order to minimize the difficulty of backing trailers over it. Although the width of the bridge depends on the size of the vessel's loading ports, a minimum of 20 ft is desirable for a two-way operation and 12 ft for a one-way operation. The minimum vertical clearance should be 14 ft to conform with the design standards of the American Association of State Highway Officials (AASHO). A minimum of 60 ft beyond the inshore end of the bridge is required for maneuvering the recommended size of tractor-trailer, with additional clearance for passing traffic. In general, the bridge should be designed for the standard AASHO H20-S16 loading, or for greater loads at ports in which increased axle loadings are permitted.

The transfer bridge at a trainship terminal should be designed to support a continuous line of maximum-weight (75-ton) cars on each track. Because the vessel must be fixed to the bridge to keep the tracks in alinement, the loads that will be placed on the bridge by vessel movement under the action of tide, wind, and current should be considered. Idler cars should be used between the locomotive and the cars being pushed onto the vessel so that the locomotive does not travel on the bridge. With an allowance for vessel trim, the transfer-bridge grade should be limited to 6%. The track spacing and number of tracks that should be provided on a transfer bridge depend on the size of the vessel's loading port and the track arrangement on the loading deck. The vertical clearance should be at least 16 ft to conform with the minimum clearance requirements of the American Railway Engineering Association (AREA).

The area required for marshalling is determined by the size of the vessel, the frequency of sailings, and the shape and arrangement of the available area. When trailers or rail cars for only one ship are expected in the marshalling area at any single time and when ships take on and discharge a full load, space should be provided, as a minimum, for the storage of land-carrier units equal to twice the ship's capacity. This permits the discharging of a full shipload of units before loading is begun. When trailers or rail cars for more than one ship are to be in the marshalling area at one time, studies of the arrival and departure pattern of the trailers or rail cars are required to determine the space needed.

The marshalling area for a trailership terminal should be surfaced with a high-quality pavement, such as concrete or asphalt, with adequate structural strength. Unit parking depth should be limited to two trailer lengths, parked at either 90° or 45°, for accessibility. Approximately 1,000 sq ft per trailer should be used for planning purposes, including provisions for unusable space around the perimeter of the area and at entrances and exits.

The marshalling area for a trainship terminal is a yard having rail connections to the transfer bridge and to the receiving facility. These connections should be made by a ladder track at each end of the yard. A passing track should be provided within the yard so that switching locomotives may operate at either end. For planning, and based on a 44-ft car and 13.5-ft track spacing, approximately 900 sq ft per car should be allowed for the marshalling yard, including provisions for a passing track and ladder tracks.

Trailers and rail cars are classified for weight (also size, if required) during the transfer of these units from the receiving facility to the marshalling facility.

LIFT–ON LIFT–OFF TERMINALS

At lift-on lift-off terminals, units are transferred between ship and shore by cranes. The units may be trailers, rail cars, or containers. At terminals where the tidal variation is great and a transfer bridge would be abnormally long, lift-on lift-off operations are more suitable than roll-on roll-off methods. Ship-berthing facilities for the former operations may be slip-type wharves, finger piers, or marginal wharves. The cranes may be ship-mounted or shore-based, and may be of either the revolving or horizontal-boom type. A ship-mounted crane permits a vessel to serve any terminal having a wharf apron that is suitable for truck and rail operations. The marshalling, receiving, and subsidiary areas for lift-on lift-off terminals are generally the same as those for a roll-on roll-off terminal. The wharf apron should be at least 40 ft wide in order to provide for two lanes of highway traffic and to allow space for the crane wheels and for ship-mooring facilities. The crane should handle loads of at least 25 tons to conform to AASHO standards, or heavier loads wherever greater axle loads are permitted. At trainship terminals, where cars are pulled onto track-equipped cradles that are lifted by crane via a well opening in the ship to an appropriate deck, the wharf apron should be at least 50 ft wide to provide for the cradles and adequate clearances. The crane should be capable of handling loads of 75 tons plus the weight of the cradle.

CONTAINER TERMINALS

At a lift-on lift-off container terminal, outbound containers are brought to a receiving facility on highway trailers or railroad cars. They may be removed from these units and placed in a marshalling area by mechanical handling equipment, such as fork-lift trucks or straddle carriers. For ship loading the containers may be brought from the marshalling area to the wharf apron by similar equipment. At terminals where demountable trailer-body containers are handled, the container remains on the highway-trailer chassis in the marshalling area and is moved to the wharf apron by a highway tractor.

The containers are usually equipped with lifting rings to which the crane hooks are attached. The crane lifts the containers and places them on board the ship. Cargo-handling equipment may also be placed on board the ship to move containers to other decks via internal ramps. Inbound containers are unloaded by the crane and placed on the wharf apron for transfer to the marshalling area by cargo-handling equipment. The width of the apron is determined by the size of the containers and the type of equipment and should be adequate for two lanes of traffic.

CONVEYER TERMINALS

Conveyer operations are an adaptation of the roll-on roll-off principle to containers. Equipment of the conveyer type is used to handle containers that are equipped with small flanged wheels and may be similar to a car puller, consisting of a chain or cable between rails on which the wheels roll and which is equipped with fittings connecting with the containers. The design of such a loading and unloading device must be coordinated closely with the method by which the containers are stowed in the ship. Conveyers can be used for stern, bow, side, or hatch loading, and any of the five ship-berthing types can be used with appropriate conveyer systems.

A marshalling facility and a receiving and delivery facility are also needed at this type of terminal. Their area requirements are influenced by the sizes of the containers and types of handling equipment.

COMBINED TERMINALS

Roll-on roll-off and lift-on lift-off procedures for either trailers or rail cars could be combined at a single terminal with the joint use of the marshalling and receiving and delivery facilities. At such a terminal both a transfer bridge and a crane would be required. In addition, terminal facilities should be specially designed and built for the most efficient operation of roll-on roll-off, lift-on lift-off, or conveyer services. However, it may be desirable to modify an existing general-cargo terminal so that it can be used by specialized vessels and conventional ships. A major advantage of the special-purpose ships is the saving of in-port vessel time made possible by the use of specialized facilities. To be effective, frequent service of this type must be offered by the shipping line. There would be no advantage in providing a terminal with the necessary facilities for one or more of the specialized operations if the berths were unavailable for from 6 days to 10 days because of conventional general-cargo operations. However, if the occupancy of the berth by general-cargo vessels ranges from 1 day to 3 days, and if ship schedules can be arranged to eliminate conflicts, it might be feasible to combine general-cargo operations with one or more of those that are specialized.

CONCLUSIONS

The application of the unit-load (container) principle to water shipping, as opposed to the method involving several handlings of individual small pieces of general cargo, can effect significant savings in vessel port time and cargo-handling costs. The special-purpose ships required for such operations can be handled most efficiently at specially designed terminal facilities. The layout of such terminals is influenced greatly by the variation of the water level in the berth. Except for a conveyer-type terminal, the facilities required for such terminals are relatively simple and considerably less expensive than for a conventional general-cargo terminal, which requires greater pier width and transit-shed construction The roll-on roll-off principle, involving the transportation of trailers on wheels, is not economical for long distances because the gain obtained by rapid loading and discharge is more than offset by the additional and unproductive cubic space that the vessel is required to provide for the trailer-wheel area. The lift-on lift-off principle, utilizing containers, appears to offer the most attraction to long-haul operators. The use of ship-mounted cranes permits such vessels to serve any terminal having a pier apron suitable for rail or truck operations.

ACKNOWLEDGMENTS

Much of the technical information contained herein was developed by Tippetts-Abbett-McCarthy-Stratton, New York (N. Y.), under its contract with the Maritime Administration.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2944

OPERATION PROBLEMS ON CONTROLLED-ACCESS HIGHWAYS

By Charles M. Noble,[1] M. ASCE

Synopsis

The advantages of the controlled-access highway are emphasized, and the necessary facilities on such highways for the comfort and safety of the public are outlined. Facilities providing for motor fuel, water, food, comfort, policing, fire department, communications, traffic operations, first aid and ambulance service, and wrecking, flat tire, and towing services are described, and the necessity for making these facilities available is stressed.

Introduction

Bitter experience in densely populated areas has demonstrated conclusively that control of access on major travel arteries is a vital necessity. Lack of such control results in highways that depreciate not only in volume but in facility of movement and safety. The economic loss due to such depreciation is enormous. Express highways that are adequate to cope with current and prospective traffic volumes and speed are so expensive that deterioration and obsolescence cannot be afforded. The highways must maintain their efficiency and safety indefinitely. The tragic loss in death and injury is reason alone for the controlled-access principle. However, in addition to this compelling reason, the controlled-access highway helps build economic health and develops the region through which it passes. This economic upbuilding has been proved over and over again and is no longer open for debate. The upsurge of development has been so swift that rural countryside has been transformed to bustling prosperity, dramatically demonstrating the urgent need of access control to preserve the economic value and safety of the traffic artery.

A major objective of the controlled-access highway is to relieve obsolete and overcrowded local highways by diverting through traffic, but without destroying business generated by the traveling public. Freeing local traffic

---

Note.—Published, essentially as printed here, in May, 1957, in the Journal of the Highway Division, as *Proceedings Paper 1249*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Secy. to the Governor for Highways, State of Ohio Dept. of Highways, Columbus, Ohio.

arteries from interference of that segment of through traffic not desiring to stop makes more street space available for those vehicles pausing for service, shopping, recreation, and overnight accommodations. The slogan, "every vehicle a customer," is meaningful if broad public interest is recognized and if controlled-access planning includes provision for services for travelers.

Control of access poses new problems in administration and operation that at first may appear formidable but will yield to informed planning and organization. The passage of the Federal Aid Act of 1956,[2] which features the National System of Interstate and Defense Highways (a 41,000-mile network), presents the operational problem to highway administrators. Intracontinental expressways traversing both rural and urban areas preclude promiscuous exit and entry, and planning for many services and necessities hitherto deemed beyond the field of the highway official must be considered carefully. In a sense the highway administrator will have the public in his care. Consequently, the operation of such highways is important.

It appears quite logical that highway departments could establish an operations division with a director. Such a division could become as important and render service as essential as similar departments of major railroads. It seems likely that it could take its place beside design and construction as a major activity in the highway department. Express-highway operations include a variety of activities handled by various and often uncoordinated organizations:

1. Traffic operations.
    - *a.* Safety surveillance.
    - *b.* Safety programs.
    - *c.* Accident investigation and analysis.
    - *d.* Development of safety measures.
    - *e.* Signs, signals, and markings.
    - *f.* Accident statistics and reporting.
2. Policing.
    - *a.* Accident prevention.
    - *b.* Enforcement.
    - *c.* Aids to motorists.
    - *d.* Crime detection.
3. Communication.
    - *a.* Radio.
    - *b.* Television.
    - *c.* Roadside public telephones.
4. Service.
    - *a.* Fuel, water, oil, and accessories.
    - *b.* Comfort.
    - *c.* Food.
    - *d.* Repairs, flat tire, and towing.

---

[2] *Public Law 627*, Federal-Aid Highway Act of 1956, 84th Cong., 2d Session, Chapter 462.

  - *e.* Fire.
  - *f.* Ambulance and medical aid.
  - *g.* Overnight accommodations.

5. Maintenance.
  - *a.* Roadway repairs.
  - *b.* Roadside care.
  - *c.* Structural repairs and painting.
  - *d.* Drainage.
  - *e.* Snow removal and ice control.

TRAFFIC OPERATIONS

Traffic operations are established (as of 1958) in nearly all highway departments, but new and challenging responsibilities will arise in the future under the leadership of trained traffic engineers. An intelligent analysis of traffic needs and constant surveillance can result in reduced accidents, the saving of lives, and comfort and satisfaction for the motoring public.

POLICE

Policing is an essential function that must be integrated with operations. Although state police usually operate under a different department of state government, there is no reason why this function cannot be closely coordinated with highway operations. For example, the integration of the eighty-eight-man state police detachment assigned to the New Jersey Turnpike has been completely successful. This detachment monitors all turnpike operations at headquarters through the microwave turnpike radio system, which, in turn, is connected to state police headquarters and the Pennsylvania Turnpike radio system. The police operate on an around-the-clock schedule and are effective in traffic regulation, enforcement, and criminal detection. They must be present at all accidents, not only to investigate the accident and apprehend violators, but to direct traffic, render first aid, and summon wreckers, fire apparatus, ambulances, and first-aid squads. Police contribute substantially to accident prevention by halting careless drivers or those who are intoxicated or sleepy, thereby saving many lives. They also protect stranded motorists by summoning aid for gasoline, flat tires, or repairs, and often help motorists start their automobiles. In 1955, 44,000 such motorists were aided by the police on the New Jersey Turnpike.

COMMUNICATIONS

Communications will become even more vital to the successful operation of the highway system. Radio with accompanying dial-radio phones, teletype, and facsimiles will be indispensable. In some urban areas with dense traffic, television surveillance may be necessary to detect instantly accidents or other disturbances in order that police, ambulances, fire department equipment, or other aid can be dispatched immediately. Protracted traffic tie-ups result in heavy economic loss when large volumes are involved. In rural areas roadside telephone stations equipped with acceleration and deceleration lanes will be of great assistance.

SERVICE

In the past controlled-access highways, except for turnpikes and certain parkways, have been limited to relatively short mileages in urban areas. In addition to being short, existing urban expressways are accessible to municipal and private-enterprise facilities, including police stations, fire departments, ambulance service, medical aid, fuel, repair service, food, and overnight accommodations. Frequent entrances and exits in built-up areas make these services readily available. On the other hand, long-distance, controlled-access highway systems currently (1958) introduce a new problem.

Because it is not necessary to develop self-liquidating revenue from highway services on the interstate and urban systems, existing private facilities can be utilized to the utmost. National highway policy could be based on the principle of causing the least disruption possible to existing private-enterprise facilities and plant. Informed planning during the design stage at the inception of each project can accomplish much. Thus, the highway administrator has the option of developing policy in each state and region tailored to local needs and in the public interest. It is believed that each motorist should be considered as a "customer" to be served.

Using existing services developed by private enterprise relieves the public highway agency of many annoying and detailed responsibilities. On the other hand, over-all leadership, coordination, supervision, and regulation will also be necessary.

In most cases it will be necessary for the state only to survey existing privately owned servicing facilities and to coordinate them, establish zones of operational responsibility, and make satisfactory arrangements with respect to the prices charged and the quality and scope of service. Continued follow-up and review will be desirable to assure adequate and satisfactory service.

The magnitude of the motorists' needs can be understood when it is realized that the services required by a city with a population of 200,000 constitute the daily requirements on a turnpike. There is an average of more than 86,000 vehicles with peaks of 125,000 vehicles per day. This city on wheels is in motion, but it must stop at intervals for essential services, such as those provided by the facilities of a modern city: Buildings, food, motor fuel, accessories, water supply, water treatment, commercial electric power, stand-by power, telephones, sewage treatment, garbage disposal, comfort facilities, radio, heating (including stand-by units), air conditioning, lighting, parking, air, and water. All the foregoing are taken for granted by the city dweller.

A cardinal principle has emerged: The public do not wish to leave a major highway in search of such facilities unless there are acceptable services along a well-maintained and marked route. Consequently, it is necessary to integrate these services into the over-all highway plan so that the motorist can be offered complete satisfaction. Searching for facilities in strange territory, particularly at night with the probability of getting lost, is distasteful and unacceptable.

Fuel, oil, air, water, comfort, and repair services must be available at reasonable intervals, and, if the highway is extremely long, food facilities are necessary. In addition, fire departments, towing services, sleeping accommodations, wreckers, ambulances, first-aid facilities, and "out-of-gas" roadside services are required. Communications are vital—many women and elderly or infirm persons drive automobiles, and servicing arrangements should be designed to meet their special requirements.

High-quality leadership is necessary in establishing policy to handle this problem, in order that the public interest can be served and procedures can be developed that will assure scrupulous honesty and impartial administration. To assure the greatest economic benefit to the public and to the business community from the Interstate Highway System, operational planning for controlled-access highways will revolve around existing services that are privately owned and operated, so that highway construction will result in improved business and healthy land development.

Acquainting motorists with the availability of roadside services, as well as shops or recreational facilities, is equally as important as placing traffic warning and destination markers. Dignified legible signs advising travelers in advance of the name of the community being approached and of the availability of comfort, food, fuel, overnight accommodations, and shops are an acceptable public service. In addition, a well-marked, adequate, and properly maintained access connection from the controlled-access highway to the community center is imperative. Parenthetically, it is necessary to develop some sort of system for advising motorists who are "on the fly" of available overnight accommodations. Certainly no motorist desires to leave the through highway only to be greeted with "No Vacancy" signs. Lack of positive information on where to find sleeping quarters tempts the motorist to drive beyond the fatigue limit, a condition which should be avoided. By the same token no motorist wishes to be directed to third-rate or criminally dominated facilities. Roadside industry and community officials should assume such responsibillty.

Wherever adequate facilities are not available at intervals that are frequent enough to satisfy public convenience, the state may then provide "service areas" adjacent to the expressway. Such areas might be planned in conjunction with an interchange. Contrary to turnpike practice, it is believed that state-financed areas could be provided on a simple recovery-of-cost basis and not used merely to raise revenue. It is good policy to provide facilities solely as a public service.

The plan developed by the New Jersey State Highway Department for freeways and parkways provided that the state design the site; acquire the necessary land; and grade, drain, pave, and landscape the area, making provision for electric power, telephone, and water. The site was laid out in lots, each suitable f or a complete "gas station." It was proposed at first to lease these lots on the basis of competitive bids to individuals, who would construct the buildings, gasoline pumps, and other accessories, at lease rates that would be sufficient to amortize the investment and provide annual maintenance. Prospective lessees objected to constructing permanent buildings on land owned by others, and the procedure was modified. The lots were offered at public sale, each going to the highest bidder, but not more than one brand of petroleum product was permitted in each area. Building architecture and signs were required to be approved by the state highway commissioner. In practice each site was laid out with four lots initially, but with provision for two more, which could be added at any future time, if necessary.

Publicly provided, fixed-plant roadside service will probably be confined to providing fuel and food. Facilities for dispensing fuel, oil, and road accessories are required on all highways, except for very short projects of less than 10 miles, although no general pattern can be, or at least, has been established. Environmental conditions on each highway are different, and a service-area policy needs to be tailored to the requirements peculiar to each project.

Food service is required on the longer projects, particularly on those through rural areas. Again no rule can be applied, but it appears that food facilities would not be required wherever service is available at about 50-mile intervals.

Deluxe-type overnight accommodations have not been provided (as of 1958) on controlled-access highways, and there is considerable doubt whether they are required. It is believed that caution should be exercised when providing sleeping accommodations. It is hoped that some workable solution to the problem can be devised without involving the highway agency in a business that is particularly well suited to construction and operation by private enterprise.

Controlled-access highways on which service facilities have been integrated can be divided into (*a*) toll-free freeways and parkways and (*b*) toll highways. In the first case, service, not revenue, is the controlling consideration. In the second, revenue and service are of equal importance because most toll projects must be fully self-liquidating.

It would also seem that on-the-road services that are not immediately essential to the safety and requirements of travelers should not be provided. Such a policy should cut short any tendency toward paternalism and restrict facilities to essential services only.

In addition to fixed-plant service areas, it is necessary to make arrangements for (*a*) patrol services for breakdowns and flat-tire and out-of-gas cases; (*b*) garages with repair, towing, and wrecking services; and (*c*) first aid, ambulances, and fire department equipment. The scope of the "gas" patrol is indicated by the yearly travel of 600,000 miles, resulting in aid to 28,000 distressed motorists on the New Jersey Turnpike. In addition, in 1955, forty-five private towing and repair garages responded to 15,000 emergency calls; twenty-two fire companies, to 80 calls; and ambulance and hospital squads, to 323 calls.

DESIGN OF OPERATIONAL FACILITIES

In general controlled-access highways will be constructed with a median separating opposite-direction traffic, and all crossings will be grade separated. In urban areas many long viaducts will be constructed. For these reasons it is necessary to provide operational facilities in the original design.

The police, towing services, wreckers, ambulances, fire department equipment, roadside gasoline patrol service, and maintenance operations require crossovers in the median. Without this flexibility, the effective operation of servicing units will not be satisfactorily feasible for both the operating forces and the public. After police and maintenance-patrol districts have been coordinated, crossovers should be placed at section limits, where snow and ice control trucks need to reverse direction, and to care for wrecking, fire-department, and ambulance services. Crossovers are usually needed at each end of an interchange area so that snow and ice equipment may reverse direction quickly to clear all entrance and exit ramps.

Extraordinarily wide medians with topography or vegetation that obscures vision from one roadway to the other present operating problems to patrols and policing personnel. Crossovers at each end of such medians would permit personnel to make "loop" operations to detect broken-down vehicles, accidents, or criminal operations. If "obscured-view" medians are extremely long, intermediate crossovers will be required.

Crossovers should be as inconspicuous as possible to prevent their being used by the public. They should be paved suitably and have adequate drainage and other design features to reduce maintenance costs. For enforcement purposes, signs prohibiting public use are required.

Between interchanges, there should be emergency "escape hatches" at appropriate sites to handle the entry and exit of emergency vehicles and where public entry is prohibited. Emergency entrances are of great importance when disaster occurs. These ramp points of modest and rudimentary design are located strategically to the local road system, fire departments, ambulance service, local police stations, garages, wreckers, and towing services. In order to prevent unlawful entry into a controlled-access facility, the ramps may be fitted with locked gates and keys distributed to dependable and responsible personnel. In addition to prompt entry of ambulances, fire trucks, towing, garage, and police vehicles, it may be necessary to bypass maintenance and police around trouble spots where many vehicles are stalled because of accidents or unusual weather conditions. Often motorists and truckers will abandon vehicles in the roadway, thus blocking all movement, and it is necessary to get to the head of the line quickly and pull stalled vehicles off the roadway.

Special features may be required on long viaducts where adjacent development precludes plowing snow over the side. Many such viaducts require the snow to be loaded on trucks and carted away.

Rest areas are important and add appreciably to the safety and comfort of the public. Sleepy and tired drivers need to pull off the highway for rest and sleep. Other motorists desire to eat a packed lunch and relax during the journey. Rest areas should be set back from the highway, preferably with shade trees, picnic tables, public telephones, water, and comfort facilities. Fencing may be required to prevent small children from venturing onto the adjacent main roadway. Acceleration and deceleration lanes are required for exit and entry to the highway.

Bus stations set off to one side, with adjacent parking yards contiguous to the street system, will be necessary on many parts of controlled-access highways. They must be equipped with acceleration and deceleration lanes. Unless mass transit is encouraged, it will not be possible for highway facilities to meet peak-hour morning and evening demand.

Pedestrians often do not understand the hazard of crossing wide modern highways, nor can they judge the speed of approaching vehicles. At sites where pedestrians will cross regardless of regulation, but where a grade-separated vehicular crossing is not justified, overhead pedestrian bridges provide a solution. These structures may be light, possibly of suspension design with an open floor to avoid snow loading and icing hazards. In urban areas it may be necessary to enclose the crossing with wire mesh to prevent stones being thrown at vehicles passing below. Where feasible, such bridges should have ramp approaches rather than steps, and these approaches should be no steeper than 10% gradient. The public do not like to use steps and will often walk across the road instead.

Bridge railings in urban areas have caused much public debate. Highway authorities, in an effort to design sightly structures that are aesthetically pleasing, have, at times, produced a railing that parents believe is hazardous to small children. Rails with openings that toddlers can crawl through or roll under and that older children can walk on have been the subject of attack.

Right-of-way fencing to define the limits of public property, prevent unlawful entry, and prevent stock from straying on the highway is receiving attention. Generally farm-type fencing is adequate for this purpose. In urban areas where there are children, the chain-link type may be required, and, in special cases, it may be necessary to provide from 6-ft-to-8-ft chain link that is surmounted by barbed wire. At deer runs, fences that are 12 ft high or more are required, and, in some cases, underpass culverts may be necessary to allow the deer to cross. Deer are becoming so numerous in the United States that they present a serious problem because of the accident potential.

The problem of utility crossings also requires mutual cooperation and planning between utility owners and the highway department. Such crossings should be planned so that it will be unnecessary for utility employees to work within the highway right of way after the initial installation. The hazards to these employees and the public are so great after the highway is opened to traffic that full cooperation should be assured. Underground crossings can be put through in outer sleeves. In cases of breakage or need for replacement, the utility line can be withdrawn from the sleeve while working outside the right of way.

During the initial construction, sufficient right of way should be purchased for interchange expansion. Property is quite inexpensive at that time, and it is surprising how quickly traffic expands to the point where additional facilities are needed. This also applies to right of way required for future widening.

MAINTENANCE

Maintenance on an expressway is different from that on local rural roads, not only because of heavier traffic volume and high-speed operation, but because drivers utilizing the facility feel they have the right of way and are intolerant of any operations on, or occupancy of, the roadway. Density and high speed have intensified the need to design maintenance into the expressway in order that men and equipment occupy the roadbed less. Full mechanization is necessary, not only for efficiency of operations and conservation of labor, but also to promote public safety. When machines are equipped properly and operated carefully, they reduce maintenance hazards. For these reasons it is necessary to re-evaluate maintenance procedures that use special measures when maintaining controlled-access roadways.

Each task should be analyzed to determine whether it can be accomplished effectively by machinery or other mechanical equipment. For example, ice-control chemicals and abrasives can be handled by one-man special-body trucks that are equipped with mechanical spreaders controlled by the driver in the cab. These units can be organized to assure the quick action necessitated by the high degree of service required on modern highways and to assure peak efficiency. The trucks can be loaded in approximately 10 min by mechanical loaders or by overhead storage bins.

Wherever standard commercial equipment is not available on the market for particular tasks, special equipment can be developed in cooperation with reliable and experienced manufacturers.

Feasible preventive maintenance must be designed into the expressway. High-quality, tough, and durable pavement requiring a minimum of surface maintenance is desired, and hard-surfaced shoulders flanked by adequate grass-covered berms eliminate scraping and blading. This increases safety to the motorist who must drive onto the shoulder at high speed, in addition to eliminating the frequency of men and equipment on the shoulders. There should be no open ditches, and, except in rocky or mountainous areas, side slopes that are to be mowed should be no steeper than 3:1 with all "breaks" rounded. Erosion control is a very important factor in the large areas involved in trunk-highway construction. All slopes and disturbed areas should be planted with grass seed, and clearing should be carried back sufficiently to avoid the encroachment of falling trees on the travel lanes. All slopes and grading at drainage structures should be streamlined, not only for increased vehicle safety, but to facilitate the movement of grass-mowing machinery. Drainage-pipe systems (including inlets) that are designed on the self-cleaning principle with a minimum velocity of 3 ft per sec when flowing one-third full will reduce maintenance. Liberal underdrainage to keep the water table down is extremely important, particularly in wide medians in which there is much underground water percolation.

All maintenance conducted within the roadbed from shoulder to shoulder must be surrounded by safeguards to reduce accidents to a minimum, particularly those involving travelers. Large signs repeated at regular intervals and traffic cones should be used to mark every operation, and flagman protection is essential. Elaborate arrangements and flagging services are expensive, but they are worth while from the standpoint of public safety.

The "following-truck" principle is utilized when traffic lines are painted, work is done within the roadway areas, and grass is mowed on narrow medians. A typical arrangement is as follows: All pavement-traffic paint lines are placed with a large truck applicator. The supervisor's truck follows immediately behind it, and behind this truck is one that places traffic cones at 50-ft intervals. The latter has an 8-ft-by-3-ft sign reading, "Caution Men Working." At the beginning of the operation where the paint has dried, there is a fourth truck on which is mounted an 8-ft-by-6-ft sign with letters ranging from 12 in. high to 20 in. high with the message, "Slow—Keep off White Line—Keep Right (or Left)—35 m. p. h." In addition the two preceding signs have red flashing blinkers on them. At the beginning of the entire operation, there is a stationary sign, 6 ft by 4 ft with 10-in. letters reading, "Single Lane—4 Miles."

The trucks and signs following the paint truck not only protect the crew on the paint truck, but, more important, they reduce the chance of rear-end collisions, even in the case of somnolent drivers, because the signs are visible for approximately 1 mile ahead. Work on the pavement on dense traffic routes usually cannot be begun until about 9:00 a.m. and must usually end by 4:00 p.m., or traffic tie-ups would be intolerable. If work is attempted beyond those hours, more traffic than can be handled by the restricted roadway causes vehicles to be backed up and stopped, thus inviting rear-end collisions.

There is evidence of motorists who continue driving on the road after reaching exhaustion and who are in a somnolent state, often actually dozing at the wheel. Although police are constantly on the alert for these drivers, enough of them remain undetected to be a menace. Therefore, maintenance operations must be planned and safeguards set up in anticipation that a driver may drive straight into a stopped vehicle, sign, or other object in the road. For example, flagmen must stand clear of signs and barricades because they would be endangered by flying pieces, and they must always stand facing traffic, watch each approaching vehicle, and be constantly on the alert. All operations on the highway should be conducted so that the safety and comfort of the motorist comes first and maintenance and construction second.

Snow and ice removal must be prompt and organized so that no more than 1 in. or 2 in. of snow accumulates. High-speed plows, which are painted for maximum visibility under murky conditions and equipped with powerful jumbo-sized flashing red lights, usually operate in echelon, "floating" in the traffic stream. A special arrangement is needed at interchanges for prompt snow and ice control. An interchange plugged with stalled or stuck vehicles during an ice or snow storm can cause a vast emergency.

Weather reports are used extensively to give the maintenance man advance knowledge of the weather. More and more states are using specialized, personalized, professional weather services. A foolproof system of dissemination is essential. Many states have made arrangements with commercial radio stations to broadcast adverse weather and road conditions as a public service, thus adding immeasurably to convenience and safety.

CONCLUSIONS

Highway transportation is entering a new era in which operations will assume new and challenging proportions. The new tasks and responsibilities generated by the expressway may seem great, but when they are viewed from the standpoint of public service involving the comfort and safety of the traveling public, they are worth while. These developments should be welcomed, not feared, because they demonstrate the dynamic character and maturity of highway transportation, especially with respect to the reduction in injuries and death. Adequate designs and alert operation and administration will help such accidents as evidenced on the New Jersey Turnpike, on which in 1957 the fatality rate was 1.99 per 100,000,000 vehicle-miles compared with the national rate of approximately 6.0, and on which there was an accident rate of 93½, which is 6% of the national rate.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2945

LABORATORY AND IN-SITU PERMEABILITY OF SAND

BY CHARLES I. MANSUR,[1] M. ASCE

SYNOPSIS

Seepage analyses and design of seepage-control measures require a reasonably accurate knowledge of the coefficient of permeability of the soil. The results of laboratory tests on remolded sand samples are compared with permeabilities obtained from field pumping tests. The permeabilities of the individual sand strata were determined from drawdown curves and from flow from the different strata.

INTRODUCTION

A main problem in the analysis and design of measures to control seepage beneath dams and levees that are underlain by strata of pervious sands is the correct determination of the vertical and horizontal coefficients of permeability of the various strata. This knowledge is necessary for drawing flow nets, determining "effective" penetration of relief wells, and computing seepage flows.

It has long been realized that there is a significant difference in the vertical and horizontal permeability of a given sand stratum as well as a variation between strata. However, the usual practice is to estimate an over-all permeability of a given sand stratum from (*a*) mechanical analyses of samples obtained from bailer or split-spoon borings; (*b*) laboratory permeability tests on remolded samples from such borings; (*c*) falling head tests[2] in a well or piezometer; or (*d*) pumping tests on partly or fully penetrating wells. The horizontal or vertical permeability of any given sand stratum is usually either guessed or ignored.

In 1950 a field sampling procedure[3] was developed that permits undisturbed sampling of sand either above or below the water table at reasonable cost. By

---

NOTE.—Published, essentially as printed here, in January, 1957, in the Journal of the Soil Mechanics and Foundations Division, as *Prodeedings Paper 1142*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Vice-Pres. and Chf. Engr., Independent Wellpoint Corp., Baton Rouge, La.; formerly Asst. Chief, Embankment and Foundation Branch, Soils Div., Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss.

[2] "Time Lag and Soil Permeability in Ground Water Observations," *Bulletin No. 36*, Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss., April, 1951.

[3] "Undisturbed Sand Sampling Below the Water Table," *Bulletin No. 35*, Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss., June, 1950.

transporting these samples to the laboratory in a vertical position and testing them while they are still in the tube, reasonably accurate vertical permeabilities of relatively fine sand samples can be obtained. The accuracy of such tests on medium or coarse sand is sometimes questionable because the drilling mud used with the sampling procedure may penetrate the sand ahead of the sample tube or follow its cutting edge as it is pushed down, thereby contaminating the sample and, possibly, reducing its permeability significantly.

The writer has determined the in-situ horizontal permeability of various sand strata in the alluvial valley of the Mississippi River by special pumping tests. These tests were made in connection with the design of relief well systems along levees in the St. Louis (Mo.) District, Corps of Engineers, United States Department of the Army. The design of dewatering and seepage-control facilities for a proposed lock to connect the Mississippi River and the Atchafalaya River at the proposed closure[4] of the Old River (Louisiana), approximately 45 miles south of Natchez (Miss.) was also involved. The horizontal in-situ permeabilities of the various sand strata encountered were determined by measuring the flow in a fully penetrating well screen by a sensitive well-flow meter at the boundary of each sand stratum as previously delineated by a boring at each site. From the flow, q, into the test well from each stratum, the thickness, d, of the stratum, the drawdown in the well, the radius of the well, and the radius of influence (or distance to known points of drawdown), the horizontal permeability, k_H, of the stratum being tested was computed from the following formula for artesian flow into a well (the nature of the formations causing essentially artesian flow):

$$k_H = \frac{2.3 \log_{10} \frac{r_2}{r_1} q}{2 \pi d (h_1 - h_2)} \qquad (1)$$

In Eq. 1, r_2 is the distance from the well to the drawdown observation, h_2; r_1 represents the distance from the well to the drawdown observation, h_1; h_1 is the drawdown at a distance, r_1, from the well; and h_2 denotes the drawdown at a distance, r_2, from the well.

Mechanical analyses and laboratory permeability tests were made on samples from different sand strata obtained by auger, bailer, and split-spoon samples, and a 3-in. Shelby tube (undistributed sampler), as well as on samples obtained during the well drilling. An attempt was also made to correlate the permeability coefficients determined in the laboratory and in the field with the effective grain size of the various strata. The tests also included a determination of the specific yield (flow per foot of drawdown in the well) of each well; drawdown versus time of pumping at a constant rate; drawdown curves to each well at various rates of pumping; flow versus drawdown; and hydraulic head losses through the filter, screen, and in the well.

The methods used for taking the samples and installing the wells, results of the laboratory tests, procedures for making the field pumping tests, and results of the pumping tests are described herein. The permeabilities obtained in the laboratory and in the field are compared.

---

[4] "Mississippi-Atchafalaya Diversion Problem," by John R. Hardin, *Military Engineer*, Vol. XLVI, 1954, pp. 87–92.

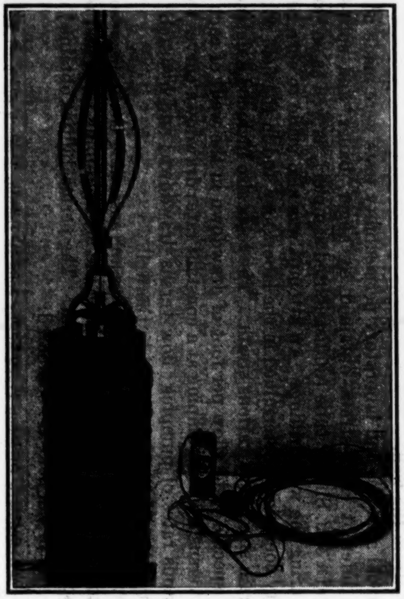


FIG. 1.—WELL-FLOW METER AND WOODEN WELL SCREEN

TABLE 1.—WELL TYPES AND METHODS OF INSTALLATION

No.	Type of well screen	Filter thickness, in inches	Depth of well, in feet	Length of well screen, in feet	SAND AQUIFER Thickness, in feet	SAND AQUIFER Penetration by screen, in percentage
H-151A[a]	Wood, $\frac{1}{16}$-in. slots[c]	6	100	66	95	70
FC-105[a]	Wood, $\frac{1}{16}$-in. slots[c]	6	120	86	95	90
OR-1[b]	Metal, No. 12 and No. 8 slots	none	148	99	118	86

[a] Installed by reverse rotary method. [b] Well OR-1 installed with a bailer and casing. [c] Open slot area in wooden screens = 27 sq in. per lin ft of screen.

TEST WELLS, PIEZOMETERS, AND BORINGS

The test wells had inside diameters of 8 in. and were of the types shown in Table 1. Fig. 1 shows a section of wooden well screen used in wells H-151A and FC-105. The well flow meter[5] used to measure the flow in the screen consisted essentially of an impeller rotating about a vertical axis, a yoke containing the impeller bearings, an adapter for centering the yoke in the well, a cable for lowering this assembly to any desired depth, and an electrical system for counting impeller revolutions. The meter was calibrated by observing the rate of impeller revolution (determined electrically) at known rates of flow in a vertical 8-in.-diameter pipe.

As holes for the wells were advanced, samples were taken from the drilling effluent or bailer, depending on the method of installation. With the equipment used, it was not possible to install wells H-151A and FC-105 to the full depth of the sand aquifer because of large cobbles near the bottom of the alluvial valley. After the wooden well screens were installed, a specially designed and prepared filter gravel was placed around the slotted wooden screen by a tremie pipe. The wells were then developed by use of a surge block and pumping.

Piezometers to measure the drawdown to the wells were installed in the upper part of the sand aquifer on ranges that were perpendicular and parallel to the river. Only data obtained from the latter ranges are included herein. Special piezometers were installed at various depths at the outer periphery of the gravel filter around wells H-151A and FC-105 to measure the loss in head through the filter and screen and in the well (Fig. 2). Others were installed on a line from well OR-1 in both the upper and lower sand strata found at the site.

Before any of the wells were installed, an exploratory boring was made at, or immediately adjacent to, each of them in order to obtain samples for laboratory tests and determine the stratification of the sand aquifer. A split-spoon sampler was used to take samples in a mudded hole at wells H-151A and FC-105, and logs of these borings are shown in Fig. 2. A 3-in. Shelby-tube sampler was used to take undisturbed samples of sand in borings LS-2 and L-8 adjacent to well OR-1. Fig. 3 contains logs of these borings.

LABORATORY TESTS

All samples obtained from the borings and well-drilling operations were classified on the basis of the Unified Soil Classification System[6] and are plotted in Fig. 2 and Fig. 3. Mechanical analyses were made on samples taken by the split-spoon, Shelby-tube, and bailer samplers, and on samples collected from the effluent of the reverse rotary well-drilling operations. Grain-size curves obtained from these tests are shown in Fig. 4. The reference numbers shown on the grain-size curves correspond to those in Figs. 2 and 3. The effective grain size for the samples tested are also shown in Figs. 2 and 3.

The permeability coefficient was determined in the laboratory on several remolded samples obtained by the various methods previously described. The

---

[5] "Waterways Experiment Station Relief Well Flow Meter," *Miscellaneous Paper 5-83*, Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss., April, 1954.

[6] "The Unified Soil Classification System," *Technical Memorandum 3-357*, Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss., March, 1953.

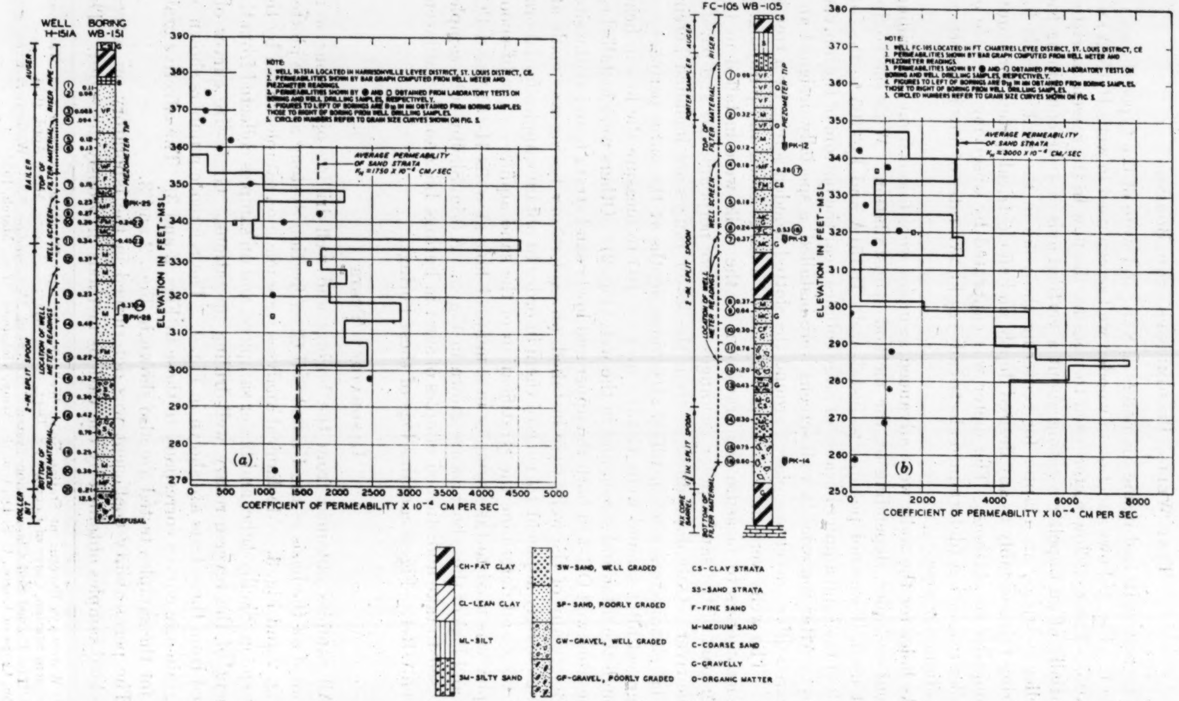
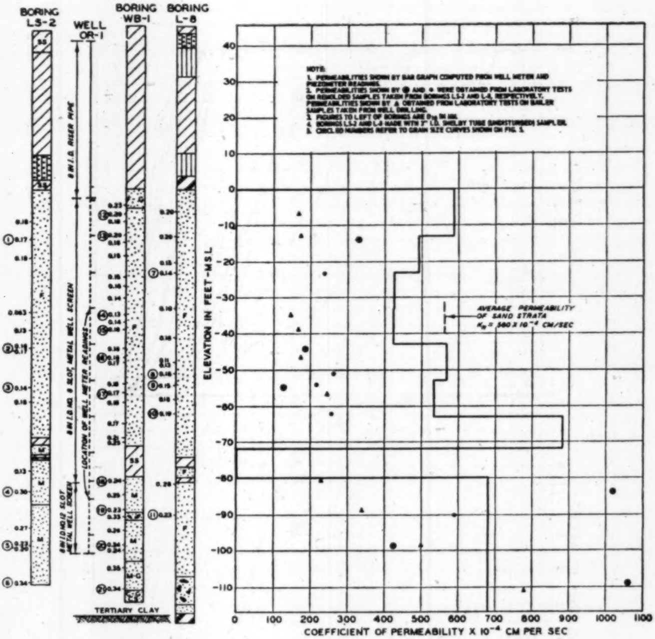


FIG. 2.—COEFFICIENT OF PERMEABILITY AND EFFECTIVE GRAIN SIZE OF INDIVIDUAL SAND STRATA



NOTE:
1. PERMEABILITIES SHOWN BY BAR GRAPH COMPUTED FROM WELL METER AND PIEZOMETER READINGS.
2. PERMEABILITIES SHOWN BY ● AND ▲ WERE OBTAINED FROM LABORATORY TESTS ON REMOLDED SAMPLES TAKEN FROM BORINGS LS-2 AND L-8, RESPECTIVELY. PERMEABILITIES SHOWN BY ▲ OBTAINED FROM LABORATORY TESTS ON BAILER SAMPLES TAKEN FROM WELL DRILLING.
3. FIGURES TO LEFT OF BORINGS ARE D_{10} IN MM.
4. BORINGS LS-2 AND L-8 MADE WITH 3" I.D. SHELBY TUBE (UNDISTURBED) SAMPLER.
5. CIRCLED NUMBERS REFER TO GRAIN SIZE CURVES SHOWN ON FIG. 5.

FIG. 3.—COEFFICIENT OF PERMEABILITY AND EFFECTIVE GRAIN SIZE OF INDIVIDUAL SAND STRATA FOR WELL OR-1

results are plotted in Figs. 2 and 3 at the corresponding elevation at which the sample was taken. All the tests were made in a constant-head permeameter. Samples obtained by a split-spoon sampler in a mudded hole were thoroughly washed before testing to remove any traces of drilling mud, and care was taken that no natural "fines" were lost during the washing.

The permeability coefficients (k_L) determined in the laboratory on samples taken at wells H-151A and FC-105 were adjusted to the estimated natural void

TABLE 2.—RELATIONSHIP BETWEEN D_{50} AND NATURAL VOID RATIO[a,b]

D_{50}, in millimeters	Void ratio, e	D_{50}, in millimeters	Void ratio, e	D_{50}, in millimeters	Void ratio, e
0.10	0.935	0.30	0.627	0.50	0.562
0.12	0.840	0.35	0.601	0.55	0.553
0.15	0.775	0.40	0.583	0.60	0.548
0.20	0.703	0.45	0.572	0.70	0.537
0.25	0.657	...	...	...	...

[a] This correlation is actually for peak grain diameter, D_p, rather than for D_{50}. However, $D_p \cong D_{50}$.
[b] "Potamology Investigations, Summary Report of Soils Studies," *Report No. 12-2*, Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss., October, 1952.

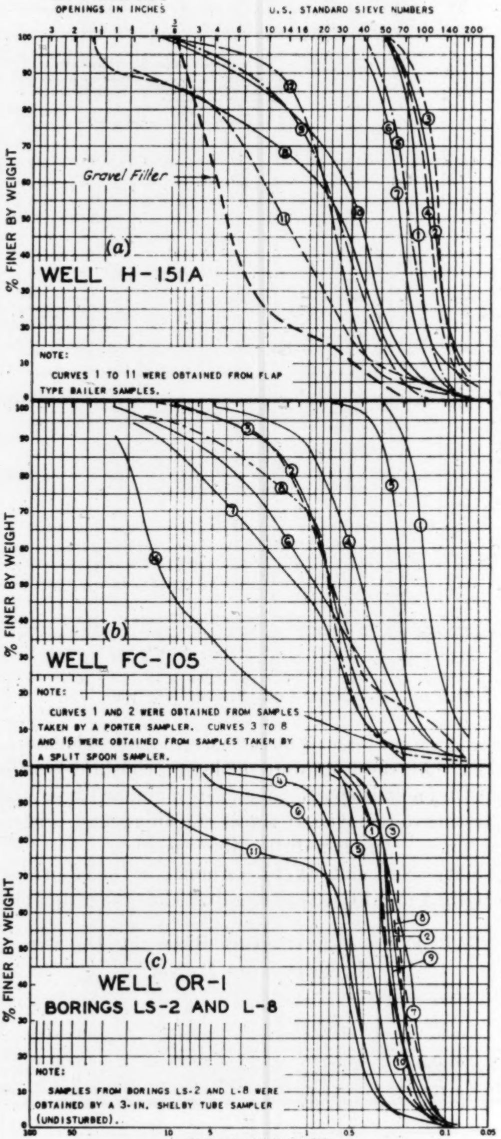


FIG. 4.—GRAIN-SIZE CURVES

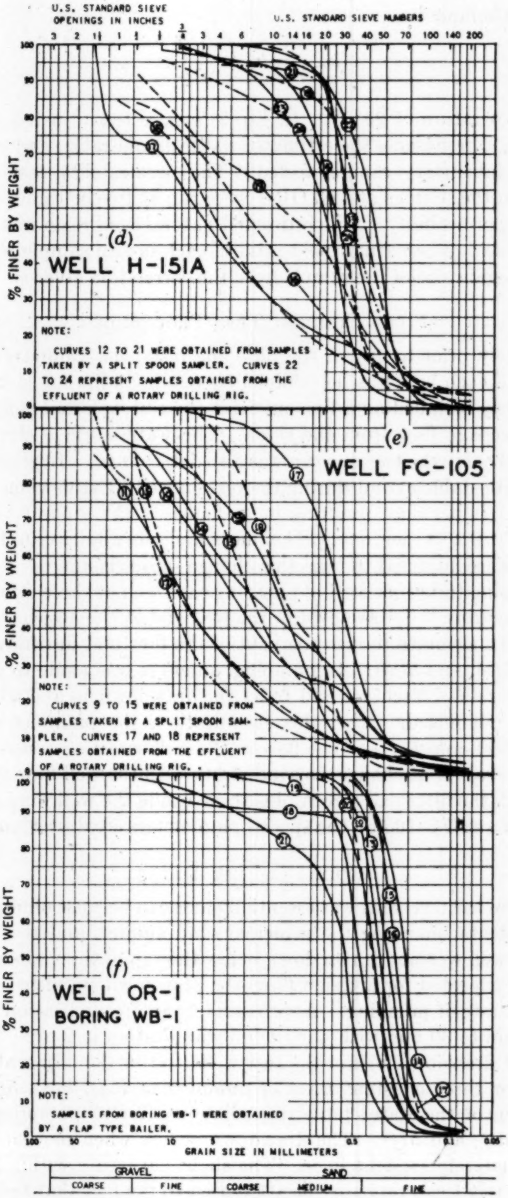

U.S. STANDARD SIEVE OPENINGS IN INCHES
3 2 1½ 1 ¾ ½ ⅜
U.S. STANDARD SIEVE NUMBERS
3 4 6 10 14 16 20 30 40 50 70 100 140 200
% FINER BY WEIGHT
(d)
WELL H-151A
NOTE:
CURVES 12 TO 21 WERE OBTAINED FROM SAMPLES TAKEN BY A SPLIT SPOON SAMPLER. CURVES 22 TO 24 REPRESENT SAMPLES OBTAINED FROM THE EFFLUENT OF A ROTARY DRILLING RIG.
(e)
WELL FC-105
NOTE:
CURVES 9 TO 15 WERE OBTAINED FROM SAMPLES TAKEN BY A SPLIT SPOON SAMPLER. CURVES 17 AND 18 REPRESENT SAMPLES OBTAINED FROM THE EFFLUENT OF A ROTARY DRILLING RIG.
(f)
WELL OR-1
BORING WB-1
NOTE:
SAMPLES FROM BORING WB-1 WERE OBTAINED BY A FLAP TYPE BAILER.
100 50 10 5 1 0.5 0.1 0.05
GRAIN SIZE IN MILLIMETERS
GRAVEL
SAND
COARSE
FINE
COARSE
MEDIUM
FINE

FIG. 4.—(CONT.)

ratio for each sample by

$$k_{e-n} = k_L \left(\frac{e_n}{e_L} \right)^2 \qquad \text{(2)}$$

and to a temperature of 20° C. In Eq. 2, k_{e-n} is the adjusted coefficient of permeability; e_n represents the estimated natural void ratio; and e_L is the void ratio of the laboratory sample tested. The permeability tests on samples obtained from the borings at well OR-1 were run at two or three different void ratios. The permeability at the estimated natural void ratio was interpolated from these data. The natural void ratio of each in-situ sample was estimated from the D_{50}-grain size of the sample from the correlation given in Table 2.

FIELD PUMPING TEST PROCEDURES

Immediately prior to any of the pumping tests, the ground-water table in the vicinity of each well was determined by reading adjacent piezometers and wells previously installed on a line out from the test well and perpendicular to the line of seepage flow from the river. The water-surface elevation of the river during the day the test was run was also determined. In general, the river stage was quite stable except during the test on well FC-105 when the river fell at a rate of 1 ft per day, resulting in a change in the ground-water table of approximately 0.1 ft per day. The measured drawdowns in the well and piezometers were adjusted to take this change into account.

The pumping tests consisted of pumping each well at three different rates of flow and measuring the drawdown in the well and in the adjacent piezometers and wells. An accurate record was kept of the flow and drawdown with time (Figs. 5(*a*), 5(*b*), and 5(*c*)). In pumping, an effort was made to set the pump at a constant rate of discharge and to let the drawdown, in and adjacent to the well, become stabilized. After this stabilization, the piezometers were read (Figs. 5(*d*), 5(*e*), and 5(*f*)). The flow in the well screen was also measured at specified intervals or at changes in sand strata with the well-flow meter. Head losses through the filter and screen plus those inside the well were measured for wells H-151A and FC-105 by the special piezometers cited previously.

PUMPING TEST DATA

The flow to the test wells was essentially artesian because of upper and lower impervious strata bounding the principal sand aquifer, and the fact that the water in the well was not drawn down below the top of the main sand stratum. As would be expected for artesian flow, the drawdown stabilized quite rapidly at a constant rate of pumping. Approximately 80% stabilization was achieved in from 15 min to 30 min with almost full stabilization in 2 hr.

Flow and drawdown within the test wells and in the adjacent piezometers are shown for three constant rates of pumping in Figs. 5(*d*), 5(*e*), and 5(*f*). The elevation of the water table immediately prior to pumping is shown in Figs. 5(*d*), 5(*e*), and 5(*f*). The drawdown curves when plotted on a semilog graph are relatively straight lines. The slight curvature within 20 ft of wells H-151A and OR-1 can probably be attributed to the fact that their screens did not completely penetrate the previous aquifer. The intersection of the draw-

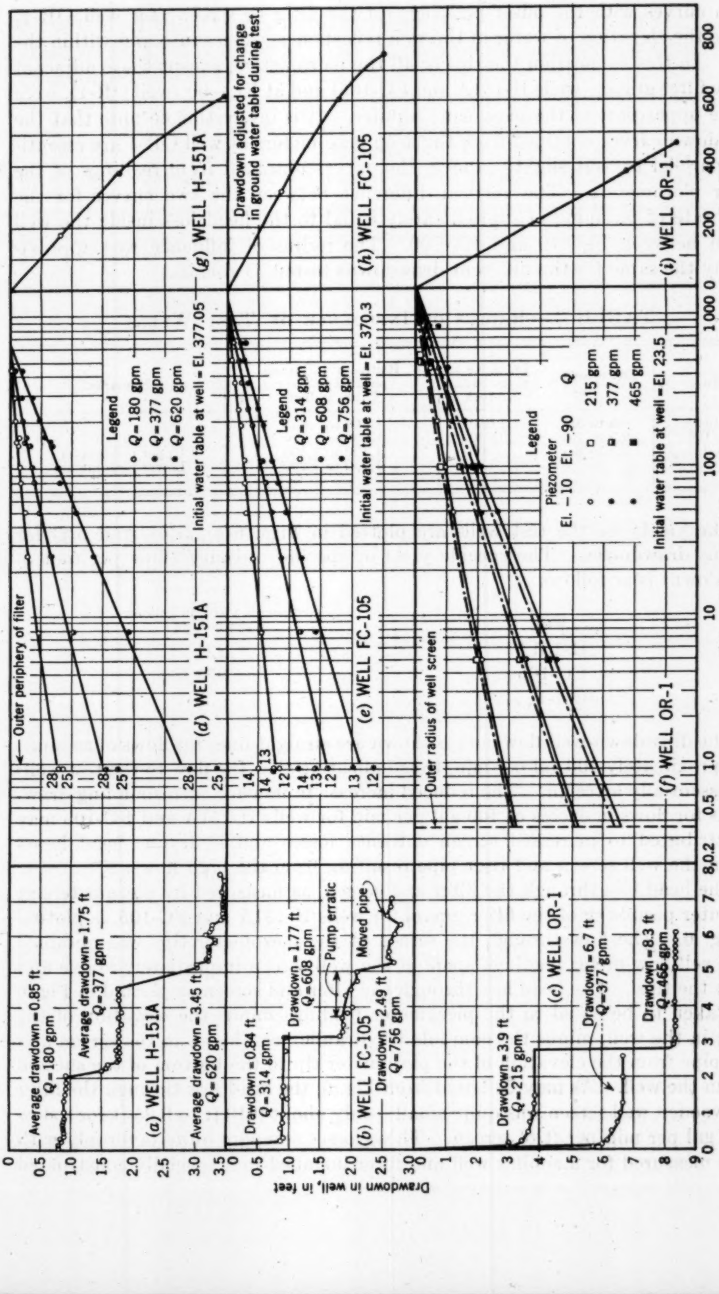


FIG. 5.—THE EFFECT OF TIME, DISTANCE, AND FLOW ON DRAWDOWN

down curves with the outer periphery of the filter or screen (for well OR-1) above the elevation of water in the well reflects head losses into and within the well. In this connection the tips of all the piezometers, except those adjacent to the filter gravel (wells H-151A and FC-105) and at El. −90 (well OR-1), were at the upper part of the main sand aquifer. It is interesting to note that the drawdown curves obtained from the deep piezometers at well OR-1 are essentially parallel to, but slightly above, the curves obtained from readings of the upper piezometers. The vertical difference between the two curves for the same rate of pumping is approximately equal to the head loss inside the well screen between El. −10 and El. −90. The radius of influence was approximately the same for the different drawdowns tested (Table 3).

TABLE 3.—RADIUS OF INFLUENCE OF TEST WELLS

No.	Drawdown, in feet	Distance to river bank, in feet	Radius of influence parallel to river bank, in feet	Remarks
H-151A	0.9 to 3.5	1,050	550 to 650	
FC-105	0.9 to 2.6	3,000	1,000 to 1,100	...
OR-1	3.9 to 8.3	900	1,400 to 1,500	Upper piezometers
			1,100 to 1,200	Lower piezometers

The yields for the test wells are plotted in Figs. 5(*g*), 5(*h*), and 5(*i*) for various drawdowns. The specific yield or specific capacity (flow per foot of drawdown) is as follows:

Well	Specific yield, in gallons per minute
H-151A	221
FC-105	378
OR-1	55

The drawdown-well-flow curves shown are straight lines for flows of as much as approximately 400 gal per min, thus indicating that the flow to the test wells was essentially artesian. The reduced increase in flow for corresponding drawdowns for flows in excess of 400 gal per min for wells H-151A and FC-105 may be attributed to increased screen entrance losses and hydraulic head losses within the well screen and riser pipe resulting from the high flows.

The head loss through the filter and screen, as measured by piezometers at the outer periphery of the filter gravel for wells H-151A and FC-105, is plotted in Fig. 6. The flow through the screen at the piezometer tips was obtained from well-flow-meter readings made in the well screen immediately above and below the tips. The head loss through the filter and screen as plotted in Fig. 6 was taken to be equal to the piezometer readings minus the elevation of the water in the well, minus the computed hydraulic head losses in the screen and riser pipe from the elevation of the piezometer tip to the bottom of the suction pipe in the well. As may be noted from Fig. 6, the head loss through the filter and wooden well screen was quite small—only about 0.10 ft to 0.25 ft for a flow of 10 gal per min per ft of screen. These losses compare quite favorably with those measured for a similar well and filter installed in a specially constructed testing tank set up in the laboratory.[7] In these tests the measured head loss through the filter and screen was approximately 0.25 ft for a well discharge of 10 gal per min per ft of screen. The slightly higher entrance loss measured in the laboratory is due to the fact that the laboratory test well had an inside diameter of 6 in. with a total open area of slots of 10.5 sq in., as compared with the 27 sq in. per lin ft of screen of wells H-151A and FC-105.

The lengths and depths of the screens for the various test wells in relation to the depth and stratification of the sand aquifer in which the wells were installed are shown adjacent to the well log in Figs. 2 and 3. The locations at which well-flow-meter readings were taken and their relationship to the stratification of the sand aquifer are also shown in Figs. 2 and 3.

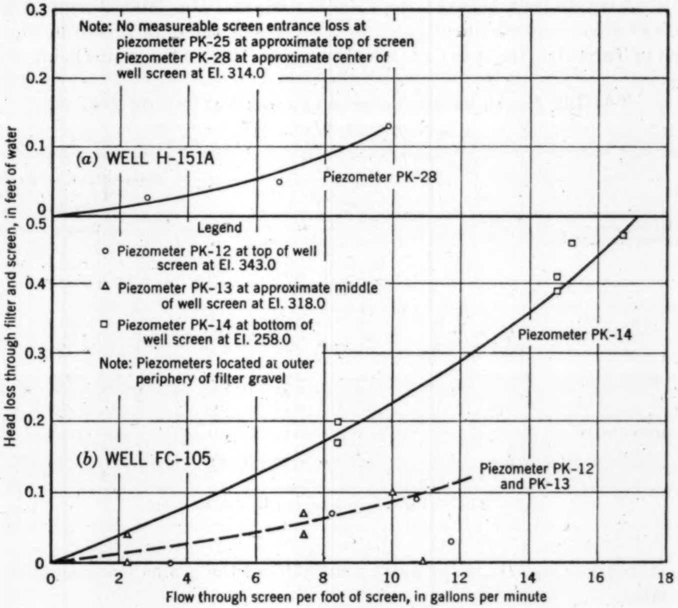


FIG. 6.—HEAD LOSS THROUGH FILTER AND WELL SCREEN

The permeability coefficient of individual sand strata, obtained from the pumping tests and well-meter readings and computed by Eq. 1, is plotted as a bar graph to the right of the well log in Figs. 2 and 3. The average permeability of the sand aquifer is also plotted as determined from the pumping tests, together with the coefficient of permeability as determined from laboratory tests on samples taken from the various sand strata.

ANALYSIS OF LABORATORY AND FIELD PUMPING TEST DATA

The mechanical analyses and laboratory permeability tests made on samples of sand from well-drilling effluent taken from wells H-151A and FC-105 were not sufficient to permit a comparison of the effective grain size, grain-size curves, or laboratory permeabilities with samples obtained from split-spoon sampling. Even if more laboratory tests had been made, comparisons probably would not have been valid because of the high degree of stratification and variation of grain size in the sand strata at the well sites.

---

[7] "Control of Underseepage by Relief Wells, Trotters, Miss.," *Technical Memorandum 3-341*, Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss., April, 1952.

At well OR-1 the variation in grain-size characteristics within any specific sand stratum was much less than at the other wells. In fact, the sands within the same general stratum were not only quite uniform with depth but also horizontally between borings at this well, which were 50 ft apart (Figs. 3 and 4(*c*)). A comparison between the values of D_{10} and D_{85} (laboratory permeabilities and field permeabilities) based on averages of the data shown in Fig. 3, is given in Table 4. D_{10} is the effective grain size (10% of the grains being smaller than this size), and D_{85} is the grain size (85% of the grains being smaller than this size).

TABLE 4.—COMPARISON OF AVERAGE VALUES OF D_{10}, D_{85}, AND k_L AT WELL OR-1

Approximate stratum elevation	Boring	Sampler[a]	GRAIN SIZE			COEFFICIENT OF PERMEABILITY, IN 10^{-4} CM PER SEC		
			No. of samples	D_{10}, in millimeters	D_{85}, in millimeters	No. of samples	k_L-value, from laboratory	Average k_L-value, from field
0 to −70	LS-2	S	9	0.15	0.31	3	215	...
	L-8	S	12	0.15	0.31	4	240	560
	WB-1	B	17	0.16	0.32	6	180	...
−80 to −103	LS-2	S	4	0.25	0.54	2	720	...
	L-8	S	2	0.25	0.52	1	590	680
	WB-1	B	3	0.26	0.56	3	360	...
−103 to −118	LS-2	S	1	0.34	1.0	1	1,060	...
	L-8	S	0	...	...	0	...	680
	WB-1	B	2	0.35	2.1	1	780	...

[a] S = 3-in. Shelby-tube sampler and B = bailer sampler.

As indicated in Table 4 and Fig. 3, for the uniform sand strata at well OR-1, there was excellent agreement between both grain size (D_{10} and D_{85}) and permeability of sand samples, as determined from samples obtained with a 3-in. Shelby-tube sampler and a bailer. This agreement is attributed to the uniformity of the sand strata, the uniform grading of the sand, and the careful sampling of the material brought out in the bailer. However, although there was little difference in the laboratory permeabilities of the Shelby-tube and bailer samples, some of the sand grains smaller than the No. 100 sieve appear to have been lost from the bailer samples (Figs. 4(*c*) and 4(*f*)).

Wherever the grain size of the sand strata varies appreciably with depth or wherever the sands are more widely graded, samples obtained with either a piston- or flap-type bailer are not as representative as those obtained with a split-spoon or Shelby-tube sampler.

Although there was considerable variation in the permeability, as measured by the pumping tests within a given sand stratum of the same classification, a definite general relationship existed between the classification (or D_{10}) and k_H. As would be expected in the alluvial valley of the Mississippi River, permeability generally increased with depth at the test-well sites.

Although the permeability of the bottom strata in well H-151A that was computed from the field pumping tests was somewhat lower than that of the upper strata, the actual permeability of the bottom strata is probably as high as that of the strata immediately above it. A possible explanation for the apparent lower permeability is that the well screen did not completely penetrate the bottom strata for this well, and, thus, there was less flow into this part of the well screen than there would have been if the screen had completely penetrated the bottom strata.

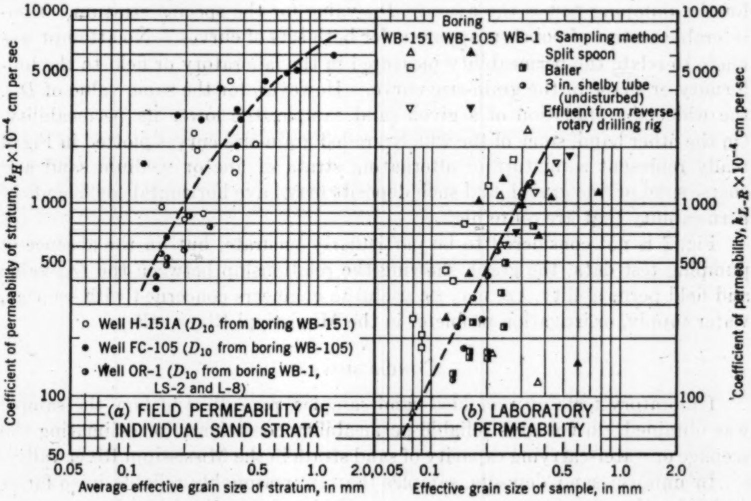


FIG. 7.—PERMEABILITY COEFFICIENT VERSUS GRAIN SIZE

Pumping tests on wells H-151A and FC-105, and on other wells installed from St. Louis south for 100 miles, show that the principal sand aquifer in the alluvial valley of the Mississippi River has a permeability (k_H) that ranges from about $1,000 \times 10^{-4}$ cm per sec to $3,000 \times 10^{-4}$ cm per sec. Similar tests in the alluvial valley at Commerce (Miss.)[8] and Trotters (Miss.)[7] (approximately 38 river miles and 70 river miles, respectively, south of Memphis (Tenn.)) showed that the principal sand aquifer at these sites had a k_H-value of $1,000 \times 10^{-4}$ cm per sec and $1,200 \times 10^{-4}$ cm per sec, respectively. Similar tests at the Old River (well OR-1, which is approximately 45 miles south of Natchez), indicated a k_H-value equaling about 600×10^{-4} cm per sec. This gradient of permeability down the Mississippi River valley conforms to the normally associated grading in an alluvial stream such as this river.

---

[8] "Investigation of Underseepage and Its Control—Lower Mississippi River Levees," *Technical Memorandum 3-424*, Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss., October, 1956.

The permeability coefficients determined in the laboratory on remolded samples are plotted at appropriate elevations on the bar graphs in Figs. 2 and 3. In general, little agreement was found between the permeabilities determined in the laboratory on remolded samples and those obtained from the field pumping tests. However, there is no reason why the permeabilities should agree, particularly wherever the aquifer is stratified and there are lenses of coarse sand and fine gravel. Generally, the field permeabilities for any given stratum exceeded the permeability determined normally in the laboratory by ← from two to four times.

Fig. 7 shows plots of effective grain size versus coefficients of permeability that were determined in the laboratory on remolded samples and from pumping tests on approximately 100% penetrating well screens. In Fig. 7 the D_{10}-value for the pumping test is the average D_{10}-value for the specific stratum. Considerable scatter of points is apparent for both sets of curves. No attempt was made to relate the permeability measured in the laboratory or field to the uniformity or shape of the grain-size curve. However, for the same value of D_{10}, the wider the gradation of a given sand sample, the lower its permeability. On the other hand, some of the widely graded grain-size curves plotted in Fig. 4 really represent a mixture of alternating strata of fine or medium sand and coarse sand or fine gravel, and such deposits may have horizontal coefficients of permeability that are quite high.

Fig. 7 is not considered to be particularly accurate, but, in the absence of pumping test data, the graph showing the relationship between the D_{10}-value and field permeability, k_H, may be useful to engineers concerned with seepage, water supply, or irrigation problems in the Mississippi River valley.

CONCLUSIONS

Laboratory tests on remolded sand samples, regardless of how the sample was obtained, do not give reliable permeability coefficients for estimating the ← seepage or water-carrying capacity of sand strata in the Mississippi River valley.

In uniform sand deposits, samples that are reasonably reliable in so far as grain-size characteristics are concerned, can be obtained with either a Shelby tube, a split-spoon sampler, a bailer, or even from the effluent of a reverse rotary well rig, provided that the sampling is properly accomplished. However, some sand grains that are smaller than a No. 100 sieve may be lost in bailer sampling or in the effluent from a reverse rotary well rig. For stratified deposits or wherever the sands are more widely graded, a Shelby-tube or split-spoon sampler should be used.

The most accurate procedure for determining the permeability of individual strata of a sand aquifer is to make a pumping test of a well with the screen penetrating all the sand strata, and to measure the flow with a well-flow meter at strata changes within the screen. Such change may be determined by a nearby boring in advance of the test, or by a log as the hole for the well is advanced.

The permeability of sand strata in the Mississippi River valley may be estimated reasonably accurately from the relationship between values of D_{10} and k_H given in Fig. 7.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2946

A FLOW CONTROLLER FOR OPEN OR CLOSED CONDUITS

By Victor L. Streeter,[1] M. ASCE

Synopsis

A general principle of flow control, utilizing a nonlinear resistance, is developed that is applicable to both open and closed conduits. The controller is infinitely adjustable over its design range and holds the flow close to the predetermined discharge for its head range. It is readily converted into a flow meter with a linear head-discharge curve and adjustable range and sensitivity. Experimental work is presented to confirm the theory.

Introduction

The combination of a disk moving within a profiled throat and a nonlinear resistance to support the disk against the pressure drop results in a principle of flow control that is infinitely variable and theoretically exact (that is, with no change in discharge or drag coefficients). By reversing the throat section and measuring the head across the disk, the device is converted into a flow meter. The underlying principles of single-orifice flow control are outlined herein before the specific relationships and design data are developed. A model has been constructed and tested in the hydraulic laboratory of the University of Michigan at Ann Arbor. The results are presented for both the flow controller and the flow meter.

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically, for convenience of reference, in the Appendix.

Concepts of Single-Orifice Flow Control

An orifice flow controller must act to hold the discharge, Q, constant in the equation,

$$Q = C_q A \sqrt{2 g h} \qquad (1)$$

Note.—Published, essentially as printed here, in August, 1956, in the Journal of the Hydraulics Division, as *Proceedings Paper 1037*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Prof. of Hydraulics, Civ. Eng. Dept., Univ. of Michigan, Ann Arbor, Mich.

by changing the orifice area, A, as the head, h, varies over its design range. The term, C_q, is the discharge coefficient and g denotes the acceleration caused by gravity. Assuming the discharge coefficient to be constant, the area must vary inversely as the square root of the head. For a control that is meant to operate at a single discharge only, this may be accomplished by use of a linear spring assembly such as that shown in Fig. 1. A head across the device displaces the disk, thereby altering the net area. Therefore, the throat must be profiled so that the discharge is held constant.

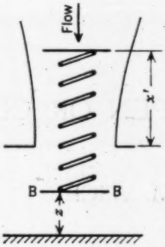


FIG. 1.—LINEAR SPRING FLOW CONTROLLER

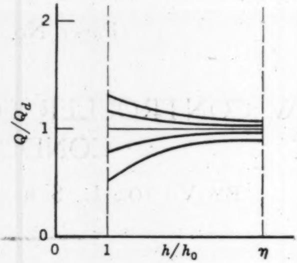


FIG. 2.—HEAD-DISCHARGE CURVES FOR FIG. 1

If one attempts to make a variable flow controller from this device by varying the position, z, of the spring base, BB (Fig. 1), the dimensionless head-discharge curves appear as shown in Fig. 2—Q_d being the design discharge; h_0, the minimum design head; and η, the ratio of maximum design head to minimum design head.

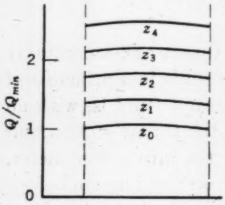


FIG. 3.—HEAD-DISCHARGE CURVES

By selecting another approach, although still retaining a linear spring, the throat profile may be constructed so that the discharge is the same at minimum and maximum heads for any setting, z, within its size range. However, the discharge at intermediate heads varies from that at the end points, as indicated in Fig. 3. In order to obtain the area required as a function of x' (Fig. 1), one can let Y be the displacement of the disk for head change from h_0 to $h_0\,\eta$—that is, from minimum to maximum design head. Therefore, the area must change from A_1 to A_2, as given by

$$A_1 \sqrt{h_0} = A_2 \sqrt{h_0\,\eta} \qquad (2)$$

which must hold for any setting, z, of the spring base, BB. Because the area of orifice opening is a function of x' only,

$$f(x') \sqrt{h_0} = f(x' - Y) \sqrt{h_0\,\eta} \qquad (3)$$

Because this relationship must hold for any initial value of x', a law of variation of A with x' of the following form is indicated:

$$A = A_0\, e^{\alpha x'} \qquad (4)$$

in which A_0 and α are undetermined constants, and e is the base of natural logarithms. Substituting Eq. 4 into Eq. 3 results in

$$A_0\, e^{\alpha x'} \sqrt{h_0} = A_0\, e^{\alpha (x' - Y)} \sqrt{h_0\, \eta} \ldots\ldots\ldots\ldots\ldots\ldots (5a)$$

or

$$\alpha = \frac{\ln \eta}{2\,Y} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5b)$$

The variation of A with x' then becomes

$$A = A_0\, e^{(x' \ln \eta)/(2\,Y)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which A_0 is the area of the opening between the disk and throat when $x' = 0$ —that is, $A_0 = A_{\min}$.

In order to improve the accuracy of this control for the same head and discharge range, two linear springs may be introduced into the assembly, as shown in Fig. 4. The first spring is altered so that the disk is displaced $Y/2$ by the first spring only as the head varies from h_0 to $h_0 \sqrt{\eta}$, and is displaced the remaining distance $Y/2$ as both springs act with the head changing from $h_0 \sqrt{\eta}$ to $h_0\, \eta$. This arrangement results in a dimensionless head-discharge curve, Fig. 5, in which the control is accurate for three heads and the maximum error has been reduced.

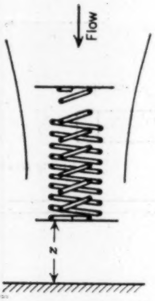


FIG. 4.—TWO-SPRING CONTROLLER

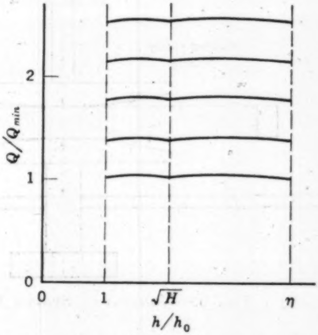


FIG. 5.—HEAD-DISCHARGE CURVES FOR FIG. 4

By taking n linear springs, the first acting over the distance, Y/n, the first and second over Y/n to $2\,Y/n$, and the first three over $2\,Y/n$ to $3\,Y/n$, . . . , with the corresponding head ranges, h_0 to $h_0\,\eta^{1/n}$, $h_0\,\eta^{1/n}$ to $h_0\,\eta^{2/n}$, and $h_0\,\eta^{2/n}$ to $h_0\,\eta^{3/n}$, . . . , the discharge is maintained the same at $n + 1$ heads, with the maximum variation in discharge decreasing as n increases. By increasing n indefinitely, a theoretically exact flow controller is obtained, and the spring system becomes nonlinear.

BASIC EQUATIONS FOR NONLINEAR RESISTANCE FLOW CONTROL

The basic equations for the flow controller have been derived by the writer in a previous paper.[2] One form of the nonlinear resistance flow control is shown in Fig. 6. The spring element is of the restrained-tip cantilever type, in which the two leaf springs are forced together by a profiled backing that shortens their effective length. The spring element is a distance $y = Y$ apart at minimum design head, and $y = 0$ for maximum design head. By rotating the milled knob the discharge setting, z, can be varied. From the geometry of Fig. 6 it is evident that

---

[2] "A Quick Response Variable Flow Control Device," by V. L. Streeter, Instrument Soc. of America, Pittsburgh, Pa., Vol. 2, February, 1955, pp. 48–51.

$$x' = y + z \qquad (7)$$

in which x' gives the disk position (the area of opening between the disk and throat being a function of x' only), and y is a function of the pressure drop across the disk only—that is, it is a function of h only for a given fluid.

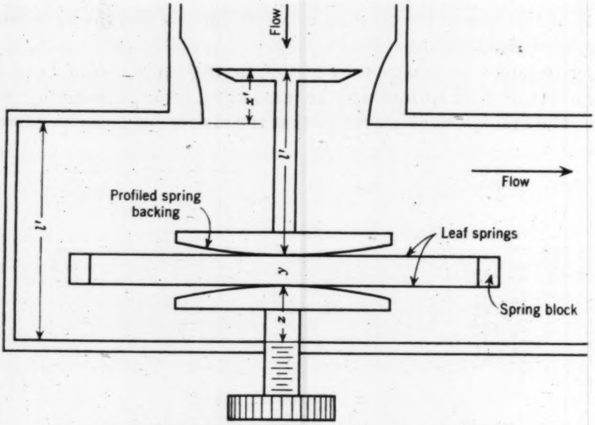


Fig. 6.—Nonlinear Spring Flow Controller

For any setting, z,

$$A\sqrt{h} = (A + \Delta A)\sqrt{h + \Delta h} \qquad (8)$$

or, because $A = f(x')$,

$$\sqrt{h}\, f(x') = \sqrt{h + \Delta h}\, f(x' + \Delta y) \qquad (9)$$

as a change in Δh causes a change in Δy and $\Delta x' = \Delta y$ for constant z. Expressing the area of opening, A, as an exponential function of x', as in Eq. 4, results in

$$f(x') = A = A_{\min}\, e^{\alpha x'} \qquad (10)$$

in which α is, as yet, undetermined. Substituting Eq. 10 into Eq. 9 and solving for α leads to

$$-\alpha = \frac{\ln \sqrt{1 + \dfrac{\Delta h}{h}}}{\Delta y} \qquad (11)$$

Expanding the logarithmic term into a series and taking the limit as Δy approaches zero,

$$-\alpha = \lim_{\Delta y \to 0} \tfrac{1}{2}\left(\frac{1}{h}\,\frac{\Delta h}{\Delta y} - \frac{1}{2\,h^2}\,\frac{\Delta h^2}{\Delta y} + \cdots\right) \qquad (12a)$$

or

$$-\alpha = \frac{1}{2\,h}\frac{dh}{dy} \qquad (12b)$$

Because α is a function of x' only and y is a function of h only, Eq. 12*b* shows a function of x' that is equal to a function of y'. This is impossible because of Eq. 7. Therefore, both sides of Eq. 12*b* must be constant. Integrating Eq. 12*b* leads to

$$-2\,\alpha\,y = \ln h + c \qquad (13)$$

Using the end conditions, $h = h_0$, $y = Y$, and $h = h_0\,\eta$, $y = 0$, both α and c are determined as

$$\alpha = \frac{\ln \eta}{2\,Y} \qquad (14a)$$

and

$$y = \frac{Y}{\ln \eta} \ln \frac{h_0\,\eta}{h} \qquad (14b)$$

Eq. 10 then becomes

$$A = A_{\min}\, e^{(x' \ln \eta)/(2Y)} \qquad (15)$$

Substituting into Eq. 1, using h from Eq. 14*b*, A from Eq. 15, and x' from Eq. 7,

$$Q = C_q\, A_{\min} \sqrt{2\,g\,h_0\,\eta}\; e^{(z \ln \eta)/(2Y)} \qquad (16)$$

which yields the discharge for the given arbitrary setting, z, and shows that Q is independent of h.

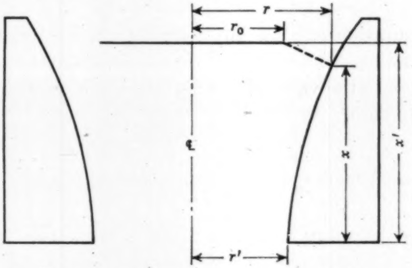


FIG. 7.—NOTATION FOR DISK AND THROAT

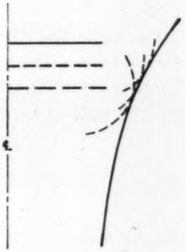


FIG. 8.—ENVELOPE SOLUTION FOR THROAT PROFILE

Eqs. 14*b*, 15, and 16 determine the characteristics of the flow controller. The nonlinear resistance must follow the head-deflection relationship of Eq. 14*b*, and the throat profile must be made so that the net area of the opening is given by Eq. 15. By providing that the base of the resistance is adjusted axially, the quantity, z, may be changed and variable flow control results, as given by Eq. 16.

EQUATION FOR THROAT PROFILE

If an axially symmetric throat section is desired, the throat profile, as given by the cylindrical coordinates (r, x) in Fig. 7, must yield the net area for any value of x' required by Eq. 15. Using Pappus' theorem,[3] this area is

$$A = 2\pi \frac{r_0 + r}{2} \sqrt{(r - r_0)^2 + (x' - x)^2} = A_{\min}\, e^{(x' \ln \eta)/(2\,Y)} \quad \text{. . . (17a)}$$

or

$$[(r')^2 - r_0^2]\, e^{(x' \ln \eta)/(2\,Y)} = (r_0 + r) \sqrt{(r - r_0)^2 + (x' - x)^2} \quad \text{. . . (17b)}$$

in which r_0 is the disk radius and r' is the minimum throat area. For a given value of x', Eq. 17*b* yields a curve of r as a function of x. Drawing segments of these curves for various values of x' (Fig. 8) shows that their envelope yields the desired throat profile. The partial differential of Eq. 17*b* with respect to x' is taken and solved simultaneously with Eq. 17*b* to yield the cylindrical coordinates of the throat profile. Dividing this partial differential of Eq. 17*b* with respect to x' into Eq. 17*b* yields

$$x' - x = \frac{Y}{\ln \eta} \left\{ 1 - \sqrt{1 - \left[\frac{(r - r_0) \ln \eta}{Y} \right]^2} \right\} \quad \text{. . . . . (18)}$$

Substituting Eq. 18 into Eq. 17*b* yields

$$x = \frac{2\,Y}{\ln \eta} \left\{ \ln \left[\frac{r_0 + r}{(r')^2 - r_0^2} \sqrt{2}\, \frac{Y}{\ln \eta} \sqrt{1 - \sqrt{1 - \left(\frac{(r - r_0) \ln \eta}{Y} \right)^2}} \right] - \tfrac{1}{2} \left[1 - \sqrt{1 - \left(\frac{(r - r_0) \ln \eta}{Y} \right)^2} \right] \right\} \quad \text{. . (19)}$$

Eq. 19 permits the direct computation of x for any value of r in terms of the design quantities, Y, η, r', and r_0.

There is an inherent limitation to the value of r in Eq. 19 that is obtained by setting the radical term equal to zero,

$$r_{\max} = r_0 + \frac{Y}{\ln \eta} \quad \text{. . . . . (20)}$$

For larger values of r the envelope solution breaks down. From Eq. 19 for $r = r_{\max}$,

$$x_{\max} = \frac{2\,Y}{\ln \eta} \left\{ \ln \left[\frac{2\, r_0 + \frac{Y}{\ln \eta}}{(r')^2 - r_0^2} \sqrt{2}\, \frac{Y}{\ln \eta} \right] - \tfrac{1}{2} \right\} \quad \text{. . . . . (21)}$$

The corresponding value of x' from Eqs. 18 and 21 is

$$x_{\max}' = \frac{2\,Y}{\ln \eta} \ln \left[\frac{2\, r_0 + \frac{Y}{\ln \eta}}{(r')^2 - r_0^2} \sqrt{2}\, \frac{Y}{\ln \eta} \right] \quad \text{. . . . . (22)}$$

---

[3] "Analytical Mechanics for Engineers," by F. B. Seely and N. E. Ensign, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., 1941, p. 157.

DISCHARGE LIMITATIONS

For the flow controller to operate, the disk must stay within the range, $0 \leqslant x' \leqslant x_{\max}'$. For control over the entire head range, this restricts z to the range $0 \leqslant z \leqslant (x_{\max}' - Y)$. To find the maximum discharge over the whole head range,

$$z = x_{\max}' - Y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

is substituted into Eq. 16 using Eq. 22,

$$Q_{\max} = C_q \, 2\, \pi \sqrt{g\, h_0}\, \frac{Y}{\ln \eta} \left(2\, r_0 + \frac{Y}{\ln \eta} \right) \ldots\ldots\ldots\ldots (24)$$

by making use of the fact that

$$A_{\min} = \pi \left[(r')^2 - r_0^2 \right] \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

The minimum discharge is obtained from Eq. 16 by setting $z = 0$ and is

$$Q_{\min} = C_q\, A_{\min} \sqrt{2\, g\, h_0\, \eta} \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Eqs. 24 and 26 permit the three design quantities, r_0, r', and Y, to be selected for a given head and discharge range. It is interesting to note that the maximum and minimum discharges are not related. The minimum throat radius, r', does not appear in Eq. 24.

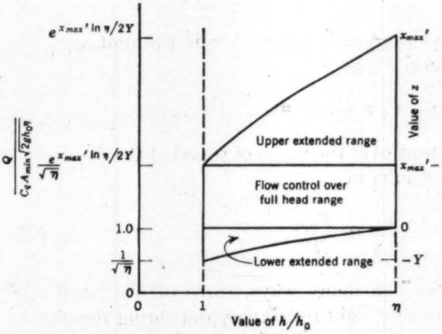


FIG. 9.—TOTAL FLOW-CONTROL RANGE

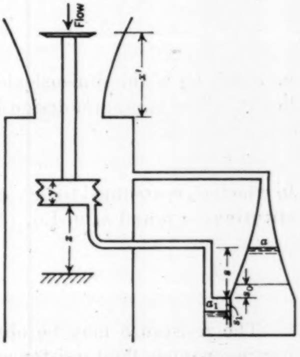


FIG. 10.—FLOW CONTROLLER WITH LIQUID SPRING

For flow control over partial head ranges, the limitation is that z varies from $-Y$ to 0 in the lower extended zone and, from $(x_{\max}' - Y)$ to $x_{\max}'$ in the upper extended zone, with the disk, x', remaining between 0 and $x_{\max}'$ at all times. The spring travel is limited to parts of the range, Y to 0, which restricts the head range to part of h_0 to $h_0\,\eta$.

For a value of z that is less than zero, the "threshold" value of y to maintain $x' = 0$ is

$$y = -z \qquad (27)$$

Substituting this value of z into Eq. 16 and using Eq. 14*b* for y leads to

$$Q = C_q\, A_{\min} \sqrt{2\, g\, h} \qquad (28)$$

Eq. 28 is the lower-limit line of the lower extended zone.

For a value of z that is greater than $(x_{\max}' - Y)$, and with the disk at its limiting position, $x' = x_{\max}'$,

$$z = x_{\max}' - y \qquad (29)$$

Again, using Eqs. 16 and 14*b*, the upper extended zone is limited by

$$Q = C_q\, A_{\min} \sqrt{2\, g\, h}\; e^{(x_{\max}' \ln \eta)/(2\, Y)} \qquad (30)$$

The total flow-control range is shown in Fig. 9 in dimensionless form, with the z-range indicated on the right.

DESIGN OF NONLINEAR RESISTANCE

The nonlinear resistance to displacement of the disk, as required by Eq. 14*b*, may be expressed in terms of force exerted on the disk. This force, F, is

$$F = C_d\, \pi\, r_0^2\, w\, h \qquad (31)$$

in which C_d is the dimensionless drag coefficient and w is the unit weight of liquid. The minimum design force, F_0, is

$$F_0 = C_d\, \pi\, r_0^2\, w\, h_0 \qquad (32)$$

in which C_d is assumed to be constant over the range of travel of the disk. Substituting for h and h_0 in Eq. 14*b* results in

$$y = \frac{Y}{\ln \eta} \ln \frac{F_0\, \eta}{F} \qquad (33)$$

The resistance may be obtained in many ways, three of which are cited herein—namely fluid resistance, dead-weight resistance, and spring resistance.

Fluid Resistance.—Referring to Fig. 10, a bellows with an area, A_b, forces manometer liquid into a manometer leg of varying cross section. The area, a, of the manometer is determined as a function of s. The necessary equations are Eq. 33,

$$-A_b\, dy = a_1\, ds_1 = a\, ds \qquad (34a)$$

and

$$F = A_b\, (w_m - w)(s + s_1) \qquad (34b)$$

in which a_1 is the constant cross-sectional area and w_m is the unit weight of manometer fluid. The terms, F and y, may be eliminated from Eqs. 33 and 34 to yield a and s in terms of s_1,

$$a = \frac{a_1}{\dfrac{a_1 \ln \eta}{A_b Y} s_0 e^{\dfrac{s_1 a_1 \ln \eta}{A_b Y}} - 1} \qquad (35a)$$

and

$$s = s_0 e^{(s_1 a_1 \ln \eta)/(A_b Y)} - s_1 \qquad (35b)$$

in which s_0 is the value of s for $s_1 = 0$ and $F = F_0$. The discharge setting is altered by changing the value of z. For vertical installations the weight of moving parts would change the setting, z, for a given discharge, but would not otherwise disturb the proper functioning of the controller.

Dead-Weight Resistance.—A weight suspended on a cable or metal tape in such a fashion that it is lifted out by a cam can be arranged easily in order to yield the nonlinear force-displacement relationship required by Eq. 33. Fig. 11 shows one method, with the cam rotating about a transverse axis. In order that the shape of the cam can be determined, reference is made to Fig. 12, in which the cam rotates through the angle, α, from 0, when $F = F_0$, to α_0 when $F = F_0 \eta$. The moment of the weight, W, about the shaft for any angle α is

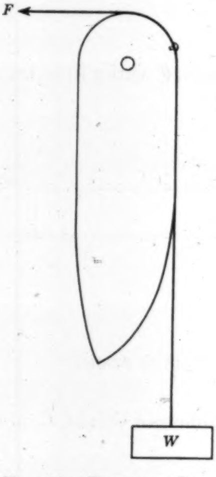


FIG. 11.—PROFILED CAM

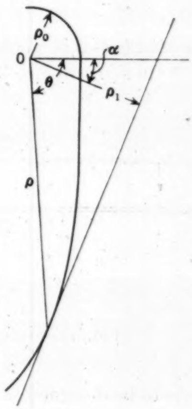


FIG. 12.—NOTATION FOR CAM EQUATION

$$W\rho \cos(\theta - \alpha) = F\rho_0 = F_0 \rho_0 \eta e^{-y(\ln \eta)/Y} \qquad (36)$$

With ρ_0 constant,

$$\frac{\alpha}{\alpha_0} = 1 - \frac{y}{Y} \qquad (37a)$$

because

$$\rho_0 \alpha = Y - y \qquad (37b)$$

Letting $W = F_0$, Eq. 36 reduces to

$$\rho \cos (\theta - \alpha) = \rho_0 \, e^{(\alpha \ln \eta)/(\alpha_0)} \qquad (38)$$

For any angle α, Eq. 38 is a straight line in polar coordinates. The envelope of the family of straight lines is the equation for the cam profile. Taking the partial differential of Eq. 38 with respect to α and solving simultaneously with Eq. 38 to eliminate α yields

$$\rho = \rho_0 \sqrt{1 + \left(\frac{\ln \eta}{\alpha_0}\right)^2} \, e^{-[(\ln \eta)/\alpha_0 \tan]^{-1}[(\ln \eta)/\alpha_0]} \, e^{(\theta \ln \eta)/(\alpha_0)} \qquad (39)$$

which is a logarithmic spiral. By dividing the partial differential of Eq. 38 with respect to α by Eq. 38 yields

$$\tan (\theta - \alpha) = \frac{\ln \eta}{\alpha_0} \qquad (40)$$

which shows that $\theta - \alpha$ is a constant. The angle, θ, varies from $\tan^{-1} (\ln \eta/\alpha_0)$ to $\alpha_0 + \tan^{-1} (\ln \eta/\alpha_0)$ in Eq. 39.

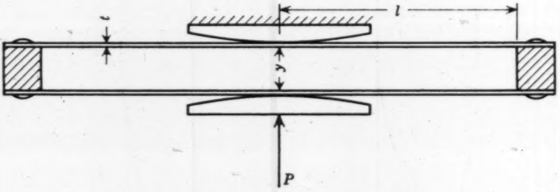


FIG. 13.—RESTRAINED-TIP CANTILEVER SPRING

If a cam is to be designed to support the submerged weight, G, of the moving parts of the controller,

$$F_0 \, \rho_0 \, e^{(\alpha \ln \eta)/(\alpha_0)} + G \rho_0 = W \rho \cos (\theta - \alpha) \qquad (41)$$

Taking the partial differential with respect to α and dividing by Eq. 41 leads to

$$\alpha_0 \tan (\theta - \alpha) = \frac{\ln \eta}{1 + \dfrac{G}{F_0} e^{-(\alpha/\alpha_0) \ln \eta}} \qquad (42)$$

For a series of values of α between 0 and α_0, corresponding values of θ are found. Using these values the corresponding values of ρ are found in Eq. 41 which yield polar coordinates of the cam profile.

Spring Resistance.—A restrained-tip cantilever spring may be designed in order to provide the required resistance law by following a method presented by

S. P. Clurman.[4] Two identical, flat springs of constant width and thickness are placed as shown in Fig. 13. As the force, P, is increased, the springs are forced against the profiled backings, thereby reducing their effective lengths and increasing their stiffness. The springs are fastened together at each end by a spacer block, which prevents rotation. Because there are actually four springs, each one takes one-half of the load and is deflected through one-half of the total displacement. The equation of the backing profile is determined as follows:

Letting $G = k\,F_0$ be the submerged weight of moving parts of the controller for vertical stem applications ($G = 0$ for horizontal applications),

$$P = F - G = F_0\left[\eta\, e^{(-y \ln \eta)/Y} - k\right] \qquad (43)$$

The sign of G has been taken for the case in which the weight of moving parts acts in the opposite direction to the pressure force on the disk. With δ as the deflection of one spring from the position for minimum design head,

$$\delta = \tfrac{1}{2}(Y - y) = \frac{Y}{2 \ln \eta} \ln\left(\frac{2\,P'}{F_0} + k\right) \qquad (44)$$

in which P' is the load on a single spring ($P\,/\,2$). For the minimum design load the spring is cantilevered from its full length, l, with the backing touching at $x_0 = 0$ (Fig. 14). The variables, x_0 and y_0, are coordinates of the backing profile. With no load on the spring it will be deflected a distance, D, from the ($\delta = 0$)-position, so that

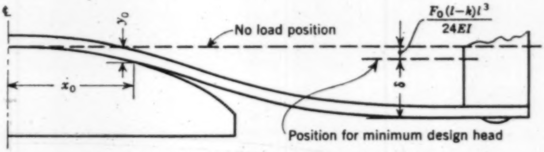


FIG. 14.—NOTATION FOR CANTILEVER SPRING

$$D = -\,\frac{F_0(1 - k)\,l^3}{24\,E\,I} \qquad (45)$$

in which E is the modulus of elasticity and I represents the section moment of inertia about the neutral axis.

With x_0 and y_0 being the coordinates of the last point of contact between spring and backing for a given active load, P', the deflection from the 0 load position is

$$\delta + \frac{F_0\,(1 - k)\,l^3}{24\,E\,I} = \frac{(l - x_0)^2}{6}\,\frac{d^2\,y_0}{(dx_0^2)} + \tfrac{2}{3}\,(l - x_0)\,\frac{dy_0}{dx_0} + y_0 \qquad (46)$$

---

[4] "The Design of Nonlinear Leaf Springs," by S. P. Clurman, *Transactions*, A. S. M. E., Vol. 73, February, 1951, pp. 155–161.

The right side of Eq. 46 is obtained by the consideration of the deflection of cantilever beams. In Fig. 15, by superposing the two simple loadings,

$$\frac{dy_0}{dx_0} = \frac{P'\,(l - x_0)^2}{2\,E\,I} - \frac{M\,(l - x_0)}{E\,I} \qquad (47a)$$

and

$$D = \frac{P'\,(l - x_0)^3}{3\,E\,I} - \frac{M\,(l - x_0)^2}{2\,E\,I} \qquad (47b)$$

Replacing M by $E\,I\,\frac{d^2y_0}{dx_0^2}$ and eliminating P' in Eqs. 47 leads to

$$D = \tfrac{1}{6}\,(l - x_0)^2\,\frac{d^2y_0}{dx_0^2} + \tfrac{2}{3}\,(l - x_0)\,\frac{dy_0}{dx_0} \qquad (48)$$

which, with the addition of y_0, is the right side of Eq. 46.

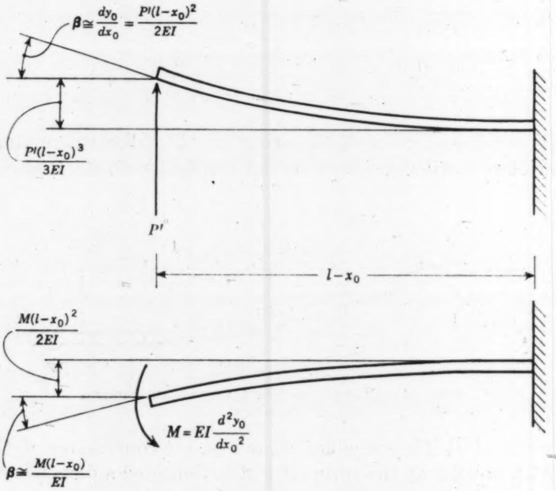

$\beta \cong \frac{dy_0}{dx_0} = \frac{P'(l-x_0)^2}{2EI}$
$\frac{P'(l-x_0)^3}{3EI}$
P'
$l-x_0$
$\frac{M(l-x_0)^2}{2EI}$
$M = EI\frac{d^2y_0}{dx_0^2}$
$\beta \cong \frac{M(l-x_0)}{EI}$

FIG. 15.—DEFLECTIONS OF CANTILEVER SPRING

The terms, δ and F_0, may be expressed in terms of the stiffness of the spring. From Eq. 44,

$$P' = \frac{F_0}{2}\,(e^{(2\,\delta\,\ln\,\eta)/Y} - k) \qquad (49)$$

Then

$$\frac{dP'}{d\delta} = \frac{F_0 \ln \eta}{Y}\,e^{(2\,\delta\,\ln\,\eta)/Y} \qquad (50)$$

which is the spring stiffness. For $\delta = 0$,

$$\left.\frac{dP'}{d\delta}\right|_{\delta=0} = \frac{F_0 \ln \eta}{Y} = K_0 = \frac{12\,E\,I}{l^3} \qquad (51)$$

The stiffness of a spring varies inversely as the cube of its length. Hence,

$$\frac{dP'}{d\delta} = \frac{l^3 K_0}{(l - x_0)^3} \ldots\ldots (52)$$

Solving Eq. 50 for δ and using Eqs. 51 and 52 results in

$$\delta = \frac{Y}{2 \ln \eta} \ln \left(\frac{Y}{F_0 \ln \eta} \frac{dP'}{d\delta} \right) = \frac{3\,Y}{2 \ln \eta} \ln \frac{l}{l - x_0} \ldots\ldots (53)$$

From Eq. 51,

$$\frac{F_0\,(1 - k)}{24\,E\,I} = (1 - k) \frac{Y}{2 \ln \eta} \ldots\ldots (54)$$

Substituting Eqs. 53 and 54 into Eq. 46 leads to

$$\frac{3\,Y}{2 \ln \eta} \ln \frac{l}{l - x_0} + (1 - k) \frac{Y}{2 \ln \eta} = \tfrac{1}{6}\,(l - x_0) \frac{d^2 y_0}{d x_0^2}$$

$$+ \tfrac{2}{3}\,(l - x_0) \frac{d y_0}{d x_0} + y_0 \ldots (55)$$

Inserting the dimensionless quantities, $\bar{x} = x_0/l$ and $\bar{y} = y_0/Y$, into Eq. 55 results in

$$\frac{3\,(1 - k)}{\ln \eta} - \frac{9}{\ln \eta} \ln\,(1 - \bar{x}) = (1 - \bar{x})^2 \frac{d^2 \bar{y}}{d\bar{x}^2} + 4\,(1 - \bar{x}) \frac{d\bar{y}}{d\bar{x}} + 6\,\bar{y} \ldots (56)$$

Eq. 56 is converted into a linear equation with constant coefficients by the substitution of $\zeta = \ln\,(1 - \bar{x})$. Then

$$\frac{d\bar{y}}{d\bar{x}} = -\,e^{-\zeta} \frac{d\bar{y}}{d\zeta} \quad \frac{d^2\bar{y}}{d\bar{x}^2} = e^{-2\zeta} \left(\frac{d^2\bar{y}}{d\zeta^2} - \frac{d\bar{y}}{d\zeta} \right) \ldots\ldots (57)$$

and Eq. 56 becomes

$$\frac{d^2\bar{y}}{d\zeta^2} - 5 \frac{d\bar{y}}{d\zeta} + 6\,\bar{y} = \frac{3}{\ln \eta}\,(1 - k - 3\,\zeta) \ldots\ldots (58)$$

The solution of Eq. 58 for the conditions, $\bar{y} = 0$, $\zeta = 0$, and $d\bar{y}/d\zeta = 0$, is

$$2\,\bar{y} \ln \eta = -\,k - \tfrac{3}{2} - 3\,\zeta + 3\,(\tfrac{1}{2} + k)\,e^{2\zeta} - 2\,k\,e^{3\zeta} \ldots\ldots (59)$$

Substituting back for $\bar{x}$ leads to

$$2\,\bar{y} \ln \eta = -\,k - \tfrac{3}{2} - 3 \ln\,(1 - \bar{x}) + 3\,(\tfrac{1}{2} + k)(1 - \bar{x})^2 - 2\,k\,(1 - \bar{x})^3 \ldots (60)$$

Eq. 60 is the dimensionless equation for the spring backing profile.

From Eq. 53 the maximum value of $\bar{x} = x_0 / l$ for $\delta = Y / 2$ is

$$\bar{x}_{\max} = 1 - \eta^{-\frac{1}{3}} \ldots\ldots (61)$$

The cross section of the spring must satisfy Eq. 51.

Consideration of stresses in the spring indicates the maximum moment at the spring extremity. From Fig. 16 the moment, M_1, is given by

$$M_1 = P' l (1 - \bar{x}) - M_2 \dots\dots\dots\dots (62)$$

The moment, M_2, may be found from Eq. 47*a* by obtaining dy_0/dx_0 from Eq. 60,

$$\frac{dy_0}{dx_0} = \frac{Y}{l}\frac{d\bar{y}}{d\bar{x}} = \frac{Y}{2\,l \ln \eta}\left[\frac{3}{1-\bar{x}} - 3\,(1 + 2\,k)(1 - \bar{x}) + 6\,k\,(1 - \bar{x})^2\right] \dots (63)$$

and

$$\frac{M_2}{E\,I}\,l\,(1 - \bar{x}) = \frac{P'\,l^2}{2\,E\,I}(1 - \bar{x})^2 - \frac{Y}{2\,l \ln \eta}$$
$$\times\left[\frac{3}{1-\bar{x}} - 3\,(1 + 2\,k)(1 - \bar{x}) + 6\,k\,(1 - \bar{x})^2\right] \dots (64)$$

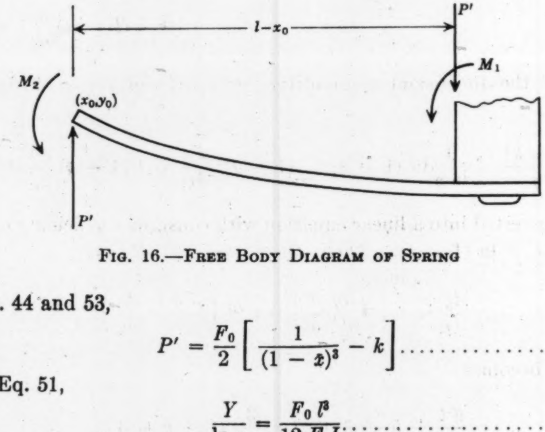


FIG. 16.—FREE BODY DIAGRAM OF SPRING

From Eqs. 44 and 53,

$$P' = \frac{F_0}{2}\left[\frac{1}{(1-\bar{x})^3} - k\right] \dots\dots\dots\dots (65)$$

and from Eq. 51,

$$\frac{Y}{\ln \eta} = \frac{F_0\,l^3}{12\,E\,I} \dots\dots\dots\dots (66)$$

Substituting Eqs. 65 and 66 into Eq. 64 and simplifying,

$$M_2 = \frac{F_0\,l}{8}\left[\frac{1}{(1-\bar{x})^2} + 1 - 2\,k\,(1 - \bar{x})\right] \dots\dots\dots\dots (67)$$

Replacing M_2 in Eq. 62 by the value from Eq. 67 leads to

$$M_1 = \frac{F_0\,l}{8}\left[\frac{3}{(1-\bar{x})^2} - 1 - 2\,k\,(1 - \bar{x})\right] \dots\dots\dots\dots (68)$$

The moment, M_1, is a maximum for $\bar{x} = 1 - \eta^{-\frac{1}{3}}$,

$$M_{1\max} = \frac{F_0\,l}{8}\,(3\,\eta^{\frac{2}{3}} - 1 - 2\,k\,\eta^{-\frac{1}{3}}) \dots\dots\dots\dots (69)$$

The maximum fiber stress, S_m, is

$$S_m = \frac{t\,M_{1\max}}{2\,I} = \tfrac{3}{4}\,\frac{F_0\,l}{b\,t^2}\,(3\,\eta^{\frac{2}{3}} - 1 - 2\,k\,\eta^{-\frac{1}{3}}) \qquad (70)$$

in which t is the spring thickness and b is the spring width. Eqs. 51 and 70

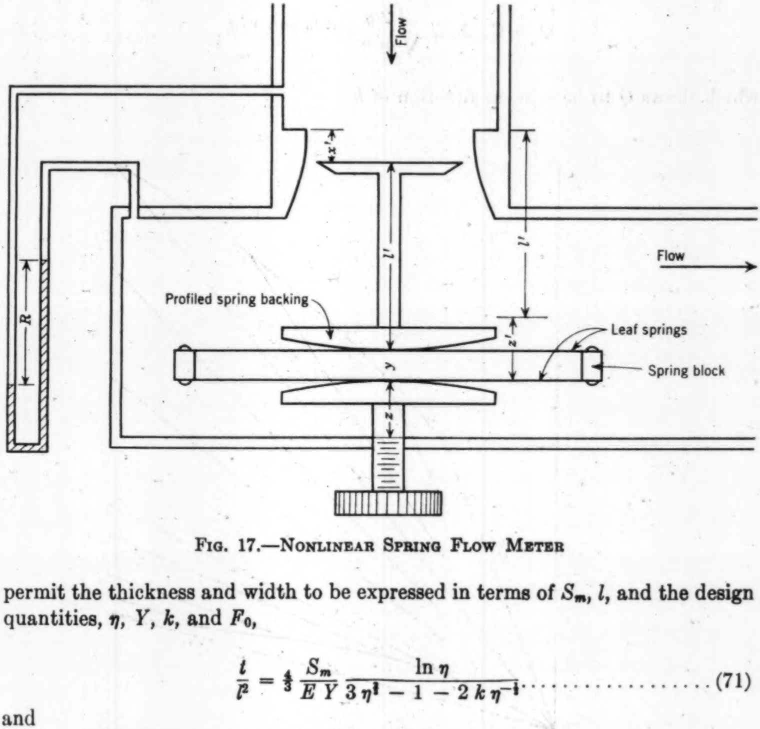


FIG. 17.—NONLINEAR SPRING FLOW METER

permit the thickness and width to be expressed in terms of S_m, l, and the design quantities, η, Y, k, and F_0,

$$\frac{t}{l^2} = \tfrac{4}{3}\,\frac{S_m}{E\,Y}\,\frac{\ln \eta}{3\,\eta^{\frac{2}{3}} - 1 - 2\,k\,\eta^{-\frac{1}{3}}} \qquad (71)$$

and

$$b\,l^3 = \frac{27}{64}\,\frac{F_0\,(E\,Y)^2}{S_m^{\,3}\,(\ln \eta)^2}\,(3\,\eta^{\frac{2}{3}} - 1 - 2\,k\,\eta^{-\frac{1}{3}})^3 \qquad (72)$$

The restrained-tip cantilever spring is relatively simple to construct and has a small hysteresis loss. The main objection to it, from the standpoint of a closed conduit controller, is its size.

FLOW METER WITH ADJUSTABLE SENSITIVITY AND RANGE

If the throat casting in the flow control unit is reversed, as it is in Fig. 17, and a manometer is connected to the two ends of the throat, the pressure drop across the disk, as given by R (the gage difference), is a linear function of the discharge. The relationship among the variables, x', y, z', is

$$x' + y = z' \qquad (73)$$

in which $z' + z$ is a constant. Substituting into Eq. 1, using Eqs. 14*b*, 15 and 73 to eliminate x', y, and A, leads to

$$Q = C_q A_{\min} \sqrt{\frac{2\,g}{h_0\,\eta}}\, e^{(z' \ln \eta)/(2\,Y)}\, h \qquad (74)$$

which shows Q to be a linear function of h.

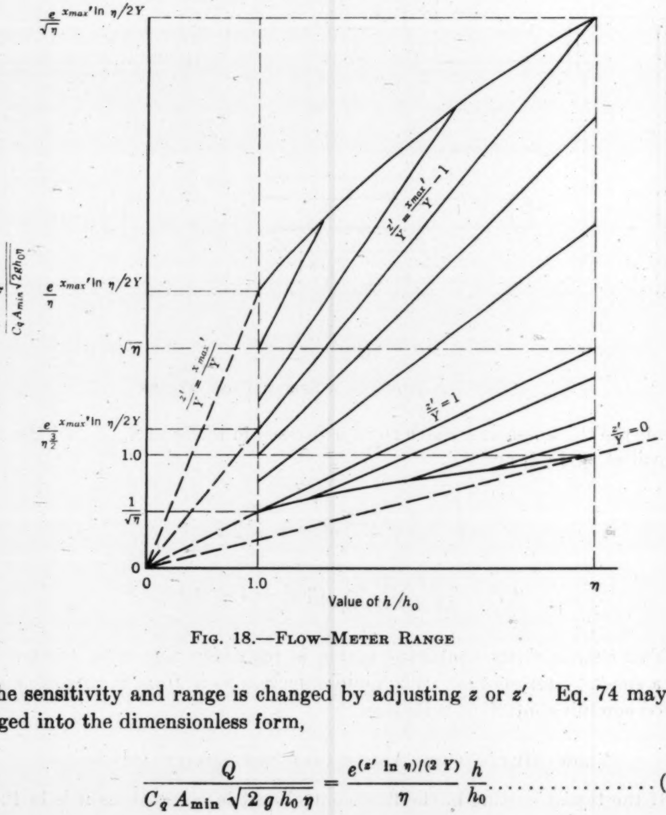


FIG. 18.—FLOW-METER RANGE

The sensitivity and range is changed by adjusting z or z'. Eq. 74 may be arranged into the dimensionless form,

$$\frac{Q}{C_q\,A_{\min}\,\sqrt{2\,g\,h_0\,\eta}} = \frac{e^{(z' \ln \eta)/(2\,Y)}}{\eta}\,\frac{h}{h_0} \qquad (75)$$

which is plotted in Fig. 18.

The upper threshold of the metering range is given by

$$\frac{Q}{C_q\, A_{\min}\sqrt{2\, g\, h_0\, \eta}} = \frac{e^{(z'_{\max}\ln \eta)/(2\, Y)}}{\eta}\sqrt{\frac{h}{h_0}} \qquad (76)$$

The lower threshold of the metering range is

$$\frac{Q}{C_q\, A_{\min}\sqrt{2\, g\, h_0\, \eta}} = \sqrt{\frac{h}{h_0\, \eta}} \qquad (77)$$

Large settings (*z*) result in sizable gage differences for a relatively small discharge range, whereas small settings permit metering over a much wider flow range. The sensitivity of the meter, as given by Q / h, is an exponential function of the setting, *z*.

EXPERIMENTAL PROGRAM

An experimental flow-control unit was designed to operate in a concrete flume in the floor of the hydraulic laboratory at the University of Michigan.

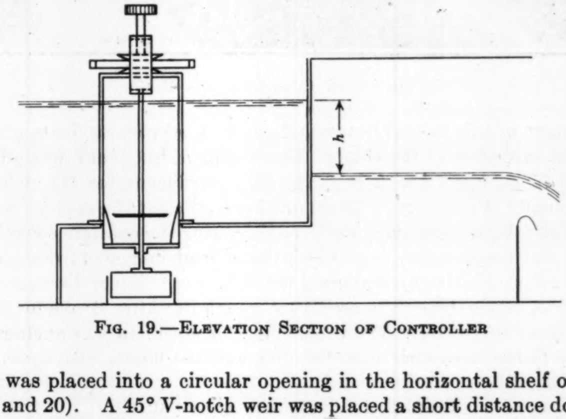


FIG. 19.—ELEVATION SECTION OF CONTROLLER

The unit was placed into a circular opening in the horizontal shelf of a barrier (Figs. 19 and 20). A 45° V-notch weir was placed a short distance downstream from the controller. Hook gages were mounted upstream and downstream from the barrier, so that the head across the controller as well as the discharge could be determined. The disk had a displacement of 0.75 in. as the head varied from a minimum of 2 in. to a maximum of 8 in. The disk diameter was 7.50 in.; the minimum throat diameter was designed to be 7.60 in.; and the minimum throat section was later measured as 7.605 in.

With the design quantities, $Y = 0.75$ in., $h_0 = 2$ in., $\eta = 4$, $r_0 = 3.75$ in., and $r' = 3.80$ in., cylindrical coordinates of the throat profile were computed from Eq. 19. The restrained-tip cantilever spring (Fig. 13) was used for the nonlinear resistance and was computed from Eqs. 60, 71, and 72 for $l = 3$ in. and $k = 0$—that is, not allowing for the weight of the disk assembly. The sub-

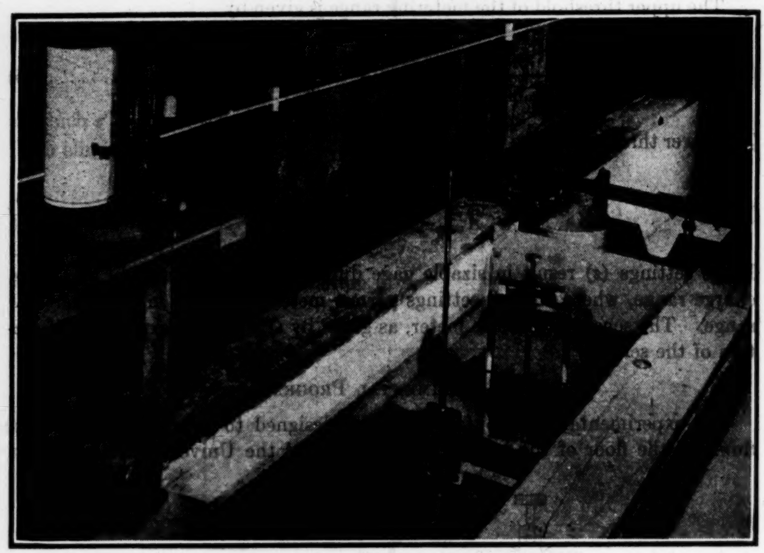


FIG. 20.—FLOW CONTROLLER IN PLACE

merged weight of this assembly was balanced by styrofoam floats placed on a downstream extension of the shaft. The degree of buoyancy was adjusted so that $y = 0.75$ in. when $h = 2$ in. The drag coefficient for the disk was assumed originally to be unity. Based on this coefficient the spring "bottomed" at $h = 6.5$ in. By adding extra width to the spring the resistance was increased so that the disk displacement was 0.75 in. for a head change of from 2 in. to 8 in.

The styrofoam floats also stabilized the disk for the larger discharges, which, without stabilization, tended to oscillate through its entire stroke for discharges greater than approximately 50 gal per min. A steel can was anchored to the floor of the flume concentric with the disk and the floats, with approximately

TABLE 1.—SUMMARY OF RESULTS

Number of turns on adjusting screw	(a) FLOW CONTROLLER			(b) FLOW METER	
	Head range, in inches	Discharge, in gallons per minute	Maximum divergence, in percentage	$m = \frac{Q}{h}$	Maximum divergence, in percentage
0	2.32 to 8.27	20.0	1.4	4.229	2.8
3	2.17 to 8.03	24.5	1.6	5.225	4.2
6	2.22 to 8.12	30.0	2.0	6.559	3.3
9	2.12 to 8.72	35.8	2.2	8.096	3.2
12	2.34 to 8.06	44.8	1.3	9.876	3.2
15	2.20 to 8.88	54.5	2.8	11.972	2.4
18	2.29 to 8.46	66.6	1.9	14.773	3.1
21	2.14 to 7.91	80.8	2.4	18.334	3.0
24	1.92 to 7.59	99.4	2.8	22.728	1.5
27	1.88 to 7.17	123.8	3.9	28.372	3.7

0.25 in. of radial clearance between the can and the floats. The resistance to forcing water into and out of the can provided adequate stabilization for all the experiments. Limitations in flume height restricted the range of heads for the larger discharges.

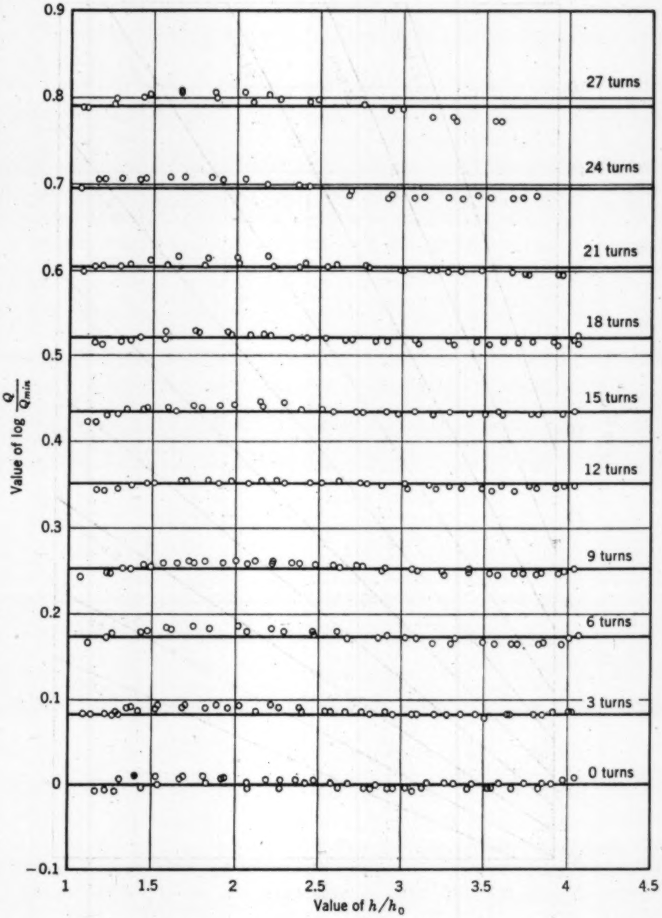


FIG. 21.—FLOW-CONTROLLER TEST DATA

Fins were placed in the throat section to guide the disk. They caused excessive friction and were cut back radially to avoid contact with the disk. A teflon bearing in a spider at the small end of the throat provided excellent guidance with a minimum of resistance.

Test results for the flow controller are shown in dimensionless form in Fig. 21, with a summary of results in Table 1(*a*).

The throat casting was inverted to convert the unit into a flow meter. The head across the disk was measured with hook gages and discharge by the weir.

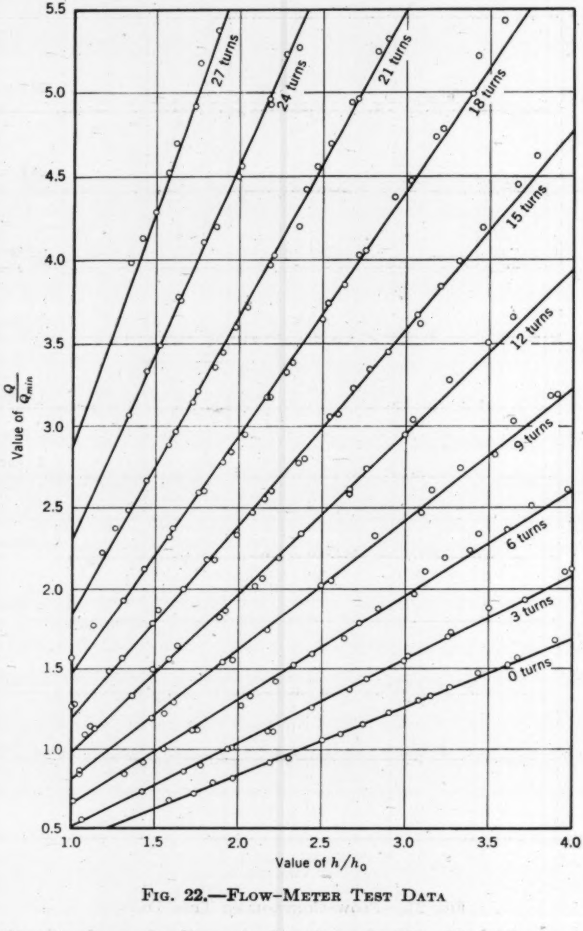


FIG. 22.—FLOW-METER TEST DATA

Test results are shown in dimensionless form in Fig. 22, with a summary of results in Table 1(*b*).

EXPERIMENTAL DATA

Because only one model was constructed, there was no opportunity to take advantage of experimental findings in order to increase accuracy. If a change in discharge coefficient occurs as a result of the change of disk position within the throat, it may be compensated for by redesigning the throat so that $C_q A$ varies with x' as prescribed by the theory. In the model there was no instrumentation to determine the exact position of the disk for a particular setting. The setting, z, could be altered either by rotating the disk on its shaft (4 threads to the inch), or by rotating the adjusting knob over the spring (13 threads to the inch). The turns referred to in Figs. 21 and 22 are for the adjusting knob above the spring.

Flow Controller.—For the flow controller the "0-turn" disk position at $h = 8$ in. was approximately at the minimum throat section. The average value of discharge for this setting, 20.05 gal per min, was taken as $Q_{\min}$. Because the exact position of the disk in the throat is not known for this setting, the area and discharge coefficient are also unknown.

Separate measurements of the discharge coefficient were made with the disk in known fixed positions and without the upper supporting framework for the spring assembly. These results indicated a coefficient of approximately 0.62 within the throat and approximately 0.63 with the disk slightly above the throat casting.

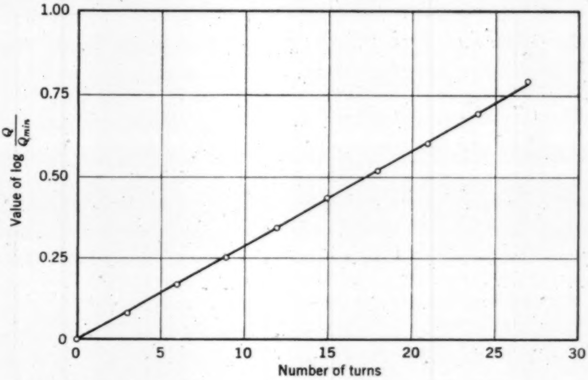


FIG. 23.—DISCHARGE RELATED TO NUMBER OF TURNS FOR FLOW CONTROLLER

In each series of observations for flow controller or flow meter, measurements were taken almost equally for increasing heads and for decreasing heads. Hysteresis effects appear to be small, much less than the S-shaped trend of the data, which apparently is caused by the spring. The width of the spring was varied in the model, as well as the quantity of styrofoam, until the displacement, Y, was 0.75 in. at $h = 2$ in. and was zero at $h = 8$ in. The S-shaped trend in the data may be a result of the spring backing being incorrect, to the extent that the spring was too stiff for the head range of from 2 in. to 5 in., thereby holding the disk higher in the throat and increasing the flow area slightly. In addition, the spring was not quite as stiff as required by the theory for the range of from 5 in. to 7.5 in., and permitted the disk to be lower than the theory requirement, thereby decreasing the discharge.

Fig. 21 is plotted with log $(\overline{Q} / Q_{\min})$ as the ordinate, so that the same relative percentage error scale occurs for all settings and the average discharge lines would be equally spaced. A change of three turns represents a displacement, Δz, of 3/13 in.

Log $(\overline{Q} / Q_{\min})$ is plotted against the number of turns in Fig. 23, in which $\overline{Q}$ is the average of the discharges for a given setting. The solid line is determined by least squares and yields

$$\frac{\overline{Q}}{Q_{\min}} = 0.994\, e^{0.06704\, t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (78)$$

Comparing the exponent with $z \ln \eta/(2\, Y)$, $z = 13.78\, t$. Because $z = 13\, t$ by direct measurement, the discrepancy can be explained as being due to a change in discharge coefficient. When the spring support stands were in place, tests with a fixed disk yielded a change in discharge coefficient that conforms generally to the relationship, $z = 13.78\, t$.

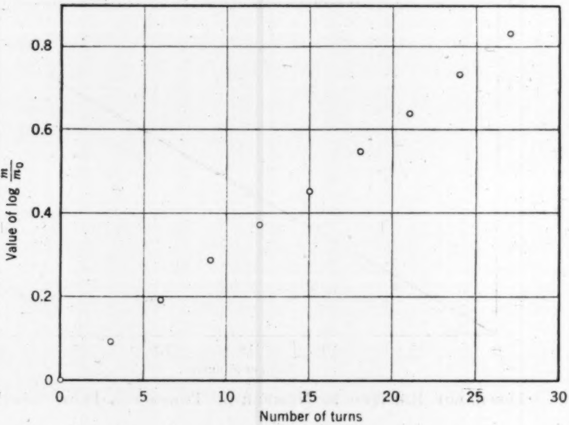


FIG. 24.—Q / h AS A FUNCTION OF NUMBER OF TURNS FOR FLOW METER

The disk was stabilized by placing the steel can around the styrofoam floats for the settings of from 15 turns to 27 turns.

Flow Meter.—The 0-turn position for the flow meter was near the minimum throat section for $h = 2$ in. The discharge, $Q_{\min}$, was taken as 20.05 gal per min, as in the case of the flow controller. The data as plotted in Fig. 22 generally confirms the relationship that discharge varies linearly with head for any setting. A slight S-shaped trend in the data, which is similar to that of the flow controller but reversed, is due to the spring being too stiff for small heads and not stiff enough for the larger heads.

The slopes, $m = Q / h$ (Q in gallons per minute and h in inches), for the best radial lines through each set of observations have been computed by the method of least squares and are shown in Table 1, with the maximum percentage divergence of any single observation. The taking of data for the flow meter was rather difficult because of the large volume of storage upstream from the meter. Water was admitted to the flume from a constant-head tank, and the level in the flume achieved equilibrium slowly. Measurements were usually taken before steady flow was established, which could introduce minor errors in the observations. The flow meter was inherently stable and did not require any form of stabilization.

It was necessary to shift the disk on its shaft between runs 0 to 12 and 15 to 27. Unfortunately, it was not adjusted so that the change from 12 turns to 15 turns was actually 3/13 in. This is evident in the plot of $\log m / m_0$ against the number of turns in Fig. 24, in which m_0 is the value of Q / h for 0 turns ($m_0 = 4.229$). By constructing the best line through the points for from 0 turns to 12 turns and for from 15 turns to 27 turns, the relationships between t and z were $z = 12.97\, t$ and $z = 12.87\, t$, respectively, which confirm the known relationship, $z = 13\, t$. A substantially constant coefficient of discharge is thus indicated over the whole range of tests on the flow meter.

Conclusions

Theory and design data for both the flow controller and the flow meter have been developed, with several forms of nonlinear resistance. An open-channel model was constructed using a restrained-tip cantilever spring for nonlinear resistance, and data was taken for ten settings each as a flow controller and as a flow meter. The results generally confirm the theory and indicate the way to the construction of accurate and relatively simple control and metering equipment.

Acknowledgments

The experimental part of the project described herein was performed with the aid of a grant from the Faculty Research Fund of the Horace H. Rackham School of Graduate Studies of the University of Michigan. The theory and design data were developed with the help of The Dole Valve Company, of Chicago, Ill. J. S. Baker and R. B. Kaiser assisted in taking data.

APPENDIX. NOTATION

The following symbols, adopted for use in this paper, conform essentially with "American Standard Letter Symbols for Hydraulics" (ASA Z10.2–1942), prepared by a committee of the American Standards Association with Society representation, and approved by the Association in 1942:

A = area of orifice:
    A_1, A_0 = area of opening between disk and throat;
    $A_{\min}$ = minimum area;
A_b = area of bellows;
a, a_1 = manometer cross-sectional areas;
b = width of leaf spring;
C_d = drag coefficient;
C_q = discharge coefficient;
D = displacement;
E = modulus of elasticity;
e = base of natural logarithms;
F = pressure force on disk:
    F_0 = minimum design force on disk;
G = submerged weight of disk assembly;
g = acceleration due to gravity;
h = dead drop across disk:
    h_0 = minimum design head loss across disk;
I = moment of inertia of spring section about neutral axis;
K_0 = stiffness of spring of length l;
k = ratio G/F_0;
l = length of spring for load $F_0/2$:
    l' = length of disk support shaft;
M = moment;
P = force on spring assembly:
    P' = force on single spring element;
Q = discharge:
    Q_d = design discharge;
R = manometer gage difference;
r = radial coordinate of throat:
    r' = minimum throat radius;
    $r_{\max}$ = maximum throat radius;
    r_0 = disk radius;
S_m = maximum fiber stress;
s, s_0, s_1 = dimensions of manometer;
t = thickness of spring;
W = weight;
w = unit weight of fluid:
    w_m = unit weight of manometer fluid;
x = axial coordinate of throat profile:
    x' = position of disk in throat;
    x_0 = coordinate of spring backing;
    $\bar{x}$ = dimensionless length, x/l;
Y = value of y for $h = h_0$;
y = position of spring:
    $\bar{y}$ = dimensionless quantity, y_0/Y;
    y_0 = coordinate of spring backing;

z = discharge setting;
α = constant, angle;
β = angular deflection;
δ = displacement of single spring element from minimum design head position;
ζ = dimensionless variable;
η = ratio of maximum design head to minimum design head;
θ = polar coordinate; and
ρ, ρ_0 = polar coordinate.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2947

HYDRAULIC MODEL STUDY OF HYPERION SEWER INTERCHANGE

BY ALFRED C. INGERSOLL,[1] M. ASCE, AND
HAJIME TANAKA,[2] A.M. ASCE

SYNOPSIS

A 1:20 scale model of the interchange structure at the junction of two outfall sewers approaching the Hyperion Sewage Treatment Plant, Los Angeles, Calif., was tested in accordance with the Froude law. The study was conducted to determine the effects of various flow distributions among the two influent sewers and channels leading to the new and existing headworks. In particular, the configuration of the throat was studied to ascertain whether the interchange could pass the required flows without causing a backing up in either of the outfall sewers.

Maximum storm, peak dry weather, average, and minimum flows were tested for normal operating conditions and for a variety of abnormal conditions with one unit or the other out of service. The best throat design was a combination of a long-pointed tail on the pier at the confluence and a cylindrical nose with a 1-ft radius on the pier facing upstream at the diversion. It was found that energy is transferred by turbulent shear from the fast-moving flow in one sewer to the flow moving slowly through the throat with the result that the energy grade line downstream from the throat is actually higher than that upstream of the throat in the other sewer.

INTRODUCTION

The enlargement program of the Hyperion Sewage Treatment Plant, Los Angeles, Calif., provides for an increase in the capacity of the plant from an average daily flow of 245 mgd to an anticipated average daily flow of 420 mgd. The plant utilizes the high-rate activated sludge process for secondary treatment (Fig. 1). It is now (1958) served by the North Outfall sewer, which carries sewage contributed by inland as well as coastal areas. The composite

---

NOTE.—Published, essentially as printed here, in June, 1957, in the Journal of the Sanitary Engineering Division, as *Proceedings Paper 1273*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Associate Prof. of Civ. Eng., California Inst. of Technology, Pasadena, Calif.

[2] Project Engr., Belt, Collins & Associates, Honolulu, Hawaii.

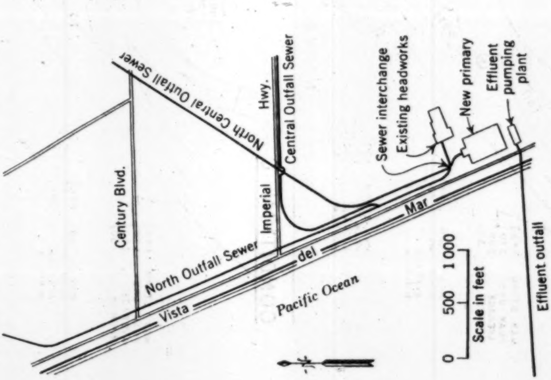

North Central Outfall Sewer
Central Outfall Sewer
Hwy.
Imperial
Century Blvd.
North Outfall Sewer
Vista del Mar
Pacific Ocean
Sewer interchange
Existing headworks
New primary
Effluent pumping plant
Effluent outfall
0
500
1 000
Scale in feet

FIG. 1.—SEWERS TO THE HYPERION TREATMENT PLANT

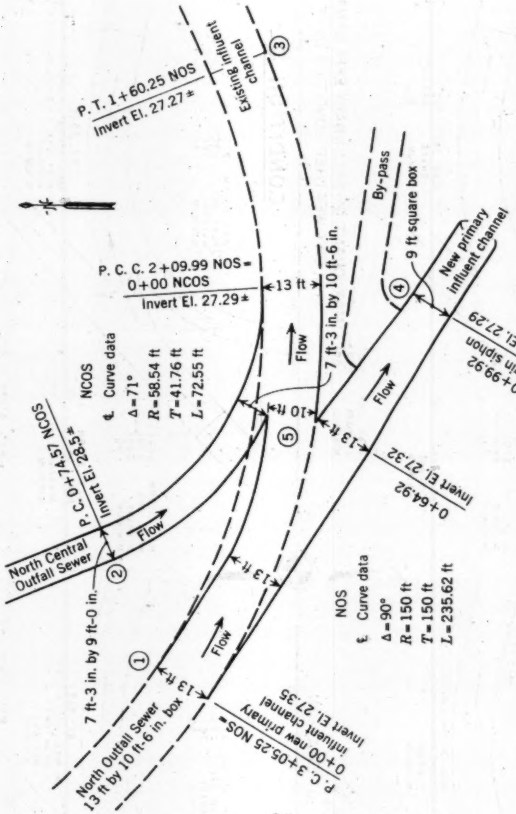

North Outfall Sewer 13 ft by 10 ft-6 in. box
7 ft-3 in. by 9 ft-0 in.
North Central Outfall Sewer
P. C. 0+74.57 NCOS
Invert El. 28.5±
Flow
13 ft
①
②
③
④
⑤
P. C. 3+05.25 NOS = 0+00 new primary influent channel
Invert El. 27.35
NOS
℄ Curve data
Δ = 90°
R = 150 ft
T = 150 ft
L = 235.62 ft
NCOS
℄ Curve data
Δ = 71°
R = 58.54 ft
T = 41.76 ft
L = 72.55 ft
10 ft
0+64.92
Invert El. 27.32
7 ft-3 in. by 10 ft-6 in.
P. C. C. 2+09.99 NOS = 0+00 NCOS
Invert El. 27.29±
0+99.92
begin siphon
Invert El. 27.29
P. T. 1+60.25 NOS
Invert El. 27.27±
Existing influent channel
By-pass
9 ft square box
New primary influent channel

FIG. 2.—PRELIMINARY DESIGN OF INTERCHANGE (ON WHICH MODEL WAS BASED) AND REFERENCE POINTS

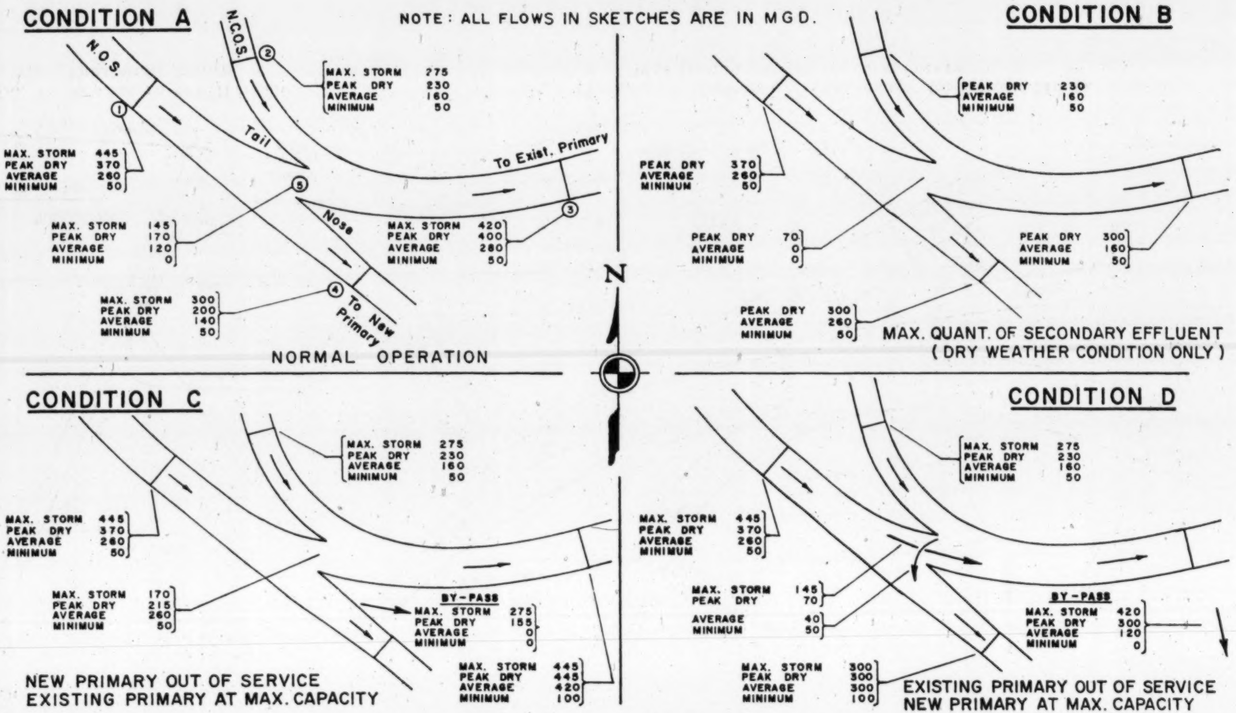

CONDITION A
NOTE: ALL FLOWS IN SKETCHES ARE IN MGD.
CONDITION B
N.O.S.
N.C.O.S.
①
②
③
④
⑤
Tail
Nose
To Exist. Primary
To New Primary
MAX. STORM 275
PEAK DRY 230
AVERAGE 160
MINIMUM 50
MAX. STORM 445
PEAK DRY 370
AVERAGE 260
MINIMUM 50
MAX. STORM 145
PEAK DRY 170
AVERAGE 120
MINIMUM 0
MAX. STORM 420
PEAK DRY 400
AVERAGE 280
MINIMUM 50
MAX. STORM 300
PEAK DRY 200
AVERAGE 140
MINIMUM 50
NORMAL OPERATION
PEAK DRY 230
AVERAGE 160
MINIMUM 50
PEAK DRY 370
AVERAGE 260
MINIMUM 50
PEAK DRY 70
AVERAGE 0
MINIMUM 0
PEAK DRY 300
AVERAGE 160
MINIMUM 50
PEAK DRY 300
AVERAGE 260
MINIMUM 50
MAX. QUANT. OF SECONDARY EFFLUENT
(DRY WEATHER CONDITION ONLY)
N
CONDITION C
MAX. STORM 275
PEAK DRY 230
AVERAGE 160
MINIMUM 50
MAX. STORM 445
PEAK DRY 370
AVERAGE 260
MINIMUM 50
MAX. STORM 170
PEAK DRY 215
AVERAGE 260
MINIMUM 50
BY-PASS
MAX. STORM 275
PEAK DRY 155
AVERAGE 0
MINIMUM 0
MAX. STORM 445
PEAK DRY 445
AVERAGE 420
MINIMUM 100
NEW PRIMARY OUT OF SERVICE
EXISTING PRIMARY AT MAX. CAPACITY
CONDITION D
MAX. STORM 275
PEAK DRY 230
AVERAGE 160
MINIMUM 50
MAX. STORM 445
PEAK DRY 370
AVERAGE 260
MINIMUM 50
MAX. STORM 145
PEAK DRY 70
AVERAGE 40
MINIMUM 50
BY-PASS
MAX. STORM 420
PEAK DRY 300
AVERAGE 120
MINIMUM 0
MAX. STORM 300
PEAK DRY 300
AVERAGE 300
MINIMUM 100
EXISTING PRIMARY OUT OF SERVICE
NEW PRIMARY AT MAX. CAPACITY

Fig. 3.—Design Criteria for Interchange Capacities

flow arriving at the plant contains slightly more salinity than is considered desirable for potential reclamation. A new trunk sewer, designated as the North Central Outfall, will supplement the existing sewer. Sewage in the North Central Outfall sewer is expected to have no objectionable salinity.

Space limitations, requirements for conformation to existing construction, and other restrictions made it necessary to provide an interchange of flows between the two trunk sewers immediately upstream of the existing headworks (Fig. 2). This interchange will permit the entire flow of low-salinity sewage from the North Central Outfall to enter the existing primary tanks. The additional flow required for the proper proportion (Fig. 3), will be "bled" from the North Outfall sewer. Thus, the saline content of this combined flow will be much less than that at present, and the resulting effluent could be reclaimed. A maximum of the flow in the North Outfall sewer will then be given primary treatment in the new headworks and settling tanks, from which it will be discharged several miles at sea.

An elaborate schedule of capacity requirements for the interchange structure has been determined (Fig. 3), specifying the flows in the outfall sewers and the proper distribution of these flows to the influent channels. Of the four conditions shown, condition A for normal operation was the criterion for the preliminary design of the interchange shown in Fig. 2. Condition B for maximum quantity of secondary effluent is the operating condition when the existing secondary tanks are at maximum capacity. Sewage in the new primary tank will be treated and discharged directly to sea. Conditions C and D were studied from the standpoint of emergency operation during a temporary shutdown of part of the plant. In addition to these flow requirements, the following specifications were to be met as closely as possible:

1. In order to minimize salinity in the sewage receiving secondary treatment, the flow to the new primary should be exclusively from the North Outfall sewer, except for abnormal condition C, which requires backflow through the throat.
2. For normal operation the North Outfall sewer should not be backed up to a hydraulic grade line at point 1 (Fig. 2) greater than 36.0 ft.
3. The water surface in the interchange structure should be as smooth as possible, to minimize release of hydrogen-sulfide gas.
4. Sedimentation in the interchange should be held to a minimum.

HYDRAULIC MODEL THEORY

Because raw sewage, such as that flowing to the Hyperion Treatment Plant, is essentially water, the use of a hydraulic model is reliable in analyzing sewer-design problems that are not amenable to theoretical solutions. In hydraulic models it is possible to establish simple dynamic similarity between a scale model and its prototype if it can be demonstrated that one type of force (for example, gravity, viscous shear, or surface tension) predominates over the other types in its relationship to the inertia forces that cause (or resist) motion in the fluid. In most cases of liquids with a free surface that involve flow around obstructions or through constrictions (as in the case of the throat of the interchange structure), the predominant force is gravity and the Froude model law is applicable.[3] Inasmuch as all the sewage flow considered is carried to, in, and from the interchange in open channels, the Froude law was indicated for this model study. That is, if the model is built to scale and operated so that the Froude number,

$$\mathsf{F} = \frac{V_p}{\sqrt{L_p\, g}} = \frac{V_m}{\sqrt{L_m\, g}} \qquad (1)$$

is the same in both model and prototype, all linear hydraulic quantities, such as water depth and velocity head, are in direct proportion as the scale ratio, and other quantities, such as velocity and discharge, are related through Eq. 1 and its derivatives. In Eq. 1, V_p and V_m are representative velocities in prototype and model, respectively, L_p and L_m are representative lengths, and g is the acceleration due to gravity. The model scale ratio is defined as

$$L_r = \frac{L_m}{L_p} \qquad (2)$$

The Froude number is a dimensionless quantity as are all similarity parameters. If the model and prototype were operated at different values of g, it would be necessary to include the different values in Eq. 1. However, for ordinary hydraulic models, g may be assumed to be constant. Eqs. 1 and 2 then establish the important relationships,

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} = \sqrt{L_r} \qquad (3)$$

and

$$\frac{Q_m}{Q_p} = \frac{V_m\, L_m^{\,2}}{V_p\, L_p^{\,2}} = \left(\frac{L_m}{L_p}\right)^{5/2} = L_r^{5/2} \qquad (4)$$

which are utilized in this study.

To establish that the Froude model law is applicable, it must be shown that the Reynolds number,

$$\mathsf{R} = \frac{L\, V}{\nu} \qquad (5)$$

indicates nonviscous or turbulent flow in both model and prototype. In Eq. 5 ν is the kinematic viscosity of the fluid. For a Froude model utilizing the same liquid in model and prototype, R varies as $L_r^{3/2}$. The lower limit on the allowable scale of a Froude model is generally established as that at which the flow in the model is just securely out of the laminar range. Surface-tension forces are significant in the Froude model in the sense that the liquid surface in the model would not have the same tendency to break up as would the surface of the prototype, but the effect on other quantities, such as depth of flow, should be negligible.

Choice of Model Scale.—The first problem in any model study is the choice of scale. The lower limit is governed by the Reynolds-number criterion cited previously, and the upper limit is set by cost considerations. The choice of a

---

[3] "Fluid Mechanics," by R. L. Daugherty and A. C. Ingersoll, McGraw-Hill Book Co., Inc., New York, N. Y., 1954, Chapter 6.

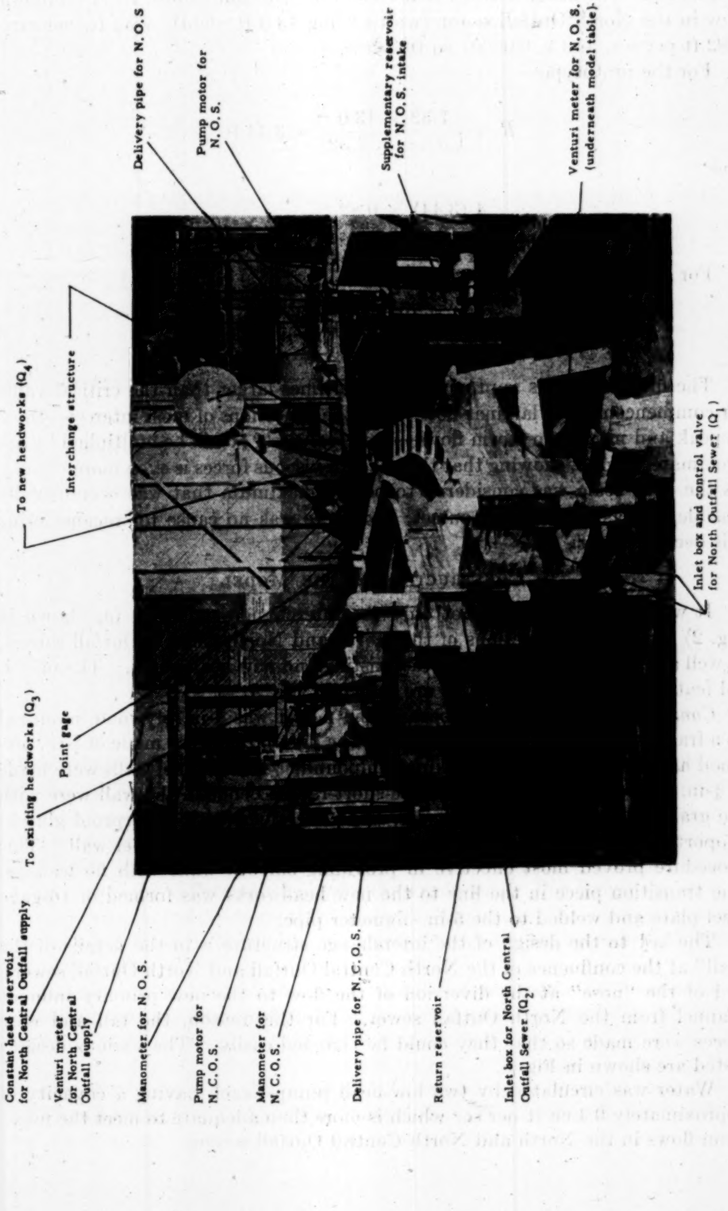

Constant head reservoir for North Central Outfall supply
Venturi meter for North Central Outfall supply
Manometer for N.O.S.
Pump motor for N.C.O.S.
Manometer for N.C.O.S.
Delivery pipe for N.C.O.S.
Return reservoir
Inlet box for North Central Outfall Sewer (Q_2)
To existing headworks (Q_3)
Point gage
To new headworks (Q_4)
Interchange structure
Delivery pipe for N.O.S
Pump motor for N.O.S.
Supplementary reservoir for N.O.S. intake
Venturi meter for N.O.S. supp (underneath model table)
Inlet box and control valve for North Outfall Sewer (Q_1)

FIG. 4.—MODEL OF THE INTERCHANGE STRUCTURE

1:20 scale for this model yields the following values of **R** (with the representative length set at the customary four times the hydraulic radius, *R*) at minimum flow in the North Outfall sewer (width being 13.0 ft, depth, 7.32 ft, velocity, 0.82 ft per sec, and ν, 0.00001 sq ft^2 per sec):

For the prototype—

$$R = \frac{7.32 \times 13.0}{1.30 + 2\ (7.32)} = 3.44 \text{ ft}$$

and

$$\mathbf{R} = \frac{4\ (3.44) \times 0.82}{0.00001} = 1.12 \times 10^6$$

For the model—

$$\mathbf{R} = \frac{1{,}120{,}000}{20^{3/2}} = 12{,}540$$

The model value is approximately five times larger than the critical value for commencement of laminar flow. For the conditions of most interest—those at peak and maximum storm flows—these values of **R** will be multiplied by approximately 7 or 8, showing that the effect of viscous forces is even more remote. As the 1:20 scale was considered to be the maximum that was economically feasible, it was fixed at the outset, and there was no cause for reconsidering this decision later.

CONSTRUCTION OF THE MODEL

It was decided to model not only the interchange structure (as shown in Fig. 2) but approach sections of the North and North Central Outfall sewers, as well as the channels leading to the existing and new headworks. The principal features of the model layout are shown in Fig. 4.

Construction Details.—The base of the model was $\frac{3}{4}$-in. plywood, mounted on a framework of 2-in.-by-4-in. joists. The inlet boxes were made of $\frac{1}{2}$-in. plywood and were sealed with caulking compound. The channel walls were made of $\frac{1}{4}$-in. plywood, oriented so that the short-radius bends in the wall were with the grain on two of the three layers, and were glued with waterproof glue to supporting strips nailed to the base at the outside of the channel wall. This procedure proved most effective in providing smooth walls with no leakage. The transition piece in the line to the new headworks was formed of 16-gage steel plate and welded to the 5-in.-diameter pipe.

The key to the design of the interchange structure is in the details of the "tail" at the confluence of the North Central Outfall and North Outfall sewers, and of the "nose" at the diversion of the flow to the new primary influent channel from the North Outfall sewer. For this reason, the tail and nose pieces were made so that they could be changed easily. The various designs tested are shown in Fig. 5.

Water was circulated by two low-head pumps, each having a capacity of approximately 0.4 cu ft per sec which is more than adequate to meet the maximum flows in the North and North Central Outfall sewers.

Discharge Measurement.—The North Central Outfall flow, Q_2, was supplied from a constant-head tank with a skimming weir 12 ft above the floor level. The supply was metered by a 2½-in. Venturi throat at the outlet of the reservoir and connected to an air-water manometer.

The North Outfall sewer flow, Q_1, was supplied directly from the other pump, which drew from the auxiliary reservoir to the right in Fig. 4. The flow was measured by a 3-in.-by-1⅝-in. Venturi meter, which was installed in the delivery line under the model table and connected to a water-mercury manometer.

The discharge to the new primary headworks, Q_4, was measured by another 3-in.-by-1⅝-in. Venturi meter. The entrance condition to the existing primary headworks was an adjustable rectangular weir. The flow, Q_3, was obtained from the relationship,

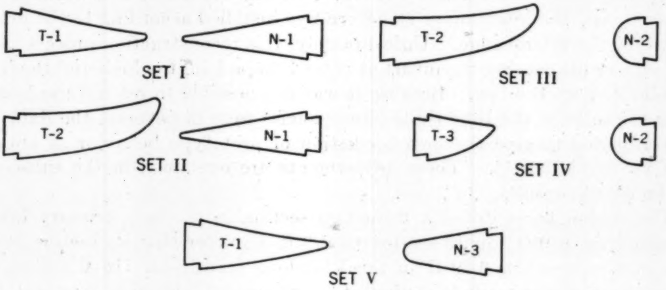


FIG. 5.—INTERCHANGE THROAT DESIGNS (T'S ARE TAIL PIECES, N'S ARE NOSE PIECES)

$$Q_3 = Q_1 + Q_2 - Q_4 \quad \text{(6)}$$

Establishing Slope of the Model.—The bed slope of a Froude model represents the effect of frictional forces, and, as such, it must be established independently of the prototype slope and Froude model law. Wherever the reach of a channel constriction under study is short, it is common to use a flat bed on the model, recognizing that the frictional forces will be small compared with the gravity forces caused by differences in hydraulic grade line. For longer reaches an attempt is made usually to establish the bed slope so that the depth is in proper scale at all stations, with the hydraulic grade line being the same in both model and prototype.

The slope of a model channel is established ordinarily by computing or measuring the slope required to produce depths that correspond to known or assumed depths in the prototype under various flow conditions. Fortunately, in the case of the Hyperion sewer interchange, there were data computed from measurements, showing the measured hydraulic grade lines at station 1 + 60.25 and station 3 + 0.525 in the existing North Outfall sewer, for two flows near the projected maximum storm and peak dry flows.

The first construction, was of the existing North Outfall sewer, extending roughly from station 1 + 22.38 to the end of the 90° curve, station 3 + 95.87. Modeled values of the measured discharge rates of 420 mgd and 320 mgd were put through the sewer, and the slope of the flat plywood bed was adjusted until the required depths were produced. These flows represent accelerating subcritical (M_2-type) surface profiles of nonuniform flow. The Froude number for a depth of 8.45 ft is 0.36. It was found that a bed slope of 0.0008 (as compared with the prototype bed slope of 0.0006) produced the required depths within approximately 0.02 ft (prototype scale), and this slope was fixed as the one that would eliminate the frictional forces from further consideration.

Discrepancies from True Model-Prototype Relationships.—Following this preliminary test, the section of the North Outfall sewer at the interchange was removed, and the interchange structure was installed according to the original design for the interchange. Unfortunately, this reconstruction process caused the bed to shift causing the invert at point 3 to be $\frac{1}{4}$ in. too high and the invert at point 4, $\frac{1}{8}$ in. too low. Because it was not possible to rebuild the bed and rerun the tests at the time these discrepancies were discovered, the data have been adjusted to give the best prediction of prototype behavior as obtained from the model study. These adjustments are explained in the subsequent section on test results.

The design necessitated a transition section in the new primary influent channel from a 9-ft square section to a 9-ft-diameter circular section, with a drop in the invert of 6.29 ft in the 12-ft-long transition. In the model the circular section was made to conform to a convenient 5-in. pipe size (prototype, 8.33 ft). However, this did not affect the hydraulics in the open channel upstream because it was necessary in any event to throttle this flow with a gate valve downstream from the Venturi meter.

The design of the North Central Outfall sewer involved a rise in the invert of approximately 0.8 ft in the curve just upstream of the junction. If properly modeled this would have meant a gradual rise in the floor of the model channel on the curve of approximately $\frac{1}{2}$ in. Inasmuch as this design was not yet firmly fixed and the losses in the North Central Outfall sewer were not critical, it was not considered worth the extra effort to install this sloping floor, with the result that the bed of this channel was at the same slope as the rest of the model. The effect was simply that the energy grade line in the prototype at point 2 would be slightly higher than its counterpart on the model for the same values of discharge and hydraulic grade line. This question is considered in detail in the section on test results.

TEST PROCEDURE

A test consisted of establishing the required flow distribution for one of the conditions and flows shown in Fig. 3, for example, condition A (normal operation) peak dry flow, and then determining the water-surface elevation at points 1, 2, 3, and 4 in Fig. 2.

The normal procedure was to set the required inflows in the North and North Central Outfall sewers, and then to adjust the gate valve to the new headworks simultaneously with the adjustable weir to the existing headworks in order to establish the proper value of Q_4 (and thus also of Q_3). To determine a reference value for the water-surface elevation, the hydraulic grade line at point 3 was made sufficient to pass the required Q_3 through the existing headworks, according to plant data supplied. This reference value was 35.72 ft for the maximum measured storm flow of 429 mgd. However, because this value applied to flow through all detritors and bar screens, it was also used for flows as low as 400 mgd. For lower values of Q_3, the hydraulic grade line at point 3 was adjusted in order that the head on the detritor weirs (set at El. 34.00) would vary as the $\frac{2}{3}$-power of the discharge.

TABLE 1.—SUMMARY OF MODEL TESTS[a]

Operating condition	Flow type	DISCHARGE, IN MGD				HYDRAULIC GRADE LINE ELEVATION, IN FT				Remarks
		North Outfall sewer, Q_1	North Central Outfall sewer, Q_2	Existing primary, Q_3	New primary, Q_4	North Outfall sewer, point 1, Fig. 2	North Central Outfall sewer, point 2, Fig. 2	Existing primary, point 3, Fig. 2	New primary, point 4, Fig. 2	
A	Maximum storm	445	275	420	300	35.87	36.23	36.16	35.94	
	Peak dry	370	230	400	200	36.09	36.29	36.15	36.25	
	Average	260	160	280	140	35.87	35.93	35.89	35.95	
	Minimum	50	50	50	50	34.67	34.67	34.72	34.65	
B	Peak dry	370	230	300	300	35.65	35.87	35.94	35.48	Must throttle Q_3, 0.3-ft pocket on south side of nose
	Average	260	160	160	260	35.57	35.67	35.81	35.33	
C	Maximum storm	445	275	445	275	35.97	36.29	36.17	36.16	0.2-ft pocket on north side of nose
	Peak dry	370	230	445	155	36.31	36.45	36.17	36.54	
	Average	260	160	420	0	36.55	36.45	36.16	36.71	0.4-ft pocket on north side of nose
	Minimum	50	50	100	0	34.91	34.87	34.90	36.91	
D	Average	260	160	120	300	36.03	36.25	36.38	35.48	Must throttle Q_3, 0.4-ft pocket on south side of nose
	Minimum	50	50	0	100	34.19	34.13	34.22	34.15	
Prototype invert elevation						27.35	27.37	27.27	27.30	

[a] All data apply to set V, Fig. 5. All discharges, velocities, and elevations are prototype values.

As explained previously, the slope of the base table shifted after construction with the result that the values of the hydraulic grade line at point 3 are approximately 0.4 ft higher than intended. This difference may be represented conveniently as an extra allowable loss across the bar screens. That is, for values of the hydraulic grade line at points 1 and 2 reported in Table 1, there is an allowable loss of approximately 0.4 ft (at the high flows) across screens or other obstructions at the entrance to the existing headworks. If there is no obstruction the water surfaces at points 1 and 2 will be lower by approximately this amount.

TABLE 2.—DEPTH, VELOCITIES AND HEAD

Operating condition	Flow type	Point 1, Fig. 2				Point 3. Fig. 2				
		Depth, D_1, in feet	Velocity, V_1, in feet per second	Specific energy, in feet	Elevation of energy grade line, in feet	Depth, D_3, in feet	Velocity, V_3, in feet per second	Specific energy, in feet	Elevation of energy grade line, in feet	Head loss from 1 to 3, in feet
A	Maximum storm	8.52	6.23	9.12	36.47	8.89	5.64	9.38	36.65	−0.18
	Peak dry	8.74	5.05	9.14	36.49	8.88	5.38	9.33	36.60	−0.11
	Average	8.52	3.64	8.73	36.08	8.62	3.88	8.85	36.12	−0.04
	Minimum	7.32	0.82	7.33	34.68	7.45	0.80	7.46	34.73	−0.05
B	Peak dry	8.30	5.31	8.74	36.09	8.67	4.13	8.94	36.21	−0.12
	Average	8.22	3.77	8.44	35.79	8.54	2.24	8.62	35.89	−0.10
C	Maximum storm	8.62	6.16	9.21	36.56	8.90	5.96	9.45	36.72	−0.16
	Peak dry	8.96	4.92	9.34	36.69	8.90	5.96	9.45	36.72	−0.03
	Average	9.20	3.37	9.38	36.73	8.89	5.64	9.38	36.65	0.08
	Minimum	7.56	0.79	7.57	34.92	7.63	1.49	7.67	34.94	−0.02
D	Average	8.68	3.58	8.86	36.23	9.11	1.57	9.15	36.42	0.19
	Minimum	6.84	0.87	6.85	34.20	6.95	0	6.95	34.22	−0.02

For certain runs involving low flow to the existing headworks and high flow to the new headworks, for example conditions B and D, it was necessary to throttle Q_3 in order to build up sufficient head to force the increased Q_4-value through the new headworks. This head was estimated on the basis of assumed losses in the Venturi meter, butterfly valve, bends, and transitions in the new primary influent channel.

TEST RESULTS

Summary data of all tests are included in Table 1, which gives the prototype values for rates of discharge and expected hydraulic grade lines. Table 2 presents detailed computations for energy grade lines and the head losses between points 1 and 3, points 2 and 3, and points 1 and 4.

Adjustments to Data in Tables 1 and 2.—The data presented in Tables 1 and 2 have been adjusted in several ways to represent, with three exceptions, the best prediction of prototype behavior as obtained from the model study. It has been noted previously that at points 3 and 4 the model as actually tested had bed elevations that were different from the proper values. This difference was 0.38 ft (prototype scale) high at point 3 and 0.21 ft low at point 4. Because these differences cause velocities at these points that are within 4% of the proper values, the average velocities between upstream and downstream points are within 2% of their proper values. In other words, in general, average velocity heads are within 4% of their proper values. Head losses, being proportional to velocity heads, should therefore also be correct within approximately 4%. However, the numerical value of head loss in almost every case is less than 0.25 ft, so that it is not reasonable to consider head-loss corrections to data showing losses to hundredths of a foot.

The foregoing explanation is the basis for the adjustment of model data—namely, head losses actually measured on the model were assumed to apply to

LOSSES (PROTOTYPE) WITH SET V, FIG. 5

POINT 2, FIG. 2					POINT 4, FIG. 2					
Depth, D_2, in feet	Velocity, V_2, in feet per second	Specific energy, in feet	Elevation of energy grade line, in feet	Head loss from 2 to 3, in feet	Depth, D_4, in feet	Velocity, V_4, in feet per second	Specific energy, in feet	Elevation of energy grade line, in feet	Head loss from 1 to 4, in feet	Operating condition
8.86	6.64	9.55	36.92	0.27	8.64	5.48	9.10	36.40	0.07	A
8.92	5.52	9.39	36.76	0.16	8.95	3.52	9.14	36.44	0.05	
8.56	4.00	8.81	36.18	0.06	8.65	2.56	8.75	36.05	0.03	
7.30	1.46	7.33	34.70	−0.03	7.35	1.07	7.37	34.67	0.01	
8.50	5.80	9.02	36.39	0.18	8.18	5.79	8.70	36.00	0.09	B
8.30	4.12	8.56	35.93	0.04	8.03	5.11	8.44	35.74	0.05	
8.92	6.59	9.60	36.97	0.25	8.86	4.89	9.23	36.53	0.03	C
9.08	5.42	9.54	36.91	0.19	9.24	2.64	9.35	36.65	0.04	
9.08	3.77	9.30	36.67	0.02	9.41	0	9.41	36.71	0.02	
7.50	1.42	7.53	34.90	−0.04	9.61	0	9.61	34.91	0.01	
8.88	3.85	9.11	36.48	0.06	8.18	5.78	8.70	36.00	0.23	D
6.76	1.58	6.80	34.17	−0.05	6.85	2.30	6.93	34.23	−0.03	

equivalent tests on a hypothetical model, which had a bed slope of 0.0008 as determined by the original slope test described previously. Therefore, working back from the measured energy grade lines, it was possible to compute all appropriate depths and velocities in the hypothetical model. These depths and velocities were then applied to the true invert elevations on the prototype in order to give the best values for hydraulic and energy grade lines and head losses. The three exceptions referred to previously are the negative head losses between points 2 and 3 for minimum flows. However, even the unadjusted data indicate this improbable situation. Therefore, it is not an indictment of the adjustment system but rather of the accuracy of the original data. These discrepancies are on the order of $\frac{1}{32}$ in.

Corrections for the Hydraulic Grade Line at Point 2.—At the peak dry flow it is desirable that the North Central Outfall sewer should not flow more than three-quarters full. Table 2 shows that the depth D_2 at this flow for normal operating condition is 8.92 ft. With the invert elevation of 27.37 ft as it is in the model, an energy grade line of 36.76 ft is produced. In the prototype sewer the invert elevation is actually 28.50 ft at point 2. Thus, to determine an effective depth for the prototype flow, one must determine the depth that will produce a specific energy (depth plus velocity head) of $36.76 - 28.50 = 8.26$ ft, with a discharge of $Q = 357$ cu ft per sec, or discharge per unit width of $q = 357/7.25 = 49.2$ cu ft per sec per ft. The relationship to be solved is

$$D_2 + \frac{\left(\frac{49.2}{D_2}\right)^2}{2\,g} = 8.26 \text{ ft}$$

The solution gives $D_2 = 7.61$ ft, or the hydraulic grade line at point 2 is 36.11 ft. As the soffit must be 10.5 ft above the invert, the depth is 72.5% of the height of the section, or under the maximum requirement of three-quarters full.

Development of Throat Design

Tail and Nose Combinations.—Throat design includes the width of the throat, the tail piece which faces downstream, and the nose piece which faces upstream. Various combinations of tail- and nose-piece shapes were tested (Fig. 5)—they are referred to as T-1 for tail piece 1 and N-1 for nose piece 1.

The initial test in the model study was made of the original interchange design submitted with the set I throat design. This design was found to pass all flows at normal operation satisfactorily to meet the requirements of the interchange. The point of interest was then the stagnation point, which was observed to point straight upstream (or a clock position of 12:00 on the nose piece) for peak dry and average flows of condition A. For condition B, average flow, the stagnation point moved to the throat side of the nose (2:00 position), and a deep depression or pocket of 0.4 ft (prototype scale) formed on the opposite side of the nose, followed by considerable turbulence. The depth, size, and turblence of the pocket became more marked for condition D, average flow.

The first effort to improve the original throat design was the insertion of a longer tail, T-2, curved away from the throat, thus restricting the North Central Outfall sewer mouth opening to 80% of the full channel width. The tail piece, T-2, produced two major effects. The hydraulic grade line at point 1 was lowered, and that at point 2 in the North Central Outfall sewer was raised. Greater energy was transferred by turbulent shear from the high velocity flow from the North Central Outfall sewer to the low velocity flow through the throat, enabling a lower water depth at the throat and, consequently, at point 1. Set II also produced a more markedly disturbed water surface downstream of the interchange, where the turbulent exchange of energy took place. Before discarding set II because of this turbulence, it was tried with a long-radius nose piece, N-2, which produced no significant difference from the set II combination.

The next step at modification was with a short tail piece, T-3, and long-radius nose piece, N-2. This combination provided the widest throat opening, but it showed no interchange-requirements improvement for normal operation. However, for condition C, average flow, where the maximum of flow passes through the throat, set IV produced a lower hydraulic grade line at point 1 than the set I throat design under similar operating conditions. The major drawback to set IV was the greater tendency for the flow from the North Central Outfall sewer to enter the new primary influent channel.

Final Throat Design.—In the long-radius nose piece, N-2, there was one desirable feature. Under abnormal operating conditions, with unnatural distribution of flows between the throat and the new primary influent channel, the stagnation point remained on the rounded nose even though it shifted in orientation. Therefore, the flow was smoother, and a much shallower pocket was formed. The nose piece finally chosen with a shorter (1-ft) radius preserves this feature. The T-1, N-3 throat design does not represent an improvement over the T-1, N-1 combination for flows at normal operations, but it is an improvement for almost all the abnormal operations.

To determine whether the flow from the North Central Outfall sewer enters the new primary influent channel, a dye test was conducted at normal operation, peak dry flow. This test revealed that a slug of dye injected into the North Central Outfall sewer did not enter the new primary influent channel, and, in fact, it took a distance of 1 ft or more in the model before the dye completely mixed with the flow in the existing influent channel.

To study the transition effects, gilsonite (an asphaltic mineral having a specific gravity of approximately 1.04) tracer particles were introduced into the model in the North Outfall sewer. At normal operation, average flow, the particles separated smoothly between the throat and the new primary influent channel. A slight eddy and depression of the water surface was observed in the lee of the nose, but the surface was not broken by turbulence at normal operation, maximum storm flow.

For condition B, average flow (for which there is no net flow through the throat), it was noted that the gilsonite particles did not pass through the throat. A smooth depression of the surface in the lee of the nose was observed during the test. Closer observation of the throat at this flow revealed that there is actually a small downstream flow at the surface and a corresponding backflow along the invert. This motion was sufficiently strong in the model to prevent any deposition of sediment.

The introduction of gilsonite during abnormal condition C, average flow (for which the flow through the throat is a maximum), resulted in a "particle cloud." This cloud revealed the manner in which the flow leaves the outer wall just before the throat, with a clockwise eddy just at the point of departure. A slight depression formed on the throat side of the nose as the flow was accelerated at this point. There is a certain amount of waviness of the surface downstream of the throat, which is inevitable.

ANALYSIS OF SPECIAL FEATURES

Mechanism of Energy Transfer.—It is instructive to investigate the mechanism by which energy is transferred from the North Central Outfall sewer (Q_2) to the slower moving flow through the throat (Q_5), so that the energy grade line at point 3 is 0.11 ft higher than that at point 1 for condition A, peak dry flow (with throat design V). Table 2 shows that the head loss from point 2 to point 3 is 0.16 ft. Because Q_2 is 357 cu ft per sec (230 mgd), this represents a power loss from point 2 to point 3 of

$$0.16 \times 357 \times 62.4 = 3{,}560 \text{ ft-lb per sec}$$

However, for Q_5 there is a gain of 0.11 ft from point 1 to point 3, and for Q_5 of 263 cu ft per sec there is a power gain of

$$0.11 \times 263 \times 62.4 = 1{,}803 \text{ ft-lb per sec}$$

By subtraction the net power loss in the two components of Q_3 between points 1 or 2 is 1,757 ft-lb per sec. For a value of Q_3 of 620 cu ft per sec, a head loss of 0.045 ft is incurred.

Such a transfer of energy must be accomplished by a velocity gradient and corresponding shear between Q_2 and Q_5. At the mouth of the North Central Outfall sewer, the corresponding mean velocities are computed to be

$$V_2 = 5.52 \text{ ft per sec}$$

$$V_5 = 2.87 \text{ ft per sec}$$

and

$$\Delta V = 2.65 \text{ ft per sec}$$

If the zone of shear can be roughly estimated to be 2 ft wide, the velocity gradient would be

$$\frac{du}{dy} = \frac{2.65}{2} = 1.325 \text{ sec}^{-1}$$

To determine the magnitude of the turbulent shear, one must consider the Prandtl equation, which includes mixing length, l,

$$\tau = \mu \frac{du}{dy} + \rho l^2 \left(\frac{du}{dy}\right)^2 \quad \text{(7)}$$

The first term in Eq. 7 represents the viscous shear and is a negligible quantity. Evaluating the second term, assuming that the mixing length is also approximately 2 ft and using $\rho = 1.94$ slugs per cu ft for the density of water,

$$\tau = 1.94 \times 2^2 (1.325)^2 = 13.6 \text{ lb per sq ft}$$

If the length of this shearing region is assumed to be on the order of 10 ft, the contact area is the length times the depth, or approximately 10 × 8.5, or 85 sq ft. The total shearing force would be on the order of 85 × 13.6, or 1,156 lb. If this force is applied to Q_5, moving at approximately 2.9 ft per sec, the rate of energy transfer would be about

$$1{,}156 \times 2.9 = 3{,}350 \text{ ft-lb per sec}$$

If it is assumed that the loss in energy of Q_2 from point 2 to point 3 owing to wall friction and turbulence is (from the foregoing computation) approximately

$$0.045 \times 357 \times 62.4 = 1{,}000 \text{ ft-lb per sec}$$

the rate of energy transfer from Q_2 to Q_5 would be on the order of 3,560 ft-lb per sec − 1,000 ft-lb per sec, or 2,560 ft-lb per sec which is almost 75% of what was indicated by the tentative shear computation. The assumed values for velocity gradient, or mixing length, or shear area are evidently too high.

The phenomenon of negative head loss from a lateral to a conduit in combining flow was observed[4] in closed conduit flow by John S. McNown, M. ASCE, but in this case the lateral flow entered at right angles to the main conduit and there could be little or no opportunity for the type of energy transfer cited previously. Therefore, the negative head loss was restricted to lateral flows of less than 15% of the combined flow and was attributed to the lateral flow moving into a region of the conduit in which the velocity was below average.

---

[4] "Mechanics of Manifold Flow," by John S. McNown, *Transactions*, ASCE, Vol. 119, 1954, p. 1117.

Deposit of Sediment.—During the model tests some fine sand was injected into the North Outfall sewer for visual observation. This sand settled on the bottom at the lower flow rates along the convex curving wall of the North Outfall sewer upstream from the interchange. At practically all flows there is separation and back flow along this wall as the flow decelerates in the widening channel.

In order to minimize this deposition, a bulge was placed in the right wall of the North Outfall sewer just opposite the interchange. It extended 2.5 ft into the stream and was smoothly shaped, so that the North Outfall sewer, in effect, did not diverge until it almost reached the interchange. The sediment was scoured away, but the bulge produced a highly disturbed flow in the new primary influent channel. The foregoing was not considered a practicable solution to the deposition problem. In fact, it should be emphasized that prototype velocities will be 4.47 times those of the model, and sand such as that used in the model will certainly be scoured. In the region of the throat, even if there is not much flow through it, there is generally enough circulatory motion to keep light material on the bottom in suspension. Moreover, if deposition does occur at the low flows it may easily be scoured by operating temporarily in order to sluice one channel or another with higher velocity flow.

Extension to Plant Operation

In order to attain an efficient operating condition at the existing plant, artificial devices for head losses must be utilized for all but maximum flows. These devices for backing up incoming flows consist of sluice gates and stop-log weirs, used separately or together, depending on the flow. The introduction herein gives the permissible depth of backwater allowed with or without the interchange at peak dry and maximum flow, normal condition A.

In the proposed operation of the Hyperion Sewage Treatment Plant, the flows to the existing and new primaries will be regulated by the throttling of a butterfly valve located immediately downstream from a Venturi tube in the new primary influent channel. The intelligence to position the valve will be furnished by a ratio controller in the control house. The information required for the proper division of flows by the controller will be furnished by a three-unit summator, which receives signals from a transmitter at the new Venturi tube and from two transmitters at the two existing Venturi tubes.

The experimental data furnished by the model showed that the proposed operation of the plant may be reasonably accomplished for normal operation. However, for abnormal conditions C and D (average flows), the existing sluice gates and stop-log weirs would need to be throttled in order to produce a sufficiently high hydraulic grade line at point 4 to pass the flows through the new primary. It is probably possible to set the new primary tanks sufficiently low, so that even these flows can be passed without throttling the existing sluice gates and stop-log weirs. However, a low weir elevation would result that would be undesirable from the viewpoint of the over-all plant enlargement.

CONCLUSIONS

Of the five throat designs tested in the 1:20 scale model of the Hyperion sewer interchange, the combination of a long slender tail and a 1-ft-radius nose (Set V, Fig. 5) approached most closely the criteria set forth in the Introduction. The results of the model study with this throat design, projected to prototype dimensions and conditions, yield the following conclusions:

1. For all conditions in which there is a positive flow through the throat, sewage in the North Central Outfall sewer is excluded from the new primary influent channel. This was demonstrated by injecting dye in the North Central Outfall sewer and observing that no color appeared in the new primary influent channel. Therefore, the maximum saline sewage from the North Outfall sewer will flow to the new primary.

2. For normal operation the North Outfall sewer is not backed up to a hydraulic grade line at point 1 greater than 36.1 ft—this value occurs at peak dry flow. Maximum storm and average flows each produce an hydraulic grade line of 35.9 ft at point 1. These values include an allowance of approximately 0.4 ft for head loss across bar screens or other obstructions at the inlet to the existing detritors.

3. The water surface in the interchange is relatively smooth. Under normal operating condition this is also accomplished with the set I throat design. However, for the abnormal conditions the cylindrical nose on the diversion pier is especially helpful.

4. If there is any deposition of sediment in the interchange, it will most likely occur along the convex or northeast wall of the North Outfall sewer, just upstream from its confluence with the North Central Outfall sewer, where there is some separation. However, a circulatory flow accompanying such separation is probably enough to keep most sediment scoured from the bottom.

5. Various combinations of throat design other than that recommended each displayed certain attributes, but in every case the accompanying disadvantages outweighed the advantages. The original design was satisfactory for normal operation, but for most of the abnormal conditions the cylindrical nose on the diversion pier produced a smoother water surface and a lower head loss across the throat.

ACKNOWLEDGMENTS

The Hyperion sewer model study was sponsored by Hyperion Engineers, a joint venture of three Los Angeles engineering firms. The writers are indebted to Vito A. Vanoni, M. ASCE, and James J. Kesler, J.M. ASCE, for their many helpful suggestions. The model was constructed by Elton Daly and Lee Carmichael.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2948

BRIDGE CLEARANCES
A SYMPOSIUM

PAGE

FOREWORD 926

POLICIES AND PRACTICE
By Eugene W. Weber, M. ASCE 927

THE INTEREST OF THE BUREAU OF PUBLIC ROADS
By Walter Kurylo 932

NEED FOR A REALISTIC APPROACH
By William E. Cleary 942

THE OPERATOR'S VIEW
By Nickles L. Caruthers 946

PROBLEMS IN NORTHEASTERN UNITED STATES
By Erhard E. Dittbrenner, M. ASCE 949

FOREWORD

This symposium is concerned with the physical characteristics of bridges crossing waterways as they affect both water and land traffic. The symposium arose from a concurrent growing awareness on the part of engineers concerned with waterway, highway, and railroad traffic of the need for rationalizing any extreme views on appropriate navigation clearances. With a national highway program (1957) of unprecedented magnitudes, these papers have great significance in their effect on the adequacy of clearances and the cost of bridge construction.

POLICIES AND PRACTICE

By Eugene W. Weber,[1] M. ASCE

Synopsis

The development of transportation facilities has of necessity resulted in intersections of the various modes of transport. In the first half of the nineteenth century there was little conflict between water and land transport and, thus, relatively few bridge-clearance problems. With the development of railroads in the latter half of the century, navigational clearance at bridges became important, and federal legislation was adopted to deal with it. During the past fifty years, increasing highway transport, resurgence of waterways traffic, and continued development of railroads have resulted in radical changes in the nature and extent of the bridge-clearance problem. The history of the problem and the steps necessary to solve it are developed herein.

The development of transportation facilities by waterway, railroad, and highway has been accompanied by constantly changing problems with respect to the necessary intersections of the various modes of transport. As speeds increased, for example, it soon became necessary to eliminate grade crossings of highways and railroads. At present, superhighways are being developed to separate the high-speed traffic as much as possible.

In considering the specific problem of bridge clearances, it is convenient to examine the evolution of the problem in the past three half-century periods:

In the first half of the nineteenth century the principal method of long-distance transport was by water, and there were relatively few bridge-clearance problems.

The last half of the nineteenth century brought about the development of the railroads as the principal means of long-distance inland transport. During this period the problem of navigational clearances at bridges became serious, and federal legislation was adopted to deal with it.

In the first half of the twentieth century a great highway system has developed, and there has been a resurgence of inland waterway traffic as well as continued rail development to meet the tremendous growth in transportation requirements. During the latter part of this most recent fifty-year period, the nature and extent of the bridge-clearance problem has changed most radically.

In tracing the evolution of this problem, one of the earliest and most interesting developments is a case which arose in 1852 concerning a bridge across the Ohio River at Wheeling (W. Va.). The state of Pennsylvania contended that the bridge was an obstruction to navigation. The United States Supreme Court analyzed the pertinent considerations involved, including the size and

---

Note.—Published, essentially as printed here, in April, 1956, as *Proceedings Paper 935*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Special Asst. to Asst. Chief of Engrs. for Civ. Works, Corps of Engrs., U. S. Dept. of the Army, Washington, D. C.

type of vessels, the feasibility of altering the vessels, the delays to navigation attributable to the bridge, the costs of bridge alterations to accommodate navigation, and the relative importance of the land and water traffic. The Supreme Court decided[2] that the bridge obstructed navigation, but the United States Congress legalized[3] the bridge and thus forced water traffic to conform to the limitations imposed by the clearances then existing. This case is noteworthy for at least two reasons: First, it is an early example of an attempt to determine, by economic analysis, a proper balance between the adverse impact of a bridge on water traffic and the adverse impact of navigation requirements on bridge costs. Second, it is an example of a situation in which the prerogative of Congress to regulate commerce was exercised in a manner which favored land transport at the expense of water transport.

Until 1890, the regulation of the construction of bridges over waterways was left to the individual states except for separate actions by Congress from time to time authorizing specific bridges and, in some cases, prescribing the navigation clearances. Meanwhile, the inland waterways were being extensively improved by the federal government to meet the growing needs for water transportation. At the same time, as the great era of railroad construction developed, more and more bridges were being built over navigable waters without effective control of their clearances for waterway traffic. Many difficult and undesirable situations which exist at land-transport crossings over navigable waterways are traceable to actions taken during this period of ineffective control of bridge construction. After an initial attempt at general bridge legislation in 1890, Congress finally passed[4] a comprehensive act, which was the foundation of the federal legislation applicable to bridges over navigable waters.

Under the Act of 1899, with subsequent amendments and related legislation, the determination of the suitability of plans and of clearances to be provided for navigation at bridges over navigable waters of the United States has been a responsibility of the Corps of Engineers (United States Department of the Army).

It is only since 1899, therefore, that procedures have been available for effective determination of the proper requirements for navigational clearances at bridges. The Corps has followed the practice of obtaining the views of interested parties on the proposed construction of bridges over navigable waters by direct inquiry, public hearings, and other appropriate procedures. At the beginning of the twentieth century, railroad and waterway operators were often the only parties expressing an interest. In any event, all pertinent information available on each proposed bridge was analyzed to determine the reasonable requirements for navigational clearances.

As the volume of traffic at railroad and highway crossings over waterways increased, it became necessary to improve and refine the methods for the analysis of bridge clearances in order to assure accurate and proper determinations.

Prior to and during the 1930's, standard bridge clearances had been established for most of the major seaport areas and inland waterways, including the

---

[2] Pennsylvania v. Wheeling and Bridgeport Bridge Co., 13 Howard 518.

[3] Act of August 31, 1852 (10 Stat. 110).

[4] Act of March 3, 1899 (30 Stat. 1121).

Mississippi River, the Ohio River, the Illinois River, the Missouri River, and the Atlantic and Gulf Intracoastal Waterways. Although establishment of such standard clearances facilitates planning by all transportation agencies, it is considered essential that they be kept constantly under review. Also, each proposed new bridge should be analyzed on its own merits to determine the clearance provisions which will result in the least total cost to the public for transportation by the most logical combination of water and land facilities.

It is necessary to look back only a few years to note the drastic changes which can take place in the interrelation of transport media. As the railroads were developed and dependence on waterways decreased, many railroad bridges were built with clearances which were thought reasonable for the navigation requirements foreseeable at that time. Within the economic life span of many such bridges, however, the need for waterway facilities for bulk inland water transport has made it necessary to undertake expensive alterations to reduce the economic losses caused by the interference with waterway traffic. Since the passage[5] of the Truman-Hobbs Act, the federal government has embarked on a program of alteration of approximately fifty obstructive bridges under the provisions of that act and under special projects consistent with the provisions. The total estimated federal cost of these alterations is more than $130,000,000. In addition, non-federal costs in connection with the program will bring the total expenditure to approximately $200,000,000.

The fact that bridge-clearance practices to date (1957) have resulted in inadequate navigation clearances which require such vast expenditures to correct has generally received less recognition than that accorded to the cases in which the clearances for navigation have proved to be no longer justified. Perhaps this is due to the fact that there are many individual motorists who have acquired a personal knowledge of the impact of waterway requirements on bridges, whereas there are relatively few individuals who have had occasion to observe or note the impact of obstructive bridges on navigation. In any event, it has become evident that efforts to improve determinations of navigational clearance requirements at bridges should be intensified. The tremendously increased volume of both land and water traffic has added to the significance of the possible adverse effects on one form of transport or another when the clearances are not satisfactory.

In view of the drastic changes which have taken place in transportation facilities in the past fifty years, it is not surprising that there are many bridges over navigable waters with clearances that are inappropriate according to present-day traffic requirements, both by land and water. In analyzing bridge-clearance problems fifty years ago, it is unlikely that many administrators or engineers envisioned the type of highway transport that exists today. Even in more recent years, when modern types of automotive transport were well developed, it is unlikely that many would have predicted the volume of traffic that is now materializing. Similarly, during the growth of the great rail network and the relative decline in waterway traffic, few analysts would have felt justified in predicting the increasingly important role that inland waterways have assumed in the transportation of bulk commodities. Of the existing drawbridges at which operation of the drawspan is no longer required because of changed conditions, about half were built fifty years ago or more and were required to have drawspans on the basis of conditions foreseen at that time. Also, practically half of the bridges which are currently considered obstructive to navigation, and which were permitted to have inadequate navigation clearances when they were built, are fifty years old or more. From 85% to 90% of all bridges now having improper clearances, from the standpoint of either land-transport or waterway-transport requirements, were built more than twenty-five years ago. It is clear from the available information that much, if not most, of the bridge-clearance problem stems from an inherent inability to forecast future developments accurately rather than from any basic inadequacy of the laws and practices that have been applicable since 1900.

---

[5] Act of June 21, 1940 (54 Stat. 497).

One of the most important recent steps toward improvement of bridge-clearance determinations had its origin in the Federal Inter-Agency River Basin Committee's benefit-cost studies, which culminated in a report.[6] This report has formed the foundation of improved practices for economic analysis in many agencies engaged in the development of water and related land use projects. The Corps was one of the participating agencies in the benefit-cost studies and has adapted the applicable parts of the recommended principles to the problem of determining bridge-clearance requirements.

In the preparation of reports on prospective navigation improvements, arrangements have been made to insure that the analysis will include a complete accounting of the impact of such improvements on land transport. The Corps' procedures for analyzing prospective navigation improvements have for many years included provisions for obtaining views and data from all affected interests. However, until the problems of the expanding highway network and its waterway crossings became more and more acute, the Corps was able to obtain relatively little definite information on highway requirements for use in analyzing waterway projects. Recent improvements in the practices of all agencies concerned with this aspect of the problem are expected to result in a better correlation of water and land-transport needs, and, in the long run, in more adequate bridge-clearance criteria.

A second way in which improved procedures for economic analysis will help in the bridge-clearance problem is through improvement of the analysis of individual bridge proposals. In a recent case involving a proposed bridge on Highway 65 across the Minnesota River near Minneapolis (Minn.), the Corps analyzed the bridge costs and the adverse effects on land transport over a range of vertical-clearance requirements and examined the economic impact of the same range of clearances on expected water traffic. The point at which there was the least total adverse economic effect on the combination of the expected land and water traffic was recommended as the proper vertical-clearance requirement for the bridge. This procedure will be effective in determining reasonable and proper bridge clearances if satisfactory data on prospective land and water traffic can be obtained for use in the analysis.

---

[6] "Proposed Practices for Economic Analysis of River Basin Projects," Report by the Subcommittee on Benefits and Costs, Federal Inter-Agency River Basin Committee, Washington, D. C., May, 1950.

Because of the pronounced changes in transport facilities in recent years, the Bureau of Public Roads, United States Department of Commerce (BPR), has become increasingly concerned with the bridge-clearance problem. Largely as a result of the initiative and concern of the BPR, the Department of Commerce, with appropriate assistance and cooperation from other agencies concerned with transportation planning and development, prepared a report,[7] which presents a comprehensive analysis of the land-transport aspects of the bridge-clearance problem. As a result of this study and report, there is a better realization of the relative interests and responsibilities of the various agencies—both federal and non-federal—that are concerned with the problem of bridge clearances over navigable waters. Further studies, particularly of the water-transport aspects of the problem, are also needed.

Conclusions

The question of the proper amount of clearance to be provided at bridges over navigable waters has become acute in recent years primarily because of changes in both land-transport and water-transport facilities and the tremendous increase in the volume of traffic. Part of the problem stems from practices prior to the beginning of the twentieth century before the adoption of adequate procedures for determining the proper clearances. There are no obstacles in existing law and practice to a realization of the goal of reasonable clearance requirements from the standpoint of all affected forms of transport. In recent years efforts have been intensified toward the improvement of procedures for determining future bridge clearances, and for the alleviation of existing problems caused by changing conditions. The continued cooperation of both federal and non-federal agencies along lines that have been developed in recent years is needed to assure satisfactory progress in solving the bridge-clearance problem.

---

[7] "Navigational Clearance Requirements for Highway and Railroad Bridges," the Office of the Under Secy. of Commerce for Transportation, U. S. Dept. of Commerce, Washington, D. C., February, 1955.

THE INTEREST OF THE BUREAU OF PUBLIC ROADS

By Walter Kurylo[1]

Synopsis

A new concept of national transportation policy concerning bridge clearances for navigational needs is being developed. Rather than protecting one form of transportation it protects the public interest. Possible solutions to problems arising under this concept are explored.

Introduction

In February, 1955, the United States Department of Commerce released the first comprehensive report[2] on the navigational clearance problem ever compiled in the transportation history of the United States. The study leading toward that report was based on four fundamental premises. Those premises were consolidated into the report, which has isolated this problem.

The first premise was that all transportation costs are ultimately borne by the general public in the cost of goods consumed, in prices paid for services received, and in taxes.

The second premise was that the expanding economy is not dependent for surface transportation solely on waterways, railways, or highways, but that all three forms are essential to a coordinated surface-transportation system. Because pipelines are assuming increasing importance in the economical movement of certain types of commodities, they also should be included.

The third premise was that sufficient factual material should be collected and evaluated to form the basis for sound conclusions and recommendations.

The fourth premise was that every federal agency having an interest in this problem, or whose activities might in any way be affected by the results of the study, should be given a real opportunity to cooperate in the undertaking and to have its views included in the report.

The study involved the cooperation of several federal agencies, and the report included information received from more than forty state highway departments and approximately sixty railroads. It contained incontrovertible facts concerning bridge-operating conditions and realistic estimates of the navigational increment to the cost of bridges.[3] The study reveals at least nine significant points concerning conditions and costs in 1950:

Note.—Published, essentially as printed here, in April, 1956, in the Journal of the Waterways and Harbors Division, as *Proceedings Paper 936*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Asst. to the Asst. to Federal Highway Administrator, Bureau of Public Roads, U. S. Dept. of Commerce, Washington, D. C.

[2] "Navigational Clearance Requirements for Highway and Railroad Bridges," the Office of the Under Secy. of Commerce for Transportation, U. S. Dept. of Commerce, Washington, D. C., 1955.

[3] *Ibid.*, pp. 30–32; pp. 49–82; pp. 136–141; and pp. 144–145.

a. In thirteen states at least 21 movable-span highway bridges and 2 railroad bridges have never been opened for navigation.

b. In thirty states no less than 375 movable-span highway bridges and 179 movable-span railroad bridges were opened on an average of once a day or less.

c. At least 424 movable-span highway and railroad bridges were not opened for a year or more.

d. Navigational clearances have been required in bridges to accommodate relatively few watercraft, including floating construction equipment and other types of craft having projections not essential for movement of the craft.

e. The added cost of constructing highway and railroad bridges due solely to navigational needs, as adjusted to 1950 price levels, is conservatively estimated at $754,200,000. This amount is 28% of the adjusted construction cost of approximately 2,200 bridges studied in detail.

f. The increased cost of maintaining and operating existing bridges solely for accommodation of watercraft is estimated at $16,900,000 annually.

g. The increased cost of vehicular and train operations due to navigation is estimated at $11,300,000 annually.

h. On a specific waterway studied in detail—the Altamaha River (Georgia)—the added cost of bridges due to navigational needs, when translated into terms of cost per ton-mile of waterway traffic, resulted in an estimate of $0.1375 per ton-mile. On the Kennebunk River (Maine), which carries very little waterway traffic under only one movable-span highway bridge, the estimate was $7.24 per ton-mile of waterway traffic.

i. According to conservative estimates, from $5,000,000 to $10,000,000 in bridge construction costs and an additional $1,000,000 in bridge-operating costs can be saved annually without unduly affecting waterway traffic. These savings would be substantially increased under an expanded highway program.

Evidence collected in the study shows that, when comparing railroads and highways, for some years the highway-transportation economy has been bearing a heavy burden of bridge costs due to navigational needs. Existing and prospective future conditions strongly suggest that highway transportation will continue to bear this burden for some time. This knowledge highlights the basis for the interest shown by the Bureau of Public Roads, United States Department of Commerce (BPR), concerning bridge clearances.

The problem has many intricate complexities. There are diverse and conflicting viewpoints as to its effective treatment. Certain waterway groups probably prefer to leave things as they are. Some overland transportation groups may hope for complete relief from costs arising from navigational requirements. The BPR believes that the answer should reflect the over-all public interest.

In striving for effective treatment of the problem, the BPR will abide by the sound premises that are basic to the report. Also, the BPR is committed to support the Department of Commerce recommendations for remedial action.[4] Summarized briefly, those recommendations provide for bridge clearances due to navigational needs based on transportation economics, except as requirements of federally owned defense watercraft, and craft engaged in foreign commerce and other deep-water transportation are controlling. When the exception applies, the federal government would finance the added cost of bridges due to navigational needs. These basic objectives would be achieved by classifying navigable waterways according to identified uses and by establishing of standard navigational clearances for bridges.

---

[4] "Navigational Clearance Requirements for Highway and Railroad Bridges," the Office of the Under Secy. of Commerce for Transportation, U. S. Dept. of Commerce, Washington, D. C., 1955, pp. 2-6.

A CHANGE IN CONCEPT CONCERNING BRIDGE CLEARANCES

The report and other developments with respect to bridge clearances clearly show that treatment of this problem is in a transitional stage. The change is from a concept that has expressly required the interests of navigation to be protected in bridge-clearance decisions, often without regard to over-all costs or transportation economics,[5] to one that expressly requires protection of the public interest with particular emphasis on over-all costs and transportation economics. The protection of the public interest also takes into account the needs of federally owned defense watercraft and other special needs of craft navigating in deep-water channels.

The Corps of Engineers (United States Department of the Army) formally recognized this concept in 1954. Following discussions with the Department of Commerce relating to the inclusion of the views of the Corps in the departmental report, the Corps issued a statement of policy, practice, and procedure governing the construction of bridges over navigable waters. This policy requires an economic analysis of comparative costs and benefits in bridge-clearance decisions.[6]

Other actions confirming the emergence of this concept are the decision of the Chief of Engineers and the Secretary of the Army on the navigational clearances for the Lyndale Avenue Bridge across the Minnesota River in Minnesota[7]; an administrative instruction issued by the BPR[8]; a statement in the Hoover Commission report on water resources and power[9]; and a recommendation made by the commission's task force which studied this problem.[10]

A NEW ERA OF NATIONAL TRANSPORTATION POLICY

In this transitional period, all surface-transportation interests stand at the threshold of a new era of national transportation policy concerning navigational clearances for bridges.[11,12] Therefore, it is possible to undertake a review in perspective. From such a review a course of action can be charted that will make it possible to appraise objectively the responsibilities that lie ahead.

---

[5] "War Department Determines Bridge Clearances Required for Public Navigation," *Civil Engineering*, Vol. 17, February, 1947, p. 71.

[6] "Bridges, Statement of Policy, Practice and Procedure," Corps of Engrs., U. S. Dept. of the Army, Washington, D. C., November, 1954.

[7] "Public Notice on the Application of the Minnesota State Highway Department for a Permit to Construct a Bridge Across the Minnesota River on Trunk Highway No. 65," Corps of Engrs., U. S. Dept. of the Army, St. Paul, Minn., June 22, 1955.

[8] "Highway-Water Resources Developments (Bridge Clearances for Navigation)," *Policy and Procedure Memorandum 50-4.1*, Bureau of Public Roads, U. S. Dept. of Commerce, Washington, D. C., July 11, 1955.

[9] "Water Resources and Power," Comm. on Organization of the Executive Branch of the Government, Vol. I, June, 1955, pp. 83–84.

[10] "Task Force Report on Water Resources and Power," *ibid.*, pp. 101–102.

[11] "Navigation Clearance Requirements for Highway Bridges," 1952 Convention Group Meetings, A.A.S.H.O., p. 28.

[12] "A Current Look at the Navigational Clearance Problem," 1953 Convention Group Meetings, A.A.S.H.O., p. 19.

It is generally agreed that the data collected for the Department of Commerce report were adequate for the purpose of policy revision. However, additional scientifically prepared information, nationwide in character, is needed to enhance the validity of the proposed standard navigational clearances for bridges. Such information also is needed for decisions concerning the navigational clearances to be required for individual structures.[13]

The data unquestionably should be prepared in conformity with the premise that the public is entitled to transportation services at the lowest possible over-all cost. This premise is sound because it does not favor any specific form of transportation. It is sound also because it recognizes the coordinate existence of all forms of transportation, and it takes into account the over-all public interest.

ACTION IS REQUIRED

Much needs to be done in the immediate future. The BPR is particularly concerned with waterways having an existing or authorized channel depth of not more than 12 ft—that is, inland streams and the intracoastal waterways. The question of bridge clearances across channels having an existing or authorized depth of more than 12 ft will not be examined herein.

In addition to the information being obtained and published by the Corps, there is a real need to identify more specifically the nature and purpose of all fixed projections and the cost of feasible watercraft alterations that extend 20 ft or more above water when the craft are light. This proposal applies to watercraft that navigate in channels having an existing or authorized depth of not more than 12 ft. The craft to be studied in detail (those having fixed projections that extend 20 ft or more above the water) represent about 20% of the total in this category. For example, of 9,953 watercraft, only 1,780 (or 18%) would be studied in detail.[14]

With respect to the craft suggested for detailed analysis, information is needed on the feasibility and cost of hinged or telescopic devices (including telescopic pilothouses) and on the monetary effect of such adaptations on watercraft operations. Information also is needed to ascertain the extent of economic losses that might result if fixed bridges restrict the operations of a few craft that have projections that cannot be modified. The evidence should show how and to what extent these losses are reflected in over-all transportation costs.

The vertical clearance for future bridges across the inland streams and intracoastal waterways should not be fixed at 20 ft above high water without regard to additional facts. Where adequate navigation exists, except in rare instances, the vertical navigational clearance for any fixed bridge across the waterway would not be less than 20 ft above water. The actual vertical navigational clearance would be based on an economic analysis of all relevant facts. In view of information which indicates that more than 80% of all watercraft operating on the inland streams and intracoastal waterways have fixed projections of less than 20 ft, there is no need for a detailed study of any of the craft in this category. The remaining craft—those having projections of 20 ft or more—are within the area of the problem that requires detailed analysis.

---

[13] "Fair Play in Navigational Clearances for Bridges," by Paul F. Royster, *American Highways*, April, 1955, p. 4.

[14] "Navigational Clearance Requirements for Highway and Railroad Bridges," the Office of the Under Secy. of Commerce for Transportation, U. S. Dept. of Commerce, Washington, D. C., 1955, pp. 49–82.

There appears to be no reason why all affected public and private waterway-transportation interests would not be willing to cooperate with the federal government by providing the additional facts.

This study of watercraft projections should clarify the extent to which federally owned craft engaged in waterway construction or maintenance, or in providing aids to navigation, may be influencing bridge clearances and bridge costs. The added bridge costs imposed on governmental agencies engaged in constructing publicly financed highway bridges to accommodate government-owned craft, without those costs being offset by equal or greater public benefits, is not sound policy. This situation can be illustrated by developments concerning navigational clearances for highway bridges across the Potomac River in Washington (D. C.). In setting forth its requirements for inclusion in the Department of Commerce report, the Coast Guard (United States Department of the Treasury) indicated that the craft it normally uses for maintenance of buoys in the channel require a vertical clearance of 97 ft.

At the public hearings held by the Corps on May 10, 1955, concerning the feasibility of the 25-ft vertical navigational clearance proposed for the Constitution Avenue Bridge as a fixed structure, the representative of the District of Columbia indicated that the inclusion of a movable span to accommodate craft requiring extreme vertical clearances would increase the construction cost of the bridge by $1,900,000. The record also revealed that a similar increase in construction costs, estimated at $1,500,000, would be expected on the anticipated replacement structure for the Old Highway Bridge if a movable span were required. Annual costs of maintaining and operating the movable spans also would be involved.

The representative of the Coast Guard at this hearing advised that by changing the type of buoys in the channel upstream from the proposed bridge, the Coast Guard could provide its services with craft that can be accommodated under the proposed bridge at an estimated increase in its operating costs of $1,000 per yr.

A similar situation existed early in 1955 with respect to the application of the state of Minnesota for a permit to construct a fixed bridge across the Minnesota River on Trunk Highway No. 65 (Lyndale Avenue). Construction of the bridge was financed with state and federal-aid funds. When the application was considered, the Second Coast Guard District recommended a vertical clearance of 50 ft above pool elevation. According to an economic analysis made by the Corps, the clearance required by the Coast Guard cutter serving the Minnesota River could be reduced by lowering the radio mast.

Because public funds pay for highway bridges and for aids to navigation, a realistic nationwide evaluation of this problem affects the public interest.

The study also should reveal whether rules prescribing the height of lights on craft need to be modified. The possibility that the public may be required to pay additional millions for highway-bridge clearances to accommodate navigational lights on relatively few watercraft, without a realistic evaluation of alternatives, is sufficient to arouse the curiosity of any taxpayer.[15,16,17]

Because certain operators have contended that lowering masts and similar projections would interfere unduly with the operation of radar and other communications media, communications experts need to explore ways and means of overcoming the difficulties. The innovations needed are no different from the improvements made in communications and transportation media in other activities. It is conceivable that in such cases a telescopic mast or telescopic tower might be more practical than a hinged mast. To ascertain what can be accomplished, some of the latest developments in communications media can be explored. The added cost of the most suitable innovation should be included with other watercraft costs that may be compared with possible reductions in bridge costs resulting from lower bridge clearances.

A special review is needed to evaluate existing minimum standard navigational clearances for bridges. The facts already available in the Department of Commerce report demonstrate the urgency of this situation. A summary of watercraft of the United States registry (except fishing and pleasure craft) that use the Gulf Intracoastal Waterway and the Mississippi River system reveals that of 9,953 craft listed, only 1,880 (or 18%) have fixed projections of 20 ft or more above water when the craft are light. Only 79 (less than 1% of the total) have fixed projections that extend 60 ft or more above the water.[18]

According to the Department of Commerce report, the Navy Department indicates it needs a vertical clearance of 59 ft across the Gulf Intracoastal Waterway.[19] The Coast Guard indicates its maximum as 61 ft on the same waterway, with lower clearances being satisfactory in certain areas.[20] However, the minimum vertical navigational clearance for fixed bridges across this waterway is 73 ft.

The evidence demonstrates that the minimum standard vertical navigational clearance for bridges across the Gulf Intracoastal Waterway is intended to serve the extremes in waterway traffic, including floating construction equipment having extreme projections not essential for movement of the craft. The evidence also shows that, in so far as this waterway is concerned, only relatively few craft need to be studied in detail to arrive at an evaluation of the effect of a lower vertical clearance for fixed bridges on waterway-transportation costs. Data concerning the number and characteristics of fishing and pleasure boats operating on these waterways also can and should be obtained.

Information is needed on the necessity of operating and maintaining movable bridges, especially in urban areas, and of requiring movable spans or extraordinary navigational clearances in new or replacement structures. The extent

---

[15] "Rules to Prevent Collisions of Vessels and Pilot Rules for Certain Inland Waters of the Atlantic and Pacific Coasts and the Coast of the Gulf of Mexico," *CG-169*, Coast Guard, U. S. Dept. of the Treasury, U. S. Govt. Printing Office, Washington, D. C., 1957, pp. 6–7.

[16] "Pilot Rules for the Western Rivers," *CG-184*, Coast Guard, U. S. Dept. of the Treasury, U. S. Govt. Printing Office, Washington, D. C., 1957, pp. 1–5.

[17] "Pilot Rules for the Great Lakes and Their Connecting and Tributary Waters," *CG-172*, Coast Guard, U. S. Dept. of the Treasury, U. S. Govt. Printing Office, Washington, D. C., 1957, pp. 1–4.

[18] "Navigational Clearance Requirements for Highway and Railroad Bridges," the Office of the Under Secy. of Commerce for Transportation, U. S. Dept. of Commerce, Washington, D. C., 1955, p. 50.

[19] *Ibid.*, p. 85.

[20] *Ibid.*, p. 90.

to which bridge costs due to navigational needs are now being offset by equal or greater savings in waterway-transportation costs should be evident. Where such savings are not indicated, corrective action may be required.[21,22]

Naval architects and marine developmental interests have a real opportunity to include adaptations in watercraft on the drawing boards or in early construction stages. Features that will enable navigation under fixed bridges having reasonable navigational clearances can be added at far less cost during construction of the craft than at a later date. Early action of this kind would be a significant step toward desirable standardization of floating units in this category.[23]

Overland-transportation interests, both highway and railroad, should ascertain more clearly their need for future bridge construction across navigable waterways, based on traffic conditions expected within the next twenty years. They also should realize that in filing an application for a permit to construct a bridge across a navigable waterway, they may request the Corps to establish navigational clearances based on transportation economics. Such requests are subject to any special needs of federally owned watercraft.

A study is needed to determine the relationship, under federal and state constitutions, between the riparian rights of a landowner abutting a waterway and the public right of navigation.

Proposals have been made in the past that a bridge might be constructed across a given waterway without regard to navigational needs if the owner of the proposed bridge acquired the riparian rights of owners of lands contiguous to the waterway and upstream from the proposed bridge. Summarized briefly, riparian rights give to the owner of land contiguous to a navigable waterway the right to (*a*) reasonable use of the water passing his property; (*b*) the flow of water past his property subject to reasonable use by other riparian owners; and (*c*) access to the waterway, including use of his banks and construction of wharves. These rights are subject to a paramount right—the public right of navigation, which, incidentally, is not a property right. The United States Congress may remove the public right of navigation on a given reach of waterway without affecting the riparian rights of a landowner. This could be accomplished by a grant of authority to construct a bridge without regard to navigational needs, or by a formal declaration that the reach of waterway is not navigable (which automatically would permit construction of a bridge without regard to navigational needs).

From a preliminary review of court decisions, it appears that under the federal constitution there is no requirement that a landowner be compensated because the public right of navigation in front of his property, or downstream therefrom, has been removed.

SIX POSSIBLE SOLUTIONS

Recognizing that action is needed in the foregoing and other areas, there are at least six possible solutions that may result from efforts to arrive at bridge

---

[21] "Navigational Clearance Requirements for Highway and Railroad Bridges," the Office of the Under Secy. of Commerce for Transportation, U. S. Dept. of Commerce, Washington, D. C., 1955, p. 47.

[22] "Are Movable Spans Economically Justified," *The American City*, September, 1955, pp. 152–154.

[23] "Navigational Clearance Requirements for Highway and Railroad Bridges," the Office of the Under Secy. of Commerce for Transportation, U. S. Dept. of Commerce, Washington, D. C., 1955, pp. 51–79.

clearances based on the lowest over-all cost to the public. A different solution may apply to different reaches of the same waterway.

Solution 1.—To obtain bridge clearances based on this concept may require the alteration and reconstruction of existing structures so as to provide greater navigational clearances than are presently available. This condition may be found on streams where waterway transportation has increased from lesser importance to considerable importance since bridges were built. In such a case, if the cost of the bridge alterations and the added cost of new bridges that are necessary because of navigational needs should be less than the expected savings in water transportation, greater clearances are economically justified and should be required.

Solution 2.—Bridge clearances that are based on this concept may require a modification of watercraft projections so that the craft can pass under fixed bridges having lower vertical clearances than are required at present (1957). In effect, the decision would be based on a comparison of added bridge costs due to watercraft needs with the cost of adjusting extreme projections so that the craft could pass under lower fixed bridge clearances than they now require. A similar method of arriving at the lowest over-all transportation cost being borne by the public also might be used; the Lyndale Avenue Bridge serves as an example.

Solution 3.—Over-all public savings might be achieved by shifting an existing waterway terminal downstream from its present location. Such a shift would permit closure or reduced operating conditions on movable-span bridges and the construction of new or replacement bridges as fixed structures affording reasonable navigational clearances.

The relative merit of such a shift in the location of a waterway terminal can be recognized when the watercraft that require excessive bridge clearances carry petroleum products. If the cost of establishing and operating a waterway terminal at a point below existing or prospective bridges (downstream from an established terminal) and the cost of pumping petroleum products to the upstream storage plant are less than the bridge costs required for continued waterway movement of these products on craft necessitating excessive bridge clearances, then these clearances should not be required.

A situation illustrating this condition exists on the Potomac River. On August 23, 1955, the Department of the Army approved the plans for constructing the proposed Constitution Avenue Bridge as a fixed structure with a vertical clearance of 27.5 ft above mean low water. As indicated previously, this fixed bridge will cost approximately $1,900,000 less than a movable-span bridge. An additional saving of $1,500,000 in construction costs can be expected through the proposed use of a fixed bridge to replace the Old Highway Bridge, an obsolete swing span. In both instances the need for operating and maintenance costs for a movable-span bridge will be eliminated.

The vertical navigational clearance approved for the Constitution Avenue Bridge and proposed for the other structure will accommodate barges and tugs, moving sand and gravel, and certain types of barges and tugs hauling petroleum products. However, the fixed structures will not permit passage of self-propelled barges and petroleum tankers that heretofore navigated this reach of the river. Testimony of waterway interests at the Corps hearing concerning this condition—and their statements that they must rely on the latter type of craft during adverse weather—led to the suggestion that the relative merits of a downstream waterway terminal and pipeline should be considered. The merits of the proposal in this instance are evident by the approval of the Constitution Avenue Bridge.

This solution would not necessarily apply solely to waterway movement of petroleum products. The movement of other products cannot always be shifted from waterways to other transportation media as readily and as economically as petroleum products can be shifted from the waterway to the pipeline. The foregoing example describes an area of transportation relationships that needs careful study in future bridge-clearance decisions.

Solution 4.—This solution involves the removal of any requirement of navigational clearance for a new bridge to cross a specific reach of waterway and allows all existing movable bridges on the reach of waterway to be permanently closed. Conditions warranting this solution may be found primarily on the upper reaches of certain navigable waterways, in urban areas where many movable bridges cross the waterway, where several new or replacement structures are contemplated, and where a relatively small volume of traffic moves on the waterway. In such cases, if expected bridge costs due solely to navigational needs cannot be offset by equal or greater savings in waterway-transportation costs, there will be no justification for navigational clearances in new or replacement bridges.

Solution 5.—The waterway can be declared nonnavigable by an act of Congress. Several such enactments are in the statute books already.[24] During its first session, the 84th Congress enacted five laws declaring parts of waterways as nonnavigable. Three of these waterways are in Connecticut,[25,26,27] the fourth is in Kenosha (Wis.),[28] and the fifth is in Boston (Mass.).[29] Such acts automatically eliminate the need for considering navigational clearances. Also, bridges can be constructed without the usual federal permits.

Solution 6.—This solution involves the retention of the status quo. Waterway interests throughout the United States can be assured that the BPR will not advocate reduction in bridge clearances when they are economically justified. As a public agency responsible for promoting the development of one means of transportation, the BPR is fully aware of the need for other forms of movement. Nevertheless, because it is charged with the responsibility of expending public funds, it must insure that every dollar spent is bringing a dollar or more in return. The profit motive that actuates private enterprise is equally applicable to the conduct of this area of the public business.

---

[24] "Navigational Clearance Requirements for Highway and Railroad Bridges," the Office of the Under Secy. of Commerce for Transportation, U. S. Dept. of Commerce, Washington, D. C., 1955, pp. 156–157.

[25] *Public Law 152* (*S. 1300*), 84th Cong., 1st Session, approved July 12, 1955, U. S. Govt. Printing Office, Washington, D. C.

[26] *Public Law 151* (*S. 1469*), 84th Cong., 1st Session, approved July 12, 1955, U. S. Govt. Printing Office, Washington, D. C

[27] *Public Law 289* (*S. 2514*), 84th Cong., 1st Session, approved August 9, 1955, U. S. Govt. Printing Office, Washington, D. C.

[28] *Public Law 169* (*S. 1250*), 84th Cong., 1st Session, approved July 26, 1955, U. S. Govt. Printing Office, Washington, D. C.

[29] *Public Law 84* (*H.R. 1816*), 84th Cong., 1st Session, approved May 13, 1955, U. S. Govt. Printing Office Washington, D. C.

JURISDICTION

The last point to be made is related to the interest of the BPR in the jurisdictional aspects of bridge clearances. It is appropriate to spell out the BPR's interest as clearly as possible.

The BPR has no interest in obtaining authority for itself to make final decisions in bridge-clearance cases. That authority is now vested by law in the Chief of Engineers and the Secretary of the Army.

The BPR's views on the jurisdictional aspects of this problem can be expressed as a continuing desire to maintain mutual respect and integrity in all the relationships concerning treatment of this problem. Under recently completed arrangements and as a part of normal operations, the BPR field offices have been instructed to furnish the Corps with needed information on expected increases in highway-bridge costs due to navigational needs. These instructions also provide for BPR action with respect to pending applications to construct highway bridges using the federal-aid system. The information being furnished in such cases should contribute toward reasonable decisions concerning navigational clearances for specific structures.

CONCLUSIONS

The BPR is especially interested in cooperating with all public and private surface-transportation agencies and groups in the development of scientifically validated techniques for bridge clearances based on transportation economics. In conjunction with the development of those techniques, it is prepared to cooperate in the determination of standard navigational clearances for highway bridges.

The services of the BPR are available to the state highway departments for economic studies of highway-bridge costs relating to the problem of navigational clearance in special cases, with a view toward determining the relationship of those costs to waterway-transportation benefits and toward recommending corrective action wherever appropriate. These studies may require the cooperation of other surface-transportation interests.

NEED FOR A REALISTIC APPROACH

By William E. Cleary[1]

Synopsis

Operators of craft on the inland waterways of the United States recognize the need for a practical approach to the problem of bridge clearances and desire to cooperate in arriving at satisfactory solutions. The agreement reached between navigation interests and the Connecticut State Highway Department with reference to clearances along the Connecticut Expressway is used as an example. Any standardized formula applied to all waterways would jeopardize water-borne commerce, and each crossing should be considered on its own merits.

Introduction

The operators of towboats, barges, and shallow-draft motor tankers on more than 28,000 miles of the navigable rivers, harbors, and inland waterways of the United States are aware of the urgent need for a practical and realistic approach to the problem of horizontal and vertical clearances on railroad and highway bridges constructed over those waterways. Although the construction of railroad bridges has remained fairly static in recent years, the tremendous increase in the number of licensed motor vehicles, used for both business and pleasure, has necessitated a comprehensive highway building program, with a resultant number of river crossings to be constructed.

The United States has most certainly become a "nation on wheels," and, indeed, a "nation on high-speed wheels." In many areas there is relocation and improvement of trunk highways and feeder roads. In particular, there is noticeable construction in state after state of the so-called "turnpikes," "thruways," or "expressways," which are toll roads constructed without federal aid.

Expressways are geared to a high-speed, uninterrupted flow of vehicular traffic. The thought of a drawspan over a navigable waterway at any point on an arterial high-speed road causes engineers to shudder because they quite naturally envision only fixed bridges to accommodate a traffic pattern of heavy density moving swiftly and without interruption.

With fixed bridges being considered "the order of the day," the inevitable controversy arises between the navigation interests, which seek clearances that will be adequate and reasonable for the present and future needs of water-borne commerce, and the agencies responsible for the design and construction of the bridges, which naturally seek to reduce costs and minimize engineering difficulties. In the past the navigation interests have been charged with being

---

Note.—Published, essentially as printed here, in April, 1956, in the Journal of the Waterways and Harbors Division, as *Proceedings Paper 937.* Positions and titles given are those in effect when the paper was approved for publication in *Transactions.*

[1] Executive Vice-Pres., The New York Tow Boat Exchange and Harbor Carriers of the Port of New York; Secy., The New York State Waterways Assn., Inc.; and Secy.-Treasurer and North Atlantic Regional Representative, The American Waterways Operators, Inc., New York, N. Y.

reactionary and arbitrary with regard to demands for excessive and unwarranted vertical and horizontal bridge clearances. Although it may be true that in isolated cases there may have been some unrealistic and selfish thinking on the part of individual vessel operators, organizations which represent the marine industry collectively have a creditable record, particularly in recent years.

It is clearly understood that each additional foot of clearance, especially of vertical clearance, adds substantially to the cost of the construction of any bridge and may well add considerably to the complexity of the engineering problems of erecting the bridge. There is complete agreement on the part of vessel owners with the desire of the designers and constructors of these bridges to hold such costs and engineering difficulties to the irreducible minimum.

However, the public right of navigation—that is, the right to proceed from place to place by water without unreasonable interference—has been clearly recognized by statute, practice, and custom since the earliest days of the United States. The proper exercise of this right is just as much in the public interest as the economical construction of highway and railroad bridges.

INDUSTRIAL SHIPPING

The ever-increasing awareness by industry of the benefits of cheap water transportation of raw materials and finished goods has resulted in a tremendous upsurge in the construction of industrial plants, powerhouses, and factories along the banks of navigable waterways, particularly in the midcontinent area. If industry is to grow and prosper, with resultant benefits to everyone, unreasonable restrictions on water-borne movements of raw materials and finished products must not be allowed to impede it.

Traffic on the inland waterways has been increased by agricultural and industrial shippers of bulk cargoes (fuels and basic raw materials and products) to the extent that the eleven principal navigation channels are saving the shippers in transportation costs more than double the total of (*a*) amortization of the cost of construction, (*b*) interest on the investment in the channels, and (*c*) the cost of maintenance and operation. One channel has a reported ratio of benefits to costs of 14.8 to 1.

The following ratios of benefits to costs prevailing on eleven principal waterways have been determined by the Corps of Engineers (United States Department of the Army):

Waterway	Ratio of Benefits to Costs	
	1953	1948
Gulf Intracoastal Waterway	14.8:1	13.5:1
Lower Mississippi River	9.5:1	5.0:1
Monongahela River	9.4:1	14.5:1
Illinois Waterway	8.1:1	5.7:1
Ohio River	7.3:1	4.1:1
Middle Mississippi River	4.4:1	3.4:1
Cumberland River	3.6:1	1.8:1
Warrior-Tombigbee Waterway	3.1:1	1.9:1
Kanawha River	2.9:1	1.9:1
New York State Barge Canal System	2.2:1	2.2:1
Upper Mississippi River	2.1:1	1.4:1

The Corps of Engineers has based the foregoing computations on direct savings in transportation costs, or on the difference between economical barge rates and the rates of competing carriers. Not included were additional savings to shippers through water-compelled railroad freight rates that are lower than the rail rates would be without barge competition. Barge transport provides another economy for the shipper in the differential in handling costs of bulk commodities. It is easier, cheaper, and faster to load or unload one 1,000-ton coal barge than twenty-five 40-ton coal cars, or one 1,000,000-gal oil barge than one hundred 10,000-gal tank cars.

Cooperation

The tug, barge, and shallow-draft motor tanker operators have been cooperating wholeheartedly with railroad and highway interests in an effort to work out reasonable and mutually acceptable vertical and horizontal navigational clearances on new bridges. These efforts have proved successful and, with increasing understanding and recognition by all interests of the problems of the other parties involved, an enlightened and constructive approach to these problems can be expected in the future.

The vessel operators in the northeastern part of the United States have reason to be proud of their record in such matters. A particular and timely example of this cooperation can be noted in the series of compromises leading to the fixing of all navigational clearances on the fixed bridges spanning the numerous rivers with a high marine-traffic-density pattern along the route of the Connecticut Expressway. Much has been accomplished through the efforts of the navigation interests and the bridge builders, but much still remains to be done.

Modification of Vessels

Vessel owners have been exploring the possibilities of the modification of the physical characteristics of their vessels in order to provide for the retraction, where feasible, of fixed projections above the normal superstructure of tugs, barges, and motor tankers in order to reduce the vertical-clearance requirements. In theory this sounds absurdly simple, and, to those unfamiliar with the practical aspects of vessel construction and operation, the caution with which the navigation interests have approached this matter appears to border on the reactionary.

However, when one seriously considers the many difficult problems involved in an industry-wide program of structural modification of the physical characteristics of vessels, the question cannot be dismissed so lightly. Historically, the waterway-transportation industry has operated on a slender margin of profit in order to attract freight. High shipyard costs of such a widespread modification program could well be a serious factor from a competitive standpoint.

Aside from the cost factor, grave doubts have arisen among many responsible and progressive vessel operators as to the safety of collapsible and retractable masts, radar antennas, and other navigational appurtenances required when a vessel is operated interchangeably in "inland" and "offshore" commerce.

The regulations, both by statute and promulgation, covering the display of certain lights are so rigid and inflexible that they make the heights of light-supporting masts entirely beyond the control of the vessel owner.

With vessels operating alternately under inland and international rules, it is interesting to note the position of the Coast Guard (United States Department of the Treasury) in this matter:

> "It might be possible to design, modify, or adjust vertical fixtures extending above the superstructures of watercraft, however, it may not be practical or safe for masts, radars, smokestacks, kingposts, antennas, etc., to be unstepped or collapsed.
>
> "The possibility also exists that the vessels, after such alterations, might violate certain provisions of the statutory international and inland Rules to Prevent Collisions of Vessels, which require that navigational lights of water-craft be placed at certain specified heights. Vessels altered in accordance with (the) conclusions * * * while operating at night might be required to alter the normal position of their navigational lights in order to pass beneath an overhead obstruction. This would be a technical violation of the aforesaid Rules. In event of collision in such cases, it is quite possible that civil liability would be predicated on the failure to show proper navigational lights at the time of casualty."

However, the marine industry is not overlooking any possibilities along these lines, and it is gratifying to report that considerable progress has already been made in lowering fixed objects on vessels, either permanently or by retraction, in order to minimize vertical-clearance requirements.

CONCLUSIONS

The problems to be solved on bridge-construction clearances vary so greatly with the location of the structure, the nature and volume of the traffic on the waterway, and other local conditions that it is felt that each crossing must be considered on its own merits, a practice which has been followed by the Corps in granting applications for the construction of any bridge over a navigable waterway.

The policies[3] of the Corps act as a continuing safeguard of the rights of navigation and other forms of surface transportation alike, with resultant benefits to all.

The navigation interests pledge their cooperation in this question, but are definitely and vigorously opposed to the setting up of constrictive so-called "standardization" formulas, which could be used to the detriment of waterborne commerce.

---

[3] "Statement of Policy, Practice and Procedure on Bridges," Corps of Engrs., U. S. Dept. of the Army, Washington, D. C., November, 1954.

THE OPERATOR'S VIEW

BY NICKLES L. CARUTHERS[1]

SYNOPSIS

Marine interests have become increasingly aware of the problem of bridge clearances across navigable waterways and recognize the need for a realistic approach to its solution. The increase in waterways traffic, the larger sizes of tows, safety of operations, and statutory requirements combine to prevent the adoption of set standards for bridge clearances. These points are developed to illustrate that each crossing should be considered on its own merits.

INTRODUCTION

For several years marine interests have become increasingly aware of the ever-growing and pressing problem relating to bridge clearances across navigable waterways. The need for a realistic approach by navigation interests to meet this issue is clearly recognized.

In the past some marine operators have asked for excessive bridge clearances. To ascertain just what mistakes may have been made in the past is to engage in hindsight and afterthought. It is not possible to analyze the circumstances and facts that prevailed at the time the decisions with respect to such bridge clearances were made.

The continuous and ever-increasing growth of land transportation is a matter of record, and with it has come the obvious demand for an extensive highway and bridge building program. It is equally true that the inland-waterway industry has experienced a phenomenal growth during the same period. Despite the fact that waterway transportation is the oldest form of transportation, the waterway industry is still in its early stages. It has provided a service geared to the modern needs of the great industries mushrooming along the waterway system. It is particularly adapted to the efficient and economical movement of large loads.

INCREASE IN TONNAGE

Inland-waterway tonnage throughout the United States has shown remarkable increases from nearly 17 billion ton-miles, excluding the Great Lakes area, in 1937 to roughly 90 billion ton-miles in 1955. Although the national economy expands at the average rate of 4% per yr, the waterways traffic has increased 15% per yr since 1946.

The need for greater efficiency has produced innovations. The size of vessels has been tripled since the early 1930's. Tows longer than the steamship *Queen Mary* ply the inland waterways. Therefore, bridge clearances pose a

---

NOTE.—Published, essentially as printed here, in April, 1956, in the Journal of the Waterways and Harbors Division, as *Proceedings Paper 938*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Asst. to the Pres., Commercial Transport Corp., New Orleans, La.

serious problem to navigation interests and to the general public. An unrealistic approach to the problem could not only greatly hamper present waterway operations, but it could prevent the development and advent of the most economical form of mass water transportation to entire areas.

CLEARANCE PROBLEMS

Navigation interests are opposed to the inauguration of set standards for bridge clearances because it is believed that any such attempt is wholly unrealistic and impracticable. In 1948 the Corps of Engineers (United States Department of the Army) established a general standard of 73 ft vertical and 125 ft horizontal to be applied to new construction and replacement of bridges across the Gulf Intracoastal Waterway. However, such a standard was to be considered merely as a guide because each application should be considered on its own merit.

As an example, the waterway operators approved a fixed bridge across the Gulf Intracoastal Waterway with a vertical clearance of 50 ft. This clearance was approved on the premise that there is a reasonable alternate route available for the movement of heavy lift and maintenance equipment. One type of problem occurs when an alternate route is available for such equipment, and another arises when there is only one access to the area involved. It is quite true that the percentage of this type of equipment is small, but it is of primary importance. Good facilities are as important to the waterway industry as fire engines and ambulances are to cities. A sinking in a waterway may completely obstruct an important channel, and the industry is dependent on heavy lift and maintenance equipment in such emergencies. The intensive offshore oil drilling program in the gulf coast area necessitates the movement of heavy equipment. It would not be advisable to permit the construction of low fixed bridges that would hamper this important operation. Similar situations may exist in various other areas.

It is interesting to examine the progress made on the Gulf Intracoastal Waterway, which reports the highest ratio of benefits to costs of any waterway in the United States. It was not until 1929 that the Corps began digging the channel at Bayou Black, just south of Lake Charles, La., and it was not until June, 1949, that the last link from Corpus Christi, Tex., to Brownsville, Tex., was completed. It was optimistically estimated that this channel would ultimately develop 5,000,000 tons of traffic per year. This "little ditch" transported approximately 44,000,000 tons in 1953. World War II caused interruptions of work on many public projects, but it was responsible for the enlargement of this canal. While tankers were being sunk by enemy submarines in sight of the coast, warships and military cargoes flowed uninterruptedly, safe from attack. In order to meet the needs of national defense, the dimensions of the waterway were increased from 9 ft by 100 ft to 12 ft by 125 ft in 1942. The need for enlarging the canal to 16 ft by 300 ft is being studied by the Corps. It is a fact that if low fixed bridges had been built across the Gulf Intracoastal Waterway it could never have developed to the industrial and economic potential that it is at present. Some sections of this coast have been transformed from a virtual wilderness to an industrial empire.

The combination of horizontal and vertical navigational clearances increase bridge costs. It is believed that the horizontal clearance for safe navigation depends on the location of a bridge. In so far as horizontal clearance is concerned, safe navigation depends on the width and depth of a waterway, currents, wind action, and various other factors. Therefore, every bridge application should be carefully scrutinized, and horizontal clearances should be determined by the channel conditions prevailing at a particular location.

The report[2] of February, 1955, is primarily concerned with the problem of vertical clearances for fixed bridges. Navigation interests would certainly prefer fixed bridges provided that such structures do not unreasonably obstruct navigation. This is particularly true on confined waterways such as the Gulf Intracoastal Waterway, where such facilities may be built with the piers placed on the banks of the channel.

When the question of fixed structures over navigable waterways arises, one must take into consideration that the clearances provided are those that will exist for the next three-quarters of a century. As noted previously, tows have increased enormously within the past few years.

The most important tools at a pilot's command are his vision and his ability to judge distance. For example, when a small empty tow 600 ft long with the pilot house 52 ft above the water line is operated, the channel buoys will disappear from sight 540 ft ahead of the tow. When loaded the same tow will have the pilot house 45 ft above the water line, and the buoys will disappear from the line of vision 355 ft ahead of the tow. When a tow 1,000 ft long (longer tows are being operated) with the pilot house 35 ft high is operated, the vision disappears 865 ft ahead of the tow, and when the pilot house is 45 ft high, the pilot loses sight of the buoys 595 ft ahead of the tow.

Much attention has been paid to the feasibility of adjusting projections, such as radio and radar antennas, masts, and other navigational appurtenances. Some of these problems may readily be solved; others present many difficulties. For example, the display of lights is covered by statute, and this question can only be answered by legislative action. The safety factors involved in such adjustments deserve the most careful scrutiny, and the problems are being earnestly studied by the operators.

CONCLUSIONS

Much has already been accomplished through meetings of waterway operators and highway officials of various states. It is believed that the realistic approach to this problem is cooperative action on the part of all concerned.

---

[2] "Navigational Clearance Requirements for Highway and Railroad Bridges," the Office of the Under Secy. of Commerce for Transportation, U. S. Dept. of Commerce, Washington, D. C., 1955.

PROBLEMS IN NORTHEASTERN UNITED STATES

By Erhard E. Dittbrenner,[1] M. ASCE

Synopsis

The development of transportation in the northeastern United States, particularly in New England and in New York State, indicates the importance of navigation in the area as a whole and illustrates the role of navigation clearances in the integration of several transportation media. Specific examples are given of marine architecture adopted to minimum clearances, and other examples are cited in which regulation could benefit the public. The ineffectiveness of zoning in view of the present concept of navigation rights is illustrated. Clearance is a problem wherever navigation and land transport exist, and there is a need for a more complete study of all waterways and watercraft leading to a determination of clearances and navigability.

Introduction

It is evident that there are many differences of opinion concerning the bridge-clearance problem and various shades of evaluation of the many facets involved. This, of course, merely illustrates the fact that when a matter is approached from a different point of view, one sees facts differently, and, not uncommonly, different facts are seen. Only in this way can they all be examined in order to serve as a basis for an objective consensus.

Navigation clearances are of importance because of their influence on the integration of the several media of transportation for the most economical overall cost to the public. A brief resume of transportation in the northeastern states (New England, New York, and New Jersey) will illustrate the point. Much of the transportation in the United States has developed in this area, and its history can illustrate many factors.

Importance of Navigation

The ports in the foregoing area dominate the shipping on the Atlantic Ocean, on which 80% of the world's shipping operates. As of 1957 these ports handle one-third of all the water-borne tonnage in the United States—namely that on inland rivers and the Great Lakes, and coastal, intraharbor, and foreign tonnage—and more than half of all the tonnage in United States seaports. This fact is graphically illustrated in Fig. 1, in which the width of the traffic route indicates the relative proportion of shipping using that track.

The region has a tidewater seaport 150 miles inland, giving access to that part of the "water-level route" which is the first break through the Appalachian

---

Note.—Published, essentially as printed here, in April, 1956, in the Journal of the Waterways and Harbors Division, as *Proceedings Paper 939*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Highway Engr., Bureau of Public Roads, U. S. Dept. of Commerce, Albany, N. Y.

Mountains north of their southern end, and which is, traditionally, the most important route from the eastern seaboard to the west. There are few other important inland waterways other than estuaries, but the history of the region and its present economy are closely interwoven with navigation in all phases. The area traditionally has been the "workshop" of the United States and is rather sparingly provided with natural resources except for waterpower, and, hence, is much concerned with all transportation to distribute its products.

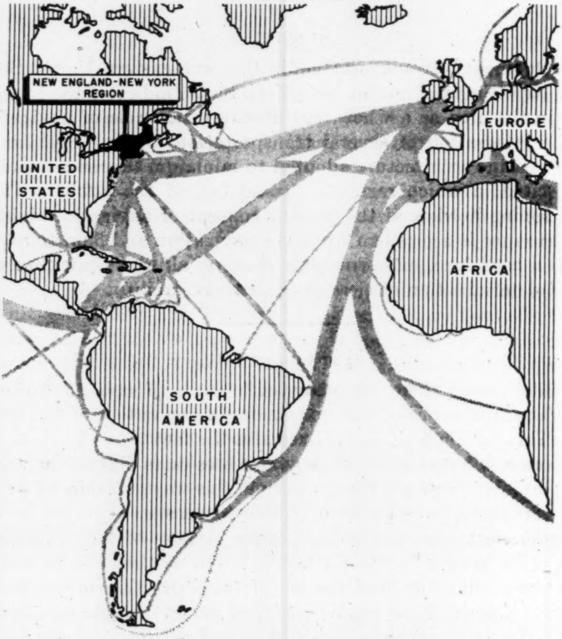


Fig. 1.—Traffic in Atlantic Ocean

Transition and Development of Transport Media

The population density is high. From the tower of the Empire State Building in New York (N. Y.), one can see the habitat of 10% of the population of the United States. Here are enough people to use all forms of transportation sufficiently to justify them; to indicate the proper place of each form of transportation in the integration that will best serve the public; and to illustrate the conflict between these forms for space, clearance, and operation.

Perhaps nowhere has there ever been the development of toll roads and canals which occurred in the state of New York in the nineteenth century. As the railroads were built westward, these toll roads and canals, with few exceptions, ceased to exist. In competition away from seaports the railroads provided a service not possible to any other media at that time. Population and industry were still too sparse in the interior to support more than one effective medium. Therefore, free highways, toll roads, and canals failed to get patronage and fell by the wayside. At the beginning of the twentieth century, no evaluation of nationally important transportation included them.

Today (1957), on the main stems of a few of these inland waterways, waterborne traffic is again in evidence. The free public highways and the toll roads are important transportation media. Population and industry support all these selectively. The famous water-level corridor contains, in addition to the sole survivor of the many canals, a major railroad system and three highways, each of them busy thoroughfares. Comparable conditions exist on waterways which are paying or have a chance to pay their way—that is, these different media follow the same general corridors, and they parallel, cross, compete with, and allegedly trespass on each other.

CONSIDERATION OF THE SEVERAL MEDIA

Many classical economists are appalled at the "economic waste" in the duplication of transportation media to be found in the United States. Because these media do not normally operate all the time at their maximum basic capacity, this duplication seems to imply to them that some of these media should be eliminated, or at least that some should not be extended. There are those who would put the inland waterways in this category; the evidence seems to contradict this theory. The return of the once outmoded transport media appears rather to have reduced the total cost of transportation. In 1929 public transportation cost the public one-thirteenth of its income; in 1952 it cost only one-nineteenth. These dates cover the period when these "outmoded" media made their "comeback." Although those in the transportation industry feel that they have not been receiving an adequate return, the difference in percentage appears too great to be explained so easily.

It seems wholly logical to assume that if public patronage supplies sufficient revenue to maintain any service, that service justifies its existence. When such support is witheld the service cannot exist for long. These statements are, of course, oversimplifications, but they state a proper basis for justification and analysis of the transportation system. That these principles operate is demonstrated by the constant decrease in railway mileage, the obsolescense and revival of some waterways, and the return of highways (especially toll roads) to a position of importance in transportation. Unquestionably, they also exercise some control on the extent to which each medium is being used and will be used. The decrease in railroad mileage does not, by any stretch of imagination, indicate obsolescence or lack of need for railroads as a whole. Neither does the resurgence of waterways traffic under certain circumstances indicate that all waterways will become economically sound arteries of transportation. Bulk commodities in quantity are the backbone of such traffic and therefore generally point only to main-stem waterways serving large population centers. Traffic on inland waterways has been growing, but so has all other traffic. Inland waterways account for only about one-sixteenth of the total transportation effort, and the ratio can hardly be considered to be changing rapidly. The changes which have been and are occurring in transportation reflect the integration of the various media into a system best adapted to present public needs. The changes also reflect the change in the relative value of each of these media, which is due, at least in part, to the fact that the public now has, can use, and does support a wider selection of transport services than it did formerly.

These considerations are of great importance to all surface-transportation media in the solution of the navigation-clearance problem. They should be and doubtless will be the factors that, in the end, will determine whether any particular watercourse will support navigation and need provision for clearances. One existing medium, or its actual or fancied possibility, should not impose a cost on other media without a corresponding benefit to the consumer.

As has been noted,[2] basic studies have been made of some of the costs incurred by highways and railways because of the provision for navigation. These studies have become public documents and are available. They are elementary but are sufficient to indicate the problem and the need for action, and to point the way to further study.

SPECIFIC ILLUSTRATIONS

A resume of the highlights of these problems in the northeastern United States will amplify the foregoing material.

The water-level corridor that has been cited applies to the Hudson River from the city of New York to Albany (N. Y.) and west from Albany up the Mohawk River, Oneida Lake, and the Ontario plains (all in New York State) to Buffalo (N. Y.). This corridor contains some of the most important surface transportation in the northeast and the United States.

The New York State Barge Canal system includes the Hudson River. A main branch goes through Lake Champlain (New York) and is a more truly water-level route than that up the Mohawk River. By way of Lake Champlain, the Richelieu River (Canada), and a small parallel canal, tidewater at Albany is connected with tidewater below Montreal (Canada). However, it should be noted that this fact in itself induces little waterway traffic. The state of New York built the barge canal in the corridor west of Albany, beginning in 1905 after traffic on the then-too-small Erie Canal had practically vanished. What is now the New York Central Railroad had occupied this corridor for many years by that time. There are (as of 1957) 309 highway and railway bridges over the canal system. Because both the highways and the canal are under a common stewardship, it would be reasonable to assume that each would be treated as equitably as possible. The state has required 15 ft of clearance above canal water level for a canal limited to a depth of 12 ft over lock sills. In a modernization program begun during the 1930's, certain sections were proposed to be and are being given 20 ft of clearance.

Under such conditions the shipper built his craft. Fig. 2 shows a typical tow operating on the canal (and on the Hudson River). This tow has the

---

[2] "Bridge Clearances: The Interest of the Bureau of Public Roads," by Walter Kurylo, *Transactions*, ASCE, Vol. 123, 1958.

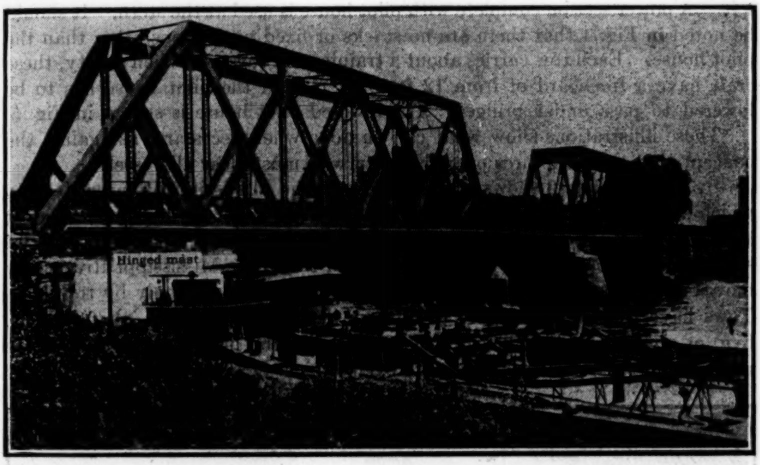


FIG. 2.—TYPICAL BARGE CANAL OIL TOW WITH HINGED MAST AT STERN

mast carrying the after range light hinged, and the funnel is just even with the pilot house. Fig. 3 shows the basic limitation of the size of tow—it fills the lock. As a matter of interest, the locks giving entrance to the old Erie Canal are visible at the right in Fig. 3.

For dry cargo a somewhat different craft is used—an articulated, self-contained connection of barges. These barges are the full width of the locks less

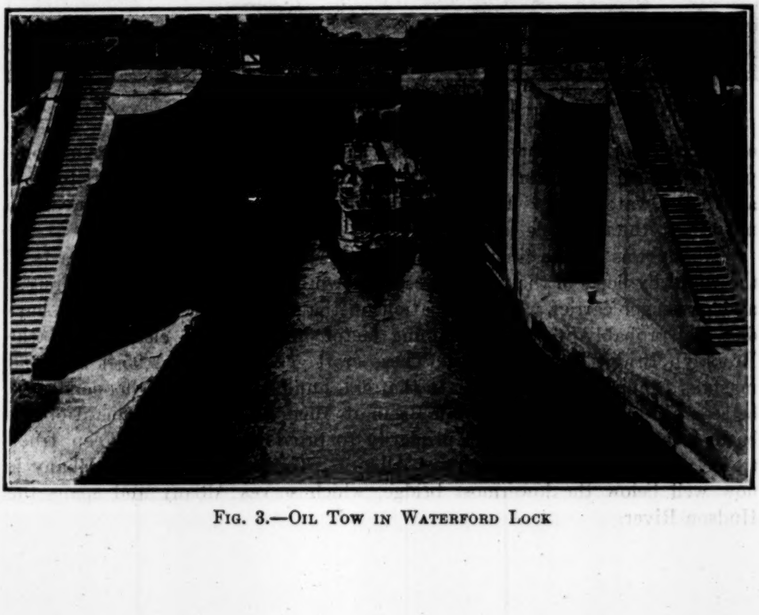


FIG. 3.—OIL TOW IN WATERFORD LOCK

1 ft. A power section complete with pilot house is used at the stern. It should be noted in Fig. 4 that there are no stacks or fixed projections other than the pilot house. Each tow carries about a trainload of cargo. When empty, these craft have a freeboard of from 12 ft to 14 ft, and the pilot house has to be lowered to pass under bridges. The lowered pilot house is shown in Fig. 5.

These illustrations show what can be done when necessary, as against the concept that the structures over the waterway must clear all vessels.

By contrast Fig. 6 shows a typical tow on the Mississippi River system, in which a 9-ft channel prevails and much of the cargo moves upstream against a stiff current. A trainload of cargo has little attraction for the operator of such a tow. Coal, once a major tonnage on the lower Mississippi River, now moves along that part of the river mostly by rail and commonly by trainload. Tows have the cargo capacity of modern deep-sea transports—that is, four, five, or six trainloads. The funnels are usually the highest projection. The modern Mississippi River towboat, even under a ruling clearance fixed by older vessels or maintenance equipment and therefore having little or no compulsion toward restriction of vessel height, has a pilot house from 25 ft to 30 ft above the water level. The tows have generous funnels, which, however, do not use up the available clearance.

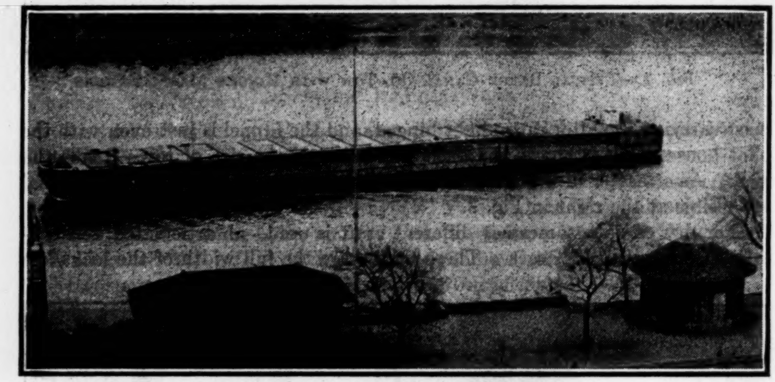


FIG. 4.—TYPICAL BARGE CANAL DRY-CARGO TOW

Albany was at one time connected with New England and other parts of the world by fleets of the type of vessel shown in Fig. 7. This was the freight and passenger carrier when navigation on the Hudson River was in its heyday and probably was a determining factor in fixing the clearance of the Brooklyn Bridge in New York. These craft generally had a 95-ft keel and masts of from 125 ft to 130 ft. In that era, hundreds of these schooners could be seen at any time in the Harbor Basin at Albany. The last remnant of this basin is being filled in (1957) primarily to provide a bypass to keep truck traffic off the downtown streets of Albany. The modern Port of Albany is now well below the lowermost bridge, which serves Albany and spans the Hudson River.

FIG. 5.—DRY-CARGO TOW WITH PILOT HOUSE LOWERED

In the 10-mile stretch of river above the present port, there are three vertical-lift bridges, giving clearances of more than 130 ft, and three swing bridges. Above that point there are only fixed bridges with barge-canal clearances. There is no substantial traffic above the port other than for the canal system. In the not-too-distant future several other bridges over the Hudson River will be needed to connect Albany and its suburbs. An important question will

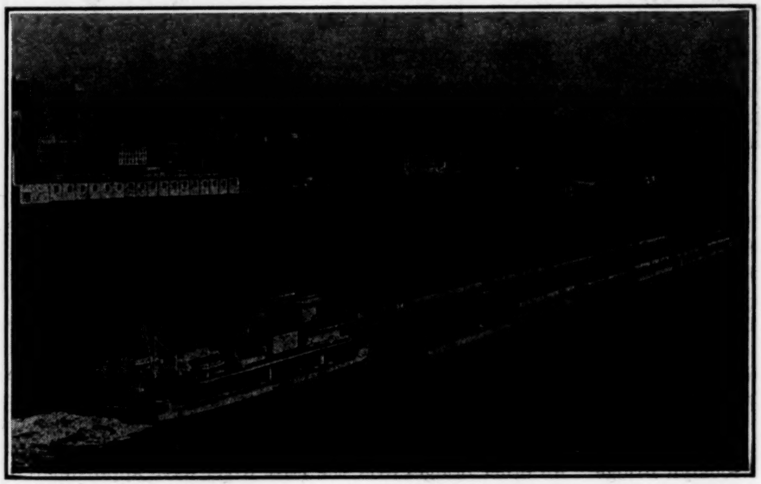


FIG. 6.—TYPICAL MISSISSIPPI RIVER TOW

then be raised: Shall this new bridge provide the same clearances as the old bridges above and below it merely because they do, or provide for present and future navigation?

At the other end of the Hudson River is New York City, one of the world's major ports. A view of this port, looking toward the Narrows, is shown in Fig. 8. It should be noted that this port has been kept clear of obstructions. The lowermost bridge over the Hudson River, the George Washington Bridge, is shown in Fig. 9; this bridge is no obstruction. The clearance of over 252 ft is more than 50 ft greater than needed by any existing or contemplated vessel. The proposed lower deck on this bridge will use only a fraction of this excess clearance.

The island of Manhattan dominates the city and the port. The island is surrounded by the Hudson River, the East River, and the Harlem River. The East River connects the Hudson River with Long Island Sound; the Harlem River connects the Hudson River and East River. The boroughs of Brooklyn, Queens, and the Bronx lie east of Manhattan, and these four boroughs surround the work area of the New York part of the port. The Brooklyn Bridge, which was the first permanent bridge to be built in the port, is the lowermost bridge over the East River. This bridge fixed the clearance over the East River at 132 ft. Nevertheless, sailing vessels, which carried the world's cargo when the bridge was built, removed their topmasts to pass under it. The Brooklyn Navy Yard was then and is now above this bridge. In Fig. 10 the *U. S. S. Saratoga* (with a hinged mast) is shown passing under the Brooklyn Bridge. All other bridges over the East River have approximately the same vertical clearance, the Triboro Bridge being the latest and uppermost.

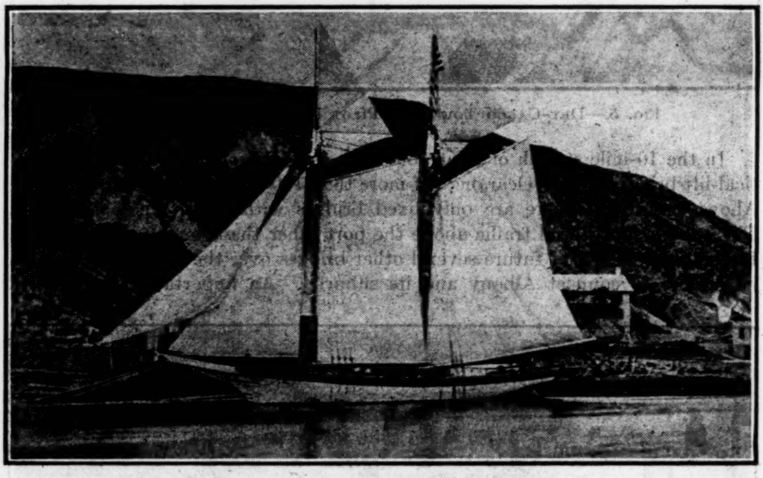


FIG. 7.—FORMER WORKHORSE ON THE HUDSON RIVER AND IN COASTAL WATERS

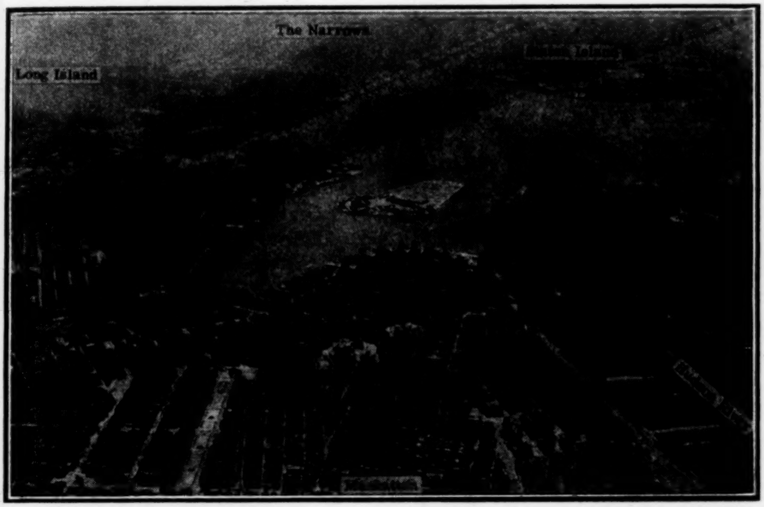

The Narrows
Long Island

FIG. 8.—PORT OF NEW YORK (N. Y.) LOOKING TOWARD THE NARROWS

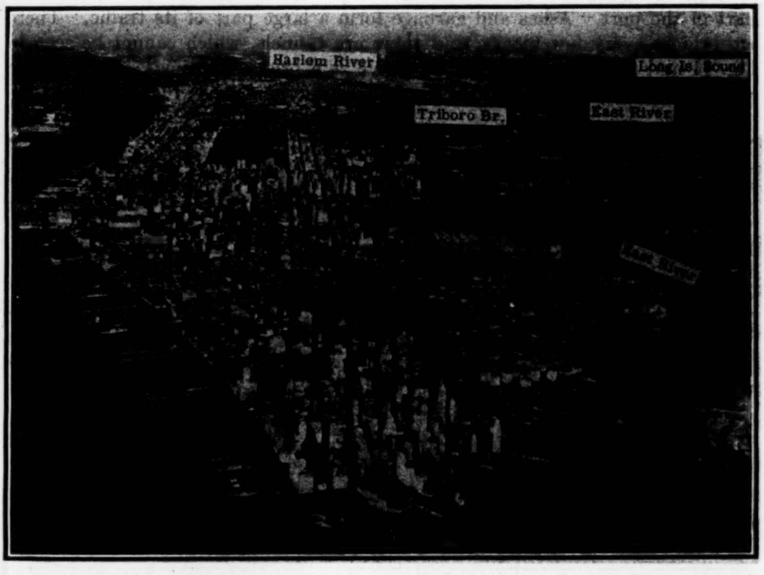

Harlem River
Long Is. Sound
Triboro Br.
East River
East River

FIG. 9.—PORT OF NEW YORK, LOOKING UP RIVER FROM STATEN ISLAND (N. Y.)

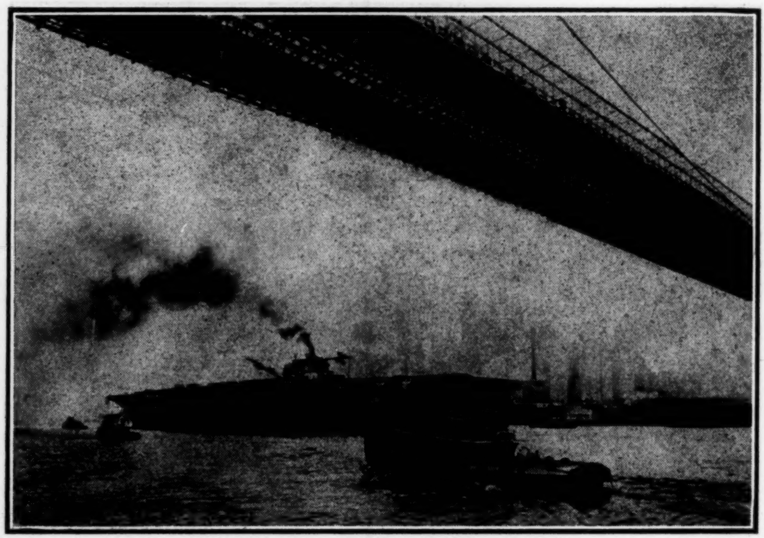


FIG 10.—*U.S.S. SARATOGA* PASSING UNDER THE BROOKLYN BRIDGE

Fig. 11 shows the Harlem River, which can be called the housekeeping part of the port. Ashes and garbage form a large part of its traffic. There are bridges every few blocks, and there are tunnels, which cannot be seen in Fig. 11 and which are twice as expensive as a bridge. Topography fixed the clearance of a few of these bridges, but most of them are swing bridges.

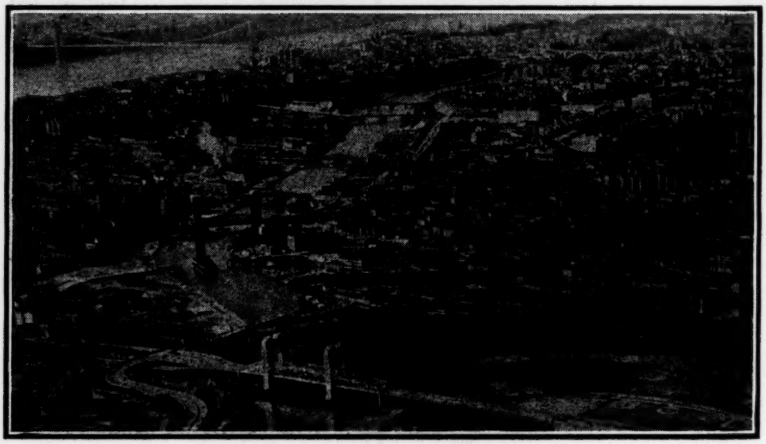


FIG. 11.—PORT OF NEW YORK, LOOKING UP THE HARLEM RIVER

Fig. 12 shows one type of craft that requires the Harlem River bridges to open. This craft is owned by a railroad, which uses this method of delivering its goods. Over an extended period of observation, only one other type of craft using this waterway did require openings—a sailing yacht with a mast 70 ft high or more; Fig. 13 shows the answer.

All craft regularly using the Harlem River are required by a regulation[3] of the Corps of Engineers (United States Department of the Army) to accommodate themselves to a clearance of 24 ft so that the bridges need not be opened. This is sufficient clearance even for the barge loads of freight cars that are freely shunted up and down the Harlem River and around the harbor.

One of the bridges shown in Fig. 11 carries the New York Central Railroad passenger trains into New York City. The total cost of this bridge was $20,000,000, of which $5,000,000 was the cost of the lift span. Because there are about 600 train movements a day over this bridge, or one every 2½ min, it can be left to the imagination as to what the opening of this bridge will do to train movements.

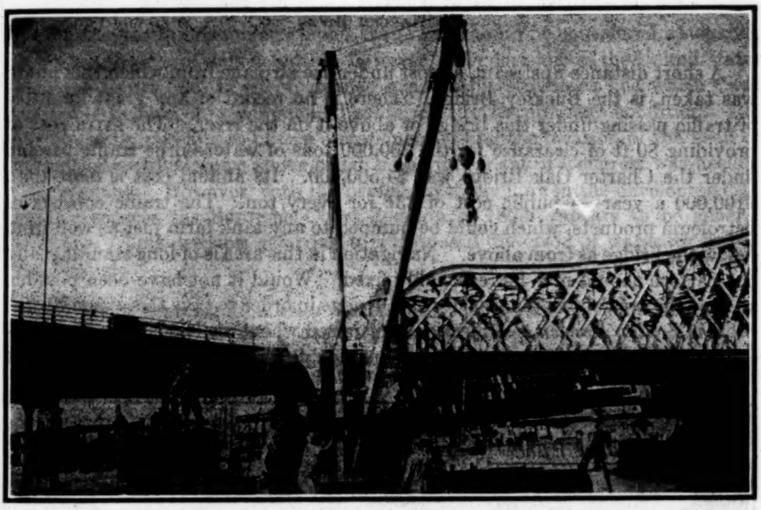


FIG. 12.—WORK CRAFT ON THE HARLEM RIVER

It is certainly reasonable to expect that an objective analysis will determine that all craft using this waterway can properly be modified to conform to the 24 ft required of other vessels and therefore eliminate movable bridges entirely.

Fig. 14 shows the Charter Oak Bridge crossing the Connecticut River at Hartford (Conn.). Fuel for the powerhouse on the right of Fig. 14 is transported by barges which need not pass under the bridge.

---

[3] "Bridge Regulation 203.160," Corps of Engrs., U. S. Dept. of the Army, Washington, D. C.

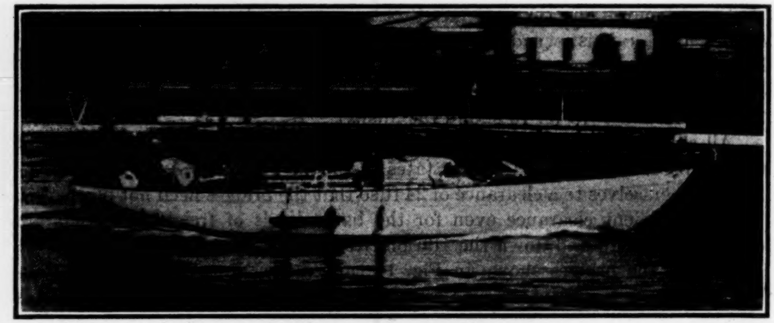


Fig. 13.—Sailing Yacht with Masts Unshipped for Transit Down the Harlem River

A short distance upstream, almost under the airplane from which this photo was taken, is the Buckley Bridge. There is no record of any great quantity of traffic passing under this bridge or above it on the river. The extra cost of providing 80 ft of clearance for the 300,000 tons of water-borne traffic passing under the Charter Oak Bridge was $4,500,000. Its annual cost is more than $100,000 a year—a public cost of 33¢ for every ton. The traffic consists of petroleum products, which could be pumped to any tank farm just as well from below the bridge as from above. Navigation in this area is of long standing, and the pattern should now be reasonably fixed. Would it not have been possible to zone the area above the Charter Oak Bridge in order to save this expenditure?

The Zoning Problem.—Objections are raised whenever it is shown that zoning restrictions would deny the use of navigable waters to anyone who might

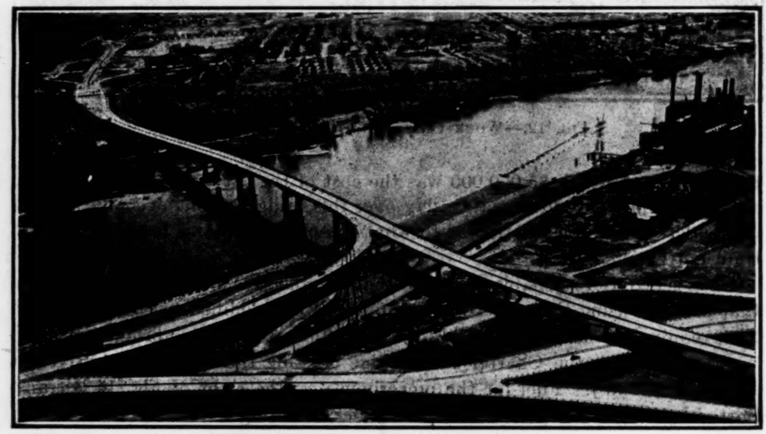


Fig. 14.—Charter Oak Bridge, Hartford (Conn.)

wish to locate a business that could, would, or might use navigation. Such objections imply that navigation is an entity having "riparian" rights on the lands bordering the waters and that no community should have the right to zone its area to prevent the establishment of a business which could impose unreasonable costs on that community. However, because under the law a navigable waterway is protected against any encroachment, such zoning would be ineffective without an act of the United States Congress. There are many instances of a single establishment located so as to require added expenditure for crossing far in excess of the value of the business to the community or to the owners. Even in cases in which the laws of the state would require full compensation to any owner (and they usually do) for the physical loss of navigation at his doorstep, the right of the community to build over navigable waters without providing for navigation can be and is still being denied.

Congress has never defined navigation, nor has it set up criteria for determining the conditions under which a waterway should be considered navigable. In the absence of such definition, however, early decisions of the United States Supreme Court had, in effect, defined a navigable stream as one which "was navigable in fact, in a commercial sense, and in its ordinary condition." It was generally possible to obtain a statement giving the location of the head of navigation on all streams above which no permit or act of Congress was needed for a crossing. However, with the passage of many laws in the development of water resources the Supreme Court decision of 1899[4] appears to be general guidance. This decision, referring to a proposed diversion of water for irrigation, stated:

> "It is not a prohibition of any obstruction to the navigation, but any obstruction to the navigable capacity, and anything, wherever done or however done, within the limits of the jurisdiction of the United States which tends to destroy the navigable capacity of one of the navigable waters of the United States, is within the terms of the prohibition."

It is quite understandable that a firm determination of the "head of navigation" can no longer be obtained except by act of Congress. The state of Connecticut has recently done so in three instances, as has been indicated by Walker Kurylo.[3] The state of Massachusetts has done the same in the case of the Fort Point Channel (in the heart of Boston, alongside its South Station) after previous experience on the Neponsett River in trying to buy out an upstream property owner so that excessive provision for navigation would not be necessary. This channel had become of no commercial value to the port and was crossed by eight movable bridges near South Station.

REVERSE RIPARIAN RIGHTS

What one may term "reverse riparian rights," for lack of a better name, is a more serious problem than any record will show. One is accustomed to think of land having riparian rights over water, not water over the adjoining land. The courts have ruled that navigation is not a property right; no landowner can acquire it by virtue of his title; and apparently no community can extinguish it by zoning or purchase. As of 1957 no agency can declare any stream nonnavigable; only Congress can do so.

---

[4] United States vs. Rio Grande Irrig. Co., U. S. Code, 174 U. S., 1899, p. 690 and p. 708; Act of September 10, 1890, Statute 26, paragraph 10, p. 426 and p. 454.

To many, navigation has become a justification for almost any expenditure that any individual wishes to make in order to use water or to protect himself against it. The effect of long-standing law has thus been magnified to the point where it is felt that the right of navigation transcends man-made statutes. This concept has been fostered by and for many honest and well-meaning interests not even remotely connected with navigation. It is one of the most serious hindrances to objective thinking that is faced in developing water resources today.

Whether the many extravagant claims purporting to benefit navigation are imaginary or substantial is immaterial in law. If Congress states that a specific act is in the interest of navigation, the courts will not entertain any question as to "fact." Congress recognized the interest of the states in their own resources in the Rivers and Harbors legislation of 1944, and declared it as policy that the states' rights to develop their own water resources were to be respected and that navigation would be considered by the Congress only if substantial. This declaration, however, even though it serves notice and encourages policy development, sheds little light on what is substantial and to whom.

CRITERIA AND APPLICATION

There are no over-all criteria for navigation clearances. They must be developed in each case by the conflicting interests and, because the existing law requires it, be in favor of navigation. If the record is accepted, then many satisfactory solutions have been reached. More likely, most of these solutions were poor compromises and satisfactory to no one. This is invariably the case when no rule or principle, justified by experience and known beforehand to all concerned, is available.

Previous studies noted[2] have examined only those situations which are physically common to waterways having navigable depths of no more than 12 ft. This comprises the major geographic area of the United States. However, such channels carry less than 10% of the total water-borne tonnage. By way of interest, more than 40% of the total water-borne commerce in rivers and ports consists of petroleum products, a prime requisite for highway construction and motor-vehicle fuel. Therefore, it is not unreasonable to assume that the rise of highways in importance as a transport medium has been a major element in the return of inland waterways traffic. This is merely another aspect of transport integration.

The foregoing conditions, particularly those of, but not confined to, the northeastern United States, clearly show that the navigation-clearance problem is not limited to channels of less than 12 ft deep but affects transportation everywhere. The public will be better served if clear and equitable criteria are established and applied, so that it is known in advance what streams are navigable and to what extent. This need not be and should not be a rigid and inflexible determination.

CONCLUSIONS

It may appear that more attention has been paid to transportation than to bridge clearances. However, the navigation-clearance problem is a transportation problem, and it is believed that a clearer concept of any facet is possible if the problem as a whole is clarified. Viewed objectively, navigation is only one medium of transportation and not by any means the major one. As the United States has developed, waterways, highways, toll roads, railroads, and air transport have been used successively. Some of these media of transportation once nearly disappeared, but they have returned. The question is not whether there should be one or the other, or that they share the traffic equally, or that one may be more important than another; the economy can use them all. Each has advantages and disadvantages; they are being integrated into their most useful pattern by public patronage. As noted previously, navigation clearances are important because of the role they play in that integration, which has a selective pattern—not a uniform one. Proper integration can occur only when the total cost to the public is at its minimum. Costs for navigation clearances are usually a major item in any specific case.

Until and unless criteria for these clearances are established and applied to every watercourse in the United States, the design of thousands of bridges will have an unsatisfactory criterion, and unnecessary costs will be borne by the public. It is certainly in the public interest that navigation be adequately provided for because it is the life blood of many communities, and in many areas, as the population increases, it will be more important. On the other hand, it is important that navigation be freed from its aura of sacredness and its implied reverse riparian right over the land. It is, and should be considered, only one of several important media of transportation.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2949

BEARING-RATIO EFFECT ON STRENGTH OF RIVETED JOINTS

By Jonathan Jones,[1] Hon. M. ASCE

Synopsis

Evidence is presented that indicates that when riveted joints of usual structural proportions are subjected to substantially static loads, the joint strength will not be reduced if the ratio of rivet bearing stress to axial stress or shearing stress is increased beyond the value that is sanctioned by most specifications for steel buildings and bridges.

Introduction

When the Research Council on Riveted and Bolted Structural Joints was organized in 1947, it designated appropriate project committees to handle seven different projects. Project No. 1 was concerned with the effect of bearing pressure on the strength of riveted joints.

The work of this project committee comprised (1) a review of prior work on the topic and (2) a series of experimental programs in the structural laboratory of the University of Illinois at Urbana.[2,3,4,5]

Review of Prior Work

At the time the project was conducted, there was little record of joint tests in which the bearing ratio had been great, or of tests directed specifically toward establishing a safe upper limit of bearing.

Note.—Published, essentially as printed here, in November, 1956, in the Journal of the Structural Division, as *Proceedings Paper 1108.* Positions and titles given are those in effect when the paper was approved for publication in *Transactions.*

[1] Chairman, Project Committee No. 1, Research Council on Riveted and Bolted Structural Joints; Retired Engr., Bethlehem, Pa.

[2] "Tests of Riveted Joints with High Rivet Bearing," by W. M. Wilson and W. H. Munse, *Progress Report*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., August 1, 1948.

[3] "The Effect of Bearing Pressure on the Static Strength of Riveted Joints," by W. K. Becker, thesis presented to the University of Illinois, at Urbana, in January, 1951, in partial fulfilment of the requirements for the degree of Master of Science.

[4] "The Effect of Bearing Pressure on the Static Strength of Riveted Joints," by J. M. Massard, thesis presented to the University of Illinois, at Urbana, in January, 1952, in partial fulfilment of the requirements for the degree of Master of Science.

[5] "The Effect of Bearing Pressure on the Static Strength of Riveted Joints," by R. C. Bergendoff, thesis presented to the University of Illinois, at Urbana, in June, 1953, in partial fulfilment of the requirements for the degree of Master of Science.

In 1938 the writer prepared a memorandum for the American Institute of Steel Construction (AISC), citing such similar German and American tests as it had been possible to locate. Information concerning the most pertinent tests may be summarized as follows:

In one case,[6] specimens contained three $\frac{5}{8}$-in.-diameter rivets in line. The rivets were in double shear, and the plates were alternately $\frac{1}{2}$ in. or $\frac{1}{4}$ in. ($S_b:S_s$ = 2:1 or 4:1). The thicker plates failed by rivet shear with a factor of safety of 3.37 and the thinner plates failed in bearing with a factor of safety of 3.02. What this failure comprised was not described. Additional tests were made,[7] and it was found that for equal safety between tension and bearing measured by ultimate load, the ratio, $S_b:S_t$, might be allowed to become as great as 3.21. At this time the German specifications adopted bearing ratios of 2.50 for static loads and 2.00 for dynamic loads, the latter being based on the concept of failure at 2,000,000 cycles.

The first fatigue tests[8] made by Otto Graf, on one-rivet connections, showed that for stress ratios of 1:0.93:2.50 a pulsating load of from 0 to 29,900 lb per sq in. of tension reached 2,000,000 cycles without failure. However, this was a "coaxing" test with initial cycles at a succession of lower and gradually increased loads.

In another series of tests,[9] the pulsating fatigue strength of a riveted joint was compared with that of a bar containing a drilled hole. A drilled bar was accepted as an upper limit to the expected resistance from a structure. It was concluded that a stress ratio of 1:0.80:2.50 would yield a pulsating fatigue strength of from 100% to 120% of that obtained from a drilled bar and therefore be satisfactory for design specifications.

Static tests were made of single-rivet specimens, with stress ratios varying in steps from 1:0.97:1.78 to 1:0.97:3.56.[10] The breaking load for the specimen with the highest bearing was within 3% of the breaking load for the specimen with the lowest bearing. The first attained 66,000 lb per sq in. in tension on the net section, as compared with the coupon ultimate of 65,000 lb per sq in.

THE COUNCIL'S PROJECT

Before presenting data from the test programs that were conducted during this project, it seems appropriate to state the following general considerations:

1. "High bearing pressure" is only a restatment of the phrase, "thin connected material" (thin in relation to the rivet used). Under working loads, because of the clamping action of the rivets, there may or may not be any actual bearing.

2. A rivet of short grip (that is, one which is driven through a certain number of plies of thin material) does not have the clamping force of a rivet of long

---

[6] "Tests on the Shearing and Bearing Strength of Riveted Connections," by H. Kayser, *Der Stahlbau*, April 17, 1931.

[7] "Further Tests on the Shearing and Bearing Strength of Riveted Connections," *ibid.*, October 13, 1933.

[8] "Fatigue Tests of Riveted Joints," by Otto Graf, *Der Bauingenieur*, Nos. 29–30, 1932.

[9] "Fatigue Strength of Riveted and Welded Joints," by K. Schaechterle, *Publications*, International Assn. of Bridge and Structural Eng., Vol. 2, 1934.

[10] "Static Tests of Riveted Joints," by Jonathan Jones, *Civil Engineering*, May, 1940, p. 287.

grip (driven through the same number of plies of thicker material). Geometrically, this should not be true, and it is presumably a function of rivet constitution and head behavior.

3. A rivet of short grip will usually fill its hole more closely than a rivet of long grip.

4. Initial slip, which represents the overcoming of friction and, consequently, brings actual bearing into existence, may, therefore, begin sooner with rivets of short grip (thin material—high bearing) than with rivets of long grip (thick material—low bearing). The initial slip should proceed over less distance for rivets of short grip than for rivets of long grip before bringing actual bearing into existence.

In static loading after initial slip has been arrested by bearing, three conditions should be noted:

a. If the critical plate is stressed in tension and there is an insufficient distance from the rivet to the unstressed end of the piece, failure can occur through the bulging out and ultimate bursting open of the plate beyond the rivet and in line with the load. This type of failure is recognized and prescribed against in the specifications for buildings of the AISC, but not in the specifications for bridges of the American Railway Engineering Association (AREA), or of the American Association of State Highway Officials (AASHO).

b. If the critical plate is stressed in tension, if there is sufficient end distance, and if the ratio of computed bearing stress to tensile stress (the bearing ratio) does not exceed that which the Council hereinafter recommends, failure will not occur as a result of the crushing of material at the surfaces in bearing, because such a failure will be preceded by the failure of the critical plate on some transverse line.

c. If the critical plate is stressed in compression or shear, after the initial slip produces actual bearing it is possible for thin material to fail by the flowing of the metal at the surfaces in bearing. The criterion is the magnitude of the stress per rivet versus the product of rivet diameter and plate thickness.

STATIC TESTS UNDER PROJECT No. 1

Summary of First, or 1949, Static Series.—A more detailed description of this series is given elsewhere.[3] The 1949 series (Table 1) yielded disappointingly low test efficiencies with respect to the ratio of breaking load to the product of the theoretical net section and the coupon ultimate strength. Moreover, the break patterns were erratic, including a few zigzag breaks through three rivets, which were contrary to the expectation under specification rules for net section, and a few rivet failures. The design expectation, as implied in Fig. 1, was a break across the two rivets in the outer row (plate-tension: rivet-shear ratio = 1:0.83).

The series was designed so that some theoretical net sections were illusory and theoretical efficiencies unattainable. For example, the theoretical efficiency of the widest and thinnest specimens was

$$\frac{13.05 - 2\ (\frac{13}{16})}{13.05} = 87.5$$

and the g/d-ratio,

$$\frac{(\frac{1}{3})\,(13.05)}{\frac{13}{16}} = 5.35$$

in which g is the distance from center to center of holes measured transversely to the line of tensile force, and d is the diameter of the holes.

TABLE 1.—SUMMARY OF 1949 STATIC SERIES

Center plate thickness, in inches	Width, w, in inches	End distance, c, in inches	Bearing ratio
	¾-IN. RIVETS		
3/4	6.46	1 1/2	1.29:1
5/8	7.40	1 1/2	1.54:1
1/2	8.82	1 3/4	1.91:1
3/8	11.17	2 3/8	2.55:1
5/16	13.05	2 1/2	3.05:1
	1-IN. RIVETS		
1	8.53	1 3/4	1.28:1
13/16	9.97	2	1.57:1
5/8	12.30	2 1/4	2.04:1
1/2	14.82	3 1/8	2.54:1
7/16	16.63	3 1/4	2.90:1

Such efficiencies are known to be illusory at such g/d-ratios, as shown in the results of the Council's program No. 2.[11] Therefore, for this series the actual values of net-section efficiency (the percentage of net area times coupon ultimate, as developed in tests) attained are not pertinent. The interesting and pertinent observation which can be made, however, from the series is: The overall efficiency (the ratio of breaking load to the product of the gross plate area and the coupon ultimate strength) was not reduced as the bearing ratios increased from 1.54 to 3.05 for ¾-in. rivets, or from 1.57 to 2.90 for 1-in. rivets.

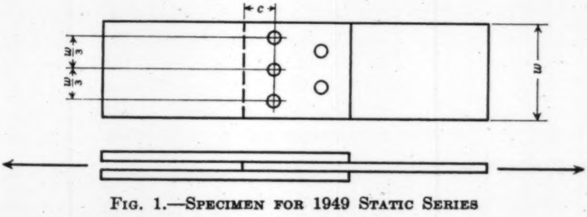


FIG. 1.—SPECIMEN FOR 1949 STATIC SERIES

Summary of Second, or 1950, Static Series.—More complete details of this series of the pattern shown in Fig. 2 have been reported elsewhere.[4] Whereas in the 1949 series the inner plate was critical and the rivet forces were delivered to it in double shear, in the 1950 series (Table 2) the inner plate was held to a constant and noncritical thickness for each of the three rivet sizes. The critical bearing was on the outer plates, in which the rivets were stressed in single shear and subjected to cantilever bending.

---

[11] "The Efficiency of Riveted Structural Joints," by F. W. Schutz, Jr., thesis presented to the University of Illinois, at Urbana, in September, 1952, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Most specimens developed a net-section efficiency of 100% or more. Three specimens developed a net-section efficiency of 99%, 98%, and 96%, respectively. One specimen (No. 50-7) developed only 89% net-section efficiency, and that was at a bearing ratio of 2.75—the greatest in its group. In this case the g/d-ratio was 6.60, so that again the theoretical efficiency could not be expected to be confirmed. There was no significant change in behavior as the bearing ratio was increased from 1.41 to 2.35.

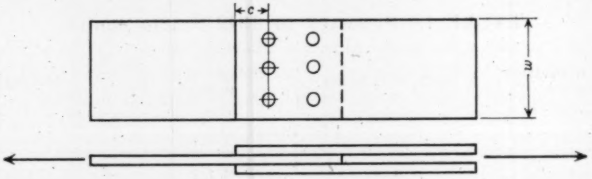


FIG. 2.—SPECIMEN FOR 1950 STATIC SERIES

TABLE 2.—SUMMARY OF 1950 STATIC SERIES

Outer plate thickness, in inches	Width, w, in inches	End distance, c, in inches	Bearing ratio
	¾-IN. RIVETS		
5/16	8.80	1 1/2	1.41:1
1/4	10.38	1 3/4	1.77:1
3/16	13.04	2 1/2	2.36:1
	⅞-IN. RIVETS		
3/8	10.02	1 3/4	1.37:1
5/16	11.48	2	1.65:1
1/4	13.64	2 1/2	2.06:1
3/16	17.24	3 1/4	2.75:1
	1-IN. RIVETS		
7/16	11.26	1 3/4	1.34:1
3/8	12.62	2	1.57:1
5/16	14.50	2 1/2	1.88:1
1/4	17.32	3	2.35:1

Unless the bearing ratio in this series exceeded 2.06, the plate developed its coupon strength, or more, and the rivets developed 45,000 lb per sq in. in shear (or more when the plate failure did not conclude the test).

Summary of Third, or 1951, Static Series.—Further details of this series are reported elsewhere.[5] Six tensile designs were tested—No. 51-1 to No. 51-6 (Fig. 3 (*a*)). Design Nos. 51-1, 51-2, and 51-3 used ¾-in. rivets and Nos. 51-4, 51-5, and 51-6 used 1-in. rivets. For each design number, three duplicate specimens were procured from one fabricator and three from another in order to check on possible differences. Such differences were usually small as measured by the breaking load, but in one instance the two stresses were 57.2 kips per sq in. and 68.1 kips per sq in., respectively. The objective in selecting this pattern was to retest it in double shear, with a more reasonable *g*/*d*-ratio (maximum 5.24) than had been used in the 1949 series. Although there was still some scatter, there was a marked improvement. For the $\frac{3}{4}$-in.-rivet series with bearing ratios of 1.56, 1.76, and 2.01, the net-section efficiencies, as previously defined, were 98%, 100%, and 97%.

For the 1-in.-rivet series with bearing ratios of 1.45, 1.71, and 2.09, the net-section efficiencies were 90%, 94%, and 85%. The latter result was the first instance of a reduction of net-section efficiency at a bearing ratio which was just slightly greater than 2.00.

Two single-shear designs (Nos. 50-x6 and 50-x7) were tested (Fig. 3(*b*)) using a single transverse row of three $\frac{7}{8}$-in. rivets. The outer or critical plates were $\frac{1}{4}$ in. and $\frac{3}{16}$ in., and the bearing ratios were 2.06 and 2.74. The latter thickness illustrates the necessity for going beyond normal design to create a single-shear bearing ratio much greater than 2.00. The six specimens with a bearing ratio of 2.06 developed a net-section efficiency of 100%. Those with a bearing ratio of 2.74 developed a net-section efficiency of 97% (specimens from one fabricator) and 82% (from the other). Efficiencies found in these tests did not approach those that would be predicted from the *g*/*d*-rule cited[11] by Frederick W. Schutz, Jr., A.M. ASCE.

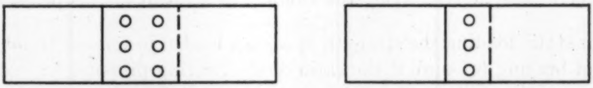

(*a*) SPECIMEN NUMBERED 51-1 TO 51-6
(*b*) SPECIMEN NUMBERED 50-x6, 50-x7

FIG. 3.—SPECIMEN FOR 1951 STATIC SERIES

The 1951 series also included six designs for testing in compression, each containing six $\frac{3}{4}$-in. rivets arranged in two longitudinal rows, or three in each line parallel to the application of stress. In three designs the inner or double-shear plate was critical, with ratios of bearing shear of 1.88, 2.36, and 4.21. In the other three designs the outer or single-shear plates were critical, with ratios of 1.88, 2.35, and 3.14.

In all compressive tests with the bearing-shear ratio not greater than 2.36, the ultimate compressive stress (maximum load divided by gross area) exceeded the coupon yield point of the critical material. In the tests with greater bearing-shear ratios, buckling intervened and established the maximum load. The compressive tests were introduced as an extension of a shorter and simpler program conducted in 1948.[2] These tests consisted of specimens with one rivet and of specimens with two rivets, in line of stress, with critical inner or double-shear plate thicknesses of $\frac{7}{16}$ in., $\frac{3}{8}$ in., and $\frac{5}{16}$ in—the rivets were $\frac{3}{4}$ in. Nominal bearing-shear ratios were 2.77, 3.13, and 3.77, respectively. All the one-rivet specimens failed by rivet shear, with no decrease of ultimate load between bearing-shear ratios of 2.77 and 3.13, and with a decrease of load not exceeding 4% from a bearing-shear ratio of 3.13 to one of 3.77. Of the six two-rivet specimens, one test was stopped at a load that was slightly greater than the yield load in order to examine conditions around the holes in the inner plate. The other five specimens failed by buckling of the inner plate, at compressive stresses of between 65,000 lb per sq in. and 88,000 lb per sq in., with no decrease of ultimate load between bearing-shear ratios of 2.77 and 3.13.

Measurements of slip at the yield point (a combination of plate strain and rivet slip) were made, showing slight differences. The slips that were measured on specimens with a bearing-shear ratio of 3.13 were slightly lower than those on specimens with ratios of either 2.77 or 3.77, and were therefore inconclusive.

Conclusions

Normal design for a tensile joint contemplates that the *g*/*d*-ratio should not exceed approximately 5 if the theoretical tensile efficiency is to be attained.[11] For joints of such design, riveted in accordance with good commercial practice in medium structural steel (comparable to that defined in Specification A7 of the American Society of Testing Materials (ASTM)), and using rivet steel comparable to that defined in ASTM Specification A141, the following conclusions have been arrived at by the Council as a result of this project:

1. For static loading the strength of a joint loaded in tension is not reduced because of bearing pressure if the ratio of the bearing pressure (S_b) to the net tensile stress (S_t) on the main material does not exceed 2.25—

$$\frac{S_b}{S_t} = \frac{\text{Net tensile area as defined by specifications}}{\text{(number of rivets) (cold rivet diameter) (plate thickness)}} \leqq 2.25 \quad (1)$$

2. For static loading the strength of a joint loaded in compression or shear is not reduced because of bearing pressure if the ratio of the bearing pressure to the shearing stress (S_s) on the rivets does not exceed 3.00—

$$\frac{S_b}{S_s} = \frac{\text{Area of total shearing surfaces of rivets}}{\text{(number of rivets) (cold rivet diameter) (plate thickness)}} \leqq 3.00 \quad (2)$$

3. In joints with no more than two rivets in the line of stress, the foregoing conclusions are correct only if the end distance, or the distance from the rivet to the unloaded end of the piece, is sufficient to avoid splitting out the rivet through the end section. The details of the latter requirement have not been explored in the project. On the contrary, such possible end weakness was avoided in the specimens tested by following the end-distance requirement of the AISC specification.[12]

4. For the ratio of rivet shear to plate tension, which is commonly found in American specifications for steel bridges and buildings, the foregoing limits may be expressed as follows:

---

[12] "Specifications for the Design, Fabrication and Erection of Structural Steel for Buildings," A.I.S.C., February, 1946, Section 23(f).

Type of stress	Stress ratio
Static tension	$S_t:S_s:S_b = 1.00:0.75:2.25$
Static compression or shear	$S_s:S_b = 1.00:3.00$

Thus, for statically loaded members,

Type of stress	Stress, in Pounds per Square Inch Buildings	 Bridges
S_t	20,000	18,000
S_s	15,000	13,500
S_b	45,000	40,500

5. No reason was found for differentiating between the allowable bearing pressure on a rivet stressed in single shear and the allowable bearing pressure on a rivet stressed in double shear.

6. At the previously recommended unit stresses, bearing in single-shear joints virtually will never be critical, nor will its computation be necessary.

7. The preceding conclusions are not applicable to high-strength steels.

Incidental Considerations

From conclusions 1 and 2 there may be derived two findings of importance in avoiding unnecessary computations as follows:

1. The shearing value of one rivet is equal to $S_s \pi d^2/4$ if in single shear, or to $S_s \pi d^2/2$ if in double shear, in which d is the rivet diameter. Also, if $S_b:S_s = 3.00:1.00$ (from conclusion 2), the value of one rivet in bearing is $3.00\ S_s\ d\ t$, in which t is the plate thickness. Thus, for single shear, $3.00\ S_s\ d\ t = S_s \pi d^2/4$, or $t/d = \frac{\pi}{(3)(4)} = 0.26$; for double shear, $3.00\ S_s\ d\ t = S_s \pi d^2/2$, or $t/d = \frac{\pi}{(3)(2)} = 0.52$. Therefore, the minimum thickness is as follows:

Rivet diameter, in inches	Minimum Thickness, in Inches Single shear	 Double shear
3/4	0.19	0.38
7/8	0.23	0.46
1	0.26	0.52

It is evident from the thicknesses that are ordinarily encountered in important bridge construction that most joints, whether in single shear or in double shear, will not require computations based on a bearing criterion.

2. Letting $g/d = 5$, which was previously mentioned as an upper limit for good design, the gross $w = 5\ d$ and the net $w = 4\ d$. Therefore, the theoretical efficiency equals $4/5 = 80\%$. Attempts to increase the efficiency by further increasing g will fail. If T and $0.75\ T$ are the allowable plate tension and rivet shear, the plate capacity equals $4\ d\ t\ T$. The required number, n, of single-shear rivets in a row may be obtained from

$$n = \frac{4\ d\ t\ T}{\frac{\pi\ d^2}{4}\ (0.75\ T)} = 6.8\ t/d$$

The bearing stress equals

$$\frac{4\,d\,t\,T}{6.8\,\frac{t}{d}\,d\,t} = 0.59\,\frac{d}{t}\,T$$

Therefore, when applied to buildings, if the maximum rivet is $\frac{7}{8}$ in. and the thinnest material for computed stress is $\frac{1}{4}$ in., the bearing stress in a single-shear connection of maximum efficiency cannot exceed 2.07 times the allowable tensile stress, $\frac{0.59\,(\frac{7}{8})}{\frac{1}{4}} = 2.07$. When applied to bridges, if the maximum rivet is 1 in. and the thinnest material for computed stress is $\frac{3}{8}$ in., the bearing stress in a connection of maximum efficiency cannot exceed 1.57 times the allowable tensile stress, $\frac{0.59\,(1)}{\frac{3}{8}} = 1.57$. Both values are less than 2.25 (conclusion 1).

In both the foregoing cases, when the theoretical efficiency is reduced to less than 80% by a closer spacing of the rivet lines, the bearing ratio will be further reduced. Therefore, in the majority of single-shear tension-member connections, it is not necessary to pay attention to bearing pressure. This is of sufficient practical importance to be stated as conclusion 6.

ACKNOWLEDGMENTS

It is generally recognized that existing practice in the design of riveted and bolted connections has been developed empirically from experience and that many of these practices and the joint capacities predicated thereon are not supported by definite scientific data. The Research Council on Riveted and Bolted Structural Joints conducts such investigations as may be necessary to determine the suitability and capacity of various types of joints used in fabricated structural frames. It is expected that such work will result in more economical and efficient practices.

The sponsoring organizations of the Research Council on Riveted and Bolted Structural Joints are as follows: The American Institute of Steel Construction, Inc.; the American Iron and Steel Institute; the Society; the Association of American Railroads; the Canadian Institute of Steel Construction, Inc.; the Engineering Foundation; the Division of Highways of the State of Illinois; the University of Illinois; the Industrial Fasteners Institute (Cleveland, Ohio); Northwestern University (Evanston, Ill.); the Bureau of Public Roads (United States Department of Commerce); and the University of Washington (Seattle).

The members of Project Committee No. 1 are: Wilbur M. Wilson, Hon. M. ASCE; Raymond Archibald, Frank Baron, Nathan M. Newmark, Thomas C. Shedd, and Lawrence T. Wyly, Members, ASCE; William H. Munse, Jr., and Cornelius Neufeld, Associate Members, ASCE; Robert B. Murphy; and the writer.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2950

THE APALACHICOLA RIVER BASIN PROJECT

By Clement P. Lindner,[1] M. ASCE

Synopsis

The primary purposes of the federal project in the Apalachicola River Basin in Florida, Georgia, and Alabama are navigation, flood control, the production of hydroelectric power, and the regulation of streamflow. Supplementary and incidental benefits are also described. Foundations were of granitoid rock; soft, porous, and cavernous limestone; low-strength, relatively nonporous limestone; and sand and clayey material. In the case of one dam, which was founded on the porous limestone, water pressures and seepage into the cofferdam were serious and unusual. Unique features and difficult problems that were encountered during the design and construction of the project and their solutions are described.

The Apalachicola River Basin

The Apalachicola River is formed by the confluence of the Flint River and the Chattahoochee River at the southwest corner of the state of Georgia near the town of Chattahoochee, Fla. From that junction, the river flows along a winding course for approximately 112 miles to Apalachicola Bay, Florida. In addition to the drainage area, which is directly tributary to the Apalachicola River, the basin includes the drainage areas of the Flint River and the Chattahoochee River. As may be seen from Fig. 1, the uppermost reach of the basin is in northeast Georgia. It extends diagonally across the state, encompassing much of the city of Atlanta, Ga., and then extends southward through Alabama and Georgia into Florida. The total drainage area is approximately 19,150 sq miles, of which approximately 8,500 sq miles are tributary to the Flint River and 8,650 sq miles are tributary to the Chattahoochee River.

The Flint River begins in the Piedmont region near Atlanta, flows through the Pine Mountain section, an upland area within the Piedmont region, and traverses the fall line to enter the coastal plain at the latitude of Columbus, Ga.

Note.—Published, essentially as printed here, in December, 1956, in the *Journal of the Waterways and Harbors Division*, as *Proceedings Paper 1120*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Chf. Engr., South Atlantic Div., Corps of Engrs., U. S. Dept. of the Army, Atlanta, Ga.

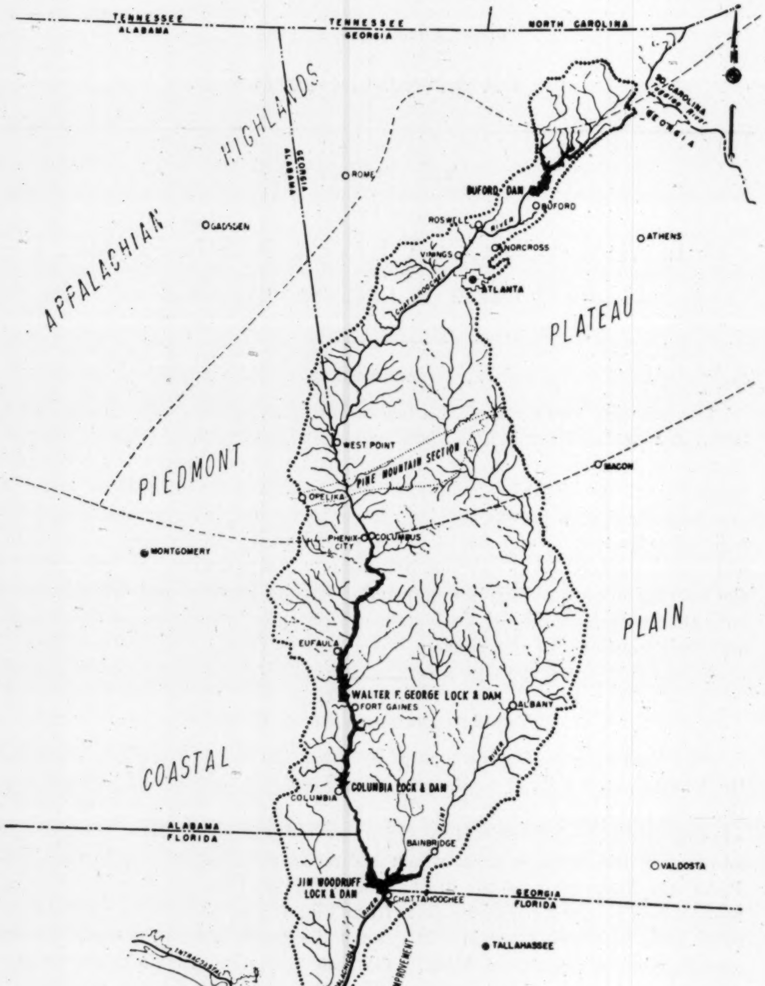

TENNESSEE
ALABAMA
TENNESSEE
GEORGIA
NORTH CAROLINA
SO. CAROLINA
Tugaloo River
GEORGIA
N
APPALACHIAN
HIGHLANDS
GEORGIA
ALABAMA
ROME
GADSDEN
BUFORD DAM
BUFORD
ROSWELL
RIVER
VININGS
NORCROSS
ATHENS
ATLANTA
CHATTAHOOCHEE
PLATEAU
WEST POINT
PINE MOUNTAIN SECTION
PIEDMONT
MACON
OPELIKA
PHENIX CITY
COLUMBUS
MONTGOMERY
PLAIN
EUFAULA
WALTER F. GEORGE LOCK & DAM
FORT GAINES
ALBANY
COASTAL
RIVER
COLUMBIA LOCK & DAM
COLUMBIA
ALABAMA
FLORIDA
FLINT
BAINBRIDGE
VALDOSTA
JIM WOODRUFF LOCK & DAM
CHATTAHOOCHEE
GEORGIA
FLORIDA
RIVER
APALACHICOLA
CHANNEL IMPROVEMENT
TALLAHASSEE
INTRACOASTAL
WATERWAY
ST MARKS
GULF
OF
MEXICO
APALACHICOLA
10 5 0 10 20 30 40 50 60 MILES

FIG. 1.—APALACHICOLA RIVER BASIN

The Chattahoochee River has its source in the Appalachian Mountain section of Georgia, and passes into and through the Piedmont region to Columbus, where it flows over a series of rapids that constitutes the fall line forming the boundary between the Piedmont region and the coastal plain. In the Piedmont and the mountain region, the valleys are generally narrow and are flanked by hills or mountains of substantial heights, the stream slopes are relatively steep, and the earth mantle is underlain with massive rock formations. The rainfall at the headwaters of the Chattahoochee River averages more than 70 in. per yr, and the mean annual runoff as far downstream as West Point, Ga., is more than 2 cu ft per sec per sq mile of drainage area, which is more than double the runoff of many eastern streams. As a result of the combination of favorable factors, there is an abundance of dam sites and reservoir sites that can potentially be developed for flood control, stream regulation, and power.

TABLE 1.—DATA ON FEDERAL PROJECT, APALACHICOLA RIVER BASIN

	Buford (Ga.) Dam	Walter F. George Lock and Dam (Ga.–Ala.)	Columbia Lock and Dam (Ala.–Ga.)	Jim Woodruff Lock and Dam (Ga.–Fla.)
Drainage area, in square miles	1,046	7,507	8,213	17,150
Normal pool elevation, in feet[a]	1,070	190	102	77
Minimum tailwater elevation, in feet[a]	915	102	77	44
Static head, in feet	155	88	25	33
Power storage, in acre-feet	1,049,000	210,000[c]	none	37,000 (pondage only)
Generating units				
Number and unit capacity	2 at 40,000 kw 1 at 6,000 kw	4 at 32,500 kw	none	3 at 10,000 kw
Total capacity	86,000 kw	130,000 kw	none	30,000 kw
Average annual output, in kilowatt-hours	170,500,000	436,000,000	none	212,000,000
Firm flood-control storage				
Between elevations, in feet[a]	1,070 and 1,085	185 and 190[d]	none	none
Quantity, in acre-feet	637,000	210,000[d]	none	none
Surcharge flood storage[b]				
Between elevations, in feet[a]	1,085 and 1,100	190 and 207	none[e]	none[e]
Quantity, in acre-feet	780,000	975,000	none[e]	none[e]
Navigation lock	none			
Size of lock chamber, in feet	...	82 by 450	82 by 450	82 by 450
Maximum lift, in feet	...	88	25	33

[a] Above mean sea level. [b] Range and amount given are for spillway design flood. [c] Used for flood-control storage during flood season (reference *d*). [d] Firm flood storage during flood season only. [e] During spillway-design flood and other extreme floods, the river stages are unaffected by the structure.

In the coastal plain region, the stream slopes are comparatively low, and the topography and geology are not favorable to the construction of dams of substantial height. However, along the Chattahoochee River the stream has eroded a shallow gorge into the limestone and the indurated earth mantle. The erosion, together with the rolling topography, makes it possible to build low and moderately high dams at sites where the foundation is adequate. The low slope is favorable for navigation development because navigation depth can be provided over many miles with fewer structures than would be necessary if the slopes were steep.

THE FEDERAL PROJECT

The existing federal project for the development of the Apalachicola, the Chattahoochee, and the Flint Rivers was adopted in the 1946 River and Harbor

Act.[2] The project provides for flood control, streamflow regulation, and power development at the Buford (Ga.) site north of Atlanta, and for a navigation channel 9 ft deep by 100 ft wide from the Gulf Intracoastal Waterway at Apalachicola, Fla., to Bainbridge, Ga., on the Flint River and to Columbus on the Chattahoochee River. The navigation channel is to be obtained by (*a*) streamflow regulation; (*b*) open-channel work in the Apalachicola River; and (*c*) construction of three locks and dams, two including power-generating facilities.

The units of the project are shown in Fig. 1. In addition to the dam at the Buford site, a lock and dam with a power installation known as the Jim Woodruff Lock and Dam is on the Apalachicola River approximately ½ mile downstream from the junction of the Flint River and the Chattahoochee River; a lock and dam without a power installation is at Columbia, Ala.; and a lock and dam with a power installation (termed the Walter F. George Lock and Dam) is at Fort Gaines, Ga. The major features of these units of the project are indicated in Table 1.

THE APALACHICOLA RIVER NAVIGATION CHANNEL

The unimproved channel of the Apalachicola River had a controlling depth at the expected minimum flow of from 4 ft to 5 ft. However, many sections had navigable depths that were equal to or more than the 9 ft provided by the adopted project, and at many sites the width at that depth was also sufficient. The preceding conditions, together with the increased low-water flow to be provided by the upstream dams and reservoirs, permitted the 9-ft depth and 100-ft width, with moderate widening at the bends to be obtained by dredging without excessive cost. The dredging required initially was approximately 2,905,000 cu yd, of which 45,000 cu yd were rock.

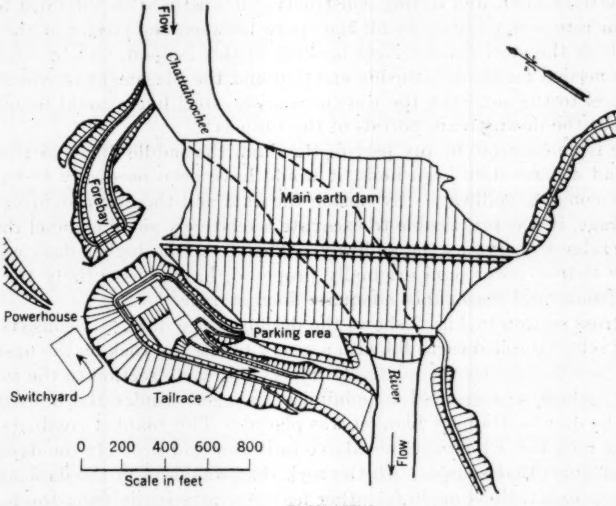
BUFORD DAM

As shown in Fig. 1, Buford Dam is on the Chattahoochee River approximately 30 miles northeast of Atlanta. It is a multiple-purpose development providing flood control, power, and streamflow regulation. The arrangement in the vicinity of the main dam and a cross section of the dam are shown in Fig. 2. The dam, which has a maximum height of approximately 200 ft, consists of three saddle dikes and the main dam, all of which are constructed of rolled-earth fill with an upstream mantle of rock. The forebay channel and tailrace channel were excavated into the hill beyond the right bank of the stream to maximum depths of almost 200 ft, of which 100 ft were in earth and 100 ft were in solid granitoid rock. Three main tunnels were driven through the rock to connect the channels. Two tunnels, each having an inside diameter of 22 ft and lined with concrete and steel, serve as penstocks. A tunnel, 10 ft in diameter and similarly lined, branches from one of the main penstock tunnels and serves the comparatively small power unit (Table 1). The third main tunnel, having an inside diameter of 13.25 ft, is the flood-control sluice and is lined only with concrete because its control is at the upstream end, and therefore, it will not be subjected to full headwater pressure. Because the forebay channel, tailrace channel, and tunnels were used for river diversion during the construction of the main dam, it was necessary to excavate the forebay channel essentially to river level and to locate the tunnels at approximately that elevation.

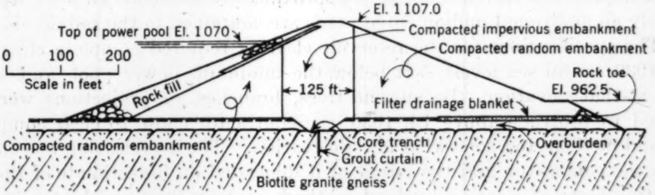
---

[2] *Public Law 525*, 79th Cong., 2d Session, July 24, 1946, Chapter 595.

The headworks were constructed against the rock face at the lower end of the forebay channel. The racks, the head gates, and the sluice gates are included in these headworks, and the gate-operating machinery surmounts them.


Flow
Chattahoochee
Forebay
Main earth dam
Powerhouse
Parking area
Switchyard
Tailrace
River
Flow
0 200 400 600 800
Scale in feet

PLAN VIEW


El. 1107.0
Top of power pool El. 1070
Compacted impervious embankment
Compacted random embankment
0 100 200
Scale in feet
Rock fill
125 ft
Rock toe
El. 962.5
Filter drainage blanket
Compacted random embankment
Core trench
Overburden
Grout curtain
Biotite granite gneiss

TYPICAL SECTION

FIG. 2.—BUFORD (GA.) DAM

The powerhouse was constructed adjacent to the vertical rock face at the head of the tailrace cut. It contains two 40,000-kw units and one 6,000-kw unit driven by Francis turbines operating at an average head of 140 ft. The average annual output of electrical energy will be approximately 170,500,000 kw-hr. The 6,000-kw unit utilizes flows that must be permitted to pass the dam for municipal and industrial needs downstream.

There are several interesting features of the powerhouse and its intake channels and discharge channels. The powerhouse is nestled in a deep rock cut; this and the forebay rock cut are approximately 100 ft deep. The total depths of these cuts, including the overburden, approach 200 ft. The depth of the open cuts was dictated by an economic study, in which it was found that the cost of tunneling plus surge tanks, which would have been required with longer tunnels, exceeded the cost of open-cut excavation. There are no steel penstocks standing free within the penstock tunnels. The steel serves simply as a watertight liner, and during construction it was used as the form behind which concrete was pumped to fill the space between the steel and the rock. By installing the steel against rigid backing in this fashion, and by using the stiffeners needed for rigidity during erection and the placement of concrete to tie the steel to the concrete, the thickness of the steel liners could be reduced except near the downstream portals of the tunnels.

There is no concrete in any part of the dam, the saddle dikes, or the spillway. Had a lower dam been built, it would have been necessary to have an expensive concrete spillway. By raising the dam and thereby providing more flood storage, it was practicable to excavate a relatively small channel through the rock ridge to serve as the spillway. Therefore, the higher dam was less expensive than a lower dam originally proposed, having essentially the same power capacity and reasonably adequate flood storage.

The cross section in Fig. 2 shows the design of the dam. It consists principally of selected rolled earth fill with a heavy blanket of rock on the upstream face. A core trench ties the relatively impervious earth section to the foundation rock, which was grouted to minimize leakage. Under the downstream third of the dam, a drainage blanket was placed. This blanket conducts seepage to the rock toe, which extends above tailwater and protects the dam from wave wash from that source. All the rock that was used in the dam was obtained from excavations made for other features, principally from the forebay cut and the tailrace cut. The total quantity of material in the main dam, which was completed in eleven months, is approximately 4,000,000 cu yd. Approximately an additional million cubic yards are contained in the saddle dikes.

The specifications for the reservoir clearing required complete clearing to El. 1030 (mean sea level), 5 ft below the minimum power pool, and a clear pool at that elevation. Because no trees, branches, or projections were permitted to protrude above El. 1030, the contractor chose to clear completely by land methods to a somewhat lower elevation. The overclearing was optional with the contractor, and the extent of it depended on his estimate of the comparative costs of clearing by land methods and by floating plant. Between the lower limits of complete clearing, the contractor topped the trees and branches at El. 1030 or lower by the use of floating equipment.

Effect of Buford Dam and Reservoir

Buford Dam and Reservoir function in conjunction with the remainder of the project downstream from Columbus by increasing the low-water flow for power production and for lockages at the downstream sites, and for maintaining navigable depths in the Apalachicola River. Its regulation was utilized in the design of the navigation channel in that stream, and, as a result, less initial dredging was required to obtain the desired low-water cross section. In addition, high tailwater during floods reduces the hydroelectric capacity at the Walter F. George powerhouse and the Jim Woodruff powerhouse. At such times, a substantial proportion of the loss can be replaced by the Buford generating units because their capability at that time will be greater than the dependable capacity during low-water periods, and, in addition, the units can be operated at overload. Therefore, Buford Dam and Reservoir constitute a fundamental part of the remainder of the project south of Columbus.

Buford Reservoir can completely control floods at the site. Its firm flood storage capacity is equal to twice the volume of the greatest flood series of record at that point. It will have a substantial effect on many floods as far downstream as West Point. With only a few exceptions, at Atlanta all floods will be restricted to bankfull stage. Table 2 shows the control that the Buford facilities could have exercised on past floods. The January, 1946, stage was the highest on record at the Vinings (Atlanta) gage. That flood could have been reduced to a no-damage stage by Buford Dam and Reservoir.

TABLE 2.—EFFECT OF BUFORD (GA.) DAM ON FLOOD STAGES

Flood date	NORCROSS (FLOOD STAGE =11.5 FT)		ROSWELL (FLOOD STAGE =11 FT)		VININGS (ATLANTA) (FLOOD STAGE =11 FT)		WEST POINT (FLOOD STAGE =16 FT)	
	Natural stage, in feet	Reduced stage, in feet	Natural stage, in feet	Reduced stage, in feet	Natural stage, in feet	Reduced stage, in feet	Natural stage, in feet	Reduced stage, in feet
March, 1942	15.2	6.5	13.8	7.8	15.1	11.5	20.1	20.1
March, 1943	11.8	3.8	11.7	5.9	13.7	9.3	20.4	20.0
March, 1944	18.0	3.5	15.8	6.2	17.6	9.0	16.0	15.5
January, 1946	27.7	5.0	23.4	6.2	28.0	7.6	19.8	17.6
February, 1946	19.4	3.2	17.0	6.3	19.2	8.4	15.6	12.3
January, 1947	21.1	9.5	17.8	10.8	20.1	13.2	19.8	19.0

However, even with the Buford facilities in operation, a few past floods would have exceeded flood stage slightly. In fact, on March 16, 1956, with only approximately 500 cu ft of water per sec being released from Buford Dam, a discharge of 15,100 cu ft per sec and a stage of 12.1 ft were recorded at Vinings. Because Buford Reservoir is a main-stem reservoir that is only a little less than 50 river miles upstream from Vinings, this emphasizes the difficulty or impracticability of furnishing a substantial degree of flood control downstream on a main river by means of small headwater storage or detention works.

The increase in low-water flow accomplished by the Buford reservoir is important and benefits not only the federal project, but also the Georgia Power Company dams, several of which are between Buford and Columbus. More important, perhaps, is the assurance it gives to industries and communities downstream, including the city of Atlanta, that there will be a dependable water supply. The minimum quantity of water to be released from the Buford reservoir will guarantee 600 cu ft per sec at Atlanta. However, the reservoir will regulate the streamflow to an average flow during low-water seasons of approximately 1,600 cu ft per sec. With a small amount of supplementary work, the communities and industries can be assured of a steady flow of at least the latter amount during low-water seasons.

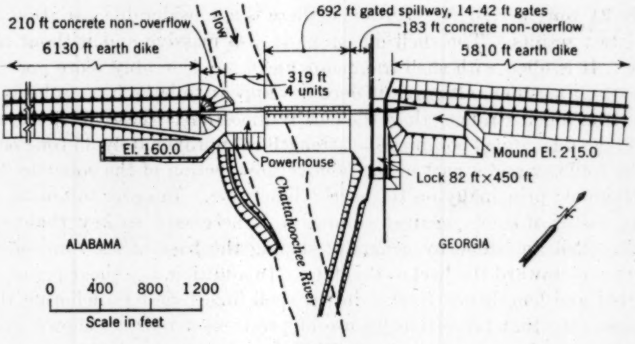
In fact, the city of Atlanta and the Georgia Power Company plan to install gates on the existing Morgan Falls Dam, a few miles upstream from Atlanta. These gates will make available sufficient pondage to raise the minimum flow to 1,000 cu ft per sec or slightly more. An application for a license has been made to the Federal Power Commission.

WALTER F. GEORGE LOCK AND DAM

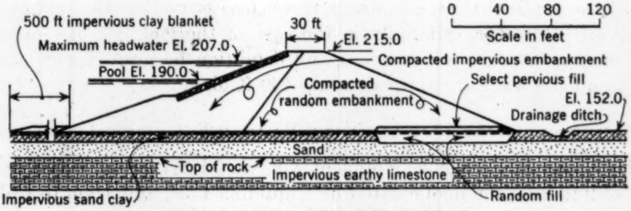
Walter F. George Lock and Dam is a multiple-purpose development that will function principally for navigation and the production of hydroelectric power, although the height of water in the reservoir will be lowered prior to the flood season, and, as a result, a measure of flood control will be provided. This development is on the Chattahoochee River approximately 85 river miles downstream from Columbus near the community of Fort Gaines (Fig. 1). The dam will consist of 11,940-ft-long earth abutments, flanking a center section containing the powerhouse, the gated spillway, the nonoverflow sections, and the lock. The arrangement and approximate lengths of individual sections are shown in Fig. 3.

The powerhouse will contain four units with a capacity of 32,500 kw each for a total capacity of 130,000 kw. The generators will be driven by high specific speed, Kaplan-type turbine runners. The estimated average annual output of the plant is 436,000,000 kw-hr. The operating net head at full load will be approximately 73 ft. During high floods the operating head will be reduced materially by tailwater rise. At such times, the reduced output will be compensated by operation at higher than dependable capacity and at overload, if necessary, of plants such as Clark Hill, Buford, Allatoona, and others that are not penalized by tailwater rise and have an excess of water when floods occur on the Chattahoochee River. As stated previously, the river channel is gorgelike for a great distance downstream as the banks are vertical or almost so. This, together with the low natural slope, accounts for the tailwater rise. For moderate increases in discharge, the rise is such that control is required of the rate at which the power units can be placed in operation and brought to full load. Otherwise, the wave that would be created would in its progress downstream constitute a navigational hazard.

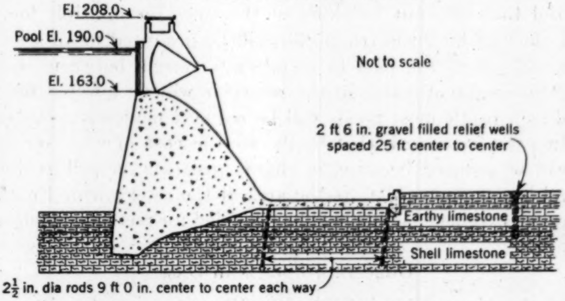
Cross sections of the gated spillway and of the west embankment are shown in Fig. 3. Materials underlying these sections, typical of the site, are also shown. Over the valley floor the general height of the earth sections of the dam will be approximately 60 ft. Immediately below the surface under the indicated embankment section and upstream and downstream from it, there is a relatively impervious blanket of varying thickness, but generally 5 ft deep or more. This is underlain by sand that is 20 ft deep or more, extending to the limestone below. Because of the impervious blanket, it was decided to utilize it to reduce seepage to acceptable limits. Therefore, the blanket was retained and strengthened by impervious fill in order to insure a blanket that is at least 5 ft thick, extending 500 ft upstream from the toe of the upstream slope of the dam. A drainage ditch will conduct the seepage to the


210 ft concrete non-overflow
6130 ft earth dike
Flow
692 ft gated spillway, 14-42 ft gates
183 ft concrete non-overflow
5810 ft earth dike
319 ft
4 units
El. 160.0
Powerhouse
Mound El. 215.0
Lock 82 ft×450 ft
Chattahoochee River
ALABAMA
GEORGIA
0
400
800
1200
Scale in feet

PLAN VIEW


500 ft impervious clay blanket
Maximum headwater El. 207.0
Pool El. 190.0
30 ft
El. 215.0
0
40
80
120
Scale in feet
Compacted impervious embankment
Select pervious fill
Compacted random embankment
El. 152.0
Drainage ditch
Sand
Top of rock
Impervious earthy limestone
Impervious sand clay
Random fill

TYPICAL SECTION-WEST EMBANKMENT


El. 208.0
Pool El. 190.0
El. 163.0
Not to scale
2 ft 6 in. gravel filled relief wells spaced 25 ft center to center
Earthy limestone
Shell limestone
2½ in. dia rods 9 ft 0 in. center to center each way

TYPICAL SPILLWAY SECTION

FIG. 3.—WALTER F. GEORGE LOCK AND DAM (GEORGIA-ALABAMA)

river. Under the downstream third of the earth section, a select, pervious drainage fill has been provided terminating in a gravel toe. If toe drains are necessary, they will be installed later.

The concrete structures (Fig. 3) are founded on rock. Two principal rock formations underlie the site. The rock nearest the surface has been designated earthy limestone, and below that there is a formation termed shell limestone. The earthy limestone is massive, without bedding planes, and relatively impervious. In so far as can be determined, it is free of solution channels. Tests indicate that its average unconfined compressive strength is approximately 21 tons per sq ft. However, there is a considerable variation in individual test results. The shell limestone is also massive and without bedding planes. It is filled with shell fragments and is considerably more porous than the earthy limestone, although it does not appear to have large solution channels. It is much stronger than the earthy limestone. Tests have shown that its average unconfined compressive strength is approximately 35 tons per sq ft.

The spillway and a part of the nonoverflow section of the concrete dam are to be founded principally on the earthy limestone. In order to obtain a satisfactory factor of safety against sliding, it is necessary to key these sections into the shell limestone by generally sloping the base of the dam downward from the toe toward the heel of the dam. In addition, the shear plane is being projected and lengthened further in the shell limestone by anchoring the stilling basin into that formation by use of prestressed rods, as shown in Fig. 3. Both the powerhouse and the lock will be founded on the shell limestone. Because of the depth of these structures, it was necessary that they extend to this formation. However, especially in the case of the lock, the use of the shell limestone appeared judicious under any condition because the maximum lift is 88 ft and the walls are approximately 130 ft high.

Results to be Obtained from the Walter F. George Development

The Walter F. George development will extend navigation with a minimum depth of 9 ft from the pool created by Columbia Lock and Dam to Columbus. The lock, which will have clear dimensions of 82 ft by 450 ft, will permit passage of substantial tows without breaking of the tows for multiple lockages. A substantial block of hydroelectric power will be produced in an area in which it has been difficult in the past to maintain a margin between capacity and demand. The height of water in the reservoir will be lowered for the flood season, and as a result most floods will be reduced modestly. An interesting fact regarding the lowering is that by its use the cost of reservoir relocations and land will be reduced because, in effect, upstream as well as downstream flood control is accomplished. As the area of the reservoir at El. 190 will be 46,000 acres, an excellent facility will be provided for recreational development and enjoyment.

Columbia Lock and Dam

Columbia Lock and Dam is located on the Chattahoochee River, approximately 29 miles downstream from Walter F. George Lock and Dam near the community of Columbia. It will consist of a low concrete dam, with a lock in the left river bank. No power installation and no flood-control storage are provided. The major part of the dam will be an uncontrolled spillway. One or two spillway gates will be provided to permit the lowering of the pool prior to the delivery of water from the Walter F. George turbines. The general layout and a cross section of the uncontrolled spillway are shown in Fig. 4. The minimum full upper pool elevation will be approximately 102 ft except when the pool is drawn down immediately prior to receiving substantial rates of discharge from the Walter F. George turbines. The maximum height of the dam from the foundation to the crest of the uncontrolled spillway will be approximately 62 ft. The maximum lift of the lock will be 25 ft, which will occur when the Jim Woodruff pool is at its static elevation of 77 ft and with no flow over the Columbia spillway. When the discharge exceeds approximately 50,000 cu ft per sec, the uncontrolled spillway will be submerged by the tailwater. For discharges of more than 100,000 cu ft per sec, vessels can pass over the dam.

Subsurface investigations have shown that there is multiple interbedding of formations of limestone, sandstone, claystone, and occasional clay with sand under artesian pressure below El. 15. Although the foundation is not ideal, it is adequate for the moderate heights and weights of structures to be placed on it.

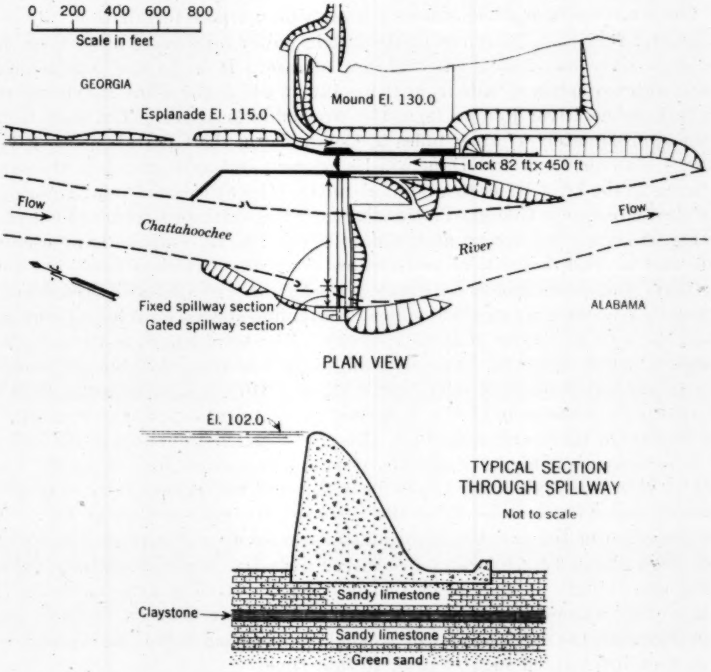

0 200 400 600 800
Scale in feet
GEORGIA
Mound El. 130.0
Esplanade El. 115.0
Lock 82 ft×450 ft
Flow
Chattahoochee
River
Flow
Fixed spillway section
Gated spillway section
ALABAMA
PLAN VIEW
El. 102.0
TYPICAL SECTION THROUGH SPILLWAY
Not to scale
Sandy limestone
Claystone
Sandy limestone
Green sand

FIG. 4.—COLUMBIA LOCK AND DAM (ALABAMA–GEORGIA)

RESULTS TO BE OBTAINED FROM COLUMBIA LOCK AND DAM

Columbia Lock and Dam will extend the navigation channel with a minimum depth of 9 ft from the Jim Woodruff pool to Walter F. George Lock and Dam, which is upstream. The lock will have a clear chamber 82 ft wide by 450 ft long, which is sufficient to permit the sized tows that are expected to use the project to pass through the lock in one lockage. By lowering the height of the Columbia pool in advance of the turbine discharge at the Walter

F. George development, the discharges will be reregulated so that the rate of rise will be reduced in the upper end of the Jim Woodruff pool and there will be less of a hazard to navigation. By proper operation the rate of rise can also be reduced in a part of the pool upstream from Columbia. This operation is desirable because of the channel characteristics that result in high increments of stage per increment of discharge change.

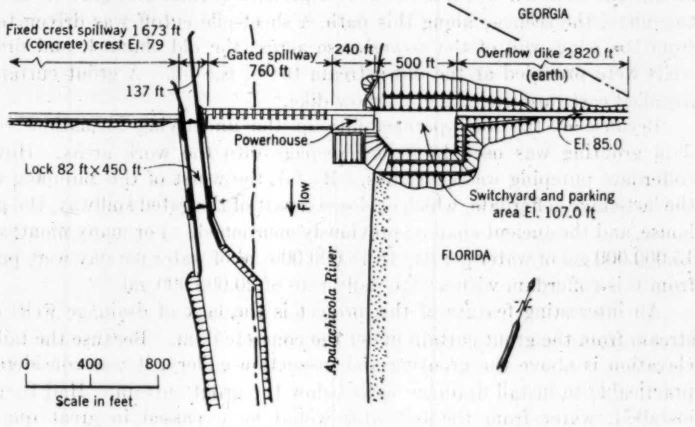
Jim Woodruff Lock and Dam

The Jim Woodruff development is a multiple-purpose installation. It provides for navigation, produces hydroelectric power, and reregulates flows for the open river navigation channel downstream. It is on the Apalachicola River, approximately ½ mile from the point at which the Flint River and the Chattahoochee River join to form the Apalachicola River. The dam backs water approximately 47 river miles to Columbia on the Chattahoochee River, where Columbia Lock and Dam will be located, and approximately the same distance up the Flint River to a point about 18 miles upstream from Bainbridge. The dam crosses the Georgia-Florida line near the southwest corner of Georgia.

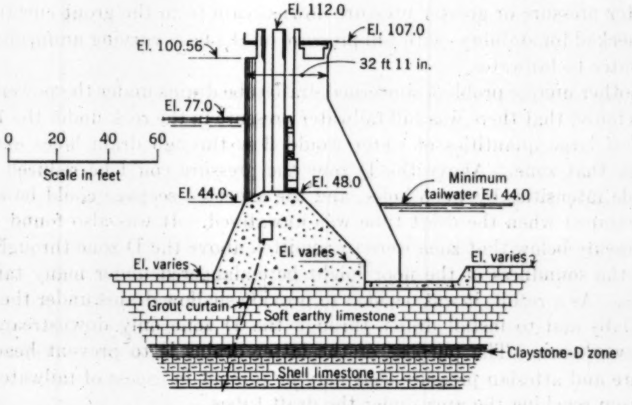
Fig. 5 shows the layout at the dam site. The development, proceeding from east to west, consists of an overflow earth dike, which is protected both upstream and downstream by grouted riprap; an elevated switchyard earth section; a powerhouse; a gated spillway containing sixteen double-leaf vertical-lift gates; a lock; and an ungated spillway. The total length of the structure is approximately 6,130 ft. The maximum height of the dam from the foundation to the gated-spillway operating deck (El. 107) is approximately 100 ft. Normal pool elevation is 77 ft. The crest of the ungated spillway is at El. 79, and the top of the overflow dike is at El. 85. The maximum lift of the lock of 33 ft occurs with minimum tailwater of 44 ft and normal pool elevation. Except in times of high water, flows pass through the power units. As flows increase beyond the capacity of the turbines, the pool is held at El. 77 by the operation of the gated spillway until its capacity is exceeded. When the pool rises above El. 79, water discharges over the ungated spillway, which has a clear length at that elevation of 1,584 ft. During extreme floods the pool and tailwater will rise so that when the overflow dike on the east bank is overtopped, the swell head will be slightly less than 2 ft. Navigation will have been interrupted slightly prior to that time.

The powerhouse contains three 10,000-kw generators driven by Kaplan-type runners. They will produce an average annual energy output of 212,000,000 kw-hr. The tailwater rise is such that during many floods the capacity of the units will be reduced, and during high floods they will be unable to operate. At such times, the lost capacity will be replaced by the operation of the Buford facilities and other plants at which an excess of capability is available beyond assigned dependable capacities. The Jim Woodruff power units normally operated on a 24-hr basis in order for sufficient water to be discharged continuously to maintain a 9-ft depth for navigation in the Apalachicola River channel downstream. Because the Walter F. George power plant, which is upstream, will not operate continuously, the operation of the power plant at the Jim Woodruff site will regulate and regularize the varying flows from upstream.

During the low-water season, especially on weekends, the foregoing may result in the Jim Woodruff pool being lowered 1 ft or 2 ft. The pool area at normal pool elevation is 37,500 acres. Therefore, a substantial pondage is available for this purpose in the top 2 ft.


GEORGIA
Fixed crest spillway 1 673 ft
(concrete) crest El. 79
Gated spillway
760 ft
240 ft
500 ft
Overflow dike 2 820 ft
(earth)
137 ft
Powerhouse
El. 85.0
Lock 82 ft × 450 ft
Flow
Switchyard and parking
area El. 107.0 ft
FLORIDA
Apalachicola River
N
0
400
800
Scale in feet

PLAN VIEW


El. 112.0
El. 107.0
El. 100.56
32 ft 11 in.
El. 77.0
0
20
40
60
Scale in feet
El. 48.0
Minimum
tailwater El. 44.0
El. 44.0
El. varies
El. varies
El. varies
Grout curtain
Soft earthy limestone
Claystone-D zone
Shell limestone

TYPICAL SECTION-GATED SPILLWAY

FIG. 5.—JIM WOODRUFF LOCK AND DAM (GEORGIA-FLORIDA)

The geologic formations underlying the structure, as shown in the cross section presented in Fig. 5, are typical of the site from the west abutment to and including the powerhouse. The rock is Tampa limestone, which is generally a soft, porous stone containing numerous solution cavities and channels. However, the stratum which is designated on the cross section as the D zone, is comparatively impervious with respect to the other layers. The stone below the D zone has the highest permeability. A deep grout curtain was required under the dam and has been provided, except for the area under the switchyard fill east of the powerhouse. The D zone does not extend across this area, and the surface of the rock is deeply depressed in what appears to be an ancient channel, which is filled with permeable sand and gravel. In order to control the seepage along this path, a sheet-pile cutoff was driven to rock from the wing wall of the powerhouse across the old channel, and drainage wells were provided at the downstream toe of the fill. A grout curtain was installed eastward under the overflow dike.

Because of the high permeability of the underlying formations, cofferdam grouting was used to reduce seepage into the work areas. However, cofferdam pumping was enormous. By far, the worst of this pumping was in the last-stage cofferdam, which enclosed a part of the gated spillway, the powerhouse, and the ancient channel previously mentioned. For many months, from 15,000,000 gal of water per day to 18,000,000 gal of water per day were pumped from this cofferdam with a high daily rate of 20,000,000 gal.

An interesting feature of this project is the lack of drainage wells downstream from the grout curtain under the concrete dam. Because the tailwater elevation is above the grouting and inspection gallery, it was considered impracticable to install drainage wells below the grout curtain. Had they been installed, water from the tailwater would have passed in great quantities through the porous rock and the drainage wells into the gallery. As a result, the dam must withstand full headwater pressure to the grout curtain and full tailwater pressure or greater pressure downstream from the grout curtain. It was checked for stability, with full pressure on the base varying uniformly from headwater to tailwater.

Another unique problem concerned draft-tube drains under the powerhouse. It was found that there was full tailwater pressure in the rock under the D zone and that large quantities of water would flow through drain holes extended through that zone. Above the D zone the pressure could be reduced to acceptable intensities by weep holes, and the expected seepage could be reasonably pumped when the draft tube was unwatered. It was also found that if the pressure below that zone were transmitted above the D zone through drain holes, the soundness of the floor might be endangered under many tailwater heights. As a result, it was decided to provide shallow drains under the draft-tube slabs and to install drains through the D zone only downstream from the powerhouse. The purpose of the latter drains is to prevent headwater pressure and artesian pressure that may be present in excess of tailwater pressure from reaching the area under the draft tubes.

Results Obtained from Jim Woodruff Lock and Dam

The Jim Woodruff development provides a minimum depth of 9 ft for navigation from the Apalachicola River channel improvement to Bainbridge on the Flint River and to the Columbia Lock and Dam site on the Chattahoochee River. The lock is of the same clear dimensions as the other locks in the project will be in order for it to be able to pass expected tows in a single lockage. The pool will regulate the varying flows from the Walter F. George development during low-water seasons, thereby providing a reasonably constant flow sufficient to maintain 9-ft navigable depths in the Apalachicola River channel downstream. The power plant will produce an annual average of 212,000,000 kw-hr of electric energy. Because it operates on a 24-hr basis, it is one of the few hydroelectric plants constructed in recent years to operate on base load during low-water seasons. In addition, the 37,500-acre lake will offer a water recreational opportunity that is unparalleled in most localities, especially for fishing and for hunting because the lake should provide a natural habitat for waterfowl.

Status of the Project and Completion Dates

Improvement of the Apalachicola River channel by dredging was completed in March, 1957. Until the channel reaches a state of relative stability and until Buford Reservoir initiates operation during low-water periods as planned, frequent maintenance dredging of the Apalachicola River channel is to be expected.

The Buford development has been completed, although the reservoir is not yet full. It is believed that it will be filled sufficiently during the calendar year, 1958, so that full operation can be initiated. The first power unit was completed and trial operation began in June, 1957, and installation of the last power unit was completed in October, 1957.

Construction began on the Walter F. George development in October, 1955, when the contractor for the west embankment initiated earth-moving operations. The gated spillway is presently (1958) under construction by contract, and substantial progress has been made toward completion of the contract. The work will continue on the development by awarding additional contracts at rates that are consistent with the completion of design and with the appropriations made. The first power unit will begin operation about September, 1962, and the entire development is scheduled for completion about June, 1963.

Construction of Columbia Lock and Dam is expected to begin in 1960. It will be completed essentially at the same time as the Walter F. George development (approximately June, 1963).

The Jim Woodruff development was essentially completed in April, 1957, when the third power unit began operating. The first power unit was placed in operation in February, 1957. The lock had been in operation for some time, but project depths were not attained throughout the project channel in the reservoir until the pool was raised to a normal elevation of 77 ft in February, 1957.

It should be noted that Buford Dam and Reservoir have been operating to control floods for about 3 yr. Power is produced presently both at Buford Dam and Jim Woodruff Dam. The dredging of the Apalachicola River channel and the filling of Jim Woodruff Reservoir have provided 9-ft navigation from the Gulf Intracoastal Waterway to Bainbridge on the Flint River and Columbia on the Chattahoochee River. Port facilities have been constructed by the state at Bainbridge, and two large companies have established separate barge terminals in the general vicinity of the state dock. Across the Chattahoochee River from Columbia, oil receiving facilities with several large storage tanks have been erected. Although only a part of the navigation project is completed, commercial utilization of the waterway appears to be developing at a more rapid rate than was anticipated. When Columbia Lock and Dam and the Walter F. George development are completed (approximately June, 1963) the entire project will be complete and all project purposes will be served. At that time, when 9-ft navigation will extend up the Chattahoochee River to Columbus, increased use of the waterway should be accelerated by the additional savings made possible by means of barge transportation over the greater distances made available thereby.

CONCLUSIONS

The federal project for the Apalachicola River Basin consists of channel improvement of the Apalachicola River; three locks and dams, one near the head of the Apalachicola River with a power installation and two on the Chattahoochee River south of Columbus (one to contain power-generating facilities); and a multiple-purpose dam and reservoir on the Chattahoochee River above Atlanta for flood control, streamflow regulation, and hydroelectric-power generation. The entire project is scheduled for completion in 1963, although all the purposes are at least partly served at the present time. In fact, streamflow regulation and flood control in the Atlanta area have been a reality for approximately 3 yr, as was flood reduction at points farther downstream, and 9-ft navigation has been available to Bainbridge and Columbia since March, 1957. When the project is completed the 9-ft navigation channel will be extended to Columbus, and hydroelectric power-generating facilities with a total capacity of 246,000 kw will have been installed that will produce an annual average of 819,000,000 kw-hr.

The three large reservoirs will offer unexcelled opportunities for water sports and recreation of all types. The two that are now essentially complete, Buford Reservoir and Jim Woodruff Reservoir, are already used extensively for these purposes. The intensive use of Buford Reservoir, which is close to the Atlanta metropolitan area, will increase greatly as the reservoir fills and with the passage of time. In addition to providing the usual recreational opportunities, Walter F. George Reservoir and Jim Woodruff Reservoir promise excellent havens for waterfowl. For that reason hunting may be popular in these reservoir areas.

In its function of regulating and conserving water supply for productive use, the project will be extremely valuable to the area served. Industries, farms, and communities, including Atlanta, will be assured of an abundant and dependable water supply. Many modern industries require a large quantity of water for cooling and processing. The water supply made available by the project should attract industries because of the economy of locating near an adequate source of water. Low-cost transportation and a satisfactory power supply at reasonable prices are also attractive to industries, and will either be provided or promoted by the project. By enabling goods to be produced and transported at less cost than elsewhere, by protecting crops and improvements from damage and destruction, and by making it possible for equal or better crops to be raised with less effort, thereby conserving energies for additional economic activities, the project will serve the economy and welfare of the entire nation.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2951

SHELLS OF DOUBLE CURVATURE

BY ALFRED L. PARME,[1] A. M. ASCE

WITH DISCUSSION BY MESSRS. TUNG AU; W. WATTERS PAGON; SANTI P. BANERJEE; MARIO G. SALVADORI; AND ALFRED L. PARME

SYNOPSIS

A comprehensive derivation of formulas for the evaluation of the membrane forces acting in any doubly curved shell is presented. For the specific case of an elliptical paraboloid shell, numerical tables are given, thus simplifying the determination of the stresses. The applicability of these tabular values to other doubly curved shells is shown together with illustrative examples.

INTRODUCTION

The great strength of doubly curved concrete shells with edges stiffened by arches or ribs is due to their ability to support any continuous load principally by direct stresses—that is, by axial compression or tension. Moreover, the stresses for these shells, including those that are extremely thin, are relatively small compared with the compressive strength of concrete. The shell is free of flexural forces except for localized bending, which may occur near the edges of a doubly curved shell, due to the effect on the shell of the displacement of the edge members. This behavior is not restricted solely to surfaces of revolution that are suitably restrained horizontally and vertically at the base, but is typical of most doubly curved shells with edge beams. As will be described subsequently, it is not necessary that the edge members be capable of resisting lateral forces.

The direct forces acting in a doubly curved shell are obtained directly from a consideration of statics only. There are innumerable coordinate systems that can be used to express the interrelationship between the internal forces acting in a shell. It has been found, however, that for the general case the Cartesian system leads to the simplest expressions.

NOTE.—Published, essentially as printed here, in September, 1956, in the Journal of the Structural Division, as *Proceedings Paper 1057*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Mgr., Structural & Railways Bureau, Portland Cement Assn., Chicago, Ill.

Adopting this coordinate system for convenience, a representative small element of a shell of double curvature is formed, as shown in Fig. 1, by two radial planes whose horizontal lines are parallel to the y-axis and by two other radial planes in which the horizontal lines are parallel to the x-axis. The direct forces, T_x and T_y, measured in pounds per unit length, are considered positive when they create tension. The shearing force, S, also measured in pounds per unit length, is positive when it creates tension in the diagonal direction of increasing values of x and y. The surface load, w, is considered positive when acting downward. The forces acting on the element are resolved into components that are parallel to the coordinate system but have their direction tangential to the surface. Thus, force T_x is parallel to the (zx)-plane but is inclined by the angle, ϕ, to the (xy)-plane.

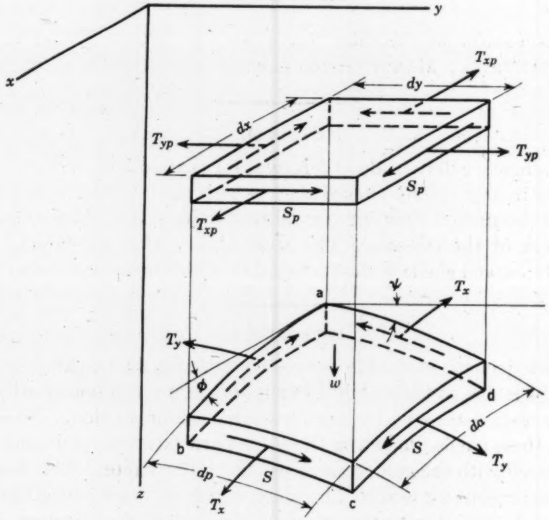


FIG. 1.—ELEMENT OF A SHELL OF DOUBLE CURVATURE

A considerable simplification[2] in the expressions for the equilibrium of forces parallel to the various axes results if the actual forces are transformed into fictitious forces acting on the projected area of the lower element in Fig. 1. From geometry it is evident that

$$dp \cos \psi = dy \qquad (1a)$$

and

$$dq \cos \phi = dx \qquad (1b)$$

The horizontal component of the normal force, T_x, acting on face ad is $T_x \cos \phi \; dp$

---

[2] "Stress Conditions in Shells Neglecting Bending," by K. W. Johansen, *Bygningsstatiske Meddelelser*, Dansk Selskab for Bygningsstatik, Copenhagen, 1938, pp. 61-84.

which, by introducing the notation of Eq. 1*a*, becomes $T_x(\cos\phi/\cos\psi)\,dy$. If the projected element is to have the same total force acting on it as the actual element,

$$T_{xp}\,dy = T_x\frac{\cos\phi}{\cos\psi}\,dy \qquad (2a)$$

or

$$T_{xp} = T_x\frac{\cos\phi}{\cos\psi} \qquad (2b)$$

Similarly,

$$T_{yp} = T_y\frac{\cos\psi}{\cos\phi} \qquad (3)$$

Equating the horizontal component of the shear acting on face *ad* to the shear on the projected element,

$$S\,dp\cos\psi = S_p\,dy \qquad (4a)$$

Substituting for the value of *dp* its value from Eq. 1*a* results in

$$S = S_p \qquad (4b)$$

Assuming that only a vertical load acts on the shell and recognizing that the forces acting on the element vary from the near face to the far face, the equilibrium of forces in the x-direction expressed in terms of T_{xp}, T_{yp}, and S_p (horizontal components of the actual forces) yields

$$\frac{\partial T_{xp}}{\partial x} + \frac{\partial S_p}{\partial y} = 0 \qquad (5)$$

Equilibrium of the forces in the y-direction results in

$$\frac{\partial T_{yp}}{\partial y} + \frac{\partial S_p}{\partial x} = 0 \qquad (6)$$

In order to establish the equations of equilibrium of forces in the z-direction, it is necessary first to obtain their vertical components. The vertical component of the normal force, T_x, acting on face *ad* is $T_x\sin\phi\,dp$. Substituting for T_x and *dp* their values as given by Eqs. 2*b* and 1*a* yields

$$T_{xp}\frac{\sin\phi}{\cos\psi}\,dy = T_{xp}\tan\phi\,dy = T_{xp}\frac{\partial z}{\partial x}\,dy \qquad (7)$$

The vertical component acting per unit of length along the y-axis is, therefore,

$T_{xp}\,(\partial z/\partial x)$. Similarly, the vertical component of T_y per unit of length along the x-axis is $T_{yp}\,(\partial z/\partial y)$. The vertical component of the shear force on face *ad* is $S\,dp \sin\psi$, which equals $S_p\,(\partial z/\partial y)\,dy$ which, per unit of length along the y-axis, equals $S_p\,(\partial z/\partial y)$. Similarly, the vertical component of shear acting on face *ab* is $S_p\,(\partial z/\partial x)$. Taking into account the variation in the magnitude of forces from one face to the other, the summation of forces in the z-direction yields

$$\frac{\partial}{\partial x}\left(T_{xp}\frac{\partial z}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yp}\frac{\partial z}{\partial y}\right) + \frac{\partial}{\partial x}\left(S_p\frac{\partial z}{\partial y}\right) + \frac{\partial}{\partial y}\left(S_p\frac{\partial z}{\partial x}\right) + w_z = 0 \ldots\ldots (8a)$$

in which w_z is the load per unit of projected area. Eq. 8*a* reduces to

$$T_{xp}\frac{\partial^2 z}{\partial x^2} + T_{yp}\frac{\partial^2 z}{\partial y^2} + 2\,S_p\frac{\partial^2 z}{\partial x\,\partial y} + \frac{\partial z}{\partial x}\left(\frac{\partial T_{xp}}{\partial x} + \frac{\partial S_p}{\partial y}\right) + \frac{\partial z}{\partial y}\left(\frac{\partial T_{yp}}{\partial y} + \frac{\partial S_p}{\partial x}\right) = -\,w_z \ldots\ldots (8b)$$

By Eqs. 5 and 6, the terms in the parentheses equal zero. Hence, Eq. 8*b* reduces to

$$T_{xp}\frac{\partial^2 z}{\partial x^2} + T_{yp}\frac{\partial^2 z}{\partial y^2} + 2\,S_p\frac{\partial^2 z}{\partial x\,\partial y} = -\,w_z \ldots\ldots\ldots\ldots\ldots (8c)$$

Eqs. 5, 6, and 8*a* can be reduced to a single equation with one unknown by introducing the function, F, so that

$$\frac{\partial^2 F}{\partial y^2} = T_{xp} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9a)$$

$$\frac{\partial^2 F}{\partial x^2} = T_{yp} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9b)$$

and

$$-\,\frac{\partial^2 F}{\partial x\,\partial y} = S_p \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9c)$$

These values satisfy the requirements of Eqs. 5 and 6 and reduce Eq. 8*c* to

$$\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 z}{\partial y^2} - 2\,\frac{\partial^2 F}{\partial x\,\partial y}\frac{\partial^2 z}{\partial x\,\partial y} = -\,w_z \ldots\ldots\ldots\ldots (10)$$

Except for a few special cases, the algebraic solution of differential Eq. 10 is difficult, and a numerical procedure such as finite differences must be used.

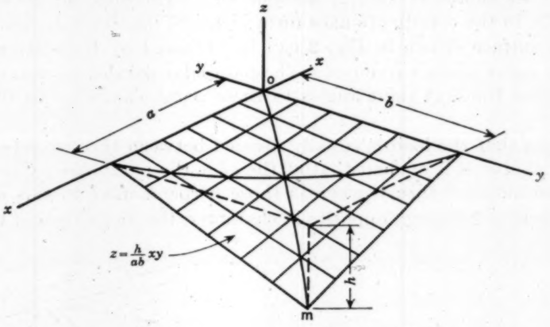


FIG. 2.—SECTIONS OF A HYPERBOLIC PARABOLOID SURFACE TAKEN AT 45° TO THE COORDINATE AXIS

One of the simpler cases to solve is the hyperbolic paraboloid shell subject to a uniform load. The surface of a hyperbolic paraboloid shell (Fig. 2) is formed by a series of straight lines parallel to the (*zx*)-plane and (*zy*)-plane and, hence, is defined by

$$z = \frac{h}{a\,b}\,x\,y \qquad (11)$$

The second differential of Eq. 11 equals zero. Therefore, for a hyperbolic paraboloid shell, Eq. 10 becomes

$$-\,2\,\frac{\partial^2 F}{\partial x\,\partial y}\,\frac{h}{a\,b} = w \qquad (12)$$

which simplifies by means of Eq. 9*c* to

$$S_p = \frac{a\,b}{2\,h}\,w \qquad (13)$$

Because the differential of S_p with respect to y and x is zero, when the direct forces normal to the edge are zero, it is seen from the relationships in Eqs. 5 and 6 that

$$T_{xp} = T_{yp} = 0 \qquad (14)$$

Eq. 14 indicates that the entire shell is subject solely to pure shear of constant intensity when uniformly loaded. Along the edges this uniform shear must be resisted by the edge member.

This state of pure shear, which actually resolves into principal stresses of equal and opposite magnitude acting on sections at 45° to shear plane, can be deduced from purely physical considerations without recourse to differential equations. As shown in Fig. 2, sections of a hyperbolic paraboloid surface taken at 45° to the coordinate axes form identical parabolic arches. In other words, the surface shown in Fig. 2 can be obtained by translating (moving) a parabolic curve along curve om. The parabolas parallel to om curve downward, whereas those at right angles to these parabolas curve in the opposite direction.

Assuming that the load is equally divided between the two sets of perpendicular parabolas, it is evident that at the edge the parabolas parallel to curve om exert an outward thrust, whereas those perpendicular to this curve exert an inward pull. Although opposite in character, the magnitude of these forces intersecting at any point on the boundary of the surface is equal because the intersecting parabolas are identical. The net effect, as shown in Fig. 3, is that the outward force acting on the edge is cancelled and only pure shear acts along the edge. This shear must be resisted by a rigid edge member. Because horizontal reactions are supplied to the ends of the parabolas by the interaction of one on the other, it is valid to assume that the load is carried by a series of parabolas.

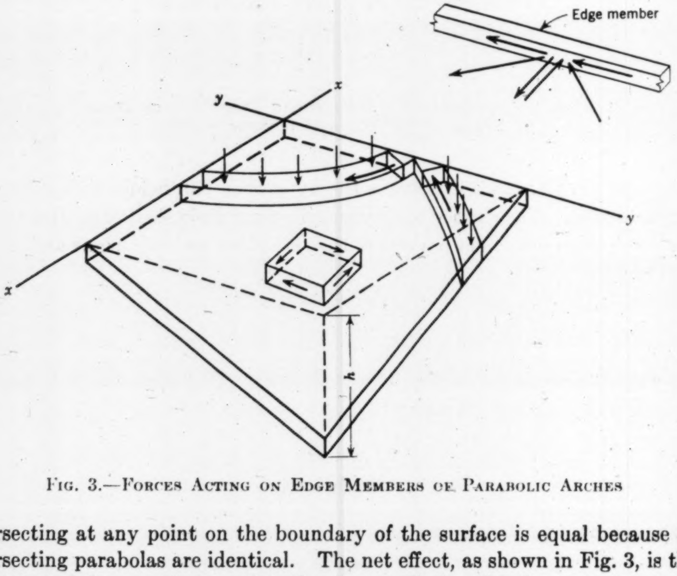


Fig. 3.—Forces Acting on Edge Members of Parabolic Arches

For most hyperbolic paraboloid shells of moderate rise, it is satisfactory to consider the load as being uniform. However, when the rise is great the dead load can no longer be considered as acting uniformly on the projected area. For this condition the dead load of the shell is

$$w_s = \frac{w}{\cos\phi \cos\psi} \qquad (15a)$$

which, by trigonometry, can be shown to equal

$$w_z = w\sqrt{\left[1+\left(\frac{h\,x}{a\,b}\right)^2\right]\left[1+\left(\frac{h\,y}{a\,b}\right)^2\right]} \qquad (15b)$$

Neglecting

$$\left(\frac{h\,x}{a\,b}\right)^2\left(\frac{h\,y}{a\,b}\right)^2$$

because it is small, Eq. 15*b* reduces to

$$w_z = w\sqrt{1+\left(\frac{h\,x}{a\,b}\right)^2+\left(\frac{h\,y}{a\,b}\right)^2} \qquad (15c)$$

From Eqs. 10 and 13,

$$-\frac{\partial^2 F}{\partial x\,\partial y}\frac{2\,h}{a\,b} = S_p\frac{2\,h}{a\,b} = w\sqrt{1+\left(\frac{h\,x}{a\,b}\right)^2+\left(\frac{h\,y}{a\,b}\right)^2} \qquad (16)$$

Differentiating Eq. 16 and integrating according to Eqs. 5 and 6 yields

$$T_{xp} = -\,w\frac{y}{2}\log\left[\frac{h\,x}{a\,b}+\sqrt{1+\left(\frac{h\,x}{a\,b}\right)^2+\left(\frac{h\,y}{a\,b}\right)^2}\right]+f(y) \qquad (17)$$

$$T_{yp} = -\,w\frac{x}{2}\log\left[\frac{h\,y}{a\,b}+\sqrt{1+\left(\frac{h\,x}{a\,b}\right)^2+\left(\frac{h\,y}{a\,b}\right)^2}\right]+f(x) \qquad (18)$$

in which $f(y)$ and $f(x)$ are constants of integration. With only one constant of integration available for each normal force and with two edges for each force—that is, at $x = 0$ and $x = a$ for T_{xp}, or at $y = 0$ and $y = b$ for T_{yp}—it is evident that, for pure membrane or direct-force action, normal reactions are required. If normal reactions are not provided along at least one of the two parallel edges, the surface is subject to bending moments.

The elliptical paraboloid is another surface that is amenable to algebraic solution, although it is slightly more involved than the solution for the hyperbolic paraboloid surface. This surface is generated by moving a parabolic curve along another parabola, as shown in Fig. 4(*a*). The equation of this surface is

$$z = \frac{h_y\,y^2}{b^2}+\frac{h_x\,x^2}{a^2} \qquad (19)$$

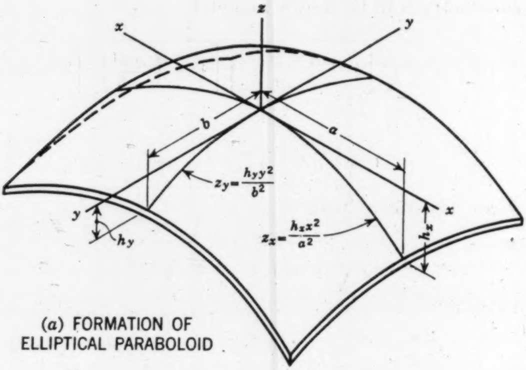
The second differentials of the foregoing expression with respect to x and y are

$$\frac{\partial^2 z}{\partial x^2} = \frac{2\,h_x}{a^2} \qquad (20a)$$

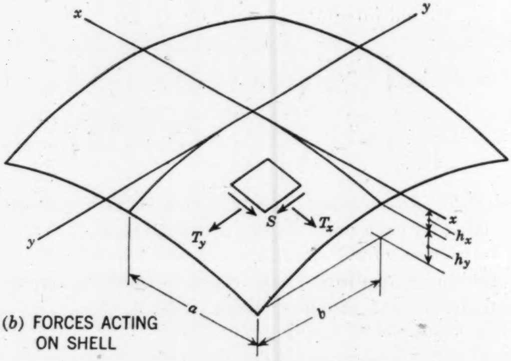
$$\frac{\partial^2 z}{\partial y^2} = \frac{2\,h_y}{b^2} \qquad (20b)$$

and

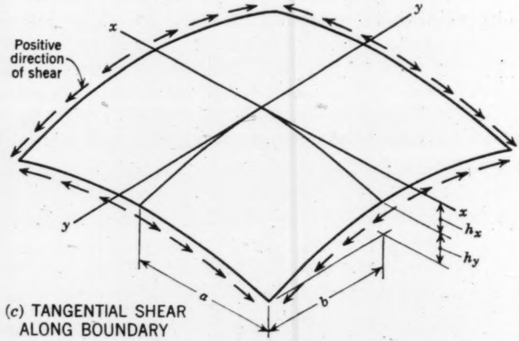
$$\frac{\partial^2 z}{\partial x\,\partial y} = 0 \qquad (20c)$$


z
x
y
b
a
$z_y = \frac{h_y y^2}{b^2}$
x
y
h_y
$z_x = \frac{h_x x^2}{a^2}$
h_x

(*a*) FORMATION OF ELLIPTICAL PARABOLOID


x
y
S
T_y
T_x
y
x
h_x
h_y
a
b

(*b*) FORCES ACTING ON SHELL


Positive direction of shear
x
y
y
x
h_x
h_y
a
b

(*c*) TANGENTIAL SHEAR ALONG BOUNDARY

FIG. 4.—ELLIPTICAL PARABOLOID SHELL

Substituting these expressions in Eq. 10, for a uniform load, $w_z = w$,

$$\frac{\partial^2 F}{\partial y^2} + \frac{h_y\, a^2}{h_x\, b^2} \frac{\partial^2 F}{\partial x^2} = -\frac{a^2}{2\, h_x} w \qquad (21)$$

Differential Eq. 21 is satisfied if

$$F = -\left[\sum_{n=1,3\ldots}^{\infty} A_n (\cosh \beta\, x)\, (\cos \lambda\, y)\right] - \frac{a^2 y^2 w}{4\, h_x} \qquad (22)$$

in which

$$\beta = \sqrt{\frac{h_x}{h_y}}\, \frac{n\, \pi}{2\, a} \qquad (23a)$$

and

$$\lambda = \frac{n\, \pi}{2\, b} \qquad (23b)$$

In the foregoing expressions, the values of n considered are the odd integers. This can be checked by differentiating Eq. 22 and substituting the resulting values in Eq. 21. If the value of F is used in accordance with Eqs. 9, the expressions for the forces are

$$T_{xp} = \left[\sum_{n=1,3\ldots}^{\infty} A_n \lambda^2 (\cosh \beta\, x)\, (\cos \lambda\, y)\right] - \frac{a^2\, w}{2\, h_x} \qquad (24a)$$

$$T_{yp} = -\sum_{n=1,3\ldots}^{\infty} A_n \beta^2 (\cosh \beta\, x) \cos \lambda\, y \qquad (24b)$$

and

$$S_p = -\sum_{n=1,3\ldots}^{\infty} A_n \beta\, \lambda\, (\sinh \beta\, x) \sin \lambda\, y \qquad (24c)$$

At the boundary, $y = \pm\, b$, $T_{yp} = 0$ because $\cos \lambda\, b = 0$ for all values of n. In order to satisfy the condition that $T_{xp} = 0$ at $x = \pm\, a$, it is necessary that $\frac{a^2\, w}{2\, h_x}$ be expressed as a Fourier series. The general expression of the trigonometric series for a constant is

$$1 = \sum_{n=1,3\ldots}^{\infty} \frac{4\,(-1)^{(n-1)/2} \cos \lambda\, y}{n\, \pi} \qquad (25)$$

Therefore, at $x = \pm\, a$, Eq. 24a becomes

$$T_{xp} = 0 = \sum_{n=1,3\ldots}^{\infty} \left\{ A_n \lambda^2 \cosh \beta\, a - \left[\frac{4\,(-1)^{(n-1)/2}}{n\, \pi} \frac{a^2\, w}{2\, h_x}\right]\right\} \cos \lambda\, y \qquad (26)$$

This expression can equal only zero for all values of y if

$$A_n = \frac{2\, a^2\, w\, (-1)^{(n-1)/2}}{n\, \pi\, h_x\, \lambda^2 \cosh \beta\, a} \qquad (27)$$

Substituting A_n in Eqs. 24 and cancelling the common terms results in

$$T_{xp} = \frac{w\,a^2}{h_x}\left[\frac{2}{\pi}\sum_{n=1,3\ldots}^{\infty}\frac{(-1)^{(n-1)/2}\cosh\beta\,x}{n\cosh\beta\,a}\cos\lambda\,y - \tfrac{1}{2}\right]\ldots\ldots(28a)$$

$$T_{yp} = -\frac{w\,b^2}{h_y}\left[\frac{2}{\pi}\sum_{n=1,3\ldots}^{\infty}\frac{(-1)^{(n-1)/2}\cosh\beta\,x}{n\cosh\beta\,a}\cos\lambda\,y\right]\ldots\ldots\ldots(28b)$$

and

$$S_p = -\frac{w\,a\,b}{\sqrt{h_x\,h_y}}\left[\frac{2}{\pi}\sum_{n=1,3\ldots}^{\infty}\frac{(-1)^{(n-1)/2}\sinh\beta\,x}{n\cosh\beta\,a}\sin\lambda\,y\right]\ldots\ldots\ldots(28c)$$

By means of Eq. 28 and Eqs. 2*b*, 3, and 4*b*, the actual internal forces can be computed as the sum of a series. If h_x/h_y is greater than unity, rapid convergence of the series is obtained for most values, and, therefore, only the first three or four terms (n = 1,3,5, and 7) are required to obtain sufficient accuracy. However, at the boundary $x = \pm\,a$ the expression for shear converges very slowly. In this case one can restate Eq. 28*c* at the boundary $x = a$ as

$$S_p = -\frac{w\,a\,b}{\sqrt{h_x\,h_y}}\left\{\frac{2}{\pi}\sum_{n=1,3\ldots}^{\infty}\left[\left(\frac{\sinh\beta\,a}{\cosh\beta\,a} - 1\right) + 1\right]\frac{(-1)^{(n-1)/2}}{n}\right\}\sin\lambda\,y\ldots(29)$$

However,

$$\sum_{n=1,3\ldots}^{\infty}\frac{(-1)^{(n-1)/2}}{n}\sin\lambda\,y = \tfrac{1}{4}\log\left(\sec\frac{\pi\,y}{2\,b} + \tan\frac{\pi\,y}{2\,b}\right)^2\ldots\ldots\ldots(30)$$

Therefore, Eq. 29 reduces to

$$S_p = -\frac{w\,a\,b}{\sqrt{h_x\,h_y}}\left[\frac{1}{2\,\pi}\log\left(\sec\frac{\pi\,y}{2\,b} + \tan\frac{\pi\,y}{2\,b}\right)^2 - \frac{2}{\pi}\sum_{n=1,3\ldots}^{\infty}(1 - \tanh\beta\,a)\frac{(-1)^{(n-1)/2}}{n}\sin\lambda\,y\right]\ldots(31)$$

For values of h_x/h_y greater than 1, for practical purposes, $\tanh\beta\,a$ is equal to 1. Therefore, the second term in Eq. 31 can be ignored; thus, the expression for shear converges rapidly.

At $y = \pm\,b$, sec $(\pi\,y/2\,b)$ and tan $(\pi\,y/2\,b)$ are infinite. Therefore, the log of these values is also infinite. Consequently, Eq. 31 indicates that the shear at the corner is infinite. This would be true if the corner were completely free of normal forces and if the shell had no bending resistance. However, because of the integral action of the supporting ribs and shell, normal forces do exist at the corner. These normal forces alter the resistance to the extent that the shear does not need to be infinite to satisfy statics. Moreover, at the corner some of the load can be, and is, resisted by flexural resistance. From studies made of cylindrical shells, it has been found that this flexural action is confined to a distance of approximately $0.4\sqrt{r\,t}$ from the rib, in which r is the radius of the shell and t is the shell thickness. Therefore, it is felt that Eqs. 28*c* and 31 do not apply within the distance $0.4\sqrt{r\,t}$ from the corner. Shear can be considered maximum at the point $y = b - 0.4\sqrt{r\,t}$.

The symbols, T_x, T_y, and S, represent forces per unit of length. In order to obtain stresses, these values must be divided by the thickness of the shell.

The trigonometric functions involved in Eqs. 2*b*, 3, and 4*b* can be readily expressed as functions of x and y. Differentiating Eq. 19 with respect to x yields

$$\frac{\partial z}{\partial x} = \frac{2\,h_x}{a}\frac{x}{a} = \tan\phi \qquad (32)$$

By utilizing

$$\tan^2\phi = \frac{1}{\cos^2\phi} - 1 \qquad (33)$$

Eq. 2*b* reduces to

$$\left(\frac{2\,h_x}{a}\frac{x}{a}\right)^2 + 1 = \frac{1}{\cos^2\phi} \qquad (34)$$

or

$$\cos\phi = \frac{1}{\sqrt{1 + \left(\frac{2\,h_x}{a}\frac{x}{a}\right)^2}} \qquad (35a)$$

Similarly,

$$\cos\psi = \frac{1}{\sqrt{1 + \left(\frac{2\,h_y}{b}\frac{y}{b}\right)^2}} \qquad (35b)$$

and, therefore,

$$\frac{\cos\phi}{\cos\psi} = \sqrt{\frac{1 + \left(\frac{2\,h_y}{b}\frac{y}{b}\right)^2}{1 + \left(\frac{2\,h_x}{a}\frac{x}{a}\right)^2}} \qquad (35c)$$

In order to avoid mathematical complications, the value of w_s was assumed to be constant in establishing Eq. 21. However, although the algebraic computations become extensive and rather formidable, the procedure outlined for the uniform load can also be applied to the case of any symmetrical loading, such as the dead weight of the shell. In this case the load is expressed in terms of the double Fourier series,

$$w_s = \sum_{m=1,3\ldots}^{\infty}\ \sum_{n=1,3\ldots}^{\infty} \beta_{mn}\cos\gamma\,x\cos\lambda\,y \qquad (36)$$

in which $\gamma = m\,\pi/2\,a$.

The resulting expressions for T_{xp} and T_{yp}, obtained by expressing w_s in this manner, indicate that any symmetrical loading can be resisted by direct forces without the necessity for lateral or normal forces at the boundaries. The behavior of the elliptical paraboloid shell under dead load therefore differs from that of the hyperbolic paraboloid shell, for which the dead load induces some bending if no lateral restraint is provided.

TABLE 1.—COEFFICIENTS FOR COMPUTING FORCE COMPONENTS OF ELLIPTICAL PARABOLOID SHELL

		VALUE OF y/b									
x/a	Force component	(a) $h_x/h_y = 1.0$					(d) $h_x/h_y = 0.8$				
		0	0.25	0.50	0.75	1.0	0	0.25	0.50	0.75	1.0
0.00	T_y	0.250	0.233	0.182	0.101	0	0.289	0.270	0.213	0.119	0
	T_x	0.250	0.267	0.318	0.399	0.500	0.211	0.230	0.287	0.381	0.500
	S	0	0	0	0	0	0	0	0	0	0
0.25	T_y	0.267	0.250	0.199	0.111	0	0.304	0.285	0.228	0.130	0
	T_x	0.233	0.250	0.301	0.389	0.500	0.196	0.215	0.272	0.370	0.500
	S	0	0.029	0.068	0.096	0.108	0	0.034	0.069	0.100	0.114
0.50	T_y	0.318	0.301	0.250	0.150	0	0.347	0.331	0.277	0.169	0
	T_x	0.182	0.199	0.250	0.350	0.500	0.153	0.169	0.223	0.331	0.500
	S	0	0.068	0.140	0.210	0.244	0	0.065	0.139	0.215	0.255
0.75	T_y	0.399	0.389	0.350	0.250	0	0.416	0.406	0.369	0.270	0
	T_x	0.101	0.111	0.150	0.250	0.500	0.084	0.094	0.131	0.230	0.500
	S	0	0.096	0.210	0.356	0.465	0	0.091	0.201	0.353	0.480
1.0	T_y	0.500	0.500	0.500	0.500	0	0.500	0.500	0.500	0.500	0
	T_x	0	0	0	0	0	0	0	0	0	0
	S	0	0.108	0.243	0.465	∞	0	0.101	0.229	0.443	∞
		(b) $h_x/h_y = 0.6$					(e) $h_x/h_y = 0.4$				
0.00	T_y	0.336	0.316	0.252	0.143	0	0.395	0.374	0.307	0.180	0
	T_x	0.164	0.184	0.248	0.357	0.500	0.105	0.126	0.193	0.320	0.500
	S	0	0	0	0	0	0	0	0	0	0
0.25	T_y	0.348	0.329	0.267	0.155	0	0.403	0.383	0.319	0.192	0
	T_x	0.152	0.171	0.233	0.345	0.500	0.097	0.117	0.181	0.308	0.500
	S	0	0.031	0.067	0.103	0.120	0	0.026	0.060	0.101	0.125
0.50	T_y	0.383	0.367	0.312	0.197	0	0.425	0.410	0.357	0.235	0
	T_x	0.117	0.133	0.188	0.304	0.500	0.075	0.090	0.143	0.265	0.500
	S	0	0.060	0.132	0.216	0.265	0	0.049	0.115	0.208	0.274
0.75	T_y	0.436	0.426	0.392	0.296	0	0.459	0.451	0.419	0.331	0
	T_x	0.064	0.074	0.108	0.204	0.500	0.041	0.049	0.081	0.169	0.500
	S	0	0.081	0.185	0.342	0.494	0	0.065	0.156	0.316	0.506
1.00	T_y	0.500	0.500	0.500	0.500	0	0.500	0.500	0.500	0.500	0
	T_x	0	0	0	0	0	0	0	0	0	0
	S	0	0.089	0.208	0.413	∞	0	0.070	0.173	0.363	∞
		(c) $h_x/h_y = 0.2$									
0.00	T_y	0.462	0.446	0.388	0.248	0					
	T_x	0.038	0.054	0.112	0.252	0.500					
	S	0	0	0	0	0					
0.25	T_y	0.465	0.451	0.396	0.261	0					
	T_x	0.035	0.049	0.104	0.239	0.500					
	S	0	0.014	0.040	0.088	0.128					
0.50	T_y	0.473	0.462	0.414	0.303	0					
	T_x	0.027	0.038	0.086	0.197	0.500					
	S	0	0.027	0.074	0.174	0.280					
0.75	T_y	0.485	0.480	0.456	0.383	0					
	T_x	0.015	0.020	0.044	0.117	0.500					
	S	0	0.034	0.098	0.246	0.510					
1.00	T_y	0.500	0.500	0.500	0.500	0					
	T_x	0	0	0	0	0					
	S	0	0.038	0.108	0.262	∞					

In order to expedite the analysis of the elliptical paraboloid shells and to obtain a better understanding of their load-carrying characteristics, Table 1 has been compiled on the basis of Eqs. 28 and Fig. 4(*b*). The expressions inside the parentheses in Eqs. 28 contain only the parameter, h_x/h_y. Therefore, the behavior of this doubly curved shell can be expressed as a function of this single parameter.

Coefficients are given for computing the three force components, T_x, T_y, and S, at the eighth points of a dome. The forces determined by multiplying the coefficients by constants are

$$T_y = -\frac{w\,b^2}{k\,h_y}\text{ (coefficient)} \qquad (37a)$$

$$T_x = -\frac{w\,a^2\,k}{h_x}\text{ (coefficient)} \qquad (37b)$$

$$S = -\frac{w\,a\,b}{\sqrt{h_x\,h_y}}\text{ (coefficient)} \qquad (37c)$$

and

$$k = \sqrt{\frac{1 + [(2\,h_x/a)(x/a)]^2}{1 + [(2\,h_y/b)(y/b)]^2}} \qquad (37d)$$

These constants are dependent only on the selected dimensions of the shell and on the load. In this connection for the sake of completeness, the factor k has been included. In practice the additional accuracy secured by the inclusion of this term is unwarranted because the stresses due to T_x and T_y are never critical. Except in the zone near the corners in which the principal stress due to the combination of the three force components is tensile, the stresses are so low in compression for spans being considered that an investigation of the stresses in a dome is of academic interest only. Therefore, the real reason and need for computing stresses in a shell with a fair degree of accuracy are to obtain a reliable determination of the tangential load which must be carried by the supporting arches.

For this purpose the tangential shear existing along the boundaries (Fig. 4(*c*)) at the tenth intervals of half the chord are shown in Table 2. Table 2 also permits a better evaluation of the tension near the corner because the principal stresses are primarily related to S.

A graphical presentation of the values in Table 1 for T_{yp} at midspan is shown in Fig. 5 for various values of h_x/h_y. The values of T_{yp} for h_x/h_y from 1.0 to 5.0 are obtained from the values of T_{xp} by symmetry. For example, the value of T_{yp} at $y = 0$ for $h_x/h_y = 5$ is the same as the value of T_{xp} at $x = 0$ for $h_x/h_y = 0.2$. At $x = \pm a$, for all values of h_x/h_y,

$$T_{yp} = -\frac{0.5\,w\,b^2}{h_y} = -\frac{0.125\,w\,L^2}{h_y} \qquad (38)$$

The last term in Eq. 38 is the thrust in a parabolic arch subject to the uniform load, w. This identity is not surprising because at the boundary the force normal to the edge was made equal to zero. Consequently, the imposed condition of restraint compels the entire load in the immediate vicinity of the edge to be carried by arch action in the y-direction. Furthermore, $0.5\, b^2/h_y$ equals the radius of the parabola at its crown. Therefore, the value T_{yp} at $x = a$ and $y = 0$ represents merely the thrust induced in a ring with the appropriate radius due to a radial load, w.

Near the crown, marked variations in the value of T_{yp} occur as h_x/h_y varies. When the rise in the x-direction is small compared with the rise in the y-direction—for example, when $h_x/h_y = 0.2$—the curves in Fig. 5 are almost horizontal, indicating that a large proportion of the load is being resisted in the y-direction. This can be anticipated from the geometry of the shell. As the curvature in one direction is flattened, thereby approaching a horizontal plane as a limit, it is natural that the load is transmitted in the other direction.

TABLE 2.—SHEAR ALONG THE EDGES OF ELLIPTICAL PARABOLOID SHELL

y/b	h_x/h_y				
	1.0	0.8	0.6	0.4	0.2
	At $x - \pm a$				
0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	0.0419	0.0389	0.0342	0.0307	0.0137
0.2	0.0854	0.0793	0.0701	0.0550	0.0286
0.3	0.1319	0.1231	0.1096	0.0872	0.0481
0.4	0.1836	0.1721	0.1546	0.1254	0.0731
0.5	0.2432	0.2294	0.2081	0.1728	0.1075
0.6	0.3204	0.3066	0.2859	0.2493	0.1818
0.7	0.4071	0.3897	0.3627	0.3173	0.2296
0.8	0.5363	0.5178	0.4887	0.4400	0.3443
0.85	0.6279	0.6090	0.5791	0.5292	0.4306
0.9	0.7570	0.7378	0.7074	0.6667	0.5659
0.95	0.9777	0.9582	0.9276	0.8763	0.7741
1.0	∞	∞	∞	∞	∞
x/a	At $y = \pm b$				
0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	0.0419	0.0444	0.0468	0.0488	0.0500
0.2	0.0854	0.0903	0.0950	0.0990	0.1014
0.3	0.1319	0.1391	0.1460	0.1519	0.1553
0.4	0.1836	0.1930	0.2019	0.2095	0.2140
0.5	0.2432	0.2545	0.2652	0.2743	0.2798
0.6	0.3204	0.3317	0.3425	0.3516	0.3571
0.7	0.4071	0.4213	0.4348	0.4463	0.4532
0.8	9.5363	0.5515	0.5659	0.5782	0.5855
0.85	0.6279	0.6434	0.6582	0.6707	0.6782
0.9	0.7570	0.7728	0.7878	0.8005	0.8081
0.95	0.9777	0.9935	1.0087	1.0215	1.0290
1.0	∞	∞	∞	∞	∞

With no normal forces along the edges, it follows that the increase in the proportion of load carried in the y-direction as h_x/h_y decreases must be accompanied by an increase in the tangential shears along the edges, $x = \pm a$. Such an increase is confirmed by the coefficients listed in Table 2. Although these coefficients diminish at $x = \pm a$ as h_x/h_y decreases, they do not diminish as rapidly as $\sqrt{h_x/h_y}$.

For large values of h_x/h_y, the values of T_{yp} become appreciably smaller as the crown is approached, and, therefore, for such shells only the exterior

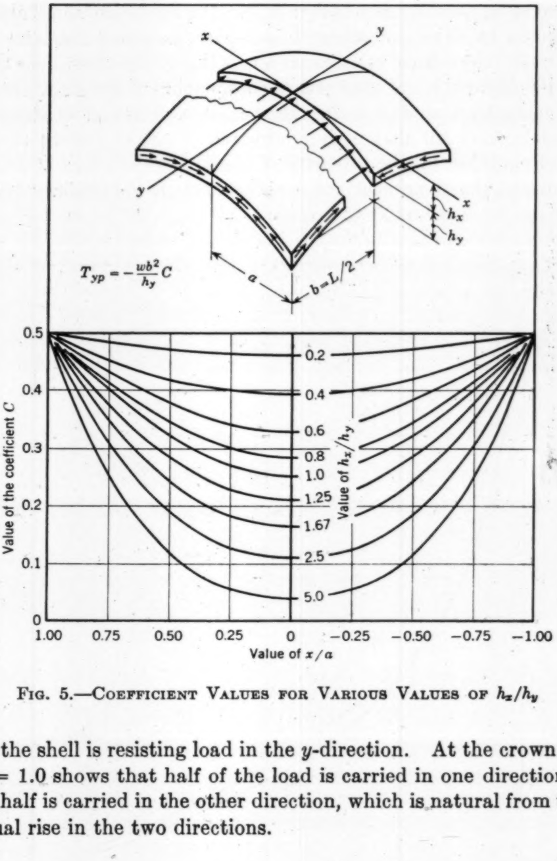


FIG. 5.—COEFFICIENT VALUES FOR VARIOUS VALUES OF h_x/h_y

portion of the shell is resisting load in the y-direction. At the crown the curve for $h_x/h_y = 1.0$ shows that half of the load is carried in one direction and the remaining half is carried in the other direction, which is natural from the condition of equal rise in the two directions.

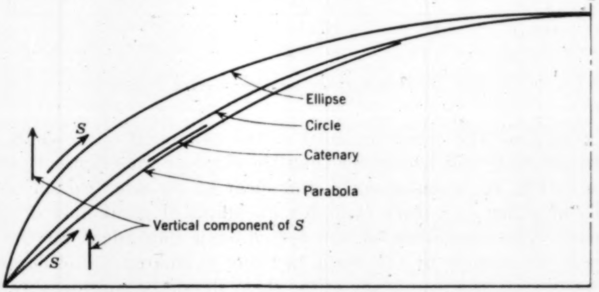

℄
Ellipse
Circle
Catenary
Parabola
S
S
Vertical component of S

FIG. 6.—SLOPE COMPARISON FOR VARIOUS CURVES

An interesting question is whether or not the coefficients in Tables 1 and 2 can be applied to domes of other shapes with an equal rise and span. As cited previously, the critical stresses are a function of the shear near the corners. However, the summation of the vertical components of the shear along an edge must equal the load on the shell. If the same variation of shear along an edge is assumed for all shapes, it is apparent that, to satisfy the foregoing condition of equilibrium, the intensity of the tangential shear is dependent on the steepness of the slope near the corner. This is particularly true because the maximum shear occurs at the corner.

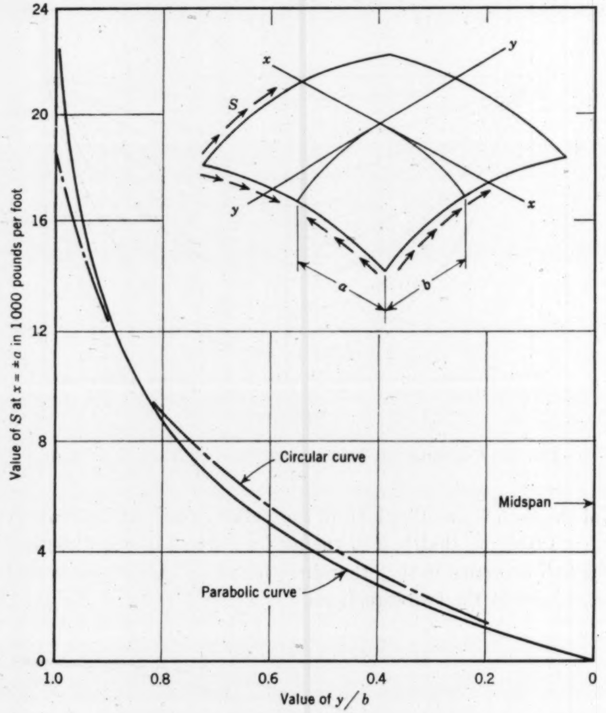


FIG. 7.—COMPARISON OF TANGENTIAL SHEAR

The slope near the corner of most of the commonly used shells of other curvature generally will be steeper than the slope of the elliptical paraboloid, as shown in Fig. 6. Consequently, the shear at the edge should be less for the shells of other curvature than for an elliptical paraboloid of the same dimensions. The magnitude of the reduction is dependent on the relative slopes near the corners of the surfaces being compared. For domes whose edges are elliptical, the magnitude of the shear should be considerably less than that for domes with other shapes. If the edge of the dome is circular, the tangential shear should be approximately the same as for an elliptical paraboloid.

To confirm this hypothesis, Fig. 7 compares the tangential shear computed[3] for domes at a factory in Brynmawr, England, and that obtained for an elliptical paraboloid of the same dimensions. The shape used for the Brynmawr domes was a surface of translation generated by moving one vertical circle on another. Fig. 7 shows good agreement between the two curves except in the immediate vicinity of the corner, in which a finite value is given for the circular curve in contrast to the infinite value implied for the parabolic curve. The reason for this apparent discrepancy is that, due to mathematical difficulties, a numerical procedure based on finite-differences equations was used to determine the forces for the Brynmawr dome. Because this procedure is based on the average value between the chosen interval, a finite value results at the corner. If a rigorous mathematical solution had been used, an infinite value for the circular curve would have resulted.

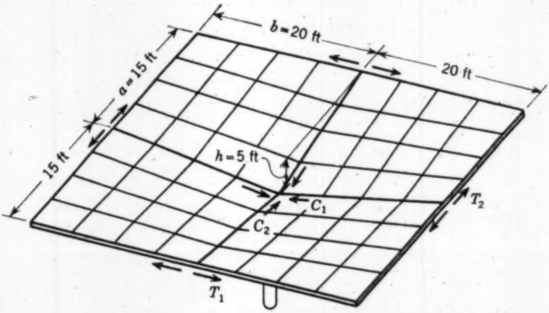


FIG. 8.—ROOF DESIGNED IN EXAMPLE 1

At $y = b - 0.4\sqrt{r\,t}$, the point previously recommended as the breakoff place for shear evaluation, the shear computed for the parabolic curve is approximately 7% higher than that for the circular curve. Whether this difference is real or merely due to dissimilarity in methods of computation is not known. However, the difference is in the proper direction.

Example 1.—A hyperbolic paraboloid shell with a column at the center is designed. The roof shown in Fig. 8 is obtained by joining four identical sections in a manner similar to the method used in Fig. 2. Many other arrangements can be used,[4] all of which are designed in the same manner by considering each quadrant of a rectangular unit individually.

Assuming $w = 60$ lb per sq ft, the internal forces at the critical points of the shell roof shown in Fig. 8 are

$$S = \frac{w\,a\,b}{2\,h} = \frac{60 \times 15 \times 20}{2 \times 5} = 1{,}800 \text{ 16-ft}$$

$$T_1 = -\,C_1\ 1{,}800 \times 20 = 36{,}000 \text{ lb}$$

---

[3] "The Design of a Reinforced Concrete Factory at Brynmawr, South Wales," by Ove Nyquist Arup and Ronald Jenkins, Pt. III, *Proceedings*, Inst. C. E., London, December, 1953, pp. 345–397.

[4] "Structural Applications of Hyperbolic Paraboloidical Shells," by Felix Candela, *Journal*, A.C.I., Vol. 26, No. 5, 1954, pp. 397–415.

and

$$T_2 = - C_2 \, 1{,}800 \times 15 = 27{,}000 \text{ lb}$$

Because the shell is subject to pure shear, the principal tensile force will also be 1,800 lb per ft. An allowable steel stress of 20,000 lb per sq in. results in a required area of steel of 0.09 sq in. per ft. Therefore, No. 2 bars, 6 in. on centers, are sufficient. This reinforcement should be placed diagonally, extending from one free edge to the other.

The shell exerts a constant shear on the edge members, which have been omitted in Fig. 8. The total thrust or pull exerted by this shear is equal to the product of the length of the edge member affected and the magnitude of the shear. In this example this equals 36,000 lb. Because there is no external reaction acting on the edge beams, either at the corners or along the edge, it is evident that the maximum tension or compression in the edge members occurs at the midspan. The tension and compression in the edge member diminish along the length to zero at the ends.

TABLE 3.—INTERNAL FORCES IN AN ELLIPTICAL PARABOLOID SHELL FOR EXAMPLE NO. 2

x/a	Force	VALUE OF y/b				
		0	0.25	0.50	0.75	1.00
0	$T_y k$	−4,300	−4,100	−3,200	−1,800	0
	T_x/k	−1,900	−2,100	−2,600	−3,500	−4,600
	S	0	0	0	0	0
0.25	$T_y k$	−4,600	−4,300	−3,400	−2,000	0
	T_x/k	−1,800	−2,000	−2,500	−3,400	−4,600
	S	0	− 400	− 800	−1,200	−1,300
0.50	$T_y k$	−5,200	−5,000	−4,200	−2,500	0
	T_x/k	−1,400	−1,600	−2,100	−3,000	−4,600
	S	0	− 800	−1,600	−2,500	−3,000
0.75	$T_y k$	−6,200	−6,100	−5,500	−4,100	0
	T_x/k	− 800	− 900	−1,200	−2,100	−4,600
	S	0	−1,100	−2,400	−4,100	−5,600
1.00	$T_y k$	−7,500	−7,500	−7,500	−7,500	0
	T_x/k	0	0	0	0	0
	S	0	−1,200	−2,700	−5,200	∞

To determine the type of force (compression or tension) present in the edge members, it is recommended that free body diagrams be drawn of the member being considered rather than relying merely on a sign convention. Thus, the possibility of making serious errors in complicated layouts will be minimized. For this case the layout is so simple that the type of force present can be ascertained by inspection. Because the shear is positive and the coordinate of each quadrant occurs at the corner, the shear is outward along the four horizontal edges and inward along the four sloping edges. Hence, the edge beams at the exterior edges are in tension, whereas those extending out from the column are in compression.

Example 2.—An elliptical paraboloid shell is designed. Table 3 shows the internal forces divided by k or $1/k$ acting in an elliptical paraboloid subject to a uniform load of 60 lb per sq ft and spanning 100 ft in one direction and 70 ft in the other with a total rise of 18 ft. These values are obtained by multiplying the coefficients for $h_x/h_y = 0.8$ shown in Table 1 by one of the following values:

For T_y—

$$\frac{w\,b^2}{h_y} = \frac{60\ (50)^2}{10} = 15{,}000 \text{ lb per ft}$$

For T_x—

$$\frac{w\,a^2}{h_x} = \frac{60\ (35)^2}{8} = 9{,}200 \text{ lb per ft}$$

For S—

$$\frac{w\,a\,b}{\sqrt{h_x\,h_y}} = \frac{60\ (50)\ (35)}{\sqrt{8\ (10)}} = 11{,}700 \text{ lb per ft}$$

Because the stresses are small the effect of k is ignored. The maximum compression due to an assumed load of 60 lb per sq ft on the shell is 7,500 lb per ft. If the thickness of the shell is assumed as 3 in., the maximum compressive stress is only

$$f_c = \frac{7{,}500}{3 \times 12} = 208 \text{ lb per sq in.}$$

which is considerably lower than the allowable stress of concrete.

TABLE 4.—SHEAR S AND PRINCIPAL STRESS S' ALONG EDGE

			$x = a$			
y/b	0	0.1	0.2	0.3	0.4	0.5
S	0	− 460	− 930	−1,440	−2,010	−2,680
S'	0	30	110	270	500	860
y/b	0.6	0.7	0.8	0.85	0.9	0.95
S	−3,590	−4,560	−6,060	−7,130	−8,630	−11,200
S'	1,440	2,150	3,380	4,300	4,880	8,060
			$y = b$			
x/a	0	0.1	0.2	0.3	0.4	0.5
S	0	− 520	−1,060	−1,630	−2,260	−2,980
S'	0	60	230	520	920	1,460
x/a	0.6	0.7	0.8	0.85	0.9	0.95
S	−3,880	−4,930	−6,450	−7,530	−9,040	−11,600
S'	2,210	3,140	4,550	5,570	7,030	9,500

To obtain knowledge of the tensile forces existing in the shell, the minimum principal stresses have been evaluated along the edges in Table 4. The value of the shear, S, is computed by using Table 2, with the multiplier in this case being 11,700 lb per ft taken from Table 3. The principal stress, S', is computed s described in most standard mechanics textbooks. The direct force at $y = b$ is 4,600 lb per ft, and the direct force at $x = a$ is 7,500 lb per ft. In most of the cases these principal values along the shell represent the maximum value in their zone.

At the corner the radius of curvature in the x-direction can be computed from

$$R_x = \frac{\left[1 + \left(\frac{\partial z}{\partial x}\right)^2\right]^{3/2}}{\frac{\partial^2 z}{\partial x^2}} \qquad \text{(39)}$$

in which

$$z = \frac{8\,x^2}{35^2} + \frac{10\,y^2}{50^2}$$

and

$$\frac{\partial z}{\partial x} = \frac{16\,x}{35^2}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{16}{35^2}$$

At the corner $x = 35$, Eq. 39 yields

$$R_x = \frac{35^2\,[1 + (16/35)^2]^{3/2}}{16} = 102 \text{ ft}$$

and, similarly,

$$R_y = 156 \text{ ft}$$

The maximum shear can therefore be expected to be at

$$\frac{x}{a} = \frac{35 - 0.4\sqrt{101 \times \frac{1}{4}}}{35} = 0.94$$

and at

$$\frac{y}{b} = \frac{50 - 0.4\sqrt{156 \times \frac{1}{4}}}{50} = 0.95$$

Therefore, from Table 4 the largest minimum principal stress along the edges is 9,500 lb per ft. Several points in the interior should be investigated also to determine the extent of the tensile area. Using the internal forces shown in Table 3, the principal stress at $y/b = x/a = 0.75$ and at $y/b = x/a = 0.5$ is

$$S' = -\left(\frac{4{,}100 + 2{,}100}{2} - \sqrt{\frac{2{,}000^2}{4} + 4{,}100^2}\right) = 1{,}100 \text{ lb per ft}$$

and

$$S' = -\left(\frac{4{,}200 + 2{,}100}{2} - \sqrt{\frac{2{,}100^2}{4} - 1{,}600^2}\right) = -\,1{,}200 \text{ lb per ft}$$

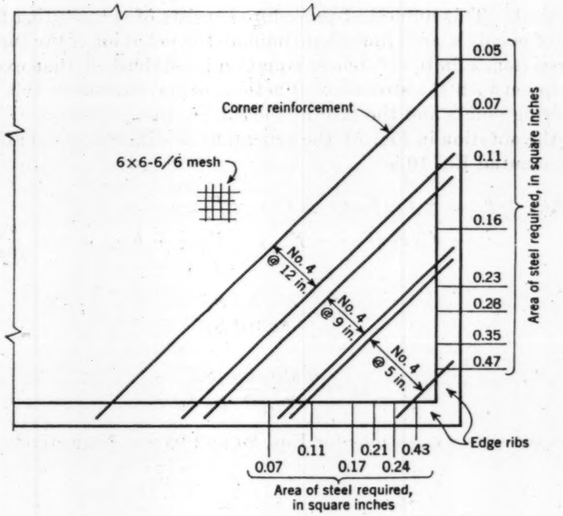

Corner reinforcement
6×6-6/6 mesh
No. 4 @ 12 in.
No. 4 @ 9 in.
No. 4 @ 5 in.
0.05
0.07
0.11
0.16
0.23
0.28
0.35
0.47
Area of steel required, in square inches
Edge ribs
0.07
0.11
0.17
0.21
0.24
0.43
Area of steel required, in square inches

FIG. 9.—REQUIRED STEEL FOR ELLIPTICAL PARABOLOID SHELL

Assuming a linear variation in principal stress between these points, zero tension would occur at $x/a = y/b = \frac{5}{8}$.

From a theoretical point of view, the reinforcement should follow the lines of principal stress. However, this is not practical, and, therefore, it is customary to place the reinforcement in the corners along diagonal lines, as shown in Fig. 9. For this particular example and probably for all instances, the controlling tension for any group of bars occurs at the edge. The amount of reinforcement, with f_s = 20,000 lb per sq in., computed from the principal stresses shown in Table 4 is shown along the edge ribs of one corner.

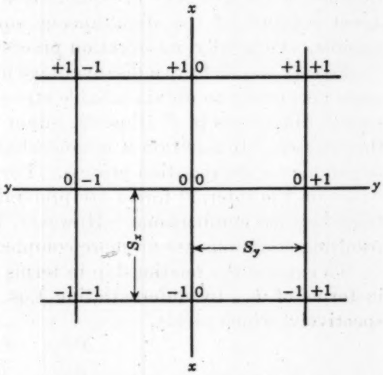

x
+1 -1
+1 0
+1 +1
y
0 -1
0 0
0 +1
y
S_x
S_y
-1 -1
-1 0
-1 +1
x

FIG. 10.—FINITE-DIFFERENCES EQUATIONS NOTATION

For most of the shells of double curvature, even for such a simple case as a translational shell formed by moving one circular curve on the other, an algebraic solution becomes extremely involved. In such cases the conversion of the various differential equations into finite-differences equation[5] is more practical. This numerical procedure consists of substituting for the surface a grid of evenly spaced lines that simulate the behavior of the surface. For each intersection, a finite-differences equation is established that expresses the relationship between the stresses or functions of the stresses at this point, and at neighboring points and the load at the intersection.

---

[5] "Solution of Difficult Structural Problems by Finite Differences," by Alfred Parme, *Journal*, A.C.I., Vol. 22, November, 1950, pp. 237–255.

Using the notation in Fig. 10, the general finite-differences equation equivalent to differential Eq. 10 is

$$F_{0,1} - 2\,F_{0,0} + F_{0,-1} + k_1\,(F_{1,0} - 2\,F_{0,0} + F_{-1,0}) - 0.5\,k_2\,(F_{1,1} - F_{-1,1} - F_{1,-1} + F_{-1,-1}) = -\frac{w_z\,S_y^2}{\partial^2 z/\partial x^2} \quad \text{(40)}$$

in which

$$k_1 = \frac{\partial^2 z/\partial y^2}{\partial^2 z/\partial x^2}\frac{S_y^2}{S_x^2} \quad \text{(41}a\text{)}$$

and

$$k_2 = \frac{\partial^2 z/\partial x\,\partial y}{\partial^2 z/\partial x^2}\frac{S_y^2}{S_x\,S_y} \quad \text{(41}b\text{)}$$

The finite-differences equations for Eqs. 9*a* and 9*b* are, respectively,

$$T_{xp} = \frac{F_{0,-1} - 2\,F_{0,0} + F_{0,1}}{S_y^2} \quad \text{(42)}$$

and

$$T_{yp} = \frac{F_{1,0} - 2\,F_{0,0} + F_{-1,0}}{S_x^2} \quad \text{(43)}$$

Because of the quantity of equations which result even with a coarse grid, a direct solution of the simultaneous equations obtained from Eq. 40 is not feasible. Generally, an iteration process called the relaxation method[6] is used.

Eqs. 42 and 43 have a disadvantage in that a value for F must be determined quite accurately to obtain reliable stress values. With the stress equal to the second differences in F (Eqs. 9), minor errors in F greatly affect the value of the stresses. In addition it is somewhat difficult to estimate the initial values to commence the iteration process. For this reason finite-differences equations based on the internal forces are preferable. For the general case these equations become cumbersome. However, for the case of translational shells, the resulting equations are no more complicated than Eq. 40.

To express the relationship in terms of the internal forces, first express T_{xp} in terms of T_{yp} by differentiating Eqs. 5 and 6 with respect to x and y, respectively, which yields

$$\frac{\partial^2 T_x}{\partial x^2} - \frac{\partial^2 T_{yp}}{\partial y^2} = 0 \quad \text{(44)}$$

Because $\partial^2 z/\partial x\,\partial y = 0$, Eq. 8*c* can be rewritten as

$$T_{xp} + T_{yp}\frac{\partial^2 z/\partial y^2}{\partial^2 z/\partial x^2} = -\frac{w_z}{\partial^2 x/\partial x^2} \quad \text{(45)}$$

---

[6] "Some Improvements in the Use of Relaxation Methods for the Solution of Ordinary and Partial Differential Equations," *Proceedings*, Royal Soc. of London, Series A-190, 1947.

Differentiating Eq. 45 twice with respect to x and subtracting Eq. 44 from the result yields

$$\frac{\partial^2 T_{yp}}{\partial y^2} + k_1 \frac{\partial^2 T_{yp}}{\partial x^2} + 2\, k_2 \frac{\partial T_{yp}}{\partial x} + k_3\, T_{yp} = -\, k_4 \qquad (46)$$

in which

$$\left.\begin{aligned} k_1 &= \frac{\partial^2 z/\partial y^2}{\partial^2 z/\partial x^2} \\ k_2 &= \frac{\partial}{\partial x}\left(\frac{\partial^2 z/\partial y^2}{\partial^2 z/\partial x^2}\right) \\ k_3 &= \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 z/\partial y^2}{\partial^2 z/\partial x^2}\right) \\ k_4 &= \frac{\partial^2}{\partial x}\left(\frac{w_z}{\partial^2 z/\partial x^2}\right) \end{aligned}\right\} \qquad (47)$$

Allowing T to equal T_{yp}, the finite-differences equation corresponding to differential Eq. 46 is

$$T_{0,1} - 2\, T_{0,0} + T_{0,-1} + \left(\frac{S_y}{S_x}\right)^2 k_1\, (T_{1,0} - 2\, T_{0,0} + T_{-1,0}) + \frac{k_2\, S_y{}^2}{S_x}\, (T_{1,0} - T_{-1,0}) + k_3\, T_{0,0} = -\, k_4\, S_y{}^2 \qquad (48)$$

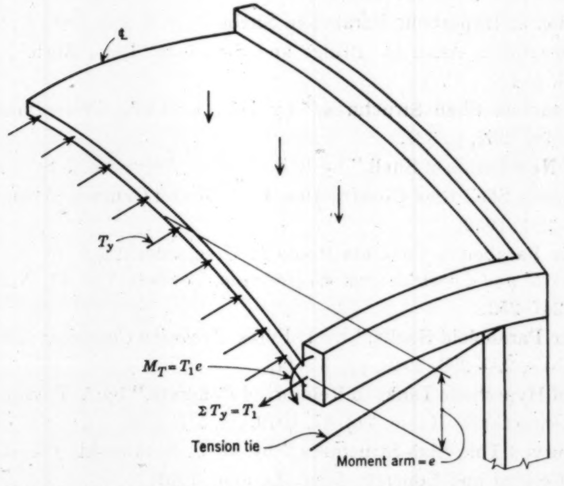


FIG. 11.—BENDING MOMENT IN SHELLS OF DOUBLE CURVATURE

The ribs supporting the arches must be designed to carry the tangential shear load imparted to them by the shell. Because this problem involves only a routine analysis of an arch, this subject will not be examined herein except to note that the analysis of the arch can be made by dealing only with the tangential shear obtained from the coefficients in Table 2, or by using directly the surface loads on the shell.[7]

If the vertical loads are used directly for shells of double curvature, consideration must be given to the bending moment created by the rise of the shell in the direction normal to the arch. This moment, as shown in Fig. 11, equals the product of the summation of the T_{xp}-forces or T_{yp}-forces from the midspan to the edge and the lever arm between the centroid of the internal forces in the shell and the centroidal axis of the arch. The tensile force, T_1, must be superimposed on the thrust due to the end reactions in order to obtain the net thrust in the arch.

APPENDIX. BIBLIOGRAPHY

"Some Aspects of the Theory of Thin Elastic Shells," by Eric Reissner, *Proceedings*, Conference on Thin Concrete Shells, Massachusetts Inst. of Technology, Cambridge, June, 1954, p. 81.

"Hyperbolic Paraboloids," by Felix Candela, *ibid.*, p. 91.

"Skew Shell Utilized in Unusual Roof," by Felix Candela, *Proceedings*, A.C.I., 1953, Vol. 49, pp. 657–664.

"Deformation of Hyperbolic Paraboloid Shells," by Shisuo Ban, *Publications*, International Assn. for Bridge and Structural Eng., Zurich, Vol. 13, 1953, p. 1.

"Simple Concrete Shell Structures," by Felix Candela, *Proceedings*, A.C.I., Vol. 48, 1952, p. 321.

"Theory of New Forms of Shell," by R. S. Jenkins, *Paper No. 7*, Symposium on Concrete Shell Roof Construction, Cement and Concrete Assn., London, July 2–4, 1952.

"Hyperbolic Paraboloid Concrete Roofs in Czechoslovakia," by K. Hruban, *Concrete and Constructional Engineering*, London, Vol. 44, No. 8, 1949, pp. 247–252.

"Hyperbolic Paraboloid Shells," by I. Fytos, *Technika Chronika*, Athens, Vol. 26, Nos. 295–296, 1949, pp. 35–44.

"Analysis of Hyperbolic Tanks in Reinforced Concrete," by A. Favini, *Giornale del Genio Civile*, Rome, Vol. 87, 1949, pp. 515–533.

"Doubly Curved Thin Slab Structures," by M. P. Borkowski, *Translation No. 31*, Cement and Concrete Assn., London, 1951.

"Calculations for Shells of Double Curvature Using Differential Equations," by A. Pucher, *Bauingenieur*, Vol. 18, 1937, p. 118.

---

[7] "Design of Cylindrical Concrete Shell Roofs," *Manual of Engineering Practice No. 31*, ASCE, 1952, p. 90.

"Treatise on Statics of Parabolic Hyperboloidal Shells Not Stiff in Bending," by F. Aimond, *Publications*, International Assn. for Bridge and Structural Eng., Zurich, Vol. 4, 1936, p. 1.

"Calculation of Thin Shells in Reinforced Concrete," by L. Issenmann Pirlaski, Dunod, Paris, 1935, Chapter 7.

"General Investigation Concerning Skew Surface Shells," by B. Laffaille, *Publications*, International Assn. for Bridge and Structural Eng., Zurich, Vol. 3, 1935, p. 295.

"Thin Shells in the Shape of Hyperbolic Paraboloids," by B. Laffaille, *Le Genie Civile*, Paris, Vol. 104, 1934, pp. 409–410.

DISCUSSION

Tung Au,[8] A.M. ASCE.—The general equations for shells of double curvature based on the membrane theory have been presented in a systematic and logical order, with special applications to hyperbolic paraboloids and elliptical paraboloids. Although only a limited number of loading conditions are amenable to an algebraic solution, the author has included the cases of practical importance. Thus, the solutions are obtained with mathematical rigor, and the structural behavior of these shells is also clearly explained.

The use of difference equations and the relaxation method are suggested for the solution of other shells of double curvature for which the classical solutions are either extremely complicated or unavailable. Therefore, it may be desirable to indicate also that such techniques can be adapted to hyperbolic and elliptical paraboloids with loadings other than uniform vertical load and with different boundary conditions. By considering w_x and w_y as components of force in the direction of the x-axis and the y-axis, respectively, acting at the center of the element in Fig. 1, Eqs. 5, 6, 8, 9, and 10 can be generalized as follows:

$$\frac{\partial T_{xp}}{\partial x} + \frac{\partial S_p}{\partial y} + w_x = 0 \qquad (49)$$

$$\frac{\partial T_{yp}}{\partial y} + \frac{\partial S_p}{\partial x} + w_y = 0 \qquad (50)$$

Eqs. 8 remain unchanged. Then the stress function, F, can be introduced so that

$$T_{xp} = \frac{\partial^2 F}{\partial y^2} - \int_{x_0}^{x} w_x \, dx \qquad (51a)$$

$$T_{yp} = \frac{\partial^2 F}{\partial x^2} - \int_{y_0}^{y} w_y \, dy \qquad (51b)$$

and

$$S_p = -\frac{\partial^2 F}{\partial x \, \partial y} \qquad (51c)$$

These values satisfy Eqs. 5 and 6 and reduce Eq. 8*c* to

$$\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 z}{\partial y^2} - 2\,\frac{\partial^2 F}{\partial x \, \partial y}\,\frac{\partial^2 z}{\partial x \, \partial y} = -\,w_z + w_x \frac{\partial z}{\partial x} + w_y \frac{\partial z}{\partial y} + \frac{\partial^2 z}{\partial x^2}\int_{x_0}^{x} w_x \, dx + \frac{\partial^2 z}{\partial y^2}\int_{y_0}^{y} w_y \, dy \qquad (52)$$

Solutions of these equations, either by algebraic or numerical methods, for hyperbolic paraboloids subjected to several types of lateral loads are well known in European literature.[9] However, they have been considered elsewhere[10] as being only of academic significance because the distribution of lateral forces is generally simplified and assumed in a mathematically convenient manner. Even though the assumption of the distribution of wind pressure in these solutions may not meet the requirements of building codes in the United States, the effect of an earthquake can be simulated by a distributed horizontal force. This latter problem is probably not academic, and a similar approach has been used to compute earthquake stresses in spherical domes and in cones.[11]

---

[8] Visiting Associate Prof., Dept. of Civ. Eng., Carnegie Inst. of Technology, Pittsburgh, Pa.

[9] "Beitrag zur Berechnung der hyperbolischen Paraboloidschale," by K. G. Tester, *Ingenieur-Archiv*, Vol. 16, 1947, pp. 39–44.

There is a minor point in the paper, which is perhaps not pertinent in practical applications but which nevertheless should be clarified. In the derivation of Eq. 14 for hyperbolic paraboloids with a moderate rise, it is stated that "because the differential of S_p with respect to y and x is zero, it is seen from the relationships in Eqs. 5 and 6 that $T_{xp} = T_{yp} = 0$." Actually, from the derivation the following equations are obtained:

$$T_{xp} = f(y) \qquad (53)$$

and

$$T_{yp} = f(x) \qquad (54)$$

The functions, $f(y)$ and $f(x)$, become zero only if the boundary conditions indicate that no normal forces are acting on the edges. Hence, it is the boundary conditions of the free edges that prescribe the constants of integration, and Eq. 14 represents only one of the many possible edge conditions. Mr. Parme has not neglected this point for hyperbolic paraboloids with a great rise because the terms, $f(y)$ and $f(x)$, are included in Eqs. 17 and 18, respectively.

W. Watters Pagon,[12] M. ASCE.—Many engineers and architects have recently become interested in the use of the hyperbolic paraboloid as a structure. The author's authoritative presentation of this subject is of great value to the profession.

However, a review of the deflections of shells of double curvature has not been included. In addition, although the hyperbolic paraboloid is considered to be rigid because of the opposing parabolic elements, no statement concerning its limiting flatness has been presented.

For roof structures, dead load and snow are usually the essential loadings because a wind load seldom will have a marked influence on domes. However, there are three other load conditions that are not included—earthquake loading, uniform radial pressure,[11] and a radial uniform pressure or suction. In 1956 the writer designed a building to house a jet-engine test facility. One of the design conditions was a large unit load per square foot with either inside pressure or inside suction. This caused large stresses and large structural members that could have been avoided had a domed structure been used.

---

[10] "Structural Applications of Hyperbolic Paraboloidical Shell," by F. Candela, *Journal*, A.C.I., Vol. 26, No. 5, January, 1955, pp. 379–415.

[11] "Earthquake Stresses in Spherical Domes and in Cones," by E. P. Popov, *Proceedings Paper 974*, ASCE, May, 1956.

[12] Cons. Engr., Baltimore, Md.

Mr. Parme has examined only a square or rectangular roof slab. Could not the hyperbolic paraboloid be used to house a cylindrical structure, or structures of hexagonal or octagonal shape?

Santi P. Banerjee,[13] A.M. ASCE.—The formulation of the equations and the method adopted for their solution have been presented.

The dead load, w_z, of a hyperbolic paraboloid shell with a reasonably great rise has been represented by Eq. 15*a*, in which w is the constant weight of shell membrane per unit area. Eq. 15*a* is also applicable to doubly curved shells of other shapes. The writer finds that, as the elements in the shell away from the origin form oblique angles, ω, between their adjacent sides, the relationship between the variable load, w_z, on the projected area and the unit weight, w, should be shown as

$$w_z = \frac{w}{\cos\phi \cos\psi} \sin\omega \qquad (55)$$

instead of Eq. 15*a*. The subsequent related equations presented in the paper also require modification.

With regard to the solution of Eqs. 5, 6, and 8*c* for the three unknowns, it is customary, as has been shown, to reduce them to a single equation with one unknown by suitably introducing a stress function, F. The mathematical solution of such an equation becomes extremely involved, and, therefore, relaxation or another similar iterative method is applied for a practical solution of F. However, these procedures also require a great amount of time because an estimate of the initial values is required to begin the iteration process.

However, the writer has found it more convenient to solve the three equations simultaneously after representing them through algebraic expressions obtained from the geometry of the shell form. A solution by this method is more direct.

To illustrate the procedure a hyperbolic paraboloid may be considered. The fundamental equations are Eqs. 5, 6, and 8*c*. The representative equation of shell form is given by Eq. 11 as

$$z = \left(\frac{h}{a\,b}\right) x\,y = C\,x\,y \qquad (56)$$

Thus,

$$\left.\begin{array}{ll} \dfrac{\partial z}{\partial x} = C\,y & \dfrac{\partial z}{\partial y} = C\,x \\[2ex] \dfrac{\partial^2 z}{\partial x^2} = \dfrac{\partial^2 z}{\partial y^2} = 0 & \dfrac{\partial^2 z}{\partial x\,\partial y} = C \end{array}\right\} \qquad (57)$$

Also,

$$\tan\phi = \left(\frac{h}{b} y\right)\frac{1}{a} = C\,y \qquad (58)$$

and

$$\tan\psi = \left(\frac{h}{a} x\right)\frac{1}{b} = C\,x \qquad (59)$$

---

[13] Cons. Engr., Calcutta, India.

Thus,

$$\left.\begin{aligned} \phi &= \tan^{-1} C\,y \\ \psi &= \tan^{-1} C\,x \\ \omega &= \cos^{-1} (\sin\phi \sin\psi) \end{aligned}\right\} \qquad (60)$$

From Eq. 8*c*, after substitution of values,

$$2\,S_p\,C = -\,w_z \qquad (61a)$$

Therefore,

$$S_p = -\,\frac{w_z}{2\,C} \qquad (61b)$$

When w_z varies, due to the constant weight, w, of the shell slab, then

$$w_z = w\left(\frac{\sin\omega}{\cos\phi\cos\psi}\right) = w\,m \qquad (62)$$

and Eq. 61*b* becomes

$$S_p = -\,w\left(\frac{m}{2\,C}\right) = -\,w\,r \qquad (63)$$

From Eq. 5 of the paper,

$$T_{xp} = -\int \frac{\partial S_p}{\partial y}\,dx + f(y) \qquad (64a)$$

and

$$T_{xp} = w \int \frac{\partial r}{\partial y}\,dx + f(y) \qquad (64b)$$

The conditions at the boundaries require that, at $x = a$, $T_{xp} = 0$. Therefore,

$$\left.\begin{aligned} f(y) &= -\,w \int_0^a \frac{\partial r}{\partial y}\,dx \\ f(y) &= -\,w\,\delta x \sum_0^a \frac{\partial r}{\partial y} \\ f(y) &= -\,w\,\delta x\,K_x \end{aligned}\right\} \qquad (65)$$

in which

$$K_x = \sum_0^a \frac{\partial r}{\partial y} \qquad (66)$$

Therefore,

$$T_{xp} = w\,\delta x\left(\sum_0^x \frac{\partial r}{\partial y} - K_x\right) \qquad (67)$$

Similarly, from Eq. 6,

$$T_{yp} = w\,\delta y\left(\sum_0^y \frac{\partial r}{\partial x} - K_y\right) \qquad (68)$$

in which $K_y = \sum_0^b \frac{\partial r}{\partial x}$.

Eqs. 63, 67, and 68 replace Eqs. 16, 17, and 18 of the paper, and, for purposes of solution, are suitable for representation in finite-differences forms.

The numerical values of r at each nodal point of the working grid on the (xy)-plane are known from the geometry of the shell form (Eqs. 60, 62, and 63). Therefore, the values of $\partial r/\partial x$ and $\partial r/\partial y$ and those of the unknowns, S_p, T_{xp}, and T_{yp} are easily obtained.

Equations similar to those derived in the foregoing can also be formed for doubly curved shells of other shapes.

MARIO G. SALVADORI,[14] M. ASCE.—Interest in translational shells that cover rectangular areas is currently increasing in the United States. The writer has had many opportunities to design shells of this kind in the shape of elliptical paraboloids in both the United States and abroad, and found that rarely, if ever, do membrane stresses create difficulties regardless of the span size. The shell thickness is never determined by membrane stresses, but by stresses due to bending, temperature, and buckling. The determination of these kinds of stresses usually presents great mathematical difficulties, but, fortunately, approximate solutions based on the Geckeler assumption[15] lead to simple formulas that are more than adequate for practical purposes.

Bending Stresses Due to Loads.—The discrepancy between the membrane displacements of the shell and the displacements of the sustaining arches introduces bending disturbances in the neighborhood of the boundary that can be determined easily by cylindrical shell theory. Assuming that the transverse shear carries the entire load in the immediate vicinity of the arches, indicating the radius by R and the thickness of the shell at the boundary by h, the shear at the boundary has the value,[16]

$$Q_o = - q\, c \qquad (69)$$

and the maximum bending moment has the value,

$$M_o = - \left(\tfrac{1}{2}\right) q\, c^2 \qquad (70)$$

in which

$$c = \frac{1}{\beta} = 0.76 \sqrt{R\, h} \qquad (71)$$

The normal stress parallel to the arch at the boundary is zero because the whole load, q, is carried by the transverse shear. Fig. 12 indicates the load carried by this shear, which diminishes rapidly as the distance from the arch is increased. In example 2 of the paper, the maximum radii and the thickness have values of $R_x = 102$ ft, $R_y = 156$ ft, and $h = 0.25$ ft. Hence,

$$c_x = 0.76 \sqrt{(102)\ (0.25)} = 3.84 \text{ ft}$$

---

[14] Prof. of Civ. Eng., Columbia Univ., New York, N. Y.; Associate, Paul Weidlinger, Cons. Engr., New York, N. Y.

[15] "Theory of Plates and Shells," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1940, Chapter 11, Section 81.

[16] *Ibid.*, Chapter 12, Section 95.

and

$$c_y = 0.76 \sqrt{(156)\ (0.25)} = 4.75 \text{ ft}$$

With a load of $q = 60$ lb per sq ft,

$$Q_x = -\ 0.23 \text{ kip per ft}$$
$$Q_y = -\ 0.287 \text{ kip per ft}$$

and

$$M_x = -\ 0.440 \text{ kip-ft per ft}$$
$$M_y = -\ 0.682 \text{ kip-ft per ft}$$

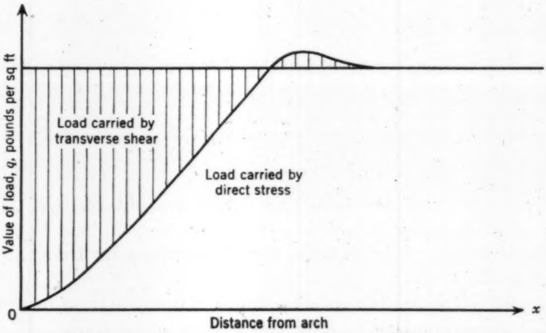


FIG. 12.—AMOUNT OF LOAD CARRIED BY TRANSVERSE SHEAR

The total load carried by the transverse shear is obtained by multiplying Q_x and Q_y by the length of the corresponding sides, which results in

$$w_Q = 2\ (0.230 \times 70 \times 2.87 \times 100) = 89.50 \text{ kips}$$

and represents 37.6% of the total load of 420 kips on the shell.

Bending Moment Due to Temperature Difference Between Shell and Arch.—If the shell is at a temperature $\Delta T°\ F$ greater than the arches and is free from them, it has a radial displacement,

$$w_o = -\ \alpha\ R\ \Delta T \qquad (72)$$

in which α is the coefficient of thermal expansion of concrete equal to 8×10^{-6} in. per in. per ° F. Cylindrical shell theory proves that if this displacement is prevented by the arches, the following boundary shear and moment are developed:

$$Q_o = \frac{\alpha\ E\ h\ c}{R}\ \Delta T \qquad (73a)$$

and

$$M_o = \frac{\alpha\ E\ h\ c^2}{2\ R}\ \Delta T \qquad (73b)$$

For the shell of example 2 these formulas yield $M_x = 63.0\ \Delta T$ and $M_y = 62.8\ \Delta T$. Therefore, it can be seen that a temperature differential of only 10° results in a considerable increase in the bending moments at the boundary. In fact, it is often impossible to absorb them because an increase in thickness of the shell at the boundary produces an increase in c. Provided that sufficient reinforcement is available at the boundaries in a direction that is orthogonal to the arches, it is often necessary and quite proper to allow plastic rotation to take care of the excess moment.

Stresses Due to the Difference in Temperature Between the Outer Surface and the Inner Surface of the Shell.—The value of these stresses can be approximated by assuming that the shell is spherical with a radius, r, equal to the average of the two maximum radii. It is found[17] that the outer and inner fiber stresses have the following values:

$$\sigma_o = (\tfrac{1}{2})\, \alpha\, E \left(1 - \frac{2}{3}\, \frac{h}{r}\right) \Delta T \ldots\ldots\ldots\ldots\ldots\ldots (74a)$$

and

$$\sigma_i = (\tfrac{1}{2})\, \alpha\, E \left(1 + \frac{2}{3}\, \frac{h}{r}\right) \Delta T \ldots\ldots\ldots\ldots\ldots\ldots (74b)$$

In the case of example 2, the following values are obtained: $\sigma_{o,i} = 12\ (1 \mp 0.0016)\ \Delta T$. Such stresses do not usually require special care but may result in an increase of temperature reinforcement that is greater than the minimum required by specifications.

Reinforcement.—The fundamental assumption in the derivation of membrane stresses is that the supporting arches cannot react horizontally. This assumption is responsible for the supposedly infinite value of the tangential shear at the corners, but, as previously mentioned, in the neighborhood of the arches the load is not carried by tangential shear. In addition, if the load carried by the transverse shear is subtracted from the total load, the tangential shear at the corners is not infinite.[18] These shears can be evaluated by the equations for the total vertical equilibrium of the shell.[19]

However, the assumption that the arches do not react horizontally is not satisfied at the corners, in which each arch is supported horizontally by the two adjoining arches at right angles. In order to make the arch as flexible as possible in a horizontal plane, it is advisable not to connect the diagonal reinforcement at the corners with the reinforcement of the arches.

Buckling.—An approximate evaluation of the buckling value of the load, q, can be found by approximating the elliptical paraboloid once more by a tangent sphere. The critical value of a uniform pressure, q, on a sphere is given[20] by

$$q_{cr} = c\, E \left(\frac{h}{r}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

in which $c = 2/\sqrt{3} = 1.155$.

---

[17] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1945, Chapter 11, Section 115.

[18] "Biegetheorie der Translations-flächen und ihre anwendung im Hallenbau," by K. Hruban, *Acta Technica*, Academiae Scientiarum Hungaricae, Budapest, Tomus XI, 1955.

[19] "Analysis and Testing of Translational Shells," by M. G. Salvadori, *Proceedings*, A.C.I., Vol. 27, No. 10, June, 1956.

[20] "Static und Dynamic der Schalen," by W. Flügge, Julius Springer, Berlin, 1957, Chapter VI.

The preceding result obtained by a first-order theory has been refined by Theodore von Kármán, Hon. M. ASCE, and H. S. Tsien,[21,22] who proved that the coefficient, *c*, should be reduced to a value, 0.312.

A recent experience of P. Csonka[23] has proved that even the last value of the coefficient is dangerously high. An elliptical paraboloid built by Csonka behaved satisfactorily for a normal snow load condition, but the paraboloid buckled two years after construction when it was subjected to an exceptional snow load. This experience allowed the evaluation of a lower bound and an upper bound for the critical load of the shell, and these bounds checked with the experimental value for the critical load of a sector of a sphere determined by Torroja. On the basis of the preceding results, a safe value for the coefficient, *c*, in Eq. 75 is 0.05.

For an average radius of 129 ft and a coefficient equal to 0.05, Eq. 75 yields q_{cr} = 81 lb per sq ft and illustrates that for shallow shells the critical load may not differ greatly from the actual load on the shell.

A knowledge of bending and membrane stresses due to loads and temperature changes will be helpful to the designing engineer, who will thus be able to cover large areas at costs less than those of any other type of structure at the present time.

ALFRED L. PARME,[24] A.M. ASCE.—Formulas have been developed by Mr. Au, facilitating to some extent the determination of the effect of lateral loads by use of an integrating factor. However, this simplification does not ease the problem of satisfying the boundary conditions. For example, the shear created in a hyperbolic paraboloid shell due to a uniform lateral load (w_x) acting in the *x*-direction is expanded by Eq. 8*b* to include the effect of a lateral force,

$$2\,S_p\,\frac{h}{a\,b} - \left(\frac{h\,y}{a\,b}\right) w_x = 0 \qquad (76a)$$

or

$$S_p = \frac{w_x\,y}{2} \qquad (76b)$$

Substituting Eq. 76*b* in Eq. 5 and modifying to include lateral surface load results in

$$\frac{\partial T_{xp}}{\partial_x} + \frac{3\,w_x}{2} = 0 \qquad (77)$$

which, when integrated, yields

$$T_{xp} = \frac{-\,3\,w_x\,X}{2} + C \qquad (78)$$

---

[21] "The Buckling of Spherical Shells by External Pressure," by T. von Kármán and H. S. Tsien, *Journal of the Aeronautical Sciences*, Vol. 7, 1939, p. 43.

[22] "A Theory for the Buckling of Thin Shells," by H. S. Tsien, *Journal of the Aeronautical Sciences*, Vol. 9, 1939, p. 379.

[23] "The Buckling of a Spheroidal Shell Curved in Two Directions," by P. Csonka, *Acta Technica*, Academiae Scientiarum Hungaricae, Budapest, Tomus XIV, 1956.

[24] Mgr., Structural & Railways Bureau, Portland Cement Assn., Chicago, Ill.

With only one available constant of integration to satisfy edge conditions for lateral forces at two boundaries, it is apparent that the membrane analysis that implies only direct axial shearing and tangential shearing in a shell is applicable or valid only if suitable reactions occur along one of the boundaries. If the latter condition is not satisfied, as in the case of free-standing umbrella-type hyperbolic paraboloids, it follows that resistance to uniform lateral load must be supplied by radial shears and bending moments in the shell. It can be shown that for a few loading cases membrane analysis is sufficient. However, in most practical cases, such as that reviewed, an analysis which includes bending moments must be used. Presently (1958) there is no simple treatment of the problem of bending in hyperbolic paraboloid shells.

It is correct that generalized differential equations expressing the equilibrium of internal forces, including bending moments in a shell, have been derived. On the other hand, direct rigorous solution of the differential equations has not progressed to a stage at which it can be applied readily. Quite frequently, approximate solutions based on limiting modes of behavior are used. For example, in the umbrella type of shell, the thickness of the shell in the valley near the column is of such magnitude that the entire bending moment can be resisted easily in this local region. The dissipation of the moment into the shell is then approximated on the basis of several limiting conditions of radial-shear distribution.

Mr. Au correctly indicated that the statement following Eq. 11 was misleading, and this error has been corrected.

Mr. Pagon stated the need for more analytical information on the subject, especially on the effect of lateral loads. As mentioned previously, an investigation of the stresses created by lateral loads requires bending analysis of the hyperbolic paraboloid shell. The magnitude of such a study is beyond the scope of this paper.

The question asked by Mr. Pagon on the degree of flatness which can be used and still have the membrane analysis valid for the hyperbolic paraboloid depends to a large extent on the magnitude of the secondary bending moments caused by axial strain. The analysis presented is based on a satisfaction solely of the equilibrium of forces, and no attention is given to the compatibility between strains and stresses. For the usual rise, $h/a = 5$ or $h/b = 5$, the effect of axial strains is unimportant and can be ignored safely. However, when the ratio, h/a, decreases, the effect of axial strains begins to exert a dominant influence on the behavior of the shell. The departure in behavior from that indicated by the simple membrane analysis in a flat shell is analogous to that occurring in a two-hinged parabolic arch subject to uniform load as the ratio of rise to span decreases. For very flat parabolic arches, it can be shown that if the rib shortening effect—that is, axial deformations—is included in the analysis, the horizontal component of the reaction for a given span decreases as the ratio of rise to span decreases. With no rise the horizontal component decreases to zero, and, thus, the secondary bending due to axial strains approaches the simple-beam bending moment as a limiting value.

The structural action of a hyperbolic paraboloid shell is due to the fact that its curved surface resists the load by two sets of parabolic arches which are perpendicular to each other, as shown in Fig. 3. Therefore, some insight into the effect of curvature can be obtained by examining a strip which is parallel to the arches as a free body. If the shearing forces and normal forces on the two opposite faces are ignored, and if it is assumed that the ends of the arches are not free to move, then the secondary bending moments due to lack of curvature can be determined as for an arch. The result of such a study is presented in Fig. 13 for various ratios of $\frac{h\,t}{a\,b}$.

The secondary bending moment at various distances from the corner, designated by the dimensionless quantity, x/t, is expressed in terms of the simple-beam bending moments occurring in a strip of length L. Fig. 13 indicates that because the ratio of rise to span approaches zero, at the corner the load is carried entirely by beam action, which is contrary to what can be expected from membrane theory. For strips farther away from the corner, the secondary moment decreases. The rate of the decrease is a function of $\frac{h\,t}{a\,b}$. The larger the ratio of $\frac{h\,t}{a\,b}$, the more rapid the decrease in the magnitude of the secondary moments. The usual value of $\frac{h\,t}{a\,b}$ for the umbrella type of hyperbolic paraboloid is approximately 0.010. Assuming that the thickness is approximately 3 in., the secondary moment becomes unimportant at a distance of approximately 5 ft from the corner.

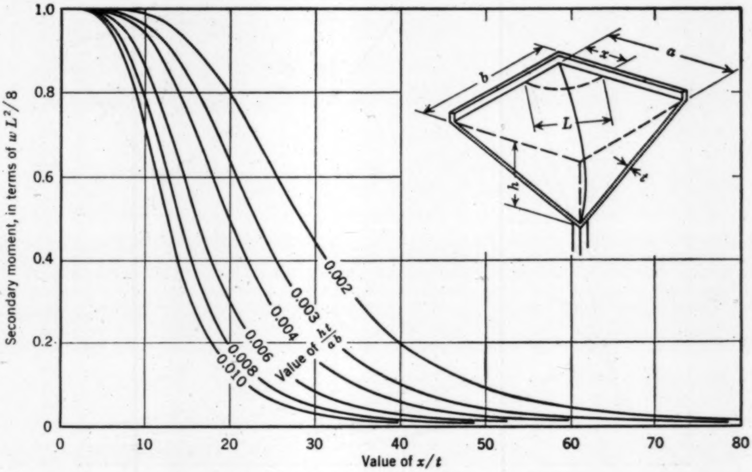


Fig. 13.—Values of Secondary Bending Moments

Fig. 13 shows another important characteristic that has been observed on some of the shells that have been built. It is seen that at the corner the load is carried mainly by ordinary beam action. Hence, the load is transmitted to the edge beams principally by radial shears. The edge beams near the corner are thus loaded vertically and act as cantilevers for a small part of their length. Consequently, the edge beams in this vicinity should not only be designed for the tension computed by membrane theory, but should also be deepened to prevent excessive deflection, and the edge beams should be reinforced for negative moment. This is especially desirable when the edge beam is upturned.

Because the value of L increases linearly in proportion to the distance from the corner, it is more expedient to show the effect of axial strains in terms of the secondary flexural stresses that are created. Such values are plotted in Fig. 14, which brings into sharper relief the importance of curvature on the magnitude of the secondary stresses. For an umbrella type of hyperbolic paraboloid that is subjected to a load of 72 lb per sq ft and with a ratio of $\frac{h\,t}{a\,b} = 0.01$, the secondary stresses occur at $x/t = 13$ and are

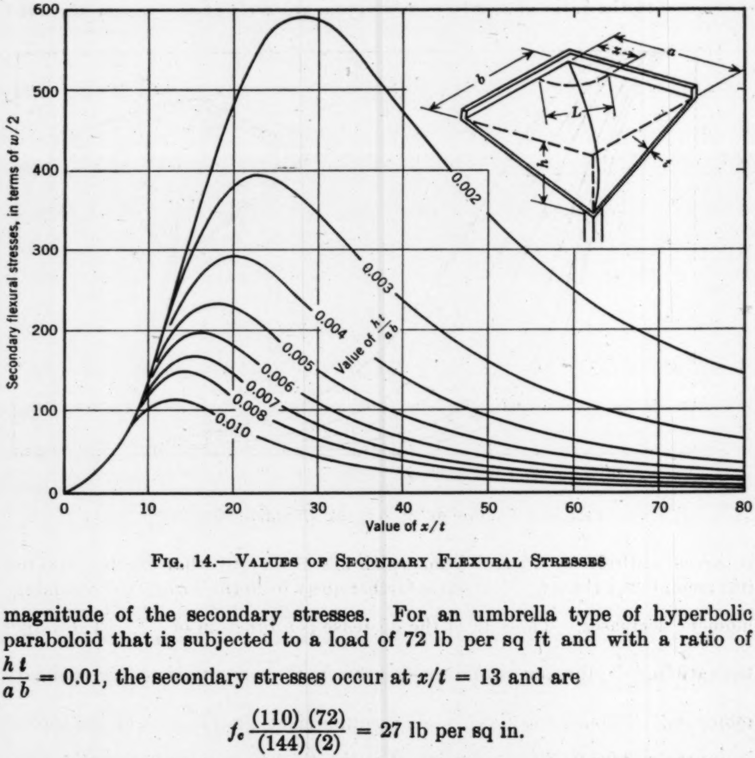


FIG. 14.—VALUES OF SECONDARY FLEXURAL STRESSES

$$f_c \frac{(110)\ (72)}{(144)\ (2)} = 27 \text{ lb per sq in.}$$

For the analysis of various types of shells with different types of boundary, as required by Mr. Pagon, no generalized solution can be given. For this reason equations based on finite differences were presented.

Mr. Bannerjee prefers to deal directly with the stresses instead of the stress function, F, which agrees with the writer's experience and with the suggested alternate finite-difference Eq. 48. His recommended modification of the formulas to include the case of oblique shape is welcome.

Mr. Salvadori's application of the well-known solution for cylindrical surfaces to the determination of the bending stresses occurring near the arches in the elliptical paraboloid is the only practical approach to estimating the bending in shells of double curvature. Nevertheless, in using this approximation the difference between the idealized state, which forms the basis of Mr. Salvadori's formulas, and the actual conditions must be considered. The magnitude of the bending moments presented in Eqs. 70 and 73*b* is that which occurs in an infinitely long cylinder subject to a radial line load that is applied uniformly about the perimeter. The formulas are also valid for cylinders subject to a radial line load which varies sinusoidally as $\sin (n \pi)$ about the perimeter, provided that $n \sqrt{t/r}$ is less than 0.50. In both cases it is implied that the resistance to the line load or the imposed deformation is developed by compression in the adjacent elements which act as fully continuous rings. When the rings are not continuous, their stiffness is greatly reduced, and, hence, the applicability of the formulas is questionable. Thus, although Eqs. 70 and 73*b* may predict the structural behavior near the crown with a fair degree of accuracy, because the horizontal strips are continuous they may exaggerate the moment near the corner.

It should also be indicated that Eqs. 70 and 73*b* assume that the edges of the shell are fully restrained against angular rotation. This is not usually the case because the arches have little torsional stiffness. With no restraint the maximum moment occurs at some distance from the edge, and its value is approximately one-third of that indicated by Eq. 70.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2952

BUREAU OF PUBLIC ROADS EXPERIENCES IN HIGHWAY SURVEYS

By William T. Pryor,[1] A. M. ASCE

Synopsis

New methods and practices in surveying are fulfilling present needs throughout all the states in the rapidly expanding highway-construction program. Ground-survey methods cannot be supplanted but can, and should be, supplemented by photogrammetry and aerial surveys in order to obtain better surveys and highways in less time with fewer engineers.

Introduction

The United States has become great through the ability of the people to change, progress, and cope with new problems of every kind, description, and magnitude. Such problems are innumerable, and for many reasons it has been questioned whether there are enough trained people to keep abreast of them.

In highway engineering the difficulties multiply and increase at an immeasurably rapid rate. Circumstances, economic forces, convenience, comfort, and safety requirements demand more and better highways. Current rates of highway construction—although much increased beyond rates in the past—do not fulfil present needs. Combined with this realization, the current (1956) but insufficient increase in highway construction reveals the alarming shortage of experienced engineers, and stresses the inadequacy of outmoded survey methods and practices in force since the railroads and before the advent of motor vehicles.

Arthur C. Clark of the Engineering Division of the Bureau of Public Roads, United States Department of Commerce, has stated[2]:

Note.—Published, essentially as printed here, in December, 1956, in the Journal of the Surveying and Mapping Division, as *Proceedings Paper 1117*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Highway Engr. and Chief of Aerial Surveys, Bureau of Public Roads, U. S. Dept. of Commerce, Washington, D. C.

[2] "Increasing Engineering Productivity for New Program Levels," *Proceedings*, A.A.S.H.O., 1955, pp. 205–219.

> "Current practices often require up to twenty-one months elapsed time, and in some instances even more time, from the date legislative authorized funds are apportioned to the actual commencement of construction. This delay is due in part to financing and right-of-way acquisition problems and in part to administrative determinations as to policies and priority of projects for programing. A substantial portion of the elapsed time, however, is consumed in performing the actual preliminary engineering operations."

Clark further stated that

> "The recently completed Highway Research Board survey of tine-saving methods in highway engineering relating to location, and road and bridge design discloses increments of progress toward modernizing highway engineering practices. It also discloses that there is no concerted effort on a broad enough scale for a full 'package' modernization of highway engineering practices."

It should be emphasized that the greatest challenges for decreasing elapsed time and saving engineering manpower lie in the preliminary engineering stages of highway work.

The many in the highway-engineering profession who question and wonder are justified in believing that the still greater increase that will occur in highway construction will overwhelm the states and all highway-engineering organizations. However, few of them feel that the equipment and materials manufacturers, the transporters of goods, or the construction contractors will fail to meet the demands of the new and larger construction programs. The greatest concern of many individuals is in the accomplishment of the engineering work that is always necessary in the preparation of plans before construction and in the performance of construction engineering once the power shovels, tractors, trucks, and other equipment are at work.

Less progress has been made in improving engineering methods than in materials, equipment, transportation, and methods of construction. Duane L. Cronk has quoted[3] that " * * * research people are * * * hard on the status quo * * * (they) change entire processes." The researchers that were referred to are learning new facts about highway administration and finance; traffic; materials, equipment, and methods of highway construction and maintenance; and ways and means of improving the appearance, durability, and safety of highways. However, the effect of research on surveys was not mentioned. Too little has been thought, done, and said about improving the old ways and devising new ways and means of making the various surveys essential in each highway engineering stage. This fact is now causing alarm as a result of the known shortage of experienced engineers and the increased highway-construction program. Even those who are reticent are much concerned because they realize that newer, faster, and fully satisfactory survey methods are necessary. Moreover, such methods must make it possible for present engineering staffs to be responsible for increased expenditures and greater highway mileages. Otherwise, it will be necessary to admit to the inability of existing staffs, which cannot be much enlarged, to cope with future highway-engineering tasks. Anticipating this situation, some individuals have been greatly concerned. Fortunately, there have been a few who have performed pioneering services in the adaptation into highway engineering of new survey methods from other professions, and in the development of new methods and the improvement of old ones. Others have begun to give the new and improved survey methods a try.

---

[3] "Road Building Begins with Research," by Duane L. Cronk, *The Highway Magazine*, June, 1956, pp. 135–137.

Such methods are aerial and photogrammetric in character and have been pioneered for several decades by a few who have had the foresight and courage to tread an unknown path full of obstacles. Regrettably, the acceptance of the new and improved methods of these pioneers has been slow, and the status quo has been adhered to tenaciously for many reasons.

The realizations and admissions by the reticent and the doubting are the greatest assurance that the aspirations of a few men will cause concern among the rest so that it will be possible to cope with the problems in their increased magnitude. In doing this it will be necessary to find new and improved methods to attain the objectives. In the highway field more and better highways that are adequate for the public's needs are one objective—engineering is only a means to that end.

The BPR has many responsibilities, among the most important of which are the administration of federal aid in all states; the engineering and construction of highways to, in, and through national forests and parks and other federal lands in thirty-six states, Alaska, and Puerto Rico; research; and the development of new methods and the improvement of old ones. In addition, the BPR has cooperated in the engineering and construction of highways in other countries. Admittedly, much of the surveying for all the engineering work has been performed, and continues to be, by ground-survey methods. Full advantage has not been taken of opportunities to use aerial methods, but, fortunately, this practice is changing. In the summer of 1954, authority was received to negotiate for the services of qualified and reliable photogrammetric engineering firms and consultants. This authority, the enlarged construction program (which will increase), and the shortage of engineers are working toward effecting the needed changes in highway-surveying practice.

Engineering and Surveying Stages

Preliminary engineering tasks are performed throughout a series of coordinated stages, and in each stage surveys are required. The surveys usually begin with the general information and proceed to the specific or detailed information. At first, their scope is broad and their characteristics are those of reconnaissance, in order that all feasible route alternatives can be determined and compared with respect to type, condition, value, and relationship. Then the surveys retain their reconnaissance characteristics, become narrow in scope, and are used to compare the alternatives in sufficient detail to determine the route having the most advantages in service; comfort; convenience; safety; initial cost of right of way and construction; motor vehicle operation expense; cost of maintenance; and cost of improvement to fulfil new requirements wherever traffic might increase beyond the design capacity. Thirdly, a preliminary survey is made of the best route for measuring dimensions accurately and obtaining sufficient information on topography, soils, drainage, land use, rights of way, and other details necessary for the design of the highway location and the preparation of plans. Fourthly, a location survey is made that includes staking the highway and its right of way and structures on the ground in readiness for construction.

Throughout the foregoing stages, consideration must be given to the multiple influences of traffic in number, weight, speed, and access to the highway; right of way and land use bordering the proposed highway site; topography; the kind and condition of soils; drainage; appearance; convenience; and comfort and safety. The complexity of these considerations has caused the replacement of the one-man highway locator by a large staff of specialists in the survey, design, and location of any major highway project.

GROUND SURVEYS VERSUS AERIAL SURVEYS

Mistakes in highway location can be avoided by sufficient reconnaissance in the first two stages. However, mistakes in reconnaissance cannot be corrected during the preliminary survey in the third stage.

Reconnaissance on the ground is difficult, and it is often piecemeal. Controls of location are at natural scale and are usually so far apart that their significance and relationships are difficult to determine and evaluate. In contrast, however, aerial photographs used as a reconnaissance tool bring the topography, land use, soil and ground conditions, drainage, and all other controls of highway location into the office at a scale that enables the engineering staff to understand the full significance and relationships of one control to another. Nothing need be missed, overlooked, or improperly evaluated. Every feasible route alternative can be determined and compared, in contrast to merely finding a way—never knowing whether it is the best or the worst—as is the case if the reconnaissance is accomplished on the ground in limited time by a one-man locator. In addition, when aerial photographs are at hand, each specialist can do his part while working with complete information. On the other hand, on the ground each individual usually obtains a different concept, and verifications or clarifications can be obtained only by repeated reconnaissance, which is costly in terms of both time and money.

For example, a reconnaissance survey was made in 1948 by aerial methods, and route alternatives were determined between two terminal points, which were 100 air miles apart. One year before this aerial survey was conducted, a ground survey had been made for a 15-mile section of the route, where it was necessary to cross eight rivers and the topography was rugged. After the aerial survey had been completed, it was necessary to decide whether the 15-mile ground survey should be adhered to or abandoned in favor of a similar length segment of the 120-mile through-route that was determined by aerial survey. Two years after the ground survey had been made and one year after the aerial survey had been completed, both routes were center-line cleared on the ground because they were in a densely vegetated, tropical region in which regrowth was rapid and in which there had been no previous clearing on the route that was determined by aerial surveys. Then, with profiles in hand, engineers walked over each route, made on-the-site estimates of highway- and bridge-construction costs, and compared the alinement, grades, curvature, and cross section of each route. Results indicated better grades on the aerial-survey route and estimated savings in construction costs of $300,000 for grading and $400,000 for bridges (an average total saving of nearly $47,000 per mile). Another comparison can be made: The total cost of the 4,000-sq-mile aerial survey was only $30,000 for all area and route photography; stereoscopic examination of the photographs; determination of more than 240 miles of route alternatives; preparation of estimates of construction cost; comparison of routes; and recommendation of the best 120-mile route for preliminary survey, design, and construction. The 15-mile partial route reconnaissance and preliminary survey that were made on the ground cost the same.

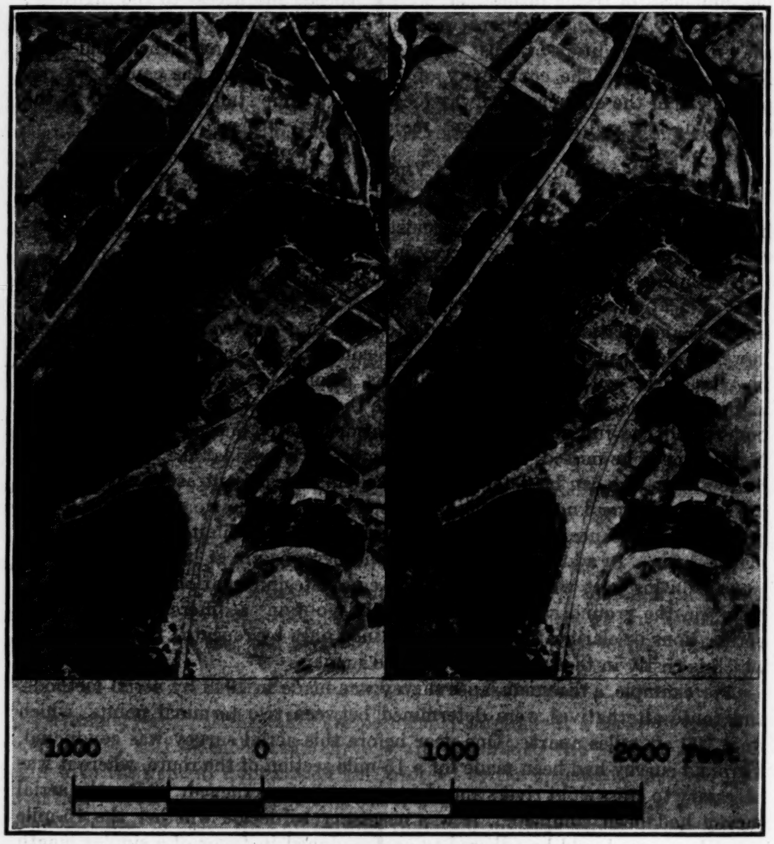


FIG. 1.—STEREOGRAM.

Once the reconnaissance has been completed by aerial methods, an exact, detailed, photographic image record is obtained in stereoscopic correspondence of all routes that were determined and of the route that was chosen for a preliminary survey. The stereogram in Fig. 1 shows possible route alternatives placed on original photographs by use of white acetate tape. This delineation of routes in stereoscopic correspondence with the topography enhances their illustrative value and is a valuable guide to field parties making a P-line survey of the route on the ground. To examine this stereogram stereoscopically, either of two methods may be used—a lens-type stereoscope or the unaided eyes. To study a stereogram without a stereoscope, the left photograph is examined with the left eye and the right photograph with the right eye. To simplify this procedure, a 10-inch card may be placed between the eyes from the face to the line between the pair of photographs of the stereogram. The observer is thus prevented from looking first at one photograph and then the other. With the card in place, the observer must look into the distance (as if he were seeing through the paper on which the photographs are printed) until the three-dimensional picture is seen beyond the pages of the book. A little practice will eliminate the necessity for the card.

Thus, with the photographs as a guide, an otherwise undirected party can make the third-stage preliminary survey of a part or all of the route at any time by either ground methods or aerial methods. However, whenever ground reconnaissance is made the only guidance records that are available are the sketches, the oral and written reports of the locator, and the flags, if any, which the locator sets while going over his route. Immediate personal attention must be given to each mile of the preliminary survey if it is to be made where intended. It would be difficult for the locator to begin anywhere and survey a section of the route.

By ground-survey methods only, it is also difficult, costly, and time consuming to make preliminary surveys of route bands that are sufficiently wide to include all the alinement, grade, and cross-sectional possibilities on the chosen route. Nor is it easy to include the changes that usually become necessary but cannot be fully anticipated.

A reduced-size reproduction of a topographic map, which was surveyed by the P-line and plane table methods, is shown in Fig. 2. The working scale was 50 ft to 1 in., and a contour interval of 5 ft was used. The narrow width mapping was insufficient, and the center line (stationed at 500-ft intervals) extended beyond the mapped area. To complete the design, additional mapping was required, for which photogrammetric methods were used. Conversely, however, a route band of topography, which is from two to twelve times as wide, can be surveyed by aerial photogrammetric methods at less cost than, or equal cost to, ground-survey methods.

Fig. 3 shows a reduced-size section of a topographic map, compiled in 1956 by stereophotogrammetric methods, of the same section of highway route that is shown in Fig. 2. The compilation scale was 100 ft to 1 in., and the contour interval was 5 ft. Ample mapping width in realistic detail was obtained easily. However, dashed-line contours are in densely wooded areas in which it was not possible to obtain the same degree of detail and accuracy as in situa-

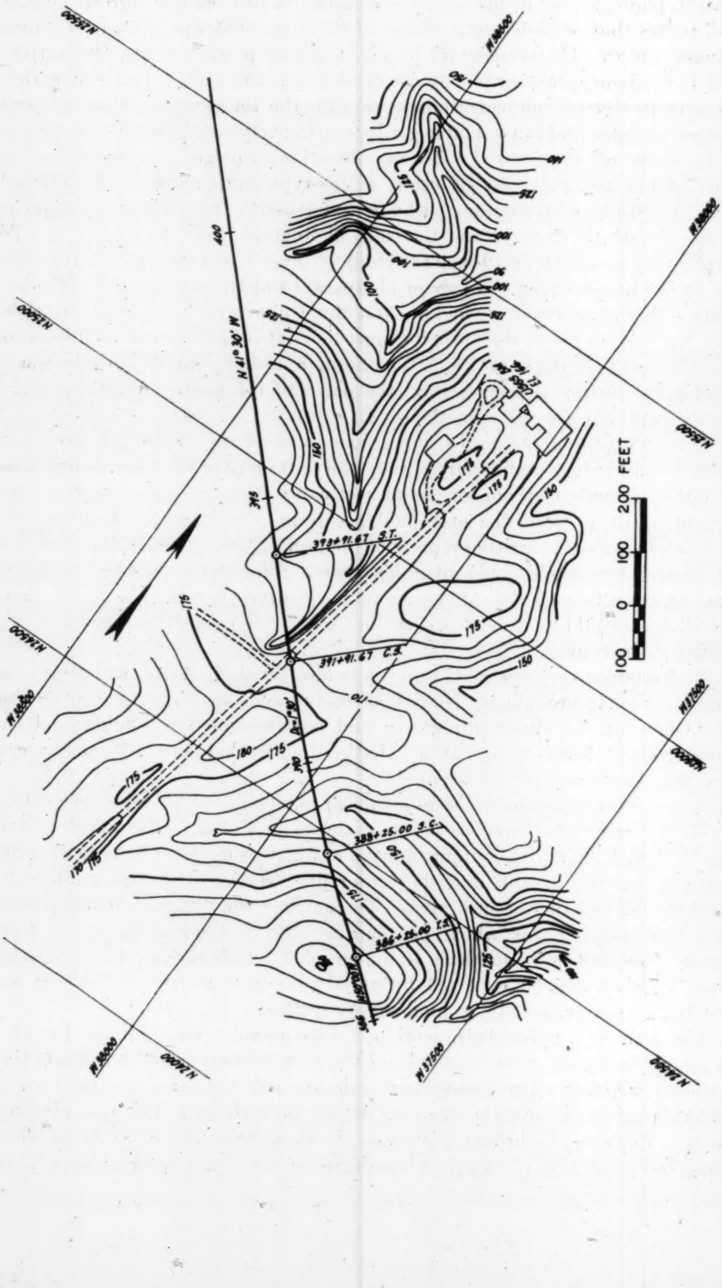

N 41° 30' W
400
395
393+91.67 S.T.
391+91.67 C.S.
388+25.00 S.C.
386+25.00 T.S.
Δ = 1° 16'
390
385
N 38500
N 38000
N 37500
N 12500
N 12000
N 12400
175
180
170
150
125
100
140
90
200 FEET
100
0
100

FIG. 2.—TOPOGRAPHIC MAP SURVEYED BY GROUND METHODS

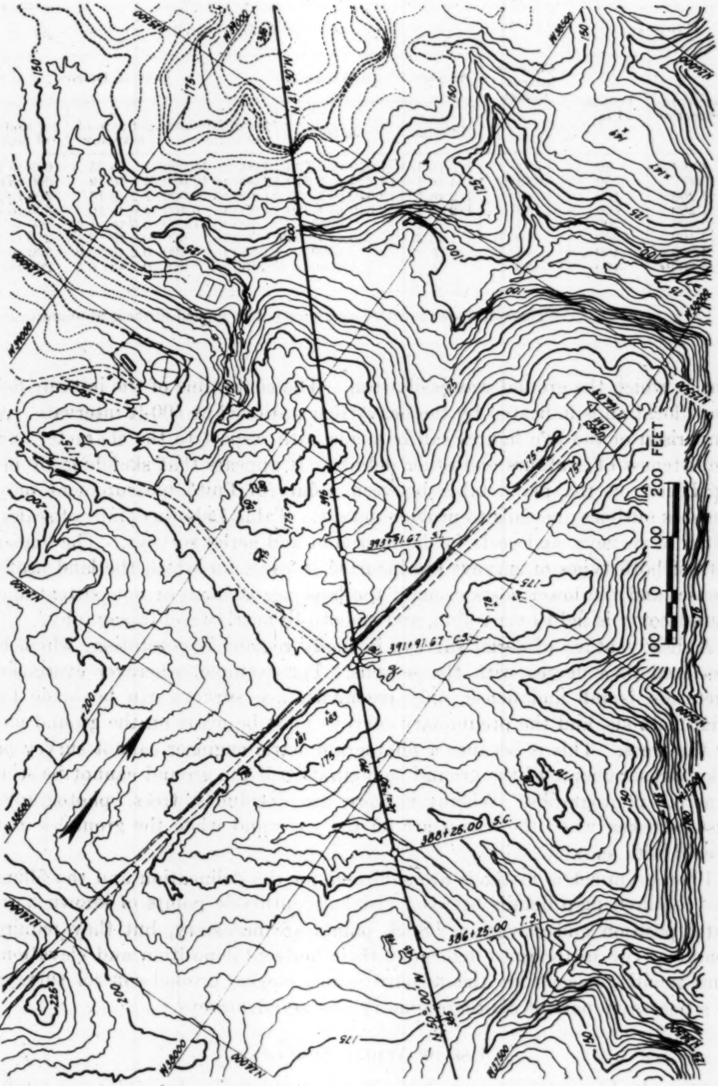

N 81° 30' W
393+91.67 S.T.
391+91.67 C.S.
388+28.00 S.C.
386+25.00 T.S.
N 50° 00' W
100 0 100 200 FEET
150
175
125
100
200

FIG. 3.—TOPOGRAPHIC MAP COMPILED BY STEREOPHOTOGRAMMETRIC METHODS

TABLE 1.—COMPARISON OF PRELIMINARY SURVEYS FOR

MAP SCALE		Contour interval, in feet	AERIAL SURVEYS				
Representative fraction	Feet to 1 in.		Average Width of Survey		Cost, in Dollars		
			in miles	in feet	per square mile of mapping	per acre of mapping	per route-mile of highway
1:480	40	1	1/5 to 1/4	1,050 to 1,320	3,000 to 16,000	5 to 25	600 to 4,000
1:600	50	2	1/4 to 1/2	1,320 to 2,640	2,500 to 12,000	4 to 19	600 to 6,000
1:1,200	100	2	1/2 to 1/4	2,640 to 1,320	1,400 to 10,000	2.2 to 16	700 to 2,500
1:1,200	100	5	1/2 to 1/4	2,640 to 1,320	1,000 to 6,400	1.6 to 10	500 to 1,600
1:2,400	200	5	1	5,280	400 to 1,000	0.6 to 1.6	400 to 1,000
1:2,400	200	10	1 to 2	5,280 to 10,560	250 to 600	0.4 to 0.9	250 to 1,200
1:4,800	400	10	2	10,560	150 to 400	0.2 to 0.6	300 to 800
1:4,800	400	20	2	10,560	100 to 300	0.2 to 0.5	200 to 600

[a] The widths are those that were surveyed by ground methods at approximately the

tions in which the ground was easily seen. The center line of the initially designed location is indicated by the heavy line stationed at 500-ft intervals. A comparison of the two figures can be made with respect to the character and completeness of contours and other details. It appears that sketching by interpolation on the plane table between points of finite measurement only furnishes outlines and fails to provide details. Table 1 shows details of scales, widths of surveys, and costs for both ground and aerial surveys. The higher costs of both types of surveys are incurred in cases for which the land use is intense, and the lower costs occur if the land use is small or nonexistent and ground cover is light over rolling and smooth-to-nearly-level topography.

A disadvantage of aerial surveys in many regions is vegetation, which is associated to a degree with the seasons. For example, wherever evergreen vegetation is tall and dense, only reconnaissance surveys can be made by aerial methods, and the preliminary surveys must be made on the ground under the trees. This is because a photogrammetric engineer cannot survey or make measurements of the ground configuration if the ground cannot be seen in aerial photographs. Wherever there are deciduous trees, photographs must be taken when the leaves are off the trees and when the ground is not covered with snow.

Photogrammetric methods are sufficient for the delineation and the filling in of required map details between visible, identifiable points of known horizontal position and elevation. Such points are necessary, but they require ground-survey methods to determine their horizontal position and elevation. Consequently, except for the reconnaissance stages, ground-control surveys are a necessary part of photogrammetry and aerial surveys.

USE OF AERIAL SURVEYS

Many highway departments have inquired about the desirability of establishing a photogrammetric and aerial-surveys unit, and of the general organization and equipment necessary to take aerial photographs, make ground-control surveys, and prepare maps by photogrammetric methods. This unit

HIGHWAYS—AERIAL SURVEYS VERSUS GROUND SURVEYS

GROUND SURVEYS					Number of times aerial survey is wider than ground survey
Average Width[a] of Survey		Cost, in Dollars			
In miles	In feet	Per square mile of mapping	Per acre of mapping	Per route-mile of highway	
1/30 to 1/16	180 to 330	12,000 to 48,000	19 to 75	400 to 3,000	6 to 4
1/24 to 1/8	220 to 660	10,000 to 32,000	15 to 50	400 to 4,000	6 to 4
1/12 to 1/8	440 to 660	7,700 to 24,000	12 to 37	640 to 3,000	6 to 2
1/12 to 1/8	440 to 660	4,800 to 13,600	7 to 21	400 to 1,700	6 to 2
1/6 to 1/8	880 to 660	2,700 to 9,600	4 to 15	450 to 1,200	6 to 8
1/6 to 1/8	880 to 660	2,100 to 8,000	3.3 to 12	350 to 1,000	6 to 16
1/5 to 1/6	1,050 to 880	1,500 to 4,800	2.3 to 7	300 to 800	10 to 12
1/5 to 1/6	1,050 to 880	1,000 to 4,200	1.6 to 6	200 to 700	10 to 12

same cost per route-mile of highway as the wider widths surveyed by aerial methods.

would supply the products of photogrammetry and aerial surveys but not use them. The users of these products should be highway engineers whenever photogrammetry is applied to highway engineering from the two stages of reconnaissance through preliminary surveys, soils exploration, and location surveys to the stages of construction and maintenance.

To apply photogrammetry effectively in highway engineering, the user of aerial photographs first must be specialized as well as qualified as a highway engineer. Secondly, he must know how to apply photogrammetry in his particular work. The scales that are most useful and the accuracy that will be required from the photographs in each stage of highway engineering must be known, as well as the means available to obtain and to use such photographs.

Each highway engineer should avail himself of every opportunity to use aerial photographs and the products that can be obtained from them, which are easily specified as required and are readily obtainable from photogrammetric engineering firms.

In photogrammetry the preparation of large-scale topographic maps, planimetric maps, and profile and cross sections from aerial photographs by use of photogrammetric equipment is a specialty similar to that of the transitmen, levelmen, rodmen, and chainmen who make both preliminary and location surveys on the ground. These specialists do not design a highway location or prepare plans and make engineering estimates by use of the qualitative information and dimensional data of their surveys. Such work is performed by highway locators, designers, and estimators who use the survey information and data furnished by the field survey parties. The topographic or planimetric maps that have been compiled and the profile and cross sections that have been measured by photogrammetric engineers can be used by highway engineers, supplemented by a stereoscopic examination of the photographs from which they were prepared. Photogrammetric engineers engaged by contract, or employed in the highway department, may be specialists who do not know fully the principles, procedures, and practices of highway engineering. Each highway engineer, however, while working on his special assignment, should be able to apply methods of photogrammetry and photographic interpretation, and should have sufficient knowledge to specify properly what the photogrammetric engineers can and should provide for his use. The limitations of photogrammetry, as well as its practical uses and advantages, must be known.

Highway engineers should obtain initial experience in aerial methods by the use of aerial photographs for reconnaissance purposes only. Such photographs may be obtained from available sources of areas in which the use of the land has not changed much since the photographs were taken, or should be newly obtained of areas in which vital changes have occurred and up-to-date information on land use is needed. Topography changes little throughout the years, but land use changes rapidly near metropolitan areas.

The reconnaissance of an area to determine feasible highway routes can be accomplished almost completely by the use of (*a*) small-scale aerial photographs and (*b*) less complex photogrammetric instruments. The latter include mirror and lens-type stereoscopes, engineer's scales, parallax bars, stereocomparagraphs, contour finders, the elevation wedge, and stereo-slope comparators. In addition, aerial photographs can be used to illustrate (1) the engineering problems caused by conditions existing prior to highway location; (2) perspective views of what the highway will look like when built, as designed on the best route; (3) a picture of the highway when constructed; and (4) the appearance of the highway after traffic use, including changed conditions.

For reconnaissance, highway engineers with experience in location and design should use the photographs stereoscopically, as well as the best available maps. For the illustrative uses, it is essential to have an artist who is familiar with highway engineering and who is skilled in the art of perspective delineation on photographs. In addition, it is advantageous to have the service of a photographic laboratory, where contact prints, photographic enlargements, and photographic mosaics of both the uncontrolled and the controlled types can be prepared from aerial negatives. For this reason the laboratory should have a contact printer, a precise photographic enlarger, a photographic-copy camera, and radial-line plot equipment.

PHOTOGRAMMETRIC ENGINEERING SERVICES

Once a highway department has initially applied photogrammetry and the use of elementary photogrammetric equipment in the reconnaissance stages, it can judge better the extent to which it should staff and equip itself to use aerial photographs and precise photogrammetric methods in the preparation of large-scale maps for the preliminary survey. However, while using photogrammetric methods for reconnaissance, the highway department may make its preliminary surveys on the ground, or contract with reliable photogrammetric engineering firms, which are equipped and staffed to perform such services for preliminary-survey photography and large-scale route mapping. There are many advantages in engaging the services of these firms, especially for aerial photography and mapping. Specifications can be written that make it practicable to obtain aerial photographs and maps to the scales needed and within the limits of accuracy required. Such specifications, when complied with, would assure results of uniform quality at a reasonable and an initially known cost.

For economic utilization of precise photogrammetric methods for mapping, there is a relationship between map scale and contour interval that should be observed. Expressed numerically, the map scale (in feet to 1 in.) should be forty times the contour interval (in feet). Some representative map scales in feet to 1 in. and the corresponding contour interval in feet are: 200 ft and 5 ft; 80 ft and 2 ft; and 40 ft and 1 ft. Wherever maps are required at scales of 100 ft to 1 in., or 50 ft to 1 in., they can be obtained by photographic reduction in map size and scale from 80 ft to 1 in. to 100 ft to 1 in., and from 40 ft to 1 in. to 50 ft to 1 in. The contour interval of 2 ft and 1 ft, respectively, is retained.

There are reasons for having a photogrammetric unit to prepare topographic maps from aerial photographs as there are reasons for an engineering organization making surveys and supervising construction instead of periodically employing consulting engineers. The principles and ideas with respect to consultants in the survey and construction field apply to photogrammetric engineering firms. When specifications are not complied with, consultants can be required to do the work over at their expense until compliance is attained. However, any incorrect work performed by the payroll employees must be done over at the employer's expense.

Testing the adequacy and the accuracy of position in map details, contour elevations, and profile and cross-sectional measurements is an arduous task. Coupled with these disadvantages are the delays that occur whenever maps or measurements made photogrammetrically fail to comply with specified accuracies.

A near-ideal situation will exist when all maps and measurements that are made photogrammetrically can be tested by the same methods rather than by traverse running, profile measuring, and position testing by ground-survey methods. The ideal will be attained when all photogrammetric work is so accurate and reliable that all tests can be eliminated. Present methods of testing maps photogrammetrically, together with a few measured spot elevations and positions, are a step forward.

A principal advantage of negotiating contracts with photogrammetric engineering firms is that payroll employees are relieved of the arduous, time-consuming task of making surveys. Thus, they will have more time to design highway locations, prepare plans, and procure rights of way by use of the aerial photographs, photographic mosaics, maps, profiles, and cross sections provided by such firms.

Another advantage is that highway stakes need not be set until staking for construction is necessary. In acquiring right of way, problems are decreased by projecting trial locations on maps without alarming the public and property owners by numerous preliminary-survey lines on the ground. Still another advantage is the continuity that can be obtained by the early completion of surveys, designs, and plans from terminal point to terminal point for right-of-way acquisition before land values increase and construction funds are available or allocated.

Photogrammetric methods that are properly used are an efficient companion to electronic methods of computing such values as end areas, earthwork volumes, mass-diagram values, and survey traverses. The profile and cross sections can be measured and their dimensions recorded photogrammetrically for making the electronic computations. Designs can be completed without plotting cross sections from ground-survey measurements or from photogrammetrically prepared maps. Similarly, the highway alinement can be computed and fully coordinated to ground-survey position in the system of state plane coordinates through the station markers set while the ground-control surveys were made for the mapping. For any projects that are more than 1,000 ft above mean sea level, the coordinates of such markers should be adjusted to apply at the average elevation of the area of survey rather than at sea-level datum. The reason for this adjustment is to make measurements of distances on the map correspond correctly with horizontal measurements on the ground, thus eliminating the need for correction of each map distance.

Conclusions

Although surveying practice has been slow to change, highway engineers must acknowledge the necessity for improvements. The next step is to take advantage of new methods that have been proved in fair trial. Finally, as more individuals take advantage of the new methods, there will be greater advances as a result of change and improvement, and the highway-engineering profession will become better qualified and capable to cope with the many existing highway problems.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2953

A PRESSURE-LINE CONCEPT FOR INELASTIC BENDING

By Frank Baron,[1] M. ASCE

With Discussion by Messrs. John A. Hribar, and Frank Baron

Synopsis

The concept of the pressure line, useful for arches and rigid frames, is developed for quickly estimating the effects of plasticity on the behavior of structural elements subjected to axial and flexural loads. An initial estimate, which is obtained by means of an elementary theory of mechanics, is adjusted to fit the conditions of plasticity.

A procedure is presented for obtaining a quick estimate of the effects of nonlinear stress-strain relationships on the behavior of structural elements subjected to axial and flexural loads. The estimate is obtained by using an elementary theory of mechanics and adjusting the estimate to fit the conditions of a theory of plasticity. In this procedure the results of both theories are available for comparisons and for further studies, as in problems of ultimate-load analyses and inelastic buckling.

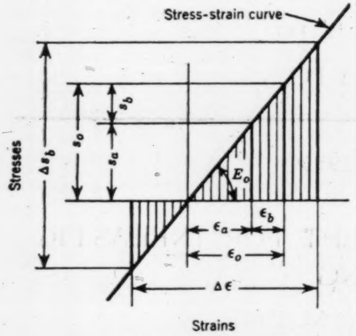
Figs. 1 and 2 show a cross section of a differential element that is subjected to an axial load, P, and a moment, M, about a centroidal axis of the section. The area of the cross section is designated by $A = \sum da$, and the moment of inertia about the centroidal axis by $I = \sum x_c^2\, da$. For convenience the cross section is assumed to have an axis of symmetry as shown. The effects of shear, Poisson's ratio, residual strains, and local instability of projecting elements of the cross section are neglected herein.

The results of the elementary theory of mechanics are summarized in Fig. 1. In Fig. 1(*a*), the stress, s_o, in a fiber is defined as

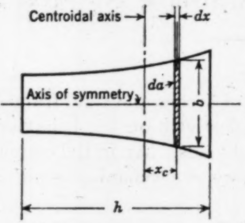
$$s_o = s_a + s_b \qquad (1)$$

Note.—Published, essentially as printed here, in January, 1957, in the Journal of the Engineering Mechanics Division, as *Proceedings Paper 1157*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

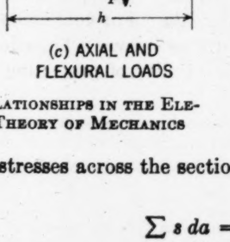
[1] Prof. of Civ. Eng., Univ. of California, Berkeley, Calif.



(a) VALUES OF STRESSES AND STRAINS



(b) PROPERTIES OF CROSS-SECTION



(c) AXIAL AND FLEXURAL LOADS

FIG. 1.—RELATIONSHIPS IN THE ELEMENTARY THEORY OF MECHANICS

in which $s_a = P/A$ and $s_b = M\,x_c/I$. The distribution of stress across the section is linear and satisfies the requirements of statics, namely—

$$\sum s_o\,da = P \ldots\ldots\ldots (2)$$

and

$$\sum s_o\,x_c\,da = M \ldots\ldots (3)$$

The linear distribution of stress is the result of assuming (1) a linear distribution of strains across the section, and (2) a linear stress-strain relationship for the material. The angle change between adjacent cross sections is stated in terms of strains, stresses, and moment as follows:

$$\Delta\phi_o = \frac{\Delta\epsilon}{h}\,ds \ldots\ldots\ldots (4a)$$

$$\Delta\phi_o = \frac{\Delta s_b}{E_o\,h}\,ds \ldots\ldots (4b)$$

and

$$\Delta\phi_o = \frac{M}{E_o\,I}\,ds \ldots\ldots (4c)$$

in which $\Delta\epsilon$ designates the difference in the strains of the extreme fibers; Δs_b, equal to $M\,h/I$, represents the difference in the stresses of the extreme fibers; and E_o is the modulus of elasticity. In Fig. 1(a), the strain in a fiber is represented by $\epsilon_o = \epsilon_a + \epsilon_b$, and the modulus of elasticity is represented by $E_o = s_o/\epsilon_o = s_a/\epsilon_a = \Delta s_b/\Delta\epsilon$.

In the theory of plasticity discussed herein, the requirements of statics and of geometry are the same as those that are considered in the elementary theory of mechanics. In each theory the distribution of stresses across the section must satisfy the requirements of statics, namely—

$$\sum s\,da = \sum s_o\,da = P \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

and

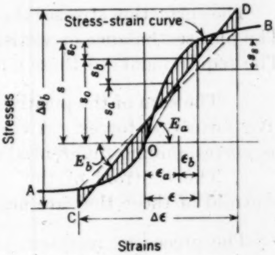
$$\sum s\,x_c\,da = \sum s_o\,x_c\,da = M \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

and in each theory the distribution of strains across the section is assumed to be linear. However, in the theory of plasticity it is assumed that the stress-strain relationships for the fibers of a member that is subjected to a simultaneous axial load and flexural load are the same as those obtained from simple coupon tests. In Fig. 2(*a*), the stress-strain relationship of the material is represented by curve AOB and is assumed to be the same for each longitudinal fiber.

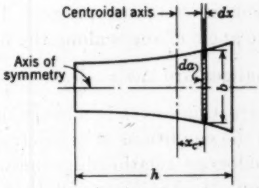
The stress in a fiber, as given by the theory of plasticity, may be interpreted as being equal to the stress given by the elementary theory plus a correction stress to account for the nonlinear stress-strain characteristics of the material. Thus,

$$s = s_o + s_e \ldots\ldots\ldots\ldots\ldots (7)$$

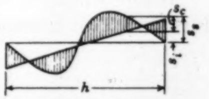
In Fig. 2 the results of the theory of plasticity are compared with those of the elementary theory of mechanics. The ordinates bounded by the stress-strain curve AOB are in accordance with the theory of plasticity, whereas those bounded by the straight line CD are in accordance with the elementary theory of mechanics. The shaded diagram of Fig. 2(*a*) and of Fig. 2(*c*) represents the differences in the results of the two theories or correction stresses as defined in Eq. 7.



(*a*) VALUES OF STRESSES AND STRAINS



(*b*) PROPERTIES OF CROSS SECTION



(*c*) CORRECTION STRESSES TO THE ELEMENTARY THEORY

FIG. 2.—THE ELEMENTARY THEORY OF MECHANICS COMPARED WITH A THEORY OF PLASTICITY

The diagram of correction stresses shown in Fig. 2(*a*) must satisfy the following conditions:

1. No load or moment should be contributed by the diagram of correction stresses; that is—

$$\sum s_e\, da = 0 \ldots\ldots\ldots\ldots (8)$$

and

$$\sum s_e\, x_e\, da = 0 \ldots\ldots\ldots\ldots (9)$$

2. A linear distribution of strains across the section must be observed.
3. The correction stresses must account for the nonlinear stress-strain characteristics of the material.

In the reverse order of the preceding listing, item 3 is satisfied because the correction stresses are represented by the ordinates bounded by line CD and curve AOB. Item 2 is satisfied because the abscissas of the stress-strain curve and the dimensions of the cross section are drawn to a linear horizontal scale. Consequently, the distribution of strains across the section is linear. The scale selected for the dimensions of the cross section agrees with the values of the strains in the extreme fibers. For values of strains in the extreme fibers other than those indicated in Fig. 2(*a*), the horizontal scale of the cross section is either reduced or magnified in proportion to the difference in the strains of these fibers.

The correction stresses that are indicated in Fig. 2(*a*) must balance or vanish. The phrase "balance or vanish" means that Eq. 8 and Eq. 9 must be satisfied. The requirements of these equations are:

a. The sum of the positive correction forces must equal the sum of the negative correction forces. A correction force is equal to a correction stress times its corresponding differential area.

b. The centroid of the positive correction forces must coincide with the centroid of those that are negative.

The preceding restatement is particularly useful in pictorial studies of the effects of a nonlinear stress-strain curve. This will be apparent to those who are already familiar with the pressure-line concept for fixed-end beams and arches. A sketching procedure may be developed herein that is similar to that used in sketching pressure lines for fixed-end beams.[2] In a fixed-end beam the angle changes along the beam caused by lateral loads must balance. These changes are $\Delta\phi = \frac{\Delta\epsilon}{h} ds = \frac{\Delta s_b}{E_b h} ds = \frac{M}{E_b I} ds$. Thus, conditions of statics for correction forces in a short element of a member, as cited herein, are analogous to the conditions of geometry for angle changes along a fixed-end beam. These analogous relationships may be summarized as was done[3] by Hardy Cross, Hon. M. ASCE, except that the column now has nonlinear stress-strain characteristics. Moreover, the correction stresses of a column are concerned herein, not the correction moments of a statically indeterminate beam or arch.

Before examples of the preceding procedure are given, Fig. 2 is studied further. Two modified moduli of elasticity are introduced, namely—

$$E_a = \frac{s_a}{\epsilon_a} \qquad (10)$$

and

$$E_b = \frac{\Delta s_b}{\Delta \epsilon} \qquad (11)$$

in which s_a is the average stress and ϵ_a is the strain of the fibers along the centroidal axis. The values of E_a and E_b are represented in Fig. 2 as slopes of corresponding straight lines and may be interpreted as reduced moduli of elasticity. They may also be interpreted as measures of stiffness to the relative translation and to the relative rotation of adjacent cross sections, respectively.

Three examples of the procedure are given in Figs. 3, 4, and 5. They are examined in the following:

---

[2] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, John Wiley and Sons, Inc., New York, N. Y., 1932, p. 289.

[3] "The Column Analogy," by Hardy Cross, *Bulletin No. 215*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1930.

EXAMPLE 1

The information for example 1 includes the stress-strain curve (Fig. 3(*a*)), the cross section (Fig. 3(*b*)), and the following values of strain in the extreme fibers:

$$\epsilon_1 = 0.001 \text{ in. per in.}$$

and

$$\epsilon_2 = 0.006 \text{ in. per in.}$$

The modulus of elasticity, E_o, is equal to 12,000 kips per sq in. The dimensions of the cross section are 18 in. by 1 in., yielding an area of 18 sq in. and a moment of inertia that is equal to 487 in.^{4} Values are required for the loads, stresses, angle change per unit of length, and the reduced moduli of elasticity.

Between the values of the extreme fiber strains, a straight line is drawn that is considered to be the best fit to the stress-strain curve. This line can be checked by observing whether the diagram of correction forces is balanced, and, if necessary, the line may be revised until the diagram of correction forces is sufficiently balanced. From Fig. 3(*a*), s_a and Δs_b are observed to be 35.7 kips per sq in. and 37.5 kips per sq in., respectively. The values of ϵ_a and $\Delta\epsilon$ are observed to be 0.0035 and 0.005, respectively. Computations yield the following values:

$$P = s_a\, A = 643 \text{ kips}$$

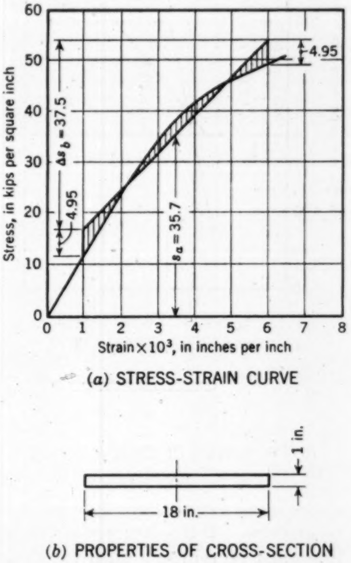
$$M = \frac{I}{h}\, \Delta s_b = 1{,}012 \text{ in.-kips}$$

$$\frac{\Delta\phi}{ds} = \frac{\Delta\epsilon}{h} = 0.000278 \text{ radians per in.}$$

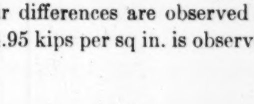
$$E_a = \frac{s_a}{\epsilon_a} = 10{,}200 \text{ kips per sq in.}$$

and

$$E_b = \frac{\Delta s_b}{\Delta\epsilon} = 7{,}500 \text{ kips per sq in.}$$



(*a*) STRESS-STRAIN CURVE



(*b*) PROPERTIES OF CROSS-SECTION

FIG. 3.—EXAMPLE 1

Stresses in accordance with each theory and their differences are observed in Fig. 3(*a*). For each extreme fiber a difference of 4.95 kips per sq in. is observed in Fig. 3(*a*).

EXAMPLE 2

The initial information for this example differs in kind from that given in the preceding example. The information for example 2 includes the stress-strain diagram (Fig. 4(*a*)), the cross section (Fig. 4(*b*)), and the following values of axial load and flexural load:

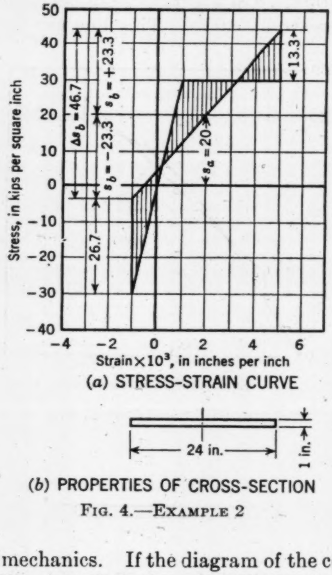
$$P = 480 \text{ kips}$$

and

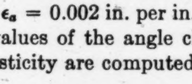
$$M = 2{,}240 \text{ in.-kips}$$

The modulus of elasticity, E_o, is equal to 30,000 kips per sq in., and the yield point is equal to 30 kips per sq in. The cross section, which is 24 in. by 1 in., has an area of 24 sq in. and a moment of inertia of 1,152 in.^{4} Values are required for the stresses, strains, angle change per unit of length, and the reduced moduli of elasticity.

The elementary theory of mechanics is used to compute the average stress and the stresses in the extreme fibers of the cross section. The elementary theory yields an average stress of 20 kips per sq in. and a flexural stress of 23.3 kips per sq in. for each extreme fiber. The difference in the stresses of the extreme fibers is computed to be equal to 46.7 kips per sq in. Horizontal lines are drawn in the stress-strain diagram consistent with the computed stresses in the extreme fibers. Between the horizontal lines, a straight line is drawn that is estimated to be the best fit to the stress-strain curve. From the intersections of the straight line and the horizontal lines, vertical lines are drawn to the stress-strain curve. Immediately below the stress-strain diagram, the cross section is drawn to a scale that corresponds with the difference in the strains of the extreme fibers. In Fig. 4(a), the ordinates between the straight line and the stress-strain curve represent the correction stresses that must be added to the stresses determined by the elementary theory of mechanics. If the diagram of the correction forces does not balance, the straight line must be redrawn until the diagram is sufficiently balanced.



(a) STRESS-STRAIN CURVE



(b) PROPERTIES OF CROSS-SECTION

FIG. 4.—EXAMPLE 2

Values of $\epsilon_a = 0.002$ in. per in. and $\Delta\epsilon = 0.006$ in. per in. are read from Fig. 4(a). The values of the angle change per unit of length and of the reduced moduli of elasticity are computed as follows:

$$E_a = \frac{s_a}{\epsilon_a} = 10{,}000 \text{ kips per sq in.}$$

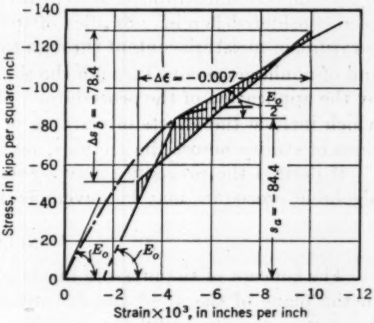
$$E_b = \frac{\Delta s_b}{\Delta\epsilon} = 7{,}780 \text{ kips per sq in.}$$

and

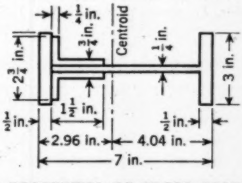
$$\frac{\Delta\phi}{ds} = \frac{\Delta s_b}{E_b\, h} = 0.00025 \text{ radian per in.}$$

EXAMPLE 3

A pin-ended column is made of a material having a stress-strain curve as shown in Fig. 5(*a*). The column is subjected to a critical axial load of 485 kips and has a cross section as shown in Fig. 5(*b*). The area of the cross section is equal to 5.75 sq in., and the moment of inertia is equal to 42.5 in.^4 A history of deformations is considered as in the double-modulus theory,[4] in which an increase in the strain of a fiber is associated with the tangent modulus and a decrease in strain is associated with the initial modulus. It is required to determine the value of the reduced modulus of elasticity and the value of L/r which is in agreement with the critical load. For illustration only the bending is assumed to occur about the centroidal axis as shown in Fig. 5(*b*).



(*a*) STRESS-STRAIN CURVE



(*b*) PROPERTIES OF CROSS-SECTION

FIG. 5.—EXAMPLE 3

The average stress is computed to be 84.4 kips per sq in. For this value of stress, a horizontal line is drawn intersecting the given stress-strain curve. At this intersection, a line is drawn tangent to the curve, and another is drawn parallel to the initial slope of the curve. These lines are considered to define the stress-strain curve associated with subsequent buckling. The values of the tangent modulus and the initial modulus are 15,000 kips per sq in. and 30,000 kips per sq in., respectively. For the critical axial load and an arbitrarily assumed moment, the stresses in the extreme fibers are computed by using the elementary theory of mechanics. The assumed moment in this example yields a value of Δs_b equal to 78.4 kips per sq in. In Fig. 5(*a*), horizontal lines are drawn consistent with the computed stresses in the extreme fibers. Between the horizontal lines, a straight line is drawn that is considered to be the best fit to the stress-strain curve associated with buckling. From the intersection of the straight line with the two horizontal lines, vertical lines are drawn to the assumed stress-strain curve. The vertical lines in Fig. 5(*a*) define the horizontal scale of the cross section as drawn in Fig. 5(*b*). The diagram of correction forces is checked to ascertain if it is balanced. If it is not balanced, the straight line must be redrawn. The slope of the straight line is the value of the reduced modulus of elasticity, $E_b = 11{,}200$ kips per sq in. The desired value of L/r is defined by the modified Euler equation for critical load as

---

[4] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1936, p. 156.

$$\frac{L}{r} = \pi \sqrt{\frac{E_b}{s_a}} = 36.3$$

The foregoing examples were restricted to a cross section having an axis of symmetry and subjected to an axial load and moment about an axis normal to the axis of symmetry. The pictorial procedure may be extended to an unsymmetrical cross section that is subjected to an axial load and to moments about the two principal axes of the section. It may be applied to certain problems of torsion, residual strains, and to histories of loading other than those that have been considered herein. Studies of the effects of nonuniform changes in temperature, of moisture content through the thickness of a long structural member, and of nonhomogeneity through the thickness of the member may also be aided by the application of the procedure. Many problems in the theory of elasticity, which include the effects of warping of cross sections or of nonlinear distributions of strains across the sections, may be studied in a similar fashion.

If desired the procedure may be stated more formally, and a numerical or algebraic procedure may be developed for similar cases.

CONCLUSIONS

The concept of the pressure line is useful for problems of an arch with loads in the plane of the arch[2] and for structures that are curved in space.[5] The concept is equally useful for problems involving the inelastic deformations of structural elements.

---

[5] "Laterally Loaded Plane Structures and Structures Curved in Space," by Frank Baron and James P. Michalos, *Transactions*, ASCE, Vol. 117, 1952, pp. 279-316.

DISCUSSION

John A. Hribar,[6] J.M. ASCE.—A pressure-line concept can be useful in estimating the effects of plasticity on the behavior of structural elements that are subjected to flexural loads. The advantages are dependent on the pictorial design procedure that is used. If it is used to obtain a quick result, that result is only an estimate unless the user is experienced in the method and has a keen judgment in balancing statics pictorially. Although only an estimate may be desired, for accuracy several revisions should be made to obtain the best fit.

This procedure may be neither more rapid nor more accurate than the methods based on the theory of plasticity that are currently used on certain materials and cross sections. Would it not be possible for members made of steel, which has an easily approximated stress-strain curve and comprises most structural material, to be analyzed by these methods with the same ease? Could not the ultimate load on a structural element of any material and cross section be as easily and more accurately computed by the theory of plasticity? These questions are raised in order to emphasize that the theory of plasticity may be as rapid, more accurate, and less artificial in some cases.

The writer agrees that as the cross section or the stress-strain curve become more unwieldy the method becomes more powerful. In addition, the by-products of the procedure are useful in a thorough analysis of a structure. The pressure-line concept also will speed up estimates on structural elements.

Frank Baron,[7] M. ASCE.—The pictorial procedure can be useful in obtaining quick estimates of the effects of plasticity on the behavior of structural elements that are subjected to axial and flexural loads. Mr. Hribar also indicates that an estimate is frequently all that is desired in studies of the inelastic behavior of structural frameworks. However, the degree of accuracy desired can depend on the needs and demands of an analyst. Such needs are not always the same, and they do not always warrant an "exact" solution to a problem, particularly in the various stages of a design.

Mr. Hribar states that the procedure results in an estimate only unless the analyst is experienced in the method and uses judgment in balancing statics pictorially. Little experience and almost no judgment are required to satisfy the requirements of statics. It is necessary only to observe that the correction diagram, as defined in the paper, vanishes or balances. The diagram is balanced when the positive areas equal the negative areas and the centroid of the positive areas coincides with that of the negative areas. This requirement is so simple that in most cases it can be judged by eye. However, a numerical check can be made of any estimate.

Several important attributes of the procedure should be emphasized. In the theory of plasticity the requirements of statics, geometry, and properties of materials are immediately apparent when they are stated pictorially and when they are compared with the results of the elementary theory of mechanics.

---

[6] Graduate Teaching Asst., Dept. of Civ. Eng., Carnegie Inst. of Technology, Pittsburgh, Pa.

[7] Prof. of Civ. Eng., Univ. of California, Berkeley, Calif.

Every relationship can be observed, including that of the theory of plasticity to the elementary theory of mechanics.

Not all members are made of steel with idealized stress-strain curves, nor are all cross sections of members simple enough to be formulated algebraically. However, if an algebraic formulation is desired for a given stress-strain curve and a shape of cross section, the pressure-line concept can be used with the same ease as currently used methods of plasticity, at times even with greater ease. For example, when considering a parabolic stress-strain curve and a simple rectangular cross section, the value of the reduced modulus, E_b, for any combination of axial load and moment about a principal axis of the section can be determined by the pressure-line concept. When the properties of a parabola are recognized, the answer is obvious.

The same statements apply to the determination of ultimate loads. Mr. Hribar agrees that as the cross section or the stress-strain curve becomes more unwieldy the pressure-line concept increases in power.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2954

SOIL MODULUS FOR LATERALLY LOADED PILES

By Bramlette McClelland[1] and John A. Focht, Jr.,[2]
Associate Members, ASCE

With Discussion by Messrs. Ralph F. Reuss; Ralph B. Peck, Melvin T. Davisson, and Vello Hansen; Raymond Lundgren; Lymon C. Reese; Eugene A. Ripperger; Hudson Matlock; and Bramlette McClelland and John A. Focht, Jr.

Synopsis

Using results from a lateral load test on a 24-in. pipe pile and laboratory tests on undisturbed clay samples, a tentative procedure for estimating the soil modulus of pile reaction is developed for problems involving transient loads. The correlation that is derived is based on the similitude on logarithmic paper of laboratory stress-strain curves and soil reaction-deflection curves from the pile test.

Introduction

The problem of determining the moments, shears, and reactions of laterally loaded piles has received increased attention in recent years. There are two distinct elements to this problem. The first is the determination of soil stress-strain characteristics as they pertain to laterally loaded piles, and the second is the mathematical determination of the pile deflection curve once the soil characteristics are known. Several mathematical procedures are available for computing the deflection curve and its accompanying moment and deflection diagrams. The most versatile of these is the difference equation solution as devised by Lawrence A. Palmer, M. ASCE, and James B. Thompson,[3] A.M.

Note.—Published, essentially as printed here, in October, 1956, in the Journal of the Soil Mechanics and Foundations Division, as *Proceedings Paper 1081*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] **Pres., McClelland Engrs., Inc., Houston, Tex.**

[2] **Chf. Design Engr., McClelland Engrs., Inc., Houston, Tex.**

[3] **"The Earth Pressure and Deflection Along the Embedded Lengths of Piles Subjected to Lateral Thrust," by L. A. Palmer and J. B. Thompson, *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Eng., Rotterdam, Vol. 5, 1948, p. 156.**

ASCE, subsequently generalized by Sol M. Gleser,[4] M. ASCE, and extended by the authors.[5] However, the correctness of the results obtained by this, or by any other mathematical procedure, depends on the validity of the stress-strain relationship assigned to the soil for the purpose of analysis.

If the pile deflection is labeled y and the depth below the soil surface is represented by x, the general differential equation of the problem is

$$E\,I\,\frac{d^4y}{dx^4} = p \qquad (1)$$

The pile moment of inertia, I, and the modulus of elasticity, E, have the usual units of inches[4] and pounds per square inch, respectively, whereas the reaction of the soil on the pile, p, has units of pounds per inch. Eq. 1 (from the theory of beams) is modified for the present problem by writing the following equation for the unit soil reaction, p,

$$p = -\,E_s\,y \qquad (2)$$

E_s is termed the soil modulus of pile reaction and is by definition the ratio between the soil reaction at any point and the pile deflection at that point. The value of E_s would be a constant only if the soil were a perfectly elastic material. However, it is known that E_s usually increases with depth and at a given depth becomes smaller as the deflection increases. Previous investigators have made various assumptions as to the variation of E_s with depth. They have assumed E_s to be constant with depth,[6] to be a linear function of depth,[7, 8] and to be an exponential function of depth.[3] It has been widely recognized that E_s varies with deflection. However, all published studies to date (1956) have accepted the erroneous assumption that E_s is constant at any given depth as a mathematical necessity.

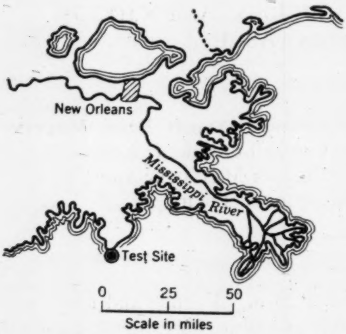


FIG. 1.—TEST LOCATION

In 1952 a lateral pile-load test was conducted for a major oil company by the Texas Agriculture and Mechanical Research Foundation at College Station.[9] The results of this study permit the computation of actual values of E_s throughout the significant depth range of the test pile, and over a wide range of pile deflections for short duration loads. For the first time these data make possible a close study of the soil modulus of pile reaction. It is the purpose of this paper to publicize the significant results of the test pile and to present a method whereby the soil modulus of pile reaction for transient loads can be predicted from laboratory tests.

---

[4] "Lateral Load Tests on Vertical Fixed-Head and Free-Head Piles," by Sol M. Gleser, *Special Technical Publication 154*, A.S.T.M., 1954, p. 75.

[5] "Analysis of Laterally Loaded Piles by Difference Equation Solution," by J. A. Focht, Jr. and Bramlette McClelland, *Texas Engineer*, Vol. 25, September–November, 1955.

[6] Discussion by Y. L. Chang of "Lateral Load Tests," by L. B. Feagin, *Transactions*, ASCE, Vol. 102, 1937, p. 272.

[7] "Die Spundwand als Erddruckproblem," by J. Rifaat, A. G. Gebr. Leeman & Co., Zürich, 1935.

[8] "Druckverteilung im Baugrunde," by O. K. Froehlich, Julius Springer, Berlin, 1934, p. 89.

[9] "An Investigation of Lateral Loads on a Test Pile," Project 31, Texas Agri. & Mech. Research Foundation, College Station, Texas, 1952.

DESCRIPTION OF TEST PILE

The pile test was conducted at a site in the Gulf of Mexico approximately four miles from the Louisiana Coast, as shown in Fig. 1. Water depth at the site is 33 ft, and the soils in the area are post-glacial deltaic deposits of the Mississippi River. The test pile was a 24-in.-diameter pipe driven to a penetration of 75 ft below the ground line. As shown in Fig. 2, the pile was driven at a point approximately 8 ft from the leg of an existing oil-well-drilling structure. A prefabricated pipe brace, which was just above the water line, positioned the pile and provided restraint against horizontal movement of the pile at that point. Loads were applied by a hydraulic jack, which was approximately 6 ft above the ground line. Loads were applied in increments in two series—a static series and a dynamic series. Loads of the static series were applied for 10 min each and ranged to a maximum jack load of 113 kips. Loads of the dynamic series were applied for a 5-sec duration and ranged to a maximum jack load of 97 kips. Electric SR-4 strain gages were installed at twenty-two different elevations on the pile for the measurement of bending strain. Seven of the gage stations were at 2-ft intervals between the jack position and a point 7 ft below the ground line; thirteen others were below these stations at 5-ft intervals extending to the bottom of the pile; and two more were between the jack and the pipe brace at the top. A more complete description of the test apparatus is given in the report of the Texas Agriculture and Mechanical Research Foundation to the test sponsor.[9]

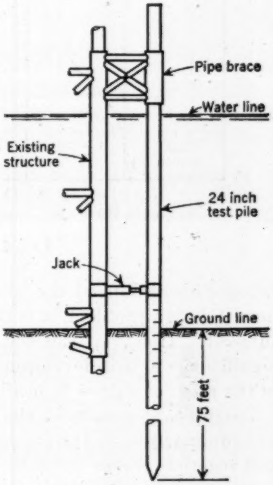


FIG. 2.—TEST LAYOUT

TEST PILE RESULTS

The bending moment in the pile at each gage station was computed by multiplying the measured strain by a constant. For each applied load these computations yielded a moment diagram as shown in Fig. 3(*a*). This curve represents results for an 80-kip static load on the jack. By graphical differentiation of the moment diagram, the shear diagram (Fig. 3(*b*)) was computed. The change in shear at the jack determined by this method differed from the actual observed load on the jack by as much as 12%. Proportionate adjustment of the shear diagram was made to correct this difference. Graphical differentiation of the adjusted shear curve produced the soil reaction curve as shown in Fig. 3(*c*). Beginning again with the moment curve, double graphical

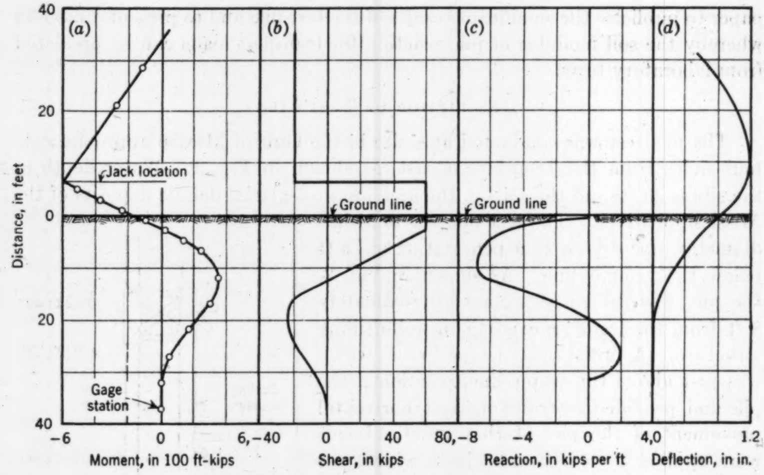


FIG. 3.—80-KIP LOAD-TEST RESULTS

integration produced the pile deflection curve as shown in Fig. 3(*d*). In an actual design problem, it is the moment curve that will probably be of greatest interest. However, the deflection curve and the reaction curve are of major significance in this development because the soil modulus, E_s, at a given depth is the ratio of values from these two curves at that depth.

Significant results of the test pile investigation with regard to this study are summarized by the curves in Fig. 4. Each curve is a plot of the computed soil reaction versus the pile deflection at a given gage station or depth. The curves are similar in form and significance to the familiar stress-strain curves obtained from laboratory shear tests. The curves have an evident family grouping, and they shift upward in a consistent manner as the depth is in-

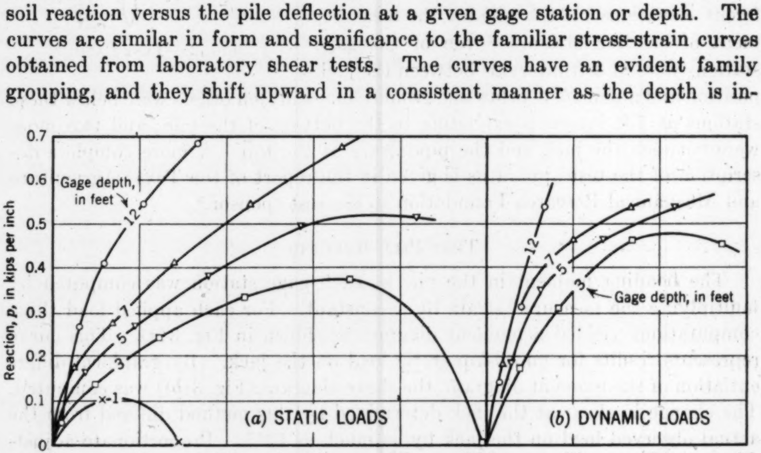


FIG. 4.—SOIL REACTION VERSUS PILE DEFLECTION FOR FIELD TESTS

creased. The shift indicates an increase in stiffness with an increase in depth for the soils at the test site.

The soil modulus of pile reaction is defined as the slope of a line connecting the origin of this plot (Fig. 4) to any point on any one of the curves. Depending on the depth and the pile deflection, it is evident that the slope of such a line could vary from zero to almost infinity. The wide range in the ratio of reaction to deflection indicates the futility of assigning any single value of E_s to a given soil. Considering any individual curve that represents only a single depth, the variation of slope shows that the usual assumption of a constant E_s-value at a given depth is less than satisfactory.

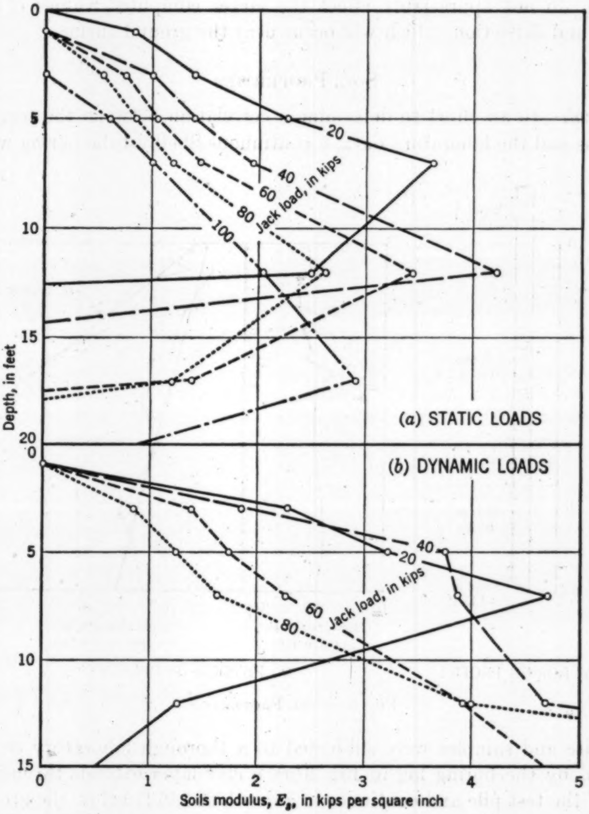
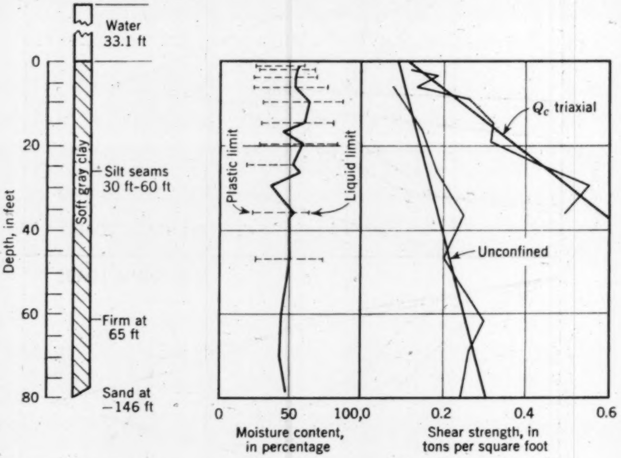


FIG. 5.—SOIL MODULUS VERSUS DEPTH FOR FIELD TESTS

To illustrate the variation in the soil modulus throughout the test, values of E_s for each load and each gage point were computed from the data in Fig. 4 and are plotted in Fig. 5. All values determined for the same load are interconnected in this plot. These curves show that for conditions attending the pile test, E_s increases almost linearly throughout the significant depth range. The fact that E_s decreases appreciably as the load increases is clearly illustrated and is perhaps more significant. The observed curves become erratic below the 12-ft depth where the deflection approaches zero due to instrumental and computational errors. These variations are not considered significant. As shown previously,[6] major errors in the value of E_s below the first point of zero deflection do not appreciably affect the larger computed values of bending moment and deflection, which will occur near the ground surface.

SOIL PROPERTIES

General.—In an effort to determine a correlation between the test pile observations and the laboratory data, a continuous Shelby-tube boring was made at the site and samples were subjected to a thorough laboratory study. As indicated by the boring log in Fig. 6(*a*), a clay layer extends throughout the depth of the test pile and continues to a depth of 146 ft below the ground line. The soil is part of a tremendous mass of clay, which is less than 5,000 yr old and was deposited by the Mississippi River in shallow marine waters. The moisture content plot (Fig. 6(*b*)), which includes plastic limit values and liquid limit values, shows that the deposit is of uniform plasticity, with the natural water content decreasing slightly with depth. The activity ratio of this deposit, the ratio of its plasticity index to its percentage of material finer than 0.002-mm grain size, is approximately 1.0, indicating a clay of medium activity.



(*a*) BORING PROFILE (*b*) PHYSICAL PROPERTIES

FIG. 6.—SOIL PROPERTIES

The shear strengths, which are plotted in Fig. 6(*c*), are values that were determined from unconfined compression tests and consolidated-undrained, or

Q_c, triaxial tests. For the latter tests each specimen was first consolidated under a lateral pressure, σ_3, which was approximately equal to its computed overburden pressure. Then the specimen was tested at a strain rate of approximately 2% per min. Overburden pressure was computed as γx, in which γ is the effective unit weight of the soil. Both groups of tests show a nearly linear increase of strength with depth. A corollary to this trend is the general reduction of the liquidity index as the depth is increased.

Results of remolded unconfined compression tests (not shown) indicate an average sensitivity ratio of 2.5 for the clay. Field vane tests at the pile test site and two other nearby sites revealed in-place shear strengths that compare closely with values determined from Q_c-triaxial tests. This comparison has resulted in the conclusion that the Q_c-triaxial data adequately represent in-place soil strength characteristics.

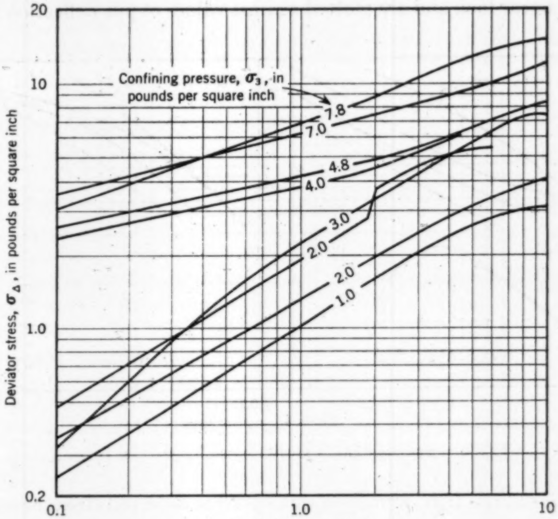


FIG. 7.—STRESS-STRAIN CURVE FOR Q_c-TRIAXIAL TESTS

All the foregoing characteristics are typical of soils found throughout a wide area of several hundred square miles off the southeastern Louisiana coast.

Stress-Strain Characteristics.—An examination of stress-strain data from all the shear strength tests revealed that only the Q_c-triaxial data exhibited the same general characteristics indicated by the test pile data. Stress-strain data from triaxial tests are given on a logarithmic plot in Fig. 7. The deviator or compressive stress, $\sigma_1 - \sigma_3$, which for simplicity is labeled σ_Δ, is shown versus the axial strain, ϵ. The number on each curve represents the confining pressure, σ_3, in pounds per square inch and corresponds approximately to the overburden pressure, γx, in each case. The rate of strain was controlled manually at approximately 2% per min. Variations from this intended rate probably account for some of the irregularities in the curves. Despite these irregularities, each curve can be approximated by a straight line. Furthermore, there is noticeable parallelism between the curves, particularly for the four lower pressures. The curves shift upward with increasing pressure, indicating an increase of soil stiffness with depth, as was noted previously from the results of the pile test.

Correlation of Test Pile and Triaxial Data

In order that the laboratory data may be compared with the test pile results, both sets of data must be expressed in a similar and dimensionally consistent manner. The test pile results presented in Fig. 4 are values of soil reaction in pounds per linear inch and are plotted against values of pile deflection in inches.

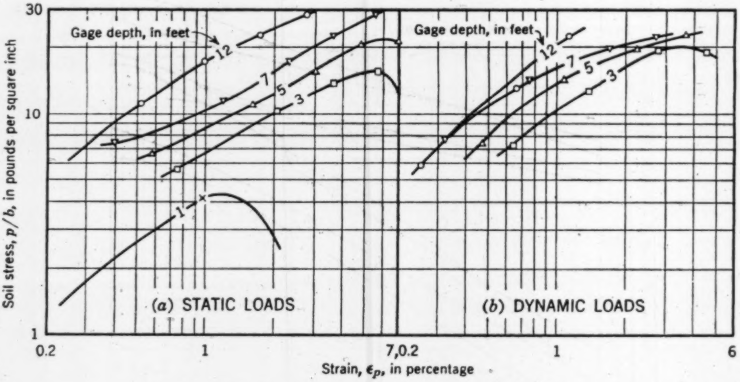


Fig. 8.—Stress-Strain Curves for Test Pile Data

The test pile soil reaction, p, may be converted to field soil stress in pounds per square inch by dividing it by the pile diameter, b, in inches. Similarly, the pile deflection, y, may be converted to dimensionless strain if divided by any satisfactory multiple or fraction of the pile diameter in inches.

In Fig. 8 the test pile data are replotted logarithmically after first being converted to stress and strain units. For this purpose, the pile radius, $b/2$ or r, was selected as the divisor for converting pile deflection to field soil strain. The pile radius was selected so that field failure strains for the more important depths in Fig. 8 would be comparable numerically to the laboratory failure strains in Fig. 7. By definition for this study, the field soil strain is

$$\epsilon_p = \frac{y}{r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{3}$$

The group similarity between the field data in Fig. 8 and the laboratory data in Fig. 7 is immediately recognizable. Both groups express a soil stress-strain function rather than a pile property. Therefore, it is reasonable to assume that the numerical differences between these two groups of curves result from two causes: First, the dimensional difference in the soil masses that are involved in the two types of tests, and second, the differences in the two systems of loading. It can be assumed further that the factor or factors necessary to transpose a laboratory curve to make it numerically equal to a pile curve are independent of the soil characteristics. Such transposition factors would therefore constitute a correlation between laboratory and field conditions. If determinable these factors should be equally applicable to triaxial data and to vertical cylindrical piles at other sites as well as at the test pile site.

A direct comparison between the individual pile curves and laboratory curves cannot be made because the depths of the laboratory test specimens do not match the depths of the pile-gage stations. Therefore, interpolation is necessary to obtain a curve from each source so that both represent the same depth. In order to facilitate interpolation, stress intercepts at 1% strain of the curves in Fig. 8 and the similar laboratory plot in Fig. 7 are replotted versus effective overburden pressure (or confining pressure) in Fig. 9. The horizontal scale in Fig. 9 is the confining pressure in the laboratory or overburden pressure in the field and may be interpreted as a depth scale. The vertical axis is the intensity of soil stress in the field or in the laboratory that is required to produce 1% strain. The open circles represent data from the static pile-load tests and the closed circles represent data from the dynamic-load tests. The upper line represents a conservative interpretation of the pile data. The slope of this line is 5.5 times greater than the slope of the lower line, which is drawn through the laboratory data.

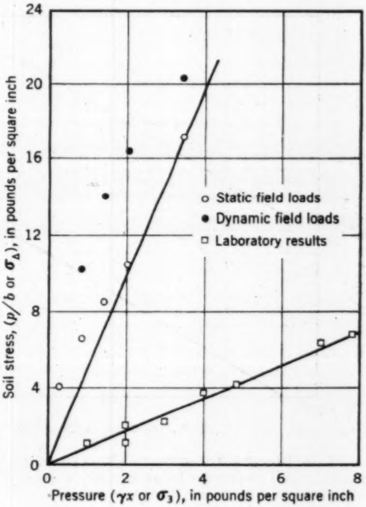


FIG. 9.—SOIL STRESS AT 1% STRAIN VERSUS PRESSURE

The curves in Fig. 10 present a combined interpretation of the pile data and laboratory data which has been given previously. They show stress-strain relationships for both field and laboratory conditions under a vertical soil pressure or confining soil pressure of 2 lb per sq in., corresponding to a depth of approximately 7 ft. The stress intercept at 1% strain for each curve was determined from Fig. 9. The slope of the Q_e-curve or laboratory curve is 0.61, which is the average slope of the four lowest curves in Fig. 7. The slope of the field curve is 0.59, which is the average slope of the static-load pile curves in Fig. 8. Because the two slopes are almost equal, it may be assumed that the soil stress on the pile, p/b, has a constant relationship to the Q_c-laboratory stress, σ_Δ, at equal soil strains and equal vertical or confining pressures. The dimensionless constant of proportionality from Fig. 9 is approximately 5.5. Thus, at $\epsilon = \epsilon_p$ and $\sigma_3 = \gamma x$,

$$\frac{p}{b} = 5.5\,\sigma_\Delta \dots\dots\dots\dots\dots\dots\dots\dots (4)$$

Eq. 4 implies that the ultimate lateral soil pressure that can be developed is eleven times the cohesive shear strength, or about twice the bearing capacity of a shallow strip footing on the same soil. A comparison of the maximum pressure indicated in Fig. 8 by the gage at a 7-ft depth and the shear strength at that depth in Fig. 2 substantiates this value, although lower ratios are indicated at lesser depths.

The soil modulus at depth x is established by combining Eqs. 2, 3, and 4, which yield

$$E_s = 11\,\frac{\sigma_\Delta}{\epsilon} \dots\dots (5)$$

The significance of Eq. 5 is that the soil modulus at a depth x is about eleven times the secant modulus from a laboratory Q_c-test on a sample from that depth, which is confined at a lateral pressure of γx. The magnitude of the secant modulus, and thereby the soil modulus, depends on the extent of the strain, ϵ, that is produced.

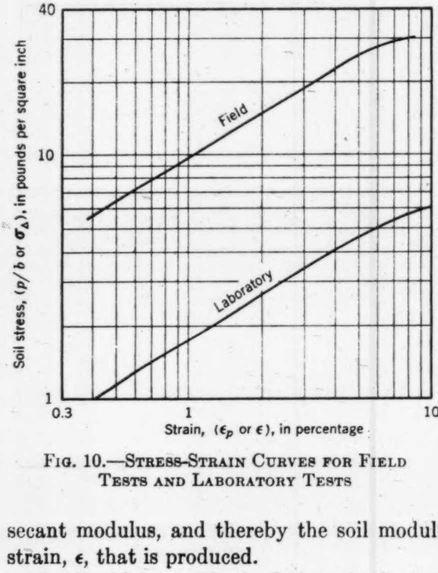


FIG. 10.—STRESS-STRAIN CURVES FOR FIELD TESTS AND LABORATORY TESTS

It should be emphasized that the linear increase of soil stiffness with an increase in depth, exhibited by the data in Fig. 9, should be expected only in uniform clay deposits that are more or less normally consolidated. Such an increase is not a necessary condition for the application of the foregoing correlation between field and laboratory data. The stiffness of stratified deposits and the stiffness of deposits that exhibit varying degrees of overconsolidation will certainly not be proportional to depth. However, the correlation coefficient just derived should still apply to Q_c-triaxial data from such deposits if the application is made only at actual sample depths.

Other curves can be drawn through the data in Fig. 9, which will yield other correlation coefficients. The coefficient that was derived is simple and conservative and based on the available data.

Several parts of the preceding derivation are substantiated by other data which are not presented. Triaxial tests for two other nearby drilling structures showed the same type of curves and grouping of curves as plotted in Fig. 7. The average slope of these other stress-strain curves on logarithmic plots is approximately equal to the average slope of the four lower pressure curves in Fig. 7. Laboratory tests on Recent clay deposits at other widely separated locations in the Gulf of Mexico have also shown similar curve groupings.

The carefully performed triaxial tests on sand by Liang-Sheng Chen[10] show relationships of stress-strain and stress-confining pressure that are similar in form to the triaxial data presented in Figs. 7 and 9. The similarity indicates that perhaps a correlation of the type that was derived herein can be determined for laterally loaded piles in sand. However, no information is available (as of 1956) as to whether the same numerical coefficient that was derived for clay soils will be applicable to cohesionless materials.

APPLICATION OF CORRELATION

General Procedure.—The correlation coefficient that has been derived does not permit direct determination of the soil modulus of pile reaction at a specific depth in a given soil condition. The coefficient provides only a means by which triaxial stress-strain data, which are applicable to a given soil depth, can be converted to pile stress-strain data at that depth. The soil modulus at a given depth will vary with deflection. Because the deflection of a laterally loaded pile at any particular depth is a function of the applied load, the manner of load application, and the pile stiffness, the soil modulus is also a function of these variables. Therefore, the correct soil modulus for a given set of conditions can be determined only by successive approximations of the pile deflection curve after the stress-strain characteristics of each soil stratum have been determined by laboratory tests.

A problem of a laterally loaded pile can be analyzed by the following steps:

1. Estimate the pile deflection curve, as in Fig. 11(*a*).
2. For several selected points above the estimated point of zero deflection, convert the depth, x, to vertical soil pressure, γx, and convert the pile deflection, y, to strain ϵ_p by Eq. 3.
3. For the same selected depths, plot logarithmic stress-strain curves from laboratory Q_c-tests (performed at lateral pressures equal to the vertical soil pressures determined in step 2), as in Fig. 11(*b*).
4. Determine laboratory stresses for values of strain ϵ, equal to the field soil strain, ϵ_p, as determined in step 2.
5. Convert each value of laboratory stress to field soil stress, p/b, by Eq. 4, as in Fig. 11(*b*).
6. The soil reaction, p, at each point may be computed in pounds per linear inch from the field soil stress.
7. Divide each soil reaction by the corresponding assumed pile deflection, following Eq. 2, to obtain the estimates of E_s at the selected depths and plot the estimates versus depth as in Fig. 11(*c*). (Eq. 5 may be used directly instead of following steps 5, 6, and 7 to secure estimates of E_s at each depth.)

---

[10] "An Investigation of Stress-Strain Characteristics of Cohesionless Soils by Triaxial Compression Tests," by Liang-Sheng Chen, *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Eng., Rotterdam, 1948, Vol. 5, p. 35.

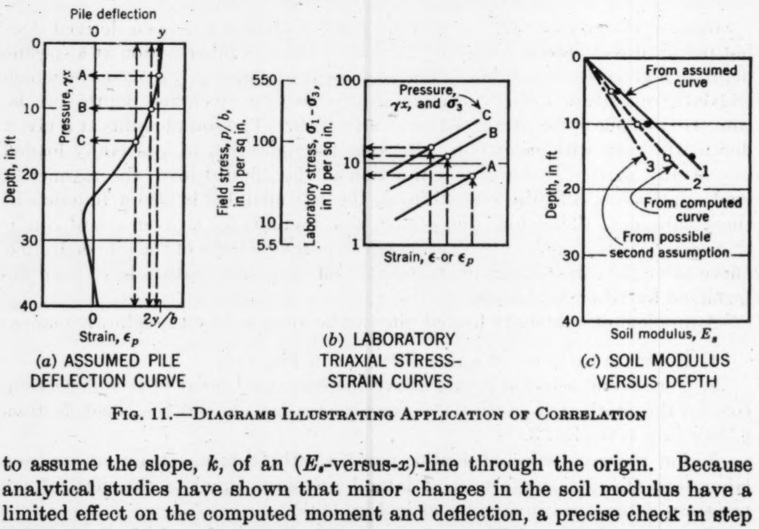
8. Make a conservative simplification of the E_s-versus-depth relationship, as in Fig. 11(*c*), for use in a difference equation computation.

9. Compute the pile deflection curve by the difference equation method[4,5]; recompute E_s as in steps 1 through 7; and compare these values with those used in the difference equation computation.

10. If necessary, assume a new E_s-versus-depth relationship and repeat step 9.

11. From the final deflection curve, compute pile bending moments, shears, and soil reactions.

If desirable, after experience with the procedure is gained, steps 1 through 7 may be omitted for the first approximation by immediately estimating a curve of E_s versus x. In normally consolidated soils, it has proved convenient to assume the slope, k, of an (E_s-versus-x)-line through the origin. Because analytical studies have shown that minor changes in the soil modulus have a limited effect on the computed moment and deflection, a precise check in step 9 is not required and a reasonable comparison will be satisfactory. If the E_s-values from the assumed deflection curve are lower than, but close to, those that were computed in step 9, the first computed deflection curve can be considered final and can be used in determining the bending moment, shear, and reaction diagrams as desired.


Pile deflection
0
y
0
10
20
30
40
A
B
C
Pressure, γx
Depth, in ft
0
$2y/b$
Strain, ϵ_p
550
100
10
5.5
Field stress, p/b, in lb per sq in.
100
10
1
Laboratory stress, $\sigma_1 - \sigma_3$, in lb per sq in.
Pressure, γx, and σ_3
C
B
A
Strain, ϵ or ϵ_p
0
10
20
30
40
Depth, in ft
From assumed curve
1
2
3
From computed curve
From possible second assumption
Soil modulus, E_s

(*a*) ASSUMED PILE DEFLECTION CURVE

(*b*) LABORATORY TRIAXIAL STRESS-STRAIN CURVES

(*c*) SOIL MODULUS VERSUS DEPTH

FIG. 11.—DIAGRAMS ILLUSTRATING APPLICATION OF CORRELATION

Illustrative Example.—The application of the correlation can be illustrated by the following example. A pipe pile with a 30-in. diameter is driven to a penetration of 75 ft in an area of soils similar to those at the test pile site. The pile has a moment of inertia of 6,223 in.^{4}, it is to be fixed against rotation at the ground line, and it is to carry a transient horizontal load of 100 kips. It is desired to determine the pile bending moments and deflections.

To solve this problem, the deflection was first assumed, as shown in Fig. 12(*a*). Following the procedure outlined previously, the (E_s-versus-depth)-

curve, labeled No. 1 in Fig. 12(*b*), was computed using the triaxial-test data in Figs. 9 and 10. This curve shows E_s increasing rapidly in the vicinity of the 20-ft depth. This is the depth where the assumed pile deflection approaches zero. A conservative simplification of this curve is a straight line, which is drawn tangent to the upper part of the soil modulus curve 1 and which was used for the first computation. In a difference equation analysis, the pile was divided into 30 equal length increments to obtain deflection curve No. 2 in Fig. 12(*a*), which is labeled "first trial". Curve 2 differs considerably from the assumed curve. The E_s-values determined from this computed curve are shown as curve 2 in Fig. 12(*b*). Although the lower part of the curve consists of greater values than those of curve 1, the upper part of the curve falls below the values used initially. Therefore, a second E_s-relationship was assumed and is represented by the dashed straight line to the left, and a second difference equation analysis was made. The second analysis yielded the deflection curve labeled "second trial," and it differs only slightly from the first computation.

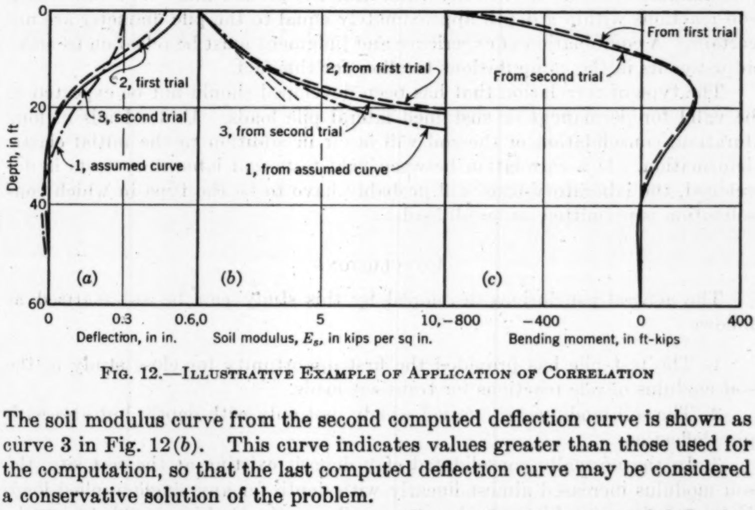


FIG. 12.—ILLUSTRATIVE EXAMPLE OF APPLICATION OF CORRELATION

The soil modulus curve from the second computed deflection curve is shown as curve 3 in Fig. 12(*b*). This curve indicates values greater than those used for the computation, so that the last computed deflection curve may be considered a conservative solution of the problem.

The bending-moment curves corresponding to the two computed deflection curves are plotted in Fig. 12(*c*) and constitute the remainder of the desired solution. In this example the first computed deflection curve produced bending-moment values that are, for practical purposes, of sufficiently high accuracy. This will often be the case, and, with experience, many solutions may be derived using the first computed deflection curve.

LIMITATIONS OF THE METHOD

There are several limitations to the suggested procedure and to the correlation coefficient that should be recognized. In review, the test pile was vertical and of cylindrical cross section. It was driven into a clay soil, and temporary loads were applied from only one direction, producing maximum deflections that were no greater than 4% of the pile diameter. The laboratory Q_c-triaxial tests were consolidated at computed overburden pressure and were loaded at a strain rate of approximately 2% per min. The pile deflections and reactions were computed from SR-4 strain-gage data.

It is reasonable to expect that the general concept of the correlation and the method that is proposed for its application can be adapted to situations other than those described in the preceding paragraphs. However, the numerical value of the coefficient would probably be different in some of the other situations. Thus, different coefficients may be found applicable to piles in sand or to piles that are not cylindrical. Pile loads that reverse or that are repeated many times may produce greater deflections than the present data would indicate. Triaxial tests performed at different rates of strain would require a different correlation factor.

The definition of field soil strain, ϵ_p, as a constant function of deflection and pile diameter (Eq. 3) is not fully valid near the ground line. For this reason soil reactions within a depth approximately equal to the pile diameter are uncertain. A combination of experience and judgment must be relied on to make adjustments in the computations to allow for this fact.

The type of correlation that has been developed should not be expected to be valid for permanent or sustained lateral pile loads. Under loads of long duration, consolidation of the soil will occur in addition to the initial elastic deformation. If a correlation between field tests and laboratory tests is developed, the laboratory tests will probably have to be the type in which consolidation is permitted under all loads.

CONCLUSIONS

The general conclusions developed by this study can be summarized as follows:

1. The test pile has provided the first opportunity for close study of the soil modulus of pile reactions for transient loads.
2. The soil modulus, E_s, varied widely, not only with depth, but also with pile deflection.
3. In the normally consolidated clay deposit existing at the test site, the soil modulus increased almost linearly with depth for any single applied load. However, the rate of increase became smaller as the load increased.
4. Logarithmic stress-strain plots of Q_c-triaxial data exhibit a family grouping similar to that of field stress-strain data in which strain is defined as a function of the pile diameter.
5. For the condition of the pile test, the similitude of pile stress-strain data and laboratory stress-strain data can be expressed by a single dimensionless coefficient, as given in Eq. 4.
6. Utilizing the tentative correlation coefficient and suitable Q_c-triaxial data, the deflection curve of a pile carrying a short-duration lateral load can be determined readily by successive approximations with a difference equation analysis.

7. Additional pile tests, adequately instrumented and accompanied by thorough laboratory tests, will be necessary to confirm the over-all validity and numerical accuracy of the proposed correlation for clay soils. Further pile tests are also needed to extend the correlation to other conditions, such as cohesionless soils, piles of different shapes, reversing or repeating loads, and sustained loads.

Although the test pile data and laboratory data on which this study was based have limitations, the tentative correlation in its present stage of development permits a more direct and rational approach to lateral pile problems than has been possible previously. The proposed procedure for the application of the correlation, using successive approximations of the pile deflection curve, should continue to prove useful and effective with improved correlations or correlations for other conditions.

ACKNOWLEDGMENTS

The pile test on which this study was based was conducted by the Texas Agricultural and Mechanical Research Foundation. The report of the test from the foundation to the test sponsor was prepared by A. L. Parrack. Figs 3 and 4 were taken directly from that report. The suggestion to plot the field deflection-reaction curves and the laboratory stress-strain curves on logarithmic paper for comparison was made by Ralph F. Reuss, A.M. ASCE.

DISCUSSION

Ralph F. Reuss,[11] A. M. ASCE.—The data and analyses that are presented are a significant contribution to the evaluation of lateral stability of piles in clayey soils. The method of correlation, based on the equality of strains, defines a possible procedure for the future correlation of field and laboratory stress-strain data.

The authors outline the field-test procedure and indicate that graphical differentiation and integration were used to obtain the soil-reaction-and-deflection values. Because these values form the basis of subsequent correlations, it would be of interest to learn the accuracy of the field measurements, the range of possible error, and the resulting influence on the computed soil modulus values.

The soil modulus curves shown in Fig. 5 indicate a decrease in soil modulus with increasing strain or lateral load, or both. The decrease in the ratio of soil pressure to deflection indicates that nonlinear stress deformation occurred at each gage depth. If linear stress strain occurred the values of E_s would be constant for increasing deflection at a given depth. The similarity of curves for the dynamic loads of 20 kips and 40 kips in Fig. 5 suggests that a linear relationship between soil stress and deformation may have occurred at each gage depth for these loads. The reaction-deflection curves shown in Fig. 4 also indicate that stress deformation was nearly linear for small deflections and that the failure of the shallow soils occurred for increasing values of lateral load. The possible occurrence of linear soil deformation and nonlinear soil deformation suggests that the limiting strain for linear deformation may be significant in the evaluation of lateral resistance and may also provide a design procedure that would preclude progressive failure of the shallow soils. If pile deflections were limited to values within the linear range of deformation, permanent soil deformation and progressive failure of the soil would be prevented for repetitive lateral loading.

The authors indicate that the sensitivity ratio, which is based on unconfined compressive strength values, has an average value of 2.5. It was also concluded that the Q_c-triaxial test value represents the in-place strength value, although it is believed that this conclusion should be qualified to apply to (*a*) Q_c-values that were determined for specimens consolidated at effective overburden pressure, and (*b*) test specimens that do not experience an appreciable volume decrease during consolidation at effective overburden pressure. If the Q_c-strength value is assumed to be representative of the in-place soil strength, the true sensitivity ratio of the soil based on maximum and minimum soil strengths will be on the order of seven or eight. Therefore, the soil that is adjacent to the test pile must be described as a sensitive clay.

The procedure outlined for the analysis of lateral stability recommends that the design be based on the ultimate soil resistance corresponding to the estimated pile deflection. Previous mathematical studies have shown that computed moments are not influenced by small changes in soil moduli. However, actual lateral deflections or loads in excess of computed values may induce failure and strength loss in the soils in the vicinity of the ground line, and may result in an equivalent increase in the height of the load application with respect to the assumed point of application. This condition occurred in the test and is demonstrated by the E_s-curves in Fig. 5, in which the point of zero soil resistance or soil modulus occurs at increasing depths below the ground line as the values of applied load are increased. The equivalent increase in the height of the load application results in a proportionate increase in the maximum deflection and moment.

---

[11] Partner, National Soil Services, Houston, Tex.

The evaluation of lateral capacity, as recommended in the paper, must be made with caution because the design in its present form does not provide for a direct application of a factor of safety to the soil resistance. It is suggested that such a factor be applied in design problems by increasing the estimated maximum lateral loading because the moments and deflections are influenced to a nearly proportionate degree by changes in the magnitude of lateral load. Increasing the design lateral loading will result in the selection of a pile section with sufficient stiffness to permit some variation in estimated deflections, lateral-loading conditions, and soil moduli without the development of excessive bending stresses.

The analysis should also include a comparison of the computed deflection and the soil-failure strains at the ground line to assure that progressive failure and an equivalent increase in the height of the load application will not occur. This phase of the analysis may result in the requirement that a larger section or added stiffness be used in the vicinity of the ground line.

The methods of analyses described in the paper permit a more accurate evaluation of lateral pile capacity in clays than was possible in the past. However, the correlation and method of analyses are tentative, and consideration must be given to all factors that will influence lateral soil resistance in a design application.

RALPH B. PECK,[12] M. ASCE, MELVIN T. DAVISSON,[13] AND VELLO HANSEN,[14] JUNIOR MEMBERS, ASCE.—The computed values of the soil modulus, E_s, are shown in Fig. 5 as a function of depth. For a given load the values of E_s increase almost linearly with the depth for a few feet below the ground line, whereupon they decrease, in some instances to zero, at depths of from 13 ft to approximately 20 ft.

It is not reasonable to assume that E_s should be zero, except possibly within a short distance below the ground line, because even in extremely soft soil there will be some resistance against deflection. The authors recognize this fact because they refer to trends within the "significant" depth, but it is not clear why only the data within approximately the upper 10 ft should be significant for a pile with an embedded length of 75 ft.

The writers have had access to the original data and have considered it desirable to study the possible reasons for this discrepancy. Detailed computations were made for the loading case for which the "static" load was equal to 93.3 kips. This case was chosen because the data seemed more consistent than the data for any other case.

---

[12] Prof. of Foundation Eng., Univ. of Illinois, Urbana, Ill.

[13] Research Asst., Univ. of Illinois, Urbana, Ill.

[14] Civ. Engr., Plant Eng. Branch, Civ. Aeronautics Administration, Fort Worth, Tex.

As a first step the values of E_s at various depths were computed by the conventional procedures of double differentiation and double integration of the moment curve, which was derived directly from the strain measurements. The results were in substantial agreement with those obtained by the authors, including a reduction in the value of E_s to a small value, although not zero, at a depth of 32 ft. Second, it was decided to determine whether a more reasonable relationship of E_s with depth could be assumed. The desired relationship would be one leading to values of moment that would not differ by more than a reasonable allowance for experimental error from the values determined from the strain readings. That is, it was considered desirable to learn whether the peculiarities in the values of E_s might be the result of a high sensitivity of the computed values to small variations in the measured quantities.

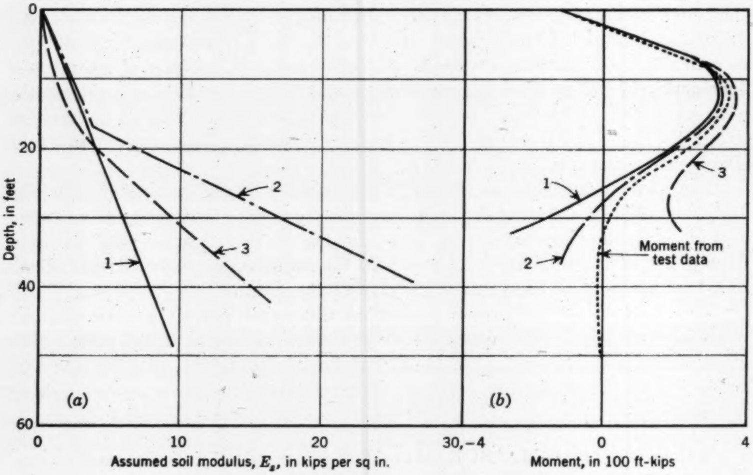


FIG. 13.—SOIL MODULUS VERSUS DEPTH AND MOMENT VERSUS DEPTH FOR A STATIC PILE LOAD OF 93.3 KIPS

The deflection curve can be determined with considerable accuracy from the measured strains because the process of double integration does not involve large errors. Therefore, the strain readings were used to compute the curve of deflection corresponding to the load of 93.3 kips. The curve of soil reaction as a function of the depth involves the inaccurate procedure of double differentiation. Hence, a relationship between E_s and the depth was assumed. The soil reactions were determined by multiplying values of E_s by the corresponding value of deflection, whereupon the moment curve was computed by the relatively accurate process of double integration. The computed moment curve was compared with the one that was derived from the measured strains to determine if discrepancies existed between the curves and if such discrepancies were within the limits of experimental error.

Three of the assumed variations of E_s are shown in Fig. 13(*a*). The corresponding moment curves are shown in Fig. 13(*b*) together with the moments that were determined directly from the strain readings. Satisfactory agreement exists in the upper 10 ft or 15 ft below the ground line, but important discrepancies appear below this level. Some indication of the possible error in the measured moment can be obtained by comparing the strain-gage readings on the front side and the back side of the pile at a given depth. These readings should be equal but of opposite sign. The numerical difference in the readings indicates the order of magnitude of the error in the moment, which was determined on the basis of the average of the two gage readings. As indicated in Fig. 14, this discrepancy is not great enough to account for the difference between the computed moment curves and the observed moment curves. Therefore, it is necessary to accept the authors' interpretation of the data, including values of E_s approximately equal to zero, or to seek other causes of the discrepancy.

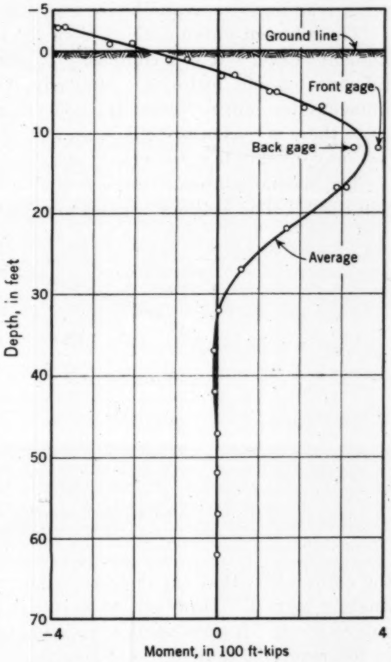


Fig. 14.—Comparison of Strain-Gage Readings for a Static Pile Load of 93.3 Kips

Two causes may be of considerable importance. The first is the rather unusual sequence of applications of the load, and the second is the lack of a positive check on the magnitude of the deflection. The first item is not apparent in the paper but must be investigated by a study of the original data. The strain gages were at twenty-two different levels. However, not enough oscillograph channels were available to permit simultaneous reading of all the gages. The experimenters had the choice of applying a given load and of successively reading the various groups of gages, or reading all the gages in one group while the load was set successively at various values and then reading another group of gages while the cycles of load were repeated. It was more expedient to vary the jack load than to switch from one set of gages to another. Therefore, the second alternative was chosen. Consequently, the data for the static load of 93.3 kips consist of four different sets of gage readings. Each set contained the readings of approximately 25% of the gages on the pile. While each group of gages was connected to the oscillograph, the load was maintained successively at four different values for the 10 min assigned to a static test. In addition, in some instances several dynamic-test measurements were also made with the same connections. Then the connections were changed to another set of gages, and the various loads were applied again in succession. Thus, three or four sets of loadings were required before a complete series of strain readings was available for any single value of jack load.

The moment curve derived from the readings does not represent one single moment curve, but is a composite of parts of several curves corresponding to different loading histories. Similarly, the deflection curve derived from the moments does not represent the deflection of the pile at any actual time. Because there is reason to believe that E_s changes with repetition of loading, this fact may invalidate considerably any conclusions that were drawn from the data.

The second source of large potential error is the magnitude of the deflection itself. The authors' studies and all the preceding comments are based on the assumption that the deflections computed from the moments are approximately correct. However, at no point in the pile is the deflection independently known. A translation or rotation of the pile without bending would result in soil reactions that were not considered in the analysis. The extent to which any such bodily movement occurred cannot be determined. An independent check of deflection would be invaluable in ascertaining the accuracy of the results of the computed deflections. According to the theory, because E_s should not equal zero at any depth, the soil reaction cannot be zero unless the deflection is zero. Hence, the curves of soil reaction and deflection, both of which are derived from the measured strains, should cross their respective zero axes at the same depths. The curves fail to do so; for that reason there is either a defect in the theory, or the data are inaccurate. A shift in the deflection curve is a possible means to restore compatibility.

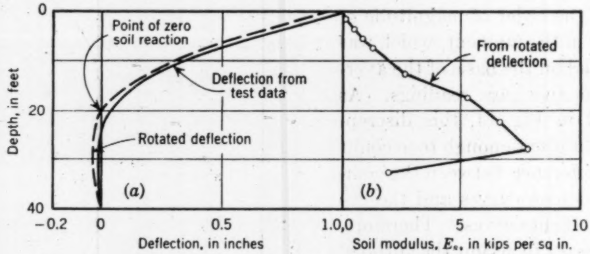


FIG. 15.—THE EFFECT ON THE SOIL MODULUS OF ROTATING THE DEFLECTION CURVE

For example, if the deflection curve for a static load of 93.3 kips is rotated slightly, as shown in Fig. 15(*a*), the points of zero soil reaction and zero deflection can be made to coincide, and a reasonable variation of E_s with depth is obtained to a depth as great as 32 ft below the ground line. This interpretation is at least as satisfactory as the data presented in Fig. 5. The values of E_s between 22 ft and 32 ft may be erratic because of the relatively large ratio of instrument error to deflection in this range. Within these depths the variation of E_s may therefore be somewhat inaccurate. Values of E_s plotted in Fig. 15(*b*) have the authors' units of kips per inch of pile length per inch of deflection.

It appears, especially in the upper part of the embedded section of the pile that the value of E_s must decrease markedly with increasing deflection. The value may even become zero in the upper few feet if the soil is pushed permanently away from the pile by repeated loading. The influence of the deflection on E_s cannot be ignored, and any theory based on the assumption that E_s is independent of deflection is likely to be erroneous.

Finally, it may be noted that the stress-strain properties of a given sample of soil may be strongly influenced by the size of the samples, the degree of sampling disturbance, the manner in which the lateral pressures are allowed to vary as the deviator stress is applied, and the time of strain. Hence, for these reasons as well as the reasons that were indicated previously, the authors' correlation of test pile data and triaxial data and its application to practical problems do not seem justified.

RAYMOND LUNDGREN,[15] A.M. ASCE.—The authors are aware of present-day construction problems and procedures. They are also familiar with modern concepts concerning the soil modulus of pile reaction, E_s.

The relationship that is developed in the paper depends on graphical differentiation and integration of a moment diagram that was developed from strain-gage readings. The correctness of the reaction and deflection diagrams, as shown in Fig. 3, will not be considered herein.

The authors believe that the value of E_s is not constant for variable loads at any given depth, which they have clearly proved by their research. The writer fully agrees with this statement. However, it should be noted that the soil modulus of pile reaction, E_s, is the same concept as the modulus or coefficient of the subgrade reaction for which Karl Terzaghi, Hon. M. ASCE, uses the symbol K_s.[16] Terzaghi considers two cases: (*a*) The K_s-value is constant, which is a fictitious case, and (*b*) the K_s-value is not constant but decreases with increasing values of the intensity of the load, which is a real case. Terzaghi's point of view is basically the same as that of the authors.

Fig. 4 indicates that as the applied lateral load is increased, the reaction, *p*, of the soil first increases, reaches a maximum, and then decreases. The deflection increases simultaneously. The decrease in reaction, *p*, only occurred close to the ground line at depths below 5 ft. At greater depths the maximum reaction value was not reached when the loads were applied. It appears that because of the application of the loads the shearing strength at shallow depths was overcome. The failure in this case consisted of a rapid decrease of the pile reaction, p, until the value of the reaction reached zero. However, the soil should have offered some resistance. The magnitude of the resistance could not be less than the remolded shearing strength of the material, or approximately the undisturbed shearing strength. Therefore, when the 100-kip load was applied the soil reaction at a depth of 3 ft should have had a definite value not equal to zero, as shown in Fig. 4.

---

[15] Partner, Woodward-Clyde-Sherard & Associates, Oakland, Calif.

[16] "Soil Mechanics in Engineering Practice," by Karl Terzaghi and Ralph B. Peck, John Wiley & Sons, Inc., New York, N. Y., 1948, pp. 214–215.

Under the heading, "Description of Test Pile," the authors state: "Loads were applied in increments in two series—a static series and a dynamic series." Apparently there was only one of each. Fig. 5 indicates that there were five horizontal loads in the static series, namely 20 kips, 40 kips, 60 kips, 80 kips, and 100 kips. The 100-kip load apparently was not applied for the dynamic series. In the actual project, for which this test probably was performed, there undoubtedly were thousands of applications of transient loads. This leads to a consideration of the possibility that the repeated applications would gradually change the physical nature of the soil and, hence, also change the E_s-values. The writer admits that transient loads would not produce a squeezing out of moisture from the soil but might produce a gradual change in the strength of the soil, thereby influencing the E_s-values. The increase in strength of a soil without a change in water content is a possibility, as laboratory observations have shown.

The authors' aim consisted in correlating their field results with the laboratory triaxial-test results on the same material. The final diagram is presented in Fig. 9, in which the pressure, p, exerted by the pile on the soil per unit of width of the pile, b (p/b), is plotted against the weight of the overburden, γx (γ being the unit weight of the soil and x denoting the depth); the resulting curve is replaced by a straight line termed "field" in Fig. 9. In the same figure the laboratory deviator stress, σ_Δ, is plotted against the confining pressure, σ_3, resulting in a straight line that has been termed "laboratory." The ratio of the slopes of the two straight lines in Fig. 9 is approximately 5.5. From these curves it was concluded that the soil modulus at a given depth is about eleven times the secant modulus from the laboratory Q_c-test on a sample of clay from that depth. Using a strain value other than 1% probably would yield different results. The writer believes that the research data described in the paper refer to the type of pile and methods of load application that were used, and their generalized application to other cases should be approached with caution. The writer also believes that the stiffness of the pile would have a considerable effect on the lateral load-deflection characteristics.

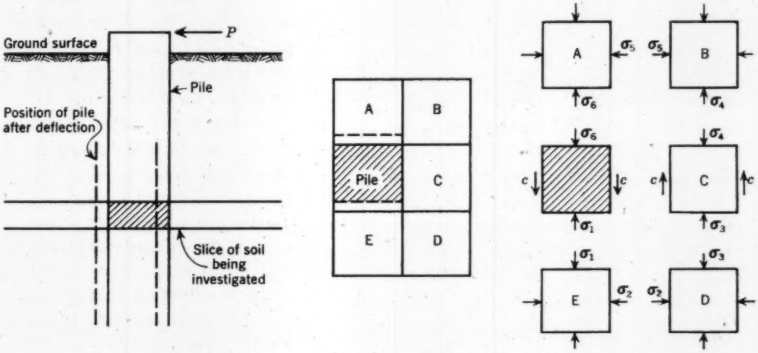
Whereas there are no major objections to the general reasoning of the authors, it would be advisable to repeat their tests on several clays and on piles having different stiffnesses. Modern laboratory apparatus could be used in the study of repetitional loads in soil.[17]

Fig. 6 shows that the Q_c-triaxial tests indicate greater shear strengths than the unconfined compression tests at equal depths. If the clay is normally consolidated and the triaxial specimens are consolidated under their natural overburden pressures, the Q_c-tests should indicate approximately the same shear strengths as the unconfined compression test. Because the field vane test revealed in-place shear strengths that compare closely with values that were determined from Q_c-triaxial tests, it is probable that the discrepancy is due to sample disturbance of the unconfined compression specimens. The shear strength as determined by the unconfined compression test or the field vane test should give values that could be used in predicting pile deflections whether or not the clay is normally consolidated.

---

[17] "Effects of Repeated Loadings on the Strength and Deformation of Compacted Clay," by H. B. Seed, Clarence K. Chan, and Carl L. Monismith, *Proceedings*, Highway Research Board, National Research Council, Washington, D. C., Vol. 34, 1955.

LYMON C. REESE,[18] A.M. ASCE.—The authors have presented a complex problem in soil mechanics involving the interaction between soils and structures, and they have applied intelligence and imagination to obtain the tentative procedure for estimating the soil modulus of pile reaction.



(a) VERTICAL CROSS SECTION OF PILE

(b) POSITION OF PILE AFTER DEFLECTION

(c) STRESS ANALYSIS

FIG. 16.—ANALYSIS OF PILE DEFLECTION

Rational methods can be applied to only one part of the problem—that is, to the determination of the ultimate resistance of the soil against the pile. If one assumes that a pile with a square cross section is moved through saturated clay at some distance below the ground surface so that there is only horizontal flow of the soil at failure, it is possible to make a rational determination of the approximate ultimate resistance that the soil will exert against the pile. Fig. 16(*a*) shows a vertical section through the pile, and Fig. 16(*b*) shows the blocks of soil that are assumed to be displaced horizontally when the pile is deflected. It is assumed that the stress, σ_1, on the side of block E next to the pile is equal to zero. Assuming a soil strength defined by the expression, $s = c$, the magnitude of the stress, σ_2, will be 2 c. By a similar analysis the stress, σ_3, is 4 c. If, in moving relative to the pile and the adjacent soil, block C is assumed to develop full resistance along each side, the magnitude of σ_4 will be 6 c. If blocks B and A fail as did blocks E and D, the value of σ_6 will be 10 c. Examining the free body of the pile section, it can be seen that the ultimate soil resistance against the pile where horizontal flow of the soil occurs is

---

[18] Associate Prof. of Civ. Eng., Univ. of Texas, Austin, Tex.

$$\left(\frac{p}{b}\right)_{ult} = \sigma_6 + 2\,c - \sigma_1 = 12\,c \qquad (6)$$

It is possible to make a rational determination of the approximate ultimate resistance that saturated clay will exert against a pile near the ground surface if one assumes that a wedge of soil is moved up and out by the pile. Fig. 17 shows a free body of such a wedge. The forces that exist on the wedge are defined as follows: F_1 is the body force; F_2 is the shear force on plane ABFE, and F_6 is the normal force on that plane; F_3 is the shear force on plane ACE; F_4 is the shear force on plane BDF; and F_5 is the shear force on plane CDFE, and F_7 is the normal force on that plane. There are also normal forces acting on planes ACE and BDF, but they are assumed to have no effect on this problem. The wedge is assumed to move along plane ABFE, and the shear forces act in a direction opposite to the movement.

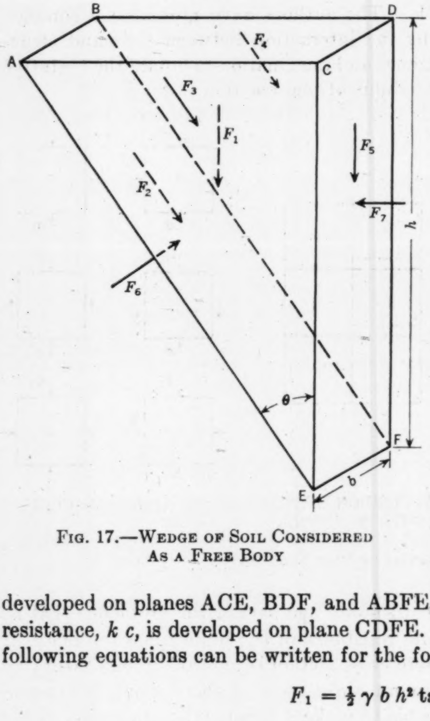


FIG. 17.—WEDGE OF SOIL CONSIDERED AS A FREE BODY

It is assumed that the full shear resistance, c, of the soil is developed on planes ACE, BDF, and ABFE, and that only a part of the shear resistance, $k\,c$, is developed on plane CDFE. Based on these assumptions, the following equations can be written for the forces:

$$F_1 = \tfrac{1}{2}\,\gamma\,b\,h^2 \tan\theta \qquad (7)$$

$$F_2 = c\,b\,h \sec\theta \qquad (8)$$

$$F_3 = \tfrac{1}{2}\,c\,h^2 \tan\theta \qquad (9a)$$

$$F_4 = \tfrac{1}{2}\,c\,h^2 \tan\theta \qquad (9b)$$

and

$$F_5 = k\,c\,b\,h \qquad (10)$$

Summing the forces in the vertical direction yields

$$F_6 = \tfrac{1}{2}\,\gamma\,b\,h^2 \sec\theta + k\,c\,b\,h \csc\theta + c\,b\,h \csc\theta + c\,h^2 \qquad (11)$$

Summing the forces in the horizontal direction results in

$$F_7 = \tfrac{1}{2}\gamma\, b\, h^2 + k\, c\, b\, h \cot\theta + 2\, c\, b\, h \sec\theta \csc\theta + c\, h^2 \sec\theta \ldots\ldots (12)$$

The soil resistance against the pile can be obtained by taking the derivative of F_7 and dividing this derivative by b. Thus,

$$\left(\frac{p}{b}\right)_{ult} = \gamma\, h + k\, c \cot\theta + 2\, c \cot\theta + \frac{2\, c\, h \sec\theta}{b} \ldots\ldots\ldots (13)$$

At the ground surface,

$$\left(\frac{p}{b}\right)_{ult} = k\, c \cot\theta + 2\, c \cot\theta \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

If the value of θ is assumed to be 45° and that of k is assumed to be zero, then

$$\left(\frac{p}{b}\right)_{ult} = 2\, c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

The value of 12 c, which was obtained by the approximate analysis for a square pile at some distance below the ground surface, compares favorably with the value of 11 c that was obtained by the authors.

However, the authors' method does not consider the fact that the soil resistance near the ground surface can be as small as 2 c. This means that errors could occur, especially if the top few feet of soil were to consist of stiff clay. Eq. 8 shows that the value of the ultimate soil resistance will reach the value of 12 c at approximately three pile diameters beneath the ground surface, and some thought should be given to modifying the method to deal with this upper zone.

The approximate values of ultimate soil resistance that were computed by the rational analysis cited previously may be of only academic interest, however, because there is no assurance that the pile can be deflected sufficiently to produce the ultimate resistance.

The nature of the soil near the ground surface is of great importance to the behavior of a laterally loaded pile. The writer has used the proposed method and the stress-strain curves in Fig. 6 to compute the soil modulus for a pile which has a length of 74 ft and a moment of inertia of 4,720 in.^{4} It was found that the computed soil modulus was entirely dependent on the soil in the top 15 ft. In this depth Fig. 7 shows five stress-strain curves, two of which are for the same depth. The scatter in the values of the soil modulus determined from the five curves was great, but the authors have indicated[5] that doubling the magnitude of the soil modulus would change the magnitude of the maximum computed moment by approximately 15%. Because the soil within a few feet of the ground surface is so important in using the proposed method, soil explorations for the design of laterally loaded structures should be especially thorough in the top several feet of soil. Multiple borings should be made, and the soil should be sampled continuously in that zone. The sampling is sometimes difficult when exploring soft, underconsolidated clay such as that near the mouth of the Mississippi River. It is possible that the vane shear test can be used to advantage in these cases.

Even though some rational verification can be made of part of the work that is presented, the method suggested by the authors is empirical and based on limited data. The authors term their method "tentative," and they suggest additional instrumented pile tests to confirm its validity.

The solution of this problem is important with regard to the design of many pile-supported structures. At the present time, probably the most important of these structures (from the economic standpoint) are the platforms that are being erected in the Gulf of Mexico by the oil industry. Several dozens of the platforms have been erected, some costing more than $1,000,000, and others are contemplated for the future. Of all the problems that are encountered in the design of offshore structures, the behavior of laterally loaded piles is the least understood, with the possible exception of the wave forces on these structures. The work cited by the authors is commendable because it tends to clarify the problem of laterally loaded piles, but it would be advisable to perform an instrumented pile test in conjunction with the construction of every major platform until relationships can be obtained between soil properties and pile behavior that will allow these structures to be designed with reasonable assurance.

It is possible that data are available that will help to verify or modify the tentative method that is proposed herein. It should be emphasized that it is not necessary to have data from an instrumented pile in order to gain insight into the problem. For example, if one assumes that the soil modulus takes the form, $E_s = k\,x$, one can compute values from which soil resistance pile deflection curves can be plotted for various depths if the investigators merely measure the deflection and rotation of the pile head for several loadings as a function of the applied load and moment. These computed soil resistance deflection curves could be compared with soil stress-strain curves. This procedure could be conducted without difficulty by using the nondimensional curves that were developed by Hudson Matlock, A.M. ASCE, and the writer.[19]

EUGENE A. RIPPERGER.[20]—The soil modulus of pile reaction, as defined by the authors, is an important concept in the computations for a laterally loaded pile, and, as such, its basic validity should be examined critically. One might appropriately ask if there really is such a factor as a soil modulus, and, if so, is it uniquely defined for a given soil? If the preceding questions can be answered in the affirmative, there is justification for trying to find a method for determining the modulus that does not require driving and testing a pile. If the answer is in the negative, some other approach should be sought to the problem of designing piles for lateral loads.

Obviously, a modulus of some sort exists. The authors have presented plots of reaction versus deflection, and by definition a secant of one of the load deflection curves is a modulus. The authors imply that for a perfectly elastic soil the modulus for a given pile and a given soil would be single valued, but for an inelastic soil there would be an infinite number of moduli. The definition in the case of an inelastic soil is made somewhat more significant by considering the modulus at an arbitrarily chosen deflection. It is by no means clear, however, that for either the elastic case or the inelastic case the modulus as defined has a unique value or that the value can be obtained from a laboratory test on soil samples.

---

[19] "Non-Dimensional Solutions for Laterally Loaded Piles with Soil Modulus Assumed Proportional to Depth," by L. C. Reese and H. Matlock, paper presented at 8th Texas Conference on Soil Mechanics and Foundation Eng., Univ. of Texas, Austin, September 15, 1956.

[20] Associate Prof. of Eng. Mechanics, Univ. of Texas, Austin, Tex.

Although soil is clearly not an elastic material, except possibly at extremely small strains, it will be assumed that it is completely elastic, and the theory of elasticity will be used to determine the ratio of load to deflection for a laterally loaded pile.

One can now consider a thin horizontal slice that has been taken from the pile-soil system at a distance equal to at least five pile diameters below the soil surface. Except for differences in the vicinity of the point of load application, the stress distribution in this slice, when the pile segment is loaded, is the same as for a concentrated load applied to a point in an infinite plate. If this is considered as a problem in plane strain, the solution for the deflection[21] is

$$\delta = \frac{p}{E} \frac{(1 - \nu^2)(3 + 2\nu)}{4\pi} \log \frac{b}{r} \quad \text{(16)}$$

in which δ is the deflection; p represents the load; r denotes the distance from the point of load application to the point at which the deflection is measured; E designates the modulus of elasticity of the medium; ν is Poisson's ratio; and b represents the distance from the point of load application to a point at which δ vanishes.

Eq. 16 can be rewritten as

$$\frac{p}{\delta} = \frac{4\pi E}{(1 - \nu^2)(3 + 2\nu) \log \frac{b}{r}} \quad \text{(17)}$$

If δ were actually the deflection at the face of the pile, Eq. 17 would be equivalent to 2 E_s, as defined by the authors. Eq. 17 indicates that as b approaches infinity, δ becomes very large and E_s approaches zero. Because the strain, ϵ_r, vanishes only when r approaches infinity, the point at which the deflection vanishes must also be at $r = \infty$. Thus, it appears that in an infinite elastic medium the deflection at a distance, r, from the point of load application will be infinite. This means that the deflection will also be infinite at the face of the pile. A semi-infinite, axially loaded elastic bar is somewhat analogous to this case. For such a bar the displacement of the end in the direction of the load is infinite for all values of the load, except that as the load becomes infinitesimally small the displacement becomes indeterminate.

If the soil is considered not as perfectly elastic, but as a material that is elastic at small strains and inelastic at large strains, it may enter the inelastic range in the vicinity of the pile. However, at great distances it will still behave elastically, and the stress distribution in this elastic zone will be the same as it was for the completely elastic soil. Hence, the deflections at the face of the pile will be infinite for this soil, too, if the soil mass extends to infinity in all directions from the pile.

---

[21] "Theory of Elasticity," by S. P. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1939, p. 110.

If the soil mass in the vicinity of the pile enters the plastic range exhibited by normal cohesive soils and if a flow pattern develops around the pile, the stress pattern can be divided into three zones, all of which move together through the soil as the pile moves. The first zone would contain the flowing soil. This zone is contained in a limited region, probably no more than three diameters from the center of the pile. The second zone is the region in which the soil behaves inelastically but does not flow. This region might extend as far as ten diameters from the pile. The third zone would be the elastic zone in which the stresses are small, but the distribution is the same as it was at that distance for the purely elastic case. The deflection at the face of the pile in that case would also be infinite if the lateral dimensions of the soil are infinite.

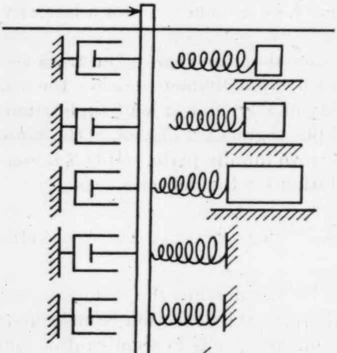


FIG. 18.—MECHANICAL MODEL OF THE PILE–SOIL SYSTEM

Thus, it appears that if a soil behaves elastically only at extremely small strains, the deflection in an infinite medium will be infinite regardless of (*a*) the soil characteristics at the higher strains and (*b*) the magnitude of the load. Obviously, under these assumptions the soil modulus can have no meaning and no unique value other than zero.

One might logically inquire at this point as to why an actual pile carrying a lateral load does not deflect an infinite distance. One limiting factor is the finite extent of the soil system. Except in mathematics, there can be no such thing as an infinite soil medium. Finite distances to the boundaries limit the deflections to finite quantities. This means that the deflection, and hence the ratio of load to deflection, E_s, depend not only on the soil properties, but also on the distance to the boundaries and, probably, the diameter of the pile. Other factors that may contribute to the limiting of the deflection are (*a*) the finite length of the pile and (*b*) the variation in load along the length of the pile. The preceding factors prevent the stress distribution around the pile from corresponding to the conditions for plane strain. If these factors limit the deflection they affect the soil modulus. If they affect the soil modulus that factor cannot be regarded as unique because it depends not only on the elastic characteristics of the soil, but on several other characteristics that cannot be evaluated easily.

The relationship between load and deflection in an actual pile is more complex than is indicated in the preceding presentation. The general nature of this relationship is illustrated graphically in Fig. 18.[19] In this illustration the dashpots indicate time dependence; the springs attached to blocks of different sizes indicate a reaction increasing with deflection and then reaching a yield point or limiting value that depends on depth; the taper in the springs indicates a nonlinear variation of load with deflection; the gap between the pile and springs indicates a molding away of the soil by repeated loadings; and the increasing stiffness of the soil is shown by the shortening of the springs as the depth below the surface increases.

In view of all the factors that contribute to the relationship between load and deflection in the actual case and the conclusions to which the simplified analysis leads, it does not seem likely that the soil modulus, even if uniquely defined, could be related to the modulus of elasticity, which was determined by laboratory triaxial tests, by a simple numerical factor having any fundamental basis.

Nevertheless, the authors have performed a valuable service in setting up this hypothesis and focusing attention on the problem. Studies and investigations of the validity of the hypothesis, which are necessary before it can be either completely accepted or rejected, will lead eventually to a clearer understanding of the problem of the laterally loaded pile.

HUDSON MATLOCK,[22] A.M. ASCE.—The use of strain gages to produce the data on which the authors' analysis is based appears to be the most promising way to develop the basic correlations that are needed for a better understanding of the complex problem of laterally loaded piles. However, the validity of such basic correlations is dependent on the reliability of the experimental data. Soil reaction values are extremely sensitive to errors in strain-gage readings and to methods of data evaluation. Although moments computed from theory are not very sensitive to variations in assumed soil modulus values, an appreciation is needed of the order of precision of proposed correlation methods or constants. It is hoped that the authors will indicate what, in their judgment, is the probable precision of the proposed constant.

Workable correlations might be developed from tests in which only the loading and movement at the top of the pile are observed, but such tests fail to indicate maximum stresses in the pile. Because of the many parameters affecting the behavior of a pile-soil system, a great number of such pile tests would be required. In view of the meager understanding of the problem at the present time, it seems more logical to concentrate first on fundamental relationships. This requires the measurement of soil resistance and pile deflection at various depths for comparison with soil characteristics which may be determined by laboratory tests.

---

[22] Associate Prof. of Civ. Eng., Univ. of Texas, Austin, Tex.

It would be highly desirable to make direct measurements of the soil resistance per unit length of the pile. Pressure cells fail to yield this quantity because their sensitive areas are limited. At each measuring station the entire perimeter of the pile would have to be fitted with such pressure cells, and, even then, the determination of the total force component acting opposite to the direction of movement of the pile would be hopelessly complex.

TABLE 1.—EXAMPLE SET OF MOMENT VALUES OBTAINED FROM THEORETICAL SOLUTION

Depth, x, in inches	Moment, M, in inch-kips
54	692.68
60	...
66	751.65
72	767.01
78	773.52
84	771.74
90	762.36
96	...
102	723.73

Several methods are available for more or less direct determinations of pile deflection or slope as a function of depth. Attempts to obtain soil reaction values from deflection measurements or slope measurements alone are certain to fail because the attainment of reasonable accuracy and resolution (with respect to depth) in soil resistance values would require impossible precision in the original measurements. This is due to the loss in accuracy that would arise from the three or four necessary differentiations.

Even the method of using strain gages suffers from the fact that measurements of moment by the gages must be differentiated twice to yield soil resistance distributions along the pile. There is no difficulty in obtaining accurate values of deflection because integration tends to smooth out the effects of irregularities. All that is required is that there be a well-established reference tangent somewhere along the pile to serve as a datum for the integrations.

To provide an example for evaluating the order of precision required in values of moment, it may be assumed that a soil modulus, E_s, exists that varies in simple proportion to the depth, x, so that $E_s = k\,x$. The following data will also be assumed:

Property	Value
k	0.015 kip per cu in.
Length of pile	600 in. (50 ft)
$E\,I$ of pile	11.66×10^6 kips $\times$ sq in.
Lateral thrust applied at the ground line	16.67 kips

A nondimensional solution,[19] based on a 100-point difference equation solution, has been used to compute "exact" values of moment at several depths in the vicinity of the maximum moment; the exact values are given in Table 1.

The value of soil resistance, p, at the central depth (78 in.) is computed from nondimensional solutions to be 0.230 kip per in. of pile length. This could be obtained from the preceding tabulation by the method of divided differences as follows:

The soil resistance, p, per unit length of pile is the second derivative of the moment-depth relationship. Thus,

$$p = \frac{d^2M}{dx^2} \qquad (18)$$

Referring to the notation for data points in Fig. 19, the method of divided differences yields

$$p = \frac{M_{i+1} - 2\,M_i + M_{i-1}}{n^2} \qquad (19)$$

One method that has been used to compute soil resistance values from experimentally determined moment values consists, in effect, of fitting a cubic polynomial by least squares to successive sets of five equally spaced points and then differentiating this polynomial at the central point.

The curve in Fig. 19 is assumed to be of the form,

$$M = a + b\,x + c\,x^2 + d\,x^3 \qquad (20)$$

in the vicinity of a set of any five equally spaced data points. When the curve is fitted by least squares and then differentiated at the central value, the following expression is obtained for the soil resistance:

$$p = \frac{1}{7\,n^2}\,(2\,M_{i+2} - M_{i+1} - 2\,M_i - M_{i-1} + 2\,M_i)_{-2} \qquad (21)$$

and a mild degree of smoothing to reduce the effect of experimental error is thus introduced.

To indicate the effect of an error in test data, the central value of moment in Table 1 is changed by 1% from 773.52 in.-kips to 781.26 in.-kips. The resulting values of computed soil resistance are compared in Table 2 for four different methods of computation. The results indicate that, even with some smoothing, errors in soil resistance values are relatively much greater than errors in moments. The effect of spacing is also illustrated. It is possible to obtain more reliable average values by increased spacing, but only by sacrificing resolution or the ability to distinguish the effects of individual soil layers. Graphical differentiation offers no improvement. Second derivatives that are obtained graphically are greatly affected by both human error and judgment, and can serve only to give a general idea of the soil resistance distribution.

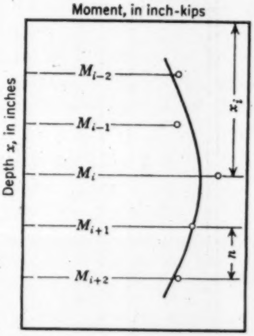


FIG. 19.—BASIS FOR PROCESSING TEST DATA

Regardless of the methods that are used for data reduction, extreme care must be taken to achieve maximum accuracy in the strain-gage data. This means that instead of using nominal gage constants and computed section properties, it is necessary to calibrate the pile and the indicating instruments with known moments applied. Furthermore, the gages selected should exhibit a minimum of creep and hysteresis. They must be waterproofed much more thoroughly than in ordinary practice, and special circuits must be used to attain maximum stability.

TABLE 2.—SOIL RESISTANCE VALUES COMPUTED AT 78-IN. DEPTH

Method	Spacing, in inches	SOIL RESISTANCE, IN KIPS PER INCH		Percentage change in computed soil resistance due to error in moment
		From "exact" moments in Table 1	With 1% error in moment at 78-in. depth	
Divided differences.....	6 12	−0.230 −0.229	−0.660 −0.337	187 47
Least squares, 5-point cubic...............	6 12	−0.229 −0.226	−0.291 −0.242	27 7

The validity of the strain-gage method was established in an extensive series of lateral load tests on an instrumented pile.[23] Although the test results are not available for publication, a sample set of curves from one loading is given in Fig. 20. The 12-in.-diameter pipe pile had an embedded length of 42 ft with gage-station spacings that varied from 6 in. near the top of the pile to 4 ft in the lower section. The pile was loaded with a horizontal thrust at

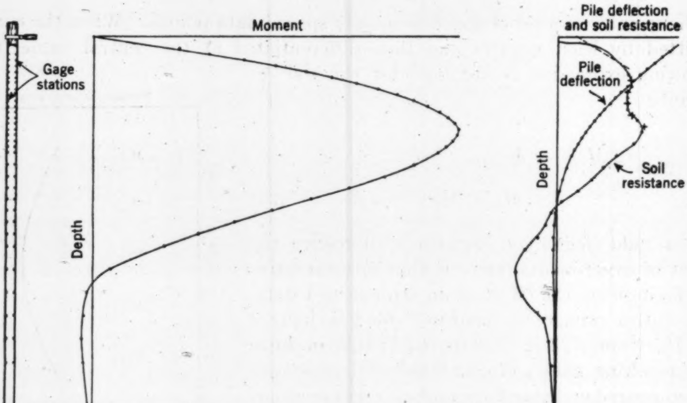


FIG. 20.—RESULTS OF A LATERAL LOAD TEST ON AN INSTRUMENTED PILE

the ground line. An individual calibration constant was obtained for each gage station by subjecting the pile to precisely known moments. Duplication of results to 0.1% or better was attained during calibration. The moment values plotted in the figure are those obtained directly from the strain-gage readings multiplied by the calibration constants.

---

[23] "Procedures and Instrumentation for Tests on a Laterally-Loaded Pile," by Hudson Matlock and E. A. Ripperger, *Proceedings*, Eighth Texas Conference on Soil Mechanics and Foundation Eng., Univ. of Texas, Austin, September, 1956.

Pile deflection curves and soil resistance curves that were computed from the moment data are also shown in Fig. 20. The soil resistance values were obtained with the least-squares, 5-point cubic fit discussed previously. In the zone of 6-in. gage spacings near the top of the pile, two independent sets of computations were made using alternate sets of gages at 12-in. spacing. The extra set of soil resistance values is indicated by using crosses for the plotted points. The agreement with the values shown by open circles in this zone indicates the degree of reliability in the results.

BRAMLETTE MCCLELLAND[24] AND JOHN A. FOCHT, JR.,[25] ASSOCIATE MEMBERS, ASCE.—The primary reason for attempting to determine the soil modulus of laterally loaded piles without a full-scale test is to permit a reasonably accurate determination of the pile bending moments, pile deflections, and soil reactions, with emphasis on the bending moments. Mr. Ripperger questions the existence of "such a factor as a soil modulus," then concedes that "obviously, a modulus of some sort exists." There definitely is no unique value of the soil modulus for a given soil because the pile test showed that the modulus varies with depth and pile deflection. Because the pile deflection varies with the pile size, pile stiffness, load magnitude, and manner of load application, the modulus will also vary with these factors. Therefore, the soil modulus exists only as a mathematically convenient expression for the ratio of soil reaction to pile deflection, p/y. The term E_s is a symbol for this ratio and should not be considered as a property exclusively of the soil.

The tentative correlation coefficient established in the paper does not in itself establish a soil modulus for a given soil, but permits the establishment of a soil-reaction pile deflection curve (stress-strain curve) for some selected pile size and at a selected depth within that soil. The proper soil modulus for a given problem becomes known only after the problem is fully solved by successive approximations, using a difference equation solution or some equally suitable method, and the pile deflection curve is determined.

Mr. Reuss and Mr. Matlock have requested the writers' estimate of the effect of inaccuracies in the pile test on the correlation coefficient. To the reasonable question of "what is the possible percentage of error in the coefficient?" should be added: "What degree of accuracy is required of the coefficient?" Mathematical analyses using idealized E_s-versus-depth relationships answer the second question. Considering the value of E_s to be constant with depth and deflection, the pile bending moment, M, varies as the fourth root of E_s.[6] If E_s is constant with deflection but increases linearly with depth, then M varies as the fifth root of E_s.[19] To compute M to within a 5% range, E_s could be in error by approximately 20% in the first case and 25% in the second case. With respect to the first question and considering the accuracy of the correlation coefficient to be affected only by the precision of the computed deflections and reactions for the test pile, the possible error in the value of the coefficient might be as much as 50%, thus permitting a possible error of 10% in a computed moment. The actual error resulting from inaccuracies in the test pile data is believed to be less than 50% because of the experimenters' statistical treatment of the data[9] and because of the partial confirmation given by the rational analysis of Mr. Reese. With intelligent use the correlation coefficient is therefore believed to be sufficiently accurate to permit the computation of pile bending moments, with an error of less than 10% for loading conditions similar to the test.

---

[24] Pres., McClelland Engrs., Inc., Houston, Tex.

[25] Chf. Design Engr., McClelland Engrs., Inc., Houston, Tex.

In most problems involving laterally loaded piles, the desired product of the analysis is the pile bending moment at or near the ground line. The "significant depth" of soil (questioned by Messrs. Peck, Davisson, and Hansen) governing pile behavior has been shown by previous analytical studies[5] to extend down to approximately the point of zero deflection. Significant changes in the assumed value of soil modulus below this point will have little effect on the computed moment and deflection at the ground line. Fig. 21 and Table 3, which were taken from a previous paper[5] by the writers, clearly demonstrate this fact. Mr. Reese also found that the upper 15 ft of soil at the test site governs the pile performance even though the embedded pile length is 75 ft. The relative stiffness of the pile soil system below the point of zero deflection has only limited effect on the deflection curve near the ground line.

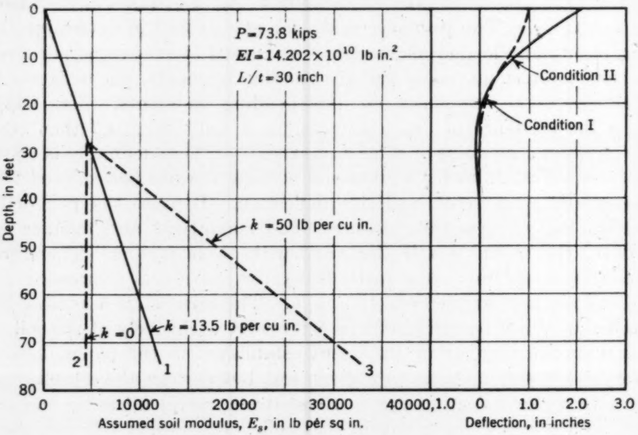


FIG. 21.—EFFECT OF SOIL MODULUS VARIATION ON MOMENT AND DEFLECTION

Mr. Peck and his associates apparently attach much importance to the irregular and small values of E_s indicated by the test pile data below 15 ft. With much amplification, they expressed a belief which the writers accepted by inspection—that much of the irregularity is due to inaccuracies in the deflection and reaction determinations in a zone which both of these values are small. They indicated that adjustment of the pile deflection curve by a minor rotation, in order to equalize the depths to zero deflection and zero reaction, will produce more reasonable E_s-values to a depth of as great as 30 ft. However, they did not acknowledge, or perhaps failed to recognize, that this same rotation produced a negligible effect on the E_s-values above a 20-ft depth, and that the latter values controlled the pile behavior and were those on which the correlation coefficient was based. The primary importance of the values at shallow depths and the relative unimportance of the values at greater depths was stated by the writers. Mr. Reese arrived independently at the same conclusion.

The possibility that the effects of the loading sequence may have had a marked influence on the results is raised by Mr. Peck and his associates. According to a study conducted[26] by Roy D. Gaul, J.M. ASCE, if the soil is not stressed to failure a small number of load applications will not cause any significant decrease in the E_s-value at a given depth. A total of only fifty-seven load applications, both static and dynamic, were made to the pile. In general, the magnitude of the loads increased progressively. Some erratic errors could have developed because of the necessity for switching between loads, but a procedure of statistical averaging of the basic moment data, as followed by the test sponsors, probably reduced most errors.

TABLE 3.—EFFECT OF CHANGES OF THE SOIL MODULUS ON THE MOMENT AND DEFLECTION

	Assumed E_s-value[a]	Deflection at ground line, in inches	Moment at ground line, in foot-kips
Condition I slope = 0	1	0.508	579
	2	0.508	579
	3	0.507	578
Condition II slope = − 0.006	1	1.073	182
	2	1.073	182
	3	1.071	181

[a] From Fig. 21.

Mr. Reese's derivation of 12 *c* as the ultimate soil resistance below the ground surface is substantiated by a value of 11.42 computed by C. G. Meyerhoff,[27] both of which compare favorably with the value of 11 *c* computed from Eq. 4. Therefore, Eq. 4 has a theoretical basis as well as an empirical one.

The zone near the ground-line boundary, as described by Mr. Reese, requires further study. In Fig. 22 the ratio of ultimate soil resistance, p/b, to the soil shear strength is plotted versus the depth-diameter ratio. The soil resistance was computed from Eq. 13, assuming $\theta = 45°$, $\gamma = 0$, and $k = 0$. Four points, which were computed from (*a*) the maximum soil stresses resulting from static loads (taken from Fig. 8) and (*b*) the average Q_c-triaxial shear strength curve in Fig. 6, are also plotted in Fig. 22. These data confirm Mr. Reese's conclusion that the ultimate soil resistance will be less than 11 c or 12 c within a depth range of approximately three pile diameters. On the basis of limited evidence, the writers believe that Eq. 4 may be valid even at shallow depths if the applied loads produce soil reactions that are less than the maximum resistance. The presence of the ground-line boundary permits failure at a lower strain than would be allowed if the soil surface were farther removed. However, it may not significantly affect the stress-strain curve up to the point of failure.

---

[26] "Model Study of a Dynamically Laterally Loaded Pile," by Roy D. Gaul, *Proceedings Paper 1535*, February, 1958, ASCE.

[27] "The Ultimate Bearing Capacity of Foundations," by C. G. Meyerhoff, *Géotechnique*, Inst. C. E., Vol. 2, No. 4, December, 1951, p. 312.

The emphasis placed by Mr. Reese on thorough exploration of shallow soils for laterally loaded structures is warranted. In addition the most careful sampling techniques and testing techniques should be observed. Messrs. Peck, Davisson, and Hansen conclude that application of the tentative correlation is not justified because of possible variations in sampling and laboratory procedures. However, because these variables can be controlled to obtain reproducible results, their conclusion does not seem warranted.

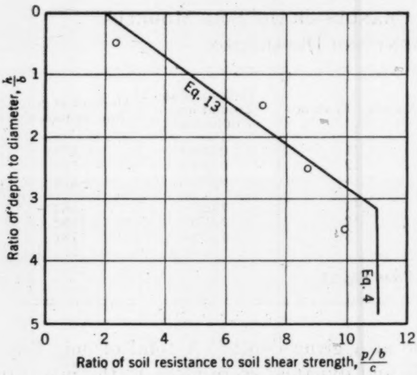


FIG. 22.—RATIO OF MAXIMUM SOIL RESISTANCE TO SOIL SHEAR STRENGTH

Lateral load tests in which only deflection and rotation of the pile head are measured, as suggested by Mr. Reese to obtain soil resistance and pile deflection curves, will yield results that are only as good as the assumption of $E_s = k\,x$. It appears preferable to instrument the test pile fully, as was done by Messrs. Matlock and Ripperger,[23] than to test uninstrumented piles. The remarkable degree of precision that was developed in their instrumentation of a laterally loaded test pile constitutes a highly desirable precedent for future pile tests. In their tests, supplemental measurements of deflection and slope at selected points, as well as measurements of the applied load, permitted minor adjustments to reduce the errors that were introduced by such effects as a slight rotation of the pile.

The ingenious diagrammatic representation of the pile-soil system given by Mr. Ripperger (Fig. 18) clearly shows the complexity of the problem. In considering a thin horizontal slice from the pile-soil system, Mr. Ripperger states that "except for differences in the vicinity of the point of load application, the stress distribution in this slice * * * is the same as for a concentrated load applied to a point in an infinite plate." He then states that deflection theoretically should be infinite for all loads, which means that the "* * * soil modulus can have no meaning and no unique value other than zero." Although

Mr. Ripperger mentions several factors that may explain why actual deflections are not infinite, the writers believe that he overlooked the prime factor which is the exception to his assumed load distribution, as stated in the preceding quotation. As established[28] previously by S. P. Timoshenko, the infinite deflection at the point of application of a concentrated load may be disregarded because it is theoretical, and a finite distributed load on the edge of an infinite plate or on the surface of a semi-infinite mass will create a finite displacement. Therefore, the prime factor producing finite deflection is the finite area of load application rather than the other factors that were mentioned. Thus, with finite displacements having a fundamental basis, there should be finite values of the soil modulus, which will depend on depth, deflection, pile size, pile stiffness, pile length, and soil type. The influence of the preceding variables on the deflection can be reasonably estimated by following the suggested method for applying the proposed correlation.

Mr. Reuss introduced the problem of a factor of safety in situations involving laterally loaded piles. This significant problem has not been solved. There are several ways in which a laterally loaded pile structure could fail. The pile could move through the soil, failing the soil for the full length of the pile; or excessive bending stresses could result in structural failure of the pile; or the deflection, or rotation, of the pile could be detrimental to the utilization of the structure. The first and third modes of failure would probably occur in piles which are relatively stiff with short penetrations. The second type of failure would probably develop in piles of relatively great penetration with low stiffness. Inasmuch as the technique of predicting at what load a failure will occur and what type of failure will develop is only tentative, a relatively large factor of safety is desirable. The inclusion of a factor of safety by increasing the design load, as suggested by Mr. Reuss, is a satisfactory way of handling the problem. In the case of laterally loaded piles for offshore structures, however, the determination of the loads to be imposed is also subject to diverse errors. The probability of the combination of conditions to produce the maximum load is also uncertain. Therefore, for offshore structures it has been necessary to use design loads without applying a safety factor and to accept a calculated risk.

The possibility of a linear (or elastic) deformation followed by a nonlinear (or plastic) deformation as the load increases is presented by Messrs. Ripperger and Reuss. Mr. Reuss' suggestion that the deflection be limited to within the elastic range to avoid progressive failure has merit because the data developed[26] by Gaul show that within the elastic range repeated loads do not cause progressive failure. There are some indications of elastic action under the dynamic loads because the stress-strain curves in Fig. 8 have an initial slope of approximately 45°. However, there is no indication of linearity for the static loads in this test, which indicates that the linear range, if any, was small. Therefore, designing for the limitation that was suggested by Mr. Reuss would restrict a static lateral load to almost zero. For overconsolidated soils a wider range of linear stress versus strain could be expected.

---

[28] "Theory of Elasticity," by S. P. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1934, p. 88 and p. 333.

Mr. Lundgren amplified several pertinent assumptions, limitations, and results. It seems reasonable to expect a soil reaction greater than zero at a 3-ft depth even for a 100-kip load. In the case of the test pile, the effects of cyclic loads and remolding effects could have caused the reaction to be truly zero. It is also possible that the true reaction was a small finite value, but, due to minor errors in moment observations, the computed reaction of zero is in error.

The effect of a large number of transient loads, particularly reversing ones, might tend to increase the strength of the soil, as suggested by Mr. Lundgren. However, substantial deflections of the soil will result during the process, so that the effect should be for the soil modulus to decrease with repeated loads if the deflections are in the plastic range—that is, the deflections of a pile probably will increase as the lateral loads are repeated. There may even be a tendency towards progressive failure. For this reason more study of the effects of repeated loads and cyclic loads is required before any definite conclusions can be made.

The strain value of 1% that was used to develop the plot of Fig. 9 was chosen for illustration only. Similar plots for strains of from 0.5% to 3% would yield correlation factors differing only slightly from the value of 5.5. Other tests to extend, verify, or disprove the tentative correlation are definitely and urgently needed.

In summation, any design procedure based on a single test, or even one test series covering a significant load range, can be compared with extrapolation from a single point. General considerations can provide guides for the extrapolation, but the procedure must be considered tentative and subject even to major revision until further data are secured. The discussions have indicated for the most part that there was a definite need for a starting point in order to obtain solutions for laterally loaded piles. As Messrs. Peck, Davisson, and Hansen correctly stated, "* * * E_s must decrease markedly with increasing deflection," and "* * * any theory based on the assumption that E_s is independent of deflection is likely to be erroneous." The tentative correlation and the proposed method for its application to practical problems constitute the only procedure that has been advanced for estimating the decrease of E_s with increasing deflection. With the application of careful thought and analysis and with additional fully instrumented lateral load tests, a more precise correlation substantiated by well-documented data can be developed by which the stress-strain relationships for laterally loaded piles can be accurately predicted.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2955

BEHAVIOR OF RIVETED TRUSS-TYPE CONNECTIONS

BY EUGENE CHESSON, JR.,[1] J. M. ASCE, AND WILLIAM H. MUNSE,[2] A. M. ASCE

WITH DISCUSSION BY MESSRS. ARTHUR J. FRANCIS; AND EUGENE CHESSON, JR. AND WILLIAM H. MUNSE

SYNOPSIS

The tests reported herein were performed to provide information on the general behavior of large, truss-type, riveted steel connections. The variables of the test program included specimen configuration, method of hole preparation, and size of rivets. A study was made of the comparative behavior of the specimens, the distribution of load to the gusset plates, the strains in the lacing bars, the effect of hole preparation, and the predicted and computed efficiencies of the connections.

INTRODUCTION

Research on riveted joints has been conducted since about 1837. Nevertheless, many problems have never been solved completely. Such research placed emphasis on flat-plate joints, probably reflecting the past but declining importance of riveted joints in vessels, tanks, and boilers. However, since the late 1800's the use of long-span bridges and the construction of many tall buildings have made large built-up members important. Nevertheless, the literature[3] yields little data on tension tests of full-size, truss-type members, although a few studies have been made on gusset plates, columns, and some related structural components, such as angles, lug angles, and tie plates. In general, the latter tests were limited in scope and involved few specimens. Since 1945 occasional

---

NOTE.—Published, essentially as printed here, in January, 1957, in the Journal of the Structural Division, as *Proceedings Paper 1150*. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Research Associate, Dept. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

[2] Prof., Dept. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

[3] "Riveted Joints, a Critical Review of the Literature Covering Their Development, with Bibliography and Abstracts of the Most Important Articles," by A. E. Richard de Jonge, *Research Publication*, A.S.M.E., 1945.

tests of large tension members have been made. However, the tests included only several specimens of similar size and shape and often were limited to, or principally concerned with, the behavior of high-strength bolts.

Flat-plate joints are sometimes referred to as "single-plane members," or those in which the loads on the fasteners are applied in one plane. In the case of double-lap joints, the loads on the fasteners are applied in two planes separated only by one or more thicknesses of material, a distance that is usually small in relation to the width of the joint. In contrast to single-plane members, many large truss specimens in general use can be termed "double-plane members"—that is, those in which the loads are applied through gussets in two planes separated by a distance that is often equal to or exceeds the width of the joint.

The behavior up to failure of full-size, truss-type members subjected to static tensile loads will be examined. Because they were tested in duplicate, the sixteen specimens represent eight variations. The latter include five distinct specimen patterns for which the rivet holes were drilled plus three for which the rivet holes were punched.

Description of Specimens

Fabrication and Description of Specimens.—The material was ordered in accordance with American Society of Testing Materials (ASTM) designation A7.[4] The gusset plates were cut from 40-in.-by-½-in. hot-rolled sheared plates; the web plates and battens (or tie plates) were cut from 16-in.-by-½-in. universal mill plates; and the lacing bars were sheared from a ¼-in. plate. The angle stock consisted of 3½-in.-by-3½-in.-by-$\frac{7}{16}$-in. material in 22-ft 6-in. lengths; 5-in.-by-3-in.-by-⅜-in. angles that were 34 ft long; and 5-in.-by-5-in.-by-⅜-in. angles that were 30 ft long.

All material was carefully identified and cut at the University of Illinois (Urbana). The batten plates and the web plates for a given specimen came from the same pieces of plate. Similarly, without exception, all four angles for any given specimen were cut from a single length. Coupons were taken from approximately the mid-length of each piece of stock that would comprise part of the critical section of the specimens. In general, the plate material was flame-cut to final dimensions, and the angles were generally saw-cut to length.

One of the principal variables of these tests was the method of hole preparation. In order to reduce the variations resulting from fabrication to a minimum, all pieces for the drilled specimens were match-drilled and fitted up completely at the university.

Punched specimens were made as follows: The plates were laid out at the university and center-punched. They were then punched full size at the fabricator's shop, using a conventional punching machine. The angles for the punched specimens, having been cut at the university, were set up and carefully punched on a standard spacing table at the fabricator's shop. Because these angles had been laid out earlier at the university, the stops or settings of the spacing table were checked in a "dry run" before actual punching began. The use of these procedures resulted in uniform spacing and constant gage distances.

---

[4] "Tentative Specification for Steel for Bridges and Buildings," *ASTM A7-50T*, A.S.T.M., 1950.

In driving the more than 1,500 rivets, only nine holes required reaming, which, however, did not appear to reduce the strength of the specimens involved because the failures did not occur at those joints in which the rivet holes had been reamed.

All rivets were from the fabricator's stock and of ASTM A141 designation[5] with cold-formed heads. The length required for the rivets was determined by the rivet-gang foreman in the usual shop fashion. New kegs of rivets were opened and used for these specimens, and four sample rivets of each diameter and length were set aside for laboratory testing. The ¾-in. rivets in specimens AD1 and AD2 and the ⅞-in. rivets for the tie plates and lacing bars of specimens DD1 and DD2 were all hand-driven. All other rivets were machine-driven in both the punched and the drilled specimens.

TABLE 1.—AREAS AND PROPERTIES OF SPECIMENS

				GROSS AREA, IN SQ IN.			NET AREA, IN SQ IN., BASED ON:				
Specimen (1)	Type (2)	Hole preparation (3)	Rivet size, in inches (4)	Handbook (5)	Measured (6)	Col. 6 ÷ Col. 5, in % (7)	AREA (8)	Modified AREA (9)	Relative gage (10)	Shear area,[a] in sq in. (11)	Tension: shear: bearing ratio, AREA (12)
AD1	Box section	Drilled	3/4	27.48	27.12	98.68	20.23	20.77	20.57	30.93	1.0:0.65:0.77
AD2	Box section	Drilled	3/4	27.48	27.24	99.13	20.23	20.77	20.57	30.93	1.0:0.65:0.77
BD1	Laced angles	Drilled	7/8	11.44	11.44	100.00	8.62	8.81	8.63	14.43	1.0:0.60:1.10
BD2	Laced angles	Drilled	7/8	11.44	11.20	97.90	8.62	8.81	8.63	14.43	1.0:0.60:1.10
BP1	Laced angles	Punched	7/8	11.44	11.24	98.25	8.62	8.81	8.63	14.43	1.0:0.60:1.10
BP2	Laced angles	Punched	7/8	11.44	11.24	98.25	8.62	8.81	8.63	14.43	1.0:0.60:1.10
CD1	I-section	Drilled	7/8	19.44	19.37	99.64	15.62	15.87	15.69	24.05	1.0:0.65:1.19
CD2	I-section	Drilled	7/8	19.44	19.17	98.61	15.62	15.87	15.69	24.05	1.0:0.65:1.19
DD1	Laced angles	Drilled	7/8	11.44	11.32	98.95	7.20	7.48	7.22	12.03	1.0:0.60:1.10
DD2	Laced angles	Drilled	7/8	11.44	11.24	98.25	7.20	7.48	7.22	12.03	1.0:0.60:1.10
DP1	Laced angles	Punched	7/8	11.44	11.16	97.55	7.20	7.48	7.22	12.03	1.0:0.60:1.10
DP2	Laced angles	Punched	7/8	11.44	11.20	97.90	7.20	7.48	7.22	12.03	1.0:0.60:1.10
ED1	Laced angles	Drilled	3/4	14.44	14.48	100.35	11.94	12.12	12.00	17.67	1.0:0.68:1.06
ED2	Laced angles	Drilled	3/4	14.44	14.44	100.00	11.94	12.12	12.00	17.67	1.0:0.68:1.06
EP1	Laced angles	Punched	3/4	14.44	14.48	100.35	11.94	12.12	12.00	17.67	1.0:0.68:1.06
EP2	Laced angles	Punched	3/4	14.44	14.44	100.00	11.94	12.12	12.00	17.67	1.0:0.68:1.06
					Average	98.99					

[a] Based on the nominal rivet diameter.

There were five basic types, which were designated alphabetically A through E and designed to give as wide a range of predicted efficiencies as possible with the usual specification requirements of gage distances (except for the type D specimens), edge distances, and spacing. A marking system identified each piece of material by specimen type, method of hole preparation, specimen number, and final location. It was thus possible for each piece to be followed from the original length of stock to the assembled specimen. All drilled specimens are identified by the letter D following the type designation, and the punched specimens are designated by the letter P. Because each type was tested in duplicate, the first and second specimens are numbered 1 and 2, respectively. Thus, BD1 signifies the No. 1 specimen of type B, which was prepared by drilling, and DP2, the second punched specimen of type D. Details of the specimens are shown in Figs. 1 and 2 (to be presented subsequently) and Table 1.

---

[5] "Standard Specifications for Structural Rivet Steel," *ASTM A141-39*, A.S.T.M., 1939.

Mechanical Properties of Materials.—Coupons from the materials were machined to a $1\frac{1}{2}$-in. width and to a standard 8-in. gage length.[6] Both surfaces of coupons from the junction of the legs of an angle were machined to provide parallel surfaces. All other coupons were tested with the flat surfaces in the "as-rolled" condition. Every coupon was carefully marked to identify its original position and its related specimen. Because all angles of a given size were from the same heat, coupons were taken from the toe, center, and fillet positions of each leg of one length of stock. Only two coupons, one from the center of each leg, were taken from other lengths of angles. From two to five coupons were tested from each of the various pieces of plate stock.

TABLE 2.—AVERAGE MECHANICAL PROPERTIES OF SPECIMEN MATERIALS[a]

Coupons for specimen	Location	UPPER YIELD	LOWER YIELD	ULTIMATE STRENGTH	ELONGATION	REDUCTION IN AREA
		Average, in pounds per square inch			Average, in percentage	
AD1	angles	45,900	45,000	69,400	26.2	49.6
AD1	webs	37,400	36,200	62,900	26.7	51.6
AD2	angles	43,900	43,500	68,900	26.2	49.7
AD2	webs	37,000	33,600	58,000	30.6	53.7
B	lacing bars	47,900	45,600	59,900	23.4	47.4
BD1	angles	40,200	39,200	60,600	25.6	51.3
BD2	angles	39,500	38,300	61,600	25.8	48.8
BP1	angles	41,200	40,200	62,500	27.3	48.6
BP2	angles	43,400	41,900	62,200	28.0	49.9
CD1	angles	38,650	38,650	61,300	28.4	49.3
CD1	web	40,000	38,000	63,400	27.9	51.4
CD2	angles	41,500	40,300	61,900	26.6	49.4
CD2	web	40,000	38,000	63,400	27.9	51.4
D	lacing bars	48,800	45,700	58,800	22.3	48.5
D	gusset plates	38,500	36,200	59,300	28.5	54.1
DD1	angles	41,000	40,100	62,200	26.3	50.9
DD2	angles	40,100	39,300	61,200	26.4	50.3
DP1	angles	39,000	39,000	61,800	28.6	51.1
DP2	angles	39,800	39,600	62,200	28.1	49.5
E	lacing bars	46,300	43,900	58,700	21.3	49.3
ED1	angles	40,200	38,900	65,300	28.5	48.9
ED2	angles	40,300	38,900	65,400	27.9	48.8
EP1	angles	40,300	39,000	65,900	27.2	49.0
EP2	angles	40,700	39,700	65,500	27.9	49.0

[a] All coupons were standard.

---

[6] "Standard Methods of Tension Testing of Metallic Materials," *ASTM Standard E8-46*, A.S.T.M., 1946.

The average mechanical properties of the coupons are listed in Table 2. The chemical composition and mechanical properties from mill reports for the angles and the plate material are listed in Table 3.

Table 2 shows that most of the material for these specimens met the requirements of ASTM A7, although some of the plate material had an ultimate strength of as low as 58,000 lb per sq in., or approximately 3% lower than the required 60,000 lb per sq in. All coupons met the minimum yield requirement of 33,000 lb per sq in. and the requirements for elongation.

The information obtained from the coupons also provided a means of checking the actual dimensions of the truss-type specimens. By taking the thicknesses of the unmachined coupons and averaging them for a given member, it was possible to compute the areas of those members for comparison with the areas obtained from the handbook of the American Institute of Steel Construction (AISC).[7] It was found that the measured areas of the specimens tended to be approximately 99% of the handbook areas. Individual specimens were as low as 97.5% and as high as 100.3%, as shown in Table 1. Such a range is within the ±2.5% allowed by AISC and ASTM specifications.[7,8] The measured area was used only in the determination of test efficiencies. All other references to area are to the handbook or nominal areas.

TABLE 3.—CHEMICAL ANALYSIS AND MECHANICAL PROPERTIES OF MATERIALS FROM MILL REPORTS

Material	Carbon, in %	Manganese, in %	Phosphorus, in %	Sulfur, in %	Silicon, in %	Tensile strength, in lb per sq in.	Yield point, in lb per sq in.	Elongation, in %
Angles 3½ in. × 3½ in. × 7/16 in. × 22 ft 6 in.	0.23	0.52	0.014	0.039	...	66,320	39,650	28.0
Angles 5 in. × 3 in. × ⅜ in. × 34 ft	0.21	0.42	0.023	0.042	0.07	63,283	38,877	27.0
Angles 5 in. × 5 in. × ⅜ in. × 30 ft	0.24	0.46	0.017	0.036	0.04	68,963	40,948	25.0
Plate, hot rolled sheared 40 in. × ½ in. × 10 ft	0.20	0.41	0.010	0.032	...	60,000	34,200	26.5
Plate, universal mill 16 in. × ½ in. × 25 ft 8 in.	0.23	0.52	0.010	0.033	...	62,280	37,500	28.0
Plate, universal mill 16 in. × ½ in. × 25 ft 8 in.	0.22	0.49	0.016	0.026	...	64,740	38,860	26.25

The four samples of each rivet length-diameter combination were tested in shear and tension. The results of the tests are shown in Table 4. The average shear strength of the ¾-in. rivets exceeded that of the ⅞-in. rivets by approximately 10%. The ultimate tensile strengths of these rivets were generally higher than those specified[5] by ASTM A141-39. However, the latter specification governs the properties of the as-rolled bars and not the manufactured rivets that were tested.

---

[7] "Steel Construction," A. I. S. C., 5th Ed., 1955.

[8] "Tentative Specification for Delivery of Rolled Steel Plates, Shapes, Sheet Piling, and Bars for Structural Use," *ASTM A6-50T*, A.S.T.M., 1950.

Instrumentation and Equipment.—All sixteen specimens had similar instrumentation involving mechanical dials, electric strain gages, and a qualitative visual indicator of the extent of yielding.

The mechanical dials had 0.001-in. divisions and a range of 1 in., and were used to (1) indicate the over-all deformation of the specimen and joint, (2) measure the relative movement of the gusset plates and angles at the critical sections or first rows of rivets in the joints, and (3) indicate the relative movement of the angles and gussets at the last row of rivets in the joints. Mechanical dial locations are shown in Figs. 1 and 2.

The electric strain gages were SR-4 (type A1 and 11/16-in. gage length) wire-resistance gages. They were placed on the various specimens, as shown in Figs. 1 and 2, and were used to give comparative strains in the angles, lacing bars, and web plates of the members. In order to determine the load distribution to the members, three pairs of gages were placed on each of the four pull plates to which the gussets were welded. The gages, one on either side of the pull plate, gave average strains at the gage locations and were used to evaluate the magnitude of the load in the gusset plates. Strain-gage locations were chosen to obtain the most representative measurements with the least number of gages.

TABLE 4.—COUPON TESTS OF RIVET STOCK

Rivet size, in inches	Average measured diameter, in inches	Average ultimate shear strength,[a] in lb per sq in.	TENSILE TEST[b]		
			Ultimate, stress, in lb per sq in.	Reduction in area, in %	Elongation, in %
¾×2¼	0.741	46,700	67,700	58.6	18
¾×2½	0.740	46,700	68,900	54.7	17
¾×2¾	0.740	49,100	70,100	62.1	19
¾×2⅞	0.743	50,300	69,600	65.9	22
Average for ¾-in. rivets		48,200	69,100		
⅞×2⅞	0.864	43,700	62,500	65.5	19
⅞×3	0.861	44,200	62,800	63.8	20
Average for ⅞-in. rivets		44,000	62,700		

[a] Average of four loadings on two rivets; two loadings on each rivet were made on surfaces from ¾ in. to 1 in. apart. Shear stress tabulated is based on nominal diameter; loading rate was 0.04 in. per min.

[b] Average from two tests on coupons machined from undriven rivets with no annealing. Coupons were 0.25 in. in diameter, had a 1.00-in. gage, and were tested at a loading rate of 0.02 in. per min.

Whitewashing the specimens provided a simple method of indicating where yield patterns (shear lines or Lüder's bands) were formed in the specimens. The whitewash spalled off the surfaces of the specimens with the mill scale as yielding took place.

DESCRIPTION OF TESTS

The specimens were tested in a 3,000,000-lb testing machine in Talbot Laboratory at the University of Illinois. The manner of installation and testing

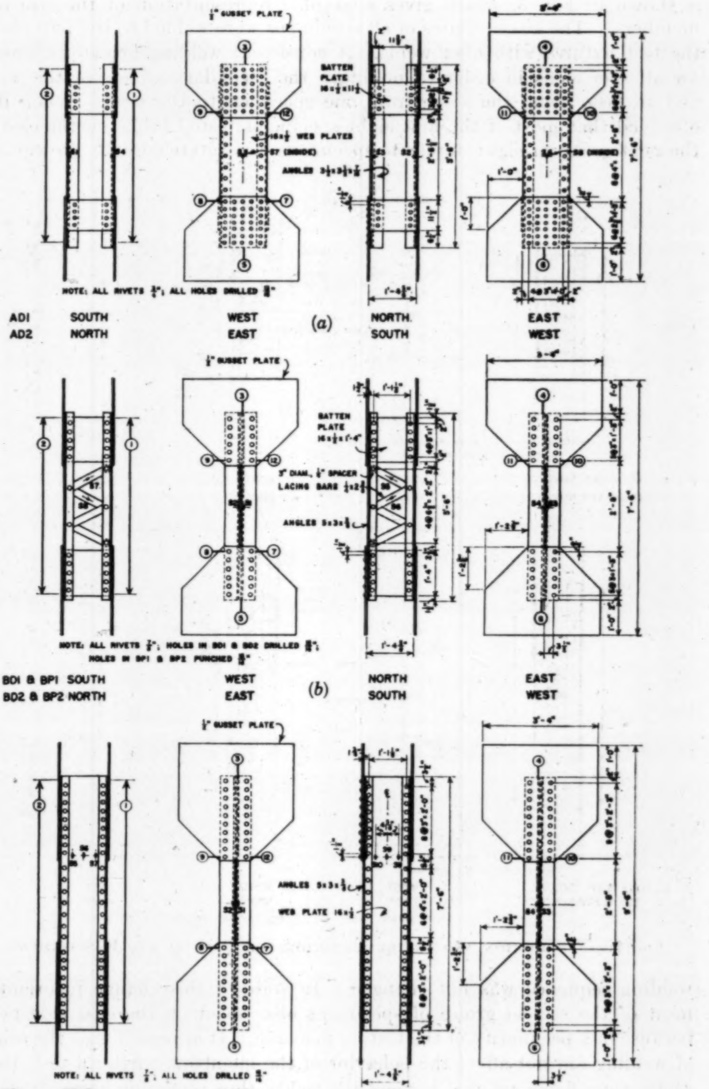

1/2" GUSSET PLATE
BATTEN PLATE 16 x 1/2 x 11 1/2
16 x 1/2 WEB PLATES
S7 (INSIDE)
ANGLES 3 1/2 x 3 1/2 x 7/16
S8 (INSIDE)
3'-4"
7'-6"
1'-4 1/2"
NOTE: ALL RIVETS 3/4"; ALL HOLES DRILLED 13/16"
AD1 SOUTH WEST NORTH EAST
AD2 NORTH EAST SOUTH WEST
(a)
1/2" GUSSET PLATE
BATTEN PLATE 16 x 1/2 x 1'-4"
3" DIAM., 1/2" SPACER
LACING BARS 1/2 x 2 1/2
ANGLES 5 x 3 x 1/2
3'-4"
7'-6"
1'-4 1/2"
NOTE: ALL RIVETS 7/8"; HOLES IN BD1 & BD2 DRILLED 15/16"; HOLES IN BP1 & BP2 PUNCHED 15/16"
BD1 & BP1 SOUTH WEST NORTH EAST
BD2 & BP2 NORTH EAST SOUTH WEST
(b)
1/2" GUSSET PLATE
ANGLES 5 x 3 x 1/2
WEB PLATE 16 x 1/2
3'-4"
9'-6"
7'-6"
1'-4 1/2"
NOTE: ALL RIVETS 7/8", ALL HOLES DRILLED 15/16"
CD1 SOUTH WEST NORTH EAST
CD2 NORTH EAST SOUTH WEST
(c)
• TYPE A-1 SR-4 STRAIN GAGES
⑩ SLIP AND DEFORMATION DIALS

FIG. 1.—FABRICATION AND INSTRUMENTATION FOR TYPE A, B, AND C SPECIMENS

is shown in Fig. 3, which gives a graphic representation of the size of the members. The gusset plates of all specimens were welded to the pull plates of the test fixtures with fillet welds. A consistent welding procedure was used for all specimens in order to minimize the secondary effects of the welding and to keep them the same from one specimen to the next. When it was observed that most of the first eight specimens tested broke on the east side, the next group of eight duplicate specimens was rotated 180°. However, the welding sequence was not changed. In spite of this change in orientation, most of the second group of specimens also failed on the east side possibly because of a peculiarity of the testing machine. It appeared that the sequence of welding did not affect the behavior of the members. After a test, the pull plates were flame-cut just beyond the welds, thus removing approximately 1½ in. from the pull plates for each specimen tested.

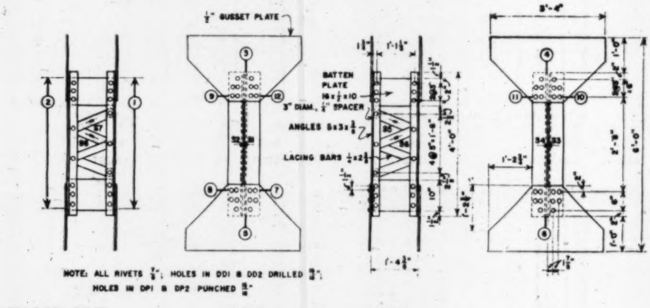

½" GUSSET PLATE
3'-4"
BATTEN PLATE 16 x ½ x 10
3" DIAM., ½" SPACER
ANGLES 5 x 3 x ½
LACING BARS ½ x 2½
1'-4⅜"
NOTE: ALL RIVETS ⅞"; HOLES IN DD1 & DD2 DRILLED 15/16"; HOLES IN DP1 & DP2 PUNCHED 15/16"
DD1 & DP1 SOUTH
DD2 & DP2 NORTH
WEST
EAST
(a)
NORTH
SOUTH
EAST
WEST
½" GUSSET PLATE
3'-4"
BATTEN PLATE 16 x ½ x 11½
3" DIAM., ½" SPACER
LACING BARS ½ x 2½
ANGLES 5 x 3 x ½
1'-4⅜"
NOTE: ALL RIVETS ¾"; HOLES IN ED1 & ED2 DRILLED 13/16"; HOLES IN EP1 & EP2 PUNCHED 13/16"
ED1 & EP1 SOUTH
ED2 & EP2 NORTH
WEST
EAST
(b)
NORTH
SOUTH
EAST
WEST
TYPE A-1 SR-4 STRAIN GAGES
SLIP AND DEFORMATION DIALS

Fig. 2.—Fabrication and Instrumentation for Type D and E Specimens

The specimens were tested to ultimate load in approximately fifteen load increments. The strain gages and deformation dials were read immediately

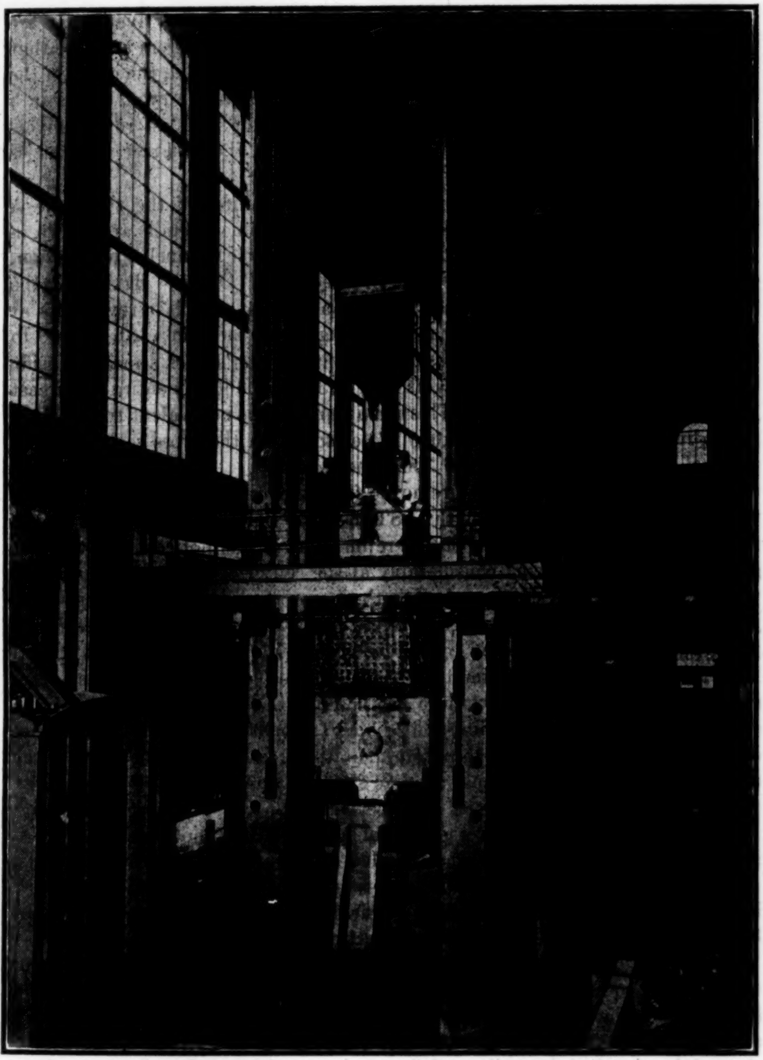


FIG. 3.—SPECIMEN ED2 IN 3,000,000-LB. TESTING MACHINE

after each increment had been applied but while the load was maintained constant.

The phrase "a row of rivets" used herein signifies those rivets that are perpendicular to the axis of loading, and "a line of rivets" refers to those that are parallel to the axis of loading. Unless otherwise specified, the order of the rivet rows refers to the member itself. That is, the first row of rivets in a joint is the one at the net section of the member, or the first row of rivets nearest the mid-length of the specimen. Similarly, the last row of rivets is that farthest from the mid-length of the specimen.

Type A Specimens.—Because of the unusual failures of the two type A specimens, their tests are described fully. The specimen details are shown in Fig. 1 (*a*). The cross section can be described as a double channel or box section.

Specimen AD1.—During the test of specimen AD1, the flaking of the whitewash of the lower east gusset around and below the last row rivets at 400,000 lb indicated the beginning of yielding in the member. At 700,000 lb (25,500 lb per sq in. on the gross area), the first Lüder's lines appeared inside the east web at the first row of rivets, and at 1,155,000 lb the entire lower east joint failed suddenly in shear. This maximum load is equivalent to 57,100 lb per sq in. on the net section, 42,000 lb per sq in. on the gross section, and a nominal average rivet shear of 37,300 lb per sq in.

The sheared rivets were then removed from the lower joint, and the connection was bolted with high-strength (ASTM A325)[9] bolts at a torque of approximately 370 ft-lb. When the bolts were installed it was noticed that the east gusset had "necked down" considerably at the center of the last row of holes.

A second test was conducted with the lower joints bolted and the upper joints still fastened with the original rivets. Failure began with the tearing of the lower east gusset followed by a similar rupture of the lower west gusset. Both tears then propagated simultaneously until the east gusset tore to an edge, as shown in Fig. 4. Prior to failure five bolts on the west side and two on the east side sheared. The maximum load was 1,235,000 lb (61,000 lb per sq in. on the net section and 44,900 lb per sq in. on the gross area). The rivets in the upper joint withstood a nominal average shearing stress of 39,900 lb per sq in. without failing.

Specimen AD2.—Both lower gussets of specimen AD2 showed signs of yielding at 400,000 lb as the whitewash spalled off around the last rows of rivets. Under a load of 500,000 lb (18,200 lb per sq in. on the gross section), Lüder's bands developed on the east web at the first-row rivets, and, 100,000 lb later, both web plates had yielded at the net section.

At a maximum load of 1,190,000 lb (58,800 lb per sq in. on the net area, 43,300 lb per sq in. on the gross section, and a nominal average shear of 38,500 lb per sq in.), the outer lines of rivets in the lower east gusset sheared suddenly and the load dropped to 400,000 lb. Because the gusset was still attached to the web by the three inner lines of rivets, the member continued to carry load until at 940,000 lb the east web tore at the first row of rivets. This fracture is shown in Fig. 5.

---

[9] "Tentative Specifications for Quenched and Tempered Steel Bolts and Studs with Suitable Nuts and Plain Washers," *ASTM A325-53T*, A.S.T.M., 1953.

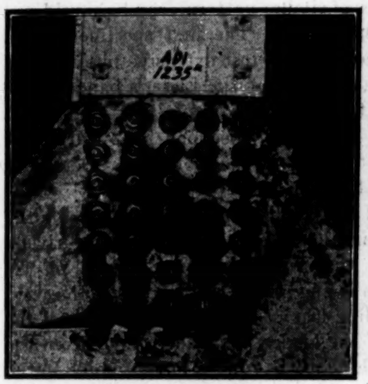


FIG. 4.—SPECIMEN AD1, EAST GUSSET AFTER RUPTURE

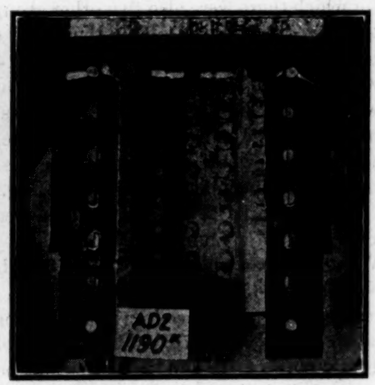


FIG. 5.—SPECIMEN AD2, BOTTOM EAST JOINT AFTER FAILURE

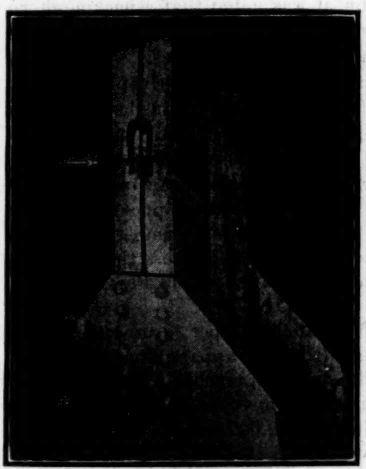


FIG. 6.—SPECIMEN BD1, LOCATION OF RUPTURE

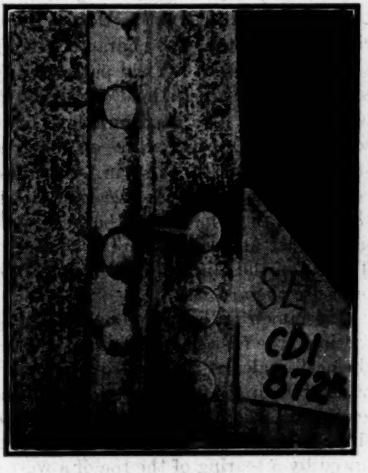


FIG. 7.—SPECIMEN CD1, NET-SECTION FAILURE IN SOUTHEAST ANGLE

Type B Specimens.—The type B specimens, being of a laced-I configuration (of which there were also two other groups of similar pattern), can be considered to be representative of all the laced specimens. The first specimen to be tested in the entire program was BD2, and in this test extensive photographs were taken of the progress of yielding as depicted by the Lüder's bands in the whitewash. Because such yielding was typical of specimens B, D, and E, and because of the more thorough coverage, the test of specimen BD2 will be examined somewhat more fully than the tests of similar members. Details of the type B specimens are shown in Fig. 1(*b*).

Specimen BD2.—As the load reached 350,000 lb (30,600 lb per sq in. on the gross section), Lüder's lines appeared in the angles at the first-row rivets. When the load was raised to 390,000 lb, the whitewash began to spall off the outstanding (5-in.) legs of the angles at a lacing rivet. At 410,000 lb the yield bands were pronounced on the outstanding legs of the angles opposite all the lacing rivets. At 475,000 lb the heel of the angles had pulled away from the gusset plates from ⅛ in. to ¼ in. This behavior was typical of that noted in the tests of all the laced-I and solid-I specimens. At 500,000 lb (58,000 lb per sq in. on the net section, 43,700 lb per sq in. on the gross section, and a nominal average shear stress of 34,700 lb per sq in.), the specimen failed on the east side at the top lacing rivet. The toes of the inside legs of the west angles also ruptured at the center lacing rivet, producing a secondary failure. This failure was at a point of high localized stress produced by the lacing bars, which bent the angles or "pinched them in." In addition, it was noted that a necking-down had occurred at the net section of the connection as well as at the other lacing rivets. This, too, was characteristic of all the laced specimens.

Specimen BD1.—Yield patterns appeared at the first-row rivets of the east angles of BD1 when a load of 250,000 lb (21,900 lb per sq in. on the gross area) was reached. At a maximum load of 498,000 lb (57,800 lb per sq in. on the net section, 43,500 lb per sq in. on the gross section, and a nominal average shear stress of 34,500 lb per sq in.), the toes of the east angle began to tear at the center lacing rivet. As the load slowly dropped, fractures appeared through the toes of the 3-in. legs of the west angles also. At the same time, the fractures in the east angles had spread through the 3-in. legs and across the 5-in. legs. Typical of the failures of the type B members is that shown in Fig. 6 for BD1 opposite the arrow.

Specimen BP1.—At a load of 300,000 lb (26,200 lb per sq in. on the gross section), the first Lüder's bands on specimen BP1 were apparent on the inside of the east angles at the first-row rivets. The specimen reached a maximum load of 462,000 lb (53,600 lb per sq in. on the net section, 40,400 lb per sq in. on the gross area, and a nominal average shear stress of 32,000 lb per sq in.), at which time failure occurred at the toe of an east angle at the center lacing rivet, followed by a tearing of the toe of a west angle at the top lacing rivet. Although tears were apparent in all the angles, final fracture occurred in the west angles at the top lacing rivet. Even though the east side of this specimen was more highly strained initially and failure was initiated on the east side, the principal failure occurred on the west side of the specimen.

Specimen BP2.—The west angles of specimen BP2 began to yield at the first-row rivets at approximately 325,000 lb (28,400 lb per sq in. on the gross section). At a load of 410,000 lb, the east angles exhibited Lüder's lines originating at the lacing rivets, and at 425,000 lb the west angles indicated similar yielding. When a maximum load of 458,000 lb (53,100 lb per sq in. on the net section, 40,000 lb per sq in. on the gross section, and a nominal average shear stress of 31,700 lb per sq in.) was reached, the east angles failed. Although of the same general character and at approximately the same ultimate load as the other type B specimens, the failure was unusual because the southeast angle ruptured at the lower lacing rivet and the northeast angle ruptured at the upper lacing rivet. At the center lacing rivet in the west angles, the toes ruptured in a secondary failure.

Type C Specimens.—The type C specimens were of the solid-I type and are illustrated in Fig. 1 (*c*). Both of them failed at the net sections in a similar fashion.

Specimen CD1.—The lower east gusset began to yield at the last-row rivets at a load of 300,000 lb. However, it was not until the load had reached 550,000 lb (28,300 lb per sq in. on the gross section) that the east angles indicated yielding at the net section. At approximately 600,000 lb, the east edge of the web developed Lüder's lines at the first-row rivets, and at 650,000 lb the web and inner legs of the east angles indicated yield bands in the whitewash at the stitch rivets. The specimen failed at a maximum load of 872,000 lb (55,800 lb per sq in. on the net section, 44,900 lb per sq in. on the gross area, and a nominal average shear stress of 36,300 lb per sq in.). Final failure occurred at the net section at the lower east side, as indicated in Fig. 7. It was interesting to note that necking-down occurred at the angles at each stitch rivet (arrow in Fig. 7). In addition, a similar yielding was noted in the web at each of the stitch rivets. The failure of CD1 was typical of the net-section failures observed in the other specimens (Table 5, Col. 5).

Specimen CD2.—At a load of 300,000 lb, Lüder's lines developed on the lower gusset plates at the last row of rivets, and by 500,000 lb (25,700 lb per sq in. on the gross section) the outstanding legs of the angles had yielded at the first three rows of rivets. At 650,000 lb the first signs of the spalling of whitewash on the web were noticed. The maximum load of 902,000 lb (57,700 lb per sq in. on the net section, 46,400 lb per sq in. on the gross section, and a nominal average shear stress of 37,500 lb per sq in.) produced the primary failure at the top west net section. A secondary failure occurred at the toes of the east angles at the lower net section.

Type D Specimens.—The type D specimens were of the laced-I design shown in Fig. 2 (*a*). The angles of these members were of the same size as those of the type B specimens. However, the section at the lacing rivets tended to be less important as points of stress concentration because the type D specimens were prepared with a smaller net area.

Specimen DD1.—At a load of 250,000 lb on specimen DD1 (21,900 lb per sq in. on the gross section), Lüder's lines appeared at the first-row rivets of the southeast angle. When the load reached 410,000 lb, yield bands became evident on the angles at the lacing rivets. The maximum load for failure was 450,000 lb (62,500 lb per sq in. on the net section, 39,300 lb per sq in. on the gross section, and a nominal average shear stress of 37,400 lb per sq in.), with the primary failure occurring at the top west side and a secondary (or partial) failure occurring in the net section at the lower east side of the member.

At the point of primary failure, an unusual break occurred. The rupture of the southwest angle passed through the two rivet holes in the outstanding leg and then to the second rivet at the batten plate, but it did not tear completely through the angle. Instead, the toe of the inner leg of the angle tore at the first rivet. However, the northwest angle tore through the two rivets in the outstanding leg and through the first batten rivet.

Specimen DD2.—The angles showed first Lüder's lines or yield bands at the toes near the first-row rivets when the load reached approximately 300,000 lb (26,200 lb per sq in. on the gross area). When the load was raised to 400,000 lb, the east angles developed Lüder's bands at the lacing rivets. A maximum load of 444,000 lb (61,700 lb per sq in. on the net section, 38,800 lb per sq in. on the gross area, and a nominal average shear stress of 36,900 lb per sq in.) was reached. At this load the east angles failed through the lower net section.

Specimen DP1.—The lower east gusset began showing Lüder's bands or yield lines in the whitewash at 275,000 lb. When the load was raised to 300,000 lb (26,200 lb per sq in. on the gross section), the outstanding legs of the east angles developed yield patterns at the net section. The maximum load was 439,000 lb (61,000 lb per sq in. on the net section, 38,400 lb per sq in. on the gross section, and a nominal average shear stress of 36,500 lb per sq in.), and rupture occurred at the lower east joint.

Specimen DP2.—By 300,000 lb (26,200 lb per sq in. on the gross section) plastic flow of the angles beneath the first-row rivets was apparent in specimen DP2. Lüder's lines developed in the angles at the lacing rivets at 400,000 lb. The maximum load reached was 449,000 lb (62,400 lb per sq in. on the net section, 39,200 lb per sq in. on the gross section, and a nominal average shear stress of 37,300 lb per sq in.) when the top west angles ruptured at the toe. As the load dropped, the lower east joint began to tear and there was final failure through the net section at the lower east side. The secondary (or partial) failure actually occurred first.

Type E Specimens.—The type E specimens were also of the laced-I design but had 5-in.-by-5-in.-by-$\frac{3}{8}$-in. angles. The details of this type are shown in Fig. 2(*b*). Because of the marked differences in behavior between the drilled and punched specimens, and because of the repeated tests that were necessary on the drilled specimens before final failure, the tests of this group of specimens will be described in somewhat greater detail.

Specimen ED1.—Three tests were conducted on specimen ED1. In the first test when the load had reached 250,000 lb, Lüder's bands appeared on the lower east gusset at the last row of rivets. By 500,000 lb (24,600 lb per sq in. on the gross section), yielding was evident at the first-row rivets of the east angles. The maximum load was 722,000 lb (60,500 lb per sq in. on the net section and 50,000 lb per sq in. on the gross section), at which point the load began to drop. At about 690,000 lb, the specimen suddenly failed by shearing all rivets of the lower east gusset. The nominal average maximum shear on the rivets had been 40,860 lb per sq in.

The lower west rivet heads were cut off, and the rivets were driven out so that both lower gussets could be reconnected using high-strength (ASTM A325) bolts installed by a torque wrench at 370 ft-lb or more. With the lower joint bolted in this fashion, the second test was run on specimen ED1. At about

625,000 lb both first-row bolts on the east side sheared off. When the load reached 762,000 lb (approximately 63,800 lb per sq in. on the net section and 52,800 lb per sq in. on the gross section), the upper west gusset suddenly sheared all rivets, which occurred at a nominal average shearing stress of 43,100 lb per sq in. on the rivets.

Again the heads of the remaining rivets were cut off, and the rivets were replaced with high-strength bolts. The two bolts that sheared in the lower east gusset during the second test were not replaced because in order to do so considerable reaming would have been necessary. With both joints bolted, a third test was run. The maximum load was 811,000 lb (67,900 lb per sq in. on the net section, 56,200 lb per sq in. on the gross section, and a nominal average shear stress of 45,900 lb per sq in.), and the tension failure occurred on the lower east side at the second-row-rivet holes (first bolts) in the outstanding legs and through the first batten rivet.

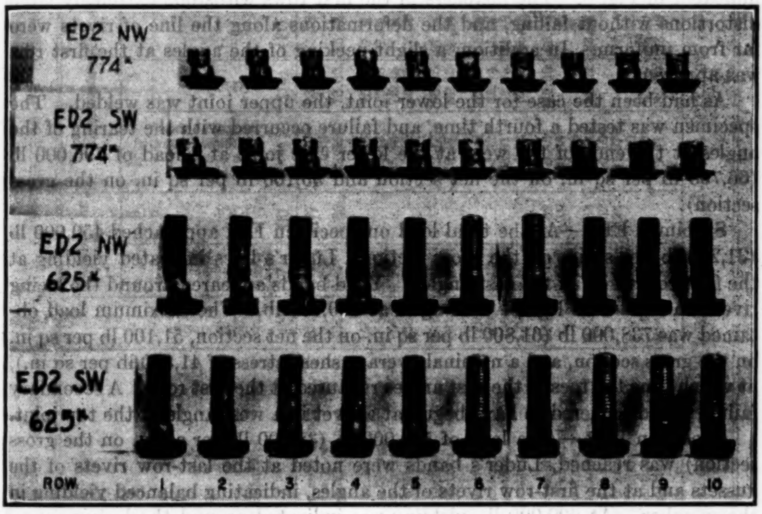


Fig. 8.—Fasteners Removed from the Unfractured Joints of ED2

Specimen ED2.—Four tests were run on specimen ED2. At 500,000 lb (34,600 lb per sq in. on the gross section) during the first test, faint Lüder's lines were noticed at the first-row rivets on the angles. By 600,000 lb, yield bands had appeared at the lacing rivets. The load was increased in stages to 700,000 lb (58,600 lb per sq in. on the net section and 48,500 lb per sq in. on the gross section), at which point the usual readings were taken. After the load was dropped to 600,000 lb to permit safe removal of the gages, the specimen failed in shear, as it was reloaded, at 698,000 lb (58,500 lb per sq in. on the net section and 48,300 lb per sq in. on the gross section). At 700,000 lb the average nominal shear stress on the rivets had been 39,600 lb per sq in.

For the second test the rivets in the lower gussets were replaced with common bolts, which were torqued to relatively high tensions. The bolts had an average ultimate strength of 66,230 lb per sq in. on the mean root area. The two first-row bolts in the east angles sheared at 525,000 lb and at 618,000 lb. When the load reached 625,000 lb, the second-row bolts in the east angles sheared, immediately after which all the bolts in the bottom east joint also sheared.

For the third test the lower joint was welded with full-length fillet welds along the toes and across the ends of the angles. No weld was placed across the edge of the gusset near the first-row holes. The "fitting-up" bolts were left in place. As the load reached 774,000 lb (64,800 lb per sq in. on the net section and 53,600 lb per sq in. on the gross section), the rivets sheared at the top east gusset at a nominal average shear stress of 43,800 lb per sq in. The relative shear deformation in the rivets and bolts along the length of this member is indicated in Fig. 8. The fasteners in the first rows withstood extremely large distortions without failing, and the deformations along the line of rivets were far from uniform. In addition, a slight necking of the angles at the first row was apparent.

As had been the case for the lower joint, the upper joint was welded. The specimen was tested a fourth time, and failure occurred with the tearing of the angles at the ends of the weld at the lower east joint at a load of 796,000 lb (66,700 lb per sq in. on the net section and 55,100 lb per sq in. on the gross section).

Specimen EP1.—As the total load on specimen EP1 approached 450,000 lb (31,200 lb per sq in. on the gross section), Lüder's lines indicated yielding at the first-row rivets of the east angles. Yield bands appeared around the lacing rivets on the inside legs of the angles at 500,000 lb. The maximum load obtained was 738,000 lb (61,800 lb per sq in. on the net section, 51,100 lb per sq in. on the gross section, and a nominal average shear stress of 41,800 lb per sq in.), at which time the toes of the east angles ruptured at the first row. A secondary failure was discovered to have begun at a rivet in a west angle at the top joint.

Specimen EP2.—As a load of 550,000 lb (38,100 lb per sq in. on the gross section) was reached, Lüder's bands were noted at the last-row rivets of the gussets and at the first-row rivets of the angles, indicating balanced yielding in the member. At 600,000 lb, yielding was indicated around the lacing rivets. The maximum load reached was 733,000 lb (61,400 lb per sq in. on the net section, 50,800 lb per sq in. on the gross section, and a nominal average shear stress of 41,500 lb per sq in.), and failure occurred through the net section at the lower east gusset.

Results and Analysis of Tests

Table 1 shows the areas and properties of the various specimens, and Table 5 indicates the ultimate loads, type and location of the failures, and specimen efficiencies.

Because stresses are used more generally than strains, the strain data obtained in these tests are presented in terms of stress. Such an analysis must be limited necessarily to the range of loads for which stress is proportional to strain. This method of presenting strains in terms of a stress level should give a clearer understanding of the behavior of the members. However, it should be remembered that a stress obtained in this fashion represents only the stress

TABLE 5.—Ultimate Loads and Efficiencies of Specimens

						Factors of Safety		Efficiencies, in Percentage					Ratio, Predicted to Test Efficiency			
Specimen (1)	Hole preparation (2)	Rivet size, in inches (3)	Ultimate load, in kips (4)	Mode of failure (5)	AREA design load, in kips[a] (6)	AREA design, Col. 4 ÷ Col. 6 (7)	Shear,[b] AREA (8)	Test, based on average ultimate strength, central coupon (9)	AREA (10)	Modified AREA (11)	Relative gage, based on average reduction, central coupon (12)	Eq. 5 (13)	AREA (14)	Modified AREA (15)	Relative gage (16)	Eq. 5 (17)
AD1	Drilled	3/4	1,155	Rivets sheared in east gusset	364.1	3.17[c]	2.77	64.89[c]	73.62	75.58	79.92	71.06	1.135	1.165	1.229	1.095
	Bolted	3/4	1,235	Lower gussets tore		3.39		69.38					1.061	1.089	1.150	1.024
AD2	Drilled	3/4	1,190	Rivets sheared, east web tore	364.1	3.27[c]	2.85	69.85[c]	73.62	75.58	79.84	71.06	1.054	1.082	1.142	1.017
BD1	Drilled	7/8	498	Angles at east center lacing rivet	155.2	3.21		71.85	75.35	77.01	80.45	72.12	1.049	1.072	1.113	1.004
BD2	Drilled	7/8	500	Angles at east top lacing rivet	155.2	3.22		72.52	75.35	77.01	79.80	72.12	1.039	1.062	1.103	0.994
BP1	Punched	7/8	462	Angles at west top lacing rivet	155.2	2.98		65.80	75.35	77.01	68.72	72.12	1.145	1.170	1.045	1.096
BP2	Punched	7/8	458	Angles at east top and bottom lacing rivet	155.2	2.95		65.52	75.35	77.01	69.07	72.12	1.150	1.175	1.050	1.101
CD1	Drilled	7/8	872	East net section	281.1	3.10		72.37	80.35	81.64	81.30	75.22	1.110	1.128	1.123	1.039
CD2	Drilled	7/8	902	West net section	281.1	3.21		75.17	80.35	81.64	81.30	75.22	1.069	1.086	1.081	1.001
DD1	Drilled	7/8	450	West net section	129.6	3.47		63.88	62.94	65.38	68.93	64.42	0.985	1.023	1.074	1.008
DD2	Drilled	7/8	444	East net section	129.6	3.43		64.52	62.94	65.38	68.79	64.42	0.976	1.013	1.063	0.998
DP1	Punched	7/8	439	East net section	129.6	3.39		63.61	62.94	65.38	59.41	64.42	0.989	1.028	0.931	1.013
DP2	Punched	7/8	449	East net section	129.6	3.46		64.42	62.94	65.38	59.11	64.42	0.977	1.015	0.919	1.000
ED1	Drilled	3/4	722	Bottom east rivets sheared	214.9	3.36[c]	3.02	76.35[c]	82.69	83.93	85.00	76.69	1.083	1.099	1.111	1.004
	Bolted	3/4	762	Top west rivets sheared		3.55[c]	3.19	80.58[c]					1.026	1.042	1.053	0.952
	Bolted	3/4	811	East net section		3.77		85.76					0.964	0.979	0.990	0.894
ED2	Drilled	3/4	700	Bottom east rivets sheared	214.9	3.26[c]	2.93	74.14[c]	82.69	83.93	84.91	76.69	1.115	1.132	1.145	1.034
	Bolted	3/4	625	Bottom east bolts sheared		2.91[c,d]	2.62	66.19[c]					1.249	1.268	1.282	1.159
	Welded	...	774	Top west rivets sheared		3.60[c]	3.24	81.97[c]					1.009	1.024	1.035	0.936
	Welded	...	796	Angles on east side above welds		3.70		84.30					0.981	0.995	1.007	0.910
EP1	Punched	3/4	738	East net section	214.9	3.43		77.35	82.69	83.93	73.27	76.69	1.069	1.085	0.946	0.991
EP2	Punched	3/4	733	East net section	214.9	3.41		77.51	82.69	83.93	73.27	76.69	1.067	1.083	0.944	0.989

[a] Based on 18,000 lb per sq in. in tension. [b] Factor of safety in shear = (ultimate load) ÷ (rivet area × 13,500 lb per sq in.). [c] Fasteners sheared. [d] Common bolts used.

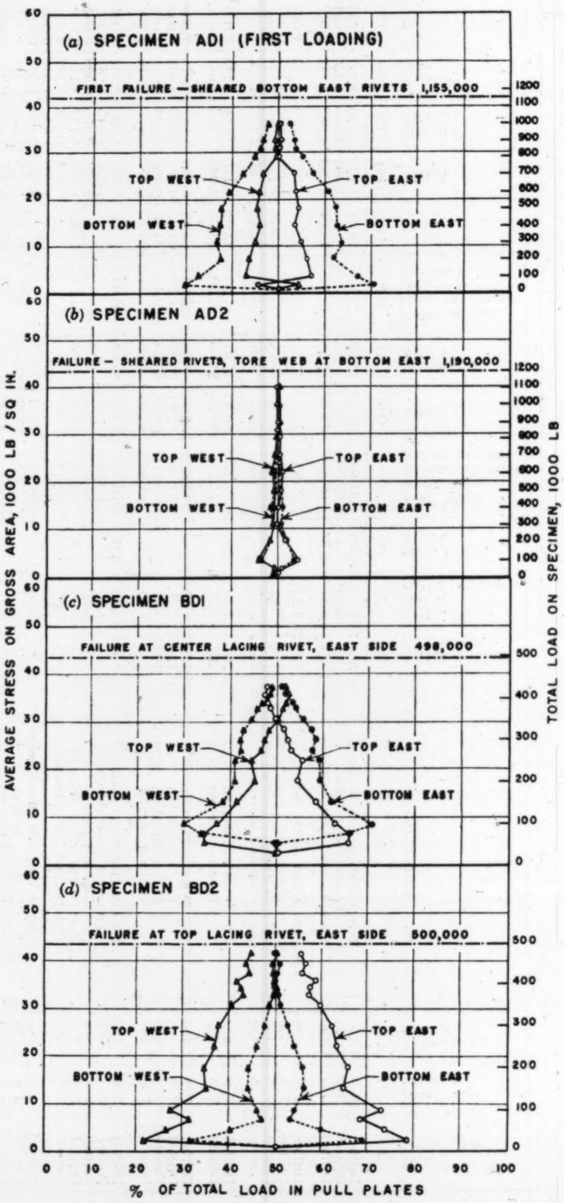


FIG. 9.—LOAD DISTRIBUTION TO PULL PLATES FOR TYPE A AND TYPE B DRILLED SPECIMENS

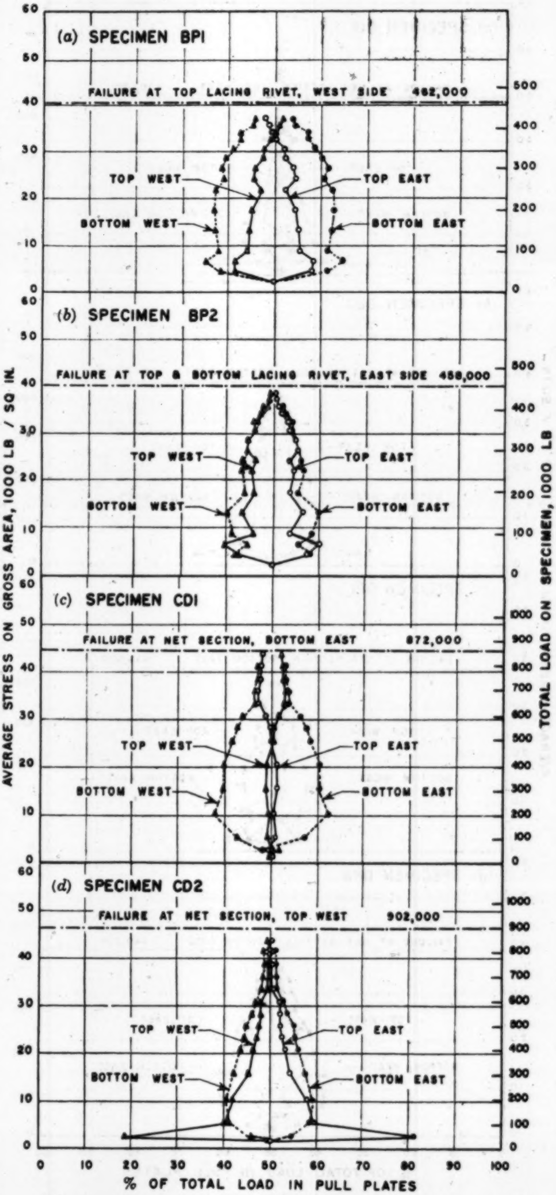


FIG. 10.—LOAD DISTRIBUTION TO PULL PLATES FOR TYPE B PUNCHED AND TYPE C SPECIMENS

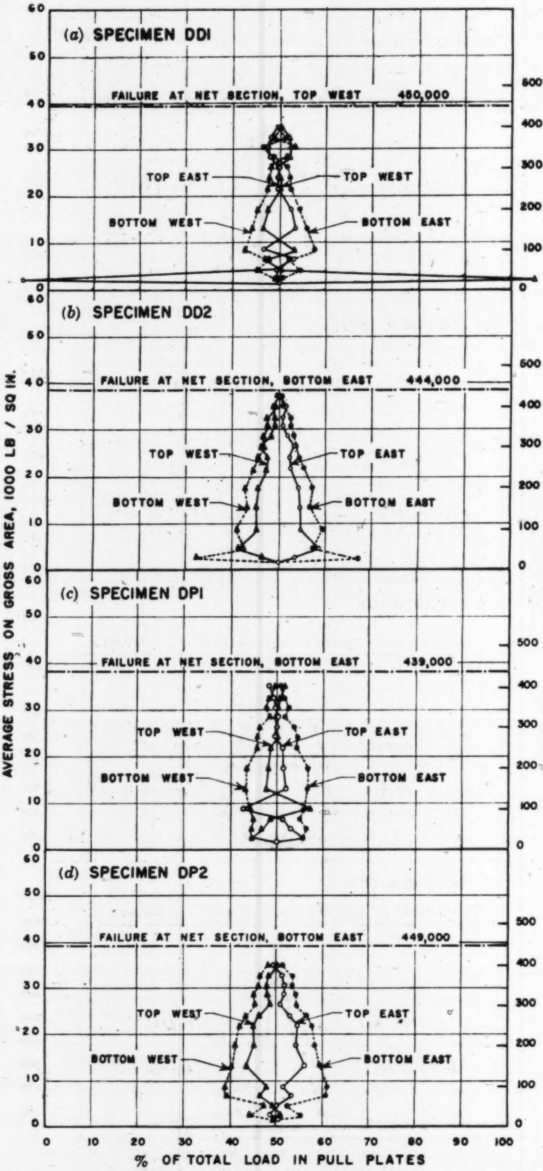


FIG. 11.—LOAD DISTRIBUTION TO PULL PLATES FOR TYPE D SPECIMENS

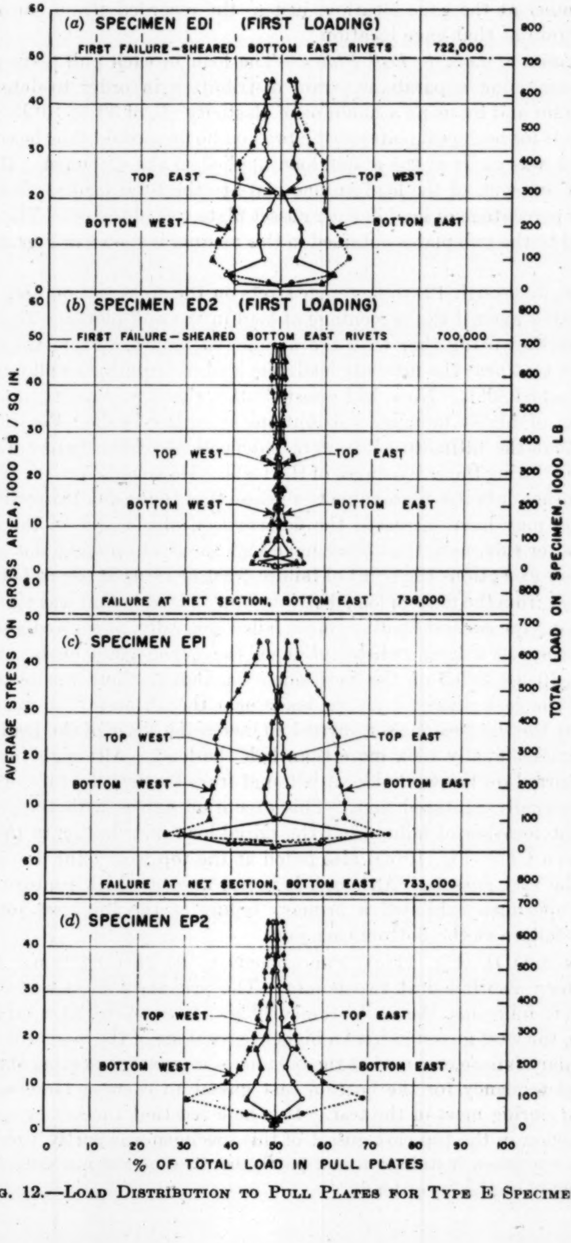


FIG. 12.—LOAD DISTRIBUTION TO PULL PLATES FOR TYPE E SPECIMENS

in the member at the gage location, just as the recorded strain can represent only the strain at that gage location.

Distribution of Load to Pull Plates.—The load in each pull plate was computed by assuming a parabolic strain distribution in order to determine an average strain and by using a modulus of elasticity, E, of 30×10^6 lb per sq in.

The loads for both pull plates at the top and bottom could thus be computed, and a check was made of the actual load applied to the specimen. By a comparison, at one end, of the load on one plate to the total load carried by both plates, the percentage of load in each gusset plate was obtained. The distribution of load to the pull plates obtained in this manner is shown in Figs. 9 through 12.

In Figs. 9 through 12, the average stress on the gross section and the total load is plotted against the percentage of load in the pull plates. The ultimate load is also shown together with the manner of failure. From these plots it can be seen that near the ultimate loads the load distribution to all four gussets had approached 50%. None had a distribution that was more than 5% different in spite of highly unequal distributions at earlier loads. For this reason it is felt that the ultimate loads were independent of the variations in load distribution during the early stages of the tests. However, from studying these plots and other data it is reasonable to surmise that the earlier inequality of load distribution may have increased the deformation of one side of the specimen over the other side, and, thus, may have had an effect on the point of failure.

With few exceptions the point of failure (east or west, or top or bottom) can be predicted from the plots of load distribution. The side that was more heavily loaded during the earliest loading stages failed generally, or showed a secondary failure in spite of a later redistribution of the gusset-plate loads. Specimen BP1 (Fig. 10(*a*)) failed on the west side even though there was heavy initial loading of the east gussets. At the loads near the ultimate, the west gussets, particularly the top west gusset, carried an increasing share of the load, and the top west gusset finally took more than half the load. Although the primary failure occurred on the west side, it is found from the test description that the east side actually ruptured first. This exception agrees with the correlation between the location of failure and the initial load distribution in the gussets.

Specimen CD2 (Fig. 10(*d*)) also failed at the top west joint despite higher loads in the east gussets. Although it is not known which rupture occurred first, this specimen exhibited a primary failure at the top west joint and a secondary failure at the bottom east joint.

Specimen DD1 (Fig. 11(*a*)) also appears to be an exception. However, this specimen also failed at two points. The primary failure was at the top west gusset, following the indication of Fig. 11(*a*) where, at approximately 350,000 lb, the west gusset carried a higher percentage of the load near ultimate. The secondary failure occurred at the bottom east gusset, reflecting the effect of the general tendency for the bottom east gusset to carry a large percentage of the load during most of the test. It is believed that the initial appearance of compression in the top east gusset of this specimen was partly a result of the manner of specimen installation and partly because of the insensitivity of the measurements at the lower loads.

Load-Strain Relationships.—That the word "stress" means that which is at the gage point and not an average stress is important in order to understand the subsequent material. The type A specimens will be considered first; the type C specimens, second; and the laced members, types B, D, and E, will be analyzed last.

Type A Specimens.—The stresses in the angles of specimen AD1 at the four strain-gage locations are summarized in Table 6, in which it may be seen that both the east angles have higher stresses than the west angles. This is as should be expected from the load distribution shown in Fig. 9(*a*), in which, to as much as 600,000 lb, the east pull plates carried about 60% of the load, or approximately one and one-half times that in the west plates. This same relative distribution is evident in the angle stresses.

TABLE 6.—ANGLE STRESSES IN SPECIMEN AD1

Load, in kips	Average stress on gross area, in kips per sq in.	STRESS, IN KIPS PER SQ IN., IN THE ANGLES BASED ON MEASURED STRAINS				Ratio, east to west	Average measured stress, in kips per sq in.
		Gage S1 SW	Gage S2 NW	Gage S3 NE	Gage S4 SE		
50	1.82	1.5	1.5	2.1	2.7	1.60	1.95
100	3.64	2.5	3.0	4.5	5.1	1.75	3.77
200	7.28	5.7	6.0	8.1	9.6	1.51	7.10
300	10.92	8.7	9.3	12.6	14.1	1.48	11.17
400	14.56	12.0	12.6	16.8	18.6	1.44	15.00
500	18.18	15.3	16.5	21.3	23.4	1.41	19.12
					Average	1.53	

Because the four angles were not equally strained, the batten plates and webs connecting them must have developed shears and thus introduced additional secondary stresses in the specimen. If the difference in total load between the east (60%) and the west (40%) sides of the specimen were assumed to be taken by the batten plates only, at 500,000 lb the 100,000-lb total shear would be distributed to the four batten plates, thus giving a computed unit shear of less than 4,500 lb per sq in. However, the load distribution to the two sides of the specimen became more nearly equal when the ultimate load was approached, so that the shear on the battens did not increase proportionately.

TABLE 7.—ANGLE STRESSES IN SPECIMEN AD2

Load, in kips	Average stress on gross area, in kips per sq in.	STRESS, IN KIPS PER SQ IN., IN THE ANGLES BASED ON MEASURED STRAINS				Ratio, east to west	Average measured stress, in kips per sq in.
		Gage S1 NE	Gage S2 SE	Gage S3 SW	Gage S4 NW		
50	1.82	2.1	2.1	1.8	1.8	1.17	1.95
100	3.64	4.5	4.0	3.6	3.6	1.18	3.92
200	7.28	8.7	7.5	7.5	7.5	1.08	7.80
300	10.92	13.2	11.1	11.7	11.4	1.05	11.85
400	14.56	17.4	15.0	15.6	15.3	1.08	15.82
500	18.18	22.2	18.9	19.5	19.8	1.05	20.10

Similarly, inspecting the differences in strain and the relative areas involved indicates that the unit shearing stresses in the webs would probably be from one-fifth to one-tenth of that in the batten plates and would not be significant.

The strain data for the angles of specimen AD2 shown in Table 7 reveal that the northeast angle was the most highly stressed, whereas the others appear to be about equally stressed (or strained). Fig. 9(*b*) shows that the loads in all pull plates of AD2 were approximately equal. As a result, the shears in the battens and web plates caused by the unequal distribution of strains in the angles of this specimen will be even less important than those in specimen AD1.

An inspection of the strains in the web plates of both type A specimens showed that little bending occurred in these plates at mid-length. Because the strains on both sides of the web plates were similar at a given load, strains at the center of each web plate have been averaged and converted to a stress at that point; these web stresses are given in Table 8.

TABLE 8.—WEB STRESSES AT CENTER OF WEB BASED ON AVERAGE MEASURED STRAINS

Load, in kips	Average stress on gross area, in kips per sq in.	Average stress on net section at center, in kips per sq in.	STRESS IN AD1, IN KIPS PER SQ IN.		STRESS IN AD2, IN KIPS PER SQ IN.	
			East, $\frac{S6+S8}{2}$	West, $\frac{S5+S7}{2}$	East, $\frac{S5+S7}{2}$	West, $\frac{S6+S8}{2}$
50	1.82	2.04	3.6	1.6	3.3	2.3
100	3.64	4.09	6.5	3.5	5.7	4.0
200	7.28	8.18	12.2	7.2	10.8	8.4
300	10.92	12.27	18.2	11.3	15.6	13.5
400	14.56	16.36	24.0	15.9	20.4	18.0
500	18.18	20.45	29.7	20.5	25.4	22.8

Table 8 seems to show that the webs of these box members tended to be more than 100% efficient—that is, to carry a higher stress (or share of the total load) than would be expected from the average stress on the net section of the specimen at that gage location. The uneven distribution of stress to the webs and angles causes this to appear thus. However, the web efficiency of these box members would be greater than that of I-beam webs[10] because of the direct transfer of load from the gussets to the webs in these box sections.

A comparison of the stresses in the angles and webs of the type A specimens indicates that the webs of specimen AD2 were probably less effective—that is, had less strain at any given total load than the specimen AD1 webs. Consequently, for AD2 there was probably less shear on the rivets connecting the gusset to the web (excluding those rivets that also connected the angles) and more shear on the rivets connecting the angles to the gussets than for AD1. This may account, at least in part, for the different modes of first failure for AD1 and AD2 (AD1 having had a complete shear failure of the joint and AD2 shearing only the outer gusset-web-angle rivets). Another factor which may have contributed to the difference in behavior was the variation in ductility of the web plates, which may be found in Table 2.

---

[10] "A Study of the Behavior of Large I-Section Connections," by J. R. Fuller, T. F. Leahey, and W. H. Munse, Jr., *Proceedings-Separate No. 659*, ASCE, April, 1955.

Type C Specimens.—Analysis of the strain data for both type C specimens reveals that the stress in the east angles was considerably higher than that in the west angles. However, because of the small number of gages and the stress concentrations caused by the stitch rivets, a thorough analysis of the strains cannot be made.

TABLE 9.—ANGLE STRESSES AT MID-LENGTH FOR TYPE C SPECIMENS

Total load, in kips	Average stress on gross area, in kips per sq in.	STRESS IN CD1, IN KIPS PER SQ IN.			STRESS IN CD2, IN KIPS PER SQ IN.		
		West, $\frac{S1+S2}{2}$	East, $\frac{S3+S4}{2}$	Ratio, west to east	West, $\frac{S3+S4}{2}$	East, $\frac{S1+S2}{2}$	Ratio, west to east
50	2.57	1.90	4.10	0.46	2.00	3.35	0.60
100	5.14	3.90	7.75	0.50	3.85	6.60	0.58
150	7.72	5.65	11.20	0.50	5.65	9.70	0.58
200	10.29	7.55	14.80	0.51	7.65	12.90	0.59
250	12.86	9.55	18.10	0.53	10.10	15.80	0.64
300	15.43	11.70	21.50	0.54	12.60	18.70	0.67
350	18.00	13.80	24.70	0.56	15.10	21.45	0.70
400	20.58	15.95	27.90	0.57	17.55	24.40	0.72
			Average	0.52			0.64

It is interesting to note that the average loads in the west sides of both members, as shown in Figs. 10(*c*) and 10(*d*), were approximately 80% of the loads in the east sides at total loads of from 50,000 lb to 400,000 lb. This ratio differs considerably from that obtained on the basis of the strains in the angles (Table 9). Although measurements are not available to explain this difference, it is believed that it may have resulted from an initial curvature in the specimens.

TABLE 10.—LOAD PROPORTION ON EAST SIDE COMPUTED FROM ANGLE STRAINS

Specimen	Load proportion	Specimen	Load proportion
BD1	0.68	BD2	0.63
BP1	0.66	BP2	0.61
DD1	0.57	DD2	0.63
DP1	0.59	DP2	0.60
ED1	0.54	ED2	0.57
EP1	0.62	EP2	0.61
Average	0.61	Average	0.61

Types B, D, and E—Laced Specimens.—The examinations of the load-strain relationships for the laced-specimen types B, D, E have been combined because their behaviors were similar. Although the lacing configuration is not the same (east to west) at mid-height for the No. 1 and the No. 2 specimens, no change was made in the numbering of the strain gages. Thus, for example, S3 and S4 will always designate those gages opposite the point on the angles at which there was a lacing rivet, and readings of these gages may always be compared with other S3 and S4 gages on similar specimens.

By converting the ratios of strains in the two sides of the laced members to the percentage of load, it is found that between 25,000 lb and 300,000 lb the east sides tended to carry approximately 60% and the west sides 40% of the total load. However, the load distributions based on the strains and given in Table 10 all appear to be slightly higher than those obtained from the pull-plate measurements shown in Figs. 9(*a*), 9(*d*), 10(*a*), 10(*b*), 11, and 12. This relationship is similar to the variation noted also for the type C specimens.

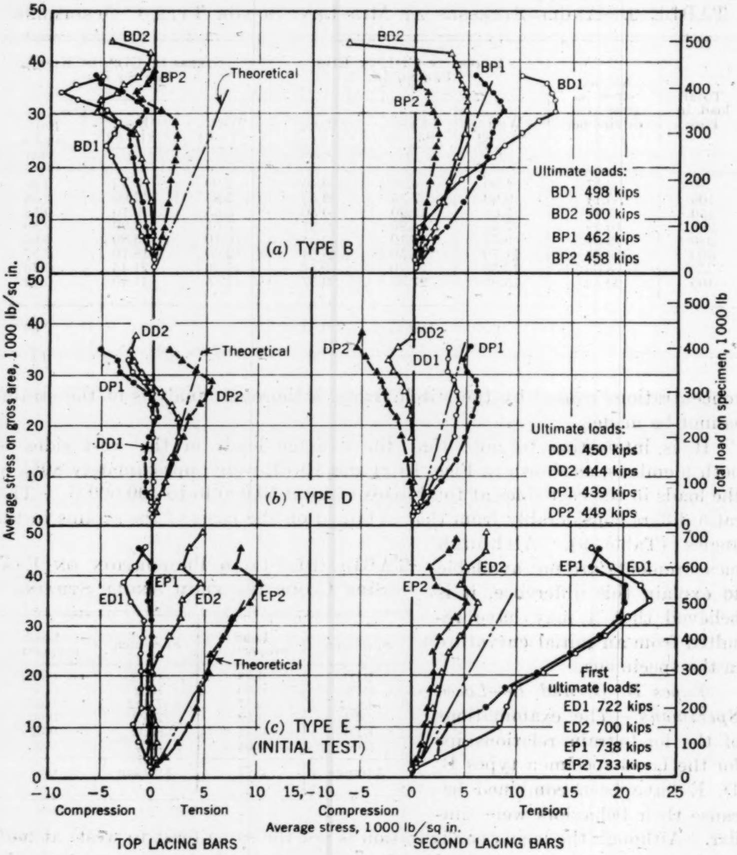


FIG. 13.—AVERAGE STRESS IN LACING BARS FOR TYPE B, D, AND E SPECIMENS

By comparing the measured strains in all laced members, it was seen that there was little difference in the average strains at the mid-length gage locations for loads of from 25,000 lb to 300,000 lb whether the specimens were drilled or punched. A comparison was also made of ultimate loads for the four pairs of laced specimens that had failures other than shear. This latter comparison gave an average of 96% for the ratios of loads of punched to drilled members versus 99% for the ratio of drilled to punched average strains in the twelve laced specimens.

The four type E specimens were not included in this ultimate-load comparison because the two drilled specimens sheared their fasteners at loads about 3% less than those at which the punched specimens failed at the net section. If these are included in the average ratio, a value of 0.985 is obtained, which is approximately the same as the value of 0.99 obtained by comparing the strains. If the strain-hardening effects of the multiple loadings on specimens ED1 and ED2 were neglected, and if their final fracture loads were used in computing the ratios, the ultimate loads of the punched specimens with lacing would be approximately 95% as great as those of the drilled specimens.

The ultimate strength of the punched specimen DP2 exceeded that of its drilled counterpart, DD2, and both failed in tension in similar fashion.

Lacing-Bar Strains.—Each of the two upper lacing bars of all the laced specimens had two strain gages mounted at mid-length and along the center line, one on either side. A determination of the magnitude of the stresses in the lacing bars could therefore be made. There was little bending of the lacing throughout the range of loading. Accordingly, the readings of the two strain gages on each bar have been averaged and converted to stress on the gross section of the bar. The results of these computations are presented in Fig. 13.

Curves are also shown in Fig. 13 for the theoretical stresses (elastic range only), which are equal to the nominal stresses in the angles multiplied by the squares of the cosines of the angles that the lacing bars made with the axes of the member. The stresses in the top lacing bars differed from the theoretical stresses because of the complex end effects and the unequal distribution of load to the members. The same factors also affected the stresses in the second lacing bars but to a lesser degree. It is felt that for a long member the central lacing bars would be stressed generally at a level approximately equal to that suggested by theory.

The lacing bars of both No. 1 specimens behaved differently from those of the No. 2 specimens. Because of this consistent variation, it is believed that orientation of the specimens in the testing machine produced such a dissimilarity.

Fig. 13 shows that at working loads the stresses on the lacing bars were less than 10,000 lb per sq in. on the gross section of the bars. At loads about twice the design load, the stresses on the gross section of the lacing bars reached from 15,000 lb per sq in. to 20,000 lb per sq in. in some cases.

A variety of failures was obtained with the laced specimens. The type B specimens failed at lacing rivets in the middle of the members; the drilled type E specimens failed initially in shear and, subsequently, in tension at the net section; and the punched type E specimens and all the type D specimens failed at the net section. Of those specimens that failed at the net section, three fractured at the side of the connection at which the lacing bar terminated. The remaining five fractured at the side of the connection opposite the lacing bar. Therefore, the position of the lacing bars did not seem to affect the location of the rupture.

Load-Deformation Relationships.—Measurements were made of the over-all deformation, the slip at the first and last rows, and the lateral movement of the upper pull plates. For the analysis of the over-all deformation, the four readings from the duplicate specimens were averaged, thus permitting a comparison of the punched and drilled specimens as well as one of the average deformations of the various types of members tested.

In general, the foregoing measurements reflected the same general pull-plate load distribution shown in Figs. 9 through 12. Where the east pull plates were more heavily loaded, the east deformation was greater. Just as the loads in the pull plates became more nearly equal as the loads approached the ultimate, the deformations of the two sides of the specimens became more nearly equal. Although the differences in loading on the two sides introduced a smaller deformation on one side than on the other, the average deformations in Fig. 9(*a*) give a general indication of the relative behavior of the various members.

Fig. 14(*a*) shows that at a given load the punched specimens of any type (shown by dashed lines) deformed less than did the similar drilled specimens. This was particularly true of the loads above the normal design range.

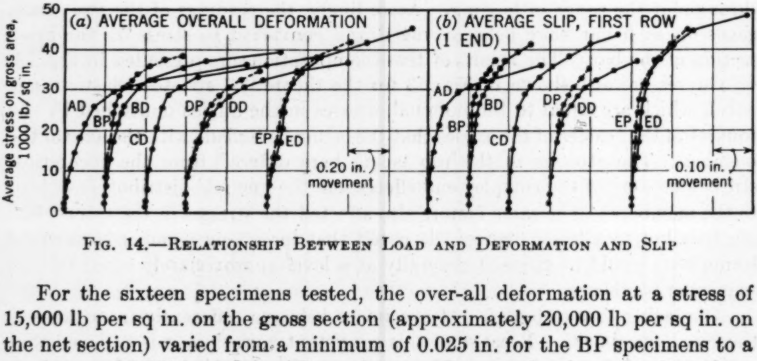


FIG. 14.—RELATIONSHIP BETWEEN LOAD AND DEFORMATION AND SLIP

For the sixteen specimens tested, the over-all deformation at a stress of 15,000 lb per sq in. on the gross section (approximately 20,000 lb per sq in. on the net section) varied from a minimum of 0.025 in. for the BP specimens to a maximum of 0.059 in. for the AD specimens. Because for these members there were only 32 in. between the first-row fasteners, the usual elastic analysis (based on the gross section) yields a computed deformation of 0.016 in. for the member itself. However, the actual deformation would be slightly larger. For this reason slip at the first rows and deformation along the length of the two joints should account for the balance of the movement, or about 0.01 in. for the BP specimens and 0.04 in. for the AD specimens. In Fig. 14(*b*) it is found that at the two first rows the slip in the joints of the BP specimens actually totalled almost 0.01 in. and in the two joints of the AD specimens, approximately 0.03 in. Therefore, the average slip and deformation in the two connections of a specimen were approximately equal to the computed deformation of an additional 4-ft length of the member. This seems partly to justify the use of the distance between panel points to determine the deformation of the members of a truss. Such an assumption, although approximate, is probably no more in error than many other assumptions made in determining the deformations of a structure.

For a given specimen, slip at the first row was obtained at six separate points. The measurements at all points reflected variations in the distribution of load and specimen configuration. By averaging all six first-row slip readings of one specimen with six from the identical specimen, Fig. 14(*b*) was obtained. Again it is evident that at any given load the punched specimens generally underwent less average deformation than the comparable drilled specimens. By taking the scale of Fig. 14 into consideration, the slip in the two joints is shown to be equal to approximately one-half of the over-all deformation obtained from these comparatively short members.

The measurement of slip at the last row of rivets yielded only relative information on the deformation. These measurements at the end of the joints (as with all "slip" measurements taken) included not only slip but local deformations in the angles and gussets. Because the measurements at the last row were affected by very large deformations in the gussets, they are not directly comparable with the other slip data. Nevertheless, the deformation of the punched specimens at the last row of rivets was again less than that of the drilled specimens.

Failures.—Only eight of the sixteen specimens failed initially at the net section. The other eight fell into two groups—those that failed in shear and those that failed in tension at points other than the net section.

It has long been impossible to analyze the behavior of riveted joints in an exact fashion. Many attempts have met with only partial success because of the variables affecting this behavior. As early as 1867 J. W. Schwedler showed that the behavior of a riveted joint at working loads was dependent on the friction, thereby extending the observations made by Edwin Clark[3] in 1850. This discovery has been reported repeatedly, and at one time, according to Arthur J. Francis,[11] German specifications used joint friction as a basis for design. However, the friction that will exist in a given joint is difficult to predict. This fact led engineers in the United States to continue using design methods that do not utilize friction but which, instead, are based on the areas required for tension, shear, and bearing.

The most common specifications,[12,13,14] which are based on many years of experience, assume that designs will be made for equal partition of the load to the fasteners in a joint and specify the use of nominal unit stresses. Some investigators have presented statements supporting this practice.[15,16] However, others have questioned the validity of the assumption when a connection is very long.[17]

Francis,[11] C. Batho,[18,19] and others[3] present observations and computations that indicate that, depending on the number of fasteners in a line, the end rivets of a connection may take from 2 to 15 times the load carried by the innermost rivets, and that this may represent actual rivet loads of from 1.2 to 2.8 times that assumed in design. Batho[18,19] further suggests that increasing the rivets in a line to more than five does not materially reduce the load on the end rivets. Despite the fact that such elastic analyses are not completely suitable for the prediction of the load partition to rivets because of joint friction, the change in joint behavior and slip throughout the range of loading, and the impossibility of obtaining an ideal joint commercially, they do emphasize the error in assuming equal load distribution. Experiments on conventional joints have indicated that load is only partly redistributed after yielding and slip occur.

---

[11] "The Behavior of Aluminum Alloy Riveted Joints," by A. J. Francis, ***Research Report No. 15***, Aluminum Development Assn., London, 1953.

[12] "Specifications for Steel Railway Bridges, for Fixed Spans Not Exceeding 400 ft. in Length," A.R.E.A., 1953.

[13] "Standard Specifications for Highway Bridges," A.A.S.H.O., 1953.

[14] "Specifications for the Design, Fabrication and Erection of Structural Steel for Buildings (Riveted, Bolted and Arc-welded Construction)," in "Steel Construction," A.I.S.C., 5th Ed., 1955 (revised 1949).

[15] "Work of Rivets in Riveted Joints," by A. Hrennikoff, *Transactions*, ASCE, Vol. 99, 1934.

[16] "Tension Tests of Large Riveted Joints," by Raymond E. Davis, Glenn B. Woodruff, and Harmer E. Davis, *ibid.*, Vol. 105, 1940.

[17] Discussion by A. E. Richard de Jonge of "Tension Tests of Large Riveted Joints," *ibid.*, p. 1284.

[18] "Riveted and Bolted Connections," by C. Batho, *First Report of the Steel Structures Research Committee*, Dept. of Scientific and Industrial Research, London, 1931, pp. 100–113.

[19] "Experimental and Theoretical Investigations on Riveted and Bolted Joints, With the Application of the Theory to Welded Joints," *ibid.*, p. 113–179.

Because of the unequal distribution of loads along the length of long connections, a shear failure of the rivets may be progressive. However, progressive failure may, in reality, be so sudden that the eye cannot detect the sequence of failure. The first rivet fails most likely after gradually reaching a large deformation. When it fails the load is suddenly shifted to the next rivet, which already has been deformed nearly enough to cause its failure. The shock or impact attending the load transfer shears this second rivet immediately, and, seemingly, the rivets all fail at the same time.

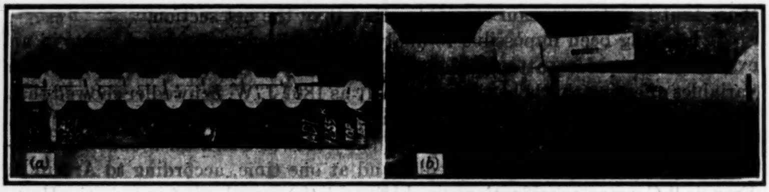


FIG. 15.—SECTION AND CLOSE-UP ALONG OUTER LINE OF RIVETS, SPECIMEN AD1

An examination was made of the deformations of the specimens tested in this program. An unfractured joint from each type of specimen with each method of hole preparation was cut from that specimen and sectioned at the rivet center lines. These sections clearly illustrated the relative deformations of the rivets in the connections and aided greatly in explaining the behavior of the members.

The unequal rivet deformation and the resulting inequality in the partition of load, which caused the premature shear failures in the type A specimens, were evident in the sections of the member of this type. All the rivets in the center line of the joint experienced small deformations. The two end rivets exhibited shear deformations of only approximately 0.08 in. The end rivets of the second line deformed approximately 0.10 in., and those of the outer line deformed about 0.32 in. The latter deformation may be seen in Fig. 10(*a*), and an enlarged view of the first rivet is shown in Fig. 15(*b*). One might note how the sharp corner of the hole in the web had deformed the rivet at the web-gusset plane and how the severe bending had begun to pry off the rivet head.

The end rivets were the most highly deformed in the sectioned connections. In addition, the inner rivets frequently had comparatively low deformations, those of the punched specimens were less deformed than those of similar drilled specimens, the holes of the latter type became enlarged more than those of the punched specimens, and the first-row rivets of specimens CD2 and EP1 (both with ten rows) were probably deformed to such an extent that the ultimate strength of the end rivets had been reached. Further loading of those two joints would probably have caused failure of the rivets.

The type A and E joints, which were refastened with ASTM A325 high-strength bolts, failed in tension and sheared several bolts. However, because the bolts were installed in a specimen that had already been distorted, direct comparisons should not be made between the riveted and bolted connections.

Shear failures occurred in the AD and ED specimens, all four of which were drilled and fastened with ¾-in. rivets. The rivets of the AD specimens were hand-driven, but those of the ED specimens were machine-driven.

TABLE 11.—APPROXIMATE MAXIMUM NOMINAL SHEAR ON RIVETS

Specimen	Percentage of load in gusset	Estimated maximum load on gusset, in kips	Shear area, one gusset, in square inches	Maximum nominal unit shear, in pounds per square inch	Rivet size, in inches
AD1	52	601	15.46	38,900[a]	3/4
AD2	50	595	15.46	38,500[a]	3/4
BD1	52	259	7.21	35,900	7/8
BD2	56	280	7.21	38,800	7/8
BP1	54	249	7.21	34,500	7/8
BP2	51	234	7.21	32,500	7/8
CD1	52	453	12.02	37,700	7/8
CD2	51	459	12.02	38,200	7/8
DD1	50	225	6.01	37,400	7/8
DD2	50	222	6.01	36,900	7/8
DP1	52	228	6.01	37,900	7/8
DP2	52	233	6.01	38,800	7/8
ED1	53	383	8.33	46,000[a]	3/4
ED2	51	357	8.33	42,900[a]	3/4
EP1	54	399	8.33	47,900	3/4
EP2	51	374	8.33	44,900	3/4

[a] Rivets failed.

From Table 4 it can be seen that the undriven ¾-in. rivets and ⅞-in. rivets exhibited an average nominal ultimate shear strength of 48,200 lb per sq in. and 44,000 lb per sq in., respectively. Several investigators[20] have shown that driving increases the shear strength based on nominal diameter by as much as 33%. After driving, similar increases have been noted also in the tensile strength of rivets. Because the properties shown in Table 4 are for undriven rivets, one would expect that the ultimate rivet strengths in the connections might be as much as from 15% to 20% higher.

A summary of the maximum nominal shearing stresses that may have existed at the ultimate are given in Table 11. These values are based on the distribution of load to the east and west pull plates just before failure.

The only specimens that failed in shear were AD1, AD2, ED1, and ED2. The nominal unit shears (Table 11) at failure of the type A and type E specimens were quite different, yet the ultimate strength in shear of the rivet stock for both was approximately 48,200 lb per sq in. Therefore, the nominal shear stresses on the type A specimen rivets were only about 0.8 of the ultimate. Furthermore, from Table 1 it is seen that the tension-to-shear ratio of 1.0:0.65 is considerably less than the ratio of 1.0:0.75 allowed by current specifications.

---

[20] "Tests of Riveted and Welded Joints in Low-Alloy Structural Steels," by W. M. Wilson, W. H. Bruckner, and T. H. McCrackin, Jr., *Bulletin 337*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., September, 1942.

Because the nominal shear stresses were not excessively high, could the cause of failure be combined tension and shear caused by the bowing of gussets and webs? William H. Munse, Hugh L. Cox, and T. R. Higgins have shown[21,22] that a rivet that is subjected to 0.8 of its ultimate shear stress is able to resist a tensile stress equal to approximately one-half its ultimate tensile stress. Therefore, it appears unreasonable to assume that tension plus shear could have been the principal cause of failure. In examining the joint deformation, it appears that the shear failures of the type E and type A specimens were due to two causes, singly or in combination—excessive deformation of the end rivets in a long joint and failure to distribute the rivets on gage lines in proportion to the corresponding cross-sectional areas. The latter point has been raised previously.[15]

Because the type A joint was only seven rows long, the effect of the length of joint was probably minor compared with the second effect. A comparison of the distribution of area to the five lines of rivets indicates that the outer rivet lines each connected 3.42 sq in. of net area, or 34.1% of the total. The three inner rows each carried 1.06 sq in., or 10.6% of the total. This crude analysis shows immediately that the material in the two outer lines of rivets contained 68% of the net area of the joints but only 40% of the rivets. From the maximum gusset load for the type A specimen as given in Table 11, the unit shear on the outer lines of rivets would be

$$\frac{600{,}000 \times 0.68}{14 \times 0.442} = 65{,}900 \text{ lb per sq in.}$$

This high unit shear undoubtedly did not exist at failure because of a partial redistribution of load after yielding. However, this computed ultimate shear stress is only approximately 37% greater than the ultimate shear strength of the undriven rivets. As noted previously, the driving could have produced an increase of as much as 33%. Only the rivets in the angles of specimen AD2 sheared and then the web tore, indicating that the shear on the outer lines of rivets was excessive. Consequently, it would be desirable to proportion the rivets in such a connection on the basis of the contributing cross-sectional areas in order to equalize more nearly the loads on the rivets.

Redesigning the type A specimens on the basis of area distribution, maintaining the same net section and approximately the same number of rivets, would give twelve rivets in each outer line and four in each inner line. Such a joint would not be acceptable to many engineers because of the excessive length of the joint and the large pitch along the inner lines of rivets which would make them less effective. One method of reducing the length of such a connection proportioned on the basis of area would be to increase the rivet diameter. An-

---

[21] "The Static Strength of Rivets Subjected to Combined Tension and Shear," by W. H. Munse and H. L. Cox, *Bulletin No. 437*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., December, 1956.

[22] "How Much Combined Stress Can a Rivet Take?" by T. R. Higgins and W. H. Munse, *Engineering News-Record*, December 4, 1952.

other would be the use of lug or clip angles. However, to use the latter to transfer some of the load from the member to the gusset may not be a satisfactory solution in view of the observations of Lawrence T. Wyly,[23] M. ASCE, Thomas C. Shedd,[24] M. ASCE, and others.[3] The use of clip angles has been examined by few investigators and might well receive additional attention.

As noted previously, drilled specimens ED1 and ED2 failed in shear at loads that were less than those at which similar punched members failed in tension, although many investigators have held that drilled joints are preferable. Table 11 shows that the average nominal unit shears on the rivets of the punched specimens actually were greater than the average nominal shears on the drilled specimens.

In literature on the strengths of joints prepared by drilling and punching, this apparently higher shear strength of punched specimens has not been noted because most of the investigators have designed their joints to assure tension failures. Similarly, by using a tension-to-shear ratio of 1.0:0.68, it was assumed that tension failures would occur also for all the type E specimens. However, the design of the latter resulted in joints that were closely balanced with respect to shear and tension. This balance was affected by the method of hole preparation. The punched holes actually provided stronger joints than the drilled holes. Interestingly enough, this same paradox had been noted in 1864 by Henry N. Maynard.[3]

As was noted previously from Fig. 14, the performance of all the punched specimens at any given load was better than that of the corresponding drilled specimens in another respect—the deformation or slip was less. This may be a clue as to why the punched specimens did not fail in shear even at nominal unit shearing stresses equal to or greater than those that caused rivet failure for the drilled specimens. One reason for this difference in shear strength may be the sharpness of the edge of the drilled holes. Another may be the keying action that is produced by the slight depressions and burrs left on the surfaces of the connected parts by the punching. These burrs, under the forces produced by tension in the rivets, may act as shear keys and impede the shearing deformation of the joints.

However, why did the drilled type E specimens fail in shear when they were overdesigned in order to assure tension failures? The highest nominal unit shear on the drilled type E specimens was 46,000 lb per sq in., whereas the undriven rivets had a coupon strength of 48,200 lb per sq in. If no allowance is made for an increase in strength due to driving, it is found that this shear was only 95% of the ultimate. However, with a moderate increase in strength of only 15% from driving, the nominal shear in the connection would have been 83% of the expected ultimate. One factor that might result in such an unanticipated failure is the length of the joint. Jonathan Jones, Hon. M. ASCE, cautioned[25] that perhaps the unit shear that was allowed for a long joint should not be as great as that permitted for a short joint. The apparent lower joint strength of the drilled type E specimens may be a result of the highly unequal distribution of rivet loads that occur along the length of such a joint. On the basis of these tests and other test data, it appears that only approximately eight rivets can be placed in a line if they are to develop their full collective shear strength. At working stresses in which the load is carried principally by friction, longer joints behave satisfactorily but may give a false sense of ultimate strength because of their reduced ultimate shearing capacity.

---

[23] "A Study of the Behavior of Floorbeam Hangers," by L. T. Wyly, M. B. Scott, L. B. McCammon, and C. W. Lindner, *Bulletin 482*, A.R.E.A., September-October, 1949.

[24] "Structural Design in Steel," by T. C. Shedd, John Wiley & Sons, Inc., New York, N. Y., 1934.

[25] Discussion by Jonathan Jones of "Tension Tests of Large Riveted Joints," by Raymond E. Davis, Glenn B. Woodruff, and Harmer E. Davis, *Transactions*, ASCE, Vol. 105, 1940, p. 1254.

Two types of tension failures warranting attention are the gusset-plate failures of specimen AD1 and the failures at the lacing-bar connections for all four type B specimens. The final gusset failures of AD1 were largely the result of the relatively high stress concentrations. Several other investigators[3,26,27] have shown the importance of these stress concentrations. The gusset-net. section area (35.63 sq in.) was 176% of the net section of the specimens and would have been more than sufficient had the gussets been narrower but thicker As shown by Lüder's lines in the whitewash, the stress concentrations caused yielding of the gusset at the last-row rivets at a load of 400,000 lb. This crude evaluation suggests a stress concentration of slightly more than 3. The gusset plate tore at a nominal average stress of about 35,000 lb per sq in., which is higher than that reached by any other specimen. In the test of AD1 it was noticed that after the first shear failure at 1,155,000 lb, the east gusset had necked down considerably at the last row of rivets. Upon reloading, the gussets failed before the rivets of the upper joint sheared. However, the upper joint of AD1 was near the point of shear failure, as shown by the joint section in Fig. 15(*a*) and the rivet in Fig. 15(*b*). The loss of this rivet head at the first row would probably have initiated a progressive shear failure throughout the remainder of the joint.

The other type of unexpected tension failure was that shown by the type B specimens. The net section of two angles at the first row of rivets was 4.31 sq in., and failure occurred in the two angles at a point having a net area of 4.97 sq in. (15% greater). The reason for this unusual failure was the effect of the lacing bars, which actually contributed to the failures. There were three laced-specimen types, but only one exhibited this unusual pattern.

The type D specimens had a small net-area-to-gross-area ratio—63%. The type B specimens with the same size of angles, rivets, and lacing bars had a net-area-to-gross-area ratio of 75%. Thus, failures at the net section would be expected to occur in the type D specimens at lower total loads because both types had the same cross section at the lacing rivets. The foregoing is one explanation of why the type D specimens were more likely to fail at the net section than the type B specimens. A second reason lies in the contribution of the lacing bars to the loading in the angles. Some of the bars in the type B specimens exhibited stresses that were somewhat greater than those in the type D bars. These tensile forces acted as concentrated loads at the lacing rivets, thereby further stressing the member angles, which were already sub-

---

[26] "Experimental Investigation of Stresses in Gusset Plates," by R. E. Whitmore, *Bulletin 16*, Eng. Experiment Station, Univ. of Tennessee, Knoxville, Tenn., May, 1952.

[27] "Comparative Test of a Structural Joint Connected with High-Strength Bolts and a Structural Joint Connected with Rivets and High-Strength Bolts," by J. W. Carter, J. C. McCalley, and L. T. Wyly, *Bulletin 517*, Report of Committee 15—Iron and Steel Structures, A.R.E.A., September-October, 1954.

jected to axial tension. The bending stress applied by the lacing increased the tensile stresses on the toes of the angles at the lacing rivets, and the combined stresses caused failure sooner than would have occurred otherwise.

By an approximate analysis, the conditions that would have had to exist to induce failures at the lacing connections can be determined. At failure it would have been necessary for the center lacing bars of the type B specimens to have had a gross stress of approximately 6,000 lb per sq in. by such an analysis. This is in reasonably good agreement with the maximum stress measured at the lacing bars and shown in the right half of Fig. 13(*a*). By a similar analysis, the type D specimens would have required a lacing-bar stress of about 8,000 lb per sq in. to produce the same type of failure. However, this stress was not reached by any of the lacing bars in the type D specimens. Because the type E specimens had angles of greater stiffness than those used in the type B and D specimens, the secondary effects from the forces in the lacing bars were comparatively small. Accordingly, none of the type E specimens failed at the lacing connections. Although the lacing bars may have affected the ultimate strength of the type B specimens by perhaps 10% or less, they did not appear to affect materially the ultimate loads of the type D and E specimens.

Lacing in a tension member is used to assist in handling in the field and in the shop, thus avoiding local buckling, and to adjust the shears in the member that result from unequal loading. Marion Boardman Scott, A.M. ASCE, and J. W. Cox indicate[28] that in actual service each lacing bar of a floor-beam hanger composed of two 12-in. channels carried a total load of less than 1,000 lb. They concluded that for working loads, continuous, properly spaced lacing appeared to be adequate to tie the main components of the hanger together. Wyly had observed[23] that hangers composed of two channels that are connected with occasional tie plates did not act as a unit, but rather as two individual members that were subjected to severe racking stress. On the basis of the data reported herein and the observations of others, when a solid web is not warranted lacing bars will provide a suitable tie; however, the web is preferable.

Although it did not produce failures of the members, a great deal of warping and bending occurred in the angles and gussets at the outer ends of the type E connections. This deformation in the long connections would have been reduced greatly if the batten plates had extended the entire length of the joint. In the type B and D specimens, which had comparatively long tie plates, this deformation was not apparent.

Analysis of Joint Efficiencies.—In the United States the three most commonly used specifications (as of 1958) present the same rule for the computation of the effective net section of a tension member. Although the wording differs slightly, the specifications of the American Railway Engineering Association (AREA),[12] the American Association of State Highway Officials (AASHO),[13] and the American Institute of Steel Construction (AISC)[14] provide what will be referred to subsequently as the AREA rule for tension net areas.

---

[28] "A Further Study of the Behavior of Floorbeam Hangers," by M. B. Scott and J. W. Cox, *Bulletin 495*, A.R.E.A., June–July, 1951.

The specifications present somewhat different requirements for angles in tension, which are connected through one leg or which may be subject to bending. However, whether the method of hole preparation is drilling, subpunching and reaming, or punching, all three specifications require that: "The diameter of the hole shall be taken as ⅛ inch greater than the nominal diameter of the rivet."

The history of the acceptance of the AREA rule has been conveniently summarized[29] by C. H. Chapin, who also mentions some other rules for net-section determination used in the 1920's, which specified that the net section along a diagonal line of holes should be from 10% to as much as 40% in excess of that along a transverse line.[24,29,30] W. G. Brady and Daniel C. Drucker, M. ASCE, have shown[31] that, based on their limit analysis and tests of flat-plate specimens with open or plugged holes, the $(s^2/4\,g)$-rule corresponds to an approximate upper bound at yielding for a riveted joint. The so-called modified AREA rule differs from the AREA rule only in that the actual hole diameter is used in computing the net width.

Wilbur M. Wilson, Hon. M. ASCE, and Frederick W. Schutz, Jr., A.M. ASCE, suggested[32,33] two other design rules, which are based on empirical studies. The first has not been used in the analysis and review of these tests. The second, known as the relative gage rule, has been used to examine the results and may be expressed as follows:

> "Effective Net Section
>
> "In the case of a chain of holes extending across a part in a zigzag, diagonal or straight line, the effective net section of the part shall be the summation of the effective net sections of all the gage strips along the chain of holes. No chain of holes shall be considered which has a gage strip with a pitch of ⅔ or more of the gage of that strip.
>
> "The critical net section of the part is obtained from that chain which gives the least effective net section.
>
> "A gage strip is the portion of the part bounded by the longitudinal center lines of two successive holes in the chain of holes being investigated. A transverse edge distance is considered as one half of a gage strip which has a gage twice the edge distance. The effective net section of a gage strip is the product of the effective net width and thickness of the strip.
>
> "The effective net width (W_{EN}) of a gage strip shall be determined by the following equation:
>
> $$W_{EN} = 1.05\ (\mathrm{g} - 0.9\mathrm{d})\ \mathrm{KH} \text{ but not more than } 0.87\ \mathrm{g\ KH}$$

---

[29] "The Net Section of Riveted Tension Members," by C. H. Chapin, *Proceedings*, A.R.E.A., Vol. 36, 1935, pp. 775–780.

[30] "Structural Engineer's Handbook," by M. S. Ketchum, McGraw-Hill Book Co., Inc., New York, N. Y., 3d Ed., 1924.

[31] "Investigation and Limit Analysis of Net Area in Tension," by W. G. Brady and D. C. Drucker, *Transactions*, ASCE, Vol. 120, 1955, pp. 1133–1154.

[32] Discussion by W. M. Wilson of "Tension Tests of Large Riveted Joints," by Raymond E. Davis, Glenn B. Woodruff, and Harmer E. Davis, *ibid.*, Vol. 105, 1940, p. 1193.

[33] "The Efficiency of Riveted Structural Joints," by F. W. Schutz, thesis presented to the University of Illinois, at Urbana, in 1952, in partial fulfilment of the requirements of the degree for Doctor of Philosophy.

where

d = Actual hole diameter

g = Transverse spacing (gage) of any two successive holes

K = 0.82 + 0.0032R but not more than 1.00

R = Reduction in area of standard control coupons in per cent

H = 1.00 for drilled holes; 0.862 for punched holes."

With respect to the foregoing rule, the following points should be emphasized:

1. It is necessary to treat each gage strip separately and then to determine a weighted effective area.
2. Actual hole diameter is used in contrast to nominal connector diameter plus $\frac{1}{8}$ in. as is customary.
3. A marked distinction is made between punched and drilled holes.
4. No effect is reflected in the formula for varying the stagger between the conditions of no stagger and $s/g = \frac{2}{3}$.
5. The rule sets an upper bound of effective net width, indicating that the use of a gage of more than approximately 5.25 times the actual hole diameter does not increase the efficiency of a joint.
6. Some estimate of the ductility of steel must be made.

Test efficiency as used herein may be defined as the ratio in percentage of the ultimate test load to the expected strength of the gross section based on the average coupon strength of the specimen at the critical section. The computed test efficiencies, the predicted efficiencies, and their ratios are shown in Table 5 together with data on the ultimate strength of the specimens, AREA design loads, and the resulting safety factors. In order to convert the design loads to those allowed by the AISC, the tabulated values must be multiplied by 20/18.

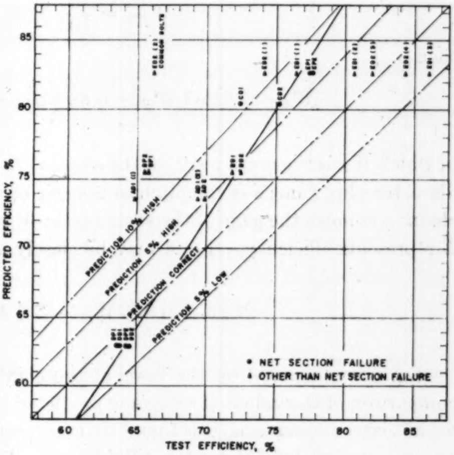


FIG. 16.—COMPARISON OF EFFICIENCY BY AREA RULE WITH ACTUAL TEST EFFICIENCY

From the ratios of predicted efficiencies, it can be seen that, on the average, the AREA rule predicted values that were 6.3% high based on initial failure loads only. The modified AREA rule gave results that averaged 8.9% too high. The relative gage rule gave results that were 11.8% too high, on the average, for all drilled specimens and 2.8% low for all punched specimens. If only those specimens initially having net-section failures are considered, it is found that the AREA rule gave averages 3.0% high, the modified AREA rule gave values 5.8% high, the results of the relative gage rule for drilled specimens were 8.5% high, and the punched-specimen predictions averaged 6.5% low. On the basis of these tests, the AREA rule gave the best agreement for truss-type members.

A comparison of the predicted AREA efficiency and test efficiency is presented graphically in Fig. 16. If the net-section failures are approximated with a straight line, the following empirical relationship for test efficiency (in percentage), E_T, results:

$$E_T = 24.5 + 0.63\, E_{\text{AREA}} \quad \ldots\ldots (1)$$

in which E_{AREA} is the AREA efficiency. However, because

$$E_{\text{AREA}} = \left(\frac{W_G - \Sigma D + \Sigma \frac{s^2}{4\,g}}{W_G} \right) 100 \quad \ldots\ldots (2)$$

and

$$W_{EN} = \frac{E}{100} \times W_G \quad \ldots\ldots (3)$$

then

$$W_{EN} = 0.245\, W_G + 0.63 \left(W_G - \Sigma D + \Sigma \frac{s^2}{4\,g} \right) \quad \ldots\ldots (4)$$

in which W_G is the gross width of the section; D represents the nominal fastener diameter plus $\frac{1}{8}$ in.; s is the pitch or stagger of any two successive holes in the chain; g denotes the gage of the same holes; W_{EN} is the effective net width; and E represents efficiency. Eq. 4 may be closely approximated by

$$W_{EN} = 0.875\, W_G - \Sigma\, \tfrac{5}{8}\, D + \Sigma \frac{s^2}{6\,g} \quad \ldots\ldots (5)$$

Eq. 5 was derived on the basis of the eight net-section failures. From a comparison of the values of efficiency predicted by Eq. 5 and the test efficiencies for all sixteen specimens, it is found that the predicted values are 2.4% too high as compared with 6.3% for the AREA rule. The eight net-section failures are predicted as 0.5% too high rather than 3.0% by the AREA rule. By using Eq. 5, all initial failures are predicted to within + 10% and −1%, whereas the predicted AREA values varied from +15% to −2%.

Eq. 5 has not been applied to any other specimens except those reported[10] by J. R. Fuller, T. F. Leahey, and William H. Munse. Nor is it presented as a recommendation for determination of the effective net width, but only as a curve of best fit for the present tests. The scarcity of full-scale tests on double-plane members will not allow an extensive statistical comparison such as has been made with other rules for predicting efficiencies of flat plates. Furthermore, Eq. 5 is limited in application to double-plane members and should not be applied to single-plane members, although if used for flat plates the results would probably be more conservative than those obtained from the AREA rule.

In order to understand more fully the significance of studies on the efficiency of riveted joints, some of the factors that are not satisfactorily reflected in the derivations or analyses of joint efficiencies should be examined:

a. The possible variation due to rolling tolerances[7,8] is ±2.5%. No method for the prediction of efficiency can remove this source of variation.

b. Wilson, William H. Munse, and M. A. Cayci have indicated[34] that identically fabricated laboratory specimens may vary in strength by as much as 10%. The variables of fabrication therefore introduce an unpredictable effect in any joint, the magnitude of which can only be determined by a destructive test.

c. Current (1958) specifications (excluding the relative gage rule[33]) do not limit the maximum efficiency of a joint. Contrary to the design formulas, the literature has reported few joints with test efficiencies of more than 88%. An upper limit of 75% has been suggested by Raymond E. Davis, Glenn B. Woodruff, and Harmer E. Davis,[16] Members, ASCE, and Wilson[16] has suggested 85%. Other recommended maximum efficiencies may be found elsewhere.[3] This absence of an upper limit is perhaps one of the most questionable points.

d. No current specifications penalize punched holes. Various tests have shown that punching may or may not reduce the strength. The results reported herein suggest that punching reduces the strength only slightly, if at all, and that in long joints a punched hole may actually increase the strength if shear is critical. The relative gage rule suggests that punched holes are only 86.2% as strong as drilled holes, thereby permitting a maximum of 75% efficiency for punched joints.

e. No specification (as of 1958) distinguishes between single-plane joints and double-plane joints, although a difference in behavior is considered in some specifications for angles connected by one leg. It is felt that a shear lag[10] reduces the effectiveness of the webs of truss-type members and accounts for much of the difference between test efficiencies and those predicted by existing design rules.

CONCLUSIONS

1. Adherence to current (1958) design stresses does not necessarily assure a balanced design (that is, a design in which at the ultimate load the member is likely to fail in either shear or tension). Shear failures can be expected in long truss-type joints of "balanced design."

2. Large connections should be proportioned so that the distribution of rivets in a joint is similar to that of areas connected by the rivets.

---

[34] "A Study of the Practical Efficiency Under Static Loading of Riveted Joints Connecting Plates," by W. M. Wilson, W. H. Munse, and M. A. Cayci, *Bulletin 402*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., July, 1952.

3. Members with drilled holes in the connections are more susceptible to shear failures than are similar punched specimens. In addition, the shear strength of the drilled member can be expected to be slightly smaller than that of the punched member.

4. Punched and drilled truss-type members of the same joint pattern and of $\frac{3}{8}$-in.-to-$\frac{1}{2}$-in.-thick material may be expected to have approximately the same efficiencies. However, for thicker materials or for members subjected to loading conditions other than those used in this investigation, the efficiencies of similar drilled and punched members may not necessarily be the same.

5. The use of lacing bars in tension members provides a secondary loading, which may reduce the strength of the members. To reduce the possibility of tensile failures at the lacing rivets, the edge distances at these rivets should be made as large as possible and the lacing bars as small as feasible.

6. Of the several design rules considered, the AREA net-section rule appears to agree best with the test efficiencies of these truss-type members.

7. In view of the lack of complete agreement between theoretical and test efficiencies, and the unpredictable variations in the materials, it is doubtful whether complicated formulas for the design of tension members are justified. Because of the simplicity of application and familiarity with the currently specified rule, it would seem desirable to retain the net-section rule as a basis for design, at the same time instituting a suitable upper limit on efficiency or effective net section. Such a procedure would correct the most serious deficiency of the current specifications for tension members, and would provide a predicted or theoretical efficiency for riveted connections that does not differ greatly from the test efficiency.

Acknowledgments

The tests described herein are part of an investigation performed as a result of a cooperative agreement between the Engineering Experiment Station of the University of Illinois, the Illinois Division of Highways, the Bureau of Public Roads (United States Department of Commerce), and the Research Council on Riveted and Bolted Structural Joints. These tests were approved by the Project II Committee of the Research Council. The investigation is a part of the structural research program of the Department of Civil Engineering of the university under the general direction of Nathan M. Newmark, M. ASCE.

DISCUSSION

Arthur J. Francis[35].—The conventional design of riveted joints is still based on the results of tests on small specimens with a few rows of rivets. The data reported by Messrs. Chesson and Munse show how dangerous it is to extrapolate such test results to the larger joints common in practice without a proper understanding of the action of joints. Fortunately, failures of joints in service seem to be rare because most large joints under axial load have a considerably smaller factor of safety when they fail by shear than designers suppose. Even though riveting is not as important for bridges as it was before the introduction of welding, more large-scale tests are still needed. Probably the only other tests in which the size of the specimens approached that of the connections common in large bridges were those conducted for the San Francisco-Oakland Bay Bridge,[16] which disclosed the possibility of premature rivet failure and, consequently, reduced shearing strength. With the large testing machines presently available, it should be possible to extend one's knowledge of joints still further. In view of the tendency toward lower margins of safety this is highly desirable.

The writer regrets that a more thorough theoretical explanation of the behavior of their specimens was not attempted by the authors. Theoretical methods[11] are available for joints in which friction may be neglected (as in aluminum alloys). Although this assumption is invalid for steel joints in the working range, it may be true when shear failure is imminent. It would be interesting to attempt a theoretical explanation because the phenomenon of progressive failure of end rivets is similar in aluminum and in steel joints.

The behavior of the type A specimens is particularly interesting and shows the importance of placing the rivets, as far as possible, according to the distribution of cross-sectional area. Another result worth noting is the better performance of the joints with punched holes compared with those having drilled holes. The "punch-versus-drill" controversy is an old one, but the arguments advanced in the past were based mainly on elastic considerations. From the plastic point of view, there seems little harm in cold-working the material around the rivet holes when it has the characteristics of structural steel. The driven rivets themselves are, as a rule, far from the annealed state. However, the explanation for the lower deformations of the punched specimen (under the heading, "Results and Analysis of Tests: Failures") is not convincing. The deformation around a punched hole containing a loaded rivet will remain elastic to a higher load than if the hole were drilled because of the cold work on the material around the hole. The total deformation of a rivet is caused, in part, by shear and bending of the rivet shank and partly by the deformation of the members around the rivet hole. The latter usually forms an appreciable part of the total deformation at the end rivets, and if this is reduced for any reason the deformation of the end rivets and the over-all movement of the joint are smaller.

---

[35] Prof. of Civ. Eng., Univ. of Melbourne, Australia.

To the writer's knowledge no experiments have been conducted on steel riveted joints under moment or eccentrically applied load. In contrast to those that are axially loaded, the strength of joints under such loading is likely to be considerably more than the conventionally computed strength if the results of tests on aluminum alloy joints[11] are any indication.

Eugene Chesson, Jr.,[36] **J. M. ASCE, and William H. Munse,**[37] **A.M. ASCE.**—Some valuable comments concerning the behavior and design of riveted joints and the deformations in joints with punched holes have been presented.

A more thorough theoretical explanation of the behavior of the truss-type joints, as suggested by Mr. Francis, will be a part of the next phase of the research program in which the large connections are being tested. Thirty additional large riveted connections are now (1958) being tested to answer many of the questions raised as a result of the initial tests. Upon completion of these tests, sufficient data should be available to permit a more sound evaluation of the applicability of previous theoretical analyses, or the development of an analysis that will explain effectively and accurately the behavior of the connections. However, because most previous theoretical studies have been based on an elastic analysis, it is doubtful whether they will provide the desired correlation with the results of the laboratory tests conducted to failure.

Several findings reported by the writers and discussed briefly by Mr. Francis are being studied further. These studies include (1) tests to confirm the importance of distributing the fasteners on the basis of the distribution of the cross-sectional area, (2) further tests of "punching versus drilling," and (3) tests to evaluate the behavior of truss-type joints assembled with high-strength bolts—some with the shear area smaller than that required by existing specifications. The data from these tests, when combined with other available data, should help to provide the information necessary for an effective re-evaluation of the design requirements for riveted and bolted connections that are subjected to static-type loadings.

---

[36] Research Associate, Dept. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

[37] Prof., Dept. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2956

OLD RIVER DIVERSION CONTROL
A SYMPOSIUM

PAGE
FOREWORD.. 1130
THE GENERAL PROBLEM
By John R. Hardin, M. ASCE........................ 1131
HYDRAULIC REQUIREMENTS
By Eugene A. Graves, M. ASCE...................... 1142
FOUNDATION DESIGN
By Willard J. Turnbull and Woodland G. Shockley,
Members, ASCE...................................... 1160
STRUCTURES REQUIRED
By Norman R. Moore, M. ASCE....................... 1172

FOREWORD

A study of the history of the major Mississippi River diversions results in the conclusion that diversion through the Atchafalaya River is imminent. Because of the consequences such an eventuality must be prevented, and the control of the Old River has become necessary. The choice of control site was influenced by several conditions, among which are the prevailing and prospective tailwater conditions. Included in this symposium is a presentation of (*a*) the general problem, (*b*) the hydraulic requirements of the control plan, (*c*) the general geology and soil conditions of the control-structure area, and (*d*) the structures that are required for the adequate control of the Old River.

THE GENERAL PROBLEM

BY JOHN R. HARDIN,[1] M. ASCE

SYNOPSIS

The Atchafalaya River, which is joined to the Mississippi River by the Old River, is the farthest upstream distributary of the Mississippi River. It provides a route to the sea that is only one-half as long as the Mississippi River's present course past Baton Rouge, La., and New Orleans, La. In recent years the Atchafalaya River has diverted increasingly larger percentages of Mississippi River flow, and in 1956 approximately 23.5% of the latter was diverted. Studies, which have been conducted by the Mississippi River Commission, Corps of Engineers (United States Department of the Army), in recent years, and observations that have been made on these streams since 1885 lead to the conclusion that the diversion of flow through the Old River and the Atchafalaya River approaches and will reach the "critical" stage (approximately 40% of the major river's flow), at which time the course of the parent stream will be uncertain. Additional exhaustive studies of past major diversions show that the present development is similar to those leading to earlier diversions, and the studies support the conclusion that diversion through the Atchafalaya River will reach the critical stage by 1975 unless preventive measures are used.

Should this diversion occur, it would be necessary to discard the present flood-control plan in the lower reaches of the Mississippi River and in the Atchafalaya River Basin; navigation would be seriously disrupted; utilities would be relocated; fresh-water supplies for New Orleans would be seriously affected; and the impact on the economy of the region would be disastrous.

In order to prevent diversion, the Mississippi River Commission has recommended the construction of two control structures on the west bank of the Mississippi River approximately ten miles upstream from the Old River; a navigation lock connecting the Mississippi River and the Old River; an earth-fill dam in the Old River; approach channels for the lock; inlet and outlet channels for the control structures; and an enlargement and extension of the main-line Mississippi River levee at that site.

The Mississippi River has played an important role throughout the history of the United States. The object of attention of many early explorers, it was the primary north-south commercial artery of the infant country and the center of battles, treaties, and diplomatic maneuvers.

This great river does not make frequent major changes in its course to the Gulf of Mexico, but there have been a few in the past. It is believed that a change of wide-reaching effect might take place in the lower 300 miles by 1975 unless methods are used to prevent it.

NOTE.—Published, essentially as printed here, in March, 1956, in the Journal of the Waterways Division, as *Proceedings Paper 906*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Major Gen. and Pres., Mississippi River Comm., Vicksburg, Miss.

The Mississippi River is truly the United States' principal river, serving a large and important segment of the nation. It is becoming even more important as a source of water and as a means of transportation, being spanned by bridges, pipelines, and cables. Great cities on its banks are growing rapidly, and the river serves these cities as well as a vast area behind them.

The preceding are physical demonstrations of the importance of the Mississippi River to the valley through which it courses and to the nation. Because this river is so vital to the national economy, it is important that it be maintained in its present location and condition so that it may continue to serve the entire United States.

The Mississippi River Basin (Fig. 1), draining 41% of the United States, is approximately funnel-shaped and converges to form a "spout" where the Old River provides a connection between the Mississippi River and the Atchafalaya River. It is through this narrow spout that floodwaters from an area of 1,250,000 sq miles must be carried.

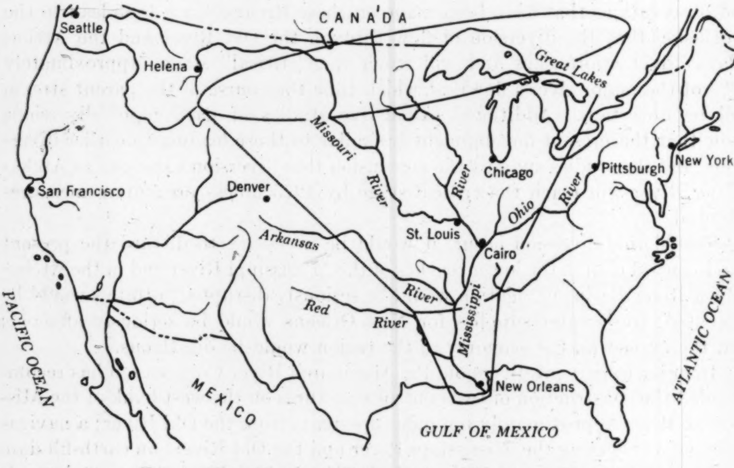

CANADA
Seattle
Helena
Great Lakes
Missouri River
River
Chicago
Pittsburgh
New York
Ohio River
San Francisco
Denver
St. Louis
Cairo
Arkansas River
Red River
Mississippi
New Orleans
PACIFIC OCEAN
ATLANTIC OCEAN
MEXICO
GULF OF MEXICO

FIG. 1.—THE DRAINAGE BASIN OF THE MISSISSIPPI RIVER

The lower alluvial valley of the Mississippi River, beginning at Cape Girardeau (Missouri) and extending to the Gulf of Mexico, was subject to overflow before protective works were erected. It is through this wide, flat valley that the Mississippi River meanders for approximately 1,000 river miles. The materials through which the Mississippi River flows were deposited by the river itself, with the result that they are easily eroded. Erosion, as well as the variations in the river's discharge and stage, creates a meandering stream whose past history is one of constant change. In building the vast delta into the gulf, it has made several major changes in course by accepting a shorter and more attractive route to gulf level.

The Atchafalaya River, the farthest upstream distributary of the Mississippi River, is the third largest river in the United States that flows into the sea.

The average annual discharge of this river is increasing, and records show that this increase is almost entirely due to the progressively greater volume of Mississippi River water being diverted to it through the Old River connection.

The Atchafalaya River flows through a low, well-defined basin extending in a general north-south direction from the latitude of the Old River through openings at Morgan City, La., and Wax Lake, La., and then out to the Gulf of Mexico, a distance of approximately 140 miles (Fig. 2). However, the distance to gulf level from the Old River, by way of the Mississippi River past Baton Rouge, La., and New Orleans, is more than twice that distance, or 320 miles.

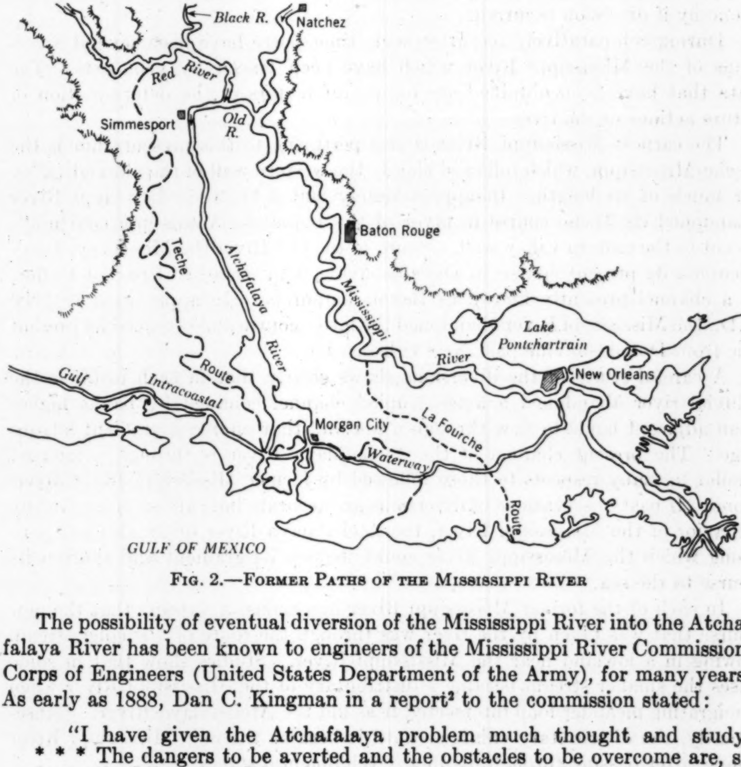

Black R.
Natchez
Red River
Old R.
Simmesport
Baton Rouge
Teche
Atchafalaya River
Mississippi River
Lake Pontchartrain
New Orleans
Gulf Intracoastal
Route
Morgan City
La Fourche
Waterway
Route
GULF OF MEXICO

FIG. 2.—FORMER PATHS OF THE MISSISSIPPI RIVER

The possibility of eventual diversion of the Mississippi River into the Atchafalaya River has been known to engineers of the Mississippi River Commission, Corps of Engineers (United States Department of the Army), for many years. As early as 1888, Dan C. Kingman in a report[3] to the commission stated:

> "I have given the Atchafalaya problem much thought and study. * * * The dangers to be averted and the obstacles to be overcome are, so far as the Mississippi River is concerned, * * * the deflection of its waters in ever-increasing quantities down the Atchafalaya which might in time, due to its shorter length, exceed in capacity the main stream and finally become the sole outlet of the Mississippi."

Since that time, and especially since 1928 when the present flood-control project was adopted, the Mississippi River Commission has studied this possibility.

---

[3] "Report of Captain Dan C. Kingman, Corps of Engineers, Upon Operations in the Fourth District," Pt. 4, *Annual Report of the Chief of Engineers, United States Army*, Appendix UU—Report of Mississippi River Comm., U. S. Govt. Printing Office, Washington, D. C., 1888, p. 2298.

However, some channel enlargement has been desired and even encouraged in order for the Atchafalaya River to be able to carry its share of the basin superflood on which the project designs are based.

By 1950 the Atchafalaya River had reached such a stage of development that a careful review of the situation was warranted. Enlargement of the upper river was occurring at a rate far in excess of the capacity of the lower river, with a consequent danger to the flood-control plan in that area. Therefore, the Mississippi River Commission began a thorough study of the complex problem of the river's tendencies and the effect of the river on the authorized flood-control and navigation programs as well as the impact on the region's economy if diversion occurred.

During comparatively recent geologic time, there have been several diversions of the Mississippi River which have been traced by geologists. The data that have been obtained are important factors in the determination of future actions of the river.

The earliest Mississippi River course pertinent to this presentation is the Teche-Mississippi, which followed closely the western wall of the alluvial valley for much of its length. In approximately 900 A.D., the Mississippi River abandoned its Teche course in favor of the Lafourche-Mississippi course adjacent to the eastern valley wall. South of the Old River the Mississippi River occupied its present course to Donaldsonville, La., where it departed to flow in a channel presently known as Bayou Lafourche. In approximately 1200 A.D., the Mississippi River abandoned the latter course and occupied its present one from Donaldsonville past New Orleans, La.

A careful study of the diversions shows clearly that in each instance the alluvial river abandoned a long-occupied channel confined by banks higher than adjacent lands to flow through a lowland that offered a gradient advantage. The present channel of the Atchafalaya River is through a lowland similar in many respects to those followed by former Mississippi River diversions. If past observations of diversions are accurate indications of any future behavior of the Mississippi River, the Atchafalaya River offers an ideal path along which the Mississippi River could steepen its gradient and shorten its course to the sea.

In each of the former Mississippi River diversions, it appears that the new course that was taken by the river was through the route of a smaller stream flowing in a lowland near the Mississippi River. Studies show that in some cases the smaller stream became a distributary of the Mississippi River when a migrating meander loop intersected it as did the Atchafalaya River. Subsequently, the distributary enlarged until it became the new Mississippi River course after passing through distinct developmental stages.

All Mississippi River diversions appear to have occurred gradually in a span of not more than 100 yr. The initial and intermediate stages are characterized by slow and progressive growth, after which a critical stage is reached in which there is an essential balance between flow down the main channel and flow down the diversion channel. Studies made under the Mississippi River Commission indicate that when a distributary of the main river carries approximately 40% of the major river's flow the critical stage has been reached, and then diversionary activity is greatly accelerated as the new channel develops rapidly in an attempt to reach equilibrium.

A study was made of the various aspects of the problem in order to determine the probable time at which diversion might occur. Consideration was given to the rate at which the Atchafalaya River had pushed through the deposits being dropped in the lower section of the Atchafalaya River Basin known as Grand Lake and Six-Mile Lake, just north of Morgan City. It was estimated that if the downstream advancement of the river channel that occurred from 1916 to 1950 continued, an efficient single river channel would be developed by 1985 or 1990. The existence of such a well-developed channel is an important consideration before the major flows of the Mississippi River choose a new route to the gulf. For reasons that will be presented subsequently, it will be necessary to expedite this channel development by dredging and by confinement of flows.

Another time element in this study concerns the discharge capacity of the Atchafalaya River in its upper reaches in the vicinity of Simmesport, La. At a stage of 40 ft at Simmesport, the discharge of the river increased 60,000 cu ft per sec between 1892 and 1932, a period of 40 yr. During the 18 yr between 1932 and 1950, the discharge capacity at the same stage increased an additional 130,000 cu ft per sec. If the discharge capacity at Simmesport continues to accelerate in the future as in the past 18 yr, by 1968 the major part of the flow of the Mississippi River will have been seized by the Atchafalaya River. However, the engineering report of the Mississippi River Commission indicated that although it is reasonable to assume that an efficient channel to the gulf might be developed at a greater rate than in the past, it is also reasonable to assume that the rate of acceleration of the upper river at Simmesport would not continue during the next 18 yr at the same rate as during the last 18 yr. The reason for the expected decrease in the rate of acceleration is due to a general flattening of the slope of the river in its leveed channel and the need for an efficient channel through the basin for moderate flood flows. In conclusion, the report indicated that the Atchafalaya River could become the main or master stream below the Old River at some time between 1968 and 1985 (approximately 1975). However, this was not the full extent of the features of the study that pertain to the element of time.

Channel dimensions of the Atchafalaya River from its head to the end of the leveed part of the river were studied in great detail, and the rate of enlargement was noted. The rate of enlargement of the Old River was noted also, as were the changes in the cross-sectional area of the Mississippi River upstream and downstream from the Old River. The changes in the stage-discharge relationship were studied intensively for all the various river channels that were considered. The changes in the slopes of the river surfaces and the deepening of the upper sections of the Atchafalaya River into the easily erodible sands had some bearing on the conclusions that were reached. The geology of the river basin, the previous changes, and the indicated future changes of the river's pattern resulted in a general insight into the sequences of flow.

River diversion into a new channel does not occur suddenly. The forces of nature must work for a considerable time in order to scour a channel that is large and efficient enough to entice a river to adopt a new course. Huge quantities of material must be transported out to sea or deposited in overbank areas.

When the critical stage is reached, the rate of enlargement may be rapid, and within a few years the channel could become so large it would be beyond control. In 1956 the Atchafalaya River carried an average of 23.5% of the Mississippi River's flow at the Old River, which was considerably less than the flow that is required for the critical stage of diversion. However, this percentage is increasing, and its rate of increase could be accelerated by a series of large floods, such as those experienced in 1927 or 1950, and could result in the critical stage occurring in 1965.

Using these exhaustive engineering and geologic studies as a background, as well as the many years of records and personal observation and the opinions of many engineering consultants, the Mississippi River Commission reported[3] to the Chief of Engineers that—

> "The Atchafalaya River, if left alone, will capture the Mississippi. * * * How soon this will occur is not susceptible of precise determination. It would be unwise to remain unprepared to take prompt and effective steps to prevent such an occurrence."

It appears that all the elements inherent in the concept of diversions are present in the Old River-Atchafalaya River situation and in a pattern that is favorable to the Mississippi River's acceptance of the Atchafalaya River channel as its own within relatively few years. If, as expected for normal conditions of the development, the Atchafalaya River becomes the master or larger stream in approximately 1975, the change will have gone practically beyond control. The problem of controlling flow and of taking corrective action should be approached on the basis of the occurrence of the critical stage rather than when diversion will likely be an accomplished fact.

Any plan for control of flows in the vicinity of the Old River must be based on several factors. One of the most important of these factors is the retention of the present proven flood-control plan, which was authorized in 1928 following the disastrous flood of 1927. The plan is a blend of many features, each performing a specific function and all, when completed, protecting the alluvial valley of the lower Mississippi River from the greatest flood that meteorologists believe will occur. The design flood is greater than that of 1927 because at the latitude of the Old River it is approximately 3,000,000 cu ft per sec; below the control structure at Morganza, La., the safe capacity of the leveed Mississippi River is 1,500,000 cu ft per sec, of which 250,000 cu ft per sec is drawn off, under the plan, before it reaches New Orleans. Therefore, 1,500,000 cu ft per sec must be disposed of at the latitude of the Old River.

During the time of the project flood, 900,000 cu ft per sec will enter the head of the Atchafalaya River Basin, 650,000 cu ft per sec of which will flow down the Atchafalaya River and 250,000 cu ft per sec down the West Atchafalaya Floodway. The remaining 600,000 cu ft per sec of the quantity to be diverted will be carried down the Mississippi River approximately 20 river miles and passed through the Morganza Floodway. All the diverted flow, a total of 1,500,000 cu ft per sec, will meet at the lower end of the Atchafalaya River guide levees and will pass through the lower Atchafalaya Basin to be discharged into the Gulf of Mexico through two outlets.

---

[3] "Atchafalaya River Study," Report by the Mississippi River Comm., Vols. 1–3, May, 1951.

The development of the comprehensive flood-control plan is in an advanced stage. On the Mississippi River below the Old River, the authorized flood-control works are essentially complete, and the Mississippi River channel, with its present confining levees, can accommodate the part of the project flood for which it is designed.

In the Atchafalaya River Basin, many features have been constructed, and approximately 85% of the entire project is completed. However, maintaining the Atchafalaya River Basin protection levees to the designed grade is extremely difficult because of poor foundation conditions and levee subsidence.

For the proper functioning of the flood-control plan, efficient channels must be developed and maintained in both the Mississippi River and the Atchafalaya River. Any appreciable deterioration in the efficiency of either stream would have serious consequences. The lower Atchafalaya River Basin is filling with sediment at a rapid rate. Siltation from the Red River waters and the Mississippi River waters contributes to this action, but the scouring of the bed and banks of the Old River and the upper Atchafalaya River is probably the principal source of deposition. The filling of the lower Atchafalaya River Basin causes the flood-flow line to rise. Because the levees in this area cannot be raised with assurance, owing to the inadequate foundation conditions in various reaches, the lower Atchafalaya River must be increased in capacity by dredging and confinement. However, to do so without controlling the diversion tendency of the Mississippi River would be disastrous. Therefore, the initiation of control works at the head of the Atchafalaya River is necessary before relief of a critical situation in the lower Atchafalaya River Basin can be considered.

The Mississippi River is becoming increasingly important as a navigation artery. It is the main stem of a vast network of inland waterways, spreading through the central United States and connecting the Great Lakes to the Gulf of Mexico. The Gulf Intracoastal Waterway crosses the Mississippi River at New Orleans. Many strategically important commodities are moved in great quantity along this route, such as theever-increasing barge movement of refined petroleum products from the Texas and Louisiana refineries to the Mississippi, Missouri, and Ohio basins, and it also provides the principal access to rapidly growing offshore petroleum exploration and development.

In recent years the Atchafalaya River, together with the Old River, has become an important navigation artery. For the new high-powered, streamlined tows seeking the shortest route from Texas to points in the upper Mississippi River Basin, this route has many advantages—it is 172 miles shorter than by way of New Orleans, and it avoids the congestion in the New Orleans harbor.

Any major change in the lower Mississippi River that would alter the efficiency of the flood-control plan or the capacity of the Mississippi River and its network of navigable streams to support the continually increasing requirements of commerce would have great impact on the regional economy.

Uncontrolled diversion of the Mississippi River through the Atchafalaya River would result in a complete change in river channels, with the present Mississippi River channel below the Old River eventually being closed except for flood flows. Existing navigation patterns would undergo great changes. New navigation structures would be required, and old ones would need to be modified. The usefulness of existing flood-control structures, notably the Bonnet Carre, La., spillway structure and the Morganza spillway structure, would be impaired or destroyed. It would be necessary to rebuild communication networks. Salt-water intrusion would exist as far upstream as Baton Rouge, with tremendous, if not disastrous, effects on the industrial and domestic use of water.

The results of such a diversion would not be restricted to the area below the Old River. Present equilibrium conditions in the Mississippi River would be disturbed as far upstream as Vicksburg, Miss., and increased meandering would no doubt result, making obsolete a large part of the presently effective bank-stabilization works and threatening the almost complete main-line levee system. In addition, navigation during the period of adjustment would be extremely difficult because of increased velocities and stream meandering.

It is evident that if the Atchafalaya River were permitted to capture the Mississippi River, the effects would have far-reaching consequences. Complete diversion would alter seriously and permanently the way of life of a large and prosperous region.

The plan for preventing the diversion does not include changing any existing condition but is designed to retain everything presently existing. Any works constructed to prevent uncontrolled diversion must provide for the regulation of flows so that the many exacting requirements to preserve present conditions can be satisfied. It appears necessary to provide permanent control structures so that flows may be regulated as required and maintenance of the desired water-sediment relationship can be assured.

Any plan for the control of flows at the Old River will require structures in the Old River, in the Atchafalaya River, on the west bank of the Mississippi River upstream from the Old River, or at a combination of these locations. Regardless of site, the structure or structures must be capable of passing the project-flood flows in the quantities already specified in the flood-control plan. They must prevent permanent capture of the Mississippi River by the Atchafalaya River and be flexible in operation in order to maintain the present distribution of flows of water and sediment as nearly as possible. The plan must provide for navigation, offer assurances of long life, and be reasonable in first cost and subsequent maintenance. The nature of the problem and its importance to the national economy require a safe and permanent solution.

Several plans were considered, among which were some that required major structures in the Old River; structures in the Atchafalaya River near the lower end of the guide levees; dams across the Atchafalaya River in its upper reaches, in conjunction with weirs, overflow dams, and notches; construction of a series of low-sill dams of stone or other nonerodible materials in the Atchafalaya River; and projects to achieve friction control by increasing channel roughness.

All the preceding plans were investigated in as much detail as necessary and discarded because they failed to meet the desired criteria.

The remaining possibility was to locate the proposed structures where flow from the Mississippi River could be controlled before it reached the Red River backwater area. The backwater area presently acts as a distributing reservoir for overbank flows at this latitude. Therefore, structures that were placed to take advantage of the distributing reservoir without altering present conditions would be ideal from both an engineering and an economic viewpoint. The plan that was selected includes control structures on the west bank of the Mississippi River above the Old River with a lock in the Old River (Fig. 3). Such a plan would satisfy the conditions of effectiveness, permanence, safety, and reasonable cost.

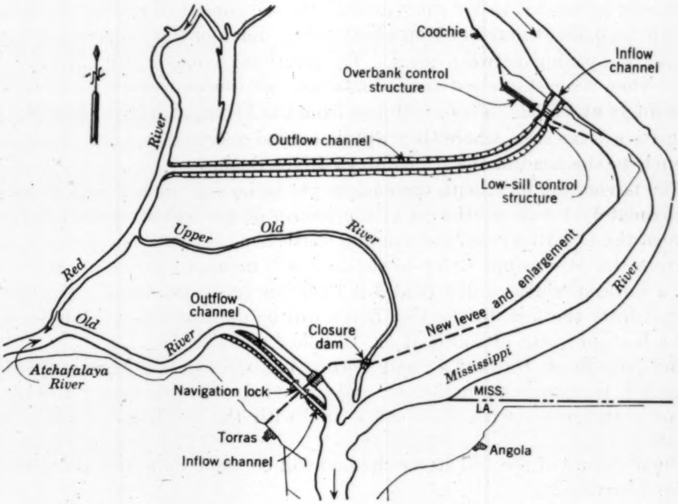

Coochie
Inflow channel
Overbank control structure
Outflow channel
Low-sill control structure
River
Red
Upper Old River
Old River
Outflow channel
Closure dam
New levee and enlargement
River
Mississippi
MISS.
LA.
Atchafalaya River
Navigation lock
Torras
Inflow channel
Angola

FIG. 3.—THE OLD RIVER CONTROL PLAN OF IMPROVEMENT

The selected plan also includes two control structures on the west bank of the Mississippi River, approximately ten miles upstream from the Old River; a navigation lock connecting the Mississippi River and the Old River; an earthfill dam in the Old River; approach channels for the lock; an inlet and an outlet channel for the control structures; and the enlargement and extension of the main-line Mississippi River levee in that location. The two control structures will accommodate a flow of approximately 700,000 cu ft per sec from the Mississippi River during the project flood.

The smaller of the two control structures will be a low-sill structure of reinforced concrete, which will be 566 ft long between abutments. It will have eleven gate bays, each having a 44-ft clear opening between piers. The weir crests will be at elevations that will permit flows at all stages to pass from the Mississippi River into an outflow channel extending to the Red River.

The inflow channel from the Mississippi River to the low-sill structure will be approximately ½ mile long, with a bottom width of 1,000 ft at El. 5 below mean sea level.

The outflow channel from the low-sill structure to the Red River will be approximately 7 miles long, with a bottom width of 900 ft at El. 8 below mean sea level near the structure and at El. 10 below mean sea level where it enters the Red River. For favorable tailwater conditions the outflow channel will have a capacity of 300,000 cu ft per sec at bankfull stage on the Mississippi River.

The second of the two structures upstream from the Old River is an overbank control structure that will resemble the Morganza control structure, which was completed in February, 1953. It will consist of reinforced concrete and will be 3,356 ft long between abutments. Each of the 73 bays will have a 44-ft clear opening between piers. The overbank control structure becomes useful when river stages get above bankfull, and the weir elevation is planned accordingly at El. 52. Flows will pass from the Mississippi River into the Red River backwater area, where they will be carried overland and through natural channels to the head of the Atchafalaya River Basin.

The navigation lock, with dimensions yet to be determined, will be located approximately 1 mile southwest of the mouth of the Old River in a land cut between the Old River and the existing south bank levee. The forebay channel from the Mississippi River to the lock will be approximately ½ mile long, with a bottom width of 250 ft at El. 10 below mean sea level. The tailbay channel from the lock to the Old River will be approximately 1½ miles long, with a bottom width of 250 ft at El. 10 below mean sea level.

Between Black Hawk, La., and Torras, La., the existing Mississippi River levees will be raised, strengthened, and extended where necessary to link the proposed structures with each other and with the existing main-line levee system.

Final closure of the Old River channel will be made by a hydraulically constructed earth dam.

After favorable channel alinement has been obtained in the Red River and the Atchafalaya River between the outflow channel and the vicinity of Simmesport, this reach will be stabilized by bank-protection works.

It was imperative that construction be initiated promptly and that it proceed in a prescribed order because the earliest date when all construction can be completed under favorable conditions almost coincides with the estimated date of the earliest probable critical stage of development in the capture of the Mississippi River by the Atchafalaya River. Any postponement may add to the construction difficulties and may increase costs. Construction was therefore begun in the latter part of 1955.

The first item planned for construction was the low-sill structure with its inflow and outflow channels and adjoining embankments. It is the most desirable control unit to have available should some unforeseen condition make it advisable to close the Old River entirely before the project is completed.

The overbank control structure and upper levee embankments should be completed next. Then the overbank gap at the Old River can be reduced in width or closed entirely, if necessary, to retard or prevent a possible dangerous acceleration in channel enlargement of the Old River and the Atchafalaya River.

The third item planned was the building of the navigation lock and its approach channels. Finally, the closure dam across the Old River and the remaining levees should be constructed. It is important that these structures be undertaken and completed during favorable low-water stages.

The estimated cost of the project, excluding the navigation lock, is $47,000,000. The United States Congress has authorized the entire plan, but has limited expenditures to the flood-control features pending a determination by the Chief of Engineers of the required size and estimated cost of the navigation lock.

CONCLUSIONS

The Old River control project is a large undertaking and will require the solution of many unusual problems in foundations and construction procedures. Its great importance to the whole region of the lower Mississippi River valley makes it necessary to execute all its features in an orderly and positive fashion. There is every reason to believe that the project can be completed in time to meet any emergencies caused by the Mississippi River.

HYDRAULIC REQUIREMENTS

By Eugene A. Graves,[1] M. ASCE

Synopsis

The streams and floodways in the Old River area form a complex system. Observed discharges and planned discharges illustrate the magnitudes involved. The control of the Old River has become necessary as a result of the possibility that the Mississippi River will adopt the course of the Atchafalaya River. The selection of a site for the control was influenced by several factors. An unusual situation exists with respect to the determination of prevailing and prospective tailwater conditions. The expected discharges through the structures are compared with natural flows. The structures permit the regulation of the flow, if required, to reduce the rate of enlargement of the Atchafalaya River, and will give assurance against the change of course of the Mississippi River.

Introduction

The Old River-Atchafalaya River system is the farthest upstream distributary of the Mississippi River. The bifurcation is at a point approximately 313 miles upstream from the mouth of the parent stream. The distance from the departure point to the mouth of the Atchafalaya River is approximately 140 miles. Thus, the slope through the Old River-Atchafalaya River system is more than twice that in the Mississippi River below their common point. A raft of logs, which blocked the Atchafalaya River from its head for a distance of approximately 20 miles downstream, was removed prior to 1855. Since 1870 the enlargement of the Atchafalaya River, a desirable development with respect to accommodating flood flows, has concerned those who feared that it might result in the change of course of the Mississippi River. Such a change seems increasingly probable unless preventive measures are taken. The hydraulic considerations controlling the design of the adopted plan will be described.

Stream and Basin Description

The Old River, which follows the lower arm of a cutoff meander loop of the Mississippi River, extends for 6 miles from its junction with the Mississippi River to a point at which it joins the Red River to form the head of the Atchafalaya River. A project plan map of the area, on which the plan has been superimposed, is shown in Fig. 1. In the Old River, channel depths vary from 60 ft to 130 ft. The enlargement and increased natural diversion at the Old River has been described elsewhere.[2] At the junction of the Old River with the Mississippi River, there is a gap of approximately 3 miles in the Mississippi

---

Note.—Published, essentially as printed here, in March, 1956, in the Journal of the Waterways Division, as *Proceedings Paper 907*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Chief, Special Studies Section, Mississippi River Comm., Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss.

[2] "The General Problem," by John R. Hardin, in "Old River Diversion Control: A Symposium," *Transactions*, ASCE, Vol. 123, 1958.

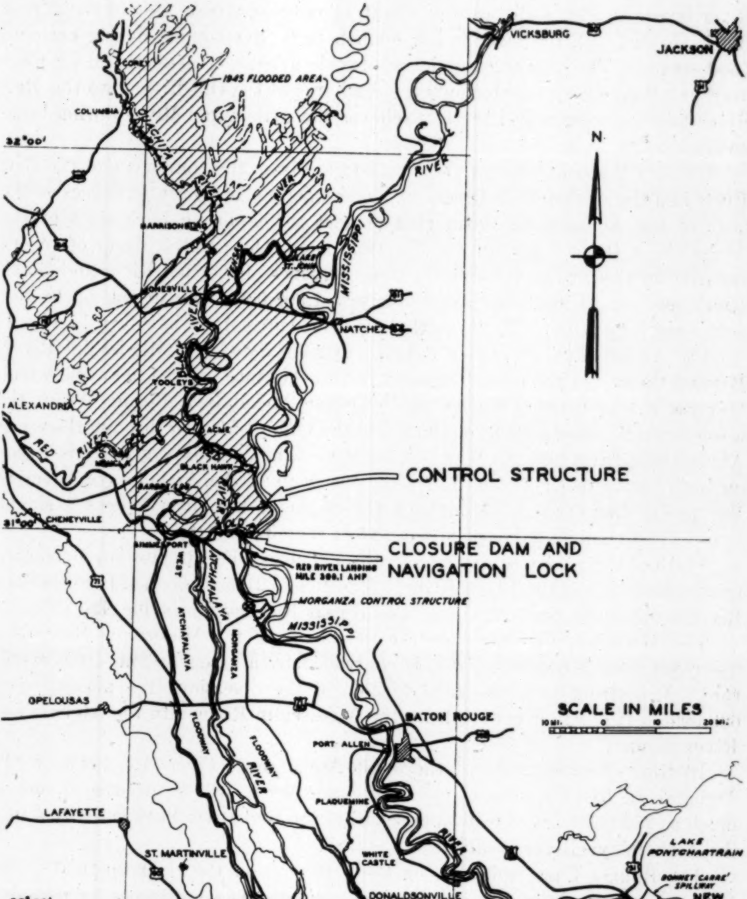

VICKSBURG
JACKSON
N
CORey
1945 FLOODED AREA
COLUMBIA
32°00'
OUACHITA
RIVER
TENSAS
RIVER
HARRISONBURG
LAKE ST. JOHN
MISSISSIPPI
RIVER
JONESVILLE
NATCHEZ
BLACK
RIVER
TOOLEYS
ALEXANDRIA
ACME
RED
RIVER
BLACK HAWK
BARBRE LDG.
CONTROL STRUCTURE
31°00'
CHENEYVILLE
SIMMESPORT
OLD
CLOSURE DAM AND NAVIGATION LOCK
RED RIVER LANDING MILE 300.1 AHP
WEST ATCHAFALAYA
ATCHAFALAYA
MORGANZA
MORGANZA CONTROL STRUCTURE
MISSISSIPPI
OPELOUSAS
FLOODWAY
FLOODWAY
RIVER
BATON ROUGE
SCALE IN MILES
10 MI.
0
10
20 MI.
PORT ALLEN
PLAQUEMINE
LAFAYETTE
RIVER
WHITE CASTLE
LAKE PONTCHARTRAIN
BONNET CARRE' SPILLWAY
ST. MARTINVILLE
DONALDSONVILLE
30°00'
NEW ORLEANS
ABBEVILLE
NEW IBERIA
WESTWEGO
CHARENTON
FRANKLIN
VERMILION BAY
MORGAN CITY
BERWICK
LOWER ATCH.
WAX LAKE OUTLET
92°00'
91°00'
GULF OF MEXICO

FIG. 1.—PROJECT PLAN MAP

River levee, and for a distance of about 14 miles upstream to the vicinity of Black Hawk (La.), the levee is low enough to be overtopped during extreme flood stages. The gap and low levee provide a large cross-sectional area for overbank flow, which supplements the discharge of the Old River into the Red River backwater area and thereby relieves the Mississippi River during large floods.

The Red River backwater area is upstream from the confluence of the Red River and the Atchafalaya River. It is bounded on the south by levees at the head of the Atchafalaya Basin that protect the lands on both sides of the Atchafalaya River. Except for the 1927 flood, the greatest flood of record occurred in 1945, when the stage at the confluence of the Red River and Black River reached El. 60.1 and approximately 1,300,000 acres in the backwater area were inundated. The area that was flooded in 1945 is shown in Fig. 1.

The Atchafalaya River is a natural distributary of the Mississippi River. Beyond the end of the leveed segment it branches into several channels which traverse a lake country known as the lower Atchafalaya Basin. From the lower basin the discharge is conducted to the Gulf of Mexico through the lower Atchafalaya River and the Wax Lake outlet. The Atchafalaya River is leveed on both banks to river mile 52 (below the head of the Atchafalaya River at Barbre Landing (La.). Throughout this segment the river follows a single, well-defined channel varying in depth from 80 ft to 180 ft.

South of the village of Acme (La.) the Red River has its flow supplemented by the discharge of the Black River. The Black River is termed the Ouachita River north of its confluence with the Tensas River at Jonesville, La. (Fig. 1).

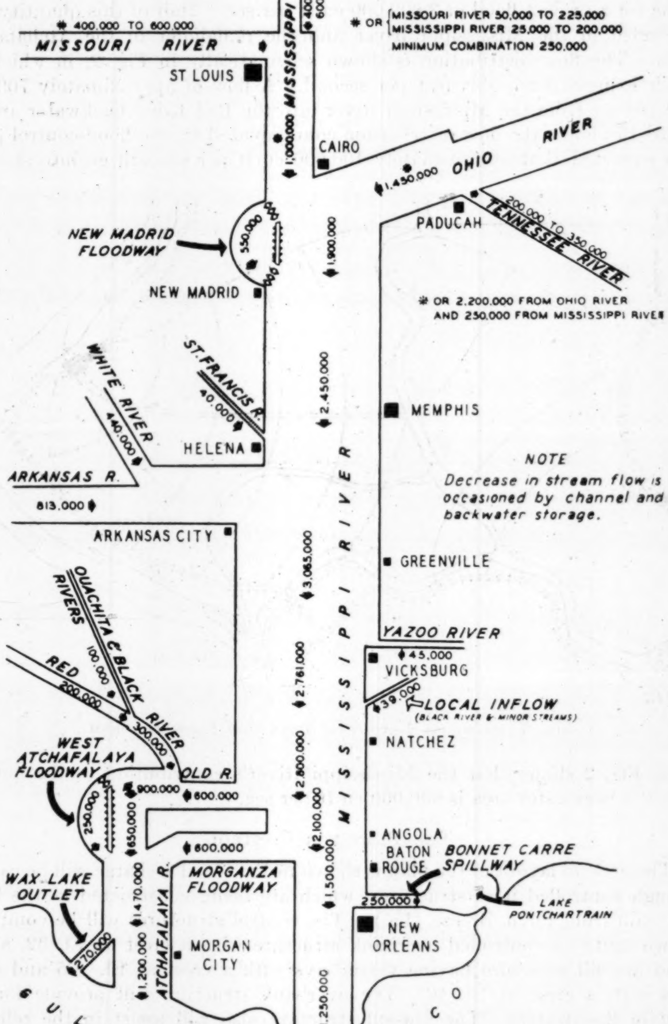
The Morganza Floodway, on the west bank of the Mississippi River just upstream from Morganza (La.), is controlled by a recently completed structure. The structure is designed to control the diversion of a maximum of 600,000 cu ft of water per sec from the Mississippi River into the Atchafalaya River Basin.

In time of extreme flood, the West Atchafalaya Floodway, between the Atchafalaya River levee and the west guide levee, will be used to divert as much as 250,000 cu ft of water per sec from the Red River backwater area into the Atchafalaya River Basin.

The Bonnet Carre spillway, on the east bank of the river upstream from New Orleans (La.), is designed to relieve the downstream levees by releasing quantities of water as great as 250,000 cu ft per sec into Lake Pontchartrain and the Gulf of Mexico.

Observed Flood Flows

Maximum recorded discharges were 1,515,000 cu ft per sec in the Mississippi River south of the Old River in 1945; 1,977,000 cu ft per sec in the Mississippi River north of the Old River in 1937; 514,000 cu ft per sec in the Old River in 1937; and 661,000 cu ft per sec in the Atchafalaya River at Simmesport (La.) in 1945. An observation of 1,595,000 cu ft per sec was made in the Mississippi River south of the Old River in 1882. However, the discharge was influenced by levee crevasses.



* 400,000 TO 600,000
MISSOURI RIVER
MISSISSIPPI R.
400,000 TO 600,000 *
* OR MISSOURI RIVER 50,000 TO 225,000
MISSISSIPPI RIVER 25,000 TO 200,000
MINIMUM COMBINATION 250,000
ST LOUIS
1,000,000
CAIRO
1,450,000
OHIO RIVER
PADUCAH
200,000 TO 350,000
TENNESSEE RIVER
* OR 2,200,000 FROM OHIO RIVER
AND 250,000 FROM MISSISSIPPI RIVER
NEW MADRID FLOODWAY
550,000
1,900,000
NEW MADRID
ST. FRANCIS R.
40,000
WHITE RIVER
440,000
2,450,000
MEMPHIS
HELENA
ARKANSAS R.
813,000
NOTE
Decrease in stream flow is occasioned by channel and backwater storage.
ARKANSAS CITY
3,065,000
GREENVILLE
MISSISSIPPI RIVER
YAZOO RIVER
45,000
VICKSBURG
39,000
LOCAL INFLOW
(BLACK RIVER & MINOR STREAMS)
NATCHEZ
OUACHITA & BLACK RIVERS
100,000
RED RIVER
200,000
300,000
2,761,000
WEST ATCHAFALAYA FLOODWAY
OLD R.
900,000
600,000
2,800,000
250,000
650,000
600,000
2,100,000
ANGOLA
BATON ROUGE
BONNET CARRE SPILLWAY
LAKE PONTCHARTRAIN
250,000
1,500,000
MORGANZA FLOODWAY
WAX LAKE OUTLET
270,000
1,470,000
ATCHAFALAYA R.
1,200,000
MORGAN CITY
NEW ORLEANS
1,250,000
GULF OF MEXICO

FIG. 2.—FLOW DISTRIBUTION FOR PROJECT FLOOD

Flow Distribution for Project Flood

The plan for flood control in the vicinity of the Old River is based on providing for a project flood of 3,000,000 cu ft per sec. Half of this quantity will be carried in the Mississippi River and the remainder in the Atchafalaya Basin. The flow distribution is shown schematically in Fig. 2, in which all flow is expressed in cubic feet per second. A flow of approximately 700,000 cu ft per sec from the Mississippi River into the Red River backwater area is required to effect the flow distribution contemplated by the flood-control plan. It is expected that approximately 100,000 cu ft per sec will go into storage. Thus, Fig. 2 shows that the Mississippi River's contribution to the outflow from the backwater area is 600,000 cu ft per sec.

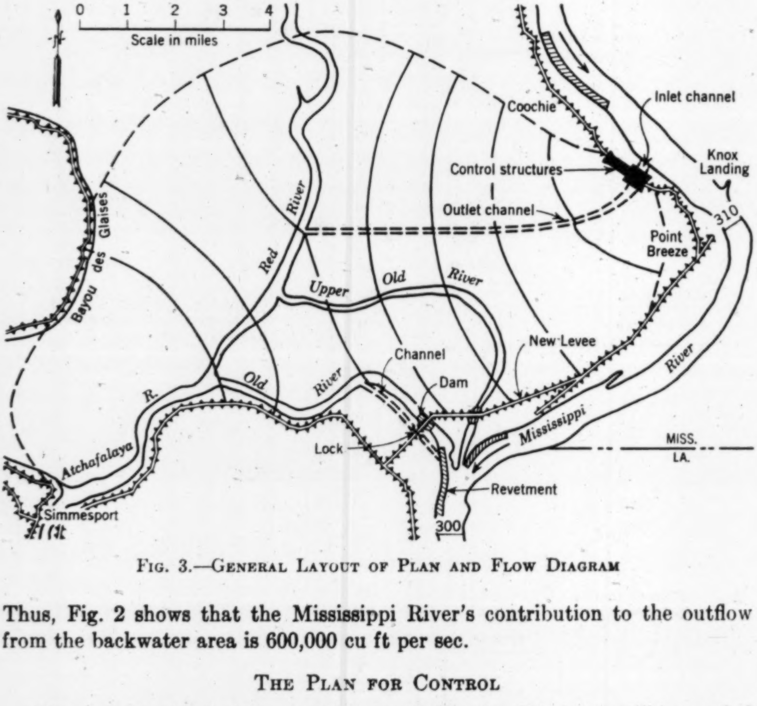

0
1
2
3
4
Scale in miles
Coochie
Inlet channel
Knox Landing
Control structures
Outlet channel
310
Point Breeze
Red River
Upper Old River
Bayou des Glaises
New Levee
Channel
Dam
Old River
R.
Atchafalaya
Lock
Mississippi River
MISS.
LA.
Revetment
Simmesport
300

Fig. 3.—General Layout of Plan and Flow Diagram

The Plan for Control

The general layout of the plan is shown in Fig. 3. Discharge will be carried through controlled inlet structures, which are being constructed in the levee upstream from Point Breeze (La.). The control structures will be composed of two units—a controlled overbank structure with a crest at El. 52, and a gated low-sill structure having three bays with a crest at El. −5 and eight bays with a crest at El. 10. The overbank structure will provide for the relief of floodwaters. The low-sill structure also will assist in the relief of floodwaters, but will be deep in order to permit discharge during moderate and low stages of the Mississippi River. The low-sill structure will have an excavated channel leading to the Red River.

Site Selection

The selection of a site was a result of making a series of separate decisions. The area near the Mississippi River was chosen because of the greater protection against the loss of the structures by degradation of the outlet channel such as has taken place in the Atchafalaya River.

After it was decided to place the structures near the Mississippi River bank, the next step was to find a site where bank caving appeared improbable. Knox Landing (La.) met this criterion. Several surveys that have been made since 1823 show that between 1823 and 1883 the river moved to the hard point (Point Breeze) near Knox Landing, and, since that time, the bankline at this site has been stable.

Another important consideration was that it was necessary for the chosen site to be of such a nature that the diverted water would contain bed sediment in about the same proportion as that for the Mississippi River. Analytical studies and model studies indicate that the selected site would be satisfactory. It is in a relatively straight stretch of river, and since 1895 the cross section (Fig. 4) has been wide, shallow, and relatively flat. Reasoning based on such considerations and confirmed by measurements indicates that the distribution of sediment is relatively uniform across the section.

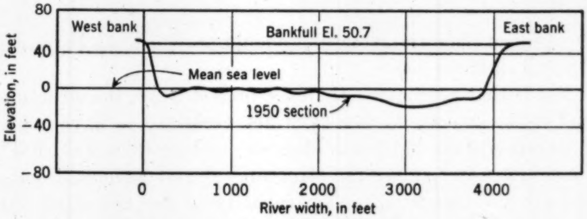


FIG. 4.—MISSISSIPPI RIVER CROSS SECTION

Two sites farther upstream were considered and discarded. One, near Coochie (La.), is subject to bank caving and appears less favorable for the diversion of bed load. Another site in the vicinity of Black Hawk is almost opposite the lower end of a point bar that has been extending slowly downstream for many years. At this site probably too much bed load would be diverted, and it is also probable that bar building will continue, thus making the maintenance of the inlet channel quite expensive.

With respect to the protection of the outlet channel against erosion, the chosen site has a marked advantage. A study of the geology of the area revealed clay-plug deposits across which the outlet channel can be made to pass, thus making it easier to protect the structures against channel raveling.

The chosen site is satisfactory in so far as the capacity of the structure to discharge water at the necessary rate is concerned. Because the water-path distance to the Atchafalaya River is almost the same for any of the considered sites and because in time of an extreme flood the water-surface slope in the backwater area is small, all the sites that were considered had tailwater heights that were approximately equal. However, the locations that were more upstream were advantageous with respect to headwater because for project-flood distribution the difference in elevation between the Mississippi River and the Red River backwater near the Old River is quite small. Thus, the Mississippi River slope from the Old River to the structure site is effective for the most part as a gain in input capacity. Therefore, the more upstream locations would have been advantageous. However, the input capacity at the chosen site, which will be examined quantitatively herein, was satisfactory, and the many important considerations cited previously outweigh the hydraulic advantage.

After the choice had been narrowed to a small area, it was necessary to select a site having the best foundation conditions. These studies have been reported elsewhere.[3]

Flow Requirements

The requirement that assurance must be given against the capture of the Mississippi River by the Atchafalaya River is, in one aspect, also one for the capacity to regulate flow. This requirement resulted in the selection of the general type of plan adopted. In addition, there are several other flow requirements:

1. For a project flood the structures should provide a discharge capacity of at least 700,000 cu ft per sec.
2. For floods that are smaller than the project flood, the structures should provide sufficient capacity so that it is not necessary to have increased frequency of operation of the Morganza Floodway and the Bonnet Carre Floodway.
3. At medium water stages the structures should have sufficient capacity to approximate the natural diversion capacity of the Old River during the period just prior to 1950. The provision of adequate capacity will assist in maintaining the regimen of the Mississippi River south of the Old River and in developing channels through the lower Atchafalaya River Basin. A change of regimen of the Mississippi River south of the Old River, if it is in the direction of causing higher stages, could necessitate higher levee grades. If the change were in the direction of causing lower stages, levee set-backs and increased maintenance of bank-protection works might be needed and the intake capacity of the Morganza Floodway would be reduced.
4. The structures should not reduce low-water flow to such a degree that navigation or water use in the Atchafalaya Basin are harmed.

Headwater Curves for the Structures

In order to appraise the discharge performance of the structures, it is necessary to consider the Mississippi River stages that will constitute the headwater on the structures. Excellent gage records and discharge records are available for Red River Landing, an important gaging station which is situated at river mile 300.1 above head of passes (AHP), approximately ¾ mile downstream from the mouth of the Old River. The rating curve for 1950 conditions was used. Assuming that the Morganza Floodway was operating, computed flow lines and the results of model tests were used to extend the rating curves of the 1950 conditions. Using stages from the rating curve and assuming that the Old River was closed, flow lines were computed to the proposed structure site. Channel friction factors were obtained from flow-line computations for observed conditions. For high stages the estimated overbank n-value was the Manning value of 0.150. The headwater curves are shown in Fig. 5.

---

[3] "Foundation Design," by Willard J. Turnbull and Woodland G. Shockley, in "Old River Diversion Control: A Symposium," *Transactions*, ASCE, Vol. 123, 1958.

For existing conditions the discharge capacity of the Mississippi River is greater upstream from the Old River than in the reaches downstream. This greater capacity is because it has been necessary for the upstream channel to accommodate itself to carry the total discharge, which, at the Old River, is divided between that stream and the lower Mississippi River. After the structures have been operating for several years, the capacity of the Mississippi River channel between the Old River and the site of the structures is expected to be reduced to approximately that presently (1958) existing downstream from the Old River. Therefore, the headwater curves, which are based on existing channel conditions, will result in slightly lower headwaters than would be expected after the structures have been in operation for an indefinite time and the downstream channel has had the opportunity to become adjusted.

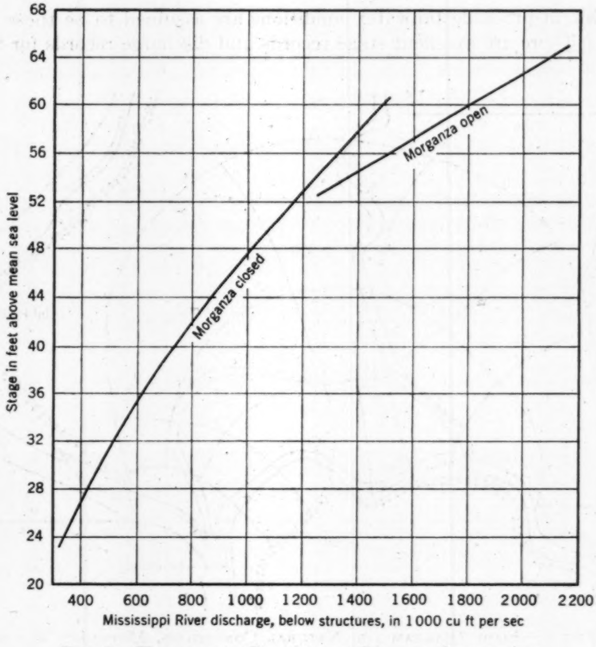


FIG. 5.—HEADWATER CURVES FOR CONTROL STRUCTURES

Mississippi River rating curves are subject to an irregular periodic fluctuation, being alternately higher and lower than average. Such a fluctuation has been observed at Red River Landing. A study of discharge records since 1893 shows that in that year and in the period from 1911 to 1912 the average rating curves were virtually the same as those that existed in 1950. The 1950 rating curves yield stages as low as any that have occurred in other years for the same discharges, and, consequently, the use of these headwater curves will result in conservative estimates of the discharge capacity of the structures.

MAXIMUM PROBABLE TAILWATER CONDITIONS

Maximum probable tailwater conditions are assumed to be those existing in 1950. There are excellent stage records and discharge records for Simmesport at the upper end of the leveed Atchafalaya River. Flow lines from Simmesport to the mouth of the Old River have established that in this river the Manning n-value averages approximately 0.030 at bankfull stages and approximately 0.035 at higher stages. A flow line for the crest of the 1950 flood indicates that the overbank n-value for the backwater area is 0.175, which was obtained simultaneously with an Old River channel value of 0.035. The flow diagram for this condition is shown in Fig. 6.

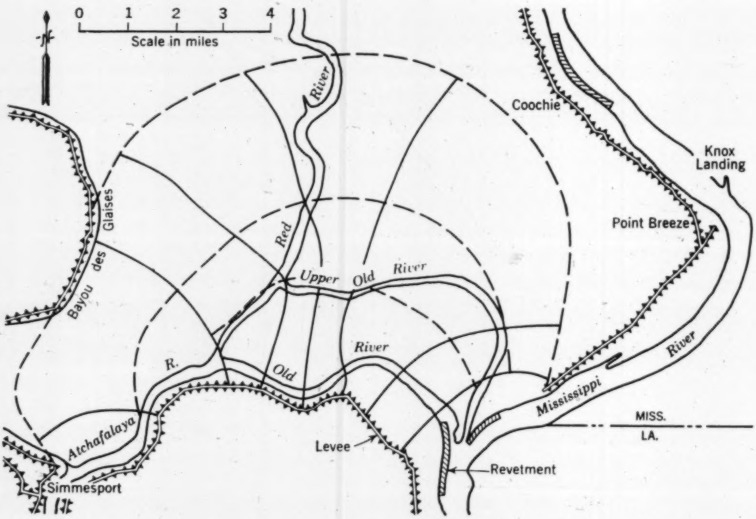


FIG. 6.—FLOW DIAGRAM FOR NATURAL CONDITIONS, MODERATE FLOOD

Assuming that the Old River is closed, flow lines were computed from Simmesport to the structures through the Red River, the outlet channel, and the overbank. The flow diagram for this condition is shown in Fig. 3. The n-values were 0.030 for the Red River, 0.030 for the outlet channel, and 0.175

for the overbank. Elements for the Red River are from a survey made in 1950. It was assumed that the outlet channel has a bottom which is 900 ft wide at El. −8 and slopes to El. −10 at the Red River, which approximates the elevation of the bottom of the Red River in this vicinity.

The nature of the backwater is such that the tailwater stage depends on the flow from the tributary streams as well as the flow from the structures. For this reason the curves take the form of a family as shown in Fig. 7. By using the data obtained from the measurements over a period of many years, the tributary inflows were plotted against the concurrent Mississippi River discharges. With this plot as a guide an envelope curve embodying the maximum probable concurrent contributions of the tributaries was drawn across the tailwater curves. This envelope curve was used to define the upper limits of tailwater for which the stilling basins were designed.

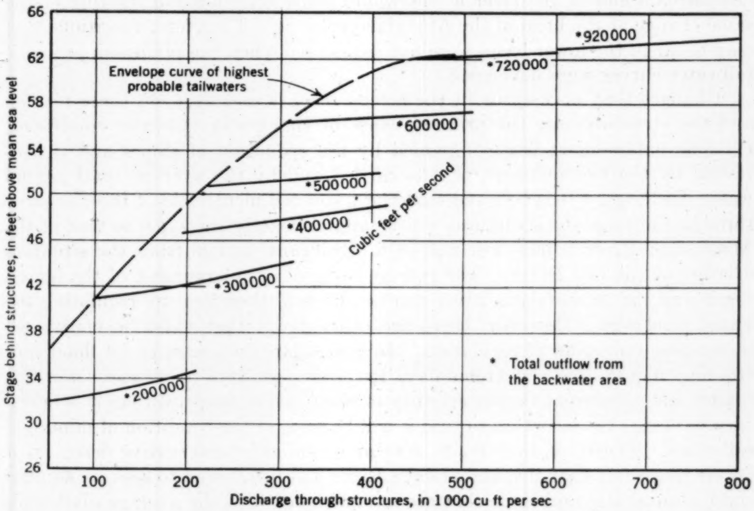


FIG. 7.—CURVES OF MAXIMUM PROBABLE TAILWATER CONDITIONS

MINIMUM PROBABLE TAILWATER CONDITIONS

In estimating future tailwater conditions, it is necessary to consider the changes that are likely to affect the tailwater. Changes that may occur in the lower Atchafalaya Basin, below the end of the river levees (Fig. 1), were considered, but it was determined that they were not likely to cause any significant variation in the tailwater conditions for the reasons that are presented subsequently. Below the end of the river levees, most of the flood flow is carried overbank within the floodway. There has been a heavy deposit of sediment in the overbank area, causing higher stages for given discharges. There has also been some local scour in the relatively small channels that lead toward the gulf. However, as a result of the deterioration of the overbank area, the overall trend has been toward the reduction of discharge capacity. Since 1932 the trend toward the reduction of discharge capacity has been especially noticeable in spite of much improvement dredging in this area, most of which was accomplished between 1932 and 1935.

The deterioration of the flood-carrying capacity of the floodway south of the end of the river levees has reached the stage at which improvement work and encouragement of channel development by scour will be required to accommodate the planned project flow of 1,500,000 cu ft per sec. However, it is not likely that a drastic lowering of stages at the end of the river levees will result. Moreover, backwater computations indicate that a large change of stage at the downstream end of the river levees would cause only a small fraction of the same change at the head of the Atchafalaya River. Therefore, variations that may occur in the lower basin were not considered when the minimum probable tailwater curves were developed.

Changes that may occur in the leveed reaches and between these reaches and the structures are the primary cause of changes in tailwater conditions. Channel enlargement is accompanied by the reduction of slopes and is controlled by the bed sediment that is carried and by the average total annual water discharge. It is believed that the water-sediment ratio of the discharge through the proposed structures will be approximately the same as that of the Mississippi River. This will not be a significant change from the situation existing at the Old River. The rate and degree of enlargement of the leveed reaches of the Atchafalaya River can be affected, therefore, by regulating the water discharge. However, there are other needs that must be considered in deciding on a plan of regulation. In particular, the necessity for flood control may require that the Atchafalaya River be permitted to enlarge. For this reason and because of the uncertainty of what future needs will be, it is necessary to design the structure so that it will be safe for the condition of minimum tailwater. Therefore, in order to develop a safe and conservative design, it is better to assume that the Atchafalaya River will be allowed to seek an equilibrium channel size and slope that are in agreement with the average discharges which will occur if there is only a minimum degree of regulation.

In the leveed section of the Atchafalaya River, a flattening of the slope has been observed for many years, and, as indicated by the foregoing, this trend will probably continue. Analytical sediment studies have indicated that for three years, during which both small floods and large floods were experienced, the equilibrium slope for the discharges occurring for natural conditions would be approximately one-half as steep as the observed slope. The lowering, which would result from a reduction to equilibrium slopes, was computed to be approximately 10 ft at high stages. Such a lowering constitutes an increase in discharge capacity for the Atchafalaya River of approximately 200,000 cu ft per sec for equivalent high stages. In developing the curves of minimum tailwater, the 10-ft lowering was assumed to be obtained from project-flood stage to bankfull stage. For discharges less than bankfull, the lowering was gradually reduced to zero at mean sea level.

Between the head of the Atchafalaya River and the structures, much of the flood flow will be overbank, at least for several years after the structures are in operation. For such circumstances the slopes will be rather flat. Because the lowering at the head of the Atchafalaya River will reduce the depth of overbank flow, thereby tending to increase the slope to the structures, there may be a considerable increase in size of the outlet channel without producing any additional lowering, and, therefore, no additional lowering has been incorporated in the curves of minimum tailwater.

The group of minimum-tailwater curves is shown in Fig. 8. An envelope curve embodying the minimum probable concurrent contributions of the backwater tributaries has been drawn across the tailwater curves in order to define the lower tailwater limits for which the stilling basins must be designed. In the routings that will be cited subsequently, the envelope curves were not used; the whole curve group was used in order to consider the effects of tributary inflows.

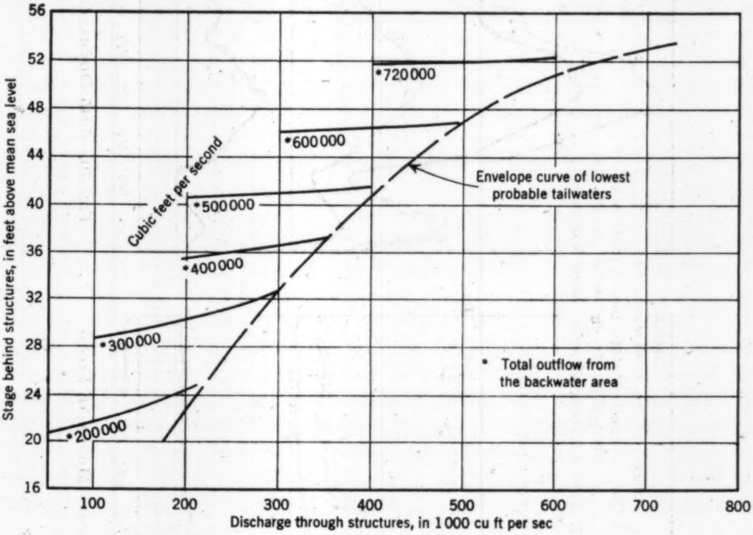


FIG. 8.—CURVES OF MINIMUM PROBABLE TAILWATER CONDITIONS

STRUCTURE PERFORMANCE AT PROJECT FLOOD

As stated previously, the project provides against a maximum outflow of 3,000,000 cu ft per sec at the latitude of the Old River. Synthetic hydrographs of the concurrent inflows of the several streams have been prepared so that the resulting outflow agrees with the project objective. For 1950 conditions and for conditions with the structures in place, routing tables have been prepared for the Mississippi River and for the backwater area. With the Old River closed the headwater curves and tailwater curves that were derived for the 1950 conditions were used. Routings of the project flood, with wide-

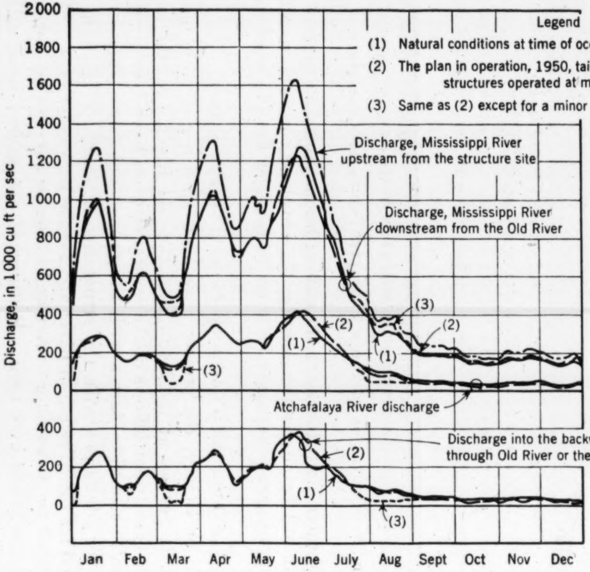


FIG. 9.—1943 DISCHARGE HYDROGRAPHS—MODERATE FLOOD

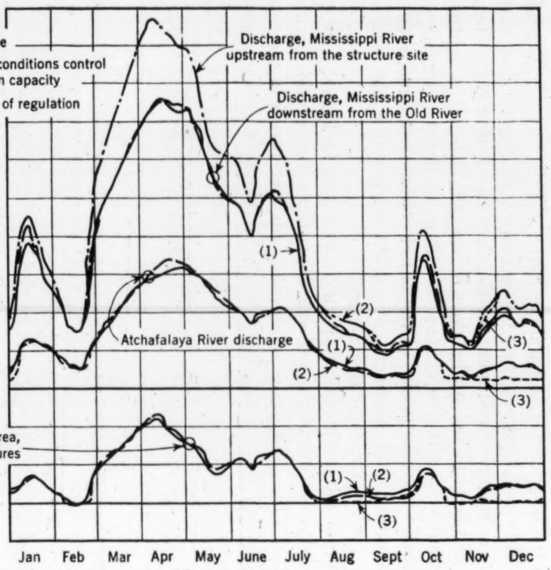


FIG. 10.—1945 DISCHARGE HYDROGRAPHS—LARGE FLOOD

open control structures (the lock being closed), show a maximum discharge of 700,000 cu ft per sec through the structures. It is indicated that if the structures are wide open they will deliver (under the project-flood conditions) approximately 100,000 cu ft per sec more to the backwater area than would the Old River. This much excess capacity is desirable in order to make the best use of the available storage during floods that are slightly smaller than the project flood, and to provide flexibility of distribution.

Large and Moderate Floods

The plan envisages an overbank structure with a crest at El. 52 having 3,200 ft of clear opening, combined with a low-sill structure having 352 ft of clear opening with a crest at El. 10 and 132 ft of clear opening with a crest at El. −5. The preceding dimensions have been determined to be the most advantageous and economical. For the project flood similar results can be obtained if the deeper structure is omitted and the overbank structure is made longer. Such an arrangement would be less expensive but would not meet the moderately large-flood, medium-stage, and low-water criteria. In investigating the requirements for satisfying these criteria, routings were made for more moderate floods. These routings are for 1943, in which moderate flooding occurred that almost forced the operation of Bonnet Carre Floodway, and for 1945, during which the second largest flood of record in this vicinity was experienced. The results are shown in Figs. 9 and 10. In each figure hydrographs numbered "(1)" refer to the Old River open condition, and hydrographs numbered "(2)" refer to the plan in operation with structures fully open. The former hydrographs represent the published discharge that occurred under natural conditions. For the hydrographs numbered "(2)" the condition of the Mississippi River is that of the time of occurrence. An exception to the foregoing statements is that during the flood periods the flows of the Atchafalaya River and the Mississippi River south of the Old River were routed for both the open and controlled conditions by tables incorporating 1950 conditions in those rivers. For the No. 2 hydrographs the "maximum probable" tailwater curves were used.

With respect to the large flood criterion, Fig. 10 shows that at the crest of a flood as great as that of 1945 the structures will approximate the performance of the Old River. For a moderate flood (Fig. 9), the proposed plan again can duplicate the discharge of the Old River. The structures can approximate the performance of the Old River at about the discharge of the Mississippi River (1,250,000 cu ft per sec downstream from the Old River), requiring the operation of one of the floodways. Thus, the plan would not force the operation of the control structure at either Morganza or Bonnet Carre whenever they would not otherwise be required to open.

Medium Stages

The performance of the structures is satisfactory during medium stages. The discharge can be made to approximate the flow that would occur under natural conditions.

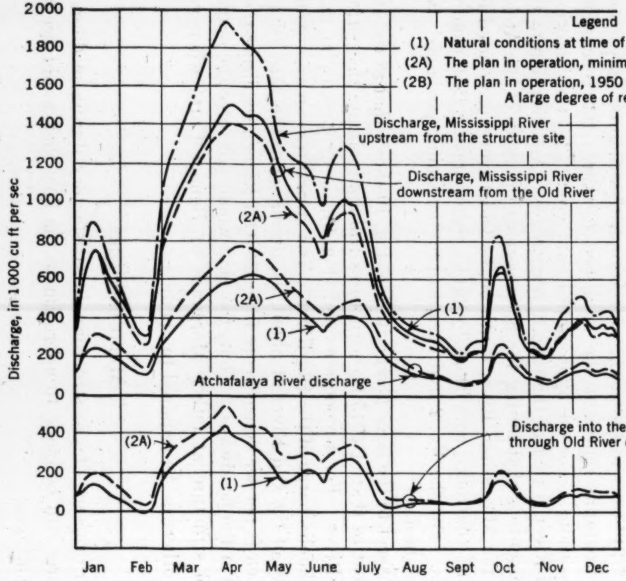


FIG. 11.—EFFECTS OF MINIMUM TAILWATER CONDITIONS (1945)

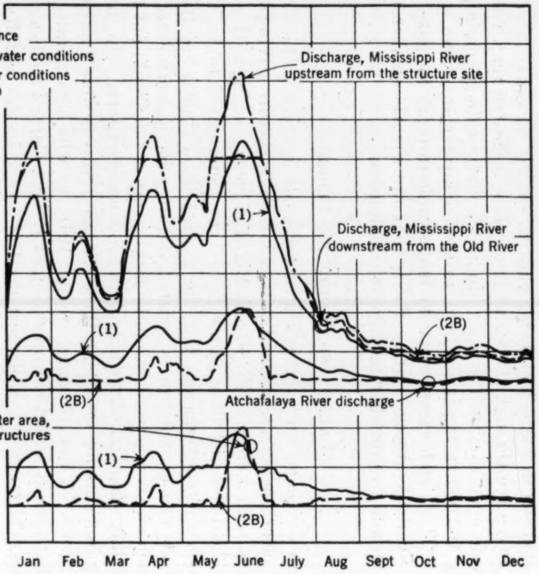


FIG. 12.—POSSIBILITIES FOR REGULATION OF FLOW (1943)

Low-Water Flow

At extreme low-water stages the flow is dependent on the slot consisting of three bays extending to El. −5 and having a clear opening of 132 ft. The lowest stage recorded at Red River Landing since 1900 was at El. 4.7 on November 3, 1939. On that date the measured discharge was 14,000 cu ft per sec through the Old River and 19,000 cu ft per sec in the Atchafalaya River at Simmesport. Under identical circumstances but with the structures in place and conditions in the Atchafalaya River channel as they were in 1950, the indicated discharge through the slot is 11,000 cu ft per sec and the Atchafalaya River discharges 16,000 cu ft per sec.

Depending on the degree of submergence by tailwater, the slot could discharge from 15,000 cu ft per sec to 22,000 cu ft per sec at the stage at which the crest at El. 10 is overtopped. The latter value is for 1950 tailwater conditions and with no flow from the backwater tributaries. El. 10 is approximately mean low water at this site.

The tailwater stages are expected to be lowered by the enlargement of the outlet channel due to scour and by the additional enlargement of the Atchafalaya River, as cited previously, thus increasing the low-water discharge capacity.

Based on records of the past 55 yr, there are on the average only twenty days per year during which the water surface at the structure would be below the sill at El. 10. In the future there will be even less time as Mississippi River flow is supplemented by additional upstream reservoirs. The additions to Mississippi River flow tend to increase the quantity of water that can be diverted into the Atchafalaya River. There will also be more low-water flow from the backwater tributaries as a result of the multiple-purpose reservoirs constructed in the tributary basins.

Effects of Minimum Tailwater

Fig. 11 shows the effects of the minimum-tailwater conditions on the 1945 flows repeated with the plan in operation. In this figure the hydrographs of the 1945 flows (Atchafalaya River and Mississippi River south of the Old River in the 1950 condition during the flood period) have been repeated in order to serve as a basis for comparison. For structures having approximately the same hydraulic characteristics as those of the proposed plan, hydrographs 2A show the effects on flows that would result from the minimum-tailwater conditions. In these routings the structures were kept fully open. The hydrographs show that a major part of the increased capacity of the Atchafalaya River would be utilized.

Regulation to Limit Enlargement of the Atchafalaya River

The enlargement of the leveed reaches of the Atchafalaya River has occurred during large floods for the most part, and filling has occurred during moderate stages. Because the water drops its sediment load at medium stages in the leveed reaches and at the same time produces bankfull stages in the relatively small channels below the end of the levees, it is believed that the enlargement of the lower reaches occurs principally during moderate stages. Therefore, from the standpoint of favorably affecting the regimen of the Atchafalaya River and reducing the problems of maintenance both in the lower basin and in the outlet channel, the best type of flow regulation consists of some reduction in ordinary flood flow with no reduction in medium-stage flow. If such regulation is instituted, it should not be extended so that the carrying capacity of the Atchafalaya River would deteriorate. There are several other factors beyond the scope of this paper that were considered in developing the plan for the operation of the structures. However, it is possible to cite what can be accomplished in the way of regulation of discharge because this affects the assurance that the structures can provide against the change of course of the Mississippi River. A minimum degree of regulation, which could be obtained with no conflict with other needs, is shown by hydrographs numbered "(3)" in Figs. 9 and 10. The hydrographs show the results of regulating the discharge whenever the New Orleans stage is below El. 8. At such times the structures were not operated at maximum capacity but were operated to maintain the Atchafalaya River discharge at 45,000 cu ft per sec. (Sometimes the structures cannot put in enough water to do so, and at other times the uncontrolled inflows exceed the desired quantity.) A much greater degree of regulation is practicable. For a repetition of the flows of 1943, Fig. 12 shows the reduction in Atchafalaya River discharge that can be accomplished under the following criteria:

1. The discharge of the Mississippi River should not by such regulation be caused to exceed 1,200,000 cu ft per sec because this discharge might force the operation of either the Bonnet Carre Floodway or the Morganza Floodway.
2. The degree of such regulation should be reduced whenever necessary in order to maintain a minimum discharge of 45,000 cu ft per sec down the Atchafalaya River for water use.

The discharges for open-river conditions are repeated for comparison. This type of regulation is not expected to be advantageous, but it is presented to show that the total flow can be reduced considerably by practicable means. Sediment studies indicate that this would be effective in reducing the further enlargement of the Atchafalaya River.

ASSURANCE AGAINST CHANGE OF COURSE OF MISSISSIPPI RIVER

Because depths of more than 100 ft below sea level are quite common in the Atchafalaya River, it would be too expensive to depend entirely on deep foundations for security against erosion. It is planned to excavate the bottom of the outlet channel at its junction with the Red River to El. −10, which corresponds generally to the elevation of the Red River bottom at this site. The channel then will slope up to El. −8 at the end sill behind the low-sill structure. A geological study has indicated that there are erosion-resistant clay plugs over which the outlet channel will pass. These plugs will delay any possible tendency of the channel to ravel back toward the structure. Revetment of the outlet channel will be provided initially in the vicinity of the structure. During the low-water period of several months each year, there will be ample opportunity to close off the structure for inspection and, if necessary, for repair of the outlet channel.

CONCLUSIONS

The primary purpose of the plan is to prevent the change of course of the Mississippi River by the interposition of safe and stable structures. By the control of flows to suitable volumes, the structures will assist in maintaining the regimen of the Mississippi River and in enlarging and maintaining at no more than a safe value the flood-carrying capacity of the Atchafalaya River. Because the situation is very complex, both from a hydraulic standpoint and from the standpoint of structure safety, there were many alternatives requiring difficult decisions. The plan presented is believed to be the most practicable and economical for meeting all objectives, and the hydraulic analysis demonstrates that these objectives are being met.

FOUNDATION DESIGN

By Willard J. Turnbull[1] and Woodland G. Shockley,[2]
Members, ASCE

Synopsis

The low-sill structure, the overbank control structure, and the navigation lock that are being constructed for the diversion control of the Old River are founded on a variety of soil types that are typical of those found in the lower Mississippi River alluvial valley. Detailed descriptions are given of the foundation conditions for the low-sill structure, which is a gated weir having its base approximately 50 ft below the ground surface and which is founded on silty soils that extend another 50 ft below the structure. Important considerations in design were excavation dewatering, seepage control, design of pile foundation, and preload fills at the abutments to minimize settlements.

Introduction

The structures for the control of the Old River are the low-sill structure, the overbank control structure, and the navigation lock. Fig. 1 shows the general locations of the various structures in the adopted improvement plan. The geology and general soils conditions for the three structures are described, with emphasis on those features that were significant in the final site selection. More detailed descriptions of foundation conditions and design features are given for the low-sill structure.

Geology and Site Selection

Before describing the details of the subsurface conditions at each structure location, it may be helpful to describe briefly the general sequence of sediments that are found in the lower Mississippi River valley. Practically the entire alluvial valley is underlain by massive sand deposits that are approximately from 100 ft to 150 ft thick. These sands are coarse and graveliferous near the base and grade into fine sands near the top of the layer. Superimposed on the underlying sands are alluvial sediments of varying character and thickness, depending on their mode of formation. The surface materials include uniform clay deposits, which vary in thickness from 10 ft to 60 ft. These deposits have been formed by the long-time deposition of river sediments in backwater areas and are termed "backswamp clays." In the areas of river meandering, the cutting of river banks and deposition downstream have formed point bar deposits consisting of alternating ridges and swales. In these deposits the top-stratum silts and clays overlying the sand ridges are relatively thin and

---

Note.—Published, essentially as printed here, in March, 1956, in the Journal of the Waterways Division, as *Proceedings Paper 908*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Chief, Soils Div., Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss.

[2] Chief, Embankment and Foundation Branch, Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss.

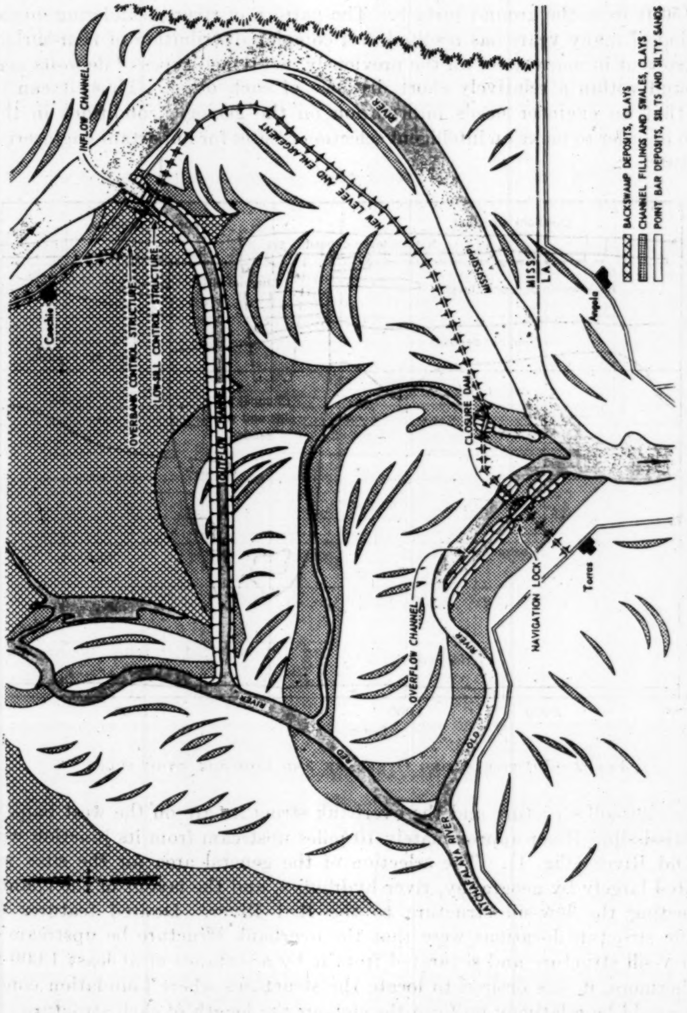
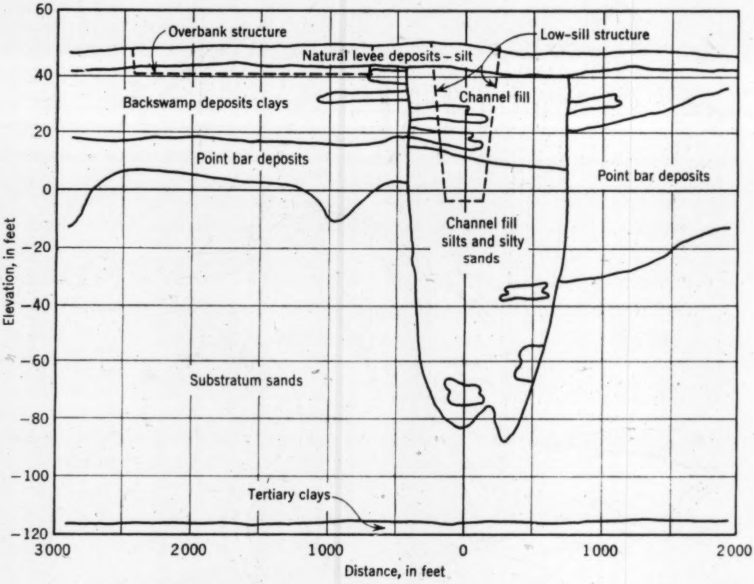

INFLOW CHANNEL
NEW LEVEE AND ENLARGEMENT
MISSISSIPPI RIVER
MISS
LA
Angola
Coochie
OVERBANK CONTROL STRUCTURE
LOW-SILL CONTROL STRUCTURE
OUTFLOW CHANNEL
CLOSURE DAM
OVERFLOW CHANNEL
NAVIGATION LOCK
Torras
RED RIVER
OLD RIVER
ATCHAFALAYA RIVER
BACKSWAMP DEPOSITS, CLAYS
CHANNEL FILLINGS AND SWALES, CLAYS
POINT BAR DEPOSITS, SILTS AND SILTY SANDS

FIG. 1.—PLAN OF IMPROVEMENT FOR OLD RIVER CONTROL

become as deep as from 20 ft to 30 ft in intervening swale areas. River cutoffs result in oxbow lakes, which eventually fill with sediments ranging from soft clays to silts and sands, depending on the mode of formation. The filled channels are sometimes as deep as the present river, or between 60 ft and 150 ft from the ground surface. The pattern of river meandering during a period of many years has resulted in a complex distribution of near-surface soils, so that in many areas all the previously mentioned types of deposits may be found within a relatively short distance of each other. Thus, it can be seen that an engineer needs information on the geologic conditions in this region in order to make an intelligent selection of sites for important engineering structures.



Overbank structure
Low-sill structure
Natural levee deposits – silt
Channel fill
Backswamp deposits clays
Point bar deposits
Point bar deposits
Channel fill silts and silty sands
Substratum sands
Tertiary clays
Elevation, in feet
60
40
20
0
-20
-40
-60
-80
-100
-120
3000
2000
1000
0
1000
2000
Distance, in feet

FIG. 2.—PROFILE ALONG OVERBANK AND LOW-SILL STRUCTURES

The low-sill structure and the overbank structure are on the west bank of the Mississippi River approximately 10 miles upstream from its junction with the Old River (Fig. 1). The selection of the general area for the sites was dictated largely by geography, river hydraulics, and the layout of the channel connecting the low-sill structure to the Red River. Desired features for specific structure locations were that the overbank structure be upstream of the low-sill structure and separated from it by a distance of at least 1,000 ft. Furthermore, it was desired to locate the structures where foundation conditions would be relatively uniform throughout the length of each structure.

In the vicinity of the low-sill structure and the overbank structure, the foundation soils consist generally of a thin layer of natural levee silts underlain by deposits of backswamp clays which vary from 10 ft thick to 20 ft thick (Fig. 2). In the upstream section of the area, the backswamp deposits are underlain by 10 ft to 30 ft of point bar deposits of silts, sandy silts, and silty sands, below which are clean sands. In the lower or downstream part of the area, the backswamp clays are underlain by an old abandoned channel of the Mississippi River that is approximately 2,400 ft wide and runs practically perpendicular to the present course of the river. The channel deposits consist of alternating strata of silts and silty sands to depths of from 100 ft to 125 ft below the ground surface. Clean sands extend below the channel fillings to a depth of approximately 165 ft below the ground, at which depth firm Tertiary clays are encountered.

It would have been possible to have located both the low-sill structure and the overbank structure on the fairly uniform foundation soils existing in the upstream section of the area. However, the deep approach channel and outlet channel that are required for the low-sill structure would have uncovered the underlying pervious sands along this reach and would have created an undesirable underseepage condition. In addition, dewatering the excavation during construction would have been a major problem. Therefore, it was decided to locate the low-sill structure approximately 2,000 ft from the Mississippi River and in the abandoned river channel, in which the top of the clean sands would be below the bottom of the approach channel and outlet channel of the structure (Fig. 3). In so doing, the severity of underseepage and the requirements for dewatering the structure excavation are greatly reduced. Inasmuch as the low-sill structure is founded on piles that penetrate into the deep underlying clean sands, the existence of the underlying silts is not considered detrimental.

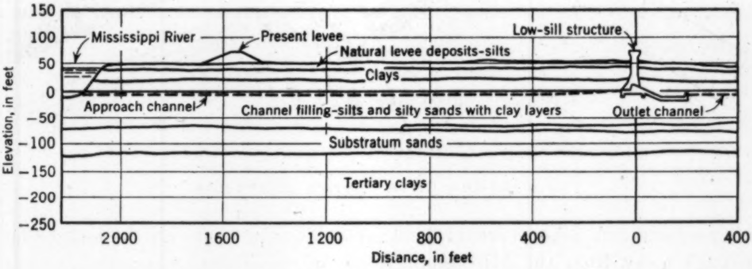

150
100
50
0
-50
-100
-150
-200
-250
Elevation, in feet
Mississippi River
Present levee
Low-sill structure
Natural levee deposits-silts
Clays
Approach channel
Channel filling-silts and silty sands with clay layers
Outlet channel
Substratum sands
Tertiary clays
2000
1600
1200
800
400
0
400
Distance, in feet

FIG. 3.—SECTION ALONG LOW-SILL STRUCTURE

Placement of the low-sill structure in the abandoned river channel made it possible to locate the overbank structure at the desired distance upstream in an area in which the sand is relatively high and foundation conditions in general are quite uniform. Fig. 4 is a rendering of the low-sill structure and the overbank structure. The excavation for the latter is only a few feet deep. Thus, the underlying backswamp clays provide protection against underseepage and thereby eliminate major dewatering during construction. Piles beneath the overbank structure were not planned because the structure loads are light and the uniform foundation conditions should permit relatively uniform settlements throughout the length of the structure.

The proposed lock is to be located near the Old River, close to the site of the closure dam. It was desired to orient the lock so that the approach channel and exit channel would have favorable alinement and the lock be far enough away from the Mississippi River as not to be endangered by any possible meandering of the river in a westward direction. It was also desired to have a firm sand foundation beneath the lock, at the same time having as much of the bottom of the approach channel and exit channel in cohesive soils as possible in order to minimize scour.

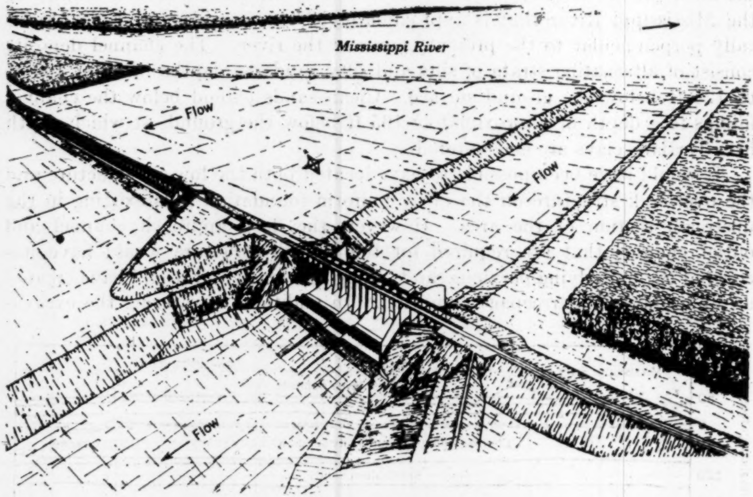

Mississippi River
Flow
Flow
Flow

Fig. 4.—Old River Control Structures

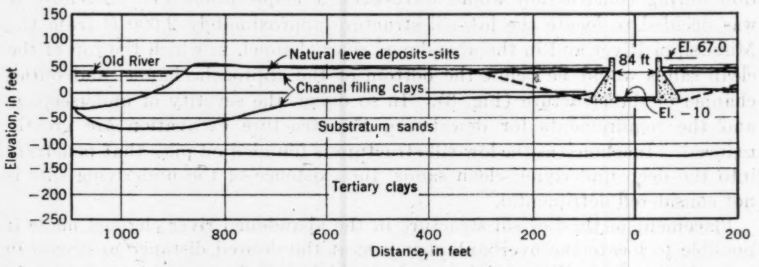

150
100
50
0
−50
−100
−150
−200
−250
Elevation, in feet
Old River
Natural levee deposits-silts
84 ft
El. 67.0
Channel filling clays
El. −10
Substratum sands
Tertiary clays
1000
800
600
400
200
0
200
Distance, in feet

Fig. 5.—Section Through Old River and Lock

Results of a comprehensive boring and geological investigation in the general area of the proposed lock showed that the most feasible location would be on the south side of the Old River (Fig. 5). In this area the Old River now occupies the northern boundary of an ancient channel of the Mississippi River. The south bank of the Old River is composed of interstratified clays and silts filling the ancient Mississippi River channel and extending approximately 100 ft below the ground surface, below which are clean sands. Southwest from the Old River the upper silts and clays gradually become thinner. At the top of the underlying sands, they become correspondingly higher until, at a distance of approximately ½ mile, the thickness of the top stratum deposits is only approximately 20 ft. In a direction parallel to the Old River, the depth to the top of clean sands along any given line is nearly constant. Thus, it was possible to select a location for the lock so that the base of the lock was on sand (approximately at El.-15) and the major section of the approach and exit channels in silts and clays. The proposed site of the lock is approximately parallel to the Old River and approximately 900 ft from the south bank, thus providing sufficient clearance to construct safe excavation slopes on the side of the lock nearest the river.

Foundation Studies, Low-Sill Structures

The principal features of the low-sill structure that were important in the foundation design are the pile foundation, dewatering the structure excavation, the drainage system beneath the structure, and settlements at the abutments of the structure. These features will be described briefly.

The field soils-exploration program in the immediate area of the structure consisted of ten split-spoon borings. Seven undisturbed-sample borings were made, in which 5-in.-diameter samples were obtained in the clays and silts and 3-in.-diameter samples were obtained in the sands. Nearly all borings penetrated several feet into the underlying clean sands, and two borings went through the clean sands into the Tertiary clays. The boring program was designed to provide soils information along the structure center line and in the adjacent channel areas. In addition, ten piezometers were installed in the area to measure hydrostatic pressures in the foundation as related to the river stage. A field pumping test also was performed at the site to determine the permeability of the deep foundation sands. A reliable estimate of permeability was considered necessary in order to design the drainage facilities and to estimate dewatering requirements during construction. The test well was an 8-in.-diameter commercial well screen with a 30-ft-long screen section installed in the sand; a solid steel riser pipe extended to the ground surface. The well was pumped at three different rates of flow. Drawdown curves were determined from piezometers, which were at various distances from the well. The average coefficient of permeability of the sand as determined by this test was $1{,}000 \times 10^{-4}$ cm per sec.

The laboratory testing program included the usual classification tests and water-content determinations on the borings. A typical boring log showing water contents and Atterberg limits data is shown in Fig. 6. Shear-strength determinations were made on the various soils that were encountered at the site by using test procedures which were considered appropriate for the soil types, and for loading conditions that were anticipated during construction

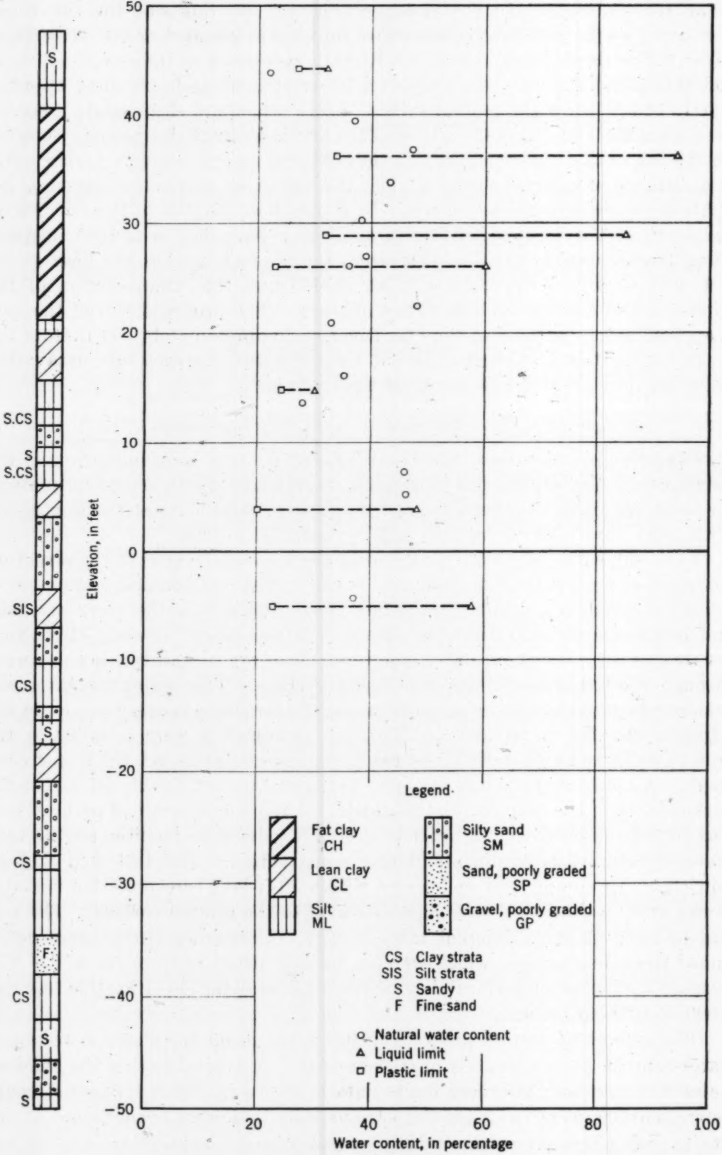


FIG. 6.—TYPICAL BORING LOG

and operation of the structure. Shear-strength tests consisted of unconfined compression tests, unconsolidated-undrained triaxial tests, consolidated-undrained triaxial tests, and consolidated-drained direct-shear tests on clays. Based on these data a strength of 0.3 ton per sq ft was selected for design for these soils. Silts, sandy silts, and silty sands were subjected to consolidated-undrained triaxial tests and consolidated-drained direct-shear tests. Design strength for these materials was selected as $\phi = 28°$ and $c = 0.1$ ton per sq ft. Consolidated-drained triaxial tests were made on the sands, and a design strength of $\phi = 33°$ and $c = 0$ was selected. Fig. 7 shows typical shear-test data on the various soil types and their relationship to selected design strengths.

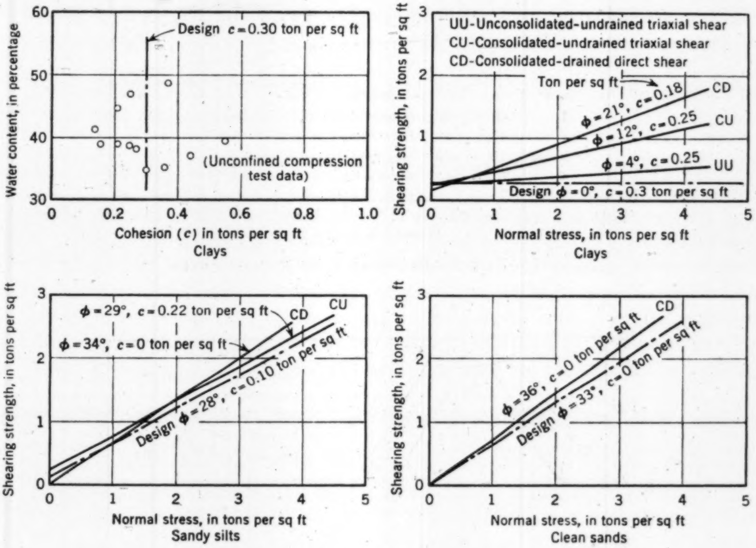


FIG. 7.—TYPICAL SHEAR-TEST DATA

Consolidation tests were performed on the clay soils, the silty soils, and the clean sand to provide data for computations concerning the settlement of the abutments and for the design of preload fills. Typical pressure-void ratio curves are shown in Fig. 8. The curves for the backswamp clays indicated preconsolidation pressures in excess of normal overburden pressures. (This phenomenon is common in backswamp clays of the lower Mississippi River valley.) Based on geologic evidence it is believed to be the result of alternate wetting and drying during deposition of the sediments.

Natural density tests and relative density tests were performed on the sands. Maximum density tests and minimum density tests were made on oven-dry samples by pouring the sand from a constant height into a 2-in.-by-4-in. mold for the minimum density test and by compacting the sand in layers

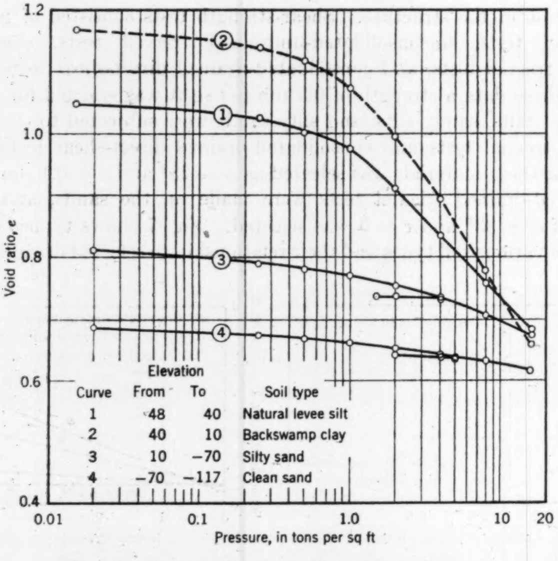


FIG. 8.—TYPICAL PRESSURE-VOID RATIO CURVES

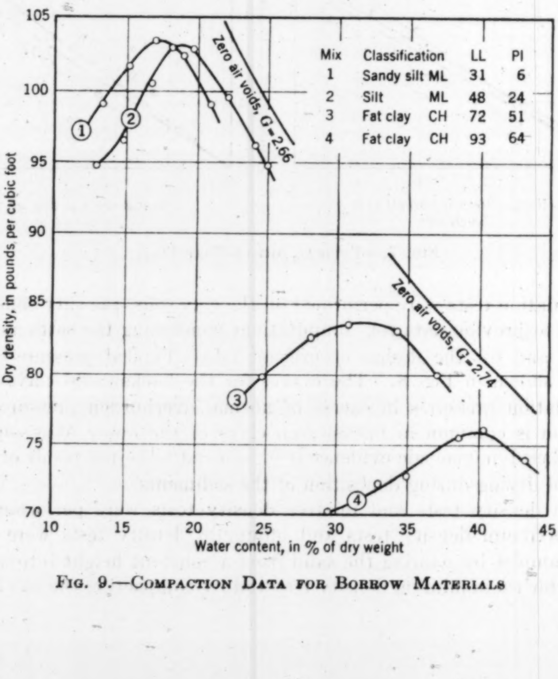


FIG. 9.—COMPACTION DATA FOR BORROW MATERIALS

in the same mold for the maximum density test.[3] The relative densities of the deep sands ranged from 55% to 96%, with an average value of 69%.

Compaction tests were made also on selected soil types for use in levees, cofferdams, and backfill. Typical compaction curves on representative soil types are shown in Fig. 9. The sandy silt mix was compacted by the Standard Proctor method, but a light compactive effort (15 blows per layer) was used on the silt and clay mixes because it was anticipated that these materials would be compacted by hauling equipment and by tractors rather than rollers.

In addition, laboratory permeability tests were performed on representative silts and sands. Vertical permeabilities of undisturbed silt samples ranged from 0.26×10^{-4} cm per sec to 34×10^{-4} cm per sec. Permeabilities of the sands were determined by tests on remolded samples. The sand samples were washed before testing to remove traces of drilling mud, and test values were corrected to the in-place void ratio. The average of laboratory permeability tests on the sands gave a value of 500×10^{-4} cm per sec as compared with a horizontal permeability of $1{,}000 \times 10^{-4}$ cm per sec.

Excavation for the low-sill structure extends from approximately 50 ft to 65 ft below the present ground surface, and, with the guide levees and cofferdams, the ultimate excavation slopes are on the order of from 70 ft high to 80 ft high. Based on the results of stability analyses, average excavation slopes of 1 on 4 to 1 on 5 have been selected. These slopes will have factors of safety ranging from 1.25 to 1.5. Excavation and construction are being conducted in the dry. Because the natural water table is high and the hydrostatic pressure in the underlying sands directly reflects the river stage, it is necessary not only to dewater the excavation on the side slopes but to reduce the hydrostatic pressure in the deep sands in order to prevent blowup of the bottom of the excavation. A multistage wellpoint system combined with deep wells is provided to accomplish this objective. In addition, excavation slopes are seeded and ditched in order to control the inflow of surface water from rains.

The gate-bay section of the structure is founded on piling driven to sand to provide positive assurance against settlement or sliding (Fig. 10). Both vertical piles and piles on a 2-on-1 batter were driven to take both vertical and horizontal loads. Design loads for the individual piles are 100 tons in compression and 40 tons in tension. Preliminary explorations at the site had indicated that there might be some difficulty in driving displacement piles through the silty soils overlying the sand. In order to solve this problem and to determine appropriate pile sizes and lengths to carry the design loads, a series of pile tests were conducted at the site in December, 1954, and in January, 1955. The tests were conducted in a deep excavation simulating actual conditions to be obtained during construction of the structure. The tests demonstrated the feasibility of driving displacement-type piles for the low-sill structure. The following alternate sizes and lengths of piles were selected to provide the design carrying capacity with tolerable settlement: A 14-in. H-beam driven 27 ft into sand, or a 20-in. pipe pile driven 15 ft into sand, or a 20-in. octagonal precast-concrete pile driven 12 ft into sand.

---

[3] "Summary Report of Soils Studies," *Potamology Report 12-2*, Waterways Experiment Station, Corps of Engrs., U. S. Dept. of the Army, Vicksburg, Miss., October, 1952.

The wing walls at each end of the low-sill structure are relatively high (on the order of 60 ft), and the base of the walls is approximately 57 ft below the ground surface. Computations show that the walls should have adequate stability without the necessity for a pile foundation. However, the deep excavation in which the walls are placed will cause some rebound of the foundation soils and recompression as the walls and backfill are constructed. Maximum computed settlements for this condition are on the order of 2 in. The top of the wall is computed to rotate approximately 1.5 in. toward the backfill. When the structure is placed in operation, the water load on the walls creates a different set of loading conditions that may result in movement of on the order of 1.0 in. at the top of the wall away from the backfill. In order to permit the anticipated differential movements, the joints between wall monoliths are provided with waterstops and with substantial thicknesses of expansion joint material.

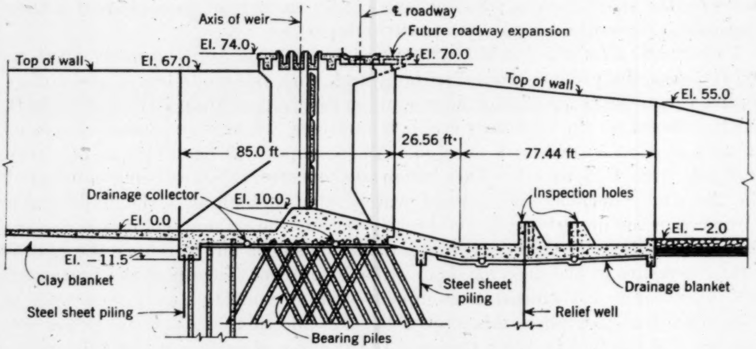

Axis of weir
℄ roadway
Future roadway expansion
El. 74.0
Top of wall
El. 67.0
El. 70.0
Top of wall
El. 55.0
85.0 ft
26.56 ft
77.44 ft
Drainage collector
El. 10.0
Inspection holes
El. 0.0
El. −2.0
El. −11.5
Clay blanket
Steel sheet piling
Drainage blanket
Steel sheet piling
Relief well
Bearing piles

FIG. 10.—TYPICAL SECTION OF THE LOW-SILL STRUCTURE

Another important consideration in the design of the low-sill structure is the control of uplift pressures beneath the gate bay and the stilling basin. Flow-net studies indicated the need for an upstream impervious blanket to reduce the seepage beneath the structure. This blanket is to be of 4-ft-thick and 175-ft-long compacted clay extending from the upstream end of the concrete apron. Beneath the gate bay and stilling basin, separate sand and gravel filter blankets, which are 14 in. thick, are provided. These blankets discharge into collector pipes, which in turn discharge into tailwater. The collector system is provided with flap gates to prevent the backflow of muddy water, manholes, and other facilities to permit access for cleaning. In addition, it was necessary to provide for control of uplift pressures in the deep sands beneath the stilling basin. This has been accomplished by seven relief wells (8 in. in diameter and surrounded with a gravel filter), which penetrate into the sands and discharge by free flow into the drainage collector system. These wells were installed during initial stages of construction in order to provide assistance in reducing uplift pressures during construction. Steel sheet piling has been provided upstream from the gate bay and between the two drainage blankets to minimize erosion and piping beneath the structure.

The foundation soils at the abutments of the structure will be subjected to significant settlements beneath the adjoining levee fills. Settlement of the foundation will result in a downward drag on the piles, which possibly could greatly overload the piles beneath the abutments piers. In addition, settlement at the abutments could cause the earth fill to pull away from curtain walls in this area and, thus, create potentially hazardous conditions with respect to seepage during high water. In order to minimize these effects, preload fills have been constructed at each end of the low-sill structure in order to reduce settlements to tolerable quantities before construction. Predicted settlements at the ends of the structure were on the order of from 10 in. to 17 in., and the preload fills were so designed and overbuilt that approximately this degree of settlement would occur in a twelve-month period. The preload fills were constructed so that a substantial part of each will remain in place as part of the permanent construction.

CONCLUSIONS

An important feature of the studies described herein was the final locations for the low-sill structure, the overbank structure, and the navigation lock for the diversion control of the Old River. It was desired that foundation conditions be uniform at each site in order to assure that settlements of the structure would be uniform and not excessive. Furthermore, the channels adjacent to the structures should be in erosion-resistant materials, and, finally, suitable and economical foundations were to be designed for each structure. By studying the geologic and soils conditions in the area, it was possible to accomplish the previously listed objectives. The low-sill structure was on silts and clays, the overbank structure on clays, and the navigation lock on sand. The adjacent channels for the structures are in silty or clayey soils. Only the low-sill structure is founded on piles. The overbank structure and the navigation lock are designed to rest directly on the underlying soils.

Of the three structures, only the foundation design for the low-sill structure has been examined in detail. This structure is founded in an old abandoned channel of the Mississippi River and rests on steel H-piles driven through the silt and clay foundation into underlying firm sands. Anticipated settlements at the abutments of the structure were minimized by the construction of preload fills. Excavation for the structure to a depth of approximately 50 ft below the natural ground surface is facilitated by a dewatering system consisting of wellpoints in the upper section of the excavation and deep wells penetrating into the underlying sands.

STRUCTURES REQUIRED

BY NORMAN R. MOORE,[1] M. ASCE

SYNOPSIS

The positive control of flow diversion from the Mississippi River into the Atchafalaya River Basin through the Old River requires structures that will permit the retention of the adopted flood-control plan. The preceding stipulation dictates a maximum discharge capacity of not less than 700,000 cu ft per sec for the project-flood stage. Also mandatory is the provision for sufficient capacity at lesser stages to make unnecessary the increased frequency of operation of the downstream Morganza (La.) Floodway and the Bonnet Carre (La.) Floodway. For medium stages the structures must have a capacity that is equivalent to the natural diversion capacity of the Old River in order that channels may be developed through the lower Atchafalaya River Basin, where such flows will result in bankfull stages. To meet navigation requirements, a waterway traversing the closure dam must be provided. The foregoing requirements are fulfilled in the design of the low-sill structure and the overbank structure described herein, and in the appurtenant channels, levees, closure dam, and navigation lock for which planning is under way.

GENERAL DESCRIPTION

The project area is on the west bank of the Mississippi River between river mile 301 and river mile 313 upstream from Head of Passes (La.).[2] The control structures will be between mile 312 and mile 313, and the navigation lock will be in a land cut south of the mouth of the Old River. Economic considerations led to the decision to provide two separate control structures (Fig. 1). Functionally, the structures are controlled spillways, differing in the elevations and lengths of the weir crests and in the type of controls. The construction cost per unit of length is considerably greater for the low-sill structure, and, consequently, the length of this structure is no greater than that which is necessary to meet the capacity criteria for lesser flows than those of the project flood. The overbank structure will assure the necessary maximum discharge capacity and will provide a degree of operative flexibility. The over-all length of the two structures, including the abutment fill that is common to both, is 5,868 ft.

The inlet channel connecting the Mississippi River with the low-sill structure will be approximately 2,200 ft long with a bottom width of 1,000 ft. The outlet channel from the structure to the Red River will be approximately 7 miles long with a bottom width of 900 ft.

NOTE.—Published, essentially as printed here, in March, 1956, in the Journal of the Waterways Division, as *Proceedings Paper 909*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Chief, Eng. Div., Mississippi River Comm., Vicksburg, Miss.

[2] "Hydraulic Requirements," by Eugene A. Graves, in "Old River Diversion Control: A Symposium," *Transactions*, ASCE, Vol. 123, 1958, Fig. 3.

The existing Old River channel will be closed by an earth dam. Levee construction will be required to connect the proposed structures to the existing main-line levee system. Channel-stabilization works will be constructed in the outflow channel after the control structures have been operated for a time so that a favorable channel alinement has developed.

Low-Sill Structure

In contrast to the Morganza (La.) spillway structure and the Bonnet Carre (La.) spillway structure, both requiring operation at infrequent, relatively short intervals, it will be necessary to operate the low-sill structure continuously (Fig. 2). To fulfil the various discharge requirements and to provide for flexible operation at the least possible construction cost, two elevations were selected for the weir crest (Fig. 3). The three center bays, with a weir crest at El. −5.0, which is the elevation of the approach-channel invert, will provide for the passage of flows at extremely low stages of the Mississippi River. The weir crests of the four bays on each side of the center bays will be at El. 10. The eight bays meet the flow requirements, with an adequate approach depth, and make it possible to provide a jump type of stilling basin. Each of the eleven bays is 44 ft wide, making a total clear width of 484 ft at the weir. It is estimated that the headwater elevation will vary from El. 67.0 to approximately El. 5.0. The tailwater stage will depend on the flow from tributary streams downstream from the structure as well as the discharge of the structure. For the project flood the computed discharge capacity of the low-sill structure, which operates together with the overbank structure with headwater elevation and tailwater elevation assumed for 1950 river conditions, is 350,000 cu ft per sec, or 50% of the total minimum discharge requirements for both structures.

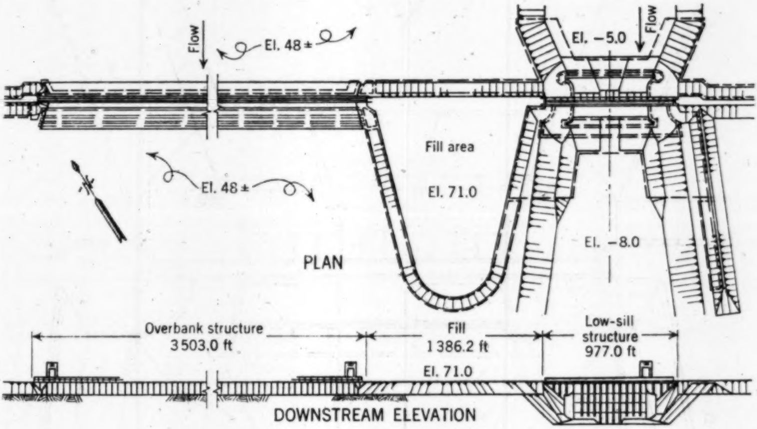


FIG. 1.—CONTROL STRUCTURES

In an attempt to hold the alinement and to prevent the approach channel from meandering, the invert will be paved with concrete for approximately 100 ft upstream from the gate-bay monoliths, and riprap protection varying in thickness will be placed on the bottom and on the 1-on-4 side slopes for an additional section of the channel. The concrete paving will be placed on a compacted fill of selected impervious material that is designed to withstand possible uplift pressures when the water-surface elevation on the downstream side is temporarily higher than on the upstream side. The thickness and gradation of riprap in the approach channel and outlet channel are proportioned conservatively according to the riprap requirements indicated by experiment and by considering the computed average velocities. The stilling basin will be a rectangular concrete channel that is 566 ft wide near the down-

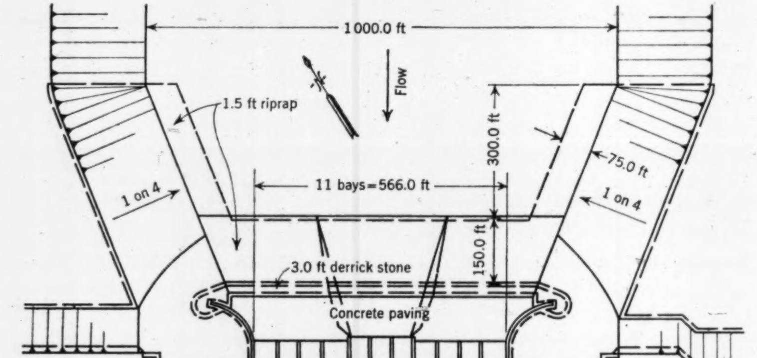

1000.0 ft
Flow
1.5 ft riprap
300.0 ft
75.0 ft
1 on 4
11 bays=566.0 ft
1 on 4
150.0 ft
3.0 ft derrick stone
Concrete paving
El. 72.0
60.0 ft
80.0 ft
El. 68.0
Guide levee
3.0 ft derrick stone
2.0 ft riprap
1.5 ft riprap
370.0 ft
1 on 4
1 on 10
1 on 10
1 on 4
El. 48.0
El. 48.0
Note: Not to scale
El. −8.0

FIG. 2.—PLAN OF LOW-SILL CONTROL STRUCTURE

stream edge of the piers and flares to a width of 592 ft at the end sill. The width of the section serving the lower weirs will be 150 ft, the floor elevation will be at El. −12.0, and the end sill will be at El. −9.0. For the higher weirs the floor elevations will be at El. −5.0, with end sills at El. −2.0. Two rows of baffle blocks, which are 10 ft high and are spaced laterally 12 ft apart,

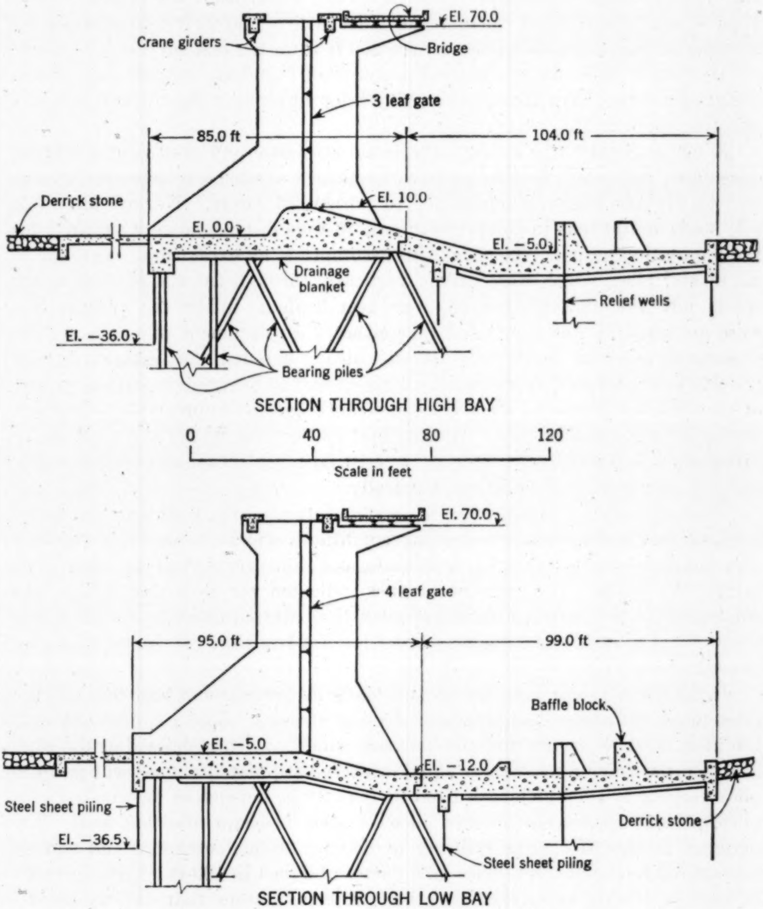

El. 70.0
Crane girders
Bridge
3 leaf gate
85.0 ft
104.0 ft
Derrick stone
El. 10.0
El. 0.0
El. −5.0
Drainage blanket
Relief wells
El. −36.0
Bearing piles
SECTION THROUGH HIGH BAY
0
40
80
120
Scale in feet
El. 70.0
4 leaf gate
95.0 ft
99.0 ft
Baffle block
El. −5.0
El. −12.0
Steel sheet piling
El. −36.5
Derrick stone
Steel sheet piling
SECTION THROUGH LOW BAY

FIG. 3.—SECTIONS OF LOW-SILL CONTROL STRUCTURE

will be placed across both sections of the stilling basin. Upstream from the first row of baffle blocks, the stilling-basin slab will be 7 ft thick. From the face of the baffle block downstream to the end sill, the thickness will taper to 4 ft. The hydraulic design of the stilling basin was based on analytical determinations that were supplemented by model studies of the structure and by considering the prototype operating characteristics of similar structures.

The outlet channel, with a rectangular section at the end sills of the two parts of the stilling basin and with invert elevations that correspond to the respective end-sill elevations, will transition to a uniform invert elevation of El. −8.0 and a bottom width of 900 ft. The side slopes of the channel will be 1 on 4, and the channel bottom and sides will be protected from scour by riprap and derrick stone for more than 600 ft from the end sill.

Hydraulic model tests on models to a scale of 1:36 have furnished structural design data and have indicated methods for improving flow conditions and effecting economy in construction.

A pile foundation to support the gated structure and abutment piers was considered necessary to give positive assurance against excessive settlement and to provide adequate resistance to horizontal forces. Normal allowable pile loads of 100 tons in compression and 40 tons in tension were assumed for the foundation design. To check this assumption, an excavation was made to El. 0, and piles, which were driven vertically through the silt and sandy-silt strata into the underlying sand, were test loaded. When the observations were analyzed, it was concluded that either a displacement pile or a nondisplacement pile can be driven. It was also concluded that either pile will provide an adequate factor of safety with respect to bearing capacity or failure of the piles by plunging, as well as assurance against appreciable settlement during a long period of time. Approximately three-fourths of the piles will be driven on a batter of 2 on 1 in order to resist both horizontal and vertical loads, and the remainder will be driven vertically.

The foundation design for the main structure used the Culmann method of analysis and considered nine loading conditions, which consisted of assumed combinations of gate openings, headwater and tailwater elevations, uplift, and bridge live loads. The number of piles indicated for each design case was computed for the battered upstream piles, the battered downstream piles, and the vertical piles. The design provides for not less than the highest number in each group.

In addition to carrying the design loads in compression and tension, the piles must withstand handling and driving stresses. The requirement that the piles must be driven into the "bearing sand," which is defined as the sand in which the driving resistances equal or exceed stipulated values, will result in piles varying in average length from 84 ft to 98 ft, depending on whether pipe piles, precast reinforced-concrete piles, or steel H-beam piles are used. Experience at the Morganza spillway in driving 20-in., octagonal and square, precast reinforced-concrete piles on a batter of 2 on 1 in 100-ft lengths or more, as well as driving experience on other projects, assures that slightly shorter piles on the same batter can be driven satisfactorily at this site.

The 44-ft width of the gate bays was selected on the basis of an economic study that involved plotting curves of the total costs of variable items, such as piers, foundation, gates, and gantry cranes, against the clear span between piers. The gated section of the structure is composed of a series of reinforced-concrete monoliths consisting of a base supporting two piers, which are tied together at the top by the crane-bridge girders. The section will be divided by joints in the base at the center of alternate bays into a low-flow monolith, transition monoliths, weir-section monoliths, and end monoliths. Three steel sheet-pile cutoff walls will be provided. One cutoff wall at the upstream end of the gate-bay monolith and one in the cutoff key at the end of the stilling basin will protect against piping and scour. An additional row at the upstream end of the stilling-basin slab will separate the two drainage blankets. Abutments, consisting of bulkhead walls which vary in height and which are supported by a series of piers, will connect the gated structure to the adjacent levee at each side. The bulkhead walls will rest in slots in the piers that are designed in order to permit minor movements that are due to temperature changes and differential settlement. The design of the upstream wing walls and downstream wing walls, which were provided to guide and facilitate the flow of water to and from the weirs, has been aided by the model studies, which have furnished data on the most favorable alinement and maximum wave heights. Access across the structure will be provided by a two-lane concrete deck bridge.

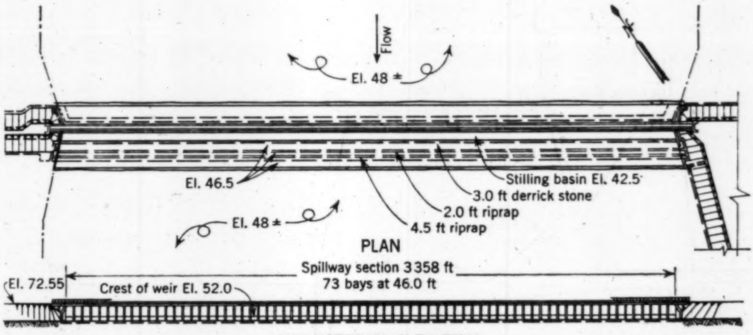


FIG. 4.—OVERBANK CONTROL STRUCTURE

Vertical-lift gates of the fixed wheel, welded structural-steel type will be provided in each of the eleven gate bays to regulate flow through the low-sill structure. Each gate is divided into sections to facilitate handling. The three center gates will have three 19-ft-high leaves and one 15-ft-high leaf, resulting in an over-all height of 72 ft. The remaining eight gates will have only the three 19-ft leaves, making an over-all height of 57 ft. The gates will be provided with rubber seals, which will be mounted in the upstream face along the sides and between the leaves. Along the sides the seals will bear on stainless-steel plates, which will be anchored in the masonry. At the lower edge of the gate, adjustable corrosion-resistant plates, which will be bolted to the skin plate, will make metal-to-metal contact with the sill.

The operating plan for the structure provides that the three center gate bays will be opened first, followed by the successive opening of alternate bays at each side for the best stilling action with minimum eddying. For a falling stage the closure will be in reverse order. When a gate bay is opened, all leaves will be removed. The leaves will be handled by a gantry crane and, except for the top three central gate leaves, will be stored in slots at the top of the piers upstream from the service slots. Provision for storing the top three central gate leaves is made at the end of the structure. Except whenever it is necessary to remove the leaves, discharge will not be passed over any of the lower gate leaves.

Overbank Structure

The overbank structure is upstream from the low-sill structure and shares a common abutment of compacted fill. This structure is needed to provide the half of the required discharge capacity that is not furnished by the low-sill structure under the project-flood conditions. For the overbank structure the headwater elevation is El. 64.0 and the tailwater is at El. 62.5 (Fig. 4). A uniform weir crest at El. 52.0 was selected because it was representative of the general bank level in the vicinity. The ground surface at the site is approximately 4 ft lower than the elevation of the weir crest. Flow regulation through this structure will be required to assure that the combined discharge of the two structures will follow the adopted plan of operation. Hence, it is necessary to provide controls.

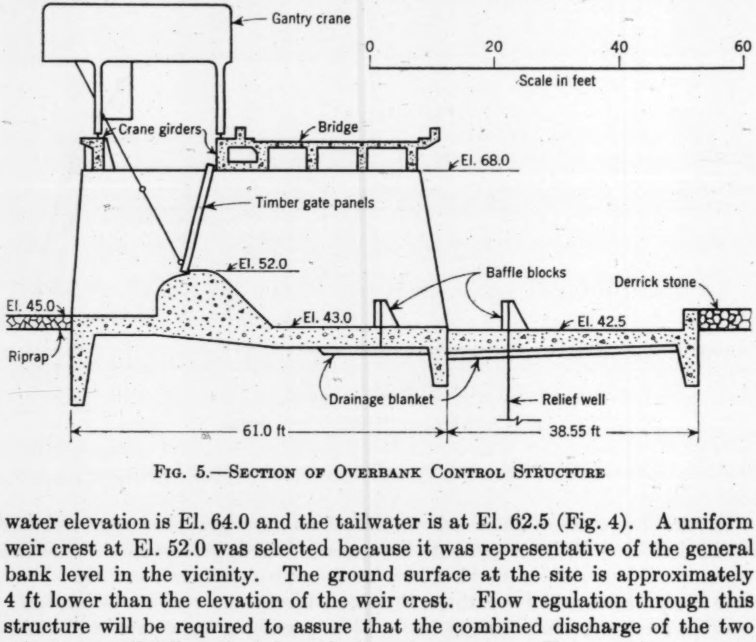


Fig. 5.—Section of Overbank Control Structure

An inlet channel will be excavated for 106 ft riverward from the overbank structure, and the remainder of the area between the river and the structure will be cleared, both upstream and downstream, to a width that is approximately 125 ft greater than the over-all length of the spillway between training walls. The average distance from the river to the structure is 3,300 ft. Riprap pavement will be placed in the excavated inlet channel at the invert elevation of 45.0 ft for 40 ft upstream from the face of the structure, and it will be extended around the wing walls and on the riverside slopes of the adjacent levee and joint abutment fill.

A stilling basin, which was designed to dissipate the energy of the design discharge of 6,780 cu ft per sec per bay, will extend downstream for 65.5 ft., measured from the heel of the weir (Fig. 5). It will contain two rows of baffle blocks, which will be 5 ft high and will terminate with a 4-ft-high end sill. A shallow outlet channel with invert at El. 46.5 will be excavated from the end sill downstream for a distance of 150 ft. The upstream two-thirds of the channel will be protected with derrick stone and riprap paving.

The same bay width of 44 ft that was used with the lower structure has been retained for the overbank spillway. A total of seventy-three bays will be provided. The piers will be 2 ft thick, and thus, the gross length between training walls will be 3,356 ft. Flow will be controlled by hinged panels.

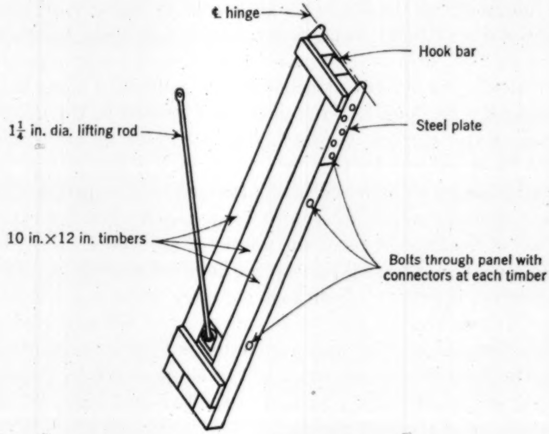

℄ hinge
Hook bar
$1\frac{1}{4}$ in. dia. lifting rod
Steel plate
10 in.×12 in. timbers
Bolts through panel with connectors at each timber

Fig. 6.—Timber Gate Panel

Because foundation conditions were favorable with respect to shear and consolidation characteristics and were relatively uniform, it was feasible to plan the overbank structure without a pile foundation. A relatively wide structure base has been provided, and the design incorporates both upstream cutoff keys and downstream cutoff keys. These features reduce bearing pressures to allowable values and assure against detrimental settlement and horizontal sliding.

Bridge piers at 46-ft centers divide the spillway structure into seventy-three bays. Expansion joints at the midpoints of alternate bays divide the structure into thirty-five typical monoliths, which are 92 ft long, and two 25-ft-long end monoliths. Two piers, the weir, the foundation slab, and the two cutoff keys are included in each monolith. Two additional bays at each end of the spillway will be 36.5 ft center to center of the piers and will be closed by reinforced-concrete bulkheads to form permanent dams connecting the structure to the abutments. The stilling-basin slab will be placed on a sand drainage blanket with perforated clay-tile drain pipes that will prevent the formation of excessive uplift pressures by seepage water. The circulation of water and the loss of foundation material will be prevented by rubber water stops placed in the expansion joints. The slab has been designed to have sufficient weight to resist flotation. As in the case of the low-sill structure, access across the structure will be provided by a two-lane concrete bridge.

Each spillway bay will have fifteen hinged panels for controlling discharge through the structure. The panels will be of treated timber and comprise three members, which are 11½ in. wide, 10 in. deep, and 18 ft long, and are fastened together as shown in Fig. 6. To allow for inaccuracies in the timbers, a ⅝-in. clearance between individual panels and a ⅞-in. clearance between the end panel and the masonry will be provided. Although such clearances will permit appreciable leakage, the quantity is not expected to be objectionable. The panel hinges are on the upstream girder of the highway bridge, which also constitutes the downstream crane girder, and in the closed position the panels will bear on an 8-in. seat in the upstream edge of the weir crest, inclined 15° from the vertical. An eye-bar attached to the panel by a hinge near the lower end, extending the full length of the panel and latched to the upper end above the maximum water surface, is provided to facilitate the underwater attachment of the hoist cable. In the open position the panels will be held in a horizontal position by a chain and hook attached to the upstream crane girder. In the event of interference with normal procedure, due for example to the accumulation of floating debris or the failure of an eye-bar, the hinge pins can be removed and the individual panels can be lifted vertically. Because of its stability under all operating conditions, a gantry crane was selected to handle the panels. It can also be used to handle most of the expected drift. Two cranes will be furnished. The upstream crane rail will be supported by a single girder, and the downstream crane rail will be supported by a girder, which was designed to carry, in addition, a section of the two-lane highway bridge and to take the reactions of the gate panels.

CONSTRUCTION MATERIALS

The required excavations for the control structures are expected to provide ample quantities of suitable materials for structure backfill and, within the limits of economic hauling distance, for levee construction. There are commercial sources of concrete aggregates within a 50-mile radius of the site. Derrick stone and riprap must be procured from sources that are 250 miles or more from the site.

CONCLUSIONS

The structures for the prevention of uncontrolled flow diversion from the Mississippi River into the Atchafalaya River Basin have been designed for the necessary flow regulation that will satisfy the many exacting requirements in assuring retention of the adopted flood-control plan. Considering the requirements for flexibility of operation and dependability of control, the estimated construction and maintenance costs are believed to be reasonable. The completed works will be a safe and permanent solution of the diversion problem.

ACKNOWLEDGMENTS

The planning and design of the Old River control structures and appurtenant works are being performed under the general supervision of the writer. Control of the Old River is a feature of the project for flood control and improvement of the lower Mississippi River and tributaries, and is under the direction of John R. Hardin, M. ASCE.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2957

TRUSS ANALYSIS BY STIFFNESS CONSIDERATIONS

By Harold C. Martin[1]

Synopsis

A general method of analysis, which is particularly advantageous for redundant structures, is developed for pin-connected trusses. The proposed method enables deflections, member forces, and reactions for any arbitrary set of applied loads to be determined.

It is shown that equilibrium and continuity requirements can be satisfied by use of a simple tabular procedure for writing the stiffness matrix of the entire structure. This is the principal task of the analyst and is sufficiently simple to be performed by nonengineering personnel. No extra complications arise from the subsequent introduction of additional redundant members or reactions. Once the table representing the stiffness of the structure has been obtained, the complete solution follows from numerical computation. This computation can be performed with a slide rule, desk calculator, or digital computer.

Changes in design are accounted for by locally correcting the stiffness matrix. Changes in external forces do not affect the analysis. Any determinate or indeterminate structure may be solved by the suggested method. The basic procedure is explained and then applied to several examples.

Introduction

The solution of any structural problem must be based on satisfying certain fundamental conditions that are well known to the structural analyst and will be listed subsequently. Although these basic conditions are not subject to change, detailed procedures for applying them to any given problem can vary considerably. The method presented herein possesses certain advantages that become particularly significant for the highly redundant structure.

A routine procedure is described herein for satisfying the fundamental requirements of equilibrium and continuity by developing the stiffness matrix

---

Note.—Published, essentially as printed here, in October, 1956, in the Journal of the Engineering Mechanics Division, as *Proceedings Paper 1070*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Prof. and Chairman, Dept. of Aeronautical Eng., Univ. of Washington, Seattle, Wash.; Structural Dynamics Unit, Boeing Airplane Co., Seattle Div., Seattle, Wash.

for the structure. This procedure is demonstrated for the truss, and applications to other structures can be made after stiffness expressions for the basic members have been determined.

The main advantage of the method is that it enables a complex truss to be analyzed with a minimum of effort. Numerical operations involving matrices are used to obtain the solution. This is also an advantage because such computations are routine and may be performed effectively on digital computers. The structural engineer will recognize that the solution (based on matrix computations) is merely an alternative to the various types of relaxation methods (such as moment distribution). However, the advent of high-speed computers has widened the horizons for the structural engineer, enabling him to undertake analyses that would have been prohibitive in the past.

Although not emphasized herein, the proposed method is useful when structural data are required for a subsequent dynamic analysis. Then it is necessary to have available an adequate set of deflection influence coefficients, which are obtained readily from the stiffness matrix.

PROPOSED METHOD

In the analysis of any structure it is generally necessary to satisfy the following conditions: (*a*) Forces must be in equilibrium; (*b*) deformations must be compatible; (*c*) forces and displacements must be related according to some prescribed rule; and (*d*) support or boundary conditions must be satisfied. The various methods of structural analysis represent the different approaches that may be used to satisfy these requirements.

The method presented herein applies to the determinate as well as to the indeterminate structure. Although it is applicable in principle to any structure, its application to the pin-connected truss will be examined specifically herein.

Prior to the consideration of external loads or the nature of supports, arbitrary displacements are imposed on each joint (node) of the truss. For example, a given node will be displaced an arbitrary distance in a given direction, whereas all other components of node displacement are held fixed. The forces required at the displaced node to produce this deflection are recorded, as are the reactions developed at all neighboring nodes (connected directly by a single member to the displaced node). These forces are known from previously determined information based on the stiffnesses of the individual truss members.

Each node is displaced in this fashion (successively in the x-, y-, and z-directions for a space truss), and the corresponding forces and reactions are recorded. Superimposing all such results leads to a set of equations relating node displacements to node forces. Because each node has been given an arbitrary displacement in each significant direction, the superposition of all such states will represent any possible truss deflections that may occur as a result of the application of any set of applied loads. Therefore, the force-deflection equations obtained in this manner can be made to fit any loaded state for the truss.

For an actual design problem, some nodes will be fixed because of the supports provided for the truss—forces at these nodes then become reactions. All other components of node force can be considered as possible applied loads. By specifying applied loads and treating node displacements as unknowns, a solution for these displacements can be obtained from the set of force-deflection equations. Once these deflections are known, the reactions at the supports can be found. Finally, because all node deflections are known, the forces induced in the individual truss members can be found. The solution will satisfy all the requirements listed previously.

Fortunately, the detailed application of the method is essentially tabular in nature. As a result, it is simple to set up the initial set of force-deflection equations. In fact, the usual practice is merely to set up the matrix of stiffness influence coefficients relating forces and deflections. As soon as this matrix has been found, the solution for node deflections, external reactions, and separate member forces proceeds from routine matrix operations.

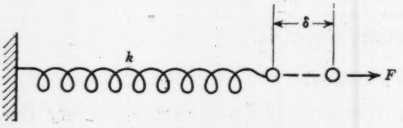


FIG. 1.—ELASTIC SPRING

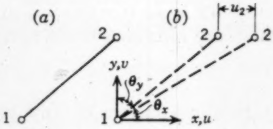


FIG. 2.—PIN-ENDED TRUSS MEMBER

Elastic Spring.—In Fig. 1 an elastic spring is deflected a distance, δ, in inches, by a force, F, in pounds. The relationship between force and displacement is

$$F = k\,\delta \qquad (1)$$

in which k is the spring "stiffness," in pounds per inch. As seen from Eq. 1, k can be interpreted as the force necessary to deflect the spring a unit distance.

From Eq. 1,

$$\delta = k^{-1}\,F \qquad (2)$$

in which k^{-1} is the spring "flexibility," in inches per pound. It is useful to interpret k^{-1} as the deflection due to a unit load. Similar expressions relate forces and displacements for complex elastic structures.

Pin-Ended Truss Member.—A pin-ended truss member is shown in Fig. 2; forces can be applied at nodes 1 and 2. In order to write the force-deflection equations, the following notation is established: L is the length of the unstrained member; A represents the cross-sectional area of the member; E is the modulus of elasticity of the member; u denotes the displacement in the x-direction; v is the displacement in the y-direction; λ represents $\cos\theta_x$ (same for unstrained and strained positions of the member); and μ is $\cos\theta_y$ (same for unstrained and strained positions of the member).

If node 1 is held fixed while node 2 is displaced a distance as shown in Fig 2(*b*), the change in length of the member is

$$\Delta L = u_2\,\lambda \qquad (3)$$

and the axial force, P, required to produce this change in length is

$$P = \frac{A\,E}{L}\,\Delta L = \frac{A\,E}{L}\,u_2\,\lambda \quad \text{(4)}$$

The components of P at node 2 are

$$F_{x2} = P\,\lambda = \frac{A\,E}{L}\,\lambda^2\,u_2 \quad \text{(5a)}$$

and

$$F_{y2} = P\,\mu = \frac{A\,E}{L}\,\lambda\,\mu\,u_2 \quad \text{(5b)}$$

Equilibrium of forces requires that

$$F_{x1} = -\,F_{x2} = -\,\frac{A\,E}{L}\,\lambda^2\,u_2 \quad \text{(5c)}$$

and

$$F_{y1} = -\,F_{y2} = -\,\frac{A\,E}{L}\,\lambda\,\mu\,u_2 \quad \text{(5d)}$$

In the same manner, the member can be subjected to displacement v_2 while all other components of node displacement remain zero. Repeating this for all components of node displacement and superimposing results will lead to the following set of force-deflection equations:

$$F_{x1} = \frac{A\,E}{L}\,(\lambda^2\,u_1 + \lambda\,\mu\,v_1 - \lambda^2\,u_2 - \lambda\,\mu\,v_2) \quad \text{(6a)}$$

$$F_{y1} = \frac{A\,E}{L}\,(\lambda\,\mu\,u_1 + \mu^2\,v_1 - \lambda\,\mu\,u_2 - \mu^2\,v_2) \quad \text{(6b)}$$

$$F_{x2} = \frac{A\,E}{L}\,(-\,\lambda^2\,u_1 - \lambda\,\mu\,v_1 + \lambda^2\,u_2 + \lambda\,\mu\,v_2) \quad \text{(6c)}$$

and

$$F_{y2} = \frac{A\,E}{L}\,(-\,\lambda\,\mu\,u_1 - \mu^2\,v_1 + \lambda\,\mu\,u_2 + \mu^2\,v_2) \quad \text{(6d)}$$

A convenient way to write Eqs. 6 is to use a matrix notation. The corresponding matrix form for Eqs. 6 is

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{x2} \\ F_{y2} \end{Bmatrix} = \frac{A\,E}{L} \begin{bmatrix} \lambda^2 & \lambda\,\mu & -\,\lambda^2 & -\,\lambda\,\mu \\ \lambda\,\mu & \mu^2 & -\,\lambda\,\mu & -\,\mu^2 \\ -\,\lambda^2 & -\,\lambda\,\mu & \lambda^2 & \lambda\,\mu \\ -\,\lambda\,\mu & -\,\mu^2 & \lambda\,\mu & \mu^2 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix} \quad \text{(7a)}$$

or, in a more compact notation,

$$\{F\} = [K]\{\delta\} \quad \text{(7b)}$$

in which $\{F\}$ is the column matrix of forces; $\{\delta\}$, the column matrix of displacements; and $[K]$, the square stiffness matrix relating forces and displacements.

Eq. 7*b* corresponds to Eq. 1 for the elastic spring, and the stiffness of the bar is represented by the square matrix, $[K]$. It should be noted that $[K]$ is symmetric—that is, if $k_{\alpha\beta}$ is the element in row α and column β of $[K]$, then $k_{\alpha\beta} = k_{\beta\alpha}$. This result exhibits the well-known reciprocal law for linearly elastic structures.

In studying the elastic truss member further, it is useful to establish sufficient supports to prevent it from moving as a rigid body. This can be accomplished by requiring at least three components of node displacement to be zero. For example, if $v_1 = u_2 = v_2 = 0$, then Eqs. 6 or 7 reduce to

$$F_{x1} = \frac{A\,E}{L}\lambda^2\,u_1 = -\,F_{x2} \qquad (8a)$$

and

$$F_{y1} = \frac{A\,E}{L}\lambda\,\mu\,u_1 = -\,F_{y2} \qquad (8b)$$

In Eqs. 8, F_{y1}, F_{x2}, and F_{y2}, are reactions, and F_{x1} is the applied load. If F_{x1} is specified, u_1 can be computed; knowing u_1, the reactions can be found. Finally, the force, P, can be found from Eq. 3, with ΔL replaced by $-\,\lambda\,u_1$.

The process explained herein for the single member can be applied to complex trusses. This possesses certain definite advantages over the more conventional types of analysis.

Stiffness Analysis of a Simple Truss

When the stiffness expressions for individual members of a structure are known, the stiffness of an assemblage of such members may be formed. A simple example is represented by the truss in Fig. 3.

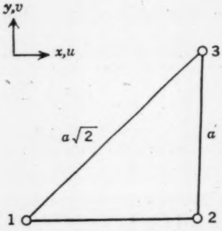


FIG. 3.—SIMPLE TRUSS

For simplicity it is assumed that all members have equal values of A and E. The stiffness matrix can be developed by first determining λ^2, μ^2, and $\lambda\,\mu$ for each member—this is done in Table 1.

It is now desirable to form the stiffness matrix for the complete truss. This is the principal task that the engineer must perform in the analysis, and, as will be seen, it is routine.

The stiffness of any single member is given by Eq. 7*a*. Because, for a truss, members have various lengths, it is advisable to bring this term inside the matrix. That is, every element in the square matrix of Eq. 7*a* must be multiplied by $1/L$. If the areas of members are different, A should also be brought inside the matrix.

Because, for the truss members, $L_{1-2} = a$, $L_{1-3} = \sqrt{2}/a$, and $L_{2-3} = a$, it is convenient to keep $A\,E/a$ outside the truss stiffness matrix. Individual member stiffness can be written in terms of $\bar{\lambda}^2$, $\bar{\mu}^2$, and $\bar{\lambda}\,\bar{\mu}$ with a common factor, $A\,E/a$, in which

$$\left(\frac{\lambda^2}{L}\right)_{i-j} = \frac{1}{a}\,\frac{(\lambda_{i-j})^2}{\dfrac{L_{i-j}}{a}} = \frac{1}{a}\,(\bar{\lambda}_{i-j})^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

For member 1–2,

$$(\lambda_{1-2})^2 = \frac{(\lambda_{1-2})^2}{\dfrac{L_{1-2}}{a}} = \frac{1}{\dfrac{a}{a}} = 1 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10a)$$

$$(\bar{\mu}_{1-2})^2 = \frac{(\mu_{1-2})^2}{\dfrac{L_{1-2}}{a}} = 0 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10b)$$

and

$$(\lambda\,\bar{\mu})_{1-2} = \frac{(\lambda\,\mu)_{1-2}}{\dfrac{L_{1-2}}{a}} = 0 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10c)$$

Table 1 also contains these values for the other members in the truss.

TABLE 1

Member	x	y	L	λ	μ	λ^2	μ^2	$\lambda\,\mu$	$\bar{\lambda}^2$	$\bar{\mu}^2$	$\bar{\lambda}\,\bar{\mu}$
1–2	a	0	a	1	0	1	0	0	1	0	0
1–3	a	a	$a\sqrt{2}$	$1/\sqrt{2}$	$1/\sqrt{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$1/(2\sqrt{2})$	$1/(2\sqrt{2})$	$1/(2\sqrt{2})$
2–3	0	a	a	0	1	0	1	0	0	1	0

Eq. 7*a* for the complete truss can be written directly from Table 1. The result will be presented first and then explained in some detail:

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{x2} \\ F_{y2} \\ F_{x3} \\ F_{y3} \end{Bmatrix} = \frac{A\,E}{a} \begin{bmatrix} 1 + \dfrac{1}{2\sqrt{2}} & \dfrac{1}{2\sqrt{2}} & -1 & 0 & \dfrac{1}{2\sqrt{2}} & -\dfrac{1}{2\sqrt{2}} \\ \dfrac{1}{2\sqrt{2}} & \dfrac{1}{2\sqrt{2}} & 0 & 0 & -\dfrac{1}{2\sqrt{2}} & -\dfrac{1}{2\sqrt{2}} \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ -\dfrac{1}{2\sqrt{2}} & -\dfrac{1}{2\sqrt{2}} & 0 & 0 & \dfrac{1}{2\sqrt{2}} & \dfrac{1}{2\sqrt{2}} \\ \dfrac{1}{2\sqrt{2}} & \dfrac{1}{2\sqrt{2}} & 0 & -1 & \dfrac{1}{2\sqrt{2}} & 1 + \dfrac{1}{2\sqrt{2}} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} \ldots (11)$$

The first element in the first column in the square stiffness matrix of Eq. 11 represents force F_{x1} due to u_1, and the second element represents force F_{y1} due to u_1. Similar explanations apply to the remainder of the column. Similarly, the second column represents these forces due to displacement v_1.

Imposing u_1, while all other node displacements remain zero, strains members 1-2 and 1-3 (Fig. 3). Noting the form of Eq. 7*a*, it is seen that for the truss,

$$F_{x1} = (\lambda_{1-2})^2 + (\lambda_{1-3})^2 = 1 + \frac{1}{2\sqrt{2}} \quad \text{(12a)}$$

$$F_{y1} = (\lambda\,\bar{\mu})_{1-2} + (\lambda\,\bar{\mu})_{1-3} = \frac{1}{2\sqrt{2}} \quad (12b)$$

and

$$F_{x2} = -\,\lambda_{1-2}{}^2 = -\,1 \quad (12c)$$

Following the form of Eq. 7*a* and using values in Table 1, all the columns in Eq. 11 can be written in turn.

Two checks can be applied to the stiffness matrix. First, it must be symmetric in the sense previously defined for the single truss member. Second, for each column the sum of the *x*-forces must vanish as must the sum of the *y*-forces.

The square matrix of Eq. 11 is the stiffness matrix for the truss. Solution for displacements, reactions, and member forces follow from a sequence of matrix operations. These are numerical and can be performed on a desk calculator or on digital computers.

Before proceeding with the solution, supports must be specified for the truss. These supports can be chosen in any desired manner; it is only necessary that rigid-body motion be prevented. For the present problem, nodes 2 and 3 will be fixed, whereas node 1 is kept free. Eq. 11 is then written as

$$\begin{Bmatrix} F_{x1}\\ F_{y1}\\ --\\ F_{x2}\\ F_{y2}\\ F_{x3}\\ F_{y3} \end{Bmatrix} = \left[\begin{array}{c|c} K_{11} & K_{12}\\ \hline K_{12}' & K_{22} \end{array}\right] \begin{Bmatrix} u_1\\ v_1\\ --\\ u_2 = 0\\ v_2 = 0\\ u_3 = 0\\ v_3 = 0 \end{Bmatrix} \quad (13)$$

in which $K_{11} \cdots K_{22}$ are obtained from the $[K]$-values of Eq. 11. For example,

$$[K_{11}] = \frac{A\,E}{a}\begin{bmatrix} 1 + \dfrac{1}{2\sqrt{2}} & \dfrac{1}{2\sqrt{2}}\\[2ex] \dfrac{1}{2\sqrt{2}} & \dfrac{1}{2\sqrt{2}} \end{bmatrix} \quad (14a)$$

$$[K_{12}] = \frac{A\,E}{a}\begin{bmatrix} -\,1 & 0 & -\,\dfrac{1}{2\sqrt{2}} & -\,\dfrac{1}{2\sqrt{2}}\\[2ex] 0 & 0 & -\,\dfrac{1}{2\sqrt{2}} & -\,\dfrac{1}{2\sqrt{2}} \end{bmatrix} \quad (14b)$$

$$[K_{12}]' = \frac{A\,E}{a}\begin{bmatrix} -1 & 0 \\ 0 & 0 \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} \end{bmatrix} \tag{14c}$$

and

$$[K_{22}] = \frac{A\,E}{a}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \\ 0 & -1 & \frac{1}{2\sqrt{2}} & 1+\frac{1}{2\sqrt{2}} \end{bmatrix} \tag{14d}$$

Eqs. 14 are submatrices of $[K]$ in Eq. 11.

In Eq. 13, F_{x1} and F_{y1} are applied loads, whereas the other forces are the unknown external reactions, and u_1 and v_1 are the unknown displacements. Solution for the unknown quantities results from expanding Eq. 13 into the following two sets of equations (by matrix multiplication):

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \end{Bmatrix} = [K_{11}]\begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix} \tag{15a}$$

and

$$\begin{Bmatrix} F_{x2} \\ F_{y2} \\ F_{x3} \\ F_{y3} \end{Bmatrix} = [K_{12}]'\begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix} \tag{15b}$$

From Eq. 15*a*,

$$\begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix} = [K_{11}]^{-1}\begin{Bmatrix} F_{x1} \\ F_{y1} \end{Bmatrix} \tag{16}$$

which gives displacements in terms of any set of applied loads. Substituting Eq. 16 into Eq. 15*b* results in

$$\begin{Bmatrix} F_{x2} \\ F_{y2} \\ F_{x3} \\ F_{y3} \end{Bmatrix} = [K_{12}]'[K_{11}]^{-1}\begin{Bmatrix} F_{x1} \\ F_{y1} \end{Bmatrix} \tag{17}$$

which gives unknown reactions in terms of any set of applied forces.

The matrix operations indicated in Eqs. 16 and 17 are standard. Eq. 16 corresponds to Eq. 2 and, as such, $[K_{11}]^{-1}$ represents the deflection or flexibility influence coefficients for the given truss. The value of $[K_{11}]$ can be obtained from the $[K]$ matrix of Eq. 11 by striking out columns and corresponding rows for which zero displacements (due to supports) have been specified.

The final step is that of determining truss member forces. By generalizing Eq. 4 it can be shown that the force in member i-j due to displacements u and v

of its nodes (at i and j) is given by

$$P_{i-j} = \left(\frac{A\,E}{L}\right)_{i-j} [\lambda_{i-j}(u_j - u_i) + \mu_{i-j}(v_j - v_i)] \ldots\ldots\ldots (18)$$

Such a relationship applies for each member of the truss. Because displacements are known in terms of applied loads from Eq. 16, the member forces can be computed from Eq. 18 by use of a matrix procedure.

For the problem at hand the following results are obtained:

(*a*) Displacements—

$$\begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix} = \frac{a}{A\,E}\begin{bmatrix} 1 & -1 \\ -1 & 1 + 2\sqrt{2} \end{bmatrix}\begin{Bmatrix} F_{x1} \\ F_{y1} \end{Bmatrix} \ldots\ldots\ldots\ldots (19a)$$

(*b*) Reactions—

$$\begin{Bmatrix} F_{x2} \\ F_{y2} \\ F_{x3} \\ F_{y3} \end{Bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \\ 0 & -1 \\ 0 & -1 \end{bmatrix}\begin{Bmatrix} F_{x1} \\ F_{y1} \end{Bmatrix} \ldots\ldots\ldots\ldots (19b)$$

(*c*) Member forces—

$$\begin{Bmatrix} P_{1-2} \\ P_{1-3} \end{Bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & -\sqrt{2} \end{bmatrix}\begin{Bmatrix} F_{x1} \\ F_{y1} \end{Bmatrix} \ldots\ldots\ldots\ldots (19c)$$

Eqs. 19 can be checked against a conventional analysis for this simple structure.

COMMENTS

Although the procedure described may seem strange and awkward, it possesses some real advantages for the structural engineer. These are as follows:

1. The procedure for setting up the stiffness matrix is entirely routine and is not complicated by redundant members or redundant reactions.

2. After the stiffness matrix has been found, the solution for deflections, reactions, and member forces follows from strictly numerical matrix operations. These computations can be efficiently performed using a desk calculator or digital computer.

3. Any set of applied loads can be inserted as the last step in the computations.

4. Changes in design only affect local sections of the stiffness matrix and subsequent numerical computations. As a result, several design configuration s can be investigated without undue effort.

5. The method applies equally well to determinate or undeterminate structures.

6. Structures containing different kinds of load-carrying members (axial-force members, beams, or torque cells) and possessing a high order of redundancy are especially suited to this method.

ANALYSIS OF AN INDETERMINATE TRUSS

The nodes of the truss in Fig. 4 are numbered 1 through 8. The lengths of the members can be determined from the dimensions in Fig. 4, and the values of *A* and *E* will be assumed as the same for each member. Supports are established that fix nodes 1, 2, 7, and 8. Therefore, the truss has three internal redundant members and five redundant components of external reactions.

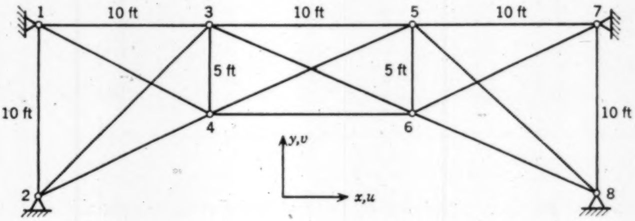


FIG. 4.—INDETERMINATE TRUSS

It is desired to find *u*- and *v*-components of displacement for nodes 3, 4, 5, and 6 as a result of any loading applied at these nodes. In addition, all internal member forces and all reactions are required. As for the simpler problem of Fig. 3, a table is determined first to give values of λ^2, $\cdots$ for each member. Conversion to $\bar{\lambda}^2$, $\cdots$ is made in this table. The common factor on all truss member stiffnesses is chosen as *A E*/10.

TABLE 2

Member	x	y	L	λ	μ	λ^2	μ^2	$\lambda\mu$	$\bar{\lambda}^2$	$\bar{\mu}^2$	$\bar{\lambda}\bar{\mu}$
1–2	0	−10	10	0	−1	0	1	0	0	1	0
1–3	10	0	10	1	0	1	0	0	1	0	0
1–4	10	−5	11.18	0.894	−0.447	0.799	0.200	−0.400	0.715	0.179	−0.358
2–3	10	10	14.14	0.707	0.707	0.500	0.500	0.500	0.353	0.353	0.353
2–4	10	5	11.18	0.894	0.447	0.799	0.200	0.400	0.715	0.179	0.358
3–4	0	5	5	0	−1	0	1	0	0	2	0
3–5	10	0	10	1	0	1	0	0	1	0	0
3–6	10	−5	11.18	0.894	−0.447	0.799	0.200	−0.400	0.715	0.179	−0.358
4–5	10	5	11.18	0.894	0.447	0.799	0.200	0.400	0.715	0.179	0.358
4–6	10	0	10	1	0	1	0	0	1	0	0
5–6	0	5	5	0	−1	0	1	0	0	2	0
5–7	10	0	10	1	0	1	0	0	1	0	0
5–8	10	−10	14.14	0.707	−0.707	0.500	0.500	−0.500	0.353	0.353	−0.353
6–7	10	5	11.18	0.894	0.447	0.799	0.200	0.400	0.715	0.179	0.358
6–8	10	−5	11.18	0.894	−0.447	0.799	0.200	−0.400	0.715	0.179	−0.358
7–8	0	10	10	0	−1	0	1	0	0	1	0

The stiffness matrix is not developed precisely as for the simple truss. The values of $\bar{\lambda}^2$, $\cdots$ in Table 2 are used, and the common factor outside the matrix will be *A E*/10. The order of the original stiffness matrix will be 16 by 16, as shown in Table 3.

For the truss in Fig. 4, zero displacements are shown at nodes 1, 2, 7, and 8. Therefore, it is convenient to rearrange the matrix of Table 3 to agree with the

TABLE 3

Node force	u_1	v_1	u_2	v_2	u_3	v_3	u_4	v_4
F_{x1}	1.715	−0.358	0	0	−1	0	−0.715	0.358
F_{y1}	−0.358	1.179	0	−1	0	0	0.358	−0.179
F_{x2}	0	0	1.068	0.711	−0.353	−0.353	−0.715	−0.358
F_{y2}	0	−1	0.711	1.532	−0.353	−0.353	−0.358	−0.179
F_{x3}	−1	0	−0.353	−0.353	3.068	−0.005	0	0
F_{y3}	0	0	−0.353	−0.353	−0.005	2.532	0	−2
F_{x4}	−0.715	0.358	−0.715	−0.358	0	0	3.145	0.358
F_{y4}	0.358	−0.179	−0.358	−0.179	0	−2	0.358	2.537
F_{x5}	0	0	0	0	−1	0	−0.715	−0.358
F_{y5}	0	0	0	0	0	0	−0.358	−0.179
F_{x6}	0	0	0	0	−0.715	0.358	−1	0
F_{y6}	0	0	0	0	0.358	−0.179	0	0
F_{x7}	0	0	0	0	0	0	0	0
F_{y7}	0	0	0	0	0	0	0	0
F_{x8}	0	0	0	0	0	0	0	0
F_{y8}	0	0	0	0	0	0	0	0

following form for the corresponding force-deflection equations:

$$\left\{\begin{matrix} F_{x3} \\ F_{y3} \\ F_{x4} \\ F_{y4} \\ F_{x5} \\ F_{y5} \\ F_{x6} \\ F_{y6} \\ -- \\ F_{x1} \\ F_{y1} \\ F_{x2} \\ F_{y2} \\ F_{x7} \\ F_{y7} \\ F_{x8} \\ F_{y8} \end{matrix}\right\} = \left[\begin{array}{c|c} K_{11} & K_{12} \\ \hline K_{12}' & K_{22} \end{array}\right] \left\{\begin{matrix} u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \\ u_6 \\ v_6 \\ -- \\ u_1 = 0 \\ v_1 = 0 \\ u_2 = 0 \\ v_2 = 0 \\ u_7 = 0 \\ v_7 = 0 \\ u_8 = 0 \\ v_8 = 0 \end{matrix}\right\} \quad \text{. . . . . (20a)}$$

or, in compact form,

$$\left\{\frac{F_{3,\,4,\,5,\,6}}{F_{1,\,2,\,7,\,8}}\right\} = \left[\begin{array}{c|c} K_{11} & K_{12} \\ \hline K_{12}' & K_{22} \end{array}\right] \left\{\frac{\delta_{3,\,4,\,5,\,6}}{0}\right\} \quad \text{. . . . . (20b)}$$

In Eq. 20*b*, $F_{3,\,4,\,5,\,6}$ represents any possible set of applied loads, and $F_{1,\,2,\,7,\,8}$ denotes the unknown external reactions. Also, $\delta_{3,\,4,\,5,\,6}$ represents all unknown

TABLE 3 (*Continued*)

u_5	v_5	u_6	v_6	u_7	v_7	u_8	v_8	Node force
0	0	0	0	0	0	0	0	F_{x1}
0	0	0	0	0	0	0	0	F_{y1}
0	0	0	0	0	0	0	0	F_{x2}
0	0	0	0	0	0	0	0	F_{y2}
−1	0	−0.715	0.358	0	0	0	0	F_{x3}
0	0	0.358	−0.179	0	0	0	0	F_{y3}
−0.715	−0.358	−1	0	0	0	0	0	F_{x4}
−0.358	−0.179	0	0	0	0	0	0	F_{y4}
3.068	0.005	0	0	−1	0	−0.353	0.353	F_{x5}
0.005	2.532	0	−2	0	0	0.353	−0.353	F_{y5}
0	0	3.145	−0.358	−0.715	−0.358	−0.715	0.358	F_{x6}
0	−2	−0.358	2.537	−0.358	−0.179	0.358	−0.179	F_{y6}
−1	0	−0.715	−0.358	1.715	0.358	0	0	F_{x7}
0	0	−0.358	−0.179	0.358	1.179	0	−1	F_{y7}
−0.353	0.353	−0.715	0.358	0	0	1.068	−0.711	F_{x8}
0.353	−0.353	0.358	−0.179	0	−1	−0.711	1.532	F_{y8}

node deflections of the truss. The submatrices, $K_{11}, \cdots$, are as follows:

$$[K_{11}] = \frac{AE}{10}\begin{bmatrix} 3.068 & -0.005 & 0 & 0 & -1 & 0 & -0.715 & 0.358 \\ -0.005 & 2.532 & 0 & -2 & 0 & 0 & 0.358 & -0.179 \\ 0 & 0 & 3.145 & 0.358 & -0.715 & -0.358 & -1 & 0 \\ 0 & -2 & 0.358 & 2.537 & -0.358 & -0.179 & 0 & 0 \\ -1 & 0 & -0.715 & -0.358 & 3.068 & 0.005 & 0 & 0 \\ 0 & 0 & -0.358 & -0.179 & 0.005 & 2.532 & 0 & -2 \\ -0.715 & 0.358 & -1 & 0 & 0 & 0 & 3.145 & -0.358 \\ 0.358 & -0.179 & 0 & 0 & 0 & -2 & -0.358 & 2.537 \end{bmatrix} \ldots (21a)$$

$$[K_{12}] = \frac{AE}{10}\begin{bmatrix} -1 & 0 & -0.353 & -0.353 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.353 & -0.353 & 0 & 0 & 0 & 0 \\ -0.715 & 0.358 & -0.715 & -0.358 & 0 & 0 & 0 & 0 \\ 0.358 & -0.179 & -0.358 & -0.179 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -0.353 & 0.353 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.353 & -0.353 \\ 0 & 0 & 0 & 0 & -0.715 & -0.358 & -0.715 & 0.358 \\ 0 & 0 & 0 & 0 & -0.358 & -0.179 & 0.358 & -0.179 \end{bmatrix} \ldots (21b)$$

and

$$[K_{22}] = \frac{AE}{10}\begin{bmatrix} 1.715 & -0.358 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.358 & 1.179 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.068 & 0.711 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0.711 & 1.532 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.715 & 0.358 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.358 & 1.179 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.068 & -0.711 \\ 0 & 0 & 0 & 0 & 0 & -1 & -0.711 & 1.532 \end{bmatrix} \ldots (21c)$$

The solution follows that for the simple truss. From Eq. 20*b*,

$$\{F_{3,\,4,\,5,\,6}\} = [K_{11}]\{\delta_{3,\,4,\,5,\,6}\} \ldots\ldots\ldots\ldots\ldots\ldots (22a)$$

and

$$\{F_{1,\,2,\,7,\,8}\} = [K_{12}]'\{\delta_{3,\,4,\,5,\,6}\} \ldots\ldots\ldots\ldots\ldots\ldots (22b)$$

From Eq. 22*a*,

$$\{\delta_{3,\,4,\,5,\,6}\} = [K_{11}]^{-1}\{F_{3,\,4,\,5,\,6}\} \ldots\ldots\ldots\ldots\ldots\ldots (23a)$$

and substituting this value into Eq. 22*b* results in

$$\{F_{1,\,2,\,7,\,8}\} = [K_{12}]'[K_{11}]^{-1}\{F_{3,\,4,\,5,\,6}\} \ldots\ldots\ldots\ldots (23b)$$

The laborious part of the foregoing analysis is that of finding $[K_{11}]^{-1}$. Because the matrix involved is 8 by 8, this is a sizable computation. However, the result presented herewith was obtained in less than 5 min by utilizing a high-speed digital computer. The numerical results are:

$$(K_{11})^{-1}=\frac{10}{A\,E}\begin{bmatrix} 0.410269 & -0.044793 & 0.060461 & -0.029113 & 0.144549 & -0.080315 & 0.105128 & -0.109534 \\ -0.044793 & 1.267086 & -0.115404 & 1.048185 & 0.080315 & 0.307294 & -0.155132 & 0.316080 \\ 0.060461 & -0.115404 & 0.430691 & -0.125972 & 0.105127 & 0.155132 & 0.178721 & 0.130841 \\ -0.029133 & 1.048185 & -0.125972 & 1.275019 & 0.109534 & 0.316080 & -0.130841 & 0.308777 \\ 0.144549 & 0.080315 & 0.105127 & 0.109534 & 0.410269 & 0.044793 & 0.060461 & 0.029113 \\ -0.080315 & 0.307294 & 0.155132 & 0.316080 & 0.044793 & 1.267086 & 0.115404 & 1.048185 \\ 0.105128 & -0.155132 & 0.178721 & -0.130841 & 0.060461 & 0.115404 & 0.430691 & 0.125972 \\ -0.109534 & 0.316080 & 0.130841 & 0.308777 & 0.029113 & 1.048185 & 0.125972 & 1.276019 \end{bmatrix} \quad ..(24a)$$

and

$$(K_{12})'(K_{11})^{-1}=\begin{bmatrix} -0.463921 & 0.502557 & 0.413503 & 0.575998 & -0.180502 & 0.082552 & -0.279754 & 0.126525 \\ 0.026856 & -0.228940 & 0.176736 & -0.273505 & 0.018029 & -0.001041 & 0.087403 & -0.008430 \\ -0.16820 & -0.724205 & -0.234451 & -0.726477 & -0.193757 & -0.304200 & -0.063292 & -0.277004 \\ -0.145447 & -0.577780 & -0.112243 & -0.543042 & -0.136619 & -0.192240 & -0.022910 & -0.175023 \\ -0.180502 & -0.082552 & -0.279754 & -0.126525 & 0.463921 & -0.502557 & -0.413503 & -0.575998 \\ -0.018029 & -0.001041 & -0.087430 & -0.008430 & -0.026856 & -0.228940 & -0.176736 & -0.273505 \\ -0.193757 & 0.304200 & 0.277004 & 0.277004 & -0.161820 & 0.724205 & -0.243451 & 0.726477 \\ 0.136619 & -0.192240 & -0.175023 & -0.175023 & 0.145447 & -0.577780 & 0.112243 & -0.543042 \end{bmatrix} \quad ..(24b$$

When used in Eq. 23 these results give all components of node deflection and all components of external reactions due to any specified set of external forces (applied at nodes 3, 4, 5, and 6).

Truss-member forces may be computed by applying Eq. 18 to each member of the truss to result in

$$\begin{Bmatrix} P_{1-3} \\ P_{2-3} \\ P_{1-4} \\ P_{2-4} \\ P_{3-4} \\ P_{3-5} \\ P_{4-5} \\ P_{3-6} \\ P_{4-6} \\ P_{5-6} \\ P_{5-7} \\ P_{6-7} \\ P_{5-8} \\ P_{6-8} \end{Bmatrix} = \begin{bmatrix} 0.41027 & -0.04479 & 0.06046 & -0.02911 & 0.14455 & -0.08032 & 0.10513 & -0.10953 \\ 0.18274 & 0.61115 & -0.02747 & 0.50954 & 0.11244 & 0.11349 & -0.02501 & 0.10328 \\ 0.06001 & -0.51160 & 0.39494 & -0.61119 & 0.04029 & -0.11233 & 0.19532 & -0.01884 \\ 0.03673 & -0.32696 & 0.29416 & 0.40963 & 0.12791 & 0.25053 & 0.09064 & 0.22818 \\ -0.03136 & 0.43780 & 0.02114 & -0.45566 & -0.05844 & -0.01758 & -0.04858 & 0.01460 \\ -0.26572 & 0.12511 & 0.04467 & 0.13865 & 0.26572 & 0.12511 & -0.04467 & 0.13864 \\ 0.04679 & -0.13978 & -0.14801 & -0.19558 & 0.21821 & 0.29213 & 0.00389 & 0.21438 \\ -0.21821 & 0.29213 & -0.00389 & 0.21438 & -0.04679 & -0.21378 & 0.14801 & -0.19558 \\ 0.04467 & -0.03973 & -0.25197 & -0.00487 & -0.04467 & -0.03973 & 0.25197 & -0.00487 \\ 0.05844 & -0.01758 & 0.04858 & 0.01460 & 0.03136 & 0.43780 & -0.02114 & -0.45566 \\ -0.14455 & -0.08032 & -0.10513 & -0.10953 & -0.41027 & -0.04479 & -0.06046 & -0.02911 \\ -0.04029 & -0.00233 & -0.19532 & -0.01884 & -0.06001 & -0.51160 & -0.39494 & -0.61119 \\ -0.03212 & 0.11349 & 0.02501 & 0.10328 & -0.18274 & 0.61145 & 0.02747 & 0.50953 \\ -0.12791 & 0.25053 & -0.09064 & 0.22808 & -0.03673 & 0.32696 & -0.29416 & 0.40963 \end{bmatrix} \begin{Bmatrix} F_{x3} \\ F_{y3} \\ F_{x4} \\ F_{y4} \\ F_{x5} \\ F_{y5} \\ F_{x6} \\ F_{y6} \end{Bmatrix} \quad ..(25)$$

CONCLUSIONS

It has been shown how complex structures can be analyzed by using the capabilities of electronic digital computers.

The paper develops and demonstrates a method of structural analysis applicable either to statically determinate or statically indeterminate structures. The procedure is shown to be especially useful for analyzing highly redundant structures, which is demonstrated for a truss containing a total of eight redundant forces.

A list of specific reasons is given showing why the method contains several advantages for structural engineers.

Finally, it is noted that the method can be extended to apply to structures other than pin-connected trusses.

ACKNOWLEDGMENTS

The detailed procedure described herein was devised at the Boeing Airplane Company, Seattle, Wash. Notable contributions to the development and application of the method were made by Marion J. Turner, Ray W. Clough, A.M. ASCE, and LeRoy J. Topp.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2958

MOBILE (ALA.) HARBOR AND SHIP CHANNEL

By Harold E. Bisbort,[1] M. ASCE

Synopsis

The Port of Mobile, Ala., is located at the head of Mobile Bay, which is an arm of the Gulf of Mexico having natural depths that are inadequate for deep-draft vessels. Federal interest in the provision of dredged channels connecting the port with the gulf began in 1826. The gradual improvement of the channel through the years is described and the history of its effect on commercial and industrial activity of the port and the inland in areas is traced.

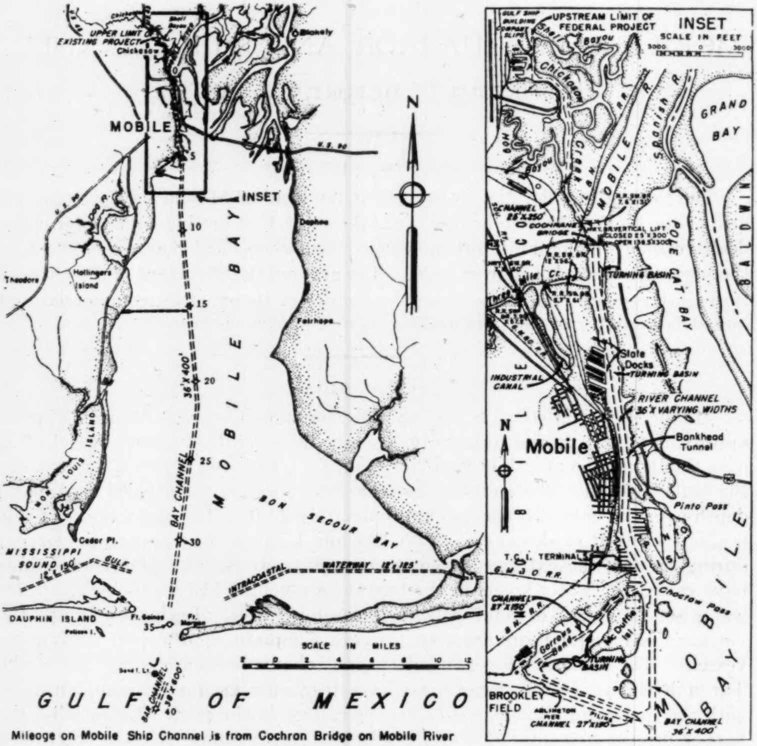
Introduction

Mobile Bay (Alabama) is very shallow, having an average natural depth of approximately 10 ft and, originally, a controlling depth of approximately 5 ft over the bar at the mouth of the river (Fig. 1). The pass between the bay and the Gulf of Mexico is about 3 miles wide, and the original natural controlling depth over the entrance bar was approximately 23 ft. The mean range of tide is about 1.2 ft at the lower end of the bay and 1.5 ft at the upper end. Except during storms, the extreme range is approximately 3.5 ft. Hurricane winds have raised the water levels in the bay to as much as 11.6 ft above mean low water and depressed it to 9.7 ft below mean low water. Both the bay and the entrance bar require dredging to provide adequate depths for ocean-going vessels. The federal government began channel improvement in 1826. Through the years the dimensions have increased gradually, commensurate with the increasing dimensions of vessels calling at the port. Essentially, the harbor project presently (1958) consists of a bar channel which is 38 ft deep and 600 ft wide, a bay channel which is 36 ft deep and 400 ft wide, and a river channel which is 36 ft deep over varying widths. The over-all length of the main channel from the gulf to the highway bridge over the Mobile River is approximately 40 miles. Depths of 42 ft on the bar and 40 ft in the bay channel and river channel have been authorized and will be dredged whenever the United States Congress appropriates funds.

Note.—Published, essentially as printed here, in May, 1957, in the Journal of the Waterways and Harbors Division, as *Proceedings Paper 1241*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Col. and Dist. Engr., Mobile Dist., Corps of Engrs., U. S. Dept. of the Army, Mobile, Ala.

In addition to Mobile, the geographical limits of the Mobile District of the Corps of Engineers (United States Department of the Army) embraces four other deep-water harbors—Gulfport (Miss.), Pensacola, (Fla.), Panama City (Fla.), and Port St. Joe (Fla.). All these harbors are actively engaged in foreign and coastwise trade. With the exception of Gulfport, the harbors are in sheltered embayments in which there are large areas with natural depths that are adequate for some of the larger freighters, and in which siltation problems (requiring expensive maintenance costs) are relatively insignificant. The harbors are important components of the national transportation system. However, in spite of their natural advantages, they have not kept pace with the phenomenal growth of Mobile, Alabama's only seaport. This gulf coastal port, on which the federal government has spent approximately $20,000,000 to overcome troublesome shoaling problems, presently accommodates nearly four times the tonnage of the other four ports combined. Mobile's growth and the factors


UPPER LIMIT OF EXISTING PROJECT
Chickasaw
Shell Bayou
Blakely
MOBILE
U.S. 90
5
INSET
N
Daphne
10
15
20
25
30
35
40
Theodore
Hollingers Island
Dog R.
MOBILE BAY
36'X400'
BAY CHANNEL
Fairhope
BON SECOUR BAY
MON LOUIS ISLAND
Cedar Pt.
MISSISSIPPI SOUND
12' × 150'
GULF
INTRACOASTAL WATERWAY 12' × 125'
DAUPHIN ISLAND
Pelican I.
Ft. Gaines
Ft. Morgan
Sand I. Lt.
BAR CHANNEL 36'X600'
SCALE IN MILES
2 0 2 4 6 8 10 12
GULF OF MEXICO
UPSTREAM LIMIT OF FEDERAL PROJECT
GULF SHIP BUILDING COMPANY SLIPS
Shell Bayou
INSET
SCALE IN FEET
3000 0 3000
Chickasaw Creek
L. & N. R.R.
MOBILE R.
Spanish R.
GRAND BAY
BALDWIN
Hog Bayou
CHANNEL 25'X250'
COCHRANE BRIDGE
HWY. BR. VERTICAL LIFT CLOSED 25'X300' OPEN 136.5'X300'
R.R. SW. BR. 7.6'X130'
R.R. SW. BR.
TURNING BASIN
Three Mile Cr.
POLE CAT BAY
State Docks
TURNING BASIN
INDUSTRIAL CANAL
RIVER CHANNEL 36'X VARYING WIDTHS
N
Mobile
MOBILE
Bankhead Tunnel
Pinto Pass
PINTO PASS
T.C.I. TERMINALS
G.M.&O. R.R.
Choctaw Pass
CHANNEL 27'X150'
L. & N. R.R.
Garrows Bend
McDuffie Isl.
TURNING BASIN
BROOKLEY FIELD
ARLINGTON PIER
PILING
CHANNEL 27'X150'
MOBILE BAY
BAY CHANNEL 36'X400'

Mileage on Mobile Ship Channel is from Cochran Bridge on Mobile River

FIG. 1.—LOCATION OF MOBILE HARBOR AND SHIP CHANNEL

resulting in its current commercial eminence may be of considerable general interest.

HISTORY

The Port of Mobile is situated strategically at the head of a virtually landlocked bay and at the mouth of a navigable river system that extends through one of the richest mining and industrial regions in the south. The city was once a peaceful Indian settlement known as Maubila, which was the capital of a large tribe of Choctaw Indians known as Maubilians, which means canoe paddlers.

Mobile Bay has been known by more than one name. The Spanish first termed it Spiritu Santo, and the English, noting its shape, designated it at one time as Gunstock Bay. The bay was first discovered and explored in 1519 by the Spaniard, Admiral Alvarez de Pineda, who recognized and noted its strategic position and other inherent advantages on the expedition map that was sent to Spain the following year. The Spanish expedition of Tristan de Luna visited Mobile Bay in 1559, but failed to establish a post there. The Spanish explorers left nothing tangible to be associated with Mobile Bay except nautical charts, which later furnished the French explorers with a geographical knowledge of the area. Under the leadership of the Canadian seaman, Jean Baptiste Le Moyne, Sieur de Bienville, who made use of the Spanish charts, a temporary post was established in 1699 on Dauphin Island opposite the entrance to Mobile Bay. In 1702 a permanent French colony, known as Fort Louis de la Louisianne, was established by Bienville on the Mobile River at Twenty-Seven-Mile Bluff, which, as the name implies, was approximately 27 miles upstream from the present city near the town presently known as Mount Vernon.

Fort Louis was settled simultaneously with the establishment of a permanent post on Dauphin Island. The French supply ships could not sail up Mobile Bay because of the shallow water. However, at that time, an excellent, crescent-shaped anchorage was available in a sheltered cove between the eastern end of Dauphin Island and a small sand island to the south. The anchorage, believed to be from 30 ft to 35 ft deep and large enough to accommodate from fifteen to thirty ships, was connected at its western end with the Gulf of Mexico by a pass that was 20 ft deep and as wide as the length of a French ship. At the little harbor, supplies destined for Fort Louis were transferred to shallow-draft vessels and freighted up the bay and river. A warehouse was built on Dauphin Island in which goods awaiting transfer were stored.

Fort Louis was the first capital of the vast Louisiana territory, and Dauphin Island, 60 miles to the south, served as its port for fifteen years. The fort at Twenty-Seven-Mile Bluff was abandoned in 1711, and the colonists moved downriver and settled at the present site of Mobile. Before 1711, the French had contemplated the move in order to reduce the distance between the fort and the warehouses on Dauphin Island, and to place the settlement within earshot of signal cannon on the island. However, the immediate cause of the change was a river flood and high tides resulting from a hurricane in August, 1711.

Mobile prospered until May, 1717, when a severe southwest storm washed sand into the narrow pass connecting the Dauphin Island anchorage with the

Gulf of Mexico, thus effectively bottling up the harbor. Communication by sea during colonial times was essential, and the disaster at Port Dauphin forced the French to shift the capital of the Louisiana territory to Biloxi (Miss.). Mobile was retained only as a supply depot for Indian trade along the waterways leading into the inland areas.

Historians have contended that it is a geographical rule that every important city in the world is built at a crossroads, or at the junction of vital trade routes. Upon examining the map, it may be seen that Mobile is a nerve center, with trade routes radiating from the port of entry northward along the Alabama River and the Coosa River, the Tombigbee River, and the Warrior River into a rich and productive land, and along sheltered intracoastal routes to Pensacola, Biloxi, and New Orleans, La. All these natural physiographical advantages combined to encourage the inevitable growth of the port. In addition, the political advantage of being the only seaport in Alabama in modern times has placed Mobile in a decidedly better competitive position from the standpoint of ocean shipping than many other gulf ports. The setback caused by the deterioration of the Dauphin Island anchorage was relatively short-lived. Shallow-draft vessels could be accommodated at that site, but it was still necessary for the larger supply ships to anchor in the lower Mobile Bay and transfer their cargo to lighters for the trip up the bay.

The control of the new colony passed from the French to the British in 1763, then to the Spanish in 1780. Mobile hoisted the United States flag in 1813, when General James Wilkinson seized the town from Spanish rule during the War of 1812. Mobile was granted a town charter in 1814 and a city charter in 1819 shortly after Alabama was admitted to the Union. In 1820 what is presently Mobile County had a population of approximately 2,700.

As is the case at present, port facilities in the early years were located on the west bank of the Mobile River, extending from its mouth for several miles upstream. Obstructive sand bars slightly downstream from the mouth prevented direct access to the city wharves, but vessels drawing 8 ft or less could bypass the shoals by sailing 12 miles up one of the deeper distributaries in the Mobile delta to a junction with the main stream, and then back down the main stream to Mobile. Vessels of deeper draft still had to be anchored in lower Mobile Bay for interchange of cargo with lighters or shallow-draft river boats. The circuity of the route to the port and the added expense of lightering cargo from the lower anchorage were responsible, at least in part, for the development of a port at Blakely, Ala., on the comparatively deep Tensaw River across the Mobile Delta, approximately 10 miles northeast of the present Mobile site. Blakely was an official port of entry of the United States, with a customs office, a shipyard, and a population in 1820 of approximately 3,500.

CHANNEL IMPROVEMENTS

It is axiomatic that the life of a port depends on deep-water access from the sea to the docks, without intermediate lighterage requirements. In 1825 an officer of the United States Engineers was stationed at Mobile and was appointed "Superintending Engineer of the construction of Fort Morgan and of the improvement of Mobile Harbor, Pass aux Herons, and the Pascagoula."

Federal interest in an adequate harbor at Mobile was inaugurated on May 20, 1826, when Congress appropriated $10,000 for removing obstructions and increasing the channel depths to the port. With these funds and later appropriations, Pinto Pass was improved, opened to traffic in 1829, and maintained until 1837. Ocean shipping was facilitated by this program, but many larger ships were still compelled to anchor in the lower bay and transfer cargo to lighters for the trip to Mobile.

Beginning in 1837 and extending to 1857, United States Army engineers dredged and maintained a depth of 10 ft over the obstructive bars at the mouth of Mobile River so that the larger vessels would have direct access to the city. This was a period of great prosperity for the port and its trade area. River traffic composed the life of the city and the state. The plantations along the Tombigbee River and the Alabama River systems furnished an abundance of products, with cotton predominating, for export through Mobile to England, France, Germany, and many other countries. The packet boats, which transported the products of inland Alabama to Mobile, also brought many wealthy planters who spent their profits in the port city. By 1860 the annual import and export trade was $40,000,000, and the city grew from a population of approximately 2,000 at the inception of United States administration to nearly 30,000 in 1860.

In 1839 Captain John Grant, a retired army engineer, opened up Grant's Pass to a depth of 6 ft between Mobile Bay and Mississippi Sound as a toll channel to connect Mobile with New Orleans by a protected inside route. This appears to have been the first step toward establishing what is presently the Gulf Intracoastal Waterway, an important trade route in the commercial life of the city.

The Port of Blakely declined in importance after the improvement of the channels approaching Mobile. However, by 1860 these channels had shoaled to a depth of 7½ ft, and during the Civil War the Confederate Army strengthened the defenses of the city by blocking the approaches through Choctaw Pass and Pinto Pass with multiple rows of piling and obsolete vessels filled with stones. When trade was resumed after the war, merchant vessels were again forced to take the devious route through one of the delta streams to reach the city docks.

By 1876 the obstructions had been removed and a channel was reopened through the bars at the mouth of the river to a depth of 13 ft. Thereafter, the improvement of the ship channel through the bay kept pace with the rapid growth and development of the interior. Statistical records show that each time the harbor channels were deepened, water-borne commerce at the port increased. Space limitations prohibit a detailed description of the extent to which the channel dimensions were increased and for what specific reasons. A good example of how the channel depth affected the economic well-being of the port city can be found by studying the history of the era immediately following the Civil War. During that period, only one railroad extended from Mobile into the interior, and most of the cotton, which comprised the principal export commodity of all southern ports, was shipped to Mobile by river steamer rather than by lighter, to the fleet anchorage in lower Mobile Bay. Other southern ports were well served by railway lines, and most of the ports had deep-water access to the docks. Rapid transit offered by the railroads induced planters and merchants in the inland areas to do business with the ports, and, because railroads were more interested in ports with deep water at the wharves, Mobile suffered the consequences. Cotton exports declined from more than $12,000,000 in 1877 to $3,000,000 in 1882. The rapid decline in cotton shipments affected every line of business. Unemployment was widespread, and there were less profits for the brokerage firms and others that handled cotton.

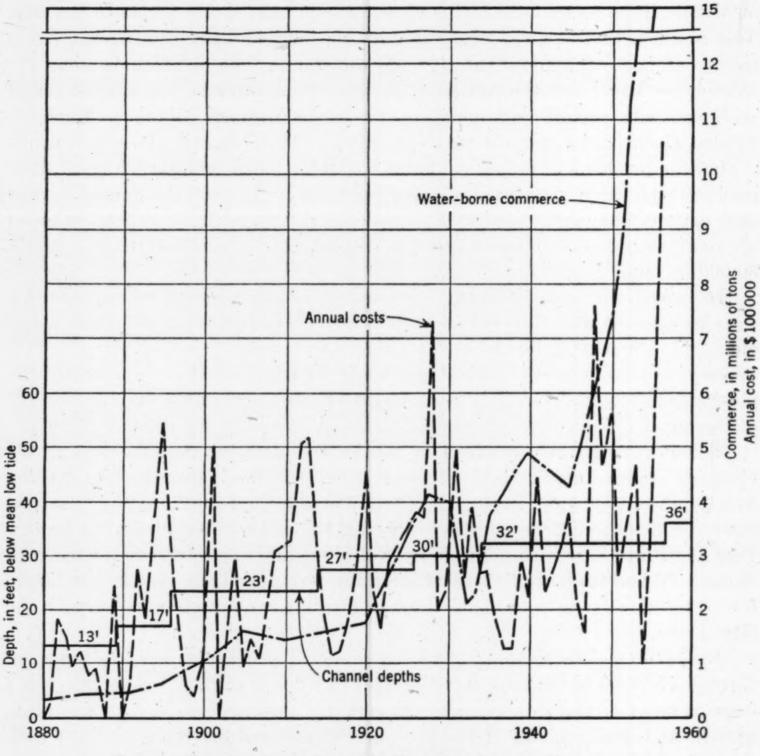


FIG. 2.—INCREASES IN CHANNEL DEPTHS, ANNUAL COSTS, AND GROWTH OF COMMERCE IN MOBILE HARBOR

As a result of the concerted efforts of local tradesmen, a channel improvement program was begun in 1870 by the federal government and was conducted until a continuous waterway was available from the wharves to the gulf that was deep enough to eliminate expensive lighterage operations. History records that the improvement in water-borne commerce which followed was quick and marked. All business firms were given new life, and the period of depression eventually ended.

Briefly, the bay-channel depths and river-channel depths were increased by the federal government from 10 ft to 13 ft, during the period from 1870 to 1876; to 17 ft, during the period from 1880 to 1889; to 23 ft during the period from 1889 to 1896; to 27 ft, from 1910 to 1914; to 30 ft, from 1918 to 1926; and to 32 ft, during the period from 1930 to 1934 (Fig. 2). The bar channel first received attention in 1902 when Congress authorized dimensions of 30 ft by 300 ft. In 1917 the authorized dimensions of the bar channel were increased to 33 ft by 450 ft, and in 1930 another increase was made to 36 ft by 450 ft. From time to time various side channels and turning basins were added to the project. In the 1954 River and Harbor Act, Congress authorized dimensions of 42 ft by 600 ft across the entrance bar, 40 ft by 40 ft in the bay, and a depth of 400 ft in the river channel as far north as the highway bridge, including a new turning basin opposite Magazine Point. The first phase of this development is completed. It consists of a 38-ft depth over the bar and a 36-ft depth in the bay and river. Actually, in order to compensate for the shoaling that was expected to take place in the bay channel and river channel during the long dredging process, the contractor was required to excavate to a depth of 38 ft, with a 1-ft allowable overdepth. This first phase of improvement required the removal of nearly 34,000,000 cu yd of material. Approximately 20,000,000 cu yd of additional dredging will be needed to provide the 40-ft project. This project will be provided when it is found warranted by shipping demands and if Congress appropriates funds.

DREDGING PROBLEMS

In the 130-yr period beginning in 1826, the United States Government has spent approximately $8,700,000 for new work dredging in Mobile Bay and Mobile River and nearly $11,300,000 for maintenance to restore authorized, periodic channel dimensions. During the period from 1870 to 1956, the quantity of material actually removed during dredging comprised nearly 320,000,000 cu yd. Bay-channel dredging was contracted for 50¢ per cu yd in 1870; for 17¢ per cu yd by 1875; and from 1875 to 1900, as the efficiency of excavating machinery increased, unit costs decreased gradually to approximately 7¢ per cu yd. In more recent years, maintenance costs have usually fluctuated from as little as 3¢ per cu yd to approximately 6¢ per cu yd, with new work costs slightly higher. Contract price per cubic yard for the most recent (1956–1957) new work in the bay was 8.27¢, and in the river, 15.47¢. Excavation in the bar channel, for which a government-owned hopper dredge is used, usually costs on the order of 18¢ per cu yd.

Until the early 1900's the bay channel and the river channel were excavated by dipper, or clam-shell dredges, the material being loaded on scows and hauled to the spoil areas. During the early history of the project, the clam-shell dredges removed on an average of from about 2,000 cu yd per day to 4,000 cu yd per day. Toward the end of the nineteenth century and during the early 1900's, the dredge capacities were from approximately 7,000 cu yd per day to 10,000 cu yd per day. Between about 1900 and 1910, the hydraulic dredges with their tremendous excavation capacities began to be used on area projects. The large hydraulic dredges used to enlarge the bay channel and river channel in 1956 and 1957 worked twenty four hours per day and removed, during an average day, quantities ranging from 28,000 cu yd to 40,000 cu yd, depending on the particular section of the channel being dredged and type of material encountered. The largest of the three dredges used had a 30-in. suction line, a 27-in. discharge, and a 2,100-hp diesel engine on the pump. The government-owned hopper dredge that is generally used on bar-channel dredging is equipped with two pumps, each with 1,150-hp motors, 30-in. suction pipes, and 27-in. discharge lines. The bin capacity is 3,000 cu yd.

Based on the twenty-two year record during which the 32-ft project was in effect, the average annual siltation rate in the bay channel and river channel, as determined from maintenance-dredging records, was 3,556,000 cu yd and 410,000 cu yd, respectively, or a total of nearly 4,000,000 cu yd. The degree to which the shoaling rate in the bay and river has varied with channel cross-sectional areas is illustrated in Table 1.

Records of maintenence dredging and allocation of yardage removed for new work and for maintenance for the 23-ft project are not clear, and the annual average derived in Table 1 is not conclusive. For the remaining projects, the available evidence does not indicate that successive increases in channel dimensions have resulted in particularly significant increases in the annual rate of filling.

TABLE 1.—COMPARISON OF AVERAGE ANNUAL MAINTENANCE DREDGING

Minimum channel dimensions, in feet	Year completed	Average annual maintenance dredging, in cubic yards
23 × 50 to 100	1896	1,127,000
27 × 200	1914	3,624,000
30 × 300	1926	3,661,000
32 × 300	1934	3,966,000

Previous investigations and studies of the Mobile Bay channel problem disclosed that shoaling is not largely a result of the caving of the banks, which appear to be relatively stable on a 1-on-5 slope, nor does the bed of the channel rise, or the sides flow in. Siltation is due apparently principally to the tidal currents, which generally do not coincide with the direction of the dredged channel. The material that is carried by the cross currents, either in suspension or as bed load, is deposited in the channel and, due to the reduction in current velocity, is not carried over the opposite bank.

Maintenance is performed each year for designated sections of the main channels, the work being programmed so that each section will be redredged in three-year to four-year cycles. The bar channel is subject to filling from littoral material moving predominately from east to west and is redredged about every two or three years. The annual average shoaling rate, based on a ten-year record for the 36-ft by 450-ft channel is approximately 178,000 cu yd.

Material from bay-dredging operations is discharged through floating pipelines to spoil dumps in open water, located about 2,000 ft from the edge of the channel, both on the east side and west side. Through these parallel spoil mounds, openings are left wherever they may be required by small boat traffic.

There is no evidence to indicate that much, if any, material from the spoil areas finds its way back to the channel. Most of the material from river dredging is discharged through a shore pipe to spoil areas along the eastern edge of Pinto Island and east of the highway on Blakely Island. Temporary easements have been obtained from property owners for this purpose, and either perpetual easements or long-term leases are being sought for some areas in order that spoil from future maintenance operations will not present disposal problems.

Growth of Commerce and Industry

Many factors have influenced the growth of commerce and industry at the Port of Mobile. The French settlers, who were essentially traders, imported blankets, cloth, and axes from France to trade with the Indians for skins and furs which were to be exported to France and England. The bulk of the water-borne commerce at Mobile during early colonial times consisted of such miscellaneous items. Then, in 1793, the cotton gin was invented, making the growth and marketing of cotton profitable and of worldwide economic importance. The greater quantity of cotton that was grown in upstate plantations was transported via the Alabama River and the Tombigbee River to Mobile for export to European mills. The strategic position of Mobile in this trade made it one of the leading ports on the Gulf of Mexico and a rich economic prize for the European countries competing for the possession.

The railroad building era after the Civil War fostered the development of the second major export commodity. The railroads tapped vast forests of virgin pine and hardwoods in the inland areas, and the lumbering business, which flourished in the ensuing years, furnished cargoes to vessels leaving Mobile for many destinations. By extending the boundaries of the port's trade area, the railroads tended to diversify the import, export, and coastwise trade.

Birmingham, Ala., was established in 1871 in a region that was rich in coal and iron-ore deposits and other minerals needed to produce iron and steel. The steel industry, which grew up in that region during the latter part of the nineteenth century, and the improvement of the Warrior-Tombigbee Waterway to provide a cheap transportation route to tidewater further enhanced Mobile's importance to the state economic welfare. During the late 1800's and the early 1900's, cotton, timber, lumber, coal, iron and steel, sulfur, bananas, sisal grass, and manufactured articles formed the bulk of the export-import trade. Tonnage handled over the city wharves increased steadily from less than 500,000 in 1880 to 3,000,000 in 1925.

Many developments described previously contributed to the growth of Mobile as a port. However, except for the deep-water channel itself, the construction of the Alabama State Docks and Terminals in 1928 was probably the most significant. In 1948 a weekly periodical[2] of the United States Department of Commerce, in commenting on the state docks, stated that

> "* * * they form the strategic link between the hinterland and the waterways of the world. They represent the State's successful effort to increase

---

[2] "Mobile Wins New World-Trade Rank and Sets Still Higher Goals," by Annie S. Howard, *Foreign Commerce Weekly*, U. S. Dept. of Commerce, Washington, D. C., June 26, 1948.

the flow of commerce through the port to the economic benefit of the whole State."

As shown in Fig. 2, statistical proof of the beneficial effect of the state docks on the port is the fact that 4,000,000 tons of commerce handled in 1928 had increased to 15,000,000 tons by 1955.

The principal units of the docks system include the following:

1. Three piers, which are 1,600 ft long, and a riverside wharf, 1,700 ft long, all with steel and concrete transit sheds and warehouses of modern design providing 2,221,000 sq ft of covered storage;
2. A shipside bulk-material handling plant with an optimum capacity of 600 tons per hr onto ships, and unloading equipment consisting of five towers, with capacities ranging from 300 tons per hr to 800 tons per hr each of bauxite, iron ore, manganese, and other bulk materials;
3. A shipside, bonded cotton warehouse and a high-density press;
4. A shipside cold-storage plant with a quick-freezer;
5. A grain elevator with a 1,600,000-bu capacity; and
6. A terminal railroad with a car-holding capacity of 1,000 cars, connecting the dock system and adjacent industries with the four trunk rail lines serving the port. Handling equipment includes heavy-duty cranes with a capacity ranging from 30 tons to 75 tons; small crawler-type cranes; numerous warehouse tractors and lift trucks; and several portable conveyers. Additional berthing and terminal facilities to accommodate three ships are being constructed, and a building to house an international trade mart is planned. When the new facilities are complete, twenty-eight ship berths will be available at the state docks, and the entire port will afford berthing space for approximately fifty-five ships. The total investment in the state docks is $35,000,000, and a $5,000,000 expansion program is presently under way.

Since the state docks system was placed in operation, several developments have promoted an additional increase in the receipt and shipment of bulk commodities through the port facilities.

The depletion of iron-ore reserves in the United States caused the steel companies to search for new sources in South America and other countries. As a result of the discovery of important deposits in Venezuela, in 1954 shipments to the Birmingham mills were begun, and a newly constructed bulk handling plant at Choctaw Point at the mouth of the river was put in operation. The new plant has an unloading capacity of approximately 1,300 tons per hr. The ore is unloaded by a grab bucket onto conveyer belts and then into either rail hopper cars or barges. Ore imports were 2,000,000 tons in 1955, and in the near future, steel interests expect to import not less than 3,000,000 tons annually. Three of the largest freighters afloat (60,000 tons deadweight) are used in this trade.

The recent discovery of crude oil in Mississippi and Alabama was followed by the construction of port outloading facilities, including a storage tank farm just upstream from the state docks' property and a crude oil pipeline extending to the Baxterville field and Eucutta field in Mississippi and the Citronelle field in Alabama. More than 1,000,000 tons of crude oil were shipped in 1955 from these facilities by coastwise tanker to Atlantic and Gulf Coast refineries, and the volume increases daily as new wells come into production.

The increasing use of aluminum and aluminum products in the aircraft and other metal industries has caused the demand for bauxite from foreign sources to rise. In 1955 nearly 2,500,000 tons of bauxite were imported through the bulk materials handling plant at the Alabama state docks in ships ranging to a deadweight tonnage of 32,000 and a draft of 34.5 ft.

Conclusions

According to the Bureau of Business Research of the University of Alabama, at University, the Port of Mobile is well balanced and exceptionally well equipped. It has an excellent combination of a good harbor, inland waterway arteries, intracoastal waterway arteries, and rail facilities to maintain its position as a leading port. The harbor has terminal and port facilities that are among the best in the United States, with all types of ship services available. A canalized waterway via the Mobile River, Tombigbee River, and Warrior River, provides adequate depths for modern barge traffic between Mobile and the rich, industrial Birmingham region in north-central Alabama. The ultimate development of the Alabama-Coosa branch of the Mobile River system, the initial development of which is authorized, will further enlarge the system of waterways connecting Mobile with the inland industrial sections of the state. A continued industrial expansion is indicated in the Mobile area, and port activity is expected to increase with the new, deep 36-ft channel and after the authorized 40-ft channel has been provided.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2959

SURFACE WATER RESOURCES

By Joseph V. B. Wells,[1] M. ASCE

Synopsis

The average annual runoff for any stream during a period of years tends to conform to a characteristic geographic pattern, whereas year-to-year runoff varies widely. Annual runoff in the United States during the 4-yr period from 1953 to 1956 is analyzed briefly. The paper describes the rapid expansion of irrigation in the eastern states and emphasizes the importance of hydrologic analyses and legislation to meet problems that will arise as a result of competition with other uses.

Introduction

The United States is becoming increasingly water conscious mainly because of water problems that have developed in many communities during recent years. The rapidly increasing use of water has caused some concern over the possibility that the nation's future economic growth may be handicapped by an inadequacy of water. As a result of diminished precipitation, since 1951 streamflow has been generally less than normal over much of the southern half of the United States. In the southwest, drought has prevailed since the middle 1940's, and many other communities in the United States have experienced temporary water shortages. Although there appears to be no significant dwindling of the total water supply, the margin between supply and demand is narrowing.

Availability of water probably will be a limiting factor in the further expansion of certain uses of water in some areas, and it is almost inevitable that conflicts between types of uses will become more frequent. There is need for a nationwide coverage of up-to-date legislation governing the use of water.

Distribution of Runoff

The total amount of water on the earth is almost limitless as compared with the needs of man, and, as far as is known, it remains practically constant. A

Note.—Published, essentially as printed here, in June, 1957, in the Journal of the Sanitary Engineering Division, as *Proceedings Paper 1272*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Chief, Surface Water Branch, Water Resources Div., Geological Survey, U. S. Dept. of the Interior, Washington, D. C.

small part of the water is distributed over the earth and becomes available to man for his use as a result of the natural phenomena known collectively as the hydrologic cycle. The hydrologic cycle is affected by many natural influences. As a result of some of these influences, such as physiography and prevailing winds, the distribution of moisture tends to follow a geographic pattern. As a result of other and more obscure influences the pattern of moisture distribution is irregular with respect to time.

A generalized geographic pattern of average annual runoff in the United States is shown in Fig. 1. The figure illustrates that water is much more plentiful in the east than in the west. The average annual runoff is less than 1 in. over large parts of many of the western states—an aggregate area of more than 25% of the total area of the United States. However, in the eastern states the average annual runoff is generally in excess of 10 in. In twelve states east of the Mississippi River, it is 20 in. or more. The nationwide total averages approximately 9 in. per yr, or approximately 30% of the average annual rainfall.[2]

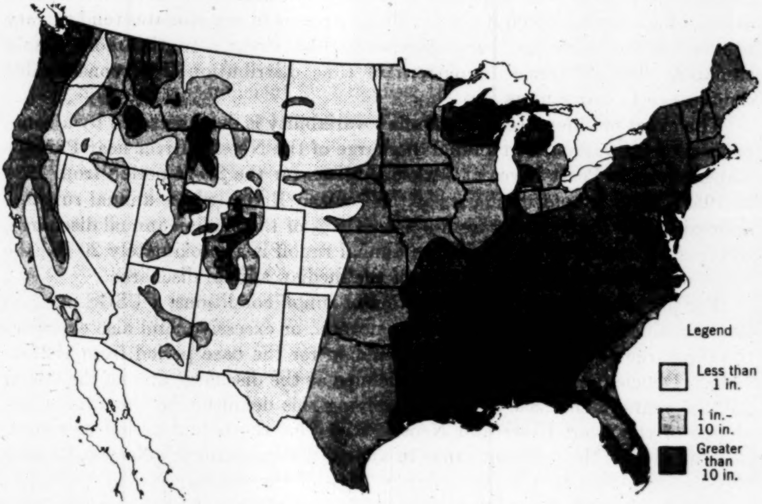


FIG. 1.—AVERAGE ANNUAL RUNOFF

Much more complex is the time distribution of moisture. In view of the rapidly mounting use of water, the water shortages experienced in recent years have resulted in some concern over the possibility of a downward trend in water supplies as a result of long-term climatic trends. Streamflow records that have been collected by the Geological Survey, United States Department of the Interior (USGS), for a period of 50 yr or more do not indicate a persistent downward nationwide trend, when adjustments are made for man-made changes in the consumptive use of water. Far-reaching climatic trends appear to be so gradual as to be scarcely discernible within a lifetime. Of much greater immediate significance is the great year-to-year variability in weather and the apparent tendency for wet years and dry years to group in sequences in which one or the other predominates.

---

[2] "Annual Runoff in the United States," by W. B. Langbein et al., *Circular 52*, Geological Survey, U. S. Dept. of the Interior, Washington, D. C., 1949.

The year-to-year variation in streamflow is due to a combination of climatic conditions and hydrologic conditions. This variation is greater than that taking place in precipitation because of the fact that streamflow is residual. With nature's "take" remaining relatively constant from year to year, the percentage difference in runoff is much greater than the percentage difference in rainfall. Furthermore, and for the same reason, the percentage difference in runoff from year to year is greater in areas of low annual precipitation than in areas of high annual precipitation. Thus, streams in dry climates tend to vary between wider extremes, percentagewise, than streams in the more humid climates. The difference in pattern of time distribution is of considerable significance to water users.

The effect of difference in streamflow variability is demonstrated by a com-, parison of the variation in annual discharge of the Neosho River near Parsons Kans., and the Pearl River at Jackson, Miss., for the 27-yr period from 1929 to 1955. The annual discharge of the Neosho River, whose annual runoff is approximately 5 in., varied from 9% to 410% of the median annual discharge, whereas for the Pearl River, whose annual runoff is approximately 20 in., the range was only from 42% to 222% of the median annual discharge.

For purposes of general classification of runoff conditions, a USGS publication[3] classifies streamflow as deficient, normal, or excessive, and also expresses the discharge as a percentage of the median for the base period from 1921 to 1945. Deficient annual discharge is defined as the discharge among the lowest 25% of years in the base period. Applying this definition to the annual discharge of the Pearl River and Neosho River for the actual period of record, it is found that the deficient range in annual discharge on the Neosho River is the discharge less than approximately 30% of the median annual discharge, and on the Pearl River, less than approximately 70%. Thus, an annual discharge of 65% of the median would be considered deficient on the Pearl River, whereas an annual discharge of 35% of the median would be within the normal range on the Neosho River. The preceding two records were chosen because they are streams in the general area that illustrated the point that was being made, but they are not extreme examples of the runoff conditions for either a humid area or an arid area.

DROUGHTS

What is a drought? Many general definitions can be found. *Webster's New International Dictionary* defines drought as "* * * such dryness of weather or climate as affects the earth, and prevents the growth of plants," which probably expresses a widely accepted concept of drought, at least from a qualitative point of view. Thus far, no satisfactory formula has been devised that defines a drought quantitatively in terms of rainfall, streamflow, or soil moisture.

---

[3] "Water Resources Review," Geological Survey, U. S. Dept. of the Interior, Washington, D. C., 1952–1956.

Droughts may be classified broadly as (1) water supply droughts and (2) agricultural droughts. Both may be the result of climatic conditions, or the result of the demand outgrowing the supply. Provided that the needs for water in a community are satisfied, no evidence of droughts is noticed, however small the supply may be. Conversely, when needs for water exceed the supply, an apparent drought exists even though the deficiency may be no greater than can be expected to recur every few years. However, when deficiencies occur simply because the supply has been overdeveloped, can such deficiencies correctly be termed droughts?

The effect of a dry period varies widely with the type of vegetation and with the type of irrigation common to an area. Deep-rooted grasses and trees can withstand a drought of several weeks, whereas shallow-rooted vegetation will be badly damaged. In areas in which long periods of no rainfall occur every year, native vegetation adapts itself to the climate and man makes provision to irrigate his crops. In these areas droughts occur only when the supply of water is insufficient for irrigating the usual acreage.

In a country as large as the United States and one that is subjected to such a diversity of climatic influences, it is only in an unusual year that streamflow is within the normal range in all parts of the nation. The occurrence of moisture deficiencies and consequent deficient runoff in some areas of the United States may be considered normal. However, the present drought in the southwest is of such a wide areal extent and of such persistence that it is undoubtedly one of the major droughts on record.

For the past several years, the USGS has been in the process of preparing a report on droughts in the southwest. The USGS had been waiting for the end of the drought before completing the report but decided to close the report with the year ending September 30, 1956, even if it resulted in an interim rather than a final report. More than one wet year will be needed before it can be stated that the southwest drought is really finished.

The southwest drought began in Arizona and New Mexico in 1943 and, except for occasional temporary relief, has persisted. It gradually spread eastward and northeastward as far as Illinois. However, runoff for the water year ending September 30, 1952, was normal or greater in all the states except the gulf states (although during the summer, runoff was deficient over large areas). Hence, attention is directed mainly to the annual runoff for the 4 yr from 1953 to 1956.

During the foregoing period, annual runoff was deficient or in the low-normal range over most of the southern United States from the Atlantic Ocean to the Pacific Ocean. The most notable exception was an area embracing Mississippi and Louisiana and parts of the adjacent states, in which runoff was greater than median and even excessive in 1953. In 1954 the areas of deficiency spread widely. Conditions improved slightly in 1955, and in the southwest the drought pattern showed signs of disintegrating. However, during 1956 no definite evidence was observed that the drought was generally abating. In fact, annual runoff reached new record lows at three key gaging stations in Arizona and one in Texas, as well as two in Florida, one in Virginia, and two in western North Carolina. These key stations are used by the USGS as indexes of runoff conditions in the general area in which they are located. Annual runoff in the state of Oklahoma for the year 1956 was the least of any calendar year since the collection of streamflow records began. The power output of Bagnell Dam (Osage River, Missouri) in 1956 was the least in 25 yr of record, and was only 12% of the average annual output for that period. Combined storage in Elephant Butte Reservoir and Caballo Reservoir (Rio Grande, New Mexico) at the end of 1956 was only 5% of the 18-yr average, and only a little more than 5% of the normal annual irrigation demands. These statistics show that the drought conditions are widespread and most serious.

The number of deficient years in the 4-yr period from 1953 to 1956 serves as an index of the deficiency of annual runoff in the United States during that period. "Contours" indicating the years of deficient runoff at each index station that was used[2] are shown in Fig. 2.

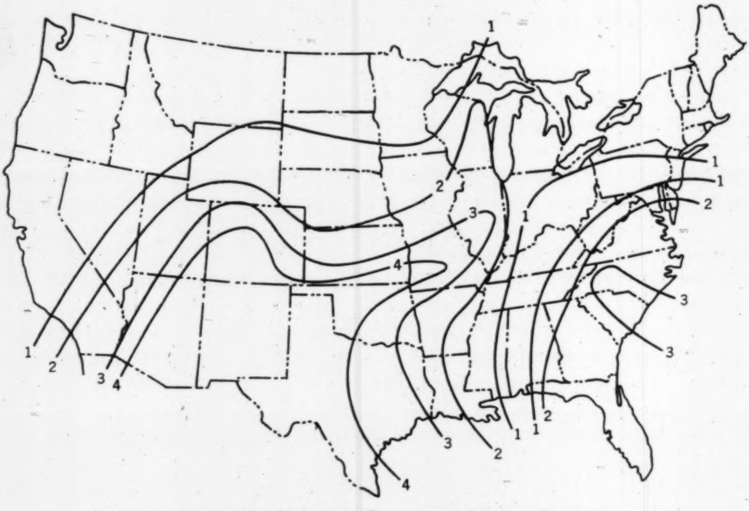


FIG. 2.—NUMBER OF YEARS OF DEFICIENT ANNUAL RUNOFF IN A 4-YR PERIOD FROM 1953 TO 1956

Deficient runoff has been defined previously as the runoff during the lowest 25% of the years in the base period from 1921 to 1945. Accordingly, a deficiency in annual runoff during 1 yr of the 4 yr could be expected on the average. Moreover, because of the fact that wet years and dry years do not follow in regular sequence, it could be considered normal to find a scattering of points of 0 yr, 1 yr, or 2 yr deficient out of 4 yr, but only in an area of protracted drought could a consistent pattern of from two to four deficient years out of four be expected. The lines joining the points of an equal number of deficient years emphasize the severity and scope of the drought.

The area of the drought during the last 4 yr is delineated by the 4-yr contour. Less severe droughts are indicated by 3-yr and 2-yr lines. One striking feature of the drought pattern is the tongue that protrudes across Missouri and into Illinois. Except for an area in the Carolinas, no persistent pattern of deficiency of annual streamflow is indicated in the southeastern and mid-Atlantic states, although there is a greater tendency toward deficient runoff than would be expected, on the average, in a 4-yr period.

Attention is focused on annual runoff instead of monthly runoff or seasonal runoff. Although seasonal deficiencies, such as the drought in northern Georgia in 1954, may thus be obscured, it is believed that a persistent deficiency in streamflow is illustrated better by annual runoff than by the much more erratic pattern that would be indicated by monthly runoff or seasonal runoff. This does not imply that water shortages of relatively short duration are any less serious to a particular area than those based on annual runoff. Just the reverse could be the case, but herein the annual runoffs are believed to be more meaningful.

Development of Water Supplies

Deficiencies as well as floods can be considered normal events because they will continue to occur. Water needs will have to be reconciled with the natural supply. Through modification in water needs and utilization practices, and through greater expansion of facilities for storage and conveyance, it is possible to stretch the present supplies.

In some of the problem areas of the west, water supplies are being taxed heavily, and further growth of the communities dependent on those supplies must involve some modification of water uses. Some of the irrigated land in the west may have to be retired in order to provide water for use with a higher cash value. Undoubtedly, much can still be accomplished toward reducing the waste of water by consumers. Also, more can be accomplished toward the elimination and the control of nonbeneficial water-loving plants, although the economic value of the water thus salvaged may not justify the expense involved. Importation of water may be the answer in some areas provided that a source of supply is available and that the expense of conveyance facilities is economically justified.

In the eastern United States the picture is generally brighter as far as the availability of surface water is concerned. The east has had an increasing number of problem areas in recent years, but most of these areas have been industrial or population centers. Additional water usually is available provided it is feasible to draw on more distant sources. In the east more than in the west, one answer to the problem is that of more storage reservoirs.

Supplemental irrigation is becoming increasingly popular in the east. According to one authority,[4] irrigated acreage increased significantly during the 5-yr period ending 1955. States showing increases of more than 1,000% were Alabama, Mississippi, North Carolina, Kentucky, Tennessee, West Virginia, and Delaware. These percentage increases may be somewhat misleading when they are viewed with the awareness that the total irrigated acreage in the east is still less than 10% of the irrigated acreage in the seventeen western states.

---

[4] "1956 Directory and Buyers Guide," *Irrigation Engineering and Maintenance*, H. L. Peace Publications, New Orleans, La., Vol. VI, No. 9, August 31, 1956.

However, the irrigation business in the east is booming. Conceivably, the ratio of irrigated acreage in the eastern United States to that in the west could be changed greatly in the next decade.

The greatest potential for further expansion of irrigation undoubtedly is in the eastern United States, where experience has proved that crop yield can be increased substantially by supplemental irrigation. This fact is valid even though crops can be grown in the east with only the rain that falls during the growing season. In the future a large share of the burden of increasing the nation's food production may fall on eastern agriculture, with the result that irrigated acreage may be increased many times. With respect to irrigation, the east has several advantages over the west: Base flow of streams is generally greater and more reliable; a given quantity of water can serve a much larger acreage through supplemental, rather than almost total, irrigation; and objectionable salts, where deposited in the soil by irrigation water in humid areas, at least are partly leached out by rains, whereas in the west the leaching process must be accomplished largely by the application of irrigation water sometimes greatly in excess of crop requirements.

One of the primary requirements for irrigation from surface-water sources is storage, and again the east has its advantages over the west. Because of the smaller irrigation requirements and the greater stability of the streams, smaller storage projects will suffice. The criterion for design commonly will be not how much water is available, but how much water is needed to accommodate the user during the deficient season. The criterion for design in the west, especially for the larger irrigation projects, is generally how much water is available. Some of the larger reservoirs in the west store a comparatively large part of a normal year's runoff—more water could be used if it were available.

A large expansion of irrigation in the eastern United States will lead to conflicts with other uses, particularly in localities in which irrigation results in the depletion of streams serving industrial and population centers. Irrigation use is consumptive, and a certain degree of depletion will occur, possibly to the extent of materially changing the low-flow regimen of some streams. Industry is well established in the northeastern states, whereas industry and irrigation are expanding rapidly in the southeastern states. Nowhere in the United States are water supplies so generally favorable for further development as in the southeastern states, with the possible exception of the more humid parts of the Pacific northwest. Although irrigation use is consumptive, it may have little effect on water quality, except in areas in which return flow carries salts that have been leached out of the soil to which irrigation was applied. This is not believed to be a matter of great concern in cases in which (*a*) irrigation is supplemental in character and (*b*) streamflows are rather large compared with this type of use. Industrial use, although essentially not consumptive, almost inevitably results in some pollution, especially when a stream is used to dispose of industrial wastes. Hence, conflicts involving both quantity of water and quality of water may be expected in areas in which industry and irrigation compete for the use of the same stream.

Need for Legislation

A real need exists for studies to anticipate these conflicts, delineate the problem areas, and determine the probable nature of the problems that may develop. Studies should include the collection of streamflow records for those cases in which adequate records do not exist, and the analyses of these records should include the effect that the various uses will have on stream regimens. Studies of the surface-water resources of an area are incomplete without similar studies of the ground-water resources because in many areas the two are closely interrelated. For example, extensive use of the ground water of an area may have a pronounced effect on the base flow of the streams in the area. Detailed studies are required to determine the degree and timing of this effect. Furthermore, the effect of diversions and uses on the quality of these waters is of equal importance, in certain cases, to the effect on quantities that are available.

In many states in which water laws do not exist or are not conducive to a logical development of the water resources, much thought is being given to developing laws governing the use of water.[5] The development of water laws that are practicable, workable, and equitable requires an understanding of the engineering and hydrologic aspects of the problems that the laws are designed to meet. This matter is being given serious consideration in Mississippi and other southern states. Even in the humid areas, water no longer can be taken for granted, or assumed to be available for all in an inexhaustible supply.

Civil engineers undoubtedly will have a prominent role in studying the future needs for water in all parts of the country and in developing wise laws to regulate its use for the maximum benefit of all. The legal profession can phrase the laws so that they will stand the test of the courts, but the engineering profession must furnish the facts and their interpretation so that such laws will conform to natural laws.

Conclusions

The nation's total water supplies probably will not be substantially greater or less in the future than in the past. Deficiencies of water, such as have been experienced in recent years, are recurrent and must be taken into consideration in planning for the future. In view of a rapidly growing population and the rapidly increasing use of water, it is important that the hydrologic characteristics of the streams and other sources of water be understood so that legislation will be directed toward an equitable distribution of water in order to meet greater requirements than presently exist.

Acknowledgments

Substantial advice and assistance in preparing the paper was received from engineers of the hydrologic studies unit of the Surface Water Branch of the USGS, especially from Conrad D. Bue, A. M. ASCE.

---

[5] "Water Resources Law," *Report of the President's Water Resources Policy Commission*, U. S. Govt. Printing Office, Washington, D. C., Vol. 3, 1950.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2960

LOAD DISTRIBUTION IN HIGHWAY BRIDGE DECKS

BY ARNOLD W. HENDRY[1] AND LESLIE G. JAEGER[2]

WITH DISCUSSION BY MESSRS. PRANAB KUMAR CHAUDHURI, AND
ARNOLD W. HENDRY AND LESLIE G. JAEGER

SYNOPSIS

A general method is presented for computing the distribution of longitudinal moments, deflections, and similar quantities in bridge decks. This method is based on the assumption that the discrete girders of the transverse system can be replaced by a uniformly spread material having the same total flexural rigidity as the original structure. Distribution coefficients are derived for each harmonic of the total bending moment or deflection curve for the span. The derivation of these coefficients is illustrated for the general case of beams with arbitrary torsional rigidity. In design work, however, it is sufficiently accurate to interpolate between zero torsional rigidity and infinite torsional rigidity by the use of a suitable interpolation function. It is shown that a bridge having a large number of longitudinals can be replaced by an equivalent five-girder structure for the purpose of determining its deflected cross section and, thence, distribution coefficients for its longitudinals. Graphs of the distribution coefficients for a five-girder bridge are included herein.

The application of this method to slabs is considered. In this case it is necessary to derive coefficients for a system in which the ends of the longitudinals remain upright. Again, the general solution for arbitrary torsional rigidity is demonstrated, but, as in the previous case, it is possible in design computations to interpolate between zero torsional rigidity and infinite torsional rigidity. Numerical examples showing the method of computation are included, and comparisons between theoretical and experimental results are made for a steel-and-concrete T-beam, a steel beam and jack arch, a beam and slab, a filler joist bridge deck, and a reinforced concrete slab.

NOTE.—Published, essentially as printed here, in July, 1956, in the Journal of the Structural Division, as Proceedings Paper 1023. Positions and titles given are those in effect when the paper or discussion was approved for publication in *Transactions*.

[1] Prof. of Building Science, Univ. of Liverpool, England.

[2] Lecturer in Eng., Univ. of Cambridge, England.

INTRODUCTION

The practice of transporting heavy, indivisible loads over roads has created the need for an easily applied and accurate method of computing the division of loads between the main girders of highway bridges. Several solutions have been proposed, the most successful being the solution by moment distribution,[3,4] or the solution using the theory of anisotropic plates.[5] The writers have developed a different method of analysis,[6] which is believed to possess significant advantages over previous theories. These advantages are that:

1. The derivation is relatively simple and may be fully understood by any engineer having a knowledge of elementary calculus and Fourier series.
2. The solution is obtained in the form of distribution coefficients, which have an immediate physical significance and which may be plotted against a certain dimensionless parameter for design purposes. Thus, the method can be used in design offices as a matter of routine even though the designer may not have followed the derivation. Furthermore, the computations for even complicated structures can be made rapidly.
3. The method has a wide field of application and has been applied to single-span bridges and continuous bridges, portal frames, skew spans, slabs, and a variety of other structures.[7,8,9,10] It has been applied successfully to actual bridge structures of widely differing form.

The application of the method to the computation of the distribution of longitudinal moments and deflections in bridges having a large number of longitudinals and in reinforced concrete slabs will be examined. Only the single-span case will be considered, but the methods will apply to all the structures analyzed by the writers in other publications.

THE METHOD OF ANALYSIS

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the text or in the illustrations, and are arranged alphabetically for convenience of reference in Appendix II.

General.—Although no attempt will be made herein to examine the method of analysis in detail, it is necessary to describe the assumptions briefly and to indicate how the distribution coefficients were obtained. In fact, the theory requires only one simplifying assumption—the actual transverse system should

---

[3] "A Distribution Procedure for the Analysis of Slabs Continuous over Flexible Beams," by N. M. Newmark, *Bulletin No. 304*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1938.

[4] "Deflections in Gridworks and Slabs," by W. W. Ewell, S. Okubo, and J. I. Abrams, *Transactions*, ASCE, Vol. 117, 1952.

[5] "Methode de calculs des ponts à poutres multiples tenant compte de leur résistance à la torsion," by C. Massonet, *Publications of the International Assn. for Bridge and Structural Eng.*, Zurich, Vol. 10, 1950, pp. 147–182.

[6] "A General Method for the Analysis of Grid Frameworks," by A. W. Hendry and L. G. Jaeger, Pt. III, *Proceedings*, Inst. C. E., London, December, 1955.

[7] "The Load Distribution in Interconnected Bridge Girders with Special Reference to Continuous Beams," by A. W. Hendry and L. G. Jaeger, *Publications of the International Assn. for Bridge and Structural Eng.*, Zurich, Vol. 15, 1955.

[8] "The Analysis of Certain Interconnected Skew Bridge Girders," by A. W. Hendry and L. G. Jaeger, Pt. III, *Proceedings*, Inst. C. E., London, 1957.

[9] "The Analysis of Interconnected Bridge Girders by the Distribution of Harmonics," by L. G. Jaeger and A. W. Hendry, *Structural Engineer*, London, 1956.

[10] "The Analysis of Grid Frameworks of Negligible Torsional Stiffness by Means of Basic Functions," by L. G. Jaeger, Pt. III, *Proceedings*, Inst. C. E., London, 1957.

be replaced by a continuous medium. The foregoing is in accordance with fact in a beam-and-slab bridge. However, even when the transversals are discrete members, it has been found that the assumption is valid for as few as three cross girders. In most bridges, with the exception of slabs, torsional effects in the transverse system are negligible and are neglected in the analysis. However, it is possible to include transverse torsion with little difficulty wherever necessary. Although distribution coefficients may be derived for any degree of torsional rigidity of the longitudinals, it is convenient to consider three limiting cases with respect to torsion of the longitudinals and to obtain intermediate results by a suitable interpolation function. The limiting conditions are (*a*) when the longitudinals are of negligible torsional rigidity; (*b*) when the longitudinals are of infinite torsional rigidity but may rotate as rigid bodies about their longitudinal axes; and (*c*) when the longitudinals are of infinite torsional rigidity but are prevented from rotating, as in case (*b*), by external torques. Case (*a*) applies to bridges in which the torsional rigidity of the longitudinals is small compared with their flexural rigidity (as in structural steel I-sections). Case (*b*) applies to most practical concrete bridges, whereas case (*c*) rarely applies to any kind of bridge but is used in the analysis.

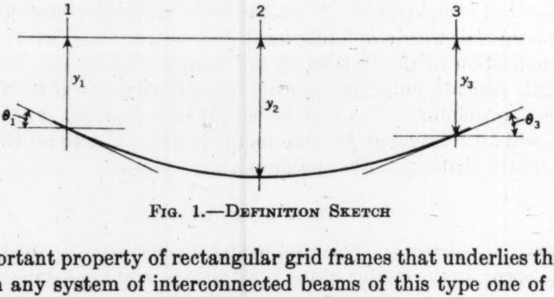


FIG. 1.—DEFINITION SKETCH

An important property of rectangular grid frames that underlies this method is that if in any system of interconnected beams of this type one of the longitudinals is loaded in such a way that the load intensity at any point is proportional to the deflection at that point, all the longitudinals in the system assume the same deflected form. The foregoing has been proved,[10] but it will readily be seen that in the case of simply supported longitudinals on a single span, the load profile that satisfies this condition is a sine curve, that clearly gives a deflection of the same form. On the assumption of a continuous transverse medium, it is relatively easy to derive distribution coefficients for the harmonic components of the applied load, free bending-moment diagram, or free deflection curve. The free bending-moment diagram and free deflection curve are, respectively, the bending-moment diagram and deflection curve that would result if the applied load were carried by the loaded girder itself.

In order to illustrate the method of analysis, one may consider the case of a girder bridge having three longitudinals loaded on the center girder. All girders are assumed to be of equal flexural and torsional rigidity ($E I$ and $G J$, respectively); E is the modulus of elasticity, I denotes the rectangular moment of inertia, G is the modulus of rigidity, and J represents the torsional constant for the section. Therefore, the bending moment, M, and the shear, V, at the edge of the transverse medium, by the usual slope-deflection equations, referring to Fig. 1, are

$$M_{12} = \frac{6\, n\, E\, I_T}{h^2\, L}\left(y_2 - y_1 - \frac{2\, h\, \theta_1}{3}\right) \qquad (1)$$

and

$$V_1 = m\left(y_2 - y_1 - \frac{h\, \theta_1}{2}\right) \qquad (2)$$

in which

$$m = \frac{12\, n\, E\, I_T}{h^3\, L} \qquad (3)$$

In Eq. 3, n is the number of transversals; I_T is the moment of inertia of the transversals; h represents the transversal spacing; L is the span; y denotes the deflection; and θ is the angle of rotation.

Considering a first harmonic deflection of the loaded girder, the deflections of girders 2 and 1 are, respectively,

$$y_2 = a_2 \cos \frac{\pi\, x}{L} \qquad (4a)$$

and

$$y_1 = a_1 \cos \frac{\pi\, x}{L} \qquad (4b)$$

in which a is the amplitude of the first harmonic component of y. The deflection of the bridge is symmetrical about midspan so that the torsional-equilibrium condition for the outer longitudinal is

$$\int_0^{L/2} M_{12}\, dx = 0 \qquad (5)$$

Assuming that the angle of rotation at distance x from midspan is given by

$$h\, \theta = c + \lambda \cos \frac{\pi\, x}{L} \qquad (6)$$

then

$$\int_0^{L/2} M_{12}\, dx = \tfrac{3}{2}\,(a_2 - a_1) - c_1 \frac{\pi}{2} - \lambda_1 = 0 \qquad (7)$$

in which c and λ are parameters defining the twist of the longitudinals—that is,

$$c_1 \frac{\pi}{2} + \lambda_1 = \tfrac{3}{2}\,(a_2 - a_1) \qquad (8)$$

Furthermore, the torque, T, in girder 1 at x from midspan is

$$T = \int_0^x M_{12}\, dx \qquad (9)$$

Thus,

$$- G J \frac{d\theta}{dx} = \int_0^x M_{12}\, dx \qquad (10)$$

That is—

$$- G J \; (\theta)_0{}^{L/2} = \int_0^{L/2} \int_0^x M_{12}\, dx\, dx \qquad (11)$$

Substituting

$$\theta = \frac{c}{h} - \frac{\lambda}{h} \cos \frac{\pi x}{L} \qquad (12a)$$

and

$$M_{12} = \frac{6\, n\, E\, I_T}{h^2\, L} \left[(a_2 - a_1) \cos \frac{\pi x}{L} - \tfrac{2}{3} \left(c_1 + \lambda_1 \cos \frac{\pi x}{L} \right) \right] \qquad (12b)$$

and integrating and inserting limits results in

$$- G J \left[\frac{c}{h} - \left(\frac{c + \lambda}{h} \right) \right] = \frac{6\, n\, E\, I_T}{h^2\, L} \left[(a_2 - a_1) \frac{L^2}{\pi^2} - \tfrac{2}{3} \times \left(c_1 \frac{L^2}{8} + \lambda_1 \frac{L^2}{\pi^2} \right) \right] \qquad (13)$$

which reduces to

$$\frac{\pi^2}{2\, n} \frac{h}{L} \left(\frac{G J}{E\, I_T} \right) \lambda_1 + 2 \left(c_1 \frac{\pi^2}{8} + \lambda_1 \right) = 3 \; (a_2 - a_1) \qquad (14)$$

Putting

$$Q = \frac{\pi^2}{2\, n} \left(\frac{h}{L} \right) \left(\frac{G J}{E\, I_T} \right) \qquad (15)$$

leads to

$$2 \left(c_1 \frac{\pi^2}{8} + \lambda_1 \right) + \lambda_1 Q = 3 \; (a_2 - a_1) \qquad (16)$$

From Eqs. 8 and 16,

$$\lambda_1 + \lambda_1 \frac{Q}{2} - \lambda_1 \frac{\pi}{4} = \tfrac{3}{2} \; (a_2 - a_1) \left(1 - \frac{\pi}{4} \right) \qquad (17)$$

and putting

$$Q' = \frac{Q}{1 - \frac{\pi}{4}} \qquad (18)$$

leads to

$$\lambda_1 = \tfrac{3}{2} \; (a_2 - a_1) \frac{2}{2 + Q'} \qquad (19)$$

Substituting in Eq. 8 results in

$$c_1 = \frac{3}{\pi} \; (a_2 - a_1) \frac{Q'}{2 + Q'} \qquad (20)$$

The loading applied to longitudinal 1 is

$$V_1 = \frac{d^4y}{dx^4} = m\left[y_2 - y_1 - \frac{3}{2\,\pi}\,(a_2 - a_1)\,\frac{Q'}{2 + Q'} \right.$$

$$\left. - \tfrac{3}{4}\,(a_2 - a_1)\,\frac{2}{2 + Q'}\cos\frac{\pi\,x}{L}\right] \quad \text{..(21)}$$

For a first harmonic solution one can replace c_1 by its first harmonic component, $(4\,c_1/\pi)\cos\,(\pi\,x/L)$, in Eq. 21. This leads to

$$a_1 = K\,(a_2 - a_1)\left[1 - \frac{6}{\pi^2}\,\frac{Q'}{2 + Q'} - \frac{6}{4\,(2 + Q')}\right] \quad \text{........(22)}$$

in which

$$K = \frac{12}{\pi^4}\left(\frac{L}{h}\right)^3 \frac{n\,E\,I_T}{E\,I} \quad \text{....................(23)}$$

Writing

$$K_g = K\left[1 - \frac{6}{\pi^2}\,\frac{Q'}{2 + Q'} - \frac{6}{4\,(2 + Q')}\right] \quad \text{............(24)}$$

Eq. 3 becomes

$$a_1 = K_g\,(a_2 - a_1) \quad \text{......................(25)}$$

The free deflection as previously defined is

$$Y = y_1 + y_2 + y_3 \quad \text{......................(26a)}$$

and the amplitude of the first harmonic of Y is

$$A = a_1 + a_2 + a_3 = 2\,a_1 + a_2 \quad \text{...............(26b)}$$

Thus, substituting in Eq. 25 yields

$$a_1 = \frac{K_g}{1 + 3\,K_g}\,A \quad \text{.....................(27a)}$$

and

$$a_2 = \frac{1 + K_g}{1 + 3\,K_g}\,A \quad \text{.....................(27b)}$$

and the distribution coefficients for girder 1 and girder 2 are, respectively,

$$R_1 = R_3 = \frac{K_g}{1 + 3\,K_g} \quad \text{..................(28a)}$$

and

$$R_2 = \frac{1 + K_g}{1 + 3\,K_g} \quad \text{.......................(28b)}$$

The derivation of distribution coefficients for the limiting cases of $Q = 0$ and $Q = \infty$, and also for the case of torsionally stiff longitudinals prevented from rotating, is performed in a similar manner.[7,9] The coefficients obtained

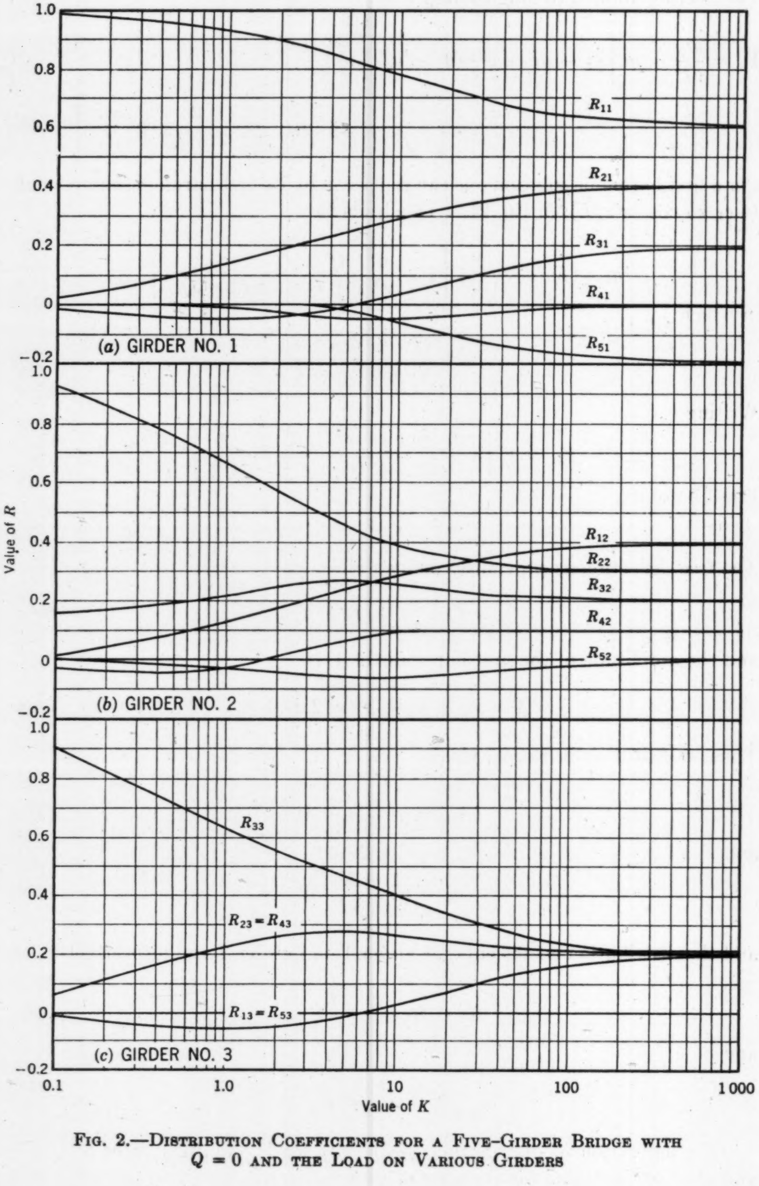


FIG. 2.—DISTRIBUTION COEFFICIENTS FOR A FIVE-GIRDER BRIDGE WITH $Q = 0$ AND THE LOAD ON VARIOUS GIRDERS

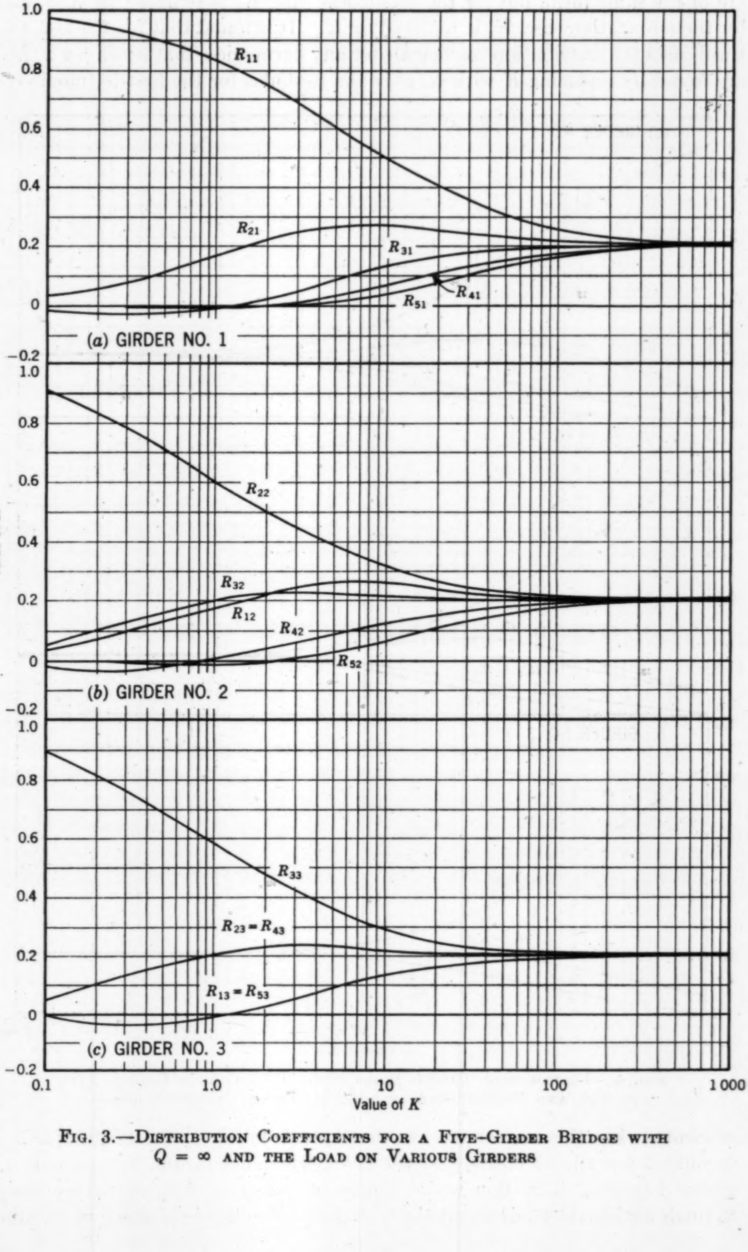


FIG. 3.—DISTRIBUTION COEFFICIENTS FOR A FIVE-GIRDER BRIDGE WITH $Q = \infty$ AND THE LOAD ON VARIOUS GIRDERS

are of the same form, but for the torsionless case, K_g is replaced by $K/4$, and for the no-rotation case, K_g is replaced by K. It is found that for the first and third cases the distribution coefficients for any harmonic—that is, $\sin p\,\pi\,x/L$—are found by replacing K with K/p^4 in the formulas for the first harmonic coefficients. For the case of $Q = \infty$, however, rotation of the longitudinals is accounted for almost entirely by the first harmonic deflection component. A second harmonic deflection would impose a system of torques on the longitudinals antisymmetrical about the midspan, which would produce no rotation

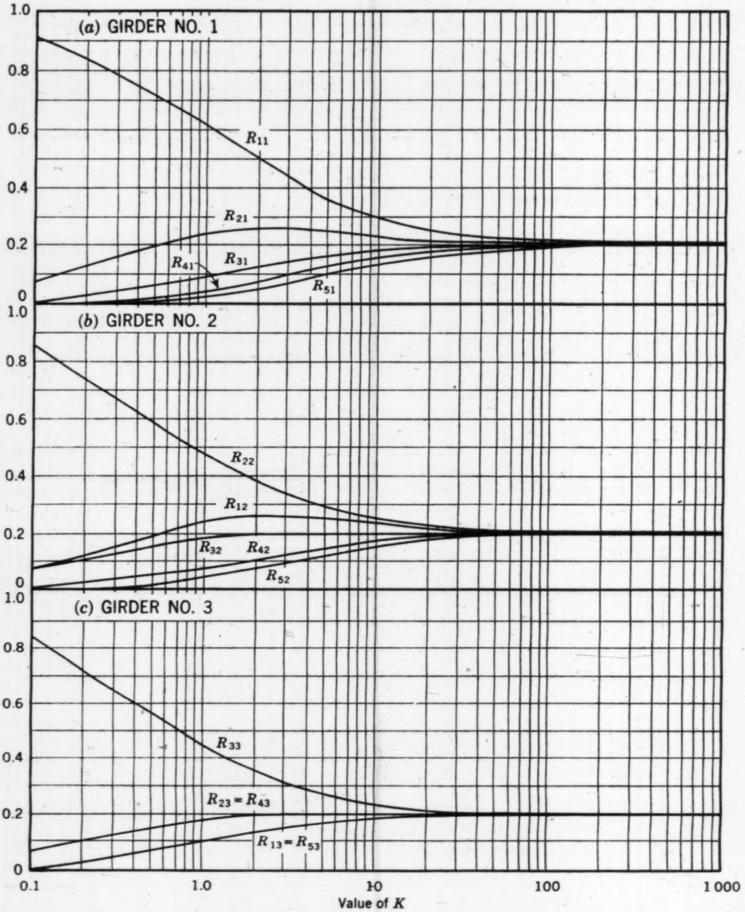


FIG. 4.—DISTRIBUTION COEFFICIENTS FOR A FIVE-GIRDER BRIDGE WITH $Q = \infty$ AND NO ROTATION AND THE LOAD ON VARIOUS GIRDERS

at all, whereas a third harmonic deflection would result in very small, unbalanced torques and correspondingly small rotations. Therefore, the writers have computed the distribution coefficients for the second and higher harmonics in this case, assuming that the rotational effects are negligible. In other words, the coefficients for the third limiting case are used.

With regard to bridges having many longitudinals, the distribution coefficients for a five-girder bridge are used. The formulas for these coefficients for $Q = 0$, $Q = \infty$, and $Q = \infty$ with no rotation are given in Appendix I and are plotted in Figs. 2, 3, and 4. For coefficients in the range between zero torsion and infinite torsion, intermediate values are most conveniently found by interpolation between the appropriate limiting values by use of

$$R_Q = R_0 + (R_\infty - R_0) \sqrt{\frac{Q\sqrt{K}}{3 + Q\sqrt{K}}} \qquad (29)$$

In analyzing the load distribution in a bridge, one must resolve the applied loading, the free bending-moment diagram, or the free deflection curve into harmonic components and distribute these harmonics separately using the appropriate distribution coefficients. In practice the bending moments and deflections are of the greatest interest, and it will rarely be necessary to distribute more than three harmonics of the bending-moment diagram and one of the deflection curve. Frequently, distribution of the first harmonic is sufficiently accurate for design purposes.

Analysis of Bridges with Many Longitudinals

The method described previously could be used to determine distribution coefficients for bridges having more than six longitudinals, but the solution would be tedious and the resulting formulas rather cumbersome. It has been found, however, that it is not necessary to determine coefficients for more than five or six girders because the transverse deflected forms of comparable bridges having more than this number of longitudinals are identical. The foregoing may be demonstrated for both the torsionless case and the torsionally stiff case as follows: If the actual bridge is replaced by an equivalent plate, as in the analysis developed[5] by C. Massonet, the plate will have an effective width of

$$B_e = N\,h \qquad (30)$$

in which N is the number of longitudinals in the bridge. The K-value is then found for a five- or six-girder bridge, which would be replaced by the same hypothetical plate. For the bridge having N longitudinals,

$$K_N = \frac{12}{\pi^4}\left(\frac{L}{h}\right)^3 \frac{n\,E\,I_T}{E\,I} \qquad (31a)$$

$$K_N = \frac{12}{\pi^4}\left(\frac{L}{\frac{B_e}{N}}\right)^3 \frac{(E\,I)_{tt}}{(E\,I)_{tl}}\,N \qquad (31b)$$

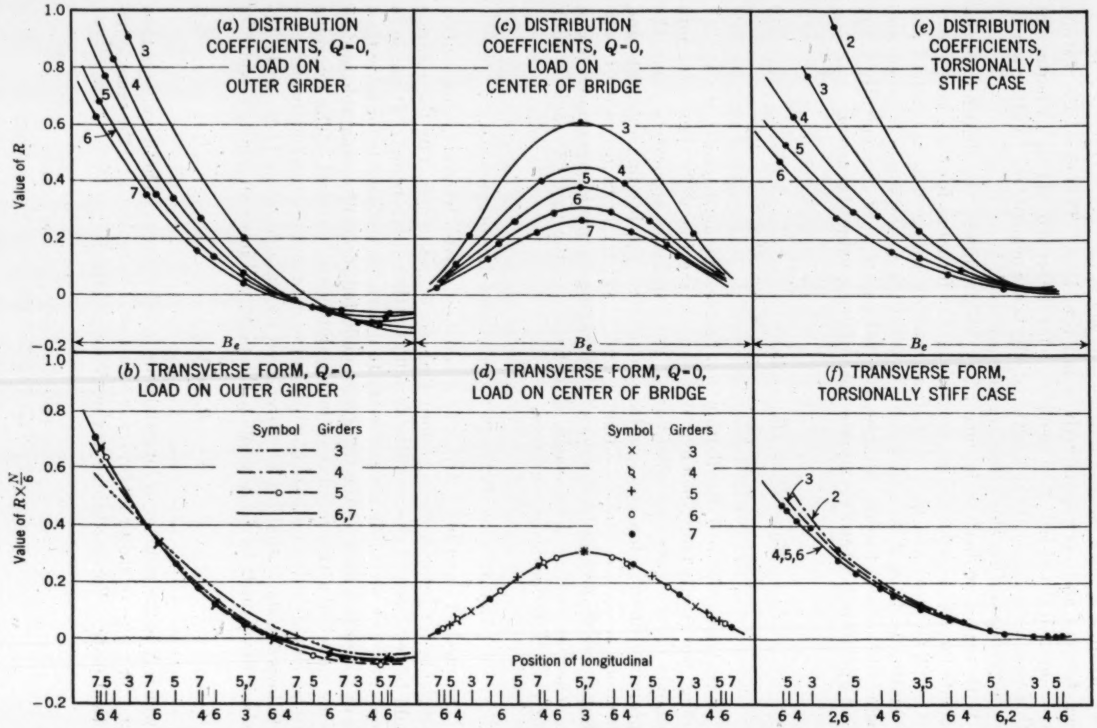


FIG. 5.—DISTRIBUTION COEFFICIENTS AND TRANSVERSE FORK FOR BRIDGES HAVING THE SAME EQUIVALENT PLATE WIDTH

and

$$K_N = \frac{12}{\pi^4}\left(\frac{L}{B_e}\right)^3 N^4 \frac{(E\,I)_{tt}}{(E\,I)_{tl}} \ldots\ldots\ldots\ldots\ldots\ldots (31c)$$

in which $(E\,I)_{tt}$ is the total transverse $E\,I$ and $(E\,I)_{tl}$ is the total longitudinal $E\,I$. Now L/B_e and the ratio of $(E\,I)_{tt}$ to $(E\,I)_{tl}$ are the same in both bridges. Therefore,

$$K_5 = K_N\left(\frac{5}{N}\right)^4 \ldots\ldots\ldots\ldots\ldots\ldots (32a)$$

and

$$K_6 = K_N\left(\frac{6}{N}\right)^4 \ldots\ldots\ldots\ldots\ldots\ldots (32b)$$

Thus, one may replace a bridge of N girders with one having five or six girders by multiplying K for the original bridge by $(5/N)^4$ or $(6/N)^4$, respectively.

Convergence of Transverse Form

Using this device one may compare the transverse forms of bridges having various longitudinals for the cases in which $Q = 0$ and $Q = \infty$. If $K = 2.0$ for a three-girder bridge, then, for other numbers of girders,

Number of girders	Value of K
4	6.32
5	15.44
6	32.0
7	58.8

Considering first the torsionless case and assuming a load on the outer girder of each bridge, the distribution coefficients for the various girders are shown in Fig. 5(*a*). The transverse deflected forms are proportional to these curves, and, in order to determine whether they are geometrically similar, it is necessary to select one—for example, the six-girder curve—as the basis of comparison and then to multiply the ordinates of the others by $N/6$. The coincidence of the curves will indicate that the deflected form is the same, as shown in Fig. 5(*b*), from which it will be observed that the curves for five-, six-, and seven-girder bridges practically coincide. The four-girder curve is slightly off the datum curve, and the three-girder curve is considerably off the datum curve. Fig. 5(*c*) shows a similar set of curves for the load placed at the center of the bridge. In Fig. 5(*d*) all five curves coincide.

Therefore, it appears that the transverse forms converge as the number of girders is increased, and it may be assumed that the transverse forms for five or more longitudinals coincide.

For the torsionally stiff case the parameter, Q, is never actually infinite although the torsional stiffness of the longitudinals may be large enough to make the assumption of $Q = \infty$ sufficiently accurate for practical purposes. Because

$$Q = \frac{\pi^2}{2\,n}\frac{h}{L}\left(\frac{G\,J}{E\,I_T}\right) = \frac{\pi^2}{2}\frac{B_e}{L}\frac{1}{N^2}\frac{N\,G\,J}{n\,E\,I_T} \ldots\ldots\ldots (33)$$

it is clear that in changing from N longitudinals to six, for example, one must adjust the value of Q by multiplying it by $(N/6)^2$. It has been found from theoretical investigation and has been confirmed experimentally that if the value of Q for a three- or four-girder bridge is greater than approximately 1.25, it is sufficiently accurate to compute the distribution coefficients as if Q were infinite. Thus, if one starts with a three-girder bridge for which $K = 2.0$ and

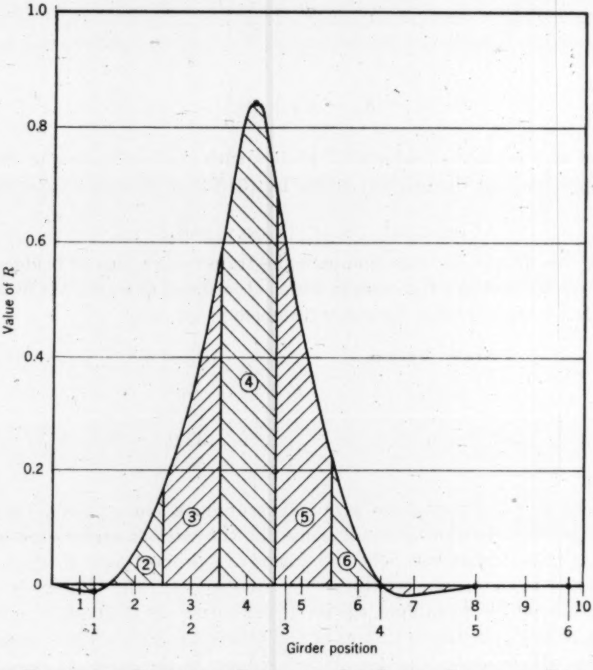


FIG. 6.—TRANSVERSE DEFLECTED FORM OF A SIX-GIRDER BRIDGE TO DETERMINE DISTRIBUTION COEFFICIENTS FOR A TEN-GIRDER BRIDGE

$Q = 2.0$, the corresponding values of K and Q for equivalent four-, five-, and six-girder bridges are:

Number of girders	Value of K	Value of Q
4	6.32	1.13
5	32.0	0.72
6	58.0	0.50

In computing the distribution coefficients for the four-, five-, and six-girder bridges, one cannot, therefore, assume that $Q = \infty$, and one must use coefficients for the appropriate value of Q. This leads to the curves of Figs. 5(*e*) and 5(*f*), which show that the transverse forms of equivalent bridges having more than four girders may be considered to be identical.

DISTRIBUTION COEFFICIENTS FOR BRIDGES HAVING MORE THAN SIX LONGITUDINALS

It follows from the foregoing that it is possible to plot the transverse deflected form of a bridge having any number of longitudinals from the coefficients for a five- or six-girder bridge using the adjusted values of K and Q. If this curve is plotted as shown in Fig. 6 and if the positions of the centers of the longitudinals in the N-girder bridge are marked along the base line, the hatched areas as fractions of the total area of the diagram represent the distribution ocefficients for the longitudinals of the N-girder bridge. Because the strips into which the diagram is divided are of equal width, it is convenient to consider the mean ordinate of the curve in each strip as representing the area. The distribution coefficients for the longitudinals are thus quickly obtained.

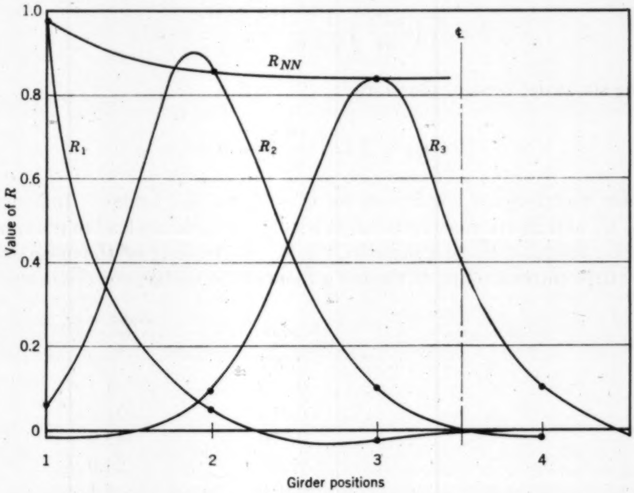


FIG. 7.—INFLUENCE LINES FOR DISTRIBUTION COEFFICIENTS

In drawing the transverse profile it will frequently be found that the loads are applied at points between the longitudinals of the equivalent five-girder bridge. In this case it is necessary to plot influence lines showing the variation of the distribution coefficients with the transverse position of the load (Fig. 7). From these curves it is possible to obtain the distribution coefficients for any load position. Again, the peak value of the transverse deflection curve will often be located between the longitudinals of the five- or six-girder bridge, and, in order to plot the curve, it is helpful to draw a curve of R_{NN}—that is, the distribution coefficient at station N for the load at N. Points on this curve are known at each of the longitudinals of the five- or six-girder bridge, and the curve may be sketched as shown in Fig. 7.

For example, one may consider the determination of distribution coefficients for a load on girder 4 of the reinforced concrete T-beam bridge shown in Fig. 8, that has ten main girders with a 64-ft span. If the moments of inertia of each main girder and of a 1-ft width of the slab are 83,134 in.^{4} and 241.7 in.^{4}, respectively, one obtains

$$K = \frac{12}{\pi^4}\left(\frac{64}{9.08}\right)^3 \frac{64 \times 241.7}{83{,}134 \times 6} = 1.33$$

assuming a modular ratio (steel to concrete) of 6. For the equivalent six-girder bridge,

$$K_6 = 1.33\left(\frac{6}{10}\right)^4 = 0.173$$

If the torsional rigidity of each main beam is 115,500 G in.-units,

$$Q = \frac{\pi^2}{2}\left(\frac{9.08}{64}\right)\frac{1}{2.5}\,\frac{115{,}500}{64 \times 241.7} = 2.12$$

For the six-girder bridge, therefore,

$$Q_6 = 2.12\left(\frac{10}{6}\right)^2 = 5.9$$

One may therefore use coefficients for $Q = \infty$ for this bridge. Influence lines for R_1, R_2, and R_3 are now plotted as in Fig. 7. The values lead to the transverse profile for the given loading shown in Fig. 6. By reading off the mean ordinates of the strips corresponding to the ten girders of the bridge, one obtains

Girder number	Ordinate	Value of R, in percentage
1	−0.001	0.0
2	0.070	4.1
3	0.370	21.7
4	0.770	45.0
5	0.425	24.9
6	0.070	4.1
7	0.002	0.0
8	0.000	0.0
9	0.000	0.0
10	0.000	0.0
Total	1.708	

It will be observed that, by the reciprocal theorem, the influence line for any girder is, in fact, the curve of distribution coefficients for the load at that girder. For example, considering the influence line for R_2, it will be seen that $R_{21} = R_{12}$, $R_{23} = R_{32} \cdots$.

APPLICATION OF THE THEORY TO SLABS

How the theory can be applied to grids with large numbers of longitudinals has been shown. Therefore, one can now examine the possibility of applying the theory to slabs. If a slab is considered to be divided into a large number of longitudinal elements, the behavior of the slab is such that the ends of these elements do not rotate. If each element were of infinite torsional stiffness, the distribution coefficients based on the infinite torsional rigidity would be applicable, assuming that there was no rotation. If the elements were of negligible torsional stiffness, the distribution would be given by the coefficients for $Q = 0$. Actual slabs will lie somewhere between these extremes, and the problem of applying this theory is that distribution coefficients must be derived for any value of Q on the assumption that the ends of the beam elements do not rotate. The coefficients are derived for longitudinal torsional stiffness only, but transverse torsional stiffness is provided for by taking $G\,J$ in the parameter, Q, as the sum of the torsional stiffnesses in the longitudinal and transverse directions. The reason for this procedure can be understood by considering the well-known differential equation for an anisotropic slab. In this equation[11] the torsional rigidities per unit length do not occur separately, and the twist component of the equation contains only the sum of these torsional rigidities. Thus, it would be possible to obtain the same deflections for an infinite number of slabs provided that the sum of the torsional rigidities is constant. In particular, a slab in which the torsional rigidity occurs in one direction (with zero torsional rigidity at right angles) will give the same deflections as all the other members of the set. This is the result of including in the parameter, Q, the sum of the longitudinal and transverse torsional rigidities.

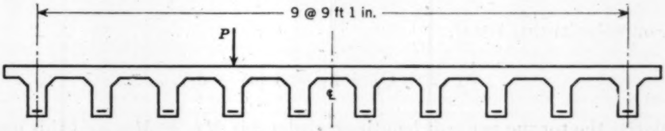


FIG. 8.—CROSS SECTION OF T-BEAM BRIDGE

The derivation of distribution coefficients for any value of Q for a five-girder bridge on the assumption that the ends of the beams do not rotate will be shown for the loading on girder 3. With the same notation used previously,

$$M_{12} = \frac{6\,n\,E\,I_T}{L\,h^2}\left(y_2 - y_1 - \tfrac{1}{3}\,h\,\theta_2 - \tfrac{2}{3}\,h\,\theta_1\right) \qquad (34)$$

In this case,

$$h\,\theta_1 = \lambda_1 \sin\frac{\pi\,x}{L} \qquad (35a)$$

and

$$h\,\theta_2 = \lambda_2 \sin\frac{\pi\,x}{L} \qquad (35b)$$

so that consideration of girder 1 leads to

$$M_{12} = \frac{6\,n\,E\,I_T}{L\,h^2}\sin\frac{\pi\,x}{L}\left(a_2 - a_1 - \frac{\lambda_2}{3} - \frac{2\,\lambda_1}{3}\right) \qquad (36)$$

---

[11] "Theory of Plates and Shells," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1940, p. 191.

Taking a unit length of girder 1 and letting the torque on this element be T,

$$T = -\,G\,J\,\frac{d\theta_1}{dx} \ldots\ldots\ldots\ldots (37)$$

and

$$\frac{dT}{dx} = -\,G\,J\,\frac{d^2\theta}{dx^2} \ldots\ldots\ldots\ldots (38)$$

Because dT/dx is clearly M_{12}, by Eqs. 36 and 38,

$$G\,J\,\frac{\pi^2}{L^2}\,\frac{\lambda_1}{h}\sin\frac{\pi\,x}{L} = \frac{6\,n\,E\,I_T}{L\,h^2}\sin\frac{\pi\,x}{L}\left(a_2 - a_1 - \frac{\lambda_2}{3} - \frac{2\,\lambda_1}{3}\right) \ldots (39)$$

and, on substituting for Q,

$$\lambda_1\,(2 + Q) + \lambda_2 = 3\,(a_2 - a_1) \ldots\ldots\ldots\ldots (40)$$

Similarly, the torque per unit length on girder 2 is $M_{21} + M_{23}$, and this leads to

$$\lambda_1 + \lambda_2\,(4 + Q) + \lambda_3 = 3\,(a_3 - a_1) \ldots\ldots\ldots\ldots (41)$$

By symmetry, $\lambda_3 = 0$, so that, on solution of Eqs. 40 and 41,

$$\lambda_1 = \frac{3\,[(a_2 - a_1)\,(4 + Q) - (a_3 - a_1)]}{7 + 6\,Q + Q^2} \ldots\ldots\ldots\ldots (42)$$

and

$$\lambda_2 = \frac{3\,[(a_3 - a_1)\,(2 + Q) - (a_2 - a_1)]}{7 + 6\,Q + Q^2} \ldots\ldots\ldots\ldots (43)$$

By denoting the loading transmitted to the longitudinals through the transverse medium by V_1, V_2, there results

$$V_1 = m\left(y_2 - y_1 - \frac{h\,\theta_1}{2} - \frac{h\,\theta_2}{2}\right) \ldots\ldots\ldots\ldots (44)$$

which becomes

$$a_1 = K\left\{a_2 - a_1 - \tfrac{3}{2}\left[\frac{(a_2 - a_1)\,(3 + Q) + (a_3 - a_1)\,(1 + Q)}{7 + 6\,Q + Q^2}\right]\right\} \ldots (45)$$

Similarly,

$$a_2 = K\left\{a_1 - 2\,a_2 + a_3 + \tfrac{3}{2}\left[\frac{(a_2 - a_1)\,(4 + Q) - (a_3 - a_1)}{7 + 6\,Q + Q^2}\right]\right\} \ldots (46)$$

Also,

$$2\,a_1 + 2\,a_2 + a_3 = A \ldots\ldots\ldots\ldots (47)$$

and, on solution of Eqs. 45 and 46, there results

$$R_1 = \frac{a_1}{A} = \frac{K^2\,(1+6\,Q+4\,Q^2)-6\,K\,(1+Q)}{K^2\,(5+30\,Q+20\,Q^2)+K\,(68+90\,Q+20\,Q^2)+4\,(7+6\,Q+Q^2)} \quad \text{(48a)}$$

$$R_2 = \frac{a_2}{A} = \frac{K^2\,(1+6\,Q+4\,Q^2)+(22+24\,Q+4\,Q^2)}{K^2\,(5+30\,Q+20\,Q^2)+K\,(68+90\,Q+20\,Q^2)+4\,(7+6\,Q+Q^2)} \quad \text{(48b)}$$

and

$$R_3 = \frac{a_3}{A} = \frac{K^2\,(1+6\,Q+4\,Q^2)+K\,(36+54\,Q+12\,Q^2)+4\,(7+6\,Q+Q^2)}{K^2\,(5+30\,Q+20\,Q^2)+K\,(68+90\,Q+20\,Q^2)+4\,(7+6\,Q+Q^2)} \quad \text{(48c)}$$

For a load on girder 1 the computation is most conveniently done in two parts. Distribution coefficients are derived for a symmetrical system consisting of equal downward loads on girder 1 and girder 5, and these coefficients are combined with the coefficients for a skew symmetrical system consisting of equal loads, one acting downward on girder 1 and the other upward on girder 5. On combination, the loads on girder 5 cancel each other, and the resulting coefficients are for the load on girder 1. Distribution coefficients for the load on girder 2 are obtained in the same manner. Using these coefficients, it was proved that the interpolation function cited previously also applies accurately to this case. For distribution coefficients for higher harmonics, K/p^4 must be substituted for K in the cases of $Q = 0$ and $Q = \infty$ with no rotation before interpolation.

THE PARAMETERS, K AND Q, FOR SLABS

It is convenient to compute K for a slab as follows: If the slab is divided into N longitudinal strips, the distance between the centers of these strips is B_e/N. Letting the ratio of $E\,I$ per foot longitudinally to transversely be r, then, for each element,

$$E\,I = r\,E\,I_T\,\frac{B_e}{N} \quad \text{(49)}$$

and

$$K = \frac{12}{\pi^4}\left(\frac{L}{h}\right)^3 \frac{L\,E\,I_T}{E\,I} = \frac{12}{\pi^4}\left(\frac{L}{B_e}\right)^4 \frac{N^4}{r} \quad \text{(50)}$$

Similarly,

$$Q = \frac{\pi^2}{2}\left(\frac{B_e}{L}\right)^2 \frac{1}{N^2}\,\frac{G}{E}\,\frac{J_T + J_L}{I_T} \quad \text{(51)}$$

in which $G\,J_T$ and $G\,J_L$ are the torsional rigidities per unit length in the transverse and longitudinal directions, respectively. If the slab is uncracked it is sufficiently accurate to assume isotropic behavior, in which case,

$$J_T = J_L = J \quad \text{(52a)}$$

and

$$G\,J = k\,d^3 \quad \text{(52b)}$$

in which d is the thickness of the slab. In an isotropic slab,

$$G\,J = E\,I = E\,\frac{d^3}{12} \quad \text{(53a)}$$

and, therefore,

$$k = \frac{E}{12\,G} \qquad \text{(53b)}$$

which gives

$$Q = \frac{\pi^2}{2}\left(\frac{B_e}{L}\right)^2 \frac{1}{N^2}\,\frac{G}{E}\,\frac{2\,\frac{1}{12}\,\frac{E}{G}\,d^3}{\frac{1}{12}\,d^3} = \frac{\pi^2}{N^2}\left(\frac{B_e}{L}\right)^2 \qquad \text{(54)}$$

The value of $G\,J$ is based on the assumption that the shear stress will vary through the depth of the slab in the manner indicated in Fig. 9. This is believed to be more correct than computing the torsional rigidity of a small rectangular element. The latter procedure would lead to a result that would depend on the width of the element selected.

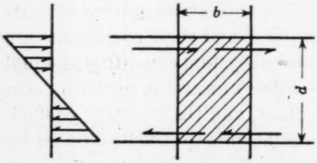


FIG. 9.—VARIATION OF SHEAR WITH DEPTH

An additional point that should be noted is that, in order to allow for the effect of lateral restraint on the deformation of the material in a slab, a modified value of E should be used to compute the deflections for a slab. This value[11] is

$$E' = \frac{E}{1 - \mu^2} \qquad \text{(55)}$$

in which μ is Poisson's ratio. The distribution of deflections is not affected, but their magnitude is reduced as a result.

The method of analysis has also been found to apply accurately to cellular plated structures consisting of an "egg box" grid of diaphragms covered by skin plating. In this case the values of K and Q are the same as those for an isotropic slab.

A NUMERICAL EXAMPLE

As an example of the method of analysis for slabs, if it were required to determine the transverse distribution of deflections and bending moments in a reinforced concrete slab of span-to-width ratio of 1 to 3, the reinforcement per foot is the same in both directions and the slab carries a single concentrated load at its center point. In this case for an equivalent five-beam grid,

$$K_5 = \frac{12}{\pi^4}\left(\frac{1}{3}\right)^4 \frac{N^4}{1}\left(\frac{5}{N}\right)^4 = 0.95$$

and

$$Q_5 = \frac{\pi^2}{N^2}\,(3)^2\left(\frac{N}{5}\right)^2 = 3.6$$

The distribution coefficients for this value of Q for a load on the center girder were interpolated between values from Figs. 2(*c*) and 4(*c*) using the interpolation function mentioned previously; the coefficients are plotted in Fig. 10. If $N = 20$ (that is, if the width of the bridge is divided into 1-ft strips), from Fig. 10 one can obtain the first harmonic distribution coefficients shown in Table 1 (*a*). Third harmonic coefficients have also been computed for the five-girder bridge and are plotted in Fig. 10. The resulting coefficients for $N = 20$ are given in Table 1(*a*).

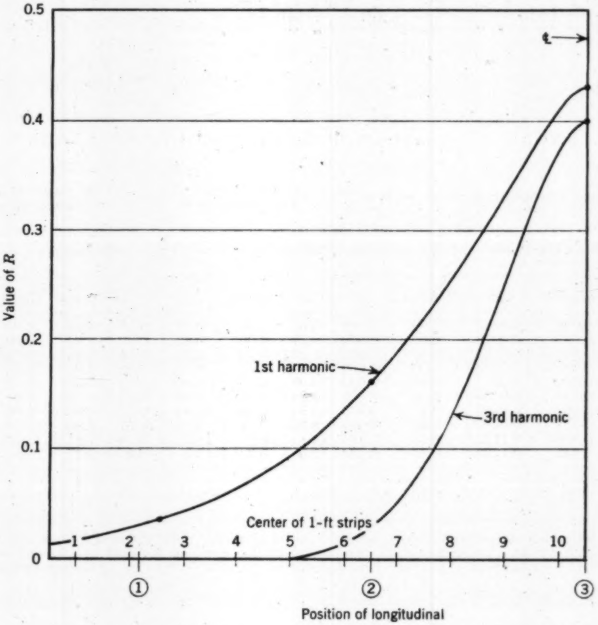


FIG. 10.—TRANSVERSE PROFILE FOR REINFORCED CONCRETE SLAB

The free deflection series for a point load on a simply supported beam that is distance *b* from the left end is

$$Y = \frac{2\,W\,L^3}{\pi^4\,E\,I}\left(\sin\frac{\pi\,b}{L}\sin\frac{\pi\,x}{L} + \frac{1}{2^4}\sin\frac{2\,\pi\,b}{L}\sin\frac{2\,\pi\,x}{L} + \frac{1}{3^4}\sin\frac{3\,\pi\,b}{L}\sin\frac{3\,\pi\,x}{L} + \cdots\right)\qquad(56)$$

In the foregoing example, $b = L/2$. Therefore, taking the first two terms,

$$Y = \frac{2\,W\,L^3}{\pi^4\,E\,I}\left(\sin\frac{\pi\,x}{L} - \frac{1}{81}\sin\frac{3\,\pi\,x}{L}\right)\qquad(57)$$

Thus, the first harmonic accounts for 98.75% of the total deflection, and the third harmonic, for the remaining 1.25%. Therefore, the harmonics greater than the third harmonic are negligible. The percentage of the total deflection of each element across the width of the slab is obtained by applying the first and third harmonic distribution coefficients to 98.75% and 1.25%, respectively, and by adding the results as in Table 1 (*b*).

The series for the free bending-moment diagram is

$$M = \frac{2\,W\,L}{\pi^2}\left(\sin\frac{\pi\,b}{L}\sin\frac{\pi\,x}{L}\,\frac{1}{2^2}\sin\frac{2\,\pi\,b}{L}\sin\frac{2\,\pi\,x}{L}\right.$$
$$\left.+\frac{1}{3^2}\sin\frac{3\,\pi\,b}{L}\sin\frac{3\,\pi\,x}{L}+\cdots\right)\ldots(58)$$

TABLE 1.—ANALYSIS OF REINFORCED CONCRETE SLAB

Point	(*a*) DISTRIBUTION COEFFICIENTS				(*b*) DEFLECTION DISTRIBUTION			(*c*) BENDING-MOMENT DISTRIBUTION			
	First Harmonic		Third Harmonic		Harmonics		TOTAL	Harmonics			TOTAL
	Ordinate	*R*	Ordinate	*R*	First[a]	Third[b]		First[c]	Third[d]	Higher[e]	
1	0.020	0.0062	0.00	0.000	0.61	0.00	0.61	0.50	0.00	0.00	0.50
2	0.025	0.0078	0.00	0.000	0.77	0.00	0.77	0.63	0.00	0.00	0.63
3	0.040	0.0125	0.00	0.000	1.23	0.00	1.23	1.01	0.00	0.00	1.01
4	0.065	0.0202	0.00	0.000	1.99	0.00	1.99	1.64	0.00	0.00	1.64
5	0.100	0.0311	0.00	0.000	3.07	0.00	3.07	2.52	0.00	0.00	2.52
6	0.140	0.0437	0.03	0.009	4.31	0.01	4.32	3.54	0.08	0.00	3.62
7	0.180	0.0561	0.11	0.033	5.54	0.04	5.58	4.54	0.30	0.00	4.84
8	0.260	0.0811	0.28	0.089	8.00	0.11	8.11	6.57	0.80	0.00	7.37
9	0.360	0.1121	0.43	0.134	11.1	0.2	11.3	9.08	1.21	1.50	11.78
10	0.415	0.1292	0.75	0.234	12.8	0.3	13.1	10.48	2.07	4.00	16.55
Sum	1.605 × 2 = 3.21	. . .	1.60 × 2 = 3.20	. . .							

[a] First harmonic component of free deflection = 98.75 units. [b] Third harmonic component of free deflection = 1.25 units. [c] First harmonic component of free bending moment at midspan = 81 units. [d] Third harmonic component of free bending moment at midspan = 9 units. [e] Higher harmonics = 10 units.

Putting $b = L/2$ results in

$$M = \frac{2\,W\,L}{\pi^2}\left(\sin\frac{\pi\,x}{L} - \frac{1}{9}\sin\frac{3\,\pi\,x}{L} + \cdots\right)\ldots\ldots\ldots\ldots(59)$$

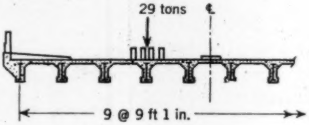
In the deflection series, harmonics greater than the third harmonic were negligible, but this is no longer the case in the bending-moment series. The first harmonic accounts for 81% of the free bending moment; the third, for 9%; and the higher harmonics, for the remaining 10%. The higher harmonics, however, are almost entirely retained in the elements in the immediate vicinity of the load, and the distribution of bending moments is shown in Table 1 (*c*) in terms of the percentage of the total moment on the span.

In design the results for several point loads may be obtained by superposition. For complicated loading it may be advantageous to resolve the free bending-moment diagram into harmonic components by one of the well-known numerical methods.

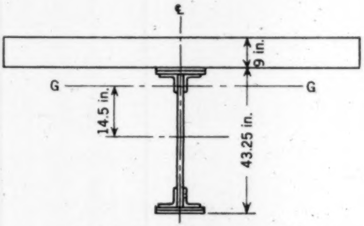
TESTS ON LARGE MODELS AND FULL-SIZED BRIDGES

It is interesting to compare the results given by the theory with those obtained from tests on large models or on full-sized structures. For this purpose the writers have used the results of tests conducted at the Building Research Station, Watford, England, and at the University of Illinois, at Urbana. Two of the field tests reported[12] by F. G. Thomas in 1955 will be examined.

Uxbridge Canal Bridge, England.—This bridge was a concrete **T**-beam bridge with steel plate girders (Fig. 11) having ten main girders and loaded by a 45-ton low-loading vehicle.

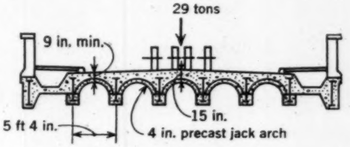


(*a*) CROSS SECTION AND VEHICLE POSITION

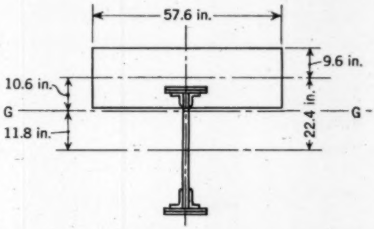


(*b*) COMPOSITE LONGITUDINAL SECTION

FIG. 11.—UXBRIDGE CANAL BRIDGE



(*a*) CROSS SECTION AND VEHICLE POSITION



(*b*) COMPOSITE LONGITUDINAL SECTION

FIG. 12.—LAUREL BRIDGE, CHADDERTON

To begin the analysis, one must estimate the parameters, K and Q, for the structure. Considering the **T**-beams as separate units, $Q = 3.1$, based on a torsional constant of 115,500 in.^4. In estimating K it is necessary to know the moments of inertia of the main beams and of the deck slab. Two difficulties are encountered at this stage: First, the uncertainty of the composite action of the steel and concrete elements, and, second, the question of the elastic modulus of concrete. The moment of inertia of the deck slab, assuming a critical percentage of steel and an effective depth of 8 in., was found by the conventional equivalent-concrete-area method to be 407.2 in.^4 (concrete units), taking a modular ratio of 15, and 241.7 in.^4 with a modular ratio of 6. Assuming that the 9-in. deck slab acts with the steel beam as a composite girder (Fig. 11(*b*)), the moment of inertia of the longitudinals is 68,664 in.^{4} and 83,134 in.^{4} (steel units) with a modular ratio of 15 and 6, respectively. These values lead to

---

[12] "Loading Tests on Bridges," by F. G. Thomas, Conference on the Correlation Between Calculated and Observed Stresses and Displacements in Structures, Inst. C. E., London, 1955.

$$K = \frac{12}{\pi^4}\left(\frac{64}{9.08}\right)^3 \frac{64 \times 407.2}{68{,}664 \times 15} = 1.09 \quad \text{(for a modular ratio of 15)}$$

and

$$K = \frac{12}{\pi^4}\left(\frac{64}{0.98}\right)^3 \frac{64 \times 241.7}{83{,}134 \times 6} = 1.33 \quad \text{(for a modular ratio of 6)}$$

Thus, the K-values corresponding to these two values of the modular ratio are quite close, and the difference in the distribution coefficients will be small.

TABLE 2.—Comparison of Distribution Coefficients

(a) Uxbridge Canal Bridge

Girder	1	2	3	4	5	6	7	8	9	10
Experimental	5	7	18	44	19	6	1	0	0	0
Theoretical	0	4	22	45	25	4	0	0	0	0

(b) Laurel Bridge, Chadderton

Girder	1	2	3	4	5	6	7	8
Experimental	5	4	18	26	26	16	4	1
Theoretical	1	5	16	27	27	16	5	1

Even a large error in the assumed value of the modular ratio will make little difference in the distribution of the loads between the girders. On the other hand, the computation of stresses is radically affected by the assumed value of the modular ratio, but this is a question that arises in the case of all concrete structures.

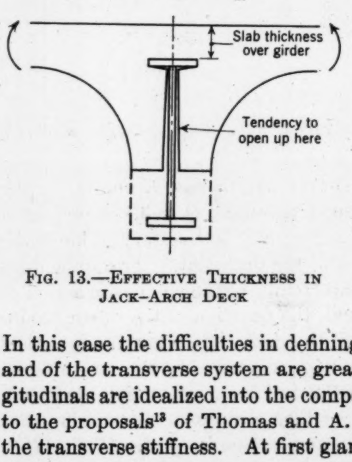


Fig. 13.—Effective Thickness in Jack–Arch Deck

Taking $K = 1.33$ for the bridge under consideration and computing the distribution coefficient for each longitudinal, the comparison in Table 2(*a*) between experimental and theoretical values is obtained.

Laurel Bridge, Chadderton.—This bridge, at Chadderton, England, is a steel-beam and jack-arch structure of the cross section shown in Fig. 12(*a*). In this case the difficulties in defining the effective sections of the longitudinals and of the transverse system are greatly increased. The dimensions of the longitudinals are idealized into the composite section shown in Fig. 12(*b*), according to the proposals[13] of Thomas and A. Short. It is still more difficult to assess the transverse stiffness. At first glance, it appears that the bridge would have a high transverse stiffness, but there is no effective continuity of the concrete through the longitudinal steel girder (Fig. 13), and a conservative view should be taken as to the effective depth of the transverse slab. A reasonable estimate of this depth would, therefore, be 9 in., giving a transverse moment of inertia of 729 in.^4 per ft. Then

---

[13] "A Laboratory Investigation of Some Bridge Deck Systems," by F. G. Thomas and A. Short, Pt. I, *Proceedings*, Inst. C. E., London, Vol. 1, No. 2, 1952.

$$K = \frac{12}{\pi^4}\left(\frac{50 \times 25}{5 \times 33}\right)^3 \frac{50 \times 25 \times 729}{37 \times 240 \times 15} = 6.9$$

Using this value of K and $Q = \infty$, the distribution coefficients for the various girders are as given in Table 2(*b*). A comparison between the experimental and theoretical results shows excellent agreement, but, as in the previous case, this agreement is partly due to chance. Not only is it possible to vary the value of K by assuming different dimensions for the sections of the beams and slab, but the experimental results are far from being precise. Nevertheless, these results are encouraging, especially when it is remembered that even large variations in K produce only a small percentage change in the values of the distribution coefficients.

TABLE 3.—FIRST HARMONIC DISTRIBUTION COEFFICIENTS FOR MODEL BEAM AND SLAB BRIDGE

Girder	LOAD ON GIRDER 1		LOAD ON GIRDER 2		LOAD ON GIRDER 3	
	Theoretical	Experimental	Theoretical	Experimental	Theoretical	Experimental
1	86	79	22	21	−4	2
2	22	21	52	56	26	22
3	−2	2	26	20	51	54
4	−4	−1	4	5	26	22
5	−2	−1	−2	−1	4	6
6	0	0	−2	0	−4	2

Tests on a Model Beam and Slab Bridge.—The results of tests on model beam and slab bridges with a 9-ft span reported[13] by Thomas and Short are considered. The model consisted of six 8-in.-by-4-in., rolled-steel **I**-sections placed at 3-ft centers on which a 3-in.-thick slab was laid, that was reinforced at the top and bottom with ¼-in. bars at 1 ⅝-in. centers with a ½-in. cover. Assuming an uncracked section, the moment of inertia of the slab per foot was 27 in.^4 (concrete units), yielding

$$K = \frac{12}{\pi^4}\left(\frac{9}{3}\right)^3 \frac{27 \times 9}{3.95 \times 6.5} = 3.14$$

The modular ratio of 6.5 was taken from data compiled by Thomas and Short. For this value of K, the first harmonic distribution coefficients for the various girders with the load placed over girders 1, 2, and 3, in turn, are as given in Table 3.

Therefore, the proportions of the total moment taken by each beam can be estimated with reasonable accuracy. The distribution theory can be used only up to this stage. Inspection of the strain measurements made by Thomas and

Short indicates that the beams transmitted a much smaller proportion of the total moment than can be easily accounted for by simply considering the flexural rigidity of the slab in the direction of the span. There is a further reduction of the moments, possibly resulting from the dispersion of applied point load both transversely and longitudinally.

Filler Joist Slab Bridge.—The second model bridge tested by Thomas and Short was a filler joist slab consisting of sixteen 6-in.-by-3-in., 12-lb-per-ft, rolled-steel I-sections at 12-in. centers embedded in a concrete slab that was 9 in. thick. The span was 9 ft, and the concrete cover on the lower flanges of the steel beams was 1 in. Light reinforcing fabric was laid above and below the steel beams. The model was loaded by a single point load at midspan, first at the center of the bridge, then near the edge.

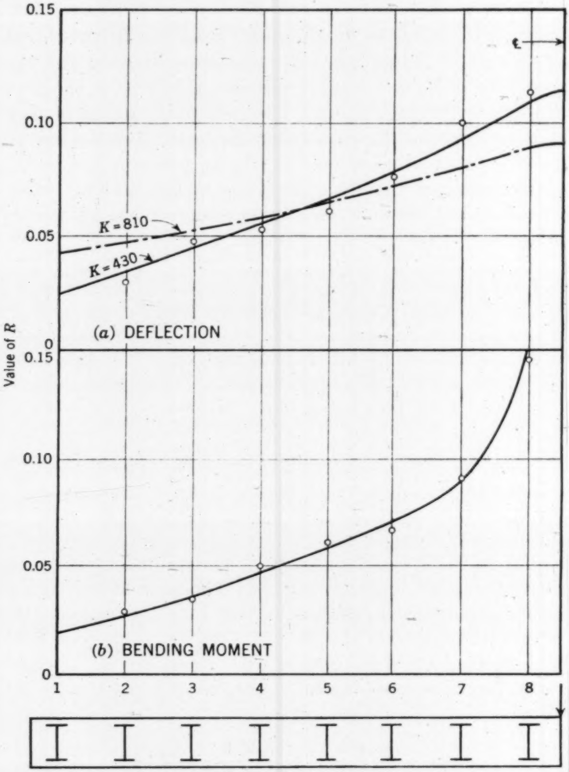


Fig. 14.—Distribution Coefficients for Filler Joist Bridge Model

The difficulty of estimating the moments of inertia longitudinally and transversely is apparent. As a first approximation, it is possible to assume that the flexural rigidity per foot is equal in both directions; this assumption gives $K = 810$, but it results in too much distribution. This fact can be seen in Fig. 14(*a*), that shows experimental and theoretical coefficients for beam deflections. Taking a cracked section transversely yields $I_T = 282$ in.^{4} per ft of width, and I for the composite-beam longitudinals as 530 in.^{4} results in $K = 430$. The value of 530 in.^{4} for I was derived from the measured free deflection and the experimental value of E cited by Thomas and Short; no attempt was made to derive a purely theoretical value for I. The deflection distribution coefficients are obtained and are shown by the solid line in Fig. 14(*a*). This line shows excellent agreement with the experimental results of Thomas and Short. The experimental points shown in Fig. 14(*a*) were taken from a small diagram in the paper by Thomas and Short. This diagram showed zero deflections on the two outermost girders, but, because this conflicts with the strain readings recorded on these beams and also because it appears to be erroneous, the two readings were discarded. It is interesting to note that a ratio of almost 2 to 1 in the value of K produces a relatively small change in the distribution coefficients.

TABLE 4.—DISTRIBUTION OF BENDING MOMENTS IN FILLER JOIST SLAB BRIDGE

Girder	First harmonic	Third harmonic	Higher harmonics	Total
1	1.97	0.00	0.0	1.97
2	2.67	0.01	0.0	2.68
3	3.58	0.02	0.0	3.60
4	4.44	0.07	0.0	4.51
5	5.47	0.23	0.0	5.70
6	6.40	0.67	0.0	7.07
7	7.37	1.41	0.0	8.78
8	8.59	2.11	5.0	15.70

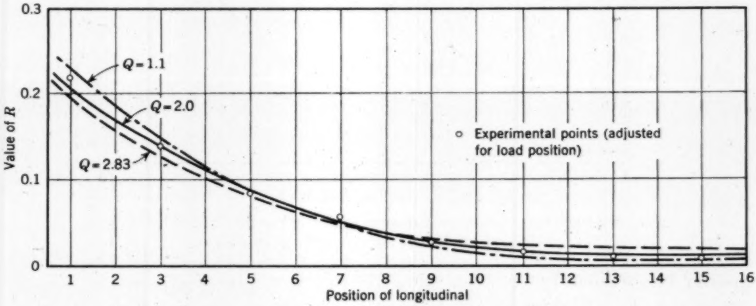


FIG. 15.—DEFLECTION DISTRIBUTION COEFFICIENTS FOR LOAD ABOVE BEAM 2 IN FILLER JOIST BRIDGE

The deflection coefficients in Fig. 14(*a*) include first and third harmonic components of the deflection curve, and this is sufficiently accurate. In dealing with the strains or bending moments, however, it is necessary to allow for the higher harmonics on the beams nearest to the load. If the free bending moment, $W L/4$ (in which W is a concentrated load), is taken as 100 units, the amplitudes of the first and third harmonics are, respectively, 81 and 9 units, which leaves 10 units to be accounted for by the higher harmonics. The distribution of these moments between the various girders is given in Table 4.

The large contribution from the harmonics higher than the third harmonic on the two girders nearest to the load results from the pointed shape of the free bending-moment diagram. These theoretical results are compared with the experimental values found from strain readings in Fig. 14(*b*), and the agreement is quite satisfactory.

The deflection distribution for this bridge model has also been investigated with the load near the edge of the slab. In this case the parameter, Q, assumes greater importance but is even more difficult to estimate accurately than the parameter, K. Various assumptions as to the effective torsional rigidity of the slab longitudinally and transversely give values of Q that are between 1.1 and 2.83 for the equivalent five-girder bridge, and it is encouraging to find that even this 3-to-1 variation of Q makes only a small difference to the resulting distribution curve, as shown in Fig. 15. The assumption of isotropy would result in $Q = 1.7$ and, in this case, would also result in an accurate estimate of the deflection distribution.

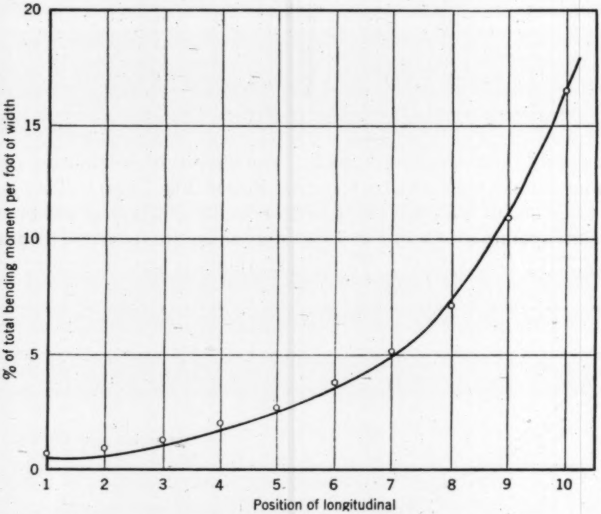


FIG. 16.—DISTRIBUTION OF LONGITUDINAL BENDING MOMENT ACROSS WIDTH OF REINFORCED CONCRETE SLAB

Model of Reinforced Concrete Slab Bridge.—A great many tests on slab-and-beam bridges and slab bridges have been performed at the University of Illinois. These tests have been described[14] by Frank E. Richart, Jr., A.M. ASCE.

---

[14] "Laboratory Research on Concrete Bridge Floors," by F. E. Richart, in "Highway Bridge Floors: A Symposium," *Transactions*, ASCE, Vol. 114, 1949.

Richart cites the results of a test on a simply supported slab with a span of 6 ft 8 in. and a width of 20 ft carrying a central point load. The slab was 6½ in. thick and was reinforced by ½-in.-sq bars at 5-in. centers longitudinally and 5½-in. centers transversely. The load distribution was computed on the assumption that the moment of inertia was the same in both directions. The slab was divided transversely into twenty elements, and the distribution of bending moments per foot across the slab at midspan is found as shown in Fig. 16. Points obtained from the experimental results are also shown, and there is excellent agreement.

CONCLUSIONS

The method described herein provides an accurate and easily applied way of computing the load distribution in many common types of highway bridge decks. The primary difficulty in applying the theory is not in the distribution theory itself but in assessing the effective moments of inertia, torsional constants, and elastic moduli of the bridge elements, particularly compound concrete and steel members. Nevertheless, even wide variations in the assumed properties of the elements result in relatively small variations of the distribution coefficients.

ACKNOWLEDGMENTS

The work described herein is part of a research program on the analysis of grid frameworks and related structures that was conducted at the University College of Khartoum, Sudan, Africa. The writers are indebted to the college for providing facilities to perform the work. They also wish to thank D. A. El Turabi for his assistance in investigating the convergence of transverse deflected forms of interconnected bridge girders.

APPENDIX I. FORMULAS FOR DISTRIBUTION COEFFICIENTS FOR A FIVE-GIRDER BRIDGE

(a) *Torsionless Case, $Q = 0$.*—

Load on Girder 1, Outer Girder.—

$$R_{11} = (224 + 654\,K + 324\,K^2 + 15\,K^3)/D_1 \qquad (60a)$$

$$R_{21} = K\,(68 + 101\,K + 10\,K^2)/D_1 \qquad (60b)$$

$$R_{31} = K\,(K - 6)\,(5\,K + 8)/D_1 \qquad (60c)$$

$$R_{41} = K\,(12 - 35\,K)/D_1 \qquad (60d)$$

and

$$R_{51} = K\,(-\,5\,K^2 + 12\,K - 2)/D_1 \qquad (60e)$$

Load on Girder 2.—

$$R_{12} = K\,(68 + 101\,K + 10\,K^2)/D_1 \qquad (61a)$$

$$R_{22} = (224 + 500\,K + 152\,K^2 + 7.5\,K^3)/D_1 \qquad (61b)$$

$$R_{32} = K\,(22 + K)\,(5\,K + 8)/D_1 \tag{61c}$$

$$R_{42} = K\,(-\,72 + 44\,K + 2.5\,K^2)/D_1 \tag{61d}$$

and

$$R_{52} = K\,(12 - 35\,K)/D_1 \tag{61e}$$

Load on Girder 3, Center Girder.—

$$R_{13} = R_{53} = K\,(K - 6)\,(5\,K + 8)/D_1 \tag{62a}$$

$$R_{23} = R_{43} = K\,(22 + K)\,(5\,K + 8)/D_1 \tag{62b}$$

$$R_{33} = (28 + 36\,K + K^2)\,(5\,K + 8)/D_1 \tag{62c}$$

and

$$D_1 = (5\,K + 8)\,(5\,K^2 + 68\,K + 28) \tag{62d}$$

For second and higher harmonics, K/p can be substituted for K in the foregoing.

(*b*) *Torsionally Stiff Case,* $Q = \infty$.—

Load on Girder 1, Outer Girder.—

$$R_{11} = (0.5 + 1.305\,K + 0.171\,K^2)/D_2 + (0.5 + 0.392\,K)/D_3 \tag{63a}$$

$$R_{21} = (0.239\,K + 0.171\,K^2)/D_2 + 0.196\,K/D_3 \tag{63b}$$

$$R_{31} = (-\,0.174\,K + 0.171\,K^2)/D_2 \tag{63c}$$

$$R_{41} = (0.239\,K - 0.171\,K^2)/D_2 - 0.196\,K/D_3 \tag{63d}$$

and

$$R_{51} = (0.5 - 1.305\,K - 0.171\,K^2)/D_2 - (0.5 - 0.392\,K)/D_3 \tag{63e}$$

Load on Girder 2.—

$$R_{12} = (0.239\,K + 0.171\,K^2)/D_2 + 0.196\,K/D_3 \tag{64a}$$

$$R_{22} = (0.5 + 0.805\,K + 0.171\,K^2)/D_2 + (0.5 + 0.145\,K)/D_3 \tag{64b}$$

$$R_{32} = (0.826\,K + 0.171\,K^2)/D_2 \tag{64c}$$

$$R_{42} = (0.5 - 0.805\,K - 0.171\,K^2)/D_2 - (0.5 - 0.145\,K)/D_3 \tag{64d}$$

and

$$R_{52} = (0.239\,K - 0.171\,K^2)/D_2 - 0.196\,K/D_3 \tag{64e}$$

Load on Girder 3, Center Girder.—

$$R_{13} = R_{53} = (-\,0.174\,K + 0.171\,K^2)/D_2 \tag{65a}$$

$$R_{23} = R_{43} = (0.826\,K + 0.171\,K^2)/D_2 \tag{65b}$$

$$R_{33} = (1 + 1.61\,K + 0.171\,K^2)/D_2 \tag{65c}$$

$$D_2 = (1 + 2.914\,K + 0.855\,K^2) \tag{65d}$$

and

$$D_3 = (1 + 1.07\,K + 0.076\,K^2) \tag{65e}$$

For second and higher harmonics, coefficients from Appendix I (*c*) must be used, with K/p^4 substututed for K.

(*c*) *Infinite Torsion and No Rotation.*—

Load on Girder 1, Outer Girder.—

$$R_{11} = \tfrac{1}{2}[(1 + 4\,K + 2\,K^2)/D_4 + (1 + 2\,K)/D_5] \qquad (66a)$$

$$R_{21} = \tfrac{1}{2}[K\,(1 + 2\,K)/D_4 + K/D_5] \qquad (66b)$$

$$R_{31} = K^2/D_4 \qquad (66c)$$

$$R_{41} = \tfrac{1}{2}[K\,(1 - 2\,K)/D_4 - K/D_5] \qquad (66d)$$

and

$$R_{51} = \tfrac{1}{2}[(1 - 4\,K - 2\,K^2)/D_4 - (1 - 2\,K)/D_5] \qquad (66e)$$

Load on Girder 2.—

$$R_{12} = \tfrac{1}{2}[K\,(1 + 2\,K)/D_4 + K/D_5] \qquad (67a)$$

$$R_{22} = \tfrac{1}{2}[(1 + K)\,(1 + 2\,K)/D_4 + (1 + K)/D_5] \qquad (67b)$$

$$R_{32} = K\,(1 + K)/D_4 \qquad (67c)$$

$$R_{42} = \tfrac{1}{2}[(1 - K)\,(1 - 2\,K)/D_4 - (1 - K)/D_5] \qquad (67d)$$

and

$$R_{52} = \tfrac{1}{2}[K\,(1 - 2\,K)/D_4 - K/D_5] \qquad (67e)$$

Load on Girder 3, Center Girder.—

$$R_{13} = R_{53} = K^2/D_4 \qquad (68a)$$

$$R_{23} = R_{43} = K\,(1 + K)/D_4 \qquad (68b)$$

$$R_{33} = (1 + 3\,K + K^2)/D_4 \qquad (68c)$$

$$D_4 = (1 + 5\,K + 5\,K^2) \qquad (68d)$$

and

$$D_5 = (1 + 3\,K + K^2) \qquad (68e)$$

For second and higher harmonics, K/p^4 must be substituted for K in the foregoing.

APPENDIX II. NOTATION

The following symbols, adopted for use in the paper and for the guidance of discussers, conform essentially with "American Standard Letter Symbols for Structural Analysis" (ASA Z10.8–1949), prepared by a committee of the American Standards Association with Society representation, and approved by the Association in 1949:

A = amplitude of the first harmonic of Y;

a = amplitude of the first harmonic of y;

B_e = effective width of the plate;

b = distance from end of span;
c = parameter defining the twist of the longitudinals;
d = thickness of the slab;
E = modulus of elasticity;
G = modulus of rigidity;
h = spacing of transversals;
I = moment of inertia;
J = torsional constant;
K = term defined in Eq. 23;
K_g = term defined in Eq. 24;
L = span length;
M = moment of force;
m = term defined in Eq. 3;
N = number of longitudinals;
n = number of transversals;
p = parameter of harmonic;
Q = term defined in Eq. 15;
Q' = term defined in Eq. 18;
R = distribution coefficient;
r = ratio of longitudinal $E\,I$ to transverse $E\,I$;
T = torque;
V = shear of load;
W = concentrated load;
Y = free deflection from Eq. 26a;
y = deflection;
θ = angle of rotation;
λ = parameter defining the twist of the longitudinals; and
μ = Poisson's ratio.

DISCUSSION

Pranab Kumar Chaudhuri[15].—The problem of assessing accurately how a concentrated load or a system of loads applied on a bridge deck is distributed among the main longitudinal girders is of great importance. However, because the interconnected structure is highly redundant, an exact elastic analysis normally becomes extremely cumbersome. During recent years several ingenious methods have been advanced for solving this problem. A new method has been added to this list. In addition to being fairly simple, the method proposed by Messrs. Hendry and Jaeger has the added advantage of being complete—that is, it is directly applicable to all kinds of interconnected bridge girders with any degree of torsional stiffness and with any variation in the relative sizes of the girders composing the grid. Although this is the chief merit of the method, certain points deserve further consideration.

Unless a bridge structure has a high torsional rigidity, the significance of a transverse girder in helping to distribute load among the various longitudinals will diminish gradually as the transversal is located increasingly far away from the center of the span. Therefore, in an ordinary steel bridge system in which the longitudinals have low torsional stiffness, the use of a great many transverse girders does not contribute to an economic distribution of the superimposed load. Fritz Leonhardt,[16] M. ASCE, contends that the lateral distribution of a concentrated load among the longitudinals of an interconnected bridge system of low torsional stiffness remains the same for more than five transverse girders as for five only. Consequently, he uses the same equivalent single cross girder at the middle to replace five or more transversals. Second, the interconnected bridge girder for long spans will have little appeal for designers because the dead weight of the structure will offset the distribution of heavy concentrated loads. Third, the designer would have to consider the added expense of extra forms for concrete bridges and of extra welding and handling of steel for a bridge system with many interconnection points between girders in two directions. On the other hand, for short-span bridges the load-distribution problem is not so important. Therefore, the design of a grid for the effective distribution of heavy concentrated loads is economical only in the case of medium-span structures (from 50 ft to 200 ft). Bridges designed and constructed deliberately as grids rarely exceed this medium-span limit.

The suitability of the authors' method for design purposes must be assessed in the light of the individual designer's practical requirements. If he must design an economical medium-span steel bridge, as shown previously, he will rarely need to exceed six longitudinals by three (intermediate) transversals. The authors' assumption of a continuous transverse medium is supposed to be applicable to as few as three cross girders. However, it is open to question whether their elaborate analysis is really a simplification of the designer's job for most practical steel bridges. Even if Leonhardt's formulas are not satisfactory for such cases, it is possible to analyze precisely such a "negligible-torsional stiffness" case without difficulty. For practical concrete bridges the formulas and curves given by the authors for use in the case of a bridge having five or more longitudinals are convenient. However, similar curves must also be provided for bridges having three and four longitudinals before the authors' method can compete on equal terms with the Massonet solution. Basically, their method is simpler and easier to understand, but Massonet's curves are easy to use in a design office.

---

[15] Structural Designer, Bridge & Roof Co. (India), Ltd., Calcutta, India.

[16] "Die vereinfachte Trägerrostberechnung," by F. Leonhardt and W. Andra, Julius Hoffman, Stuttgart, 1950.

Methods in which the torsional stiffness of the grid can be taken into account if required lead to a study of the importance of torsion in the analyses of practical bridge grids. Experiments on actual bridge grids have been conducted by Massonet in Europe and by M. Naruoka[17] (among others) in Japan. A definite conclusion cannot yet be made, but the writer feels that the effect of torsion in a practical bridge case has often been overemphasized. Thus, the cautious designer conducting an "exact" analysis probably does not realize that more concrete grids than he imagines could be analyzed on the simple assumption of negligible-torsional stiffness in Leonhardt's method. The effect of torsion is not always negligible, but a designer should develop a practical structural sense to serve as a guide in the selection of a method of analysis.

The results in the paper seem to be drawn entirely from tests performed on bridges, in which the transverse distribution is effected only with the help of slabs or jack arches but not by discrete cross girders. In such structures the simplicity and completeness of the authors' method is evident. There are other methods available, the most notable being the numerical procedure developed[3] by Nathan M. Newmark, M. ASCE, and the conventional orthotropic plate analysis and its variants,[12,17,18,19] but the authors' solution is a neat and general approach. The fact that the difficulty in computing the effective second moments of area and torsion constants of the bridge elements and the consequently wide variations in the assumed properties of the elements result in relatively small variations of the distribution coefficients lend special weight to the authors' contention of the advantage of their method applied to slab bridges. In attempting to gage the practical utility of their method to designers, it was not intended herein to detract from its acknowledged merits.

The method is probably most convenient for analyzing bridges that were not designed consciously as grids but which, nevertheless, act as such. A knowledge of the load distribution in such structures is of great importance because on such knowledge will depend whether the existing bridge can be expected to respond satisfactorily to the ever-increasing demands of the present and future.

ARNOLD W. HENDRY[20] AND LESLIE G. JAEGER[21].—The writers appreciate the generous assessment by Mr. Chaudhuri of their method for the analysis of highway bridge decks.

---

[17] "A Study of Composite Grillage Girder Bridge," by M. Naruoka and I. Hirai, *Technical Report No. 30*, Eng. Research Inst., Kyoto, Japan, 1956.

[18] "Die Berechnung der ebenen Flächentragwerke mit Hilfe der Theorie der orthogonal-anisotropen Platte," by Wilhelm Cornelius, *Der Stahlbau*, Heft 2, February, 1952, p. 21.

[19] "On the Application of the Theory of the Orthotropic Plate to the Continuous Beam Bridge," by M. Naruoka and Y. Yonezawa, *Proceedings*, 5th Cong. of the International Assn. for Bridge and Structural Eng., Lisbon, 1956.

[20] Prof. of Building Science, Univ. of Liverpool, England.

[21] Lecturer in Eng., Univ. of Cambridge, England.

In reply to the points he has raised, it is suggested that one of the main advantages of this method is that nearly all the factors he has mentioned can be assessed quantitatively and easily by the theory. However, it was not possible to examine all these factors in the paper. The effect of the positioning of cross girders in a torsionless bridge can, for example, be allowed for by taking the effective value of the number of cross girders in computing the parameter, K, as

$$n + \Sigma \cos \frac{2 \pi e}{L}$$

in which the transversals are placed at distances e_1, $e_2 \cdots$ from midspan. It will be seen that the effective value of K for a given total transverse inertia is twice as great when the transverse medium is concentrated in a single beam at midspan as when it is uniformly spread over the span. Thus, there is no difficulty in allowing for cross-girder positioning and no necessity to avoid the issue by an approximation. It should be noted, however, that cross girders are not usually put in only to secure the distribution of heavy loads. More often they are intended in the first instance to provide support for the road slab or trough floor, which, in some designs, spans longitudinally between the cross girders.

Whether it is worth considering the effects of interconnection in any given case is also easily investigated. On long spans it is obvious that dead loads will predominate and that even heavy vehicle loads will not be of prime importance in relation to the design of the main members. On the other hand, the writers do not agree that load distribution is unimportant in small bridges. It is quite impossible to lay down any range of spans over which interconnection may be considered with advantage. Comparison of dead and live loads, computation of parameters K and Q, and inspection of the graphs of distribution coefficients will usually show in a few minutes whether it is necessary to make detailed computations. In practice, bridges are designed for the span fully loaded with some particular design live load, and this will always be the starting point for obtaining the sections of the main girders, deck slab, and similar structural components. Distribution theory has little relevance in relation to such a loading because the main girders will deflect more or less by the same degree, and there will be little redistribution of the load by the transverse system. The design resulting from this procedure must be re-examined in order to assess its capacity with respect to large concentrated loads such as industrial trailers or army tanks. For these cases the theory will be of considerable assistance to the designer. He will be able to undertake this work quickly to ascertain whether any economically feasible modifications would permit the carrying of higher excess loads, or whether the design can be lightened in any way while still meeting the requirements of the specification. As Mr. Chaudhuri has indicated, the theory can be applied similarly to existing bridges.

With respect to torsion, computation of parameters K and Q and the use of the interpolation function provide an immediate method of assessing the importance of torsion in any given case. As a rule it is safe to neglect torsion, but by doing so one may overestimate the bending moments in a longitudinal by as much as 20% or 30%, which is economically significant.

Mr. Chaudhuri has referred to the design curves based on Massonet's analysis. The writers have prepared[22] a comprehensive work that shows the application of this method to many different problems and includes curves of distribution coefficients for all cases of from two to six longitudinals. As compared with the Massonet curves, these curves give a much clearer and more direct impression of the behavior of interconnected systems. Because the writers' theory deals with actual girder spacings, the interpolations required in the Massonet method are avoided. In fact, the proposed method helps the designer to get the "feel" of the problem and assists him in developing his structural sense in relation to this type of structure.

---

[22] "The Analysis of Grid Frameworks and Related Structures," by A. W. Hendry and L. G. Jaeger, Chatto & Windus, London, 1958.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2961

FRICTION MEASUREMENTS IN THE APALACHIA TUNNEL

By Rex A. Elder,[1] M. ASCE

Synopsis

The Apalachia power tunnel of the Tennessee Valley Authority was in continuous operation from its opening in 1943 until October, 1953, at which time it was unwatered and inspected. The tunnel surfaces were found to be covered with a uniform, $\frac{1}{16}$-in. deposit of a black mucilaginous material. Friction measurements were made in the tunnel in 1944, five months after the initial filling, and in 1954, five months after the inspection.

The paper presents the methods used in reducing both sets of data. The results are presented in terms of the friction coefficient, f, in the Weisbach equation. Values of n for the Manning equation are also shown. A description is given of each type of surface for which coefficients could be determined.

Description of the Tunnel

Note.—The paper supersedes and supplements one prepared[2] by George H. Hickox and Alvin J. Peterka, Members, ASCE, and the writer. An error in the use of the basic data was found which changes, in varying degrees, the results published in the prior paper. This error consisted of the transposition of Cols. 7 and 8 in Table 2 of that paper. Correction of the error results in a major change in the friction coefficient for the steel pipe, changing the f-value in Weisbach's equation from 0.0085 at maximum gate to 0.0138. The concrete-tunnel values changed from an average of 0.0133 to 0.0138 and 0.0127 for the two test reaches, and the unlined-rock-tunnel values only changed from 0.094 to 0.096. Thus, the only major effect was in the steel-pipe data.

General.—The Apalachia project of the Tennessee Valley Authority has been described[3] by Howard W. Goodhue, A.M. ASCE, R. L. Smart, and Adolf A.

Note.—Published, essentially as printed here, in June, 1956, in the Journal of the Hydraulics Division, as *Proceedings Paper 1007*. Positions and titles given are those in effect when the paper was approved for publication in *Transactions*.

[1] Head, Hydr. Lab., Div. of Water Control Planning, Hydr. Data Branch, Tennessee Valley Authority, Norris, Tenn.

[2] "Friction Coefficients in a Large Tunnel," by George H. Hickox, Alvin J. Peterka, and Rex A. Elder, *Transactions*, ASCE, Vol. 113, 1948, p. 1027.

[3] "The Design of Recent TVA Projects: VIII. Apalachia and Ocoee No. 3," by H. W. Goodhue, R. L. Smart, and A. A. Meyer, *Civil Engineering*, October, 1943, pp. 465–468.

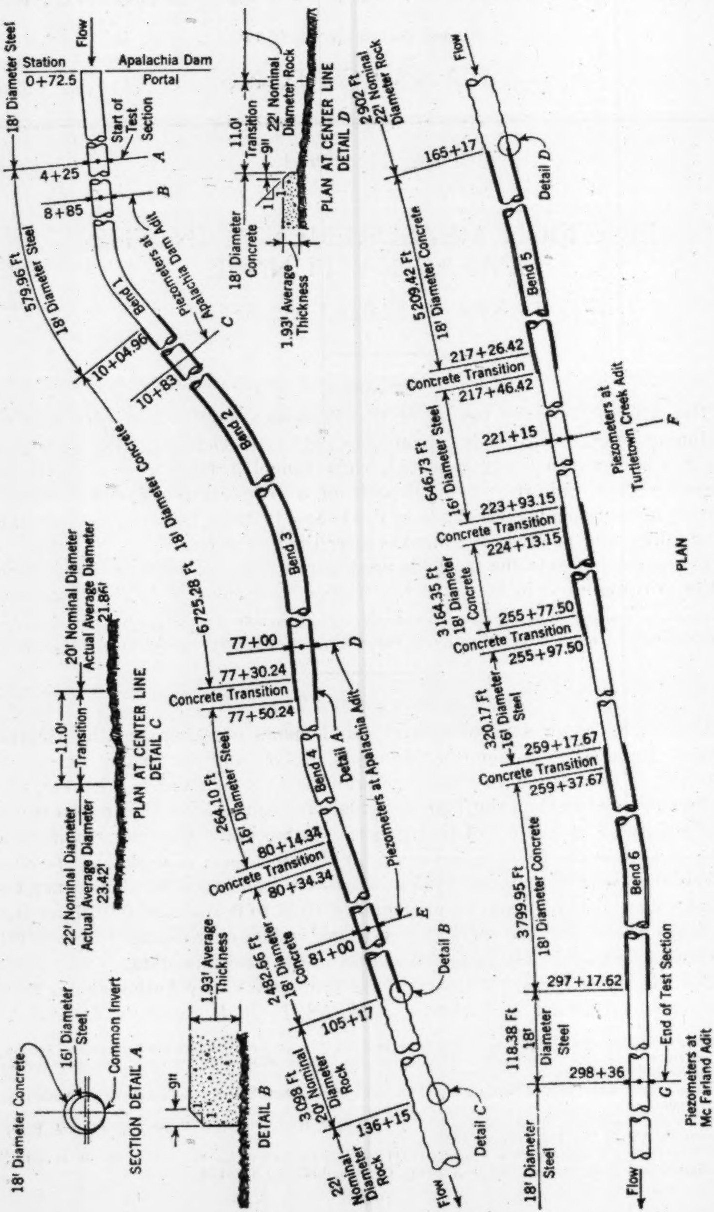

Flow
Station 0+72.5
Apalachia Dam Portal
18' Diameter Steel
Start of Test Section
A
4+25
B
8+85
Piezometers at Apalachia Dam Adit
579.96 Ft 18' Diameter Steel
Bend 1
C
10+04.96
10+83
Bend 2
18' Diameter Concrete
Bend 3
6725.28 Ft
77+00
D
77+30.24
Concrete Transition
77+50.24
Detail A
Bend 4
264.10 Ft 16' Diameter Steel
80+14.34
Concrete Transition
80+34.34
Piezometers at Apalachia Adit
E
81+00
2482.66 Ft 18' Diameter Concrete
Detail B
105+17
3098 Ft 20' Nominal Diameter Rock
136+15
Detail C
22' Nominal Diameter Rock
Flow
18' Diameter Concrete
16' Diameter Steel
Common Invert
SECTION DETAIL A
1.93' Average Thickness
9"
1
1
DETAIL B
22' Nominal Diameter Actual Average Diameter 23.42'
11.0' Transition
20' Nominal Diameter Actual Average Diameter 21.8'
PLAN AT CENTER LINE DETAIL C
18' Diameter Concrete
1.93' Average Thickness
11.0' Transition
9"
1
1
22' Nominal Diameter Rock
PLAN AT CENTER LINE DETAIL D
2902 Ft 22' Nominal Diameter Rock
Flow
165+17
Detail D
5209.42 Ft 18' Diameter Concrete
Bend 5
217+26.42
Concrete Transition
217+46.42
646.73 Ft 16' Diameter Steel
221+15
F
Piezometers at Turtletown Creek Adit
223+93.15
Concrete Transition
224+13.15
3164.35 Ft 18' Diameter Concrete
255+77.50
Concrete Transition
255+97.50
320.17 Ft 16' Diameter Steel
259+17.67
Concrete Transition
259+37.67
3799.95 Ft 18' Diameter Concrete
Bend 6
297+17.62
118.38 Ft 18' Diameter Steel
298+36
End of Test Section
G
Piezometers at Mc Farland Adit
18' Diameter Steel
Flow
PLAN

FIG. 1.—PLAN OF TUNNEL LAYOUT

Meyer, M. ASCE. The principal structures are a low ponding and diversion dam, a pressure conduit approximately 8.3 miles long, a surge tank, and a powerhouse containing two turbines.

The pressure conduit consists primarily of a concrete-lined tunnel, 18 ft in diameter. The conduit also has two long reaches of unlined rock, which have nominal diameters of 20 ft and 22 ft. Just below the dam there is a short length of steel pipe measuring 18 ft in diameter. At several other places, where there is not sufficient cover for the pressure tunnel, short lengths of steel pipe, 16 ft and 18 ft in diameter, are used. The alinement is nearly straight, but topographical considerations necessitated several bends. In general, the grade is quite flat with a slight slope toward the powerhouse. Fig. 1 shows a plan of the test reach, which comprises approximately the upper two-thirds of the conduit length.

Piezometers.—During construction seven sets of piezometers were installed at the sites shown in Fig. 1 so that tests could be made to determine the various hydraulic losses.

The installations at A, B, C, D, and E consisted of a ring of four piezometers spaced at 90° intervals around the conduit and manifolded together so that an average pressure could be obtained. In 1954 two of the four piezometer openings at A were unusable. Piezometers F and G consisted of a single opening at the horizontal center line.

Fig. 1 also shows that the piezometers are placed so that friction-loss measurements could be obtained on sections containing three types of conduit—(1) 18-ft steel pipe, (2) 18-ft concrete-lined tunnel, and (3) unlined rock tunnel. It was also possible to measure the loss in one of the bends.

CONDUIT SURFACES AT COMPLETION OF CONSTRUCTION

When construction was completed several different and distinctive tunnel surfaces were present. These surfaces are described in some detail because it is necessary to have a clear understanding of the characteristics of each surface in order to interpret the data intelligently.

Steel Pipe.—The various sections of steel pipe were formed of plates rolled to the proper curvature and butt welded. After the pipes were completed, the inner surfaces were covered with bituminous paint applied hot with swabs. Each application of the swab covered an area that was approximately 6 in. wide and 2 ft long, and deposited a layer of bituminous material that was approximately $\frac{1}{16}$ in. thick. The cold steel cooled the paint so rapidly that the resulting surface was quite irregular and included ridges and bumps as high as $\frac{1}{8}$ in. However, the surface of the paint itself was almost glossy. Fig. 2 shows the interior of one of the steel pipes treated in this manner.

Between Sta. 255 + 97.50 and Sta. 259 + 17.67, the steel liner was not painted in this manner because of the lack of ventilation in this section at the time of painting. Instead, the bituminous paint was applied cold in a slightly diluted condition, which produced a smoother surface than that in the other pipe sections.

Concrete Tunnel.—The 18-ft concrete tunnel lining was poured in two stages. An 80° invert section was poured first. The surface of the invert, which was

FIG. 2.—STEEL PIPE COATED WITH HOT BITUMINOUS PAINT

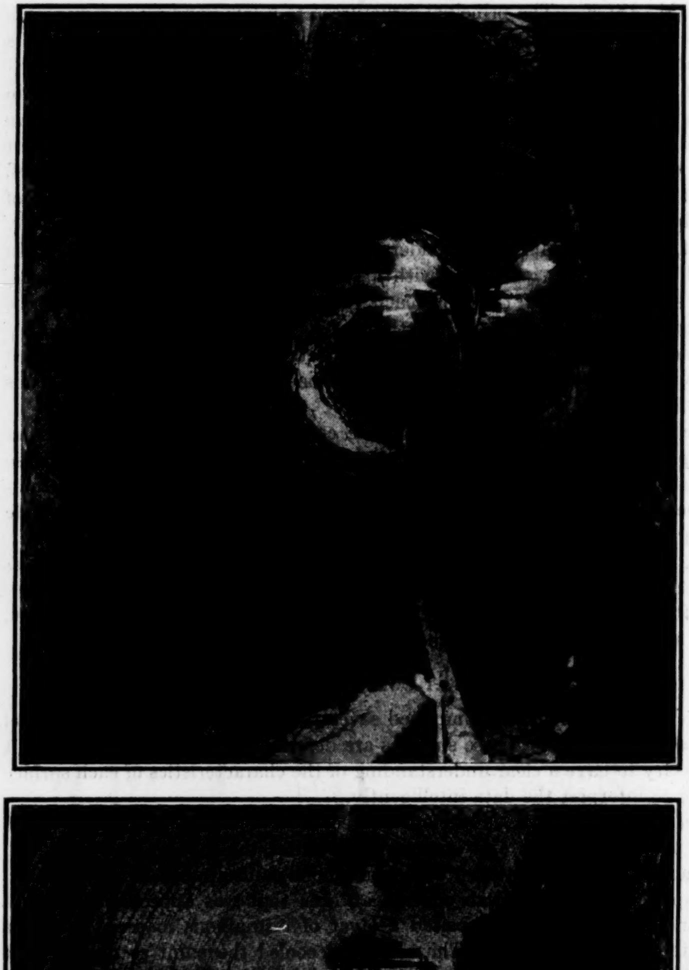


FIG. 3.—UNLINED TUNNEL SECTION

formed by a screed traveling along the top of the longitudinal invert forms, was floated with steel floats immediately after the screed had passed. The quality of the concrete was controlled very carefully so that the best possible surface would result. In addition, the concrete was poured continuously so that no joint irregularities would occur.

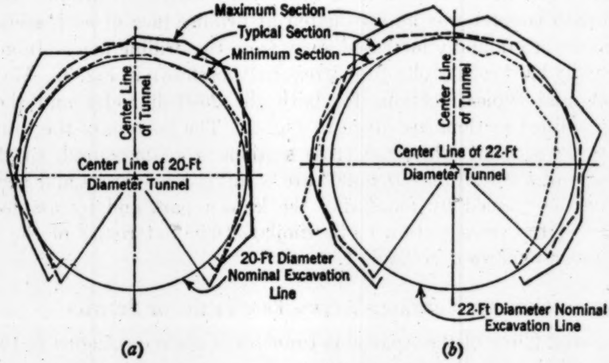

Maximum Section
Typical Section
Minimum Section
Center Line of Tunnel
Center Line of 20-Ft Diameter Tunnel
20-Ft Diameter Nominal Excavation Line
Center Line of Tunnel
Center Line of 22-Ft Diameter Tunnel
22-Ft Diameter Nominal Excavation Line
(a)
(b)

FIG. 4.—DIMENSIONS FOR UNLINED TUNNEL SECTIONS

The arch section was poured against a three-piece steel form, of which the top piece covered a 90° arc and the two side pieces, 95° each. Hinged at each joint, these form sections, each 5 ft long, could be collapsed and moved ahead through the other secrions. Thus, irregularities due to transverse form joints occurred every 5 ft on the final surface. For convenience in erection four form sections were kept bolted together as a unit during each change. There is no reason to believe that irregularities at the unit joints were greater than any others because the units were alined by drift pins in the same manner as were the sections composing the units. Each piece of each section was equipped with an 18-in.-by-24-in. inspection door and two or three 2-in. grout-pipe holes. Each door and grout-pipe opening left a slight irregularity on the surface, averaging not more than $\frac{1}{32}$ in. high. Approximately 400 ft of forms were used so that concrete could be placed continuously without resorting to transverse bulkheads. After the forms were removed, the surface was given a coat of curing compound. This compound disintegrates with time and was probably completely washed off by the time the tests were made.

TABLE 1.—CHARACTERISTICS OF UNLINED-ROCK-TUNNEL SECTIONS

Nominal diameter, in feet	AREA, IN SQUARE FEET			Diameter, equivalent circle, in feet	HYDRAULIC RADIUS, IN FEET		Average overbreak, in feet
	Maximum	Minimum	Average		Equivalent	Measured	
20	419	335	375.3	21.86	5.465	5.36	0.93
22	504	368	430.8	23.42	5.855	5.68	0.71

The concrete for the tunnels was obtained from two mixing plants, with the aggregate for both plants coming from the same source. All concrete downstream from Turtletown Creek adit was obtained from a mixing plant located at the McFarland adit. The concrete upstream from Turtletown Creek was placed after that downstream from that point.

Unlined Rock Tunnel.—The unlined-rock-tunnel sections are of two nominal diameters but are otherwise similar. The tunnels were drilled by removing 11-ft-long sections of rock with each blasting operation. Although the nominal diameter was approximated at drilling face of each section, the actual diameter gradually increased away from the drilling face. In addition, the unusually hard rock broke quite irregularly as shown in Fig. 3. Maximum, minimum, and typical sections for both the 20-ft-diameter and the 22-ft-diameter unlined sections are given in Fig. 4. The bottom of the tunnel had not been cleaned out when the cross sections were measured, so that the exact shape of the bottom 80°-to-90° arc is unknown. Areas and perimeters have been determined by measuring the known part and by assuming the remainder of the cross section to be similar. Characteristics of the unlined sections are summarized in Table 1.

Conduit Surfaces After Ten Years of Service

The initial filling of the Apalachia tunnel was made on August 6, 1943. It remained filled until October 16, 1953, when the tunnel was unwatered for inspection. The October, 1953, inspection revealed that the tunnel was in the following condition:

General.—All tunnel surfaces were coated with a $\frac{1}{16}$-in. layer of black mucilaginous material. This deposit was exceedingly uniform both in depth and in surface-roughness characteristics. It covered and eliminated all small original surface roughnesses, but it reproduced all major surface shapes, such as joint misalinements. The material eliminated the original surface-roughness characteristics so effectively that the steel and concrete sections were indistinguishable.

Spectrographic X-ray analyses showed that the coating material was primarily manganese in the forms, MnO and Mn_2O_3. It is assumed that deposition was a result of the adsorption of the manganese by algal polysaccharides through the process determined[4] after a similar deposition was found in a tunnel 35 miles north of the Apalachia project.

Downstream from the unlined rock section a strip, which was approximately 1½ ft wide along the bottom, was free of the deposited material. The cause was believed to have been the continual movement of small quantities of sand and gravel picked up in the unlined section.

Areas were found throughout the tunnel in which the material had peeled off at some time prior to the inspection. New deposition had begun, but the edges at the break remained well defined and varied in size from less than 1 sq ft to 20 sq ft. No particular surface seemed to peel more than the others, and probably not more than from 2% to 3% of the total tunnel surface was affected. Fig. 5 shows the general surface texture of the coating and illustrates the reproduction of major surface shapes and the sharp edges at a point where peeling had occurred.

---

[4] "The Adsorption of Manganese by Algal Polysaccharides," by Arthur Z. Pillard and Peter Byrd Smith, *Science*, Vol. 114, No. 2964, October 19, 1951, pp. 413–414.

Steel Pipe.—The irregularities created by the bitumastic paint were eliminated by the manganese coating.

The tunnel was unwatered for two and one-half days for the 1953 inspection. At the end of that time it was noted that considerable flaking of the deposited coating occurred along the overhead parts of the exposed steel pipes as a result of the drying out of the material. Probably from 20% to 30% of the total surface was affected. Thus, the exposed pipe surfaces at the dam and at the Turtletown Creek adit became considerably roughened from the hydraulic standpoint. These exposed lengths extended between Sta. 1 + 28 and Sta. 9 + 15 and between Sta. 218 + 66 and Sta. 221 + 01. The liners at the Apalachia adit and McFarland adit and the one between Turtletown Creek adit and McFarland adit were all buried and not exposed to the sun, with the result that they did not become roughened.

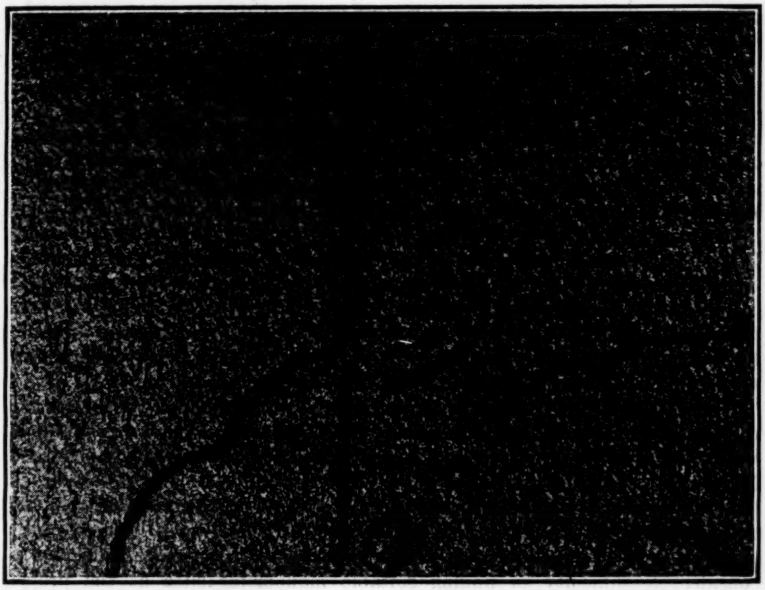


FIG. 5.—NORMAL 1954 COATING SHOWING REPRODUCTION OF MAJOR SURFACE SHAPES AND PEELED AREAS

Concrete Tunnel.—The texture of the coating on the concrete surfaces was uniform. All irregularities due to the form joints and inspection doors were completely obliterated by the coating.

Unlined Rock Tunnel.—The $\frac{5}{16}$-in. coating could not materially change the shape of the unlined rock surfaces. It did tend to round over the edges of the protruding rock. The bottom in this section may have changed slightly from the standpoint of hydraulic roughness. Originally, some of the holes were filled with sand and gravel. Ten years of use washed the fine material out of the unlined sections and worked the larger particles downstream. Most of these larger stones have become worn until they have the appearance of river gravel. In many places, especially in the 22-ft-diameter reach, they have created pot holes, some of which are 6 in. deep and from 8 in. to 10 in. in diameter. There are not enough of them, however, to have any appreciable roughness effect.

1944 Field Tests

The 1944 field observations were made at the same time as the turbine-index tests. This procedure was possible because both types of tests required that the turbines be operated over a wide range of gate openings, with each gate position held constant for a sufficient length of time to allow the flow to become steady.

On January 25, 1944, unit 1 was operated alone. The day began clear but became partly cloudy by late afternoon. On January 26, which was dark and rainy, both units were operated identically.

Manometer Installations.—At the Apalachia Dam adit two water-air differential manometers and one open U-tube mercury manometer were used, the latter being attached to piezometer C, and the others to piezometers A and B and B and C, respectively.

At Apalachia adit one water-air differential manometer and one open U-tube mercury manometer were used. In this location the open mercury manometer was connected to piezometer E, and the water-air differential manometer to piezometers D and E.

At both the Turtletown Creek adit and McFarland adit, single-column, 10-ft mercury manometers were used. These manometers were made in two sections to facilitate carrying and were carefully calibrated.

At the Apalachia Dam adit the pressures were sufficiently low that the gages could be installed nearly at tunnel level. Each manometer was equipped with a blowoff cock in order that the piezometer lines could be flushed frequently.

Maximum and minimum pressures at the other three locations were too great to allow the manometers to be installed at tunnel level. Therefore, it was necessary to place them on the hillsides above the tunnel. There was considerable difficulty in finding suitable manometer sites at the proper elevations because of the steep terrain. Data from all manometers were tied together through the level system established for the tunnel construction.

Connection of the manometers to the piezometer lines was by $\frac{3}{4}$-in. pipe. The long lines used for most of these connections could have caused considerable trouble as a result of differential temperature effects. Fortunately, however, the tests were conducted when the air temperatures were within a few degrees of the water temperature and did not vary materially.

On January 25, 1944, solar heating may have affected some of the pressure data slightly, but this effect should have been negligible because at Apalachia, where the small differentials were measured, the vertical length subjected to differential heating was not more than 3 ft. Because it was January and a partly cloudy day, there was not enough available solar energy to affect appreciably the larger differential between the other piezometers.

Observations.—Because each of the four locations was entirely isolated from the others and from the powerhouse, it was necessary to take readings at 1-min intervals continuously throughout the test period to assure a complete record. All observers' watches were carefully compared at the beginning and end of each day's observations, and all time readings were corrected to correspond with a master electric clock at the powerhouse. At each location the manometers were operated and read by an experienced member of the engineering staff of the Tennessee Valley Authority's hydraulic laboratory.

Although every effort was made to obtain continuous records at each site, several breaks occurred. For example, on January 26, rain interfered several times with the recording of the measurements. The data paper became so wet that a legible record could not be kept. At one gate opening on January 26, the top of the mercury column at Turtletown Creek was behind a rubber connector which united the two 5-ft gage sections.

1954 Field Tests

The 1954 field tests were made on March 27 and 28, the single-unit test being performed on unit 1 on March 28 and the double-unit tests on March 27. The first day was dark and rainy, and the second was clear. At the time of these tests the wicket gates were equipped with blocks, which prevented the gates from exceeding a 90% opening. A test was made at this point on March 27, but time did not permit a similar reading on March 28.

Piezometer Cleaning.—At the time of the October, 1953, inspection, an area of at least 3 ft in diameter was cleaned around all piezometer openings.

Manometer Installations.—At Apalachia Dam three single-column, 5-ft water gages were mounted on the hillside at such an elevation that all readings would be within the 5-ft range. At Apalachia adit the same equipment was used as in the 1944 tests. At Turtletown Creek adit a 5-ft, U-tube mercury manometer was mounted, and at McFarland a 5-ft, single-column water gage was used at multiple locations. A series of temporary benchmarks was set on the steep hillside at the McFarland site, and as the hydraulic grade line varied, the water-column gage was moved to keep the reading within the range of the gage.

All gages were connected to the piezometer outlets by ½-in. translucent polyethylene plastic tubing. Solar radiation probably caused an appreciable effect on the piezometer A to B differentials on March 28; otherwise no effect should have occurred.

Observations.—Portable two-way radio equipment was available for use at each adit and at the powerhouse. A 1-min time signal was broadcast from the powerhouse to initiate a reading at each gage site. Time-signal checks were broadcast approximately every 20 min and at the beginning of each steady test period. The radio made it possible to obtain complete data at every station for every gate position. As in the 1944 tests, experienced personnel were used at all gage sites.

Tunnel Discharges

Tunnel discharges for both test periods were based on records obtained at a gaging station immediately below the powerhouse. Because steady river flow did not occur during the actual testing period, as a result of the short time each gate position was held, the actual discharges were computed on the basis of differential-pressure measurements obtained from turbine-scroll-case piezometers. These pressures were correlated to the river discharges by measurements obtained at periods when the gate openings were held constant for a sufficient time for the river flow to stablize and, thus, to allow the determination of the discharge from the gage-rating curves. Discharge measurements made at the station by the Geological Survey (United States Department of the Interior) were carefully examined and plotted to give the best rating curve for the time of the friction measurements.

1944 Tests.—The actual gate opening depends on the movement of the servomotor piston. In turn, the discharge varies with the gate opening, and the differential pressure, with the discharge. Thus, the differential pressure should vary smoothly with the observed movement of the servomotor piston.

It is probable that in 1944 the measurement of piston travel was more accurate than the measurement of differential pressure. Accordingly, the turbine-gate openings were recorded in terms of the movement of the servomotor piston that opens the gates, and the differential pressure for each gate opening was taken from smooth curves drawn through the points determined by the observed relationship between the piston movement and differential pressure. The piston movement, the observed differential pressure, and the adjusted value taken from the smooth curve for each gate opening on January 25 and January 26, 1944, are summarized in Tables 2(*a*) and 2(*b*).

River stages at the gaging station during the 25% gate opening and 100% gate opening indicated river discharges of 1,150 cu ft per sec and 3,290 cu ft per sec, respectively. These discharges included both the flow through the tunnel and the flow in the river above the powerhouse. The latter discharge was measured during the tests and was 84 cu ft per sec. However, because the river discharge can be determined only to the nearest 10 cu ft per sec, the net discharges through the turbines can be considered as 1,070 cu ft per sec and 3,210 cu ft per sec.

The known discharge of 3,210 cu ft per sec for both units at a gate opening of 100% on January 26 was divided between the two units in proportion to their power outputs. The index test data gave values of 39,790 kw and 40,000 kw for unit 1 and unit 2, respectively. Assuming that both units were operating at the same head and efficiency, the corresponding discharges were 1,600 cu ft per sec and 1,610 cu ft per sec. Discharges for each unit at other gate openings were then computed by assuming the discharge to be proportional to the square root of the differential pressure. These values for unit 1 and unit 2 on January 26 are also given in Table 2(*b*), which shows the computed discharge of 1,060 cu ft per sec for a 25% gate opening. This value compares favorably with that of 1,070 cu ft per sec obtained from the rating curve. Discharges for unit 1 on January 25 were computed from the same relationship and are also given in Table 2(*a*).

1954 Tests.—The method used in measuring the scroll-case differentials for the 1954 tests was more accurate than that used in 1944. Therefore, it was not necessary to adjust these values on the basis of the servomotor-piston-movement measurements. Tables 2(*c*) and 2(*d*) give the measured scroll-case differential pressures.

Steady river stages were not obtainable on the days of the tests, but arrangements were made for discharge tests shortly after the friction-measurement tests. Single-unit tests were made at gate openings of 60% on unit 1 and 58% for unit 2. The rating curve gave a discharge of 1,300 cu ft per sec for both units. These discharges include the flow in the river above the powerhouse and the leakage through the shutdown unit. Subsequent field measurements gave a value of 50 cu ft per sec to these flows. The net turbine flows were 1,250 cu ft per sec. The scroll-case differentials were

TABLE 2.—GATE POSITIONS, DIFFERENTIAL PRESSURES, AND DISCHARGES

Gate opening, in percentage	UNIT 1				UNIT 1			UNIT 2			DISCHARGE, IN CUBIC FEET PER SECOND		
	Piston movement, in inches	Differential Pressure		Discharge, in cu ft per sec	Piston movement, in inches	Differential Pressure		Piston movement, in inches	Differential Pressure		Unit 1	Unit 2	Total
		Observed	Adjusted			Observed	Adjusted		Observed	Adjusted			
	(*a*) January 25, 1944				(*b*) January 26, 1944								
		In. of mercury				In. of mercury			In. of mercury				
25	4.22	1.32	1.32	525	4.25	1.345	1.35	3.19	1.360	1.36	530	530	1,060
30	4.69	1.99	1.97	640	4.75	1.926	1.90	3.72	1.964	1.99	628	640	1,270
35	5.19	2.76	2.74	755	5.25	2.545	2.57	4.25	2.726	2.70	730	745	1,475
40	5.69	3.56	3.56	860	5.72	3.405	3.35	4.78	3.516	3.50	833	850	1,685
45	6.25	4.60	4.59	975	6.25	4.255	4.25	5.28	4.313	4.31	939	943	1,880
50	6.75	5.56	5.58	1,075	6.75	5.112	5.12	5.78	5.188	5.19	1,030	1,033	2,060
55	7.28	6.67	6.72	1,180	7.22	5.938	5.90	6.31	6.163	6.14	1,106	1,124	2,230
60	7.75	7.77	7.73	1,265	7.72	6.819	6.75	6.81	6.978	6.98	1,182	1,198	2,380
65	8.25	8.84	8.80	1,350	8.22	7.455	7.58	7.31	7.883	7.83	1,252	1,269	2,520
70	8.75	9.85	9.87	1,430	8.75	8.500	8.45	7.81	8.528	8.62	1,322	1,331	2,650
75	9.25	10.93	10.93	1,500	9.25	9.330	9.26	8.28	9.598	9.36	1,383	1,387	2,770
80	9.75	12.00	11.98	1,575	9.75	10.020	10.03	8.81	9.971	10.15	1,440	1,443	2,880
85	10.22	13.09	12.95	1,640	10.25	10.620	10.73	9.31	10.905	10.86	1,490	1,495	2,980
90	10.72	13.82	13.97	1,700	10.72	11.305	11.33	9.81	11.700	11.53	1,530	1,540	3,070
95	...	...	...	...	11.25	12.110	12.92	10.37	12.253	12.18	1,570	1,582	3,150
100	...	...	...	...	11.73	12.195	12.36	10.81	12.425	12.61	1,600	1,610	3,210
	(*c*) March 28, 1954				(*d*) March 27, 1954								
		In. of water				In. of water			In. of water				
25	...	...		...	4.25	14.08		3.50	18.41		481	564	1,045
35	...	...		...	5.22	29.65		4.34	32.68		699	751	1,450
40	5.72	40.46		826	5.75	38.88		4.75	39.56		800	826	1,626
45	6.25	51.95		935	6.44	52.59		5.38	52.80		930	955	1,885
50	6.75	64.50		1,040	6.97	63.67		5.91	63.65		1,024	1,048	2,072
55	7.25	77.62		1,140	7.50	74.97		6.44	75.07		1,111	1,138	2,249
60	7.75	92.35		1,243	7.97	85.31		6.94	86.54		1,185	1,222	2,407
65	8.25	104.68		1,323	8.50	96.46		7.62	101.05		1,260	1,321	2,581
70	8.75	119.00		1,410	9.00	107.10		7.97	107.88		1,328	1,365	2,693
75	9.25	131.93		1,484	9.47	116.71		8.44	117.89		1,386	1,427	2,813
80	9.72	144.56		1,553	10.00	125.78		8.97	126.64		1,439	1,479	2,918
85	10.25	159.36		1,630	10.44	135.05		9.44	135.65		1,491	1,530	3,021
Open	...	...		...	11.12	144.88		10.31	149.73		1,544	1,608	3,152

TABLE 3.—ELEVATIONS OF HYDRAULIC GRADE LINE AND MEASURED HEAD LOSSES

Gate opening, in percentage	GRADE-LINE ELEVATION (FEET ABOVE EL. 1200) AT PIEZOMETERS:					MEASURED LOSS, IN FEET, IN SECTIONS:		
	...	C	E	F	G	A-B	B-C	D-E
	Reservoir[a]	Apalachia Dam[a]	Apalachia[a]	Turtle-town Creek[a]	McFarland[a]	18-ft steel pipe	Bend 1	Apalachia transition
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				(a) One unit, January 25, 1944				
45	77.65	76.96	75.85	...	...	0.076	0.047	0.120
50	77.75	76.98	75.59	70.75	69.50	0.096	0.064	0.148
55	77.82	76.99	75.23	69.39	67.91	0.110	0.073	0.177
60	77.84	76.80	74.77	68.07	66.17	0.128	0.089	0.211
65	77.91	76.83	74.48	66.80	64.64	0.145	0.098	0.232
70	77.93	76.79	74.07	65.44	63.04	0.166	0.110	0.260
75	77.00	76.74	73.75	64.15	61.64	0.183	0.123	0.287
80	77.01	76.61	73.34	62.92	60.13	...	...	0.312
				(b) Two units, January 26, 1944				
25	78.38	77.64	76.29	71.46	70.51	0.088	0.051	0.145
30	78.47	77.50	75.48	68.76	67.23	0.131	0.075	0.199
35	78.55	77.31	...	65.13	62.62	0.188	0.108	...
40	78.58	77.02	73.15	61.10	57.54	0.241	0.144	0.368
45	78.62	76.72	71.93	57.13	52.98	0.297	0.177	0.450
50	78.61	76.40	70.62	52.78	47.65	0.357	0.210	0.546
55	78.66	76.08	69.32	48.61	42.59	0.411	0.251	0.634
60	78.68	75.78	67.96	...	37.24	0.475	0.290	0.724
65	78.69	75.48	66.63	39.96	31.94	0.535	0.325	0.820
70	78.70	75.11	65.33	35.86	27.02	0.592	0.363	0.899
75	78.70	74.78	64.12	31.81	22.02	0.645	0.396	0.994
80	78.69	74.46	62.84	27.91	17.28	0.699	0.432	1.066
85	78.70	74.18	61.75	24.43	13.10	0.747	0.458	1.147
90	78.69	73.92	60.76	21.28	9.14	0.788	0.489	...
95	78.67	73.71	...	18.45	5.70	0.834	0.521	...
100	78.66	73.51	...	16.72	3.68	0.862	0.536	...
100	78.65	...	59.28	16.69	3.67	...	...	1.315
				(c) One unit, March 28, 1954				
40	78.06	77.789	77.059	74.156	73.538	0.070	0.026	0.070
45	78.08	77.773	76.792	73.158	72.316	0.089	0.040	0.095
50	70.12	77.698	76.408	71.993	70.896	0.107	0.048	0.124
55	78.14	77.618	76.066	70.744	69.248	0.135	0.059	0.149
60	78.20	77.510	75.654	69.454	67.616	0.161	0.075	0.173
65	78.22	77.359	75.164	68.103	65.925	0.166	0.083	0.198
70	78.16	77.125	74.595	66.623	64.096	0.178	0.092	0.225
75	78.07	76.892	74.057	65.200	62.352	0.195	0.099	0.250
80	77.92	76.652	73.564	63.829	60.725	0.219	0.108	0.274
85	77.84	76.419	73.050	62.301	58.932	0.242	0.117	0.299
				(d) Two units, March 27, 1954				
25	78.21	77.656	76.328	71.920	70.546	0.095	0.047	0.115
35	78.23	77.222	74.606	66.227	63.629	0.186	0.095	0.228
40	78.17	76.994	73.702	63.062	59.804	0.234	0.116	0.289
45	78.01	76.433	72.030	57.707	53.339	0.321	0.162	0.396
50	78.00	75.996	70.655	53.412	48.174	0.386	0.195	0.478
55	77.90	75.563	69.258	49.019	42.819	0.455	0.231	0.565
60	77.80	75.104	67.867	44.589	37.498	0.521	0.261	0.650
65	77.70	74.546	66.226	39.616	31.472	0.588	0.306	0.741
70	77.66	74.180	65.131	36.145	27.288	0.644	0.330	0.809
75	77.67	73.790	73.931	32.384	22.755	0.699	0.359	0.886
80	77.63	73.453	62.788	28.665	18.220	0.757	0.391	0.959
85	77.59	73.164	61.786	25.246	14.138	0.812	0.412	1.024
Open	77.64	72.708	60.335	20.641	8.589	0.880	0.449	1.115

[a] **Names of adit where piezometers were located or of section where loss was measured.**

measured as 94.92 in. and 90.52 in. of water for unit 1 and unit 2, respectively. By using these discharges and differential pressures, the coefficients, K, in

$$Q = K \sqrt{\Delta P} \qquad (1)$$

were computed to be 128.3 for unit 1 and 131.4 for unit 2. These values compare with coefficients of 128.3 for unit 1 and 127.9 for unit 2 determined in 1944. This change in coefficient is reasonable because one of the scroll-case piezometers for unit 2 was modified when the unit was shut down in 1953. Tables 2(*c*) and 2(*d*) give the discharges computed on the basis of the 1954 coefficients.

REDUCTION OF TEST DATA

Manometer readings were reduced to elevations of the pressure grade line. These elevations, as well as the differential pressures between piezometers A and B, B and C, and D and E, are shown in Tables 3(*a*) and 3(*b*) for the 1944 data and in Tables 3(*c*) and 3(*d*) for the 1954 data. Reservoir elevations for the test periods were taken from the chart of the headwater recording gage at the dam and are also indicated in Table 3.

The hydraulic-grade-line elevations and differential pressures given in Table 3, together with the discharge rates in Table 2 and the general data in Fig. 1, were sufficient to compute the various losses in the test sections of the tunnel.

The tunnel conditions in 1944 were such that the data in Tables 3(*a*) and 3(*b*) could be reduced to give friction-loss coefficients for three general tunnel-surface roughnesses—bitumastic painted steel, concrete, and unlined rock. The data also made it possible to determine the bend-loss coefficient for bend 1.

The tunnel conditions in 1954 also resulted in three general tunnel-surface roughnesses for which friction-loss coefficients could be computed from the data of Tables 3(*c*) and 3(*d*). Two of the three 1944 surface-roughness types were changed materially by the $\frac{5}{16}$-in. manganese coating, and only the rock was essentially similar in 1954 and 1944. The loss coefficient of bend 1 could also be computed from the 1954 data.

The section between piezometer D and piezometer E contained one contraction and one expansion from 18 ft to 16 ft. The losses proved to be so small for both sets of test data that they could not be separated.

Friction losses for each type of surface were expressed in terms of the friction coefficient, f, in the Weisbach equation,

$$h_f = f \frac{L}{D} \frac{V^2}{2 g} \qquad (2)$$

and the results are plotted as a function of the Reynolds number, R, in which

$$\mathsf{R} = \frac{V D}{\nu} \qquad (3)$$

In Eqs. 2 and 3, L is the length of any test reach; D denotes the diameter of the pipe; V is the average flow velocity; h_f represents the energy loss over the length, L; and ν is the kinematic viscosity of the flowing water.

TABLE 4.—Friction-Loss Computations

Gate opening, in percentage (1)	Velocity in feet per second (2)	Reynolds number, **R** (3)	Weisbach's *f*-factor for 18-foot steel pipe (4)	Bend 1 Loss		18-Ft Concrete-Lined Tunnel[a]		
				Net loss, in feet (5)	Coefficient, C_B (6)	Computed Loss in Feet		Weisbach's *f*-factor (9)
						Bend (7)	Friction (8)	
(*a*) One unit, January 25, 1944								
45	3.83	4.6×10^6	0.0130	0.017	0.0746	0.07	0.92	0.0110
50	4.22	5.1×10^6	0.0136	0.026	0.0939	0.08	1.16	0.0114
55	4.63	5.6×10^6	0.0129	0.026	0.0781	0.10	1.48	0.0121
60	4.97	6.0×10^6	0.0130	0.034	0.0850	0.11	1.71	0.0121
65	5.30	6.4×10^6	0.0130	0.035	0.0800	0.13	1.99	0.0124
70	5.61	6.7×10^6	0.0133	0.038	0.0778	0.14	2.32	0.0130
75	5.89	7.1×10^6	0.0133	0.044	0.0816	0.16	2.54	0.0128
80	6.19	...	...	...	...	0.17	2.79	0.0127
(*b*) Two units, January 26, 1944								
25	4.16	4.8×10^6	0.0128	0.014	0.0520	0.08	1.13	0.0114
30	4.99	5.8×10^6	0.0132	0.020	0.0517	0.11	1.71	0.0120
35	5.79	6.7×10^6	0.0141	0.031	0.0595	...	...	...
40	6.61	7.7×10^6	0.0139	0.042	0.0619	0.20	3.30	0.0132
45	7.38	8.6×10^6	0.0137	0.050	0.0590	0.25	4.09	0.0131
50	8.09	9.4×10^6	0.0137	0.058	0.0570	0.30	4.93	0.0132
55	8.75	10.2×10^6	0.0135	0.071	0.0597	0.34	5.79	0.0137
60	9.35	10.9×10^6	0.0137	0.084	0.0618	0.40	6.70	0.0134
65	9.89	11.5×10^6	0.0138	0.094	0.0618	0.44	7.59	0.0136
70	10.40	12.1×10^6	0.0138	0.107	0.0636	0.49	8.39	0.0136
75	10.88	12.7×10^6	0.0137	0.116	0.0630	0.53	9.14	0.0135
80	11.31	13.2×10^6	0.0137	0.129	0.0648	0.58	9.97	0.0136
85	11.70	13.6×10^6	0.0137	0.135	0.0634	0.62	10.66	0.0136
90	12.07	14.0×10^6	0.0136	0.145	0.0640	...	...	...
95	12.38	14.4×10^6	0.0137	0.159	0.0667	...	...	...
100	12.61	14.7×10^6	0.0136	0.160	0.0647	0.72	2.19	0.0134
(*c*) One unit, March 28, 1954								
20	1.56	1.9×10^6	0.0197	...	...	0.01	0.16	0.0115
30	2.44	2.9×10^6	0.0190	...	...	0.02	0.41	0.0121
40	3.25	3.9×10^6	0.0167	0.003	0.0207	0.04	0.62	0.0102
45	3.67	4.4×10^6	0.0166	0.011	0.0523	0.05	0.84	0.0108
50	4.09	4.9×10^6	0.0162	0.012	0.0463	0.06	1.10	0.0116
55	4.48	5.4×10^6	0.0169	0.016	0.0513	0.08	1.33	0.0116
60	4.88	5.9×10^6	0.0170	0.024	0.0649	0.09	1.59	0.0117
65	5.20	6.2×10^6	0.0155	0.025	0.0595	0.10	1.90	0.0123
70	5.54	6.6×10^6	0.0146	0.026	0.0545	0.12	2.19	0.0125
75	5.83	7.0×10^6	0.0144	0.026	0.0493	0.13	2.46	0.0127
80	6.10	7.3×10^6	0.0148	0.029	0.0502	0.14	2.68	0.0126
85	6.40	7.7×10^6	0.0149	0.029	0.0455	0.15	2.92	0.0125
(*d*) Two units, March 27, 1954								
25	4.11	4.9×10^6	0.0142	0.011	0.0420	0.06	1.15	...
35	5.70	6.8×10^6	0.0144	0.026	0.0515	0.12	2.27	0.0119
40	6.39	7.7×10^6	0.0144	0.029	0.0458	0.15	2.85	0.0122
45	7.41	8.9×10^6	0.0147	0.045	0.0528	0.20	3.80	0.0122
50	8.14	9.8×10^6	0.0147	0.053	0.0514	0.25	4.62	0.0121
55	8.84	10.6×10^6	0.0147	0.064	0.0526	0.29	5.45	0.0122
60	9.46	11.4×10^6	0.0147	0.070	0.0503	0.33	6.25	0.0122
65	10.14	12.2×10^6	0.0144	0.086	0.0538	0.38	7.20	0.0122
70	10.58	12.7×10^6	0.0145	0.090	0.0517	0.42	7.82	0.0122
75	11.05	13.3×10^6	0.0144	0.098	0.0516	0.46	8.52	0.0122
80	11.46	13.8×10^6	0.0145	0.110	0.0538	0.49	9.22	0.0122
85	11.87	14.2×10^6	0.0145	0.110	0.0502	0.53	9.83	0.0122
Open	12.38	14.9×10^6	0.0144	0.121	0.0507	0.57	10.69	0.0122

[a] From Apalachia Dam adit to Apalachia adit.

The water temperature on January 25, 1944, and on March 27 and March 28, 1954, was 8°C, whereas the temperature on January 26, 1944, was 7°C. A value of ν of 1.498×10^{-5} was used at 8°C, and a value of 1.542×10^{-5}, at 7°C.

The values of the roughness coefficient, n, in the Manning equation,

$$V = \frac{1.486}{n} R^{2/3} S^{1/2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

can be directly related to f in Eq. 2 and are also shown on the plotted data for the benefit of those more familiar with the use of n than f.

The bend losses were expressed in terms of the velocity head by

$$H_B = C_B \frac{V^2}{2g} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which H_B is the head loss due to the bend and C_B is the loss coefficient.

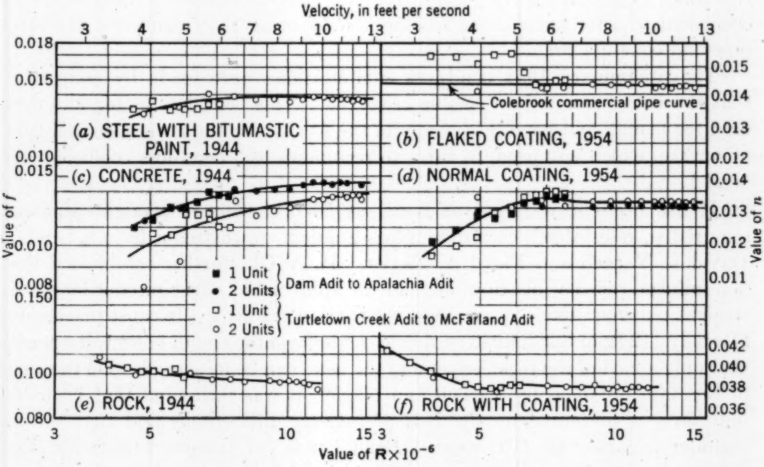


FIG. 6.—FRICTION COEFFICIENTS

1944 Friction Loss in 18-Ft Steel Pipe.—The friction coefficients for the 18-ft steel pipe were computed by Eq. 2 from the discharges in Tables 2(*a*) and 2(*b*), the observed pressure drops in Col. 7 of Tables 3(*a*) and 3(*b*), and from the dimensions of the test section in Fig. 1. These values are given in Tables 4(*a*) and 4(*d*) and are plotted in Fig. 6(*a*).

1954 Friction Loss for Flaked-Coated Surface.—In 1954 the manganese coating in the steel pipe, which contains piezometer reach A-B, was roughened by flaking during the inspection unwatering period. Friction coefficients for this type of surface were computed by Eq. 2 from the discharges of Tables 2(*c*) and 2(*d*), the observed pressure drops in Col. 7 of Tables 3(*c*) and 3(*d*), and the dimensions of the section. These values are given in Tables 4(*c*) and 4(*d*) and are plotted in Fig. 3(*b*).

Bend Losses.—In 1944 the pressure drop measured between piezometer B and piezometer C was the result of the loss due to bend 1 and the friction loss in 198 ft of tunnel, of which 120 ft were 18-ft steel pipe and 78 ft were 18-ft concrete-lined tunnel. In 1954 the 198-ft reach of tunnel was normal-coated. Because both test sections of the 18-ft, concrete-lined tunnel contained bends, it was necessary to determine the bend-loss coefficients and the friction coefficients for the concrete tunnel by successive approximations.

In reducing the 1944 data the tunnel between piezometer B and piezometer C was first assumed to be composed entirely of 18-ft steel pipe for which the friction coefficient was known. The friction loss in this reach was then approximated by assuming it to be proportional to the measured loss between piezometer A and piezometer B. Subtracting the computed friction loss from the total measured loss gave the loss for bend 1, from which the preliminary bend-loss coefficient, C_B, was computed by Eq. 5. The value of C_B determined was applied to bend 2 and bend 3 between piezometer C and piezometer D in order to obtain an approximate f-value for the concrete tunnel. The bend-loss coefficient, C_B, for bend 1 was recomputed by using the f-values for the steel pipe and concrete tunnel.

The preliminary 1954 bend-loss computations were made by estimating that the B-C reach had an f-value of 0.0125. This bend-loss factor was then used for bend 2 and bend 3 to compute the f-value for the normal-coated type of surface, which was used, in turn, to recompute the bend-loss coefficient for bend 1.

It has been proved that as many as 50 diameters of straight pipe are required to re-establish the friction-loss gradient downstream from a bend.[5,6] David L. Yarnell and Floyd A. Nagler,[5] M. ASCE, further established that nonuniform velocity distribution had an appreciable effect on the length of straight pipe required to re-establish the friction gradient. Because piezometer C is only 112 ft, or 6.2 diameters, downstream from the end of bend 1, it appears that only a part of the total loss due to the bend can be measured. On the basis of the data obtained by Yarnell and Nagler, it was estimated that probably only 50% is measured by the B-C piezometer differentials and that the remainder as part of the C-D losses. Bend 3 ends 141 diameters upstream from piezometer D. The loss from bend 2 and bend 3 should therefore be entirely contained in the C-D measurements. Bend 4 ends 10 diameters upstream from piezometer E; approximately 80% of the bend loss was measured between piezometer D and piezometer E, and 20% remained in the E-F section. Bend 5 ends 231 diameters upstream from piezometer F. Therefore, the loss should be entirely contained in the E-F section. Similarly, the loss for bend 6 should be entirely contained in the F-G section because it ends 208 diameters upstream from piezometer G.

By using this assumed distribution of measured bend loss and the measured loss for bend 1 as the base value, bend-loss coefficients were determined for the remaining five bends. Table 5 lists the six bends included in the test sections, together with the deflection angle, the radius, and the pipe diameter of each. Available information concerning bend losses indicates that the value of C_B varies with the deflection angle and the ratio of bend radius to pipe diameter. Relative values of the coefficient in terms of these variables are given by William P. Creager and Joel D. Justin,[7] whose values of K, reflecting the effect of deflection angle, are given in Col. 5 of Table 5. Col. 6 of Table 5 gives values of r/D for each bend, and Col. 7 indicates the corresponding values of the bend-loss coefficient for a 90° bend as recommended by Creager and Justin. The values of C_B for bend 1 through bend 6 are given in Col. 8 and Col. 9 of Table 5. For bend 1 this value was found by doubling the value determined from the second set of bend-loss computations. For bend 2 through bend 6, the value of C_B is the value for bend 1 multiplied by the ratios of appropriate factors in Col. 5 and Col. 7. For example, in 1944

---

[5] "Flow of Water Around Bends in Pipes," by D. L. Yarnell and F. A. Nagler, *Transactions*, ASCE, Vol. 100, 1935, p. 1018.

[6] "Loss in 90-Degree Pipe Bends of Constant Circular Cross-Section," by A. Hofmann, *Bulletin 3*, Transactions of the Hydr. Inst. of the Munich Technical Univ., Germany (1929), A.S.M.E., 1935.

TABLE 5.—BEND-LOSS COEFFICIENTS

Bend No.	Deflection angle	Radius r, in feet	Pipe diameter D, in feet	Deflection factor,[a] K	Ratio, r/D	90° bend-loss coefficient,[a] F	Coefficient, C_B	
							1944	1954
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	34°15′34″	90	18	0.571	5.00	0.28	0.12	0.10
2	15°43′05″	250	18	0.311	13.9	0.45	0.11	0.09
3	18°18′53″	250	18	0.359	13.9	0.45	0.12	0.10
4	12°40′35″	90	16	0.257	5.63	0.30	0.05	0.05
5	4°41′51″	250	18	0.100	13.9	0.45	0.03	0.03
6	8°43′25″	250	16	0.179	13.9	0.45	0.06	0.05

[a] Footnote reference 7.

$$C_{B2} = C_{B1} \times \frac{K_2}{K_1} \times \frac{F_2}{F_1} = 0.12 \times \frac{0.311}{0.571} \times \frac{0.45}{0.28} = 0.11$$

The values for bend 2 and bend 3 given in Table 5 were used to recompute the f-values for reach C-D. The latter values were then used to recompute the loss at bend 1. The final C_B-values are shown in Col. 6 of Table 4 and in Fig. 7. The change from the average values used in Table 5 was not sufficient to warrant any further refinement of f or C_B.

1944 Friction Loss in 18-Ft, Concrete-Lined Tunnel, Apalachia Dam Adit to Apalachia Adit.—Hydraulically between piezometer C and piezometer D, the test section contained 6.617 ft of concrete-lined, 18-ft-diameter tunnel, bend 2 and bend 3, and one half of bend 1. Elevations of the hydraulic grade line at piezometer C and piezometer E and the differential pressures between piezometer D and piezometer E are recorded in Tables 3(*a*) and 3(*b*). Adding the differentials between piezometer D and piezometer E to the hydraulic-grade-line elevations for E gave the grade-line elevations at D. Subtracting these elevations from those at C gave the hydraulic losses between piezometer C and piezometer D.

---

[7] "Hydro-Electric Handbook," by William P. Creager and Joel D. Justin, John Wiley & Sons, Inc., New York, N. Y., 1950, pp. 104–105.

Losses for bends 1, 2, and 3 were computed by Eq. 5 from the values of C_B given in Table 5, and values of f were computed from Eq. 2. The results are summarized in Tables 4(*a*) and 4(*b*) and are plotted in Fig. 6(*c*).

1954 Friction Loss for Normal-Coated Surface, Apalachia Dam Adit to Apalachia Adit.—In 1954 the surface in the section from Apalachia Dam adit to Apalachia adit was normal-coated. Identical reduction procedures were used for the 1954 data as for the 1944 data. Tables 4(*c*) and 4(*d*) summarize the computations, and Fig. 6(*d*) shows the plotted f-values.

1944 Friction Loss in 18-Ft, Concrete-Lined Tunnel, Turtletown Creek Adit to McFarland Adit.—Between Turtletown Creek adit and McFarland adit, the tunnel was largely concrete-lined and 18 ft in diameter, similar to that between Apalachia Dam adit and Apalachia adit. Therefore, it should have the same friction and roughness coefficients. The computations are somewhat more difficult because the test section included not only 6,994 ft of the 18-ft concrete lining but also 118 ft of 18-ft-diameter steel pipe, 598 ft of 16-ft-diameter steel pipe, 60 ft of concrete transitions having an average diameter of 17 ft, one contraction, two expansions, and bend 6. In addition, the tunnel did not have the same diameter at the two adits, so that the measured drop in the hydraulic grade line is not equal to the drop in the energy grade line.

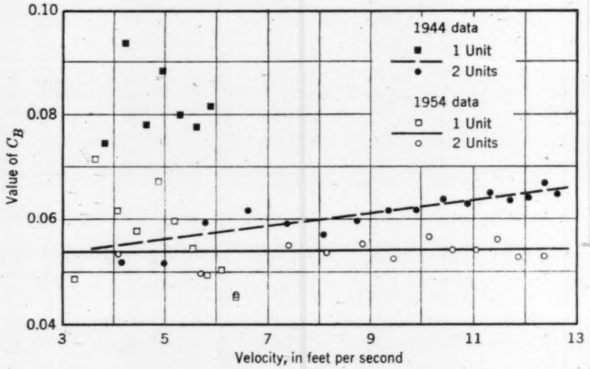


FIG. 7.—BEND-LOSS COEFFICIENTS

The drop in the energy grade line can be determined by adding the velocity head at each piezometer to the elevation of the hydraulic grade line at that piezometer and taking the difference of these sums. Thus, if the energy grade line is represented by E and the hydraulic grade line by H, and if the Turtletown Creek and McFarland adits are denoted by the subscripts T and M, respectively, the drop in the energy grade line is

$$\Delta E = \left(H_T + \frac{V_T^2}{2\,g}\right) - \left(H_M + \frac{V_M^2}{2\,g}\right) \ldots\ldots\ldots\ldots (6a)$$

which can be rewritten as

$$\Delta E = (H_T - H_M) + \left(\frac{V_T^2}{2\,g} - \frac{V_M^2}{2\,g}\right) = (H_T - H_M) + \frac{\Delta V^2}{2\,g} \ldots (6b)$$

The friction loss in the 18-ft, concrete-lined section of the tunnel is the latter value of ΔE less the other listed losses.

Losses in the steel-pipe sections were computed from Eq. 2 by using f-values from Fig. 6(*a*), assuming that f did not vary with the change in diameter. Losses in the transition sections were ignored because reduction of the data for the piezometer section D to E indicated there was no measurable loss due to the transitions. The bend loss was based on C_B-values taken from Table 5 for bend 6. Values of f were computed for the combined concrete-surface sections on the basis of the remaining loss. The computations are summarized in Tables 6(*a*) and 6(*b*), and the results are plotted in Fig. 6(*c*).

1954 Friction Loss for Normal-Coated Surface, Turtletown Creek Adit to McFarland Adit.—The tunnel surfaces in the reach from Turtletown Creek adit to McFarland adit were all normal-coated. The only loss other than friction loss due to this surface roughness was that caused by bend 6. The computed f-value is based on a tunnel section of which 91.5% is 18 ft in diameter, 7¾% is 16 ft in diameter, and 0.75% is 17 ft in diameter. On the basis of the relative-roughness concept advanced[8,9,10] by J. Nikuradse, these values should be slightly larger than those for the section from Apalachia Dam to Apalachia adit.

The total energy loss for the latter reach was computed in the same manner as for the 1944 data, and the losses for bend 6 were based on the C_B-value in Table 5. The computations are summarized in Tables 6(*c*) and 6(*d*), and the results are plotted in Fig. 6(*d*).

1944 Friction Loss in Unlined Rock Tunnel.—Between the Apalachia and Turtletown Creek adits, the tunnel consisted of 3,098 ft of unlined tunnel, 20 ft in nominal diameter; 2,902 ft of unlined rock tunnel, 22 ft in nominal diameter; 7,626 ft of 18-ft, concrete-lined tunnel; 20 ft of concrete-lined transition averaging 17 ft in diameter; and 369 ft of 16-ft steel pipe. Losses were also caused by bend 5, the remaining 0.2 of the loss at bend 4, the expansions from 18-ft concrete to 20-ft rock, the contraction from 20-ft rock to 18-ft concrete, and the contraction from 18-ft concrete to 16-ft steel pipe. The expansion from 20-ft rock to 22-ft rock and the contraction from 18-ft concrete to 16-ft steel were so gradual that losses due to them were neglected.

The energy loss in this section was not equal to the fall in the hydraulic grade line between the piezometers because of the smaller diameter, with correspondingly higher velocity, at piezometer F. The energy loss was determined by adding the difference between velocity heads at piezometer E and piezometer F to the elevation of the hydraulic grade line at piezometer F,

---

[8] "Widerstandsgesetz und Geschwindigkeitsverteilung von turbulenten Wasserströmung in glatten und rauhen Röhren," *Proceedings*, 3d International Cong. on Technical Mechanics, Stockholm, 1930.

[9] "Strömungsgesetz in rauhen Röhren," by J. Nikuradse, *Forschungsheft*, Verein Deutscher Ingenieure, No. 361, 1933.

[10] "Gesetzmässigkeiten der turbulenten Strömung in glatten Röhren," by J. Nikuradse, *Zeitschrift*, Verein Deutscher Ingenieure, Vol. 77, 1933, p. 48.

TABLE 6.—FRICTION LOSS IN 18-FT, CONCRETE-LINED TUNNEL, TURTLETOWN CREEK ADIT TO MCFARLAND ADIT

Gate opening, in percentage	Computed friction losses, in feet, in steel-pipe sections	Computed bend loss, in feet	Net loss in concrete section, in feet	Weisbach's f-factor	Reynolds number, R
(*a*) One unit, January 25, 1944					
50	0.24	0.02	1.16	0.0107	5.1×10^6
55	0.29	0.02	1.37	0.0106	5.6×10^6
60	0.34	0.02	1.77	0.0118	6.0×10^6
65	0.39	0.03	2.00	0.0117	6.4×10^6
70	0.44	0.03	2.23	0.0117	6.7×10^6
75	0.48	0.03	2.33	0.0111	7.1×10^6
80	0.54	0.03	2.57	0.0110	7.4×10^6
(*b*) Two units, January 26, 1944					
25	0.23	0.02	0.86	0.0082	4.8×10^6
30	0.34	0.02	1.40	0.0093	5.8×10^6
35	0.47	0.03	2.33	0.0115	6.7×10^6
40	0.61	0.04	3.32	0.0125	7.7×10^6
45	0.76	0.05	3.85	0.0117	8.6×10^6
50	0.92	0.06	4.77	0.0120	9.4×10^6
55	1.08	0.07	5.60	0.0121	10.2×10^6
60	1.23	0.08	...	...	10.9×10^6
65	1.38	0.09	7.74	0.0126	11.5×10^6
70	1.53	0.10	8.23	0.0126	12.1×10^6
75	1.67	0.11	9.10	0.0127	12.7×10^6
80	1.81	0.12	9.90	0.0128	13.2×10^6
85	1.93	0.13	10.56	0.0127	13.6×10^6
90	2.06	0.14	11.30	0.0128	14.0×10^6
95	2.16	0.14	11.88	0.0128	14.4×10^6
100	2.25	0.15	12.13	0.0126	14.7×10^6
(*c*) One unit, March 28, 1954					
20	...	0.00	0.31	0.0177	1.9×10^6
30	...	0.00	0.50	0.0118	2.9×10^6
40	...	0.01	0.71	0.0095	3.9×10^6
45	...	0.01	0.96	0.0100	4.4×10^6
50	...	0.01	1.24	0.0105	4.9×10^6
55	...	0.02	1.67	0.0117	5.4×10^6
60	...	0.02	2.04	0.0121	5.9×10^6
65	...	0.02	2.41	0.0126	6.2×10^6
70	...	0.02	2.79	0.0128	6.6×10^6
75	...	0.03	3.14	0.0130	7.0×10^6
80	...	0.03	3.42	0.0130	7.3×10^6
85	...	0.03	3.72	0.0128	7.7×10^6
(*d*) Two units, March 27, 1954					
25	...	0.01	1.52	0.0127	4.9×10^6
35	...	0.02	2.88	0.0125	6.8×10^6
40	...	0.03	3.61	0.0124	7.7×10^6
45	...	0.04	4.84	0.0124	8.9×10^6
50	...	0.05	5.81	0.0124	9.8×10^6
55	...	0.06	6.87	0.0124	10.6×10^6
60	...	0.07	7.86	0.0124	11.4×10^6
65	...	0.08	9.03	0.0124	12.2×10^6
70	...	0.09	9.82	0.0124	12.7×10^6
75	...	0.10	10.68	0.0123	13.3×10^6
80	...	0.10	11.59	0.0124	13.8×10^6
85	...	0.11	12.32	0.0123	14.2×10^6
Open	...	0.12	13.37	0.0123	14.9×10^6

and by subtracting the sum from the elevation of the hydraulic grade line at piezometer E. This process is similar to that used in finding the energy loss between Turtletown Creek adit and McFarland adit. The loss in the unlined rock tunnel was found by subtracting from the total energy loss, not only the friction losses in the 18-ft section and 17-ft concrete section and in the 16-ft steel pipe, but also the losses due to bend 4 and bend 5 and the expansions and contractions.

What value of f to use in computing the losses due to the two concrete sections caused some concern because, as can be seen in Fig. 6(c), the f-values for the section from Apalachia Dam adit to Apalachia adit differ materially from those for the section from Turtletown Creek to McFarland. The construction records disclosed that two separate mixing plants were used for the tunnel concrete. A plant at the dam mixed all concrete placed upstream from Turtletown Creek adit, whereas a plant at McFarland adit was used for the sections below Turtletown Creek. The same source of aggregate was used at both plants, and the same type of agitator cars was used to haul the concrete to the pour sites. Identical equipment was used to place the concrete, but different types of batching equipment and mixers were used at the mixing plants. It would not seem that these differences in equipment could have caused the difference in the f-value. However, the 1954 data for these two sections, as shown in Fig. 6(d), produced nearly identical values. Because essentially the same reduction method was used for each set of data, a difference must have existed in the surface characteristics of the original concrete surfaces. Therefore, it was concluded that such a difference did exist and that it must be due to the source of concrete. Because the concrete for the section from Apalachia adit to Turtletown Creek adit came from the dam mixer, the f-values for the concrete in the reach were taken from the Apalachia Dam to Apalachia adit curve shown in Fig. 6(c).

The steel-pipe f-values were taken from the data shown in Fig. 6(a). This assumed that the change in diameter did not affect the f-value. Hydraulic losses due to bend 4 and bend 5 were computed from the C_B-values of Table 5, using 0.2 of the computed loss for bend 4.

The loss due to the enlargement from the 18-ft lined section to the 20-ft unlined section was computed from the equation given[11] by Hunter Rouse, M. ASCE:—

$$H_E = \left[1 - \left(\frac{D_{18}}{D_{20}}\right)^2\right]^2 \frac{(V_{18})^2}{2\,g} \qquad (7)$$

which reduces to

$$H_E = 0.104\,\frac{(V_{18})^2}{2\,g} \qquad (8)$$

At the entrance to the 18-ft lined tunnel, the concrete lining has a 9-in. chamfer, as shown in detail A of Fig. 1. Tests on concrete box culverts with a similar beveled entrance[12] indicate that the loss at the entrance may be expressed in the form of Eq. 5 with a coefficient of 0.15.

---

[11] "Elementary Mechanics of Fluids," by Hunter Rouse, John Wiley & Sons, Inc., New York, N. Y., 1946, p. 265.

[12] "The Flow of Water Through Culverts," *Bulletin No. 1*, Studies in Eng., Univ. of Iowa, Iowa City, 1926, p. 119.

TABLE 7.—FRICTION LOSSES IN UNLINED ROCK TUNNEL

Gate opening, in percentage	COMPUTED FRICTION LOSSES, IN FEET					Computed miscellaneous losses, in feet	Net loss in unlined tunnel, in feet	Weisbach's *f*-factor	Reynolds number, **R**
	18-ft concrete tunnel	17-ft concrete tunnel	16-ft steel pipe	Normal-coated surface	Flaked-coated surface				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(a) One unit, January 25, 1944									
50	1.35	0.00	0.13	...	...	0.06	3.25	0.104	4.0×10^6
55	1.68	0.01	0.16	...	...	0.08	3.87	0.103	4.4×10^6
60	2.00	0.01	0.19	...	...	0.09	4.37	0.100	4.7×10^6
65	2.30	0.01	0.22	...	...	0.10	5.00	0.101	5.1×10^6
70	2.63	0.01	0.24	...	...	0.11	5.58	0.101	5.4×10^6
75	2.92	0.01	0.27	...	...	0.12	6.20	0.102	5.6×10^6
80	3.26	0.01	0.30	...	...	0.14	6.63	0.098	5.9×10^6
(b) Two units, January 26, 1944									
25	1.31	0.00	0.13	...	...	0.06	3.17	0.109	3.8×10^6
30	2.02	0.01	0.19	...	...	0.09	4.18	0.100	4.6×10^6
40	3.74	0.01	0.34	...	...	0.16	7.39	0.100	6.1×10^6
45	4.74	0.02	0.42	...	...	0.20	8.91	0.097	6.8×10^6
50	5.74	0.02	0.51	...	...	0.24	10.71	0.097	7.5×10^6
55	6.76	0.02	0.60	...	...	0.27	12.33	0.096	8.1×10^6
65	8.70	0.03	0.77	...	...	0.35	15.90	0.097	9.1×10^6
70	9.69	0.03	0.85	...	...	0.38	17.50	0.096	9.6×10^6
75	10.60	0.04	0.93	...	...	0.43	19.20	0.096	10.0×10^6
80	11.55	0.04	1.00	...	...	0.46	20.68	0.096	10.4×10^6
85	12.35	0.04	1.08	...	...	0.49	22.07	0.096	10.8×10^6
90	13.15	0.05	1.14	...	...	0.52	22.36	0.095	11.1×10^6
100	14.35	0.05	1.25	...	...	0.57	24.88	0.093	11.6×10^6
(c) One unit, March 28, 1954									
20	...	...	...	0.17	0.01	0.01	0.40	0.098	1.5×10^6
30	...	...	...	0.39	0.03	0.02	1.00	0.100	2.3×10^6
40	...	...	...	0.71	0.06	0.04	1.99	0.112	3.1×10^6
45	...	...	...	0.99	0.07	0.05	2.39	0.105	3.5×10^6
50	...	...	...	1.28	0.09	0.06	2.83	0.101	3.9×10^6
55	...	...	...	1.62	0.11	0.07	3.33	0.099	4.3×10^6
60	...	...	...	1.98	0.13	0.09	3.78	0.094	4.7×10^6
65	...	...	...	2.30	0.14	0.10	4.27	0.094	5.0×10^6
70	...	...	...	2.62	0.16	0.11	4.79	0.093	5.3×10^6
75	...	...	...	2.92	0.18	0.12	5.32	0.093	5.6×10^6
80	...	...	...	3.17	0.20	0.13	5.89	0.094	5.8×10^6
85	...	...	...	3.47	0.22	0.15	6.53	0.095	6.1×10^6
(d) Two units, March 27, 1954									
25	...	...	...	1.30	0.09	0.06	2.79	0.098	3.9×10^6
35	...	...	...	2.79	0.17	0.12	5.00	0.092	5.4×10^6
40	...	...	...	3.45	0.22	0.15	6.44	0.094	6.1×10^6
45	...	...	...	4.61	0.29	0.20	8.71	0.094	7.1×10^6
50	...	...	...	5.56	0.35	0.24	10.47	0.094	7.8×10^6
55	...	...	...	6.55	0.41	0.28	12.27	0.093	8.4×10^6
60	...	...	...	7.50	0.47	0.32	14.15	0.094	9.0×10^6
65	...	...	...	8.63	0.54	0.37	16.11	0.093	9.7×10^6
70	...	...	...	9.39	0.59	0.40	17.56	0.093	10.1×10^6
75	...	...	...	10.25	0.64	0.44	19.08	0.093	10.5×10^6
80	...	...	...	11.03	0.70	0.47	20.69	0.094	10.9×10^6
85	...	...	...	11.82	0.75	0.51	22.14	0.093	11.3×10^6
Open	...	...	...	12.87	0.81	0.55	24.02	0.093	11.8×10^6

It was not possible to determine roughness coefficients for the 20-ft and 22-ft unlined tunnels separately. However, values of f were computed from Eq. 2 by assuming that the same value applies equally to both diameters. In Fig. 6(*e*), f is plotted against a value of R based on the average diameters and areas of the two sections. Values of n and v are also indicated. The computed factors are summarized in Tables 7(*a*) and 7(*b*). Col. 7 of Tables 7(*a*) and 7(*b*) includes the bend, contraction, and expansion losses.

In making the computations for f, the diameter was taken as that of a circle whose area was equivalent to the average area of the section. The hydraulic radius used was one-fourth of the equivalent diameter. This assumption is not quite correct because of the irregularity of the tunnel cross sections. For example, the average hydraulic radii of the 20-ft and 22-ft tunnels as measured from the cross sections are actually 5.36 ft and 5.60 ft instead of the equivalent diameter values of 5.465 ft and 5.855 ft, respectively. Use of the measured values in Eq. 2 produces values that are 2.3% smaller than those given in Tables 7(*a*) and 7(*b*). However, the differences are minor, and, in view of the uncertainty of actual tunnel shapes (which depend on the character of rock and the manner in which it breaks, as well as on the care taken in measuring the irregular sections), it is better to use the simpler assumption of an equivalent circular area and the corresponding hydraulic radius.

1954 Friction Loss in Unlined Rock Tunnel.—The composition of the test reach from Turtletown Creek to McFarland was essentially the same in 1954 to 1944, except that it was necessary to split the steel-pipe sections into two categories because 234 ft were exposed to the sun and became roughened, as did the piezometer A-B section, whereas the remaining 135 ft were covered and therefore were of the normal-coated-surface type.

The total energy loss in the section was obtained in the same manner as in 1944. There was no question as to which values of f to use because the two sections with normal-coated surfaces gave essentially identical results. Therefore, the f-value was taken from the curves of Fig. 6(*d*) for the normal-coated surfaces and from Fig. 6(*b*) for the flaked coating in the 234 ft of exposed steel pipe. Hydraulic losses due to bend 4 and bend 5 were computed from the C_B-values of Table 5. The expansion and contraction losses were computed in the same manner as for the 1944 data, and identical equivalent-area dimensions were used to compute the f-values. Tables 7(*c*) and 7(*d*) indicate the computations, and the f-values are plotted in Fig. 6(*f*).

CONCLUSIONS

Tests on the Apalachia tunnel were made at discharges of between 826 cu ft per sec and 3,210 cu ft per sec. Friction coefficients were determined for five widely different types of surfaces, including steel coated with bituminous paint, concrete placed against steel forms, unlined rock, a fairly uniform deposited coating on the concrete and steel surfaces, and a roughened coating on a steel surface. Discharges were based on a rating curve established by current-meter measurements supported and supplemented by observations of turbine-scroll-case differential pressures.

Although some of the methods used for obtaining the 1954 test data differed in detail from those for the 1944 tests, the data are equally reliable. Both sets of data obtained with single-unit operation were taken on clear or partly cloudy days, so that some slight inaccuracy may have resulted from the solar heating of some of the long connecting lines. The effect should have been more pronounced in 1954 because it was clear all day and because the gages were mounted so that considerable vertical line could be affected. The two-unit tests were performed on rainy days and should not have been affected in such a fashion. Fig. 6 indicates that the apparent spread in the single-unit test data appears to follow this reasoning. In 1944 the single-unit data spread more than the two-unit data, but not so much as the single-unit 1954 data, and, as might be expected, the 1954 two-unit data had little spread.

The two sets of data taken ten years apart cannot be compared because the surfaces were essentially different. However, it is interesting to note from Fig. 6 that the manganese coating resulted in a decrease in friction coefficient for all surfaces except the concrete lining from Turtletown Creek to McFarland, which remained almost constant, and the flaked-coated surfaces in the exposed steel pipes. It is also interesting to note that the shapes of the curves changed. The original concrete and the normal-coated surfaces produced curves that were somewhat comparable with those found in laboratory tests on artificially roughened pipes.[8,9,10,13] The original bitumastic-enamel, painted-steel surface gave a curve with a shape somewhat between an artificially roughened pipe curve and an average commercial pipe curve,[14,15,16] whereas the 1954 flaked coating in the exposed pipe seemed to indicate a commercial-surface type of curve. The wide spread of the single-unit data for this surface is not very serious because errors of from 0.008 ft to 0.010 ft in the observed losses would account for the spread at Reynolds numbers less than 5×10^6. Density variations due to solar heating of the piezometer lines could have resulted in errors of this magnitude.

Bend losses were approximated in order that more accurate computation of the friction losses in the tunnel could be made. These losses were not recomputed after the final determination of friction loss had been made. However, if there had been any change, it would have been negligible. Because these losses are a relatively small part of the total loss, the effect of a small change in the roughness coefficient used in determining them would change them only slightly, and the final effect on the friction coefficients for the tunnel surfaces would be insignificant. Fig. 7 shows that the measured bend loss was essentially the same for both sets of data.

Some question may be raised as to the adequacy of the method used for the bend-loss computations. In order to determine the significance of the bend-loss factor, the data were reduced by using the bend-loss values given by Rudolph Wasielewski.[17] These data give values of C_B for bend 1 through

---

[13] "Experiments with Fluid-Friction in Roughened Pipes," by C. F. Colebrook and C. M. White, *Proceedings*, Royal Soc. of London, Vol. 161, 1937.

[14] "Modern Conceptions of the Mechanics of Fluid Turbulence," by Hunter Rouse, *Transactions*, ASCE, Vol. 102, 1937, p. 463.

[15] "Turbulent Flow in Pipes with Particular Reference to the Transition Region Between the Smooth and Rough Pipe Laws," by C. F. Colebrook, *Journal*, Inst. C. E., London, Vol. 11, 1938–1939, p. 133.

[16] "Evaluation of Boundary Roughness," by Hunter Rouse, *Proceedings*, 2d Hydraulics Conference, Studies in Eng., *Bulletin No. 27*, Univ. of Iowa, Iowa City, 1943.

[17] "Loss in Smooth Pipe Bends with Bend Angles Less Than 90 Degrees," by Rudolph Wasielewski, *Proceedings*, Hydr. Inst. of the Technical College of Munich, Vol. 5, 1932, pp. 53–67 (in German).

bend 6 of 0.064, 0.030, 0.036, 0.025, 0.009, and 0.018, respectively. By using these values it was found that the f-values for the section from Apalachia Dam adit to Apalachia adit were increased approximately 4%, whereas those for the reach from Turtletown Creek to the McFarland adit were increased only 1%. If Wasielewski's data are correct, the 1944 concrete data will show an even greater spread between the two test sections and the 1954 data will have approximately the same spread, but the f-values for the reach from Apalachia Dam to Apalachia adit will be greater. The Nikuradse relationship would indicate that the Turtletown to McFarland reach should have a slightly larger f-value because it has a slightly smaller diameter, due to 7½% of the test reach being 16 ft in diameter. Therefore, the method used is probably more nearly correct, and, in any case, the error due to the bend-loss computation cannot be too great.

The friction coefficient, the pipe diameter, and the size of the surface roughness can be equated on the basis of the von Kármán equation and the Nikuradse experiments—

$$\frac{1}{\sqrt{f}} = 1.74 + 2 \log \frac{r}{k} \qquad (9)$$

in which f is the friction coefficient, r denotes the radius of the pipe, and k is the diameter of the sand grains composing the surface. Eq. 9 is applicable only to the region of complete turbulence in which f remains constant with increasing Reynolds number. Although this condition was not quite attained in the tests, the constant value of f may be estimated by inspecting Fig. 6. Table 8 lists the estimated values of f and r for each type of tunnel surface, together with the computed values of r/k and k.

TABLE 8.—COMPUTATION OF SURFACE ROUGHNESS

Tunnel surface	Friction coefficient, f	Pipe radius r, in feet	Ratio, r/k	Grain diameter, k, in inches
18-ft steel	0.0138	9.0	2,430	0.0445
18-ft concrete				
Apalachia Dam adit to Apalachia adit	0.0137	9.0	2,510	0.0430
Turtletown Creek adit to McFarland adit	0.0128	9.0	3,550	0.0304
22.6-ft rock	0.096	11.3	5.50	24.7
Normal-coated surface	0.0123	8.974	4,360	0.0247
Flaked-coated surface	0.0145	8.974	1,900	0.0565
Coated rock	0.093	11.3	5.89	23.0

Upon inspection in 1944, the interior of the steel pipe was found to be almost glassy smooth, except for the ⅛-in.-high irregularities caused by the method of application of the bitumastic coating. The concrete lining was slightly grainy. Visual estimates of the mean sand grain diameter varied from 0.03 in. to 0.06 in. No accurate estimate of average roughness for the unlined rock can be made, but the value of 24 in. seems reasonable. No good visual determination of surface-roughness height or grain diameters could be made on the coated surfaces because this material was soft and deformed on touch.

ACKNOWLEDGMENTS

The investigations of the friction loss in the Apalachia tunnel were performed under the general direction of Albert S. Fry, M. ASCE, and under the immediate direction of the Hydraulic Laboratory of the Tennessee Valley Authority.

Because the error indicated under the heading, "Description of the Tunnel: Note," was felt to have a significant effect on the previous paper,[2] the authors and the discussers were specifically invited to comment on this paper. Those contacted were Mr. Hickox, Mr. Peterka, Wilbur R. Barrows, Joseph N. Bradley, Edward J. K. Chapman, Stephen H. Haybrook, Julian Hinds, Karl R. Kennison, and Mr. Rouse, Members, ASCE; Gustavus S. Tapley, A.M. ASCE; and Weston Gavett.

None of the foregoing felt that the error altered their previous contributions, and, therefore, they declined to submit any discussion.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2962

THE ENGINEER'S PLACE IN THE SECOND CENTURY OF TECHNOLOGY

ANNUAL ADDRESS AT THE SUMMER CONVENTION
PORTLAND, ORE., JUNE 25, 1958

BY LOUIS R. HOWSON,[1] PRESIDENT, ASCE

It was only one and a half centuries ago that Malthus reached the conclusion that "the power of population is indefinitely greater than the power of the earth to produce subsistence for man." But Malthus failed to foresee the great advances in technology and their effectiveness in increasing both industrial and agricultural output per unit of human effort. The population of the world is now more than three times what it was when Malthus made his prediction, and the people who are living in areas in which technology has made the greatest advances are doing so under conditions that are vastly improved over those of Malthus' day. Countries such as the United States and Canada where technology is furthest advanced are producing surpluses of foods which contribute materially to the welfare of the undeveloped areas of the world.

Technological progress has brought with it, as an integral part, improvement in human welfare through such developments as water purification, sanitation, and cleaner surroundings, which have a direct bearing upon mortality. Labor saving devices and improved, less hazardous working conditions have also contributed toward longer life. As a result the rate of world population growth is rapidly accelerating.

The rate of increase in the last half of the nineteenth century was such that the world's population was doubling each one hundred years. In the first half of the twentieth century, this rate increased to such an extent that the population was doubling in seventy-five years. The United Nations forecasts that in the next thirty years the rate of growth will be such as to double the population in fifty years. Of the accumulated sum of all people born since the world began, it is estimated that one of every twenty is living today.

According to these forecasts, the present world population of 2,700,000,000 will be increased to 6,000,000,000 by the end of this century and from 13,000,000,000 to 15,000,000,000 within one hundred years.

---

[1] Cons. Engr., Alvord, Burdick & Howson, Chicago, Ill.

Such forecasts, of course, are based on the assumption that advanced technology in food, metals, and energy will produce enough for this greatly expanded population. This assumption raises many questions which are of interest to engineers as technologists and as citizens of the world's most advanced and richest country. Among these questions are:

1. Can science and engineering produce the intellectual resources to meet this world-wide growth challenge?
2. Are the natural resources available for such a scale of development?
3. Should engineers simply serve as "developers" in this vast program, or should they enter into the policy-making phases of it through participation in public affairs?
4. Should engineers be concerned with the economic as well as the technical aspects of the world-wide development and our government's participation in it?

England was the first area in the world to shape an industrial economy not dependent wholly on agriculture. There the change began to take place nearly two hundred years ago and was quite definite by 1800.

Industrial progress can be measured by the use of coal and iron. In England by 1850 annual coal production had reached 1 ton and iron, 175 lb per capita. These figures were not equaled in the United States until thirty years later, and it was 1900 before this country used as much coal and iron per capita as England. However, from that time on the United States forged ahead and now produces almost 1,300 lb of steel per capita. At the present time the United States, with about 6% of the world population, is consuming approximately 50% of the world's raw materials.

Annual steel production per capita at the present time is: 1,300 lb in the United States, 700 lb in England, 107 lb in Japan, and 1 lb in India.

About 40% of all steel produced in the world since the beginning of its manufacture has been in the United States and of that amount about two-thirds is still in use. The use of other metals, such as copper, lead, and zinc, has increased about the same as steel.

The energy requirements of a society such as ours substantially parallel the steel in use. The per capita consumption of energy in the United States has doubled in the past fifty years and may double again by the year 2000. Petroleum has now displaced coal as the largest source of energy. United States production of oil in 1955 was five times what it was in 1920, yet it has been predicted (in a paper presented before the Petroleum Branch of the American Institute of Mining, Metallurgical & Petroleum Engineers in February, 1956) that the energy requirements that would be met by petroleum in the United States would increase 55% by 1965.

To bring the entire world to the level of the United States, as indicated by the barometer of tons of steel in use per capita, would require six times the total steel now in use outside the United States. To produce that tonnage of steel would require two hundred years' operation of steel-making capacity equal to that of the United States. Energy and minerals other than iron would be required in comparable quantities. This makes no provision for the 100% population growth of the next fifty years.

It is obvious that such progress cannot be made "overnight"—it is very doubtful that it can ever be attained. Certainly, even to accomplish a small percentage of what the United States has done will require the location of new sources of materials, economically practicable for use, the training of scientists and engineers, and the development of techniques essential to their utilization.

Population growth requires more food. Agricultural technology now developed should make it possible to produce readily the food for double the present population. The difficulties and deficiencies in food supply are largely economic. People in undeveloped countries could not purchase more food without increased income such as is provided by industry. Therefore, technological advancement in agriculture and industry must proceed together in such countries.

The best-fed people are those who live where the greatest technological progress has been made and where their food supply is provided by the smallest number of workers. In the United States the percentage of agricultural workers to all workers has decreased from 64 to 9 in the past one hundred years—and they are now producing food surpluses.

Population growth, increased food requirements, greater demand for steel and other basic materials for industrial expansion, and the necessity of finding new sources of raw materials and economic methods of handling materials more difficult to process all point up the necessity for more attention to the training and development of scientists and engineers. The country that does not value trained intelligence is doomed. Science and engineering are ever moving forward in other countries as well as in the United States. There is no appeal from the judgment of progress. It is believed that in this country the expansion of technical training is in large measure being taken care of by the operation of "supply and demand." In the United States the number of scientists and engineers has increased tenfold in each of the past three fifty-year periods. This has been at a rate from two to five times as rapid as the growth in population. This rate has accelerated in the past fifty years: There was 1 graduate scientist or engineer for every 1,800 people in 1900, decreasing to 1 for each 300 in 1950, and estimated to be less than 1 for every 100 by 1980.

In the ten-year period following the end of World War II, the number of graduates from American colleges receiving Bachelors degrees increased 75%, and those receiving Masters increased 200%. The Doctors increased 340%, and the proportion of engineers and scientists to the total receiving degrees is reported as being relatively constant.

It is believed no artificial stimulus is needed to provide the number of engineers required by our changing and expanding economy.

In recent months considerable hysteria has been manifested following the launching of the first artificial satellite by the Russians. There was quick political and public reaction. Some recommended various types of subsidies for education and any means of getting larger numbers of young people enrolled to study engineering and science. Numbers, particularly if augmented artificially, will not materially alter present conditions under which this country has reached the highest standard of living ever attained. Quality rather than numbers is needed. It is believed that our horizons will be extended most rapidly by making teaching, advanced study, and research more practicable and attractive to those with special aptitudes. This, too, is being accomplished by our free economy, in which industry and government have increased their combined outlay for research, which was $1,000,000,000 in 1941, tenfold since then. In recent years industrial researchers have been given greater latitude to pursue knowledge for its own sake instead of being directed to the improvement of specific products.

Science and engineering always operate as a team. The scientist is concerned with the study of the forces of nature and the discovery of new knowledge. The engineer utilizes this knowledge and the forces and materials of nature for the welfare and advancement of mankind. As scientific discoveries have been made and incorporated into engineering practice, material things have been made to serve higher ends.

Why should this procedure be altered by the atomic and hydrogen bombs and artificial satellites? These developments hold great possibilities for good as well as evil. By their very nature they make us think of wider areas, the whole of mankind, ultimate values, and ultimate uses for betterment of human welfare. Science and technology have always worked for beneficial applications of their developments.

The alternative possibilities of recent technical and scientific advances for good and evil have been well stated by Edgar B. Schieldrop of the University of Oslo, Norway, in a lecture before the Norwegian Engineering Society on November 9, 1956:

> "Modern science and technology rouse in us all a confused mixture of fear and hope. We are forced again and again to face the question of the destiny of the human race in a future as yet only dimly conceived but clearly containing fantastic possibilities of good and evil.
>
> "It hardly needs saying that scientific and technical advances have often been accepted as mixed blessings. What is new in our day is the widespread fear of advance as such. A persistent and universal fear of technical science has never before dominated our thoughts and emotions to such an extent as now.
>
> "We are afraid that the forces we ourselves have created will one day destroy human life leaving our earth to drift on, a lifeless planet, through space. Even if our fears are exaggerated the nightmare is huge enough to retain its paralyzing power.
>
> "It is a most disturbing fact that the techniques of peace have always lagged behind the techniques of war. The two techniques have always been in a state of phase displacement, and the time lag has today grown large enough to seal our fate in this country.
>
> "The process began more than five hundred years ago with the roar of the first cannon. Nevertheless, the cannon with its cylinder-barrel, piston-projectile, explosive mixture and spark is an internal combustion engine. With the invention of the cannon the technique of war thus entered its motor age five hundred years in advance of the technique of peace. Also then the voices were heard that are so familiar today. In 1687 a French journal addressed a plea to engineers of all countries that they should not use gunpowder in the dreadful cannon that would surely sooner or later blow the whole world to pieces. The gunpowder should be taken out of the cannon and put to peaceful use.
>
> "The problem is essentially the same today only more complex, more dramatic and more dangerous."
>
> ***
>
> "One day in 1919 the famous physicist, Ernest Rutherford, was carrying out shooting experiments in his laboratory, shooting on a miniature scale. The particles he was using as projectiles were, to say the least, of small calibre and the nitrogen atom he was aiming at was the tiniest target ever

used on any shooting range. Strangely enough he scored a hit with the depressing result that the nitrogen atom—in spite of its much advertised indivisibility—simply broke in pieces.

"So many things are broken to pieces in this world that an atom more or less should not seem to be of much significance. The end of that nitrogen atom could scarcely be feared to influence the fate of humanity.

"Only twenty-six years later, however, in 1945, the world was shaken by another roar which proclaimed with dramatic force the beginning of a new epoch just as the roar of cannon had done in the fourteenth century. With the Hiroshima bomb the technique of war entered its atomic age, once again in advance of the technique of peace.

"To release the vast store of energy in the atom was not Rutherford's primary aim. His was not the kind of research program that is carried out on behalf of an industry or a special fund. Neither was he experimenting with his atoms and particles as an expert of the Ministry of Fuel and Power. Rutherford was goaded on by the instinct of curiosity. He wanted to find the answer to a question he had posed himself. Could the atom for all its supposed indivisibility be split?

"The discoveries of science follow one upon the other because men are curious, eager for knowledge and gifted. In the wake of the primary discoveries of science the technical advances follow more and more closely. This has the character of an almost uncontrollable automatic process. It is also an irreversible one because a true idea can never die.

"At this critical stage we are bound to ask if the human race with the vast power Techno-Science has placed in its hands really understands how watchful it must be if the world is not to be plunged into a disaster surpassing all our nightmares.

"The outcome depends on self-control, unity of purpose and ethics. A final decisive question emerges: Is the human race with all its qualities of intelligence, spirit and genius on the one hand and its social and ethical instincts on the other, really a species that has a chance of survival in its present environment?

"Should we not then—we men of science and technology in all countries—make every effort to find the means to enable a team of the élite minds among us to prepare a blueprint that in broad but concrete terms will show all mankind what a wonderful place could be made of this planet. At this dramatic juncture we should feel it to be our duty to prevent the world from making its momentous choice blindly. There is in fact an obligation upon us to give to people of all nations a picture of the inspiring alternative offered by modern science and technology that can become a reality if the world is so resolved.

"We scientists and technologists are in duty bound to raise the banner of hope and to hold it aloft as a symbol of the world's ever-dawning life."

The Executive Committee of the Norwegian Society of Engineers, in making copies of this lecture available to others, prepared a condensed summary, which contains the following:

"Modern science and technology have given man power over the forces of nature, have given man the skill and knowledge to build and create, and the world today seems faced with two alternatives for using this power, skill, and knowledge. The one alternative—that of war and destruction—is fairly well known by the general public as a result of numerous statements and constant discussions. The terrible consequences which this alternative represents are kept vividly alive in people's minds.

"Presumably this acts as a kind of deterrent. But at the same time it has had the result that a constantly growing body of opinion in all countries points to the scientists and technicians as the guilty men. It is we who have brought the world to the brink of boundless misfortune.

> "In this situation, engineers and scientists the world over must see it as their duty—a duty to humanity and a debt they owe themselves—to present in some form or other a comprehensible picture of the other alternative. It should be possible to present, soberly and realistically and yet in an interesting way, a picture of what modern technology and science could achieve within the framework of existing possibilities. In other words, we should draft a blueprint of what we could do with this earth of ours if, as expressed in the lecture, 'the world will give us the years that are left of this frightened century for a nobly and constructively conceived expansion.'
>
> "In the profoundest sense this must also imply a moral appeal. The perspectives which are opened up will stimulate hopes and thoughts and endeavors for a worthy purpose in the service of man. This ethical aspect shines all the brighter against the dark background of the other alternative."

Certainly, in the United States there must always be, in the future as in the past, an atmosphere favorable to technological progress. So long as that is maintained I believe America will always be in the forefront of technological developments, and that will afford the greatest assurance of their utilization for human progress.

If it be granted that we have and can maintain and develop the skills and techniques essential to keep this country in the vanguard of technological progress, what are engineers and scientists doing to influence the use of their developments toward beneficial ends and to acquaint the public of what technology has contributed to better living? On this point the engineering profession does not rate so high. Technology is not just a means of displacing labor. It produces more things for all, with less labor and more relaxation for all. In some degree the impression that engineers and scientists have created monsters they cannot control results from scientific aloofness from participation in public affairs. A recent poll of members of the American Society of Civil Engineers by Opinion Research, Inc., disclosed that only 10% of the Society's members participates actively, and less than 50% participate at all, in local, civic, and community affairs. Yet engineering training deals with exact sciences. Engineers perform many public functions, and their work is essential to the operation of our public economy. Their habits of accurate thinking and precise analysis promote intellectual integrity and make for truth and conscience, characteristics which can contribute to community as well as individual living. As a profession we are failing to accept and discharge our responsibilities in American society.

The solution of most engineering problems involves economic studies of alternative methods. In 1929, engineering was described by Anson Marston, Past President, ASCE, as follows:

> "Engineering projects begin as a dream; whether and when they progress beyond that stage depends in many cases upon economic practicability and political expediency. Projects originally uneconomic may later become practical. There will always be a great many projects awaiting development—from which it is the Engineers responsibility to select 'first things first.'"

Most civil engineering projects affect human welfare.

Certainly, members of a profession trained to clear objective thinking, with a knowledge of the exact sciences and a record of their application to the advancement of human progress, should be interested in public affairs. It is not enough that their works influence economic and social conditions. Credit for that frequently goes to the politicians rather than to the engineer whose dreams, designs, execution, and ingenuity made possible a development contributing to higher standards of living. As engineers fail to communicate to others the glamor of their professional work, they lose deserved recognition and lessen their opportunity to serve humanity.

Our competitive economy and the advances which make it succeed are largely the result of engineering achievement.

Where can engineering training and experience be better utilized and produce greater results in the public interest than by participation in public affairs at all levels?

Does not the truly professional engineer have a responsibility to participate in public affairs?

Some engineering societies, including ASCE, are interested in national affairs along more or less restricted lines, but individual members are largely shirking their civic responsibilities and complaining about the status of engineers and the public relations efforts of the Engineering Societies. Public relations are but the composite of many individual human relations. The Engineers Joint Council (EJC) has recently adopted a concise definition of public relations as: "Good Performance, publicly appreciated because adequately communicated." The first and last parts of that definition, from which the other part, "publicly appreciated," results, are in our power and are an individual responsibility.

It is an engineering responsibility to participate in public affairs at all levels and to see to it that the public knows of the profession's contributions. From that knowledge will develop a better appreciation of the profession.

Technical progress recognizes no political boundaries. Engineers have contributed toward the shrinkage of distances and space which has so vitally affected world-wide contacts and economics. Engineers with their analytical approach to problems and their training in the evaluation of alternative procedures have a background valuable in directing into the best channels public expenditures both at home and abroad. The activity of the EJC in its two reports on a "National Water Resources Policy" is illustrative of the application of technical study to governmental expenditures. Such contributions should be more frequent and eventually receive more consideration by the government.

In summary it is believed technology can and will meet the challenges of the present and future; that the developments of science and engineering should and will be devoted primarily to beneficial uses rather than to destruction; that engineers should assume their responsibilities in presenting in understandable terms the great possibilities inherent in utilization of our expanded techniques for humanitarian ends; and that engineers should participate and thus be recognized in public affairs so as to guide in the programming of development along sound economic lines.

Engineering and science have brought our civilization to its present stature. They have a responsibility to participate in seeing to it that it is used for creative rather than destructive ends.

MEMOIRS OF DECEASED MEMBERS

ABSTRACTS

Honorary Member

Frederick Ohrt 1285

Other Members

Elmer Ellsworth Adams 1286
Benjamin Franklin De Barros Barreto 1286
Raymond Wentworth Brooks 1287
Roy Dayton Burdick 1288
Chesley Allen Chipley 1288
Lothrop Crosby 1289
Arthur Donald Edmonston 1290
Olaf John S. Ellingson 1290
Farley Gannett 1291
Jay Ankeny Given 1292
Allen Storr Hackett 1292
George Foster Harley 1293
Carl Hinshaw 1294
Horace Sinclair Hunt 1294
Joseph Maurice Kane 1295
Paul Cage Klyce 1296
Robert Talbot Knapp 1296
Saul Landman 1297
Thomas R. Lawson 1298
Edwin Andrew McDougle 1298
Clyde Myers 1299
Alden Gallup Roach 1300
Delbert Walton Robinson 1300
Harry Stanley Rogers 1301
William Japhia Schlick 1302
Irven Clarence Shafer 1302
Hymen Shifrin 1303
Lowell O. Stewart 1304

Walter Pearce Stine 1304
Paul Lloyd Stuenkel 1305
Donald Wood Taylor 1306
Herbert Albert Van Der Goot 1307
William James Van London 1307
William Horace Williams 1308
William Munsey Wilson 1309
John Stephen Worley 1310

FREDERICK OHRT, HON. M. ASCE[1]

DIED MARCH 13, 1957

Frederick Ohrt, the son of Peter and Bessie (Borba) Ohrt, was born on May 28, 1889, in Spreckelsville, Maui, Hawaii. After attending St. Louis College (Honolulu) and the University of Oregon (Eugene), he was graduated from Cornell University at Ithaca, N. Y., with a degree in civil engineering in 1911. In 1918 he attended the Harvard-M.I.T. School for Health Officers in Cambridge, Mass. In 1952 he received the degree of Doctor of Science from the University of Hawaii (Honolulu).

From 1912 to 1913 he was employed by the Madeira-Mamore Railway in Brazil. When he returned to Hawaii, he became resident engineer of the Waiahole Tunnel Project in Honolulu. In the period from 1915 to 1917 he held the offices of highway inspector and assistant city engineer. His next term of service was spent as sanitary engineer for the Board of Health of the Territory of Hawaii, and from 1919 to 1923 he was chief engineer of the city and county of Honolulu.

In 1924 he joined the firm of Libby, McNeill and Libby as chief engineer engaged in harbor development on the island of Molokai. In 1925 Mr. Ohrt was appointed chief engineer for the Honolulu Sewer and Water Commission. From 1929 to 1952, when he retired to accept an appointment as a trustee of the Estate of James Campbell (Honolulu), he served as manager and chief engineer of the Board of Water Supply. From 1952 until his death, Mr. Ohrt contributed greatly to the improvement and development of the Campbell estate's land holdings, the second largest in Hawaii. In his capacity as civil engineer, he was responsible for the establishment of Honolulu's interceptor sewer system and the transformation of the city's water plant and system from a series of unrelated and inadequate units into a coordinated, efficient system.

Mr. Ohrt was a member of the American Water Works Association and the New England Water Works Association. He served on the Board of Directors and was a past president of the Engineering Association of Hawaii; he was a trustee of the Territorial Employees' Retirement System from 1936 to 1945 and from 1948 to 1957; and he served as chairman of the Territorial Retirement and Pension Commission from 1946 to 1949. Mr. Ohrt was a member of three labor mediation boards; he was elected a delegate to the Hawaii State Constitutional Convention in 1950; and he served on the Territorial Salary Standardization Board from 1951 to 1953. He took an active part in the Pacific Club, the Outrigger Canoe Club, and the Oahu Country Club, and was also a member of Phi Gamma Delta.

Mr. Ohrt was elected a Junior of the Society on February 6, 1912, an Associate Member on October 8, 1918, and a Member on October 8, 1951. He was elected an Honorary Member on June 12, 1951.

NOTE.—Complete manuscripts have been filed in the Engineering Societies' Library, 29 West 39th Street, New York, N.Y., and are available for consultation. Many of these memoirs have been considerably abbreviated for printing in *Transactions*.

[1] Abstract of memoir prepared by Edward J. Morgan, M. ASCE.

ELMER ELLSWORTH ADAMS, M. ASCE [1]

DIED APRIL 14, 1957

Elmer Ellsworth Adams, the son of Abraham and Lydia (Bennett) Adams, was born on May 10, 1883, in Brainard, Minn. He was graduated from the University of Minnesota (Minneapolis) with the degree of Bachelor of Civil Engineering in 1906.

Upon graduation Mr. Adams entered the employ of the Great Northern Railway as instrumentman and head of party on the expansion of a branch line in North Dakota. In 1907 he was assigned the position of office engineer in Seattle, Wash., and was responsible for line improvements in that area. In 1921 Mr. Adams was appointed district engineer at the Great Falls (Mont.) headquarters. When he was transferred to Spokane, Wash., in 1925, he took part in the construction of many main-line track relocation programs. In 1932 he was transferred to Seattle and, in 1939, to Duluth, Minn., as district engineer, where he remained until his retirement in 1953.

During World War II he played an important role in the iron ore industry by planning for the continued flow of this vital war material. Mr. Adams was a member of Delta Upsilon.

In 1909 he was married to Marion Frost in New York, N.Y. He is survived by two sons, Edward F. and John B.; two brothers; a sister; and six grandchildren.

Mr. Adams was elected an Associate Member of the Society on April 7, 1915, and a Member on October 11, 1920. He became a Life Member in 1950.

BENJAMIN FRANKLIN DE BARROS BARRETO, A.M. ASCE [2]

DIED JANUARY 9, 1958

Benjamin Franklin de Barros Barreto, the son of Ignacio and Maria Thereza (Cavalcanti) de Albuquerque de Barros Barreto, was born in Recife, Pernambuco, Brazil, in February 26, 1898. He was graduated from Cornell University (Ithaca, N.Y.) in 1922 with the degree of Civil Engineer.

Mr. Barros Barreto began his professional career in northeastern Brazil, where he worked on drought projects. Following this he helped construct sewage and water-supply networks for the city of Fortaleza, Brazil. In February, 1925, he joined the Hydroelectric Construction Department of the São Paulo (Brazil) Light Company, becoming superintendent of the Concessions Division in 1950.

---

[1] Abstract of memoir prepared by John B. Adams, A.M. ASCE.

[2] Abstract of memoir prepared by Leonor B. Barros Barreto.

Mr. Barros Barreto collaborated on many company projects, including the extensive Cubatão plants, the Nilo Peçanha underground plant, and the Paraíba-Piraí diversion. Since 1926 he devoted much of his time to the study of the utilization of the hydraulic resources of the Paraíba Valley, a subject of vital importance to Brazil.

Mr. Barros Barreto was President (1953–1956) of the Brazilian Section of the Society. He also held membership in the Engineering Institute of São Paulo and the Engineering Club of Rio de Janeiro, Brazil.

In 1924 he was married to Leonor Cordeiro. He is survived by his widow; two sons, Carlos Cordeiro and Claudio; and two daughters, Vera Maria and Lucia Maria.

Mr. Barros Barreto was elected an Associate Member of the Society on June 10, 1929.

RAYMOND WENTWORTH BROOKS, A.M. ASCE [1]

DIED JANUARY 7, 1958

Raymond Wentworth Brooks, the son of John Pascal and Maude (Pepperill) Brooks, was born on August 30, 1889, in Red Bank, N. J. He was graduated from the University of Illinois (Urbana) in 1911 with the degree of Bachelor of Science in civil engineering.

After preliminary railroad experience, Mr. Brooks was associated (1915–1920) with the Interstate Commerce Commission in Kansas City, Mo., as Chief of Roadway Party for the Bureau of Valuation. Following this he was employed by the Missouri State Highway Commission as project engineer on highway construction (1921–1928), and chief designer (1929–1942) in the Bureau of Surveys and Plans at Jefferson City, Mo. From June, 1942, to December, 1942, he was chief specification engineer with the Widmer Engineering Company. From 1943 to 1949 he worked for the office of the United States Engineer, Corps of Engineers (United States Department of the Army), supervising final airfield installation plans. From 1949 until his death, he was associated with the firm of Black & Veatch, Consulting Engineers, of Kansas City.

Mr. Brooks was a member of the Missouri Society of Professional Engineers, the National Society of Professional Engineers, and the Engineers Club of Kansas City. He was also a member of the A.F. & A.M., a Shriner, and was active in St. George's Episcopal Church.

On June 28, 1918, he was married to Nettie Hester in El Paso, Tex. He is survived by his widow and a sister.

Mr. Brooks was elected a Junior of the Society on January 2, 1912, and an Associate Member on March 11, 1919. He became a Life Member in 1954.

---

[1] Abstract of memoir prepared by Arlow V. Ferry, M. ASCE.

ROY DAYTON BURDICK, M. ASCE [1]

DIED MARCH 2, 1954

Roy Dayton Burdick, the son of Lee and Rosemary Burdick, was born on June 21, 1892, in Homer, N. Y. He was graduated from Cornell University in Ithaca (N. Y.) with the degree of Bachelor of Science in civil engineering in 1914.

He was a captain in the Corps of Engineers, United States Army, during World War I and retired from World War II with the rank of colonel.

Among the various stations in the Corps at which he served were the office of the Assistant Secretary of War, and the Engineer Districts in Honolulu (Hawaii), Philadelphia (Pa.), Memphis (Tenn.), and Little Rock (Ark.). At Memphis, Mr. Burdick was chief of operations during the 1937 flood on the Mississippi River and was in direct charge of the flood fight at Cairo, Ill. At the time of his retirement (1946), he was district engineer of the Little Rock District.

During most of his thirty years of service with the Corps, Mr. Burdick was actively engaged in the civil works program. This interest caused him to remain in Little Rock after retirement to be an engineering consultant on dams, bridges, and flood-control matters.

Mr. Burdick was a member of the Little Rock Engineers' Club, the Arkansas Engineers' Club, the National Society of Professional Engineers, the Rotary Club, and certain civic groups.

On May 10, 1942, at West Palm Beach, Fla., he was married to Jewel T. Puckett. He is survived by his widow; a daughter, Mary Jean (Mrs. J. S. Dillahunty); a brother; a sister; and several grandchildren.

Mr. Burdick was elected a Member of the Society on November 18, 1946.

CHESLEY ALLEN CHIPLEY, A.M. ASCE [2]

DIED APRIL 15, 1956

Chesley Allen Chipley, the son of Henry and Nan (Goldsborough) Chipley, was born on October 20, 1904, in Eufaula, Okla. He was graduated from Texas Agricultural and Mechanical College (College Station) with the degree of Bachelor of Science in civil engineering in 1926.

After preliminary experience, he worked for the Texas Highway Department (1930–1939), R. W. Briggs and Company (1939–1946), and, later (1946–April,

---

[1] Abstract of memoir prepared by Charles A. Long, A.M. ASCE.

[2] Abstract of memoir prepared by Francis M. Davis, Mance R. Mitchell, and Willard E. Simpson, Sr., Members, ASCE.

1956), as vice-president and general manager for the Fordyce Gravel Company. He was also president of the Victoria Ready-Mix Concrete Company, Inc. (1949–1956), and a director in the National Sand and Gravel Association (1952–1956).

Mr. Chipley was a member of numerous engineering societies and of the Texas Society of the Sons of the American Revolution. He was a director in The Aggie Club, a steward in the Alamo Heights Methodist Church, and an ardent student of the history of the American frontier.

Mr. Chipley rendered memorable service to the San Antonio Branch of the Texas Section of the Society in helping to promote student interest and participation in all professional activities, both social and technical.

On December 17, 1939, at Stephenville, Tex., he was married to Lucile Hearon. He is survived by his widow and three daughters, Nan Elizabeth, Sue Rivers, and Jane Lucile.

On February 15, 1937, Mr. Chipley was elected an Associate Member of the Society.

LOTHROP CROSBY, A.M. ASCE[1]

DIED MAY 6, 1957

Lothrop Crosby, the son of Benjamin Lincoln and Carrie Elizabeth Lothrop (Ames) Crosby, was born on March 30, 1883, in Bismarck, N. Dak. He was graduated from the Sheffield Scientific School of Yale University, New Haven, Conn., with the degree of Bachelor of Philosophy in Mining Engineering in 1904.

After graduation Mr. Crosby was employed by the United States Reclamation Service, the Northern Pacific Railway, and Lyman E. Bishop, consulting civil and hydraulic engineer of Denver, Colo. After serving as chief engineer for the Idaho Irrigation Company of Richfield, Idaho, he entered private practice for a short time as an irrigation consultant.

His employment with the City of Tacoma (Wash.) Water Division began in 1923. He became supervisor of the construction and development of the Green River gravity water system, and he was chief assistant to the superintendent of the division in the development of the city's well system.

Mr. Crosby served in World War I as a first lieutenant and was active in many fraternal, civic, and professional societies. He was President of the Tacoma Section of the Society in 1942.

On June 21, 1906, he was married to Annalaura Terrel Rhoades in St. Joseph, Mo. He is survived by his widow; a son, Benjamin L.; a daughter, Elizabeth (Mrs. Oliver Hatch); a brother; six grandchildren; and three great-grandchildren.

Mr. Crosby was elected an Associate Member of the Society on September 11, 1917. He became a Life Member in 1952.

---

[1] Abstract of memoir prepared by Willibald A. Kunigk, M. ASCE.

ARTHUR DONALD EDMONSTON, M. ASCE[1]

DIED FEBRUARY 22, 1957

Arthur Donald Edmonston, the son of Donald and Margaret (McCombie) Edmonston, was born on November 12, 1886, in Ferndale, Calif. He was graduated from Stanford University (Calif.) with the degree of Bachelor of Arts in Civil Engineering in 1910.

During the first fourteen years of his engineering career, he was employed on the location, design, and construction of irrigation, hydroelectric, and municipal water projects in California, with time out to serve as a second lieutenant of Engineers in World War I.

He entered the service of the state of California in 1924 and soon thereafter became deputy state engineer in charge of investigations of the state water resources. In 1945 he became assistant state engineer and, in 1950, state engineer, in charge of water-resources developments, water rights, safety of dams, water quality, flood control, and beach erosion. As state engineer he was a member or executive officer of many commissions. He was also a member of professional and honorary societies. He retired on November 2, 1955.

Mr. Edmonston was largely responsible for the formulation of the State Water Plan of 1931, including the Central Valley project, and originated and directed the studies for the California Water Plan. He also conceived and directed the planning of the Feather River Project.

In 1923 he was married to Dell Loveless of Marysville, Calif. He is survived by his widow; two sons, Robert M. and Donald A.; a sister; and a brother.

Mr. Edmonston was elected an Associate Member of the Society on September 10, 1918, and a Member on October 16, 1944. He became a Life Member in 1953.

OLAF JOHN S. ELLINGSON, A.M. ASCE[2]

DIED AUGUST 30, 1956

Olaf John S. Ellingson, the son of Elling A. and Christi Ellingson, was born on March 31, 1884, in McPherson, Kans. He was graduated from the University of Texas (Austin) in 1906 with the degree of Bachelor of Science in civil engineering.

In 1908 he became construction engineer for the Midland Bridge Company of Kansas City, Mo. In 1915 he was city engineer for Sherman, Tex., until 1916 when he was appointed city manager, in which capacity he served until

---

[1] Abstract of memoir prepared by Gerald H. Jones, Percy H. Van Etten, and Thomas B. Waddell, Members, ASCE.

[2] Abstract of memoir prepared by Eugene A. Ellingson.

1932. In 1931 he was named president of the Texas City Manager's Association.

In 1932 he joined the Texas Prison System as assistant general manager of construction. After he was appointed general manager in 1935, Mr. Ellingson directed the largest building program in the history of the system. He was also responsible for the first automobile license-plate plant in Texas being built at the system's main unit in Huntsville, Tex.

During World War II he worked for the War Production Board in San Antonio, Tex. From 1945 to 1950 he served as city manager of Brownsville, Tex., Key West, Fla., and Edinburg, Tex.

Mr. Ellingson was a member of the Texas Section of the Society as well as of many other professional and civic organizations. He was a 32nd degree Mason and a member of the Presbyterian Church.

On October 30, 1912, Mr. Ellingson was married to Enid Smith. He is survived by his widow; two sons, Jack and Eugene A.; and three grandchildren.

He was elected an Associate Member of the Society on October 29, 1912, and became a Life Member in 1947.

FARLEY GANNETT, M. ASCE[1]

DIED JANUARY 20, 1958

Farley Gannett, the son of Henry and Mary (Chase) Gannett, was born in Washington, D.C., on May 6, 1880. He was graduated from the Massachusetts Institute of Technology (Cambridge) with the degree of Bachelor of Science in Sanitary Engineering in 1902.

From 1905 to 1915 he served as chief engineer of the Water Supply Commission of Pennsylvania, and in 1915 organized the firm of Gannett, Seelye, and Fleming, which became the firm of Gannett, Fleming, Corddry, and Carpenter. At the time of his death he was chairman of the Board of Directors. The record of Mr. Gannett's accomplishments is synonymous with the engineering firm he founded.

Mr. Gannett was a member of the American Road Builders Association; the American Water Works Association; the Pennsylvania Society of Professional Engineers; the Engineers Society of Pennsylvania; the Cosmos Club of Washington, D.C.; the Beaufort Hunt Club of Harrisburg, Pa.; and he was a director of the Cayuga Rock Salt Company in Myers, N. Y.

On June 14, 1905, he was married to Janet Rand Sanders in Haverhill, Mass. He is survived by his widow; three daughters, Muriel (Mrs. William R. Jones), Jane (Mrs. Gannett Behney), and Alice (Mrs. George R. Booth); a sister; seven grandchildren; and two great-grandchildren.

Mr. Gannett was elected an Associate Member of the Society on April 4, 1906, and a Member on May 15, 1917. He became a Life Member in 1955.

---

[1] Abstract of memoir prepared by James D. Carpenter, M. ASCE.

JAY ANKENY GIVEN, M. ASCE[1]

DIED NOVEMBER 13, 1956

Jay Ankeny Given, the son of Welker and Maude (Ankeny) Given, was born on July 5, 1883, in Peoria, Ill. He was graduated from Iowa State College (Ames) with the degree of Civil Engineer in 1908.

While an undergraduate, he served an apprenticeship with the Rock Island and Pacific Railroad and after graduation began his railroading career with the Union Pacific Railroad. In June, 1909, he entered the service of the Southern Pacific Railroad in Sacramento, Calif., as a draftsman and assistant engineer. In October, 1910, he became assistant division engineer of the Sacramento Division.

In June, 1917, Mr. Given entered the United States Army as Major and served in France.

In December, 1918, he returned to the Southern Pacific Railroad as division engineer of the Shasta Division in Dunsmuir, Calif. In November, 1937, Mr. Given became district engineer and acted as the representative of the railroad in connection with the relocation of the San Francisco-Portland main line. In April, 1943, he moved to San Francisco, Calif., where, as location division engineer, he engaged in surveys for line changes.

After his retirement in February, 1947, he became a consultant to the Corps of Engineers, United States Department of the Army.

On April 23, 1911, he was married to Eva Cecil Cunningham in Sacramento. He is survived by his widow.

Mr. Given was elected a Member of the Society on July 9, 1923. He became a Life Member in 1954.

ALLEN STORR HACKETT, M. ASCE[2]

DIED APRIL 23, 1957

Allen Storr Hackett, the son of John Alexander and Anna (Storr) Hackett, was born on May 30, 1880, in Shreveport, La. He was graduated from Tulane University at New Orleans, La., in 1903 with the degree of Bachelor of Engineering in civil engineering.

From 1903 to 1904 he engaged in railroad location, construction, and maintenance with the Southern Pacific Railroad in Texas and the New Orleans and Northeastern Railroad in Mississippi. He later became associated with Vaccaro Brothers and the Quiamel Fruit Company in Honduras, Central America, as location and construction engineer.

---

[1] Abstract of memoir prepared by William F. Turner, M. ASCE.

[2] Abstract of memoir prepared by Charles M. Kerr, A.M. ASCE, Charles S. Williamson, Jr., and Allan T. Dusenbury, M. ASCE.

During World War I he served in the 23rd Regiment of Engineers in the United States Army in France with the rank of captain. After the war until 1923, he and a partner, Marcel Garsaud, engaged in general contracting. In 1927, with the late George A. Hero, he organized the New Orleans-Gretna Bridge Company and sponsored the construction of a bridge across the Mississippi River, using his patented spiral approach.

Mr. Hackett was president of the Louisiana Engineering Society in 1936, and became a life member of that society in 1951.

In 1916 he was married to Madeleine Bourne of Cleveland, Ohio. He is survived by his widow; a son, Richard A.; four daughters, Betsey (Mrs. Albert B. Paterson, Jr.), Mary (Mrs. Harold V. Cummings), Ruth (Mrs. John C. East), and Jane (Mrs. Edward P. Munson, Jr.); a sister; and two brothers.

Mr. Hackett was elected a Member of the Society on May 12, 1930. He became a Life Member in 1955.

GEORGE FOSTER HARLEY, M. ASCE[1]

DIED JUNE 22, 1957

George Foster Harley, the son of James Alexander and Anne (Toombs Pierce) Harley, was born on January 6, 1875, in Sparta, Ga. He attended Mercer University, Macon, Ga., and was graduated from George Washington University, Washington, D.C., with a degree in civil engineering in June, 1907.

From 1903 to 1905 he worked for the United States Geological Survey, and from 1906 to 1907, for the United States Bureau of Reclamation. From 1908 to 1911 he was employed by J. G. White & Company of New York (N.Y.). From 1911 to 1931 he was associated with Stone and Webster of Boston (Mass.) and New York as superintendent of construction and as district engineer.

In 1933 Mr. Harley became senior engineer in the Power Division of the United States Public Works Administration in Washington. In 1939 he moved to Austin (Tex.) to undertake construction for the Lower Colorado River Authority. In 1941 he became regional director of the United States War Public Works Eighth Region in Fort Worth (Tex.). He then served as regional consulting engineer until his retirement in 1945.

Mr. Harley was a member of Sigma Nu and of St. Peter's Episcopal Church in Kerrville (Tex.).

On April 16, 1917, he was married to Roberta Slade in Columbus, Ga. He is survived by his widow; a daughter, Mrs. Francis M. Eagle; a son, George F., Jr.; a brother; a sister; and four grandchildren.

Mr. Harley was elected an Associate Member of the Society on January 3, 1911, and a Member on April 13, 1942. He became a Life Member in 1948.

---

[1] **Abstract of memoir prepared by Uel Stephens, M. ASCE.**

CARL HINSHAW, M. ASCE[1]

DIED AUGUST 5, 1956

Carl Hinshaw, the son of William and Anna (Williams) Hinshaw, was born on July 28, 1894, in Chicago, Ill. He was graduated from Princeton University in Princeton (N. J.) as a civil engineer in 1916. He received the degree of Master of Business Administration in 1917 from the University of Michigan (Ann Arbor).

After service as a captain in the United States Army, employment as a laborer, and experience in various managerial capacities, Mr. Hinshaw entered the investment-banking field (1927). In 1929 he moved to Pasadena, Calif., and entered the real estate and insurance business. In 1934, as president of the Pasadena Realty Board, he spearheaded the construction of the Arroyo Seco Parkway of Los Angeles (Calif.) County.

In 1938 Mr. Hinshaw was elected to the Seventy-Sixth Congress and reelected to succeeding Congresses from the Twentieth Congressional District until his death. He received the Citation of Honor from the Air Force Association (1948) and the Wright Brothers Memorial Trophy from the National Aeronautic Association (1953).

Mr. Hinshaw was a member of numerous professional and scientific societies and an adviser to the Radio Technical Commission for Aeronautics, director of the National Aeronautic Association, and president of the Aero Club of Southern California (1953–1954).

On January 1, 1932, Mr. Hinshaw was married to Wilberta Ripley. His first wife, Helen Veeder, died in 1929. He is survived by his widow and two sons, John and William.

He was elected an Associate Member of the Society on September 13, 1943, and a Member on January 12, 1948.

HORACE SINCLAIR HUNT, M. ASCE[2]

DIED NOVEMBER 10, 1956

Horace Sinclair Hunt, the son of Charles P. and Anna (Sinclair) Hunt, was born on October 10, 1883, in Jackson, Mich. He was graduated from Michigan State University (East Lansing) with the degree of Bachelor of Science in civil engineering in 1905.

After graduation, Mr. Hunt was employed as resident engineer by the Fargo Engineering Company of Jackson. From 1910 to 1913, after taking leave from the company, he served as superintendent of construction for the Eastern Michigan Power Company in Grand Rapids, Mich. In 1913 he was employed by the Detroit (Mich.) Edison Company. In 1914 Mr. Hunt returned to the Fargo Engineering Company as a member of the firm. He became its president in 1925 and continued in that capacity until his death.

---

[1] Abstract of memoir prepared by Frank L. Weaver, M. ASCE.

[2] Abstract of memoir prepared by Charles A. Hunt, A.M. ASCE.

During his lifetime he helped design and construct more than forty hydroelectric plants. As a result of his extensive experience in the power and river-development fields, he served as a consultant to the Tulsa (Okla.) and Vicksburg (Miss.) districts of the Corps of Engineers (United States Department of the Army), as well as to the Tennessee Valley Authority, the Lower Colorado River Authority of Texas, and the Central Nebraska Public Power and Irrigation district. During World War I he was director of conservation for Michigan under the United States Fuel Administration.

Mr. Hunt was a member of many professional and civic organizations, as well as of St. Paul's Episcopal Church of Jackson.

On June 20, 1912, Mr. Hunt was married to Mary Cecile Alden in Grand Rapids. He is survived by his widow; two sons, Horace S., Jr., and Charles A.; a daughter, Florence (Mrs. Burdette Peterson); a brother; and seven grandchildren.

Mr. Hunt was elected an Associate Member of the Society on October 7, 1914, and a Member on April 14, 1919. He became a Life Member in 1949.

JOSEPH MAURICE KANE, A.M. ASCE[1]

DIED JANUARY 23, 1957

Joseph Maurice Kane, the son of James and Jane (Jones) Kane, was born in Genoa, Nev., on April 29, 1892. He was graduated from the University of California (Berkeley) with the degree of Bachelor of Science in engineering in 1918.

After service in the United States Army (World War I), Mr. Kane was employed as a draftsman and testing engineer with the Nevada Department of Highways in Carson City, Nev. He subsequently became chief testing engineer, chief draftsman, and office engineer for this department.

Leaving Nevada in 1927, Mr. Kane worked for the California Division of Highways. He directed the preparation of the "Standard Specifications," which govern the execution of all highway construction by or for the division, and have served as a model for other organizations and an extensive reference for state and federal agencies for more than a quarter of a century. Mr. Kane was cited (1949) by the American Association of State Highway Officials for his accomplishments in highway engineering. During his twenty-nine-year career with the California division, he supervised the preparation of job specifications on more than two billion dollars' worth of highway construction. He retired (1956) as assistant office engineer in the highway division.

---

[1] Abstract of memoir prepared by Richard H. Wilson, M. ASCE, Richard R. Norton, and Edwin W. Warmby, Associate Members, ASCE.

On August 26, 1927, at Sacramento, Calif., he was married to Doris Thomas Kane. He is survived by his widow and two sisters.

On August 31, 1925, Mr. Kane was elected an Associate Member of the Society.

PAUL CAGE KLYCE, A.M. ASCE[1]

DIED APRIL 5, 1956

Paul Cage Klyce, the son of William Henry and Mary Elizabeth (Winniford) Klyce, was born in Woodbury, Tenn., on August 4, 1897. He received the degrees of Bachelor of Arts (1920) and Bachelor of Engineering (1921) from Vanderbilt University in Nashville (Tenn.).

Mr. Klyce's early experience included employment as resident engineer on the design and construction of municipal projects, consulting engineer, test engineer, and assistant engineer on road construction. Between 1929 and 1933, he was city engineer for Laurel, Miss. For more than twenty years (1933–1956) he served as office engineer in the Maps and Surveys Branch of the Water Control Planning Division of the Tennessee Valley Authority in Chattanooga.

Among the various professional and civic groups in which he was a member are the American Society of Municipal Engineers, the American Road Builders Association, the Chattanooga Engineers' Club, the Elks Club, and the Tennessee Valley Section of the Society.

On October 22, 1925, at Biloxi, Miss., Mr. Klyce was married to Mary Proctor Fleming. He is survived by a son, Paul C.; a daughter, Mary (Mrs. Robert G. Isibel); two brothers; a sister; and two grandchildren.

Mr. Klyce was elected a Junior of the Society on January 18, 1926, and an Associate Member on June 7, 1926.

ROBERT TALBOT KNAPP, M. ASCE[2]

DIED NOVEMBER 7, 1957

Robert Talbot Knapp, the son of Herman and Almira S. (Talbot) Knapp, was born on January 5, 1899, in Loveland, Colo. He was graduated from the Massachusetts Institute of Technology (Cambridge) with the degree of Bachelor of Science in mechanical engineering in 1920, and received the degree of Doctor of Philosophy in mechanical engineering from the California Institute of Technology (Pasadena) in 1929.

---

[1] Abstract of memoir prepared by Henry C. Peeples, M. ASCE, and Robert H. Nagel. A.M. ASCE.

[2] Abstract of memoir prepared by Vito A. Vanoni, M. ASCE.

Mr. Knapp joined the staff of the California Institute of Technology in 1922 as an instructor and subsequently became professor of hydraulic engineering. His interest in research and in a wide range of subjects in the field of hydraulics was reflected in numerous publications. During his many productive years he was responsible for developing the Hydrodynamics, Hydraulic Machinery, and Hydraulic Structures Laboratories of the Institute. Many of his experimental techniques and devices have been adopted by the engineering profession throughout the world.

Mr. Knapp also served as a consultant on many civic and governmental engineering projects, including the Colorado River aqueduct (California) and Grand Coulee Dam (Washington). During World War II he was a consultant to the War Department. He received the United States Navy Bureau of Ordnance Development Award for his outstanding work on torpedoes and other underwater ordnance for the Office of Scientific Research and Development.

Mr. Knapp was a member of many professional and honorary societies. In 1952 he was the James Clayton Lecturer on Cavitation for the Institution of Mechanical Engineers of London. He was the National Lecturer on Cavitation for the American Society of Mechanical Engineers (of which he was a member) in the period from 1953 to 1955; he was awarded the ASME Melville Medal in 1955 for one of his many significant papers on cavitation.

On June 14, 1925, Mr. Knapp was married to Pearl M. Gilliland in Los Angeles, Calif. He is survived by his widow and a nephew.

Mr. Knapp was elected a Member of the Society on November 18, 1935.

SAUL LANDMAN, J.M. ASCE[1]

DIED NOVEMBER 24, 1955

Saul Landman, the son of Saul and Gertrude (A.) Landman, was born on February 10, 1920, in Boston, Mass. He attended grade school and high school in Boston and was graduated from Northeastern University (Boston) with the degree of Bachelor of Science in civil engineering in 1942.

Immediately following graduation, Mr. Landman joined the Tennessee Valley Authority in Chattanooga, where he worked in surveying and topographic mapping. He served in the United States Army from July 9, 1943, to February 19, 1946, after which he returned to the TVA.

Mr. Landman was an active member of B'Nai Zion Synagogue and of the Quarter Century Club of Chattanooga.

---

[1] Abstract of memoir prepared by Gene M. Wilholte and Robert H. Nagel, Associate Members, ASCE.

On November 6, 1949, he was married to Helen Lefkoff of Boston. He is survived by his widow; his mother; a daughter, Karen; a son, Mark; and a brother.

Mr. Landman was elected a Junior of the Society on October 19, 1942.

THOMAS R. LAWSON, M. ASCE[1]

DIED MARCH 14, 1954

Thomas R. Lawson, the son of Joseph and Sarah (Brice) Lawson, was born in Wheeling, W. Va., on December 24, 1872. He was graduated from Rensselaer Polytechnic Institute (Troy, N.Y.) with the degree in civil engineering in 1898.

In 1898 he accepted the position as instructor in mechanics at Rensselaer Polytechnic Institute, becoming assistant professor in 1903 and associate professor in 1906. In 1909 Mr. Lawson was appointed professor of Rational and Technical Mechanics, and became Head of the Department of Civil Engineering in 1921, which position he held until his retirement in 1939.

Mr. Lawson was a member of Sigma Xi and Tau Beta Pi. He was active in the Rotary Club, the Vocal Society, and the Chamber of Commerce of Troy. He was a president of the American Society for Testing Materials, the Clay Products Institute of America, the Society of Engineers of Eastern New York. He was also a member of the American Welding Society, the American Concrete Institute, the New York Society of Professional Engineers, the National Society of Professional Engineers, and the Society for the Advancement of Science. In addition to his professional activities, Mr. Lawson also found time to write many books and articles of interest to the profession.

On August 23, 1899, he was married to Mary A. Lawrence of Cohoes, N.Y. He is survived by his widow and two daughters.

Mr. Lawson was elected an Associate Member of the Society on June 3, 1903, and a Member on January 3, 1907. He became a Life Member in 1938.

EDWIN ANDREW McDOUGLE, A.M. ASCE[2]

DIED MARCH 2, 1956

Edwin Andrew McDougle, the son of W. and Sarah (Arthur) McDougle, was born on December 3, 1905, at Black Mountain, N. C.

---

[1] Abstract of memoir prepared by Nancy T. Greer.
[2] Abstract of memoir prepared by William D. Painter, A.M. ASCE.

Mr. McDougle's professional career began in 1925 as a concrete inspector on highway construction for the North Carolina Highway Commission. From 1928 to 1933 he was employed by the Iowa State Highway Commission. In 1934 Mr. McDougle joined the Tennessee Valley Authority, where he assisted on many hydroelectric projects, including Wheeler Dam, Pickwick Dam, Chickamauga Dam, Watts Bar Dam, and Fontana Dam. From 1944 to 1947 he was employed by the Tennessee Eastman Company as personnel supervisor, and in 1947 he returned to the TVA. As general materials engineer for the Division of Construction, he planned and directed the central materials testing laboratory at Knoxville, Tenn.

Mr. McDougle was a talented vocalist and sang in Methodist Church choirs where he resided. In addition to music he enjoyed fishing, golfing, and hiking.

On January 28, 1928, Mr. McDougle was married to Mary Lou Glascoe. He is survived by his widow and three sons, Warren Ray, James Thomas, and Edwin Andrew, Jr.

Mr. McDougle was elected an Associate Member of the Society on June 12, 1950.

CLYDE MYERS, M. ASCE[1]

DIED FEBRUARY 4, 1957

Clyde Myers, the son of Marion Bird and Cordelia Antoinette (Combs) Myers, was born on December 10, 1884, in Pullman, Wash. He was graduated from the State College of Washington (Pullman), from which he received the degrees of Bachelor of Arts in English in 1909, Master of Science in civil engineering in 1916, and Civil Engineer in 1933.

He was city engineer for Pullman from 1916 to 1917 and again in 1921. Mr. Myers served in the Corps of Engineers, United States Department of the Army, during World War I. In 1928 he became head of the engineering department at Phoenix College, Ariz. From World War II to the Korean conflict, he assisted many governmental programs. From 1947 to 1956 he worked for the city of Phoenix, the state of Arizona, and Maricopa County.

In 1937 he became president of the Phoenix Chapter of the American Association of Engineers, and in 1938 he was elected president of the Arizona Section of the Society. He was also a member of the Arizona Society of Professional Engineers.

On August 20, 1911, Mr. Myers was married to Dora Jeannette Jensen in Sprague, Wash. He is survived by his widow; two daughters, Dora Antoinette Furedy and Martha May Glauthier; a son, Harold Combs; three brothers; two sisters; and six grandchildren.

Mr. Myers was elected an Associate Member of the Society on April 19, 1920, and a Member on March 11, 1935. He became a Life Member in 1955.

---

[1] Abstract of memoir prepared by D. Antoinette Furedy.

ALDEN GALLUP ROACH, M. ASCE[1]

DIED DECEMBER 20, 1956

Alden Gallup Roach, the son of Harry F. and Mary (Gallup) Roach, was born in St. Louis, Mo., on March 22, 1901. He was graduated from the University of Illinois (Urbana) with a degree in civil engineering in 1923.

After being employed by the Union Pacific and Missouri Pacific Railroads, Mr. Roach began his career in the steel industry with the Laclede Steel Company of Alton, Ill. In 1927 he moved to California, where he became associated with the Union Iron Works of Los Angeles, which, in 1929, became the Consolidated Steel Corporation. In 1934 he was named vice-president in charges of sales and engineering. In 1938 he became a director of the corporation and then executive vice-president. In 1941 he was elected president, in which capacity he served until 1955. In 1948 he also became president of the Columbia Steel Company, which, in 1953, became the Columbia-Geneva Steel Division of the United States Steel Corporation.

Mr. Roach was an officer and a member of many professional, civic, and national organizations.

On March 26, 1956, he was married to Genevieve Dudley. He is survived by his widow; a son, Alden G. Roach, Jr.; two daughters, Judith (Mrs. Michael Millikan) and Jennifer; three brothers; and two sisters.

Mr. Roach was elected a Junior of the Society on December 15, 1924, an Associate Member on December 3, 1928, and a Member on July 10, 1940.

DELBERT WALTON ROBINSON, M. ASCE[2]

DIED JUNE 12, 1957

Delbert Walton Robinson, the son of Burt Lloyd and Etta (Walton) Robinson, was born on July 7, 1899, in Buffalo Center, Iowa. He was graduated from the State University of Iowa (Iowa City) with the degree of Bachelor of Science in civil engineering in 1925.

Immediately after graduation Mr. Robinson became associated with the consulting firm of Malcolm Pirnie in Florida. From 1925 to 1928 he represented the firm as resident engineer supervising the installation of the waterworks system in the city of Palm Beach (Fla.) and adjacent small towns.

In 1929 Mr. Robinson joined the Texas-Louisiana Power Company (later known as the Community Public Service Company) with headquarters in Fort Worth, Tex. As chief water and gas engineer he designed and supervised the construction of water and gas system improvements and extensions in many cities in Texas, Louisiana, Alabama, Oklahoma, and New Mexico. In addition, his duties included supervising the water and gas properties owned by the foregoing companies.

---

[1] Abstract of memoir prepared by the Columbia-Geneva Steel Public Relations Division of the United States Steel Corporation.

[2] Abstract of memoir prepared by Joseph J. Rady, M. ASCE.

In 1953 Mr. Robinson was employed by the consulting engineering firm of Joe J. Rady & Company of Fort Worth. During the period from 1953 to 1956 he was design engineer on water and sewer projects.

In spite of an extremely busy work schedule, Mr. Robinson found time for professional society activities. He served as President (1948) of the Fort Worth Chapter of the Society, and as president (1941) of the Southwest Waterworks Association.

On November 9, 1927, he was married to Vivian Gretchen Martin in West Palm Beach, Fla. He is survived by his widow; two daughters, Joy and Bunny; and two brothers.

Mr. Robinson was elected an Associate Member of the Society on July 6, 1936, and a Member on September 10, 1945.

HARRY STANLEY ROGERS, M. ASCE[1]

Died June 5, 1957

Harry Stanley Rogers, the son of Samuel and Martha (Hall) Rogers, was born in Detroit, Mich., on August 7, 1890. He was graduated from the University of Wyoming (Laramie) with the degree of Bachelor of Science in civil engineering in 1914, and in 1926 received the degree of Civil Engineer. He subsequently received the degrees of Doctor of Science in 1935 (Northeastern University, Boston, Mass.), Doctor of Laws in 1942 (University of Wyoming), and Doctor of Engineering in 1950 (Rensselaer Polytechnic Institute, Troy, N.Y.).

Upon graduation he served as instructor at the Universities of Iowa (Iowa City), Wyoming, Washington (Seattle), and at Lafayette College (Easton, Pa.). In 1919 he became designing engineer with the Truscon Steel Company (Youngstown, Ohio) but returned to education in 1920 as professor of hydraulics and irrigation engineering at Oregon State College (Corvallis). In 1927 he was appointed dean of engineering at Oregon State College, and in 1928, director of the college's Engineering Experiment Station. In 1933 he accepted the appointment of president of the Polytechnic Institute of Brooklyn (N.Y.), which position he held until his death.

Mr. Rogers was a member of many honorary societies. He was also a member of the American Institute of Consulting Engineers, the American Society of Mechanical Engineers, and the American Society for Engineering Education, receiving the ASEE Lamme Award on June 25, 1953. He was president, director, and trustee of numerous civic and charitable organizations.

---

[1] Abstract of memoir prepared by Charles E. Schaffner, A.M. ASCE.

During World War II he served the Office of Production Management, the War Production Board, and received the President's Certificate of Merit for his work on the National Engineers Committee.

On August 29, 1916, he was married to Grace Larsen in Rock Springs, Wyo. He is survived by his widow; a son, Robert Hall; a sister; and a brother

Mr. Rogers was elected a Member of the Society on November 14, 1927.

WILLIAM JAPHIA SCHLICK, M. ASCE[1]

Died February 5, 1957

William Japhia Schlick, the son of Wilhelm and Phebe (Condit) Schlick, was born on January 24, 1887, in Liberty Center, Iowa. He was graduated from Iowa State College (Ames) with the degrees of Bachelor of Civil Engineering in 1909 and Civil Engineer in 1914.

He gained considerable industrial experience with the architectural firm of Wetherell and Gage in Des Moines, Iowa, and with the Purdy and Henderson Company in Chicago, Ill. From 1910 to 1913 he was employed by the United States Department of Agriculture as a drainage engineer.

In 1914 Mr. Schlick joined the Engineering Experiment Station of Iowa State College as a drainage engineer, afterwards becoming a civil engineer in 1937, research professor of civil engineering in 1938, and assistant director of the station from 1939 to 1947.

He was a member of many professional, scientific, and fraternal organizations, as well as the First Methodist Church of Ames.

In 1914 he was married to Pearle Passwater of Indianola, Iowa. He is survived by his widow; a daughter, Dorothy Ruth (Mrs. Francis Starr); a son, William T.; and seven grandchildren.

Mr. Schlick was elected an Associate Member on January 14, 1918, and a Member on December 6, 1920. He became a Life Member in 1953.

IRVEN CLARENCE SHAFER, A.M. ASCE[2]

Died February 21, 1957

Irven Clarence Shafer, the son of Charles N. and Mary (Sanders) Shafer, was born in Fredonia, Kans., on December 24, 1898.

After service in the United States Army during World War I, he was appointed assistant county engineer for Wilson County, Kans., from 1925 to

---

[1] Abstract of memoir prepared by Merlin G. Spangler, M. ASCE.

[2] Abstract of memoir prepared by Edmund Wilkes, Jr., M. ASCE.

1929. From 1929 to 1943, Mr. Shafer was associated with the engineering staff of the Kansas State Highway Commission as resident engineer in charge of surveys and construction for several highway and bridge projects. During World War II, Mr. Shafer served with the United States Navy as a construction engineer in the 60th Battalion Seabees. Following this, he was again engaged as a civil engineer by the Kansas State Highway Commission in which he served for five years. In 1950 Mr. Shafer and Philip K. Kline formed the partnership of Shafer and Kline at Overland Park, Kans.

Mr. Shafer was a member of the Kansas Engineering Society, the National Society of Professional Engineers, the Overland Park Presbyterian Church, the Dwight Cowles Post of the American Legion, and the Overland Park Masonic Lodge.

On December 8, 1923, in Chicago, Ill., he was married to Louise L. Jackson of Lecompton, Kans. He is survived by his widow; two daughters, Mrs. Robert Wyss and Mrs. Robert Tompson; a son, William I.; and eleven grandchildren.

Mr. Shafer was elected an Associate Member of the Society on July 11, 1938.

HYMEN SHIFRIN, M. ASCE[1]

DIED NOVEMBER 22, 1955

Hymen Shifrin, the son of Jacob and Sadie Shifrin, was born in St. Louis, Mo., on April 14, 1892. He was graduated from Washington University (St. Louis) in 1913 with the degree of Bachelor of Science in civil engineering.

Mr. Shifrin began his professional career in 1913 as an engineer on the staff of the Illinois State Highway Department. From May, 1917 to July, 1919, he served with the 314th Engineers, 89th Division, United States Army, as a second and first lieutenant. From 1919 to 1922, he was employed by the city of St. Louis as engineer on sewer design in the Department of the President of the Board of Public Service. From 1922 to 1933, he was assistant chief engineer of the Department of Sewers and Paving. In 1933 Mr. Shifrin, together with W. W. Horner, formed the firm of Horner & Shifrin, specializing in airport plan and design and in general municipal engineering.

Mr. Shifrin was a member of numerous professional societies and of the Chamber of Commerce of East St. Louis. At the time of his death, he was a director of the St. Louis Metropolitan Chamber and president of the YMHA-YWHA and the Jewish Community Centers Association. He served as chairman of the Illinois Transport Division of the Society.

In 1932 Mr. Shifrin was married to Helen Goodman. He is survived by his widow; a son, Walter G.; and a daughter, Jean.

Mr. Shifrin was elected an Associate Member of the Society on August 14, 1924 and a Member on July 15, 1929.

---

[1] Abstract of memoir prepared by Erwin E. Bloss, M. ASCE.

LOWELL O. STEWART, M. ASCE [1]

DIED AUGUST 24, 1957

Lowell O. Stewart, the son of Arch M. and Sarah (Endrick) Stewart, was born in Watervliet, Mich., on April 4, 1895. He was graduated from Michigan State University (East Lansing) with the degree of Bachelor of Science in civil engineering in 1917. In 1927 he received the degree of Master of Science in civil engineering and, in 1928, the degree in Professional Engineering from Iowa State College (Ames).

Mr. Stewart's initial engineering experience was in hydrographic surveying with the Coast and Geodetic Survey (United States Department of Commerce) in Alaska and Seattle (Wash.) from 1917 to 1920. In 1920 he went to the Philippine Islands as a surveyor, and in 1923 he became a building-construction foreman for a firm in Albion (Mich.). In 1924 he was chief of party for the Michigan State Highway Department for six months. In that year he was appointed to the civil engineering staff at Iowa State College, where he subsequently became a full professor and head of the department in 1938. From 1935 to 1938 he was also the first full-time engineering personnel officer at the college. From July, 1946, to March, 1947, Mr. Stewart was acting dean of the Engineering Division.

In addition to his professorial duties, Mr. Stewart participated in many professional, civic, and honorary organizations. He was secretary-treasurer of the Iowa Section of the Society from 1940 to 1957, and chairman of the civil engineering division of the American Society for Engineering Education in 1948 and 1949. He was also a member of the national council of the ASEE from 1950 to 1956.

In 1922 he was married to Gladys Comfort of Kosevisko, Miss. He is survived by his widow and a daughter, Lois Ann.

Mr. Stewart was elected an Associate Member of the Society on March 11, 1929, and a Member on May 19, 1941.

WALTER PEARCE STINE, M. ASCE [2]

DIED JANUARY 23, 1957

Walter Pearce Stine, the son of Joseph Aby and Christine Ann (Pearce) Stine, was born in Grand Haven, Mich., on April 2, 1883. He was graduated from the University of Michigan (Ann Arbor) in June, 1904, with the degree of Bachelor of Science in civil engineering.

---

[1] Abstract of memoir prepared by Cornie L. Hulsbos, A.M. ASCE.

[2] Abstract of memoir prepared by Harold A. Barr, M. ASCE.

Immediately after his graduation, Mr. Stine took a position as engineer on the Panama Canal, and the following year he became Director of Public Works for the Republic of Panama. He returned to the United States in 1912 and established a practice in Beaumont (Tex.). In 1913 Mr. Stine was employed by the Gulf Oil Corporation. In 1917 he went to Tampico, Mexico, on an assignment, and after its completion he returned to the Houston (Tex.) office where he served until 1924. At this time he was made an assistant division superintendent and assigned to the Coastal Division in Beaumont. From 1925 until his retirement in 1945, Mr. Stine was superintendent of the division. After retirement he again engaged in private practice until his death.

Mr. Stine was a member of numerous professional societies and was one of the organizers of the Southeast Branch of the Texas Section of the Society, in which he served frequently as a director. Mr. Stine was also a member of the Presbyterian Church of Beaumont and the Knights of Pythias.

On September 28, 1905, he was married to Ethel Looker of Ann Arbor. He is survived by his widow; two sons, Walter Douglas and Joe Carl; two daughters, Ruth Winifred and Dorothy Pearce; and five grandchildren.

Mr. Stine was elected an Associate Member of the Society on October 1, 1912, and a Member on August 15, 1938. He became a Life Member in 1947.

PAUL LLOYD STUENKEL, A.M. ASCE [1]

DIED NOVEMBER 15, 1957

Paul Lloyd Stuenkel, the son of William A. and Dora (Niehaus) Stuenkel, was born on October 19, 1902, in Lenora, Kans. He was graduated from Kansas State Agricultural College (Manhattan) with the degree of Bachelor of Science in civil engineering in 1927.

After graduation, he worked for the Phillips Petroleum Company, and then served as assistant county engineer of Pottawatomie County, Kans., before he was employed by the State Highway Commission of Kansas in 1929. The remainder of his career was served with the Commission.

From 1929 to 1937 he was chief of a bridge foundation survey party throughout the state before going into the headquarters office of the State Highway Commission at Topeka as a bridge designer. He was chief of a bridge design squad from 1947 until his death.

Mr. Stuenkel developed a systematic method for the analysis of continuous bridges of structural steel and reinforced concrete, and a set of tables for the rapid preliminary proportioning of these structures.

Mr. Stuenkel was a member of the Central Congregational Church of Topeka, the A.F. & A.M. Lodge No. 257 of Westmoreland, Kans., and the Kansas Engineering Society.

---

[1] Abstract of memoir prepared by Edwin S. Elcock, M. ASCE.

On October 18, 1936, he was married to Marguerite Knauer in Topeka, Kans. He is survived by his widow; a son, Paul, Jr.; a daughter, Judy; two brother; and two sisters.

Mr. Stuenkel was elected an Associate Member of the Society on August 9, 1943.

DONALD WOOD TAYLOR, A.M. ASCE[1]

Died December 24, 1955

Donald Wood Taylor, the son of Walter B. and Clara (Wood) Taylor, was born in Worcester, Mass., on December 2, 1900. In 1922 he was graduated from Worcester Polytechnic Institute with the degree of Bachelor of Science in civil engineering. In 1942 he received the degree of Master of Science in civil engineering from the Massachusetts Institute of Technology (Cambridge).

After graduation, Mr. Taylor was employed by the Coast and Geodetic Survey, United States Department of Commerce. He then worked as an engineer for the city of Los Angeles (Calif.), the Edward Miner Construction Company, and the New England Power Company. In 1932, after outstanding accomplishments on a cooperative research project between the New England Power Company and M.I.T., he joined the research staff of the Department of Civil and Sanitary Engineering of the institute.

From 1939 until his death, he was an associate professor and head of the Soil Mechanics Division at the institute. In addition to teaching, Mr. Taylor performed and directed extensive institutional and sponsored research in soil mechanics and acted as a consultant to many governmental and private organizations.

For many years Mr. Taylor was an active member of the Boston Society of Civil Engineers, in which he served as vice-president; the American Society for Engineering Education; the Highway Research Board; and the International Society of Soil Mechanics and Foundation Engineering. He was also a member of Sigma Xi and Chi Epsilon.

On October 13, 1928, at Marlborough, Mass., he was married to Beulah Elizabeth Nyman. He is survived by his widow; his parents; and a sister.

Mr. Taylor was elected an Associate Member of the Society on August 12, 1935.

---

[1] Abstract of memoir prepared by Charles H. Norris, A.M. ASCE.

HERBERT ALBERT VAN DER GOOT, A.M. ASCE [1]

DIED JUNE 21, 1957

Herbert Albert van der Goot, the son of Petrus Marten and Emmy (Jacob) van der Goot, was born on August 10, 1909, in Glendora, Calif. He attended the California Institute of Technology (Pasadena), the University of Southern California, and the University of California (both in Los Angeles).

During World War II he received the commission of lieutenant in the United States Navy and rose to be group commander of a mine-sweeping squadron.

Mr. van der Goot was appointed assistant to the city engineer of Azusa, Calif., in 1926. In 1929 he entered the service of the Los Angeles County Flood Control District as dam tender. He eventually became head of the Dams and Runoff Record Section of the district and, later, head of the Conservation and Ground Water Section of the Hydraulic Division. As head of this section, Mr. van der Goot pioneered and supervised the West Basin Barrier Test to prevent sea-water intrusion into fresh-water aquifers.

He was an ardent fisherman and took an active interest in community affairs. He was also Master of the Glendora Masonic Lodge 404 and a member of E Clampus Vitus, a Southern California Historical Society.

On September 2, 1933, he was married to Marianne Orton in Riverside, Calif. He is survived by his widow; a daughter, Mariechen (Mrs. Norman Lester); and a son, Petrus Marten II.

Mr. van der Goot was elected an Associate Member of the Society on December 21, 1943.

WILLIAM JAMES VAN LONDON, M. ASCE [2]

DIED JUNE 1, 1957

William James Van London was born on January 22, 1888, in Cache Bay, Ontario, Canada. He attended the University of Utah (Salt Lake City).

In 1917 he began his career in highway engineering with the Arizona State Highway Department. Later that year he entered the United States Army, in which he served as sergeant with the Fifth Engineers.

---

[1] Abstract of memoir prepared by Ernest J. Koch, Jr., A.M. ASCE.

[2] Abstract of memoir prepared by James C. Dingwall, M. ASCE.

After his military service he worked for private engineering firms until his employment as a highway engineer with the Texas Highway Department in Tarrant County in May, 1921. Among his numerous achievements in the department were the conception, design, and construction of the Houston (Tex.) Urban Expressway System. After a leave of absence in 1951, during which time Mr. Van London conducted a world-wide transportation survey, he returned to the department as engineer-manager, which position he occupied until his resignation on March 31, 1955. He then organized the Vlando Mining Company of which he was president and general manager until his death.

On March 26, 1919, he was married to Molliebel Sellers of Fort Worth, Tex. He is survived by his widow; a son, Gregory S.; and a daughter, Vonia.

Mr. Van London was elected an Associate Member of the Society on November 10, 1930, and a Member on November 30, 1953.

WILLIAM HORACE WILLIAMS, M. ASCE[1]

DIED FEBRUARY 6, 1957

William Horace Williams, the son of William Morrow and Eugenie Lelia (Simon) Williams, was born on July 18, 1882, at Fort McIntosh, Laredo, Tex. He was educated at Denison University at Granville, Ohio.

His professional career began in 1902 with the engineering department of the Pennsylvania Railroad. He was later (1903) associated with the Corps of Engineers, United States Army. From 1904 to 1908 he was resident engineer for the firm of Christie & Lowe in New Orleans, La., on construction of Mississippi River jetties.

In 1908 he and his partner, M. P. Doullut, formed the contracting company of Doullut & Williams. From 1918 to 1921 a subsidiary firm, Doullut & Williams Shipbuilding Company, Inc., designed and constructed steel ships and barges for the United States Government. From 1924 on, the company, under the name of W. Horace Williams Company, Inc., engaged in many large projects, including marine terminal facilities in the southern United States, Mexico, the West Indies, and South America. Under Mr. Williams' management and direction, the company also constructed many Mississippi River wharves, steel drilling platforms in the Gulf of Mexico, rail terminals, bridge foundations, and industrial plants.

Mr. Williams took an active part in community affairs and lent his talents to many civic and charitable projects. He was a member of the American Society of Mechanical Engineers, the Society of American Military Engineers, and a member and past-president of the Louisiana Engineering Society. He was also a member of many honorary, civic, social, and business organizations.

---

[1] Abstract of memoir prepared by Herrick J. Lane, A.M. ASCE, C. Glenn Capell, M. ASCE, and Robert E. Gosa, A.M. ASCE.

In 1909 he was married to Ruby Ionia Mugnier (deceased) in New Orleans. In 1923 he was married to Viola Bloch in New York, N.Y. He is survived by his widow; two daughters, Mrs. Elizabeth Ionia Seavey and Mrs. Eugenie Lorrance Livaudais; and three sons, William Horace, Jr., John Wesley, and Robert Milton.

Mr. Williams was elected an Associate Member of the Society on October 29, 1912, and a Member on June 12, 1917. He became a Life Member in 1947.

WILLIAM MUNSEY WILSON, M. ASCE[1]

DIED NOVEMBER 2, 1957

William Munsey Wilson, the son of James Calvin and Mattie Belle (Russell) Wilson, was born on October 26, 1894, in Hallettsville, Tex. He was graduated from the University of Texas (Austin) with the degree of Bachelor of Science in architectural engineering in 1923.

From 1924 to 1931 Mr. Wilson was associated with the architectural firm of Giesecke and Harris (in Austin) as chief engineer. From 1931 to 1932 he was chief engineer for Texas Agricultural and Mechanical College (College Station). In the period from 1933 to 1936, Mr. Wilson undertook several short-term projects. He also held at various times the positions of resident engineer inspector, office engineer, and assistant state engineer inspector on various projects in and around Fort Worth, Tex.

In 1937 he opened a consulting office in Austin, and during a period of approximately three years (1942–1945) he took time off to act as chief engineer for the firm of Gieg-LaRoche-Dahl-Chappell and as special instructor of structural design at the University of Texas. In 1945 he formed a partnership with Worth Cottingham. Under the title of Wilson and Cottingham, this firm was responsible for the design of many structures in the Austin area and other Texas cities.

Mr. Wilson was an active member of the Texas Section of the Society, the Texas Society of Professional Engineers, and the American Concrete Institute. He was also active with the Boy Scouts of America.

On August 5, 1923, he was married to Alberta Howell. He is survived by his widow; two sons, Robert Munsey and Albert Lane; a daughter, Elizabeth (Mrs. Donald R. Davis); and six grandchildren.

Mr. Wilson was elected an Associate Member of the Society on August 27, 1928, and a Member on July 31, 1933.

---

[1] Abstract of memoir prepared by John A. Focht, M. ASCE.

JOHN STEPHEN WORLEY, M. ASCE [1]

Died May 25, 1956

John Stephen Worley was born in Jackson County, Mo., on April 19, 1876. He was graduated from the University of Kansas (Lawrence) with the degrees of Bachelor of Science and Master of Science in 1904 and was awarded the degree of Civil Engineer in 1922. He was admitted to the Missouri Bar in 1919.

His engineering career initially consisted of a variety of assignments on railroad and public utility projects, as senior partner in the consulting office of Worley and Black in Kansas City, Mo. From 1913 to 1920, Mr. Worley served as a member of the Engineering Board, Bureau of Valuation, Interstate Commerce Commission in Washington, D.C. From 1921 to 1927, he was associated with the Habirshaw Electric Cable Company of New York (N.Y.) first, as federal receiver, and then as executive vice-president and general manager.

In 1922 he also became a member of the teaching staff at the University of Michigan (Ann Arbor), where he pioneered in the development of transportation engineering courses, and served as curator of the Transportation Library at the university. In 1945 Mr. Worley retired from teaching but continued his professional activities as a consultant principally in highway finance and taxation. In Ann Arbor Mr. Worley served as police commissioner for a number of years in the 1930's. He was an active member of the First Methodist Church.

He was married to Mayme Lee Baker. He is survived by his widow; a daughter, Mary Louise (Mrs. James Symons); and two grandsons.

Mr. Worley was elected an Associate Member of the Society on June 5, 1907, and a Member on September 3, 1913. He became a Life Member in 1942.

---

[1] Abstract of memoir prepared by the Civil Engineering Department of the University of Michigan, Ann Arbor, Mich.

TRANSACTIONS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

INDEX

VOLUME 123

1958

SUBJECT INDEX, PAGE 1312
AUTHOR INDEX, PAGE 1352

Titles of papers are in quotation marks when given with the author's name.

VOLUME 123

SUBJECT INDEX

ACCELERATION

See VELOCITY; *see also* VIBRATION

ACCIDENTS

See HIGHWAYS AND ROADS—Safety

ACCOUNTS AND ACCOUNTING

See COSTS . . . ; *also* subheading Financing under relative subject

ADDRESSES

See AMERICAN SOCIETY OF CIVIL ENGINEERS—Addresses; *also* under subject of address

AERATION

See also OXYGENATION

"Effects of Storage Impoundments on Water Quality," Milo A. Churchill, 419.

Mathematical development of the theory of re-aeration, 648.

"Mechanism of Reaeration in Natural Streams," Donald J. O'Connor and William E. Dobbins, 641. *Discussion:* Thomas R. Camp; Alexander N. Diachishin; Aale Pasveer; Myrton C. Rand; and Donald J. O'Connor and William E. Dobbins, 667.

Bibliography

Re-aeration in natural streams, 663.

AERIAL . . .

See also AIR . . .

AERIAL NAVIGATION

See AIR TRANSPORT (cross reference thereunder)

AERIAL SURVEYS AND SURVEYING

See SURVEYS AND SURVEYING, AERIAL

AGGREGATES AND AGGREGATION

Problem of chemically reactive aggregates of central California, 229.

AGREEMENTS

See CONTRACTS (cross reference thereunder)

AGRICULTURE

See EROSION, LAND; FLOOD . . . ; IRRIGATION; RAINFALL; RUNOFF; SOIL . . . ; WATER, CONSUMPTIVE USE OF; WATER RIGHTS

AIR . . .

See also AERIAL . . .

AIRCRAFT

See also BUCKLING

Plastics as lightweight and light transmitting materials, 520, 522.

AIRPORTS (structures and localities)

Development outline of New York International Airport, 368.

"Transportation Planning": A Symposium, Roger H. Gilman, Wilfred M. Post, Jr. and Burton W. Marsh, 359.

Financing

Types of financing for different size airports, 371, 376.

AIR TRANSPORT

See TRANSPORTATION

ALLOYS

See under name of relative metal

ALLUVIATION

See BARS (alluvia); EROSION . . . ; RIVER DELTAS; SILT AND SILTING . . . ; VALLEYS

ALUMINUM

Usage of aluminum as a model material, 825, 828.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Addresses

1958—Address at the Summer Convention, Portland, Ore., June 25, 1958, Louis R. Howson, 1275.

Memoirs of Members. *See* name of member in Author Index. (*See also* p. 1283)

ANALYSIS, DESIGN

See under relative subject, e.g., STRUCTURES, THEORY OF

ANALYSIS OF DATA
See EQUATIONS; MATHEMATICS; STRUCTURES, THEORY OF; *also* under relative subject

ANALYSIS, STRUCTURAL
See EQUATIONS; STRESS AND STRAIN; STRUCTURES, THEORY OF; TRUSSES...

ANALYSIS, WATER
See WATER ANALYSIS

ANCHORAGES, TAINTER GATE
See TAINTER GATES (cross reference thereunder)

APPARATUS
See under relative subject; *also* INSTRUMENTS (cross references thereunder); *also* under general types of apparatus, e.g., VIBRATION RECORDING APPARATUS

APPRAISAL
See VALUATION

AQUIFERS
See WELLS

ARCHES
See STRUCTURES, THEORY OF —Arches

ARTESIAN WELLS
See WELLS

ASSOCIATIONS
See AMERICAN SOCIETY OF CIVIL ENGINEERS

AUTHORITIES
See RIVER VALLEY AUTHORITIES

AUTOMOBILE...
See also MOTOR... (cross references thereunder)

AUTOMOBILE ACCIDENTS
See HIGHWAYS AND ROADS—Safety

AUTOMOBILES
See TRAFFIC...

AUTOMOTIVE ENGINEERS AND ENGINEERING
See ENGINEERS AND ENGINEERING

BACKWATER
"Old River Diversion Control": A Symposium, John R. Hardin; Eugene A. Graves; Willard J. Turnbull and Woodland G. Shockley; and Norman R. Moore, 1129.

BACTERIA
Storage impoundment effects on water quality, 435.

BAFFLE PIERS
See SPILLWAYS

BANKS AND BANK PROTECTION, RIVER
See RIVER BANKS AND BANK PROTECTION (cross references thereunder)

BARGES
"Bridge Clearances": A Symposium, Eugene W. Weber, Walter Kurylo, William E. Cleary, Nickles L. Caruthers, and Erhard E. Dittbrenner, 925.

BARS (alluvia)
See also RIVER DELTAS
"Mechanics of Streams with Movable Beds of Fine Sand," Norman H. Brooks (with discussion), 526.

BASINS (depression in earth's surface)
See DRAINAGE; RIVER VALLEY AUTHORITIES; VALLEYS; *see also* RIVER BASINS (cross references thereunder)

BASINS, SPREADING
See SPREADING BASINS

BASINS, STILLING
See WATER, FLOW OF, IN OPEN CHANNELS

BEACHES
See EROSION, BEACH; SHORES AND SHORE PROTECTION

BEAMS (General)
See also BUCKLING; TORSION; *also* under structure or structural part, e.g., BRIDGES; GIRDERS
"Simplification of Design by Ultimate Strength Procedures," Phil M. Ferguson (with discussion), 602.

BEAMS, CONTINUOUS
See BRIDGES; STRUCTURES, THEORY OF—Beams and Girders, Continuous; WHEEL LOADS

BEARING CAPACITY (foundations, rocks, soils)
For more general interpretation *see* cross references under LOAD
California Bearing Ratio, water content, density tests and test developed by American Association of State Highway Officials and Harvard University compared, 1.
"Experiences with Loess As Foundation Material," William A. Clevenger (with discussion), 151.
"Pile Tests, Low-Sill Structure, Old River, Louisiana," Charles I. Mansur and Robert I. Kaufman (with discussion), 715.

BEARING CAPACITY (*Continued*)

"Soil Modulus for Laterally Loaded Piles," Bramlette McClelland and John A. Focht, Jr. (with discussion), 1049.

"Stabilization of Materials by Compaction," Willard J. Turnbull and Charles R. Foster (with discussion), 1.

BED LOAD

See CANALS; CHANNELS; RIVERS; SILT AND SILTING... ; TURBULENCE; WATER, FLOW OF, IN OPEN CHANNELS

BENDING

See BUCKLING; MOMENTS; STRESS AND STRAIN; *also* under relative structure, structural part or material, e.g., FRAMES, RIGID (cross reference thereunder)

BENTS

See STRESS AND STRAIN—Frames...

BIBLIOGRAPHY

See subheading Bibliography under relative subject. (Comprehensive bibliographical footnotes existing in individual papers in which books and other material are cited)

BIOGRAPHIES OF DECEASED MEMBERS

See cross reference under MEMOIRS OF DECEASED MEMBERS. (*See also* p. 1283)

BOGS

See MARSHES

BOND STRESS

See STRESS AND STRAIN

BORINGS

"Pile Tests, Low-Sill Structure, Old River, Louisiana," Charles I. Mansur and Robert I. Kaufman (with discussion), 715.

BOUNDARIES (land ownership)

See RIGHTS OF WAY (land strips)

BOUNDARY LAYER, THEORY OF (fluid flow)

See WATER, FLOW OF...

BRACING

See TRUSSES...

BREAKWATERS

See also JETTIES; SHORES AND SHORE PROTECTION; WAVES

"Wave Run-Up on Shore Structures," Thorndike Saville, Jr., 139.

BRIDGE FLOORS AND FLOORING

"Load Distribution in Highway Bridge Decks," Arnold W. Hendry and Leslie G. Jaeger, 1214. *Discussion:* Pranab Kumar Chaudhuri, and Arnold W. Hendry and Leslie G. Jaeger, 1245.

BRIDGE LAW

Early law pertaining to bridges as crossings obstructing navigation, 927.

BRIDGES (General)

See also BEAMS... ; COLUMNS; COSTS, BRIDGE; IMPACT; PILES AND PILE DRIVING; STRUCTURES, THEORY OF—Bridges; TRUSS... ; VIBRATION; WATER, FLOW OF, IN OPEN CHANNELS; WHEEL LOADS

Bridge clearance problems too individualized for initiation of standardization, 946.

"Bridge Clearances": A Symposium, Eugene W. Weber, Walter Kurylo, William E. Cleary, Nickles L. Caruthers, and Erhard E. Dittbrenner, 925.

BRIDGES, GIRDER

See also STRESS AND STRAIN—Bridges, Girder

"Dynamic Stresses in Continuous Plate-Girder Bridges," Roy C. Edgerton and Gordon W. Beecroft (with discussion), 266.

"Load Distribution in Highway Bridge Decks," Arnold W. Hendry and Leslie G. Jaeger (with discussion), 1214.

BUBBLES

See VISCOSITY

BUCKLING

"Bearing-Ratio Effect on Strength of Riveted Joints," Jonathan Jones, 964.

"A Pressure-Line Concept for Inelastic Bending," Frank Baron (with discussion), 1039.

"Shells of Double Curvature," Alfred L. Parme (with discussion), 989.

BUILDING (process)

See CONSTRUCTION (cross reference thereunder)

BUILDING MATERIALS

See MATERIALS OF CONSTRUCTION

BUILDINGS

See also CHIMNEYS AND FLUES; COSTS... ; FAILURES... ; FLOORS AND FLOORING (cross

BUILDINGS (*Continued*)
reference thereunder); FOUNDATIONS...; MATERIALS OF CONSTRUCTION; STRENGTH OF MATERIALS; STRESS AND STRAIN...; STRUCTURES...; *also* under type of building, e.g., POWER PLANTS
"Wave Run-Up on Shore Structures," Thorndike Saville, Jr., 139.

BULKHEADS
See also RETAINING WALLS; SEA WALLS; SHORES AND SHORE PROTECTION; WAVES
"Wave Run-Up on Shore Structures," Thorndike Saville, Jr., 139.

CANALS (General)
See also CHANNELS; EROSION, STREAM; HEAD GATES; LOCKS; SEDIMENT AND SEDIMENTATION; SEEPAGE; SILT AND SILTING, CHANNEL; WATER, FLOW OF, IN OPEN CHANNELS; WATER TRANSPORTATION; WAVES
"Design of Stable Canals and Channels in Erodible Material," Pete W. Terrell and Whitney M. Borland, 101.
"Sediment-Transport Mechanics in Stable-Channel Design," Emmett M. Laursen (with discussion), 195.

CARRIERS
See AIRCRAFT; CONVEYERS AND CONVEYANCE; COSTS, HARBOR IMPROVEMENT; MOTOR... (cross references thereunder); RAIL... (cross references thereunder); TRANSPORTATION; WATER...

CARS
See AUTOMOBILE... (cross references thereunder); MOTOR... (cross references thereunder); TRAFFIC...; WHEEL LOADS

CARTOGRAPHY
See CHARTS (cross reference thereunder); MAPS AND MAPPING (cross reference thereunder); SURVEYS AND SURVEYING

CAR WHEELS
See WHEEL LOADS

CHANNEL RECTIFICATION
See CHANNELS; WATER DIVERSION; WATER, FLOW OF, IN OPEN CHANNELS

CHANNELS (waterways)
See also COSTS, CHANNEL; EROSION, STREAM; RIVERS; SILLS; SILT AND SILTING, CHANNEL; SLUICES; WATER DIVERSION; WATER, FLOW OF, IN OPEN CHANNELS; WATER POLLUTION
Channel stability problems and phenomena influencing them, 114.
"Design of Stable Canals and Channels in Erodible Material," Pete W. Terrell and Whitney M. Borland, 101.
"Developing Port Facilities on Houston's Ship Channel," Frank H. Newnam, Jr. (with discussion), 239.
"A Floodway for Houston, Tex.," Kenneth Heagy, 181.
"Mobile (Ala.) Harbor and Ship Channel," Harold E. Bisbort, 1195.
"Sediment-Transport Mechanics in Stable-Channel Design," Emmett M. Laursen, 195. *Discussion:* Sam Shulits, and Emmett M. Lausen, 204.
"Transition Profiles in Nonuniform Channels," Francis F. Escoffier (with discussion), 43.

CHARTS
See HYDROGRAPHS; *also* under relative subject, e.g., STRENGTH OF MATERIALS

CHEMISTRY
See under relative technical subject, e.g., CORROSION AND PROTECTION OF...

CHIMNEYS AND FLUES
Stacks designed for wind and seismic loadings, 222, 236, 237.

CHUTES, WATER FLOW THROUGH
See SPILLWAYS; WATER, FLOW OF, IN OPEN CHANNELS

CITIES
See also AIRPORTS; BUILDINGS; COSTS...; DOCKS AND WHARVES; EMPLOYEES AND EMPLOYMENT; EXPRESSWAYS (cross reference thereunder); GOVERNMENT (cross references thereunder); HOUSES; INDUSTRIAL...; PUBLIC HEALTH; SEWAGE...; SEWERS; TERMINALS (cross reference thereunder); WATER SUPPLY; *also* geographical subheadings under relative subject, e.g., FLOODS—Houston, Tex.
Engineering facilities of Houston, Texas, 182.
Knoxville, Tennessee sewage and industrial waste treatment plant ordinances, 595.

CIVIL ENGINEERS AND ENGINEERING
See AMERICAN SOCIETY OF CIVIL ENGINEERS; ENGINEERS AND ENGINEERING (and cross references thereunder)

CLARIFICATION, WATER
See WATER TREATMENT

CLAY
See also LOESS; PILE AND PILE DRIVING
Scant knowledge existing concerning resistance to movement of clays, 202.
"Soil Modulus for Laterally Loaded Piles," Bramlette McClelland and John A. Focht, Jr. (with discussion), 1049.
"Stabilization of Materials by Compaction," Willard J. Turnbull and Charles R. Foster (with discussion), 1.

COAST
See SEACOAST (cross references thereunder)

COATINGS, PROTECTIVE
See CORROSION AND PROTECTION...

COLUMNS
See also STRUCTURES, THEORY OF—Columns
"Simplification of Design by Ultimate Strength Procedures," Phil M. Ferguson (with discussion), 602.

COMMERCE
See AIRPORTS; CANALS; CHANNELS; CITIES; DOCKS AND WHARVES; HARBORS; HIGHWAYS AND ROADS; RAILROAD...(cross reference thereunder); RIVERS; TERMINALS (cross reference thereunder); TRANSPORTATION; WATER TRANSPORTATION

COMPACTION
See BEARING CAPACITY; SOILS

COMPRESSION
See STRESS AND STRAIN

COMPRESSION MEMBERS
See COLUMNS

COMPRESSION TESTS, SOIL
See SOILS—Tests and Testing

COMPUTERS, ELECTRONIC
See ELECTRONIC INSTRUMENTS

CONCRETE (General)
See DAMS, MASONRY AND CONCRETE; SAND

Corrosion and Protection of. *See* CORROSION AND PROTECTION OF CONCRETE

Cracks and Cracking
California aggregates as a problem in California, 229.

Design
"Simplification of Design by Ultimate Strength Procedures," Phil M. Ferguson (with discussion), 602.

Slabs. *See* SLABS (cross reference thereunder)

CONCRETE—METAL
See cross reference under REINFORCED CONCRETE

CONDENSERS AND CONDENSATION
See also EVAPORATION
"Morro Bay Steam Electric Plant," J. George Thon and Gordon L. Coltrin (with discussion), 207.

CONDUITS
See also FLUMES; PIPE LINES; TUNNELS
"A Flow Controller for Open or Closed Conduits," Victor L. Streeter, 883.
"Friction Measurements in the Apalachia Tunnel," Rex A. Elder, 1249.
"Trends in Hydraulic Gate Design," Dow A. Buzzell (with discussion), 27.

CONNECTORS AND CONNECTIONS
See also JOINTS... ; STRESS AND STRAIN—Connectors and Connections
"Behavior of Riveted Truss-Type Connections," Eugene Chesson, Jr. and William H. Munse, 1087. *Discussion:* Arthur J. Francis; and Eugene Chesson, Jr. and William H. Munse, 1127.

CONSERVATION
See GROUND WATER; EVAPORATION; WATER CONSERVATION (cross references thereunder)

CONSOLIDATION TESTS, SOIL
See SOILS—Tests and Testing

CONSTRUCTION
See BUILDINGS; CONTRACTS (cross reference thereunder); COSTS... ; FAILURES...; MATERIALS OF CONSTRUCTION; SOILS—Construction; STRESS AND STRAIN... ; STRUCTURES, THEORY OF... ; *also* under type of construction, e.g., FOUNDATIONS

CONSTRUCTION MATERIALS
See MATERIALS OF CONSTRUCTION

CONSTRUCTION, UNDERWATER
See UNDERWATER CONSTRUCTION (cross references thereunder)

CONSUMPTIVE USE (water)
See WATER, CONSUMPTIVE USE OF

CONTAINERSHIPS
See SHIPS AND SHIPPING (cross reference thereunder)

CONTAMINATION, STREAM
See WATER POLLUTION

CONTINUOUS BEAMS
See BEAMS, CONTINUOUS (cross references thereunder)

CONTINUOUS FRAMES
See STRESS AND STRAIN—Frames, Continuous; *see also* BEAMS . . . ; COLUMNS; GIRDERS . . .

CONTINUOUS GIRDERS
See GIRDERS, CONTINUOUS (cross references thereunder)

CONTRACTS
See ENGINEERS AND ENGINEERING

CONTROL AND CONTROLLERS, FLOW
See WATER, FLOW OF, THROUGH ORIFICES

CONVENTIONS (American Society of Civil Engineers)
See AMERICAN SOCIETY OF CIVIL ENGINEERS—Addresses

CONVEYERS AND CONVEYANCE
"Shoreside Facilities for Special-Purpose Ships," Howard J. Marsden, 851.

CORROSION AND PROTECTION OF CONCRETE
"Friction Measurements in the Apalachia Tunnel," Rex A. Elder, 1249.

CORROSION AND PROTECTION OF METALS
"Friction Measurements in the Apalachia Tunnel," Rex A. Elder, 1249.

COSTS (General)
See also subheading Financing under relative subject

COSTS (of work)
See COSTS . . .

COSTS, BRIDGE
"Bridge Clearances": A Symposium, Eugene W. Weber, Walter Kurylo, William E. Cleary, Nickles L. Caruthers, and Erhard E. Dittbrenner, 925.

COSTS, CHANNEL
"Developing Port Facilities on Houston's Ship Channel," Frank H. Newnam, Jr. (with discussion), 239.

COSTS, DOCK AND WHARF
"Developing Port Facilities on Houston's Ship Channel," Frank H. Newnam, Jr. (with discussion), 239.

COSTS, DREDGING
Mobile, Alabama dredging cost fluctuations since 1826, 1201.

COSTS, FLOOD
Houston, Texas rectification and division costs specified, 186, 194.

COSTS, HARBOR IMPROVEMENT
"Developing Port Facilities on Houston's Ship Channel," Frank H. Newnam, Jr. (with discussion), 239.
Facilities for cargo transfer between land and sea carriers as they relate to costs, 851.
"Mobile (Ala.) Harbor and Ship Channel," Harold E. Bisbort, 1195.

COSTS, HIGHWAY AND ROAD
"Control of Highway Access": A Symposium, Curtis J. Hooper, Bertram D. Tallamy, Dewitt C. Greer, Carl E. Vogelgesang, Hugo H. Winter, and Ralph A. Moyer, 465.
Indiana State Highway System cost estimate, 478.
Original cost of 16 miles (and a 45 and 185 mile estimate) of California freeways, 494, 495, 498.

COSTS, INDUSTRIAL WASTE
Knoxville, Tennessee approximate bid and other costs of system begun in 1953, 595.

COSTS, MULTI-PURPOSE PROJECT (RIVER PROJECTS)
Allocation methods, principles involved and major weaknesses, 85.
"Cost Allocation for Multiple-Purpose Water Projects," Newcomb B. Bennett, Jr., 85. *Discussion:* Frederick L. Hotes; Eugene W. Weber; Wendell E. Johnson and Charles A. Cocks; and Newcomb B. Bennett, Jr., 93.

COSTS, POWER PLANT
Morro Bay Steam Plant itemized costs, 208, 210, 217.

COSTS, RESERVOIR (MULTIPURPOSE RESERVOIRS)
"Cost Allocation for Multiple-Purpose Water Projects," Newcomb B. Bennett, Jr. (with discussion), 85.

COSTS, RIGHT OF WAY (land strip)
"Control of Highway Access": A Symposium, Curtis J. Hooper, Bertram D. Tallamy, Dewitt C. Greer, Carl E. Vogelgesang, Hugo H. Winter, and Ralph A. Moyer, 465.

COSTS, RIVER DIVERSION
See COSTS, WATER DIVERSION

COSTS, SAND SAMPLING
Reasonable cost of sampling procedure for usage below or above water table, 868.

COSTS, SEA WALL
Mileage and cost of sea wall in Harrison County, Mississippi, 817.

COSTS, SPREADING (WATER SPREADING)
Usage of laboratory lysimeters and field experiments compared in waste water reclamation studies, 134, 137, 138.

COSTS, SURVEY AND SURVEYING (General)
Ground versus aerial surveys, 1029, 1030, 1034, 1035.

COSTS, WATER DIVERSION
Mississippi River improvement estimated costs, 1141, 1159, 1173, 1176, 1181.

COSTS, WATER GATE (MOVABLE)
Tainter gate anchorage of economic type and estimated savings, 409, 416.

COSTS, WATERWAY
Gulf Intracoastal Waterway ratio of benefits to costs in relation to other waterways in United States, 947.

COURT DECISIONS
See under relative subject (under the subject LAW heading, e.g., RIPARIAN RIGHTS LAW)

CRACKS AND CRACKING
See under relative subject, e.g., CONCRETE—Cracks and Cracking

CREEP
See STRESS AND STRAIN

CREST GATES
See WATER GATES, MOVABLE

CURRENT METERS
See METERS AND METERING, CURRENT

CURVED STRUCTURES
See STRUCTURES, THEORY OF —Curved Structures

CURVES (design curves)
See under relative subject of design, e.g., STRENGTH OF MATERIALS

CURVES (elastic curves)
See STRUCTURES, THEORY OF

CYLINDRICAL SHELLS
See CHIMNEYS AND FLUES; SHELL STRUCTURES

DAMS (General)
See also LOCKS; SPILLWAYS; WATER DIVERSION; WATER PRESSURE

DAMS, EARTH
See also SEEPAGE
Buford Dam (Chattahoochee River, Georgia) in Apalachicola River Basin project, with outline of characteristics and construction details, 975, 976, 987.

DAMS, GRAVITY
See DAMS, MASONRY AND CONCRETE

DAMS, MASONRY AND CONCRETE
Jim Woodruff Lock and Dam on the Apalachicola River with outline of characteristics and construction details, 984, 987.
Walter F. George Lock and Dam (formerly Fort Gaines Dam) and the Columbia Lock and Dam, both on the Chattahoochee River, Georgia, in Apalachicola River Basin project, with outline of characteristics and construction details, 980, 982, 987.

DAMS, ROCK-FILL
Deformation developed by earthquake vibration, 795, 801.
"Earthquake Resistance of Rock-Fill Dams," Ray W. Clough and David Pirtz, 792. *Discussion:* Nicols N. Ambraseys; John V. Spielman; and Ray W. Clough and David Pirtz, 811.

DEFINITIONS
See TERMINOLOGY

DEFLECTIONS
See DEFORMATION (cross references thereunder); MOMENTS; STRESS AND STRAIN; STRUCTURES, THEORY OF; *also* under relative structure or structural part, e.g., BRIDGES...

DEFORMATION
See FAILURES . . . ; STRESS AND STRAIN; STRUCTURES, THEORY OF; *also* under specific type of stress, e.g., TORSION; *also* under type of material, e.g., CLAY; *also* under type of structure, e.g., DAMS, ROCK-FILL

DELTAS
See RIVER DELTAS

DEMINERALIZATION OF WATER
See WATER TREATMENT

DESIGN
See under relative subject, e.g., FRAMES; WATER GATES, MOVABLE

DESIGN ANALYSIS
See under relative subject, e.g., STRUCTURES, THEORY OF

DESIGN CURVES
See CURVES (design curves) (cross reference thereunder)

DIAGRAMS
See under relative subject, e.g., MOMENTS

DISCHARGE
See METERS AND METERING, CURRENT; PUMPS AND PUMPING; SLUICES; WATER, FLOW OF . . .

DISCHARGE COEFFICIENTS
See under relative subject, e.g., WATER, FLOW OF . . .

DISEASE
See PUBLIC HEALTH

DISINTEGRATION OF MATERIALS
See CORROSION AND PROTECTION OF . . . ; *also* under type of material

DISTORTION
See STRESS AND STRAIN; TORSION

DIVERSION
See WATER DIVERSION

DOCKS AND WHARVES
See also BULKHEADS; COSTS, DOCK AND WHARF; HARBORS; PILES AND PILE DRIVING; WATER TERMINALS
"Developing Port Facilities on Houston's Ship Channel," Frank H. Newnam, Jr. (with discussion), 239.
"Shoreside Facilities for Special-Purpose Ships," Howard J. Marsden, 851.

DRAINAGE
See also IRRIGATION; RAINFALL; RUNOFF; SEWERS; VALLEYS; WELLS; *also* under type of drainage, e.g., HOUSES—Drainage
Atchafalaya River Basin structures in the Mississippi River non-course changing diversion plans, 1129.
The drainage basin of the Mississippi River, 1132.

DRAINAGE BASINS
See DRAINAGE

DRAWDOWN
See WELLS

DRAWINGS
See under relative subject

DREDGES AND DREDGING
See also COSTS, DREDGING
"Harrison County (Mississippi) Artificial Beach," Francis F. Escoffier, 817.
"Mobile (Ala.) Harbor and Ship Channel," Harold E. Bisbort, 1195.

DRILLS AND DRILLING
See WELL DRILLING . . .

DROP-DOWN
See SPILLWAYS; WATER, FLOW OF, IN OPEN CHANNELS

DROP STRUCTURES
See DAMS

DROUGHTS
See RAINFALL

DUNES
See SAND DUNES (cross references thereunder)

DYNAMICS OF FLUIDS
See HYDRODYNAMICS

DYNAMICS OF STRUCTURES
See STRUCTURAL DYNAMICS (cross references thereunder)

EARTH . . .
See also GROUND . . . ; LAND . . . (cross references thereunder); SOIL . . .

EARTH DAMS
See DAMS, EARTH

EARTH MOVEMENTS
See EARTHQUAKES

EARTH PRESSURE
"Pile Tests, Low-Sill Structure, Old River, Louisiana," Charles I. Mansur and Robert I. Kaufman (with discussion), 715.

EARTH PRESSURE (*Continued*)
"Soil Modulus for Laterally Loaded Piles," Bramlette McClelland and John A. Focht, Jr. (with discussion), 1049.

EARTHQUAKES
See also VIBRATION; STRUCTURES, THEORY OF—Earthquake Problems
"Earthquake Resistance of Rock-Fill Dams," Ray W. Clough and David Pirtz (with discussion), 792.

EARTHS
See SOILS

EARTHWORK
See DAMS, EARTH; DRAINAGE; FOUNDATIONS...; LEVEES; REVETMENT; SOILS

ECONOMICS
See CONTRACTS (cross reference thereunder); COSTS...; VALUATION; *see also* under relative subject, subheading Financing

EDDIES
See WATER, FLOW OF, IN OPEN CHANNELS

EDUCATION
"The Engineer's Place in the Second Century of Technology": Address at the Summer Convention, Portland, Ore., June 25, 1958, Louis R. Howson, 1275.

EFFLUENTS, SEWAGE
See SEWAGE DISPOSAL

ELASTIC CURVES
See CURVES (elastic curves) (cross reference thereunder)

ELASTICITY
See also PLASTICITY; STRESS AND STRAIN
"Moment-Distribution Constants from Models," Otakar Ondra (with discussion), 318.
"Plastics: Engineering Materials," C. Howard Adams, 511.
"A Pressure-Line Concept for Inelastic Bending," Frank Baron (with discussion), 1039.
"Soil Modulus for Laterally Loaded Piles," Bramlette McClelland and John A. Focht, Jr. (with discussion), 1049.
"Truss Analysis by Stiffness Considerations," Harold C. Martin, 1182.

ELECTRIC POWER PLANTS
See POWER PLANTS

ELECTRONIC COMPUTERS
See ELECTRONIC INSTRUMENTS

ELECTRONIC INSTRUMENTS
Digital computers of particular value in analysis of redundant structures, 1182.

EMPLOYEES AND EMPLOYMENT
Toxic wastes and other harmful byproducts in relation to employee protection in Knoxville, Tennessee, 595.

ENERGY
See HYDRODYNAMICS; WATER, FLOW OF...

ENERGY, LOSS OF
See EVAPORATION; FRICTION ...; WATER, FLOW OF...

ENGINEERING
See ENGINEERS AND ENGINEERING

ENGINEERING AND WAR
See cross references under WAR AND ENGINEERING

ENGINEERING BIBLIOGRAPHY
See BIBLIOGRAPHY (cross references thereunder)

ENGINEERING EDUCATION
See EDUCATION

ENGINEERING GLOSSARIES
See TERMINOLOGY

ENGINEERING HISTORY
See AMERICAN SOCIETY OF CIVIL ENGINEERS; ENGINEERS AND ENGINEERING—History; *see also* under relative subject, e.g., HARBORS (Geographical)

ENGINEERING MECHANICS
See MECHANICS... (cross references thereunder)

ENGINEERING SOCIETIES
See SOCIETIES, TECHNICAL (cross references thereunder); *see also* AMERICAN SOCIETY OF CIVIL ENGINEERS

ENGINEERS AND ENGINEERING (General)
See also AMERICAN SOCIETY OF CIVIL ENGINEERS. (For memoirs of deceased members, *see* name of member in Author Index.) (*See also* p. 1283); EDUCATION; RESEARCH; SOCIETIES, TECHNICAL (cross references thereunder); TERMINOLOGY

ENGINEERS AND ENGINEERING (General) (*Continued*)

History

"The Engineer's Place in the Second Century of Technology": Address at the Summer Convention, Portland, Ore., June 25, 1958, Louis R. Howson, 1275.

Present and Future Trends

"Bureau of Public Roads Experiences in Highway Surveys," William T. Pryor, 1026.

"The Engineer's Place in the Second Century of Technology": Address at the Summer Convention, Portland, Ore., June 25, 1958, Louis R. Howson, 1275.

Professional Relationships

Negotiating contracts with photogrammetric engineering firms as an ideal procedure in highway surveys, 1037.

ENGINES

See POWER PLANTS

ENTRANCEWAYS

See PORTALS AND ENTRANCEWAYS (cross reference thereunder)

EQUATIONS

See also under relative subject

"Shells of Double Curvature," Alfred L. Parme (with discussion), 989.

ERECTION

See CONSTRUCTION (cross references thereunder)

EROSION, BEACH

"Harrison County (Mississippi) Artificial Beach," Francis F. Escoffier, 817.

EROSION, LAND

Control accomplished by planting of "ice plant," 214.

EROSION, STREAM

"Design of Stable Canals and Channels in Erodible Material," Pete W. Terrell and Whitney M. Borland, 101.

"Sediment-Transport Mechanics in Stable-Channel Design," Emmett M. Laursen (with discussion), 195.

ESTIMATES

See COSTS

ETHICS

See ENGINEERS AND ENGINEERING—Professional Relationships

EVALUATION

See VALUATION

EVAPORATION

See also CONDENSERS AND CONDENSATION; WATER, CONSUMPTIVE USE OF (in agriculture)

"Evaporation from Free Water Surfaces at High Altitudes," Harry F. Blaney, 385. *Discussion:* Carl Rohwer; Kenneth M. Turner; Irvin M. Ingerson and John W. Shannon; and Harry F. Blaney, 396.

EVAPORATION STATIONS

Monthly evaporation data for pans at high altitudes, United States Weather Bureau stations in New Mexico, California, Utah and Colorado, 391.

EXCAVATION (General)

See FOUNDATIONS...; TUNNELS...

EXCAVATION, HYDRAULIC

See DREDGES AND DREDGING

EXPERIMENTS AND EXPERIMENTATION

See HYDRAULIC LABORATORIES; MODELS...; TESTS AND TESTING (cross references thereunder); *also* under relative subject; *also* under material, structure or structural part tested, e.g., FLUMES

EXPLORATION

See BORINGS; FOUNDATIONS...

EXPRESSWAYS

See HIGHWAYS AND ROADS

FABRICATION

See under relative subject, e.g., PLASTIC MATERIALS

FACT FINDING

See RESEARCH

FACTOR, ROUGHNESS

See cross references under ROUGHNESS FACTOR

FAILURES (General)

See also BEARING CAPACITY; SAFETY (cross reference thereunder); STRENGTH OF MATERIALS; STRESS AND STRAIN

FAILURES, FOUNDATION

"Experiences with Loess As Foundation Material," William A. Clevenger (with discussion), 151.

FAILURES, JOINT

"Bearing-Ratio Effect on Strength of Riveted Joints," Jonathan Jones, 964.

FASTENERS AND FASTENING

See CONNECTORS AND CONNECTIONS

FATIGUE

See under relative subject, e.g., STRESS AND STRAIN—Joints; *see also* FAILURES

FILLING, HYDRAULIC

See HYDRAULIC FILLING

FILTERS AND FILTRATION, SEWAGE

See also SEWAGE DISPOSAL

"An Improved Dilution Method for Flow Measurements," William A. Cawley and Jack W. Woods, 405.

FINANCE

See COSTS . . . ; VALUATION; *also* subheading Financing under relative subject, e.g., AIRPORTS—Financing

FINITE DIFFERENCES, THEORY OF

See MATHEMATICS

FIRE PROTECTION

See WATER SUPPLY

FIRE STREAMS

See WATER, FLOW OF, THROUGH ORIFICES

FISH INDUSTRY

See under relative technical subject, e.g., WATER POLLUTION

FLEETS

See BARGES

FLEXURE

See STRESS AND STRAIN; STRUCTURES, THEORY OF

FLOOD CONTROL

See FLOOD ROUTING; FLOODS; RESERVOIRS, FLOOD CONTROL

FLOOD HYDROGRAPHS

See HYDROGRAPHS, STREAM FLOW

FLOOD ROUTING

See also WATER DIVERSION

"Old River Diversion Control": A Symposium, John R. Hardin; Eugene A. Graves; Willard J. Turnbull and Woodland G. Shockley; and Norman R. Moore, 1129.

FLOODS (General)

See also COSTS, FLOOD; DRAINAGE; HYDROGRAPHS, STREAM FLOW; LEVEES; MULTI-PURPOSE PROJECTS (cross references thereunder); RAINFALL; RESERVOIRS, FLOOD CONTROL; RUNOFF; SPILLWAYS

Floods (Geographical)

Houston, Tex.

"A Floodway for Houston, Tex.," Kenneth Heagy, 181.

Mississippi River

"Old River Diversion Control": A Symposium, John R. Hardin; Eugene A. Graves; Willard J. Turnbull and Woodland G. Shockley; and Norman R. Moore, 1129.

FLOODWAYS

See FLOODS

FLOOR BEAMS

See BEAMS . . .

FLOORS AND FLOORING

See BRIDGE FLOORS AND FLOORING; STRESS AND STRAIN—Floors and Flooring

FLOW

See FLOODS; RUNOFF; SEEPAGE; SEWAGE . . . ; SEWERS; TURBULENCE; UNDERFLOW (cross references thereunder); WATER, FLOW OF . . .

FLOW CONTROL AND CONTROLLERS

See WATER, FLOW OF, THROUGH ORIFICES

FLOW, LAMINAR

See WATER, FLOW OF, IN OPEN CHANNELS; WATER, FLOW OF, IN PIPES

FLOW METERS

See METERS AND METERING

FLOW NETS

See HYDRAULIC FLOW NETS

FLOW OF FLUIDS

See TUNNELS, WATER; WATER, FLOW OF . . .

FLOW OF LIQUIDS

See VISCOSITY; WATER, FLOW OF . . .

FLOW OF SOLIDS

See SOLIDS, FLOW OF (cross references thereunder)

FLOW OF WATER

See WATER, FLOW OF . . .

FLUES

See CHIMNEYS AND FLUES

FLUID DYNAMICS

See HYDRODYNAMICS

FLUID FRICTION

See FRICTION

FLUID FRICTION COEFFICIENTS

See FRICTION COEFFICIENTS, FLUID

FLUID MECHANICS
See WATER, FLOW OF...

FLUIDS, FLOW OF
See TUNNELS, WATER; WATER, FLOW OF...

FLUMES
See also SLUICES; WATER, FLOW OF, IN OPEN CHANNELS
Accuracy experiments on the Palmer-Bowlus flume, 749, 772, 774.
"Design of Venturi Flumes in Circular Conduits," Edwin A. Wells, Jr. and Harold B. Gotaas, 749. *Discussion:* Keeno Fraschina, and Edwin A. Wells, Jr. and Harold B. Gotaas, 772.
"A Flow Controller for Open or Closed Conduits," Victor L. Streeter, 883.
"Mechanics of Streams with Movable Beds of Fine Sand," Norman H. Brooks (with discussion), 526.

FLUVIAL MODELS
See MODELS, HYDRAULIC

FORMULAS
See under relative subject, e.g., PILES AND PILE DRIVING; SILT AND SILTING, CHANNEL

FOUNDATIONS (General)
See also BEARING CAPACITY; BORINGS; EARTH PRESSURE; FAILURES, FOUNDATION; PILES AND PILE DRIVING; SEEPAGE; SOILS; WATER PRESSURE; *also* under special material, e.g., LOESS; SOILS
"Experiences with Loess As Foundation Material," William A. Clevenger (with discussion), 151.

FOUNDATIONS, PILE
See PILES AND PILE DRIVING

FOUNDATIONS, UNDERWATER
"Old River Diversion Control": A Symposium, John R. Hardin; Eugene A. Graves; Willard J. Turnbull and Woodland G. Shockley; and Norman R. Moore, 1129.

FRAMES (General)
See also BEAMS...; COLUMNS; GIRDERS...
"Minimum-Weight Design of a Portal Frame," William Prager, 66. *Discussion:* Robert L. Ketter, and William Prager, 75.

FRAMES, CONTINUOUS
See STRESS AND STRAIN—Frames, Continuous; *see also* BEAMS...; COLUMNS; GIRDERS...

FRAMES, RIGID
See STRUCTURES, THEORY OF —Frames, Rigid

FRAMEWORKS
See STRUCTURES, THEORY OF

FREEWAYS
See HIGHWAYS AND ROADS (General)—Location

FREIGHT
See BARGES

FREIGHT TERMINALS (structures and localities)
Roll on, roll off and lift on, lift off cargo vessels, and container handling facilities, 851.
"Shoreside Facilities for Special-Purpose Ships," Howard J. Marsden, 851.

FRICTION
See also TURBULENCE
"Open-Channel Flow at Small Reynolds Numbers," Lorenz G. Straub, Edward Silberman and Herbert C. Nelson (with discussion), 685.
"Pile Tests, Low-Sill Structure, Old River, Louisiana," Charles I. Mansur and Robert I. Kaufman (with discussion), 715.

FRICTION COEFFICIENTS, FLUID
"Friction Measurements in the Apalachia Tunnel," Rex A. Elder, 1249.
"Mechanics of Streams with Movable Beds of Fine Sand," Norman H. Brooks (with discussion), 526.

GAGES (General)
See also METERS AND METERING...; *also* cross references under INSTRUMENTS

GAGES, PRESSURE
Manometer installations in Apalachia Tunnel of TVA, 1256.
Piezometers used to measure drawdown and loss of head in wells, 871.
Tensiometers used in saturated soil column pressure measurements, 118.

GAGES, STRAIN
"Dynamic Stresses in Continuous Plate-Girder Bridges," Roy C. Edgerton and Gordon W. Beecroft (with discussion), 266.
"Moments in Flat Slabs," Mark W. Huggins and Watone L. Lin (with discussion), 824.
"Soil Modulus for Laterally Loaded Piles," Bramlette McClelland and John A. Focht, Jr. (with discussion), 1049.

GATES

See under general types of gates, e.g., WATER GATES, MOVABLE; *see also* under specific type of gate, e.g., HEAD GATES; TAINTER GATES (cross reference thereunder)

GIRDER BRIDGES

See BRIDGES, GIRDER

GIRDERS (General)

See also BEAMS (General); BRIDGES; CONNECTORS AND CONNECTIONS; *also* under name of material, e.g., STEEL

"Anchorages for Large Tainter Gates," Alexander H. Kenigsberg (with discussion), 409.

GIRDERS, CONTINUOUS

See BRIDGES, GIRDER; STRUCTURES, THEORY OF—Beams and Girders, Continuous

GLOSSARIES

See TERMINOLOGY

GOVERNMENT

See AUTHORITIES (cross reference thereunder); PUBLIC...; *also* under LAW subject heading under related topic, e.g., INDUSTRIAL WASTE LAW

GRAPHICAL CHARTS

See HYDROGRAPHS...; *also* under relative subject

GRAVITY DAMS

See DAMS, MASONRY AND CONCRETE

GRIDWORKS

See BRIDGE FLOORS AND FLOORING

GROINS

"Harrison County (Mississippi) Artificial Beach," Francis F. Escoffier, 817.

"Wave Run-Up on Shore Structures," Thorndike Saville, Jr., 139.

GROUND...

See also EARTH...; LAND... (cross references thereunder); SOIL...

GROUND MOVEMENTS

See EARTHQUAKES

GROUND WATER

See also SEEPAGE; WATER SUPPLY

Ground water requirements for United States, 1900, 1955, and estimated increase by year 1975, 778.

"Importance of Ground Water in the National Economy," Albert G. Fiedler, 776.

"Laboratory and In-Situ Permeability of Sand," Charles I. Mansur, 868.

GROUND WATER LAW

New Mexico statute a model for similar laws in several states, 780, 781.

GROUNDWORK

See EARTHWORK (cross references thereunder); FOUNDATIONS

GROYNES

See GROINS

GUSSET PLATES

See PLATES (cross references thereunder)

HARBORS (General)

See also BARGES; BREAKWATERS; BULKHEADS; CHANNELS; COSTS, HARBOR IMPROVEMENT; DOCKS AND WHARVES; FREIGHT...; JETTIES; PILES AND PILE DRIVING; RETAINING WALLS; SHORES AND SHORE PROTECTION; TOWBOATS AND TOWING (cross reference thereunder); TRAFFIC, WATERWAY; WATER POLLUTION; WATER TERMINALS; WATER TRANSPORTATION; WAVES

HARBORS (Geographical)

Houston, Tex.

"Developing Port Facilities on Houston's Ship Channel," Frank H. Newnam, Jr., 239. *Discussion:* Ersel G. Lantz; Austin E. Brant, Jr.; and Frank H. Newnam, Jr., 258.

Mobile, Ala.

History of channel improvement since 1826 and its effect on commerce and industry, 1195, 1197, 1203.

"Mobile (Ala.) Harbor and Ship Channel," Harold E. Bisbort, 1195.

New York, N. Y.

Lack of obstruction in the port of New York, 956.

"Transportation Planning": A Symposium, Roger H. Gilman, Wilfred M. Post, Jr. and Burton W. Marsh, 359.

United States

"Developing Port Facilities on Houston's Ship Channel," Frank H. Newnam, Jr. (with discussion), 239.

HEAD GATES

See also INTAKES

"Trends in Hydraulic Gate Design," Dow A. Buzzell (with discussion), 27.

HEAD, LOSS OF
See AERATION; FRICTION; WATER, FLOW OF...
HEALTH, PERSONAL
See EMPLOYEES AND EMPLOYMENT
HEALTH, PUBLIC
See PUBLIC HEALTH
HIGHWAY AND ROAD LAW
Controlled access legislation enacted in regard to the Indiana State Highway System, 477, 481, 485.
Federal Aid Highway Act of 1956 featuring National System of Interstate and Defense Highways and additional needed government coordination of operation cited, 859.
HIGHWAY BRIDGE FLOORS
See BRIDGE FLOORS AND FLOORING
HIGHWAY BRIDGES
See BRIDGES
HIGHWAY ENGINEERS AND ENGINEERING
See ENGINEERS AND ENGINEERING—Present and Future Trends; HIGHWAYS AND ROADS (General)
HIGHWAY LAW
See HIGHWAY AND ROAD LAW
HIGHWAYS AND ROADS (General)
See also BRIDGE...; COSTS, HIGHWAY AND ROAD; IMPACT; RIGHTS OF WAY (land strips); SOILS; TRAFFIC...; WHEEL LOADS
"Bureau of Public Roads Experiences in Highway Surveys," William T. Pryor, 1026.
Financing
"Control of Highway Access": A Symposium, Curtis J. Hooper, Bertram D. Tallamy, Dewitt C. Greer, Carl E. Vogelgesang, Hugo H. Winter, and Ralph A. Moyer, 465.
Location
"Control of Highway Access": A Symposium, Curtis J. Hooper, Bertram D. Tallamy, Dewitt C. Greer, Carl E. Vogelgesang, Hugo H. Winter, and Ralph A. Moyer, 465.
Operation
Design of operational facilities of controlled access highways, 863.
"Operation Problems on Controlled-Access Highways," Charles M. Noble, 858.
Roadbeds. *See* SOILS
Safety
High speed and accident study of California freeways, 490, 494, 505.
"Operation Problems on Controlled-Access Highways," Charles M. Noble, 858.
HIGHWAYS AND ROADS (Geographical)
California
User benefits of freeway system, 499.
Indiana
Benefits of controlled access in a bypass and dual lane system, 477.
Los Angeles Area
Development and construction of the Los Angeles metropolitan freeway system, with resulting benefits, 486, 495, 498.
New York (state)
New York State Thruway as a controlled access highway with economic development facts enumerated, 467.
Texas
Economic effects of the Gulf Freeway on adjacent properties, 472.
United States
Analysis of large scale California freeway project, a forerunner of 1954 and 1955 national highway program, 499.
"Control of Highway Access": A Symposium, Curtis J. Hooper, Bertram D. Tallamy, Dewitt C. Greer, Carl E. Vogelgesang, Hugo H. Winter, and Ralph A. Moyer, 465.
HIGHWAY TRAFFIC
See TRAFFIC, HIGHWAY AND ROAD
HIGHWAY TRANSPORTATION
See also HIGHWAYS AND ROADS—Location; TRANSPORTATION
"Bridge Clearances": A Symposium, Eugene W. Weber, Walter Kurylo, William E. Cleary, Nickles L. Caruthers, and Erhard E. Dittbrenner, 925.
"Control of Highway Access": A Symposium, Curtis J. Hooper, Bertram D. Tallamy, Dewitt C. Greer, Carl E. Vogelgesang, Hugo H. Winter, and Ralph A. Moyer, 465.
"Operation Problems on Controlled-Access Highways," Charles M. Noble, 858.
HINGED STRUCTURES
See JOINTS

HISTORY, ENGINEERING

See AMERICAN SOCIETY OF CIVIL ENGINEERS; ENGINEERS AND ENGINEERING—History; *see also* under relative subject, e.g., HARBORS (Geographical)

HOISTING MACHINERY

See under specific types of machines, e.g., DREDGES AND DREDGING

HOUSE DRAINAGE

See HOUSES—Drainage

HOUSES (places of abode)

See also PLANTS (cross references thereunder); POWERHOUSES (cross reference thereunder)

Denver residence foundation failures related to loess, 164.

"Experiences with Loess As Foundation Material," William A. Clevenger (with discussion), 151.

Drainage. *See also* SEWERS

"Developments in Septic Tank Systems," John E. Kiker, Jr., 77.

HURRICANES

See WIND... (cross reference thereunder)

HYDRAULIC EXCAVATION

See EXCAVATION, HYDRAULIC (cross reference thereunder)

HYDRAULIC FILLING

See also DAMS, EARTH

"Harrison County (Mississippi) Artificial Beach," Francis F. Escoffier, 817.

HYDRAULIC FLOW NETS

"Laboratory and In-Situ Permeability of Sand," Charles I. Mansur, 868.

HYDRAULIC GATES

See WATER GATES, MOVABLE

HYDRAULIC LABORATORIES

See also EVAPORATION STATIONS; MODELS, HYDRAULIC

"Mechanics of Streams with Movable Beds of Fine Sand," Norman H. Brooks (with discussion), 526.

HYDRAULIC MACHINERY

See under type of machinery, e.g., PUMPS AND PUMPING

HYDRAULIC MODELS

See MODELS, HYDRAULIC

HYDRAULICS

See also AERATION; BREAKWATERS; BULKHEADS; CANALS; CHANNEL...; CONDUITS; COSTS...; DAMS...; DOCKS AND WHARVES; DRAINAGE; DREDGES AND DREDGING; EROSION...; EVAPORATION; FILTERS AND FILTRATION...; FLOOD...; FLOW... (cross references thereunder); FLUID... (cross references thereunder); FLUMES; FOUNDATIONS...; FRICTION...; GAGES...; GROUND WATER; HARBORS...; HEAD GATES; HYDRAULIC...; HYDRO-...; INTAKES; IRRIGATION...; JETTIES; LAKES; LEVEES; LOCKS; METERS AND METERING...; MODELS, HYDRAULIC; PILES AND PILE DRIVING; PIPE...; POWER...; PUMPS AND PUMPING; RAINFALL; RESERVOIRS...; RETAINING WALLS; REVETMENT; RIVER...; RUNOFF; SEA WALLS; SEDIMENT AND SEDIMENTATION; SEEPAGE; SEWAGE...; SEWERS; SHORES AND SHORE PROTECTION; SILLS; SILT AND SILTING...; SLUICE...; SPILLWAY...; SPREADING BASINS; SPREADING, WATER; STILLING BASINS (cross reference thereunder); STRESS AND STRAIN...; TANKS...; TERMINOLOGY; TUNNELS, WATER; TURBULENCE; VALLEY...; VISCOSITY; WATER...; WAVES; WELLS

"A Flow Controller for Open or Closed Conduits," Victor L. Streeter, 883.

HYDRAULIC STRUCTURES

See under type of structure, e.g., DAMS; SPILLWAYS

HYDRODYNAMICS

"Open-Channel Flow at Small Reynolds Numbers," Lorenz G. Straub, Edward Silberman and Herbert C. Nelson (with discussion), 685.

HYDROGRAPHS, RIVER

See HYDROGRAPHS, STREAM FLOW

HYDROGRAPHS, STREAM FLOW

1943 Mississippi River discharge hydrographs for moderate flow and 1945 for large flow, 1154, 1156, 1157.

HYDROLOGY

See also DRAINAGE; EVAPORATION; FLOOD...; GROUND WATER; HYDROGRAPHS; INFILTRATION (cross reference thereunder); LAKES; MARSHES; RAINFALL; RIVER...; RUNOFF; WATER...

HYDROLOGY (*Continued*)
Hydrologic cycle as affected by natural influences, 1207.

HYDROMETRY
See WATER, FLOW OF...

HYDROSTATIC PRESSURE
See HYDROSTATIC UPLIFT (cross reference thereunder)

HYDROSTATIC UPLIFT
See WATER PRESSURE

IMPACT
See also VIBRATION; WHEEL LOADS
"Dynamic Stresses in Continuous Plate-Girder Bridges," Roy C. Edgerton and Gordon W. Beecroft (with discussion), 266.

IMPOUNDING, WATER
See RESERVOIRS, WATER STORAGE; WATER STORAGE

INDUSTRIAL PLANTS
See also under specific type of plant, e.g., POWER PLANTS; *also* cross references under WORKS
"Municipal Ordinances for Industrial Wastes," Julian R. Fleming, 595.

INDUSTRIAL WASTE
See also COSTS, INDUSTRIAL WASTE; SEWAGE DISPOSAL; SEWAGE SLUDGE
Manganese used as a trace element, 408.

INDUSTRIAL WASTE LAW
"Municipal Ordinances for Industrial Wastes," Julian R. Fleming, 595.

INDUSTRIAL WATER SUPPLY
See IRRIGATION; WATER SUPPLY

INDUSTRY
See under relative technical classification, e.g., HIGHWAYS AND ROADS; *see also* EMPLOYEES AND EMPLOYMENT; *also* under type of industry or industrial plant, e.g., WELL DRILLING INDUSTRY

INFILTRATION
See FLOODS; GROUND WATER; LAND...; RUNOFF; SEEPAGE

INFLUENCE LINES
See MOMENTS

INLAND WATERWAYS
See WATERWAYS

INSECT CONTROL
See PUBLIC HEALTH

INSTRUMENTS
See GAGES; METERS AND METERING...; *also* under general types of instruments, e.g., ELECTRONIC INSTRUMENTS; *also* under usage; *also* cross reference under specific type of instrument, e.g., LYSIMETERS; *also* cross references under APPARATUS

INTAKE GATES
See INTAKES

INTAKES
See also HEAD GATES
"Effects of Storage Impoundments on Water Quality," Milo A. Churchill, 419.
"Trends in Hydraulic Gate Design," Dow A. Buzzell (with discussion), 27.

IRON
See also CORROSION AND PROTECTION OF METALS; STEEL
Impoundment effect on iron and manganese in water systems, 452.

IRRIGATION (General)
See also WATER, CONSUMPTIVE USE OF; WATER SUPPLY
Mosquito propagation and control as related to irrigation structures and practices, 847.
"Relationship of Irrigation to Public Health," Lloyd E. Myers, Jr., 846.

IRRIGATION (Geographical)
United States
"Surface Water Resources," Joseph V. B. Wells, 1206.
Western States
Water losses due to evaporation at high altitudes affecting crop irrigation, 385.

JETTIES
See also BREAKWATERS; SHORES AND SHORE PROTECTION; WATER PRESSURE; WAVES
"Wave Run-Up on Shore Structures," Thorndike Saville, Jr., 139.

JOINTS (General)
See also FAILURES, JOINT; STRESS AND STRAIN—Joints

JOINTS, RIVETED
"Bearing-Ratio Effect on Strength of Riveted Joints," Jonathan Jones, 964.
"Behavior of Riveted Truss-Type Connections," Eugene Chesson, Jr. and William H. Munse (with discussion), 1087.

LABORATORIES

See under specific type of laboratory, e.g., HYDRAULIC LABORATORIES; *also* under relative subject; *also* under MODELS...; TESTS AND TESTING (cross references thereunder); *also* under material, structure, or structural part tested

LAKES (General)

See also EVAPORATION; SHORES AND SHORE PROTECTION; WATER...; WAVES

LAKES (Geographical)

Sierra Nevada Mountains

Evaporation from free water surfaces in Huntington Lake area, California, 390, 394, 403, 404.

LAMINAR FLOW

See WATER, FLOW OF, IN OPEN CHANNELS; WATER, FLOW OF, IN PIPES

LAND...

See EARTH...; EROSION, LAND; GROUND...; MARSHES; SOIL...; SURVEYS AND SURVEYING...; VALUATION

LAND SUBMERGENCE

See SPREADING, WATER

LATERAL FORCES

See EARTHQUAKES

LATERAL PRESSURE

See EARTH PRESSURE

LAW

See LAW subject heading under related subject, e.g., WATER LAW; *see also* CONTRACTS (cross reference thereunder); WATER RIGHTS

LEGISLATION

See under relative subject (under the subject LAW heading, e.g., HIGHWAY AND ROAD LAW)

LEVEES

"Old River Diversion Control": A Symposium, John R. Hardin; Eugene A. Graves; Willard J. Turnbull and Woodland G. Shockley; and Norman R. Moore, 1129.

LIFT GATES

See WATER GATES, MOVABLE

LIFTING MACHINERY

See HOISTING MACHINERY

LIMIT DESIGN, THEORY OF

"Minimum-Weight Design of a Portal Frame," William Prager (with discussion), 66.

LININGS

See under relative type, e.g., TUNNEL LININGS

LIQUIDS, FLOW OF

See VISCOSITY; WATER, FLOW OF...

LITIGATION

See under relative subject (under the subject LAW heading)

LOAD

See BEARING CAPACITY; BUCKLING; EARTH PRESSURE; FAILURES...; IMPACT; STRESS AND STRAIN; STRUCTURES...; VIBRATION; WATER PRESSURE; WHEEL LOADS; *also* under structure, structural member or part, e.g., FOUNDATIONS; FRAMES

LOAD, BED

See CANALS; CHANNELS; RIVERS; SILT AND SILTING...; TURBULENCE; WATER, FLOW OF, IN OPEN CHANNELS

LOAD, SUSPENDED

See SEDIMENT AND SEDIMENTATION; SILT AND SILTING...

LOCKS

See also HEAD GATES

"Old River Diversion Control": A Symposium, John R. Hardin; Eugene A. Graves; Willard J. Turnbull and Woodland G. Shockley; and Norman R. Moore, 1129.

LOESS

"Experiences with Loess As Foundation Material," William A. Clevenger, 151. *Discussion:* Harry R. Cedergren; Ralph B. Peck and Herbert O. Ireland; and William A. Clevenger, 170.

Loess deposits in the United States, 172.

LOSS OF ENERGY

See EVAPORATION; FRICTION...; WATER, FLOW OF...

LOSS OF HEAD

See AERATION; FRICTION; WATER, FLOW OF...

LYSIMETERS

See WATER INSTRUMENTS

MACHINERY

See under general types of machinery, e.g., CONVEYERS AND CONVEYANCE; *also* under specific type of machine, e.g., DREDGES AND DREDGING; *also* under usage

MANGANESE

Dilution of manganese to measure flow in sewers, 405.

Impoundment effect on manganese and iron in water systems, 452.

MANOMETERS

See GAGES, PRESSURE

MAPS AND MAPPING (General)

See under relative subject

MARINE . . .

See SEA . . . ; WATER . . .

MARSHES

Swamps and sloughs as a cause of mosquitoes regarded a fallacy, 849.

MATERIALS OF CONSTRUCTION

See also AGGREGATES AND AGGREGATION; ALUMINUM; CLAY; CONCRETE; CORROSION AND PROTECTION OF . . . ; EROSION . . . ; FAILURES . . . ; IRON; LOESS; METALS (cross reference thereunder); PERMEABILITY OF MATERIALS (cross reference thereunder); SAND; SEEPAGE; SILT . . . ; SOILS; STEEL; STRENGTH OF MATERIALS; STRESS AND STRAIN—Materials of Construction; STRUCTURES, THEORY OF—Materials and Tests; *also* under types of materials, e.g., PLASTIC MATERIALS; *also* under usage

Physical properties of glass, steel, concrete and wood, and major types of plastics, 512, 514.

MATERIALS, PERMEABILITY OF

See PERMEABILITY OF MATERIALS (cross references thereunder)

MATERIALS, STRENGTH OF

See STRENGTH OF MATERIALS

MATERIALS, TESTING OF

See TESTS AND TESTING (cross references thereunder)

MATERIALS, WASTE

See cross references under WASTE DISPOSAL

MATHEMATICS

See also EQUATIONS; GRAPHICAL CHARTS (cross references thereunder); *also* under relative subject

Numerical procedure based on finite difference equations in analysis of shells of double curvature, 989.

"Shells of Double Curvature," Alfred L. Parme (with discussion), 989.

"Simplification of Design by Ultimate Strength Procedures," Phil M. Ferguson (with discussion), 602.

MEANDERS AND MEANDERING

See RIVERS

MEASURING INSTRUMENTS

See FLUMES; GAGES; METERS AND METERING; WATER INSTRUMENTS

MECHANICAL ENGINEERS AND ENGINEERING

See under relative subject, e.g., AUTOMOBILE . . . (cross references thereunder); MACHINERY (cross references thereunder)

MECHANICS (General)

See STRUCTURES, THEORY OF

MECHANICS, FLUID

See WATER, FLOW OF . . .

MECHANICS, SOIL

See SOIL . . .

MEMBERS (ASCE)

See AMERICAN SOCIETY OF CIVIL ENGINEERS. (For memoirs of deceased members, *see* name of member in Author Index.) (*See also* p. 1283)

MEMBERS, COMPRESSION

See COLUMNS

MEMBERSHIP (ASCE)

See AMERICAN SOCIETY OF CIVIL ENGINEERS

MEMBERS, STRUCTURAL

See BEAMS; COLUMNS; FRAMES . . . ; GIRDERS; STRUCTURES; *also* under name of material, e.g., CONCRETE; STEEL; *also* under structure, e.g., BRIDGES

MEMOIRS OF DECEASED MEMBERS

See name of member in Author Index. (*See also* p. 1283)

METALLURGY

See under specific metal, e.g., STEEL

METAL PROTECTION

See CORROSION AND PROTECTION OF METALS

METALS

See CORROSION AND PROTECTION OF METALS; MATERIALS OF CONSTRUCTION; REINFORCED CONCRETE (cross reference thereunder); STRENGTH OF MATERIALS; STRESS AND STRAIN . . . ; STRUCTURES, THEORY OF . . . ;

METALS (*Continued*)
also under specific metal or its alloy, e.g., ALUMINUM; STEEL

METEOROLOGY
See EVAPORATION; RAINFALL; WIND... (cross reference thereunder)

METERS AND METERING (General)
See also GAGES...; INSTRUMENTS (cross references thereunder)

METERS AND METERING, CURRENT
"A Flow Controller for Open or Closed Conduits," Victor L. Streeter, 883.

MILITARY ENGINEERS AND ENGINEERING
See under relative technical subject, e.g., AIRPORTS; BRIDGES

MINERALOGY
See under relative technical subject, e.g., SOILS

MINERALS
See under relative technical subject, e.g., CONCRETE; MATERIALS OF CONSTRUCTION; WATER, DEMINERALIZATION OF (cross reference thereunder)

MODELS, HYDRAULIC
"A Flow Controller for Open or Closed Conduits," Victor L. Streeter, 883.
"Hydraulic Model Study of Hyperion Sewer Interchange," Alfred C. Ingersoll and Hajime Tanaka, 908.
"Mechanics of Streams with Movable Beds of Fine Sand," Norman H. Brooks (with discussion), 526.
"The Viscous Sublayer Along a Smooth Boundary," Hans A. Einstein and Huon Li (with discussion), 293.

MODELS, STRUCTURAL
"Earthquake Resistance of Rock-Fill Dams," Ray W. Clough and David Pirtz (with discussion), 792.
"Load Distribution in Highway Bridge Decks," Arnold W. Hendry and Leslie G. Jaeger (with discussion), 1214.
"Moment-Distribution Constants from Models," Otakar Ondra (with discussion), 318.
"Moments in Flat Slabs," Mark W. Huggins and Watone L. Lin (with discussion), 824.
Usage of aluminum as a model material, 825.

MOISTURE
See EVAPORATION; LOESS; SEEPAGE; SOILS

MOMENT DISTRIBUTION
See MOMENTS; STRUCTURES, THEORY OF

MOMENTS
See also STRESS AND STRAIN; STRUCTURES, THEORY OF; *also* under specific type of stress, e.g., BUCKLING; TORSION
"Load Distribution in Highway Bridge Decks," Arnold W. Hendry and Leslie G. Jaeger (with discussion), 1214.
"Moment-Distribution Constants from Models," Otakar Ondra, 318. *Discussion:* John C. Murphy; Thomas D. Y. Fok; Frank J. Cain and Gerald Luck; and Otakar Ondra, 344.
"Moments in Flat Slabs," Mark W. Huggins and Watone L. Lin, 824. *Discussion:* James Chinn, and Mark W. Huggins and Watone L. Lin, 842.
"Simplification of Design by Ultimate Strength Procedures," Phil M. Ferguson (with discussion), 602.
"Soil Modulus for Laterally Loaded Piles," Bramlette McClelland and John A. Focht, Jr. (with discussion), 1049.

MOORINGS AND MOORAGES, SHIP
See DOCKS AND WHARVES

MOSQUITOES
See MARSHES; PUBLIC HEALTH

MOTOR TRAFFIC
See TRAFFIC...

MOTOR TRANSPORTATION
See HIGHWAY TRANSPORTATION

MOTOR TRUCKING
See HIGHWAY TRANSPORTATION; STREET TRANSPORTATION (cross references thereunder)

MOTOR VEHICLE ACCIDENTS
See HIGHWAYS AND ROADS—Safety

MOTOR VEHICLES
See TRAFFIC...; WHEEL LOADS

MOTORWAYS
See HIGHWAYS AND ROADS

MOVABLE WATER GATES
See WATER GATES, MOVABLE

MULTI-PURPOSE PROJECTS (RIVER PROJECTS)
See COSTS, MULTI-PURPOSE PROJECT (RIVER PROJECTS); RIVER VALLEY AUTHORITIES

MULTI-PURPOSE RESERVOIRS
See RESERVOIRS, MULTI-PURPOSE (cross reference thereunder)

MUNICIPAL ENGINEERS AND ENGINEERING
See ENGINEERS AND ENGINEERING; *see also* AIRPORTS; BRIDGES; CITIES; COSTS...; DRAINAGE; FILTERS AND FILTRATION...; HARBORS; INDUSTRIAL...; POWER...; PUBLIC HEALTH; SANITATION (cross references thereunder); SEWAGE...; SEWERS; TRAFFIC...; WATER...; and similar relative subjects

MUNICIPALITIES
See CITIES

NAVIGATION
See BRIDGES; CANALS; CHANNELS; DAMS...; ELECTRONIC INSTRUMENTS; FLOODS; HARBORS; LAKES; LOCKS; MULTI-PURPOSE PROJECTS (cross references thereunder); RIVERS; RIVER VALLEY AUTHORITIES; WATER RIGHTS; WATER TRANSPORTATION; WATERWAYS

NAVIGATION LOCKS
See LOCKS

NOMENCLATURE
See TERMINOLOGY

NONUNIFORM FLOW
See WATER, FLOW OF...

NOZZLES
See WATER, FLOW OF, THROUGH ORIFICES

NUISANCES, ABATEMENT OF
See INDUSTRIAL WASTE; INSECT CONTROL (cross reference thereunder); PUBLIC HEALTH

OBITUARIES OF MEMBERS
See name of member in Author Index. (*See also* p. 1283)

OCEAN...
See also SEA...; WAVES

OCEAN BEACHES
See EROSION, BEACH; SHORES AND SHORE PROTECTION

OCEAN CURRENTS
See EROSION, BEACH

OCEAN WAVES
See WAVES

OPEN CHANNELS
See CHANNELS; WATER, FLOW OF, IN OPEN CHANNELS

ORDINANCES
See LAW subject heading under related subject, e.g., INDUSTRIAL WASTE LAW

ORGANIZATIONS
See AMERICAN SOCIETY OF CIVIL ENGINEERS

ORIFICES
See WATER, FLOW OF, THROUGH ORIFICES

OSCILLATION
See VIBRATION

OSCILLOGRAPHS
See VIBRATION RECORDING APPARATUS

OXIDATION
See CORROSION AND PROTECTION OF METALS

OXYGEN
See OXYGENATION

OXYGENATION
See also AERATION
"Effects of Storage Impoundments on Water Quality," Milo A. Churchill, 419.
"Mechanism of Reaeration in Natural Streams," Donald J. O'Connor and William E. Dobbins (with discussion), 641.
Bibliography
Re-aeration in natural streams, 663.

PERCOLATION
See GROUND WATER; SEEPAGE; SEWAGE DISPOSAL; SOILS

PERMEABILITY OF MATERIALS
See CORROSION AND PROTECTION OF...; SEEPAGE; *also* under type of material, e.g., SAND; SOILS

PERSONNEL
See EMPLOYEES AND EMPLOYMENT

PHOTOGRAMMETRY
See SURVEYS AND SURVEYING, AERIAL

PIERS (waterway structures)
See DOCKS AND WHARVES

PIERS, BAFFLE
See SPILLWAYS

PIEZOMETERS

See GAGES, PRESSURE

PILE DRIVERS (machinery)

See PILES AND PILE DRIVING

PILES AND PILE DRIVING

See also BEARING CAPACITY; FOUNDATIONS, UNDERWATER; STRESS AND STRAIN—Piles

Formulas used in pile testing on structure on Old River, Louisiana and their relation to failure loads, 737.

Loess formations and their effect on pile driving efficiency, 177.

"Old River Diversion Control": A Symposium, John R. Hardin; Eugene A. Graves; Willard J. Turnbull and Woodland G. Shockley; and Norman R. Moore, 1129.

"Pile Tests, Low-Sill Structure, Old River, Louisiana," Charles I. Mansur and Robert I. Kaufman, 715. *Discussion:* Yoshichika Nishida; Stanley F. Gizienski; and Charles I. Mansur and Robert I. Kaufman, 744.

"Soil Modulus for Laterally Loaded Piles," Bramlette McClelland and John A. Focht, Jr. (with discussion), 1049.

PIN-CONNECTED TRUSSES

See TRUSSES, PIN-CONNECTED

PIPE LINES

See also PIPES AND PIPING (cross references thereunder); PUMPS AND PUMPING; TRANSPORTATION; WATER, FLOW OF, IN PIPES

Plastics applied in pipe lines used for corrosive liquids and their advantages, 518.

PIPES AND PIPING (fluid conveyance)

See CONDUITS; CORROSION AND PROTECTION OF...; PIPE LINES; SEWERS; TUNNELS, WATER; WATER, FLOW OF, IN PIPES

PLANNING

See under relative subject, e.g., TRANSPORTATION

PLANTS (industrial buildings and equipment)

See INDUSTRIAL PLANTS; *see also* under type of plant, e.g., POWER PLANTS; *also* HOUSES; WORKS (cross references thereunder)

PLASTICITY

See also ELASTICITY; STRESS AND STRAIN

"Minimum-Weight Design of a Portal Frame," William Prager (with discussion), 66.

"A Pressure-Line Concept for Inelastic Bending," Frank Baron (with discussion), 1039.

"Simplification of Design by Ultimate Strength Procedures," Phil M. Ferguson (with discussion), 602.

PLASTIC MATERIALS

See also STRESS AND STRAIN—Plastics

Design considerations and problems in designing plastic structures, 520.

"Plastics: Engineering Materials," C. Howard Adams, 511.

Bibliography

Comprehensive sources available on plastics, 519.

Plastics as engineering materials, 523.

PLASTIC THEORY

See PLASTICITY

PLATES

See BUCKLING; JOINTS...; SLABS; STRESS AND STRAIN—Plates

POLLUTION, STREAM

See WATER POLLUTION

PONDAGE

See WATER STORAGE

POPULATION

"The Engineer's Place in the Second Century of Technology": Address at the Summer Convention, Portland, Ore., June 25, 1958, Louis R. Howson, 1275.

POROSITY

See SEEPAGE

PORTALS AND ENTRANCEWAYS

See FRAMES (General)

PORTS

See AIRPORTS; HARBORS

POWER (General)

See DAMS; PUMPS AND PUMPING; WATER POWER (cross references thereunder)

POWERHOUSES

See POWER PLANTS

POWER PLANTS (General)

See also CHIMNEYS AND FLUES; COSTS, POWER PLANT

Unusual design features described in detail, 207.

POWER PLANTS (Geographical)
California
"Morro Bay Steam Electric Plant," J. George Thon and Gordon L. Coltrin, 207. *Discussion:* Glenn B. Woodruff; Jacob Feld; William W. Moore; George L. Jordy; and J. George Thon and Gordon L. Coltrin, 233.
Thermal mussel control problem in Pacific Ocean coastal waters, 211.
Tennessee River Valley
"Friction Measurements in the Apalachia Tunnel," Rex A. Elder, 1249.

PRECIPITATION
See RAINFALL

PRESERVATION OF...
See CORROSION AND PROTECTION OF...

PRESIDENTIAL ADDRESSES (American Society of Civil Engineers)
See AMERICAN SOCIETY OF CIVIL ENGINEERS—Addresses; *see also* under subject of address

PRESSURE
See EARTH PRESSURE; IMPACT; PUMPS AND PUMPING; STRESS AND STRAIN; WATER PRESSURE; WAVES

PRESSURE GAGES
See GAGES, PRESSURE

PRESSURE TRANSMISSION THROUGH SOLIDS
See EARTH PRESSURE

PRESSURE TUNNELS
See TUNNELS, WATER

PRIME MOVERS
See ENERGY (cross references thereunder)

PROFESSIONAL STANDARDS
See ENGINEERS AND ENGINEERING—Professional Relationships

PROPERTY (landed property)
See LAND ... (cross references thereunder); RIGHTS OF WAY (land strips); SURVEYS AND SURVEYING; VALUATION

PROTECTIVE COATINGS
See CORROSION AND PROTECTION OF...

PROTECTIVE WORKS
See under relative type, e.g., GROINS; *see also* WAVES

PROTOTYPES AND MODELS
See MODELS

PUBLIC HEALTH
See also SANITATION (cross references thereunder); WATER SUPPLY
Mosquito control especially in relation to its economic importance, 847.
"Relationship of Irrigation to Public Health," Lloyd E. Myers, Jr., 846.

PUBLIC SERVICES
See CITIES; GOVERNMENT (cross references thereunder); *see also* under type of service, e.g., SEWAGE DISPOSAL

PUBLIC UTILITIES
See POWER PLANTS

PUBLIC WORKS
See COSTS...; GOVERNMENT (cross references thereunder); RIVER VALLEY AUTHORITIES; *also* under type of structure or project

PUMPS AND PUMPING (General)
"Morro Bay Steam Electric Plant," J. George Thon and Gordon L. Coltrin (with discussion), 207.

PURIFICATION
See WATER TREATMENT

RAILROAD BRIDGES
See BRIDGES

RAILROADS (General)
See BRIDGES; FREIGHT...; TRAIN... (cross reference thereunder); TRANSPORTATION

RAILROAD TERMINALS (structures and localities)
See WATER TERMINALS

RAILROAD TRACKS
See EARTHWORK (cross references thereunder); WHEEL LOADS

RAILROAD TRAINS
See WHEEL LOADS

RAIL TRANSPORTATION
See TRANSPORTATION

RAINFALL (General)
See also HYDROLOGY; RUNOFF; SEEPAGE
"Surface Water Resources," Joseph V. B. Wells, 1206.

RAINFALL (Geographical)
Houston, Tex.
"A Floodway for Houston, Tex.," Kenneth Heagy, 181.

RAINFALL (Geographical) (*Continued*)

Major storm localities, with statistics, 183.

United States

Drought conditions in the 1950's and area ranges, 1208.

Loess deposits as affected by rainfall, 174.

RE-AERATION

See AERATION

RECLAMATION, SEWAGE

See SEWAGE DISPOSAL; SEWAGE, UTILIZATION OF (cross reference thereunder)

RECLAMATION, WASTE WATER

See WASTE WATER, UTILIZATION OF (cross reference thereunder)

RECORDING APPARATUS

See under general types of apparatus, e.g., VIBRATION RECORDING APPARATUS; *see also* under relative subject

RECORDING INSTRUMENTS

See under relative subject

RECORD KEEPING

See HYDROGRAPHS... ; *see also* cross references under RECORDING... ; *also* under relative subject

RECREATIONAL FACILITIES

See BEACHES (cross references thereunder)

REDUNDANCE, STRUCTURAL

See STRUCTURES, THEORY OF

REGIME THEORY

See SILT AND SILTING...

REGIONAL PLANNING

See HIGHWAYS AND ROADS—Location

REINFORCED CONCRETE

See CONCRETE...

RELIEF WELLS

See WELLS

REOXYGENATION

See OXYGENATION

RESEARCH

See also HYDRAULIC LABORATORIES; MODELS, HYDRAULIC; STRUCTURES, THEORY OF; TESTS AND TESTING (cross references thereunder); *also* under relative subject, e.g., RIPARIAN RIGHTS; TANKS, SEPTIC

Researchers and their effects on industrial processes, 1027.

RESERVOIRS (General)

See also COSTS, RESERVOIR; DAMS; EVAPORATION; LAKES; SEDIMENT AND SEDIMENTATION; SPILLWAYS; WATER DIVERSION; WATER STORAGE

RESERVOIRS, FLOOD CONTROL

Buford Reservoir's function at Apalachicola River Basin project, 978.

RESERVOIRS, MULTI-PURPOSE

See COSTS, RESERVOIR (MULTI-PURPOSE RESERVOIRS)

RESERVOIRS, WATER STORAGE

"Effects of Storage Impoundments on Water Quality," Milo A. Churchill, 419.

Temperature aspects in relation to water quality, 427.

RESONANCE

See VIBRATION

RETAINING WALLS

See also BULKHEADS; EARTH PRESSURE; EARTHWORK (cross references thereunder); REVETMENT; SEA WALLS

"Wave Run-Up on Shore Structures," Thorndike Saville, Jr., 139.

REVETMENT

See also LEVEES; RETAINING WALLS

"Wave Run-Up on Shore Structures," Thorndike Saville, Jr., 139.

RIGHTS OF WAY (land strips)

See also COSTS, RIGHT OF WAY; HIGHWAYS AND ROADS

Proper time of purchase for highway construction and expansion, 865.

RIGHTS, RIPARIAN

See RIPARIAN RIGHTS

RIGHTS, WATER

See WATER RIGHTS

RIGID FRAMES

See STRUCTURES, THEORY OF —Frames, Rigid

RIPARIAN RIGHTS

Need of study on relationship of landowner abutting waterways and public right of navigation, 938.

RIPARIAN RIGHTS LAW

Court rulings affecting riparian rights problems, 961.

RIPRAP (stone protection layer)

See WAVES

RIVER BANKS AND BANK PROTECTION
See EROSION, STREAM; FLOODS; GROINS; LEVEES; REVETMENT; RIPARIAN RIGHTS; SILT AND SILTING, CHANNEL

RIVER BASINS
See DRAINAGE; RAINFALL; RESERVOIRS... ; RIVER VALLEY AUTHORITIES; VALLEYS

RIVER CHANNELS
See CHANNELS

RIVER DELTAS
"A Floodway for Houston, Tex.," 181.

RIVER DIVERSION
See WATER DIVERSION

RIVER MEANDERING
See RIVERS; *see also* RIVER DIVERSION (cross reference thereunder)

RIVER POLLUTION
See WATER POLLUTION

RIVER RECTIFICATION
See RIVER REGULATION (cross reference thereunder); *see also* other related RIVER... subject headings

RIVER REGULATION
See BARS (alluvia); FLOOD ROUTING; RESERVOIRS, WATER STORAGE; REVETMENT; RIVERS; RIVER VALLEY AUTHORITIES; WATER DIVERSION; WATER, FLOW OF, IN OPEN CHANNELS

RIVERS (General)
See also BARS (alluvia); BRIDGE ... ; CHANNELS; COSTS... ; DRAINAGE; DREDGES AND DREDGING; EROSION, STREAM; EVAPORATION; FLOOD ROUTING; FLOODS; HARBORS; HYDRAULIC LABORATORIES; HYDROGRAPHS, STREAM FLOW; HYDROLOGY; LEVEES; LOCKS; MODELS, HYDRAULIC; MULTI-PURPOSE PROJECTS (cross references thereunder); RAINFALL; RESERVOIRS... ; RETAINING WALLS; RUNOFF; SEDIMENT AND SEDIMENTATION; SILT AND SILTING, CHANNEL; TRAFFIC, WATERWAY; VALLEYS; WATER...
Two main classes of rivers differentiated, 106.

RIVERS (Geographical)
Atchafalaya River
"Old River Diversion Control": A Symposium, John R. Hardin; Eugene A. Graves; Willard J. Turnbull and Woodland G. Shockley; and Norman R. Moore, 1129.
Colorado River
Hoover Dam erection and its effect on sediment load, 544, 558.
Mississippi River
"Old River Diversion Control": A Symposium, John R. Hardin; Eugene A. Graves; Willard J. Turnbull and Woodland G. Shockley; and Norman R. Moore, 1129.
"Pile Tests, Low-Sill Structure, Old River, Louisiana," Charles I. Mansur and Robert I. Kaufman (with discussion), 715.
Missouri River
Sediment transport characteristics near Omaha, Nebraska, 543, 560, 594.
Old River, La.
"Old River Diversion Control": A Symposium, John R. Hardin; Eugene A. Graves; Willard J. Turnbull and Woodland G. Shockley; and Norman R. Moore, 1129.
"Pile Tests, Low-Sill Structure, Old River, Louisiana," Charles I. Mansur and Robert I. Kaufman (with discussion), 715.
United States
"Bridge Clearances": A Symposium, Eugene W. Weber, Walter Kurylo, William E. Cleary, Nickles L. Caruthers, and Erhard E. Dittbrenner, 925.

RIVER TRANSPORTATION
See WATER TRANSPORTATION

RIVER VALLEY AUTHORITIES
See also RIVERS (under relative river); VALLEYS
"The Apalachicola River Basin Project," Clement P. Lindner, 973.
"Effects of Storage Impoundments on Water Quality," Milo A. Churchill, 419.
"Friction Measurements in the Apalachia Tunnel," Rex A. Elder, 1249.

RIVER VALLEYS
See VALLEYS

RIVETED JOINTS
See JOINTS, RIVETED

RIVETS AND RIVETING
See CONNECTORS AND CONNECTIONS

ROADBEDS
See under specific type, e.g., HIGHWAYS AND ROADS—Roadbeds (cross reference thereunder)

ROADS
See HIGHWAYS AND ROADS

ROCK-FILL DAMS
See DAMS, ROCK-FILL

ROLLING STOCK
See WHEEL LOADS

ROOFS AND ROOFING
See CHIMNEYS AND FLUES; TRUSSES

ROUGHNESS
See FRICTION

ROUGHNESS FACTOR
See FRICTION COEFFICIENTS ...; TUNNELS, WATER

ROUTING, FLOOD
See FLOOD ROUTING

RUNOFF (General)
See also FLOODS; RAINFALL

RUNOFF (Geographical)
United States
"Surface Water Resources," Joseph V. B. Wells, 1206.

SAFETY (General)
See EARTHQUAKES; FIRE PROTECTION (cross reference thereunder); HIGHWAYS AND ROADS (General)—Safety

SAFETY, PERSONAL
See EMPLOYEES AND EMPLOYMENT; SAFETY (cross references thereunder)

SAMPLING
See under relative material, e.g., SAND

SAND
See also BARS (alluvia); COSTS, SAND...; PILES AND PILE DRIVING; SEDIMENT AND SEDIMENTATION; SEEPAGE; SILT AND SILTING, CHANNEL; SOILS; WATER, FLOW OF, THROUGH SAND (cross references thereunder)
"Laboratory and In-Situ Permeability of Sand," Charles I. Mansur, 868.
"Mechanics of Streams with Movable Beds of Fine Sand," Norman H. Brooks (with discussion), 526.
Sampler usage for varied grain size strata and widely graded sands, 880, 882.
"Soil Lysimeters in Waste Water Reclamation Studies," Gerald T. Orlob and Robert G. Butler (with discussion), 116.

SAND BARS
See BARS (alluvia)

SAND DUNES
See BARS (alluvia); BED LOAD (cross references thereunder)

SANITATION
See COSTS, INDUSTRIAL WASTE; FILTERS AND FILTRATION...; HOUSES—Drainage; INDUSTRIAL WASTE; PUBLIC HEALTH; SEWAGE DISPOSAL; SEWAGE SLUDGE; SEWERS; TANKS, SEPTIC; WATER LAW; WATER POLLUTION; WATER SUPPLY; WATER TREATMENT

SCIENCE
See also under specific science, e.g., HYDRAULICS; MATHEMATICS
History and progress of science and technology, 150 year period, 1275.

SCIENTIFIC SOCIETIES
See AMERICAN SOCIETY OF CIVIL ENGINEERS; SOCIETIES, TECHNICAL (cross references thereunder); *see also* ENGINEERS AND ENGINEERING

SCOUR
See EROSION...

SCREENS, WELL
See WELLS

SEA...
See also OCEAN (cross references thereunder); WATER...; WAVES

SEACOAST
See OCEAN CURRENTS (cross reference thereunder); SHORES AND SHORE PROTECTION; WAVES

SEAPORTS
See HARBORS

SEASHORE
See SHORES AND SHORE PROTECTION

SEA WALLS
See also COSTS, SEA WALL; RETAINING WALLS; SHORES AND SHORE PROTECTION; WATER PRESSURE; WAVES
"Harrison County (Mississippi) Artificial Beach," Francis F. Escoffier, 817.

SEA WALLS (*Continued*)
"Wave Run-Up on Shore Structures," Thorndike Saville, Jr., 139.

SEDIMENT AND SEDIMENTATION
See also SILT AND SILTING...
"Design of Stable Canals and Channels in Erodible Material," Pete W. Terrell and Whitney M. Borland, 101.

SEEPAGE
See also GROUND WATER
"Laboratory and In-Situ Permeability of Sand," Charles I. Mansur, 868.
"Soil Lysimeters in Waste Water Reclamation Studies," Gerald T. Orlob and Robert G. Butler (with discussion), 116.

SEISMOLOGY
See EARTHQUAKES

SEPTIC TANKS
See TANKS, SEPTIC

SETTLEMENT OF STRUCTURES
See BEARING CAPACITY; EARTH PRESSURE; FOUNDATIONS; SOIL...

SEWAGE COLLECTION
See SEWERS

SEWAGE DISPOSAL (General)
See also FILTERS AND FILTRATION, SEWAGE; INDUSTRIAL WASTE; PIPES AND PIPING (cross references thereunder); SEWAGE SLUDGE; SEWERS; TANKS...; WATER POLLUTION—Sewage Pollution
"Developments in Septic Tank Systems," John E. Kiker, Jr., 77.
"Hydraulic Model Study of Hyperion Sewer Interchange," Alfred C. Ingersoll and Hajime Tanaka, 908.

SEWAGE DISPOSAL (Geograpical)
Knoxville, Tenn.
"Municipal Ordinances for Industrial Wastes," Julian R. Fleming, 595.

SEWAGE DISPOSAL LAW
"Municipal Ordinances for Industrial Wastes," Julian R. Fleming, 595.

SEWAGE FILTERS AND FILTRATION
See FILTERS AND FILTRATION, SEWAGE

SEWAGE RECLAMATION
See SEWAGE DISPOSAL; SEWAGE, UTILIZATION OF (cross reference thereunder)

SEWAGE SLUDGE
See also SEWAGE DISPOSAL; TANKS...
"Hydraulic Model Study of Hyperion Sewer Interchange," Alfred C. Ingersoll and Hajime Tanaka, 908.

SEWAGE SPREADING
See SPREADING, WATER

SEWAGE TREATMENT
See SEWAGE DISPOSAL

SEWAGE, UTILIZATION OF
See WATER STORAGE

SEWAGE WORKS
See SEWAGE DISPOSAL

SEWERAGE
See SEWERS

SEWER LAW
"Municipal Ordinances for Industrial Wastes," Julian R. Fleming, 595.

SEWERS (General)
See also HOUSES (places of abode)—Drainage; PIPES AND PIPING (cross references thereunder); RAINFALL; RUNOFF; WATER, FLOW OF, IN PIPES
"An Improved Dilution Method for Flow Measurements," William A. Cawley and Jack W. Woods, 405.

SEWERS (Geographical)
Los Angeles, Calif.
"Hydraulic Model Study of Hyperion Sewer Interchange," Alfred C. Ingersoll and Hajime Tanaka, 908.

SHEAR
"Bearing-Ratio Effect on Strength of Riveted Joints," Jonathan Jones, 964.
"Hydraulic Model Study of Hyperion Sewer Interchange," Alfred C. Ingersoll and Hajime Tanaka, 908.
"Mechanics of Streams with Movable Beds of Fine Sand," Norman H. Brooks (with discussion), 526.
"Shells of Double Curvature," Alfred L. Parme (with discussion), 989.
"Simplification of Design by Ultimate Strength Procedures," Phil M. Ferguson (with discussion), 602.
"Soil Modulus for Laterally Loaded Piles," Bramlette McClelland and John A. Focht, Jr. (with discussion), 1049.
"The Viscous Sublayer Along a Smooth Boundary," Hans A. Einstein and Huon Li (with discussion), 293.

SHEARING STRESS
See SHEAR

SHEDS

See under relative type, e.g., WATERSHEDS (cross references thereunder)

SHELL STRUCTURES

See also CHIMNEYS AND FLUES; FLUMES; PIPE LINES; PIPES AND PIPING (cross references thereunder); STRUCTURES, THEORY OF—Shell Structures

"Shells of Double Curvature," Alfred L. Parme (with discussion), 989.

Use of hyperbolic paraboloid as a structure, 989, 1015, 1016, 1018.

Bibliography

Shells of double curvature, 1012.

SHIP MOORINGS AND MOORAGES

See DOCKS AND WHARVES

SHIPS AND SHIPPING

See WATER TRANSPORTATION

SHOCK

See EARTHQUAKES; IMPACT; VIBRATION

SHORES AND SHORE PROTECTION (lands adjacent to lake, ocean or sea water)

See also BREAKWATERS; EROSION, BEACH; JETTIES; RETAINING WALLS; SAND DUNES (cross references thereunder); WAVES; *see also* RIVER BANKS AND BANK PROTECTION (cross references thereunder)

"Harrison County (Mississippi) Artificial Beach," Francis F. Escoffier, 817.

Sanitizer usage for beach cleaning, 823.

SIDEWALKS (General)

See CONCRETE

SILLS

"Old River Diversion Control": A Symposium, John R. Hardin; Eugene A. Graves; Willard J. Turnbull and Woodland G. Shockley; and Norman R. Moore, 1129.

"Pile Tests, Low-Sill Structure, Old River, Louisiana," Charles I. Mansur and Robert I. Kaufman (with discussion), 715.

SILT AND SILTING (General)

See also AGGREGATES AND AGGREGATION; PILES AND PILE DRIVING; RIVER DELTAS; SEDIMENT AND SEDIMENTATION

SILT AND SILTING, CHANNEL

See also RIVER DELTAS

"Design of Stable Canals and Channels in Erodible Material," Pete W. Terrell and Whitney M. Borland, 101.

Formulas and their usage in stable channel design, 195.

"Mechanics of Streams with Movable Beds of Fine Sand," Norman H. Brooks, 526. *Discussion:* Thomas Blench; James R. Barton; Hans A. Einstein and Ning Chien; Enos J. Carlson and M. Gamal Mostafa; Hsin-Kuan Liu; Pin-Nam Lin; and Norman H. Brooks, 550.

Misunderstandings of the regime theory, 550.

"Sediment-Transport Mechanics in Stable-Channel Design," Emmett M. Laursen (with discussion), 195.

SINGULAR POINTS, THEORY OF

See WATER, FLOW OF, IN OPEN CHANNELS

SKIN FRICTION

See FRICTION

SLABS

See STRESS AND STRAIN—Slabs; STRUCTURES, THEORY OF—Slabs

SLIDE GATES

See WATER GATES, MOVABLE

SLIPS AND SLIPPAGE (movement of surfaces)

See JOINTS, RIVETED

SLOPES, EARTH

See DAMS, EARTH; EARTHWORK (cross references thereunder)

SLOPING CHANNELS

See CHANNELS

SLUDGE, SEWAGE

See SEWAGE SLUDGE

SLUICE GATES

See SLUICES

SLUICES

See also FLUMES

"Trends in Hydraulic Gate Design," Dow A. Buzzell (with discussion), 27.

SMOKESTACKS

See CHIMNEYS AND FLUES

SOCIETIES, TECHNICAL

See AMERICAN SOCIETY OF CIVIL ENGINEERS; ENGINEERS AND ENGINEERING; *also* under relative subject

SOIL . . .
See also EARTH . . . ; GROUND . . . ; LAND . . . (cross references thereunder)

SOIL CORROSION
See CORROSION AND PROTECTION OF . . .

SOIL MECHANICS
See SOIL . . .

SOIL PRESSURE
See EARTH PRESSURE

SOILS (General)
See also BEARING CAPACITY; CLAY; DRAINAGE; EARTH PRESSURE; EROSION . . . ; EVAPORATION; FOUNDATIONS . . . ; FRICTION; GROUND . . . ; LAND . . . (cross references thereunder); LOESS; SAND; SEDIMENT AND SEDIMENTATION; SEEPAGE; SILT AND SILTING . . . ; SPREADING BASINS; STRESS AND STRAIN —Soils; *also* under name of soil structure, e.g., DAMS, ROCK-FILL
Behavior of five typical pervious agricultural soils under water and sewage spreading, 116, 119, 132, 135.

Compaction. *See* BEARING CAPACITY

Construction
Mississippi River improvement structures as related to subsurface conditions, 1160.
"Pile Tests, Low-Sill Structure, Old River, Louisiana," Charles I. Mansur and Robert I. Kaufman (with discussion), 715.

Tests and Testing
"Experiences with Loess As Foundation Material," William A. Clevenger (with discussion), 151.
"Old River Diversion Control": A Symposium, John R. Hardin; Eugene A. Graves; Willard J. Turnbull and Woodland G. Shockley; and Norman R. Moore, 1129.
Percolation test procedure and purpose, 79.
"Soil Lysimeters in Waste Water Reclamation Studies," Gerald T. Orlob and Robert G. Butler (with discussion), 116.
"Soil Modulus for Laterally Loaded Piles," Bramlette McClelland and John A. Focht, Jr., 1049. *Discussion:* Ralph F. Reuss; Ralph B. Peck, Melvin T. Davisson, and Vello Hansen; Raymond Lundgren; Lymon C. Reese; Eugene A. Ripperger; Hudson Matlock; and Bramlette McClelland and John A. Focht, Jr., 1064.
"Stabilization of Materials by Compaction," Willard J. Turnbull and Charles R. Foster, 1. *Discussion:* Gerald A. Leonards; John A. Focht, Jr.; Edmund J. Zegarra; Kazuo B. Hirashima; and Willard J. Turnbull and Charles R. Foster, 16.

SOIL TRANSPORTATION
See SEDIMENT AND SEDIMENTATION; SILT AND SILTING . . .

SOLIDS, FLOW OF
See SEDIMENT AND SEDIMENTATION: SILT AND SILTING . . . ; WATER, FLOW OF, IN OPEN CHANNELS; WATER, FLOW OF, IN PIPES

SOLIDS, PRESSURE TRANSMISSION THROUGH
See EARTH PRESSURE

SPACE STRUCTURES
See under specific type of structure

SPECIFICATIONS
See under relative subject; *see also* CONTRACTS (cross reference thereunder)

SPEED
See HIGHWAYS AND ROADS—Safety; VELOCITY

SPILLWAY GATES
See SPILLWAYS

SPILLWAYS (General)
See also DAMS; DROP-DOWN (cross references thereunder)
"Anchorages for Large Tainter Gates," Alexander H. Kenigsberg (with discussion), 409.
"Trends in Hydraulic Gate Design," Dow A. Buzzell (with discussion), 27.

SPILLWAYS (Geographical)

Old River, La.
"Old River Diversion Control": A Symposium, John R. Hardin; Eugene A. Graves; Willard J. Turnbull and Woodland G. Shockley; and Norman R. Moore, 1129.
"Pile Tests, Low-Sill Structure, Old River, Louisiana," Charles I. Mansur and Robert I. Kaufman (with discussion), 715.

SPREADING BASINS
Soil and gas condition variances existing between field basin and laboratory lysimeter testing, 137, 138.
SPREADING, WATER
See also COSTS, SPREADING (WATER SPREADING)
"Soil Lysimeters in Waste Water Reclamation Studies," Gerald T. Orlob and Robert G. Butler, 116. *Discussion:* Ralph Stone; and Gerald T. Orlob and Robert G. Butler, 137.
STABILITY
See BUCKLING; HYDRODYNAMICS; SOIL . . . ; STRUCTURES, THEORY OF
STACKS
See CHIMNEYS AND FLUES
STANDARDS, PROFESSIONAL
See ENGINEERS AND ENGINEERING—Professional Relationships
STATICALLY INDETERMINATE STRUCTURES
See STRUCTURES, THEORY OF
STATIONS
See under relative type of station, e.g., EVAPORATION STATIONS; *also* cross reference under TERMINALS; *see also* under relative subject
STATISTICS
See under relative subject
STEAM POWER PLANTS
See POWER PLANTS
STEEL (General)
See also ALLOYS (cross reference thereunder); CORROSION AND PROTECTION OF METALS; MATERIALS OF CONSTRUCTION; METALS (cross references thereunder); REINFORCED CONCRETE (cross reference thereunder); STRENGTH OF MATERIALS; STRESS AND STRAIN —Steel; *also* under special structure or structural part, e.g., BRIDGES . . .
"Bearing-Ratio Effect on Strength of Riveted Joints," Jonathan Jones, 964.
"Plastics: Engineering Materials," C. Howard Adams, 511.
"Simplification of Design by Ultimate Strength Procedures," Phil M. Ferguson (with discussion), 602.
STIFFNESS
"Load Distribution in Highway Bridge Decks," Arnold W. Hendry and Leslie G. Jaeger (with discussion), 1214.
"Moment-Distribution Constants from Models," Otakar Ondra (with discussion), 318.
"Plastics: Engineering Materials," C. Howard Adams, 511.
"Truss Analysis by Stiffness Considerations," Harold C. Martin, 1182.
STILLING BASINS
See WATER, FLOW OF, IN OPEN CHANNELS
STORAGE
See RESERVOIRS . . . ; WATER STORAGE
STORMS
See RAINFALL
STORM WATER
See SEWAGE DISPOSAL; SEWERS
STRAIN
See STRESS AND STRAIN
STRAIN GAGES
See GAGES, STRAIN
STREAM CONTAMINATION
See WATER POLLUTION
STREAM EROSION
See EROSION, STREAM
STREAM FLOW
See WATER, FLOW OF, IN OPEN CHANNELS
STREAMS
See RIVERS and cross references thereunder
STREAMS, FIRE
See WATER, FLOW OF, THROUGH ORIFICES
STREETS (General)
See BRIDGES; CONDUITS; IMPACT; MOTOR . . . (cross references thereunder); PIPES AND PIPING (cross references thereunder); SIDEWALKS (cross reference thereunder); WHEEL LOADS
STREET TRANSPORTATION
See TRANSPORTATION; WATER FRONT (cross references thereunder)
STRENGTH OF MATERIALS
See also FAILURES . . . ; STRESS AND STRAIN; *also* under specific material (*see* list of materials under MATERIALS OF CONSTRUCTION); *also* under fabricated structure or structural part
"Behavior of Riveted Truss-Type Connections," Eugene Chesson, Jr. and William H. Munse (with discussion), 1087.

STRENGTH OF MATERIALS (*Continued*)
"Plastics: Engineering Materials," C. Howard Adams, 511.
"Simplification of Design by Ultimate Strength Procedures," Phil M. Ferguson, 602. *Discussion:* Tung Au; Herman S. Schick; Zdenek Sobotka; Milan Spanovich; Sylvester M. Ulicny; and Phil M. Ferguson, 629.

STRESS AND STRAIN (General)
See also EARTH PRESSURE; EARTHQUAKES; ELASTICITY; FAILURES...; GAGES, STRAIN; IMPACT; MOMENTS; PLASTICITY; STIFFNESS; STRENGTH OF MATERIALS; STRUCTURES, THEORY OF; TEMPERATURE; VIBRATION; WATER PRESSURE; WAVES; WHEEL LOADS; *also* under specific type of stress, e.g., BUCKLING; SHEAR; TORSION

STRESS AND STRAIN

Bridges, Girder
"Dynamic Stresses in Continuous Plate-Girder Bridges," Roy C. Edgerton and Gordon W. Beecroft, 266. *Discussion:* Robert K. L. Wen; and Roy C. Edgerton and Gordon W. Beecroft, 287.

Connectors and Connections
"Behavior of Riveted Truss-Type Connections," Eugene Chesson, Jr. and William H. Munse (with discussion), 1087.

Curved Structures. *See* STRUCTURES, THEORY OF—Curved Structures

Floors and Flooring
"Moments in Flat Slabs," Mark W. Huggins and Watone L. Lin (with discussion), 824.

Frames, Continuous
"Moments in Flat Slabs," Mark W. Huggins and Watone L. Lin (with discussion), 824.

Joints
"Bearing-Ratio Effect on Strength of Riveted Joints," Jonathan Jones, 964.

Materials of Construction
"Plastics: Engineering Materials," C. Howard Adams, 511.
"Simplification of Design by Ultimate Strength Procedures," Phil M. Ferguson (with discussion), 602.

Piles
"Soil Modulus for Laterally Loaded Piles," Bramlette McClelland and John A. Focht, Jr. (with discussion), 1049.

Plastics
Creep characteristics, 511.

Plates. *See also* BUCKLING
"Behavior of Riveted Truss-Type Connections," Eugene Chesson, Jr. and William H. Munse (with discussion), 1087.

Slabs
"Moments in Flat Slabs," Mark W. Huggins and Watone L. Lin (with discussion), 824.

Soils
"Soil Modulus for Laterally Loaded Piles," Bramlette McClelland and John A. Focht, Jr. (with discussion), 1049.

Solids. *See* EARTH PRESSURE

Steel. *See also* Plates (hereunder)
"Behavior of Riveted Truss-Type Connections," Eugene Chesson, Jr. and William H. Munse (with discussion), 1087.

Structural Materials. *See* Materials of Construction (hereunder)

Structures (General)
"Pile Tests, Low-Sill Structure, Old River, Louisiana," Charles I. Mansur and Robert I. Kaufman (with discussion), 715.

Trusses and Trussed Structures
"Behavior of Riveted Truss-Type Connections," Eugene Chesson, Jr. and William H. Munse (with discussion), 1087.

Water Gates, Movable
"Anchorages for Large Tainter Gates," Alexander H. Kenigsberg (with discussion), 409.

STRESS, SHEARING
See SHEAR

STRUCTURAL ANALYSIS
See EQUATIONS; STRESS AND STRAIN; STRUCTURES, THEORY OF; TRUSSES...

STRUCTURAL DYNAMICS
See EARTHQUAKES; VIBRATION

STRUCTURAL ENGINEERS AND ENGINEERING
See STRUCTURES... ; *also* under relative structure, e.g., BRIDGES ... ; BUILDINGS

STRUCTURAL JOINTS
See JOINTS...
STRUCTURAL MATERIALS
See MATERIALS OF CONSTRUCTION
STRUCTURAL MEMBERS
See MEMBERS, STRUCTURAL (cross references thereunder)
STRUCTURAL MODELS
See MODELS, STRUCTURAL
STRUCTURAL REDUNDANCE
See STRUCTURES, THEORY OF
STRUCTURAL STEEL
See STEEL
STRUCTURES (General)
See also BUILDINGS; CONSTRUCTION (cross references thereunder); FOUNDATIONS...; MATERIALS OF CONSTRUCTION; MODELS, STRUCTURAL; SAFETY (cross references thereunder); STRENGTH OF MATERIALS; STRESS AND STRAIN; STRUCTURES, THEORY OF; VIBRATION; *also* under specific type of structure or related subject; *also* under general types of structures, e.g., CURVED STRUCTURES (cross reference thereunder); SHELL STRUCTURES
"Wave Run-Up on Shore Structures," Thorndike Saville, Jr., 139.
STRUCTURES, CURVED
See CURVED STRUCTURES (cross reference thereunder)
STRUCTURES, DROP
See DAMS
STRUCTURES, HYDRAULIC
See HYDRAULIC STRUCTURES (cross reference thereunder)
STRUCTURES, SETTLEMENT OF
See BEARING CAPACITY; EARTH PRESSURE; FOUNDATIONS; SOIL...
STRUCTURES, SHELL
See SHELL STRUCTURES
STRUCTURES, SPACE
See SPACE STRUCTURES (cross reference thereunder)
STRUCTURES, STATICALLY INDETERMINATE
See STATICALLY INDETERMINATE STRUCTURES (cross reference thereunder)
STRUCTURES, THEORY OF (General)
See also STRESS AND STRAIN; *also* under type of structure or structural part
Concept of the pressure line as a method of solving problems, with specific applications cited, 1039, 1046, 1047.
"Moment-Distribution Constants from Models," Otakar Ondra (with discussion), 318.
A procedure of analysis for complex structures with its advantages evaluated, 1182, 1183, 1190, 1194.
Beams and Girders, Continuous
"Load Distribution in Highway Bridge Decks," Arnold W. Hendry and Leslie G. Jaeger (with discussion), 1214.
Bridges
"Load Distribution in Highway Bridge Decks," Arnold W. Hendry and Leslie G. Jaeger (with discussion), 1214.
Columns
"A Pressure-Line Concept for Inelastic Bending," Frank Baron (with discussion), 1039.
Curved Structures
"A Pressure-Line Concept for Inelastic Bending," Frank Baron (with discussion), 1039.
"Shells of Double Curvature," Alfred L. Parme (with discussion), 989.
Earthquake Problems
"Earthquake Resistance of Rock-Fill Dams," Ray W. Clough and David Pirtz (with discussion), 792.
Frames (General). *See also* BEAMS; COLUMNS; GIRDERS; *see also* Beams and Girders... (hereunder); Columns (hereunder)
Frames, Rigid
"A Pressure-Line Concept for Inelastic Bending," Frank Baron, 1039. *Discussion:* John A. Hribar, and Frank Baron, 1047.
Girders, Continuous. *See* Beams and Girders, Continuous (hereunder)
Materials and Tests
"Simplification of Design by Ultimate Strength Procedures," Phil M. Ferguson (with discussion), 602.
Shell Structures
"Shells of Double Curvature," Alfred L. Parme, 989. *Discussion:* Tung Au; W. Watters Pagon; Santi P. Banerjee; Mario G. Salvadori; and Alfred L. Parme, 1014.

STRUCTURES, THEORY OF (General) (*Continued*)

Slabs

"Load Distribution in Highway Bridge Decks," Arnold W. Hendry and Leslie G. Jaeger (with discussion), 1214.

Trusses and Trussed Structures

"Truss Analysis by Stiffness Considerations," Harold C. Martin, 1182.

STRUCTURES, TRUSSED

See STRUCTURES, THEORY OF —Trusses and Trussed Structures; TRUSSES...

STRUCTURES, UNDERGROUND

See UNDERGROUND STRUCTURES (cross references thereunder)

SUB-...

See also UNDER-... (cross references thereunder)

SUBMERGENCE, LAND

See LAND SUBMERGENCE (cross reference thereunder)

SUBSTRUCTURES

See CONCRETE; EARTHWORK (cross references thereunder); FOUNDATIONS...; *also* under type of substructure, e.g., PIPE LINES; TUNNELS...

SUBWAYS

See EXCAVATION... (cross references thereunder)

SURFACE EVAPORATION

See WATER SURFACE EVAPORATION (cross reference thereunder)

SURFACE PROFILES

See WATER SURFACE PROFILES (cross references thereunder)

SURFACE RUNOFF

See RUNOFF

SURFACES, SLIPPAGE OF

See SLIPS AND SLIPPAGE (movement of surfaces) (cross reference thereunder)

SURVEYS (research data)

See under relative subject; *see also* RESEARCH

SURVEYS AND SURVEYING (General)

See COSTS, SURVEY AND SURVEYING; PROPERTY (landed property) (cross references thereunder); *see also* under relative subject, e.g., HIGHWAYS AND ROADS (General)

SURVEYS AND SURVEYING, AERIAL

"Bureau of Public Roads Experiences in Highway Surveys," William T. Pryor, 1026.

SUSPENDED LOAD

See SEDIMENT AND SEDIMENTATION; SILT AND SILTING ...

SWAMPS

See MARSHES

SYMBOLS

See under relative subject, e.g., HYDRAULICS; MATHEMATICS; STRUCTURES, THEORY OF

TAINTER GATES

See WATER GATES, MOVABLE

TANKS (General)

See also STILLING BASINS (cross reference thereunder); WATER, FLOW OF, IN OPEN CHANNELS; WATER STORAGE

TANKS, SEPTIC

Detergent effects on septic tank systems, and need of research, 81.

"Developments in Septic Tank Systems," John E. Kiker, Jr., 77.

TECHNICAL SOCIETIES

See AMERICAN SOCIETY OF CIVIL ENGINEERS; ENGINEERS AND ENGINEERING; *also* under relative subject

TEMPERATURE (General)

See also EVAPORATION

Effect of temperature on the re-aeration coefficient of molecular diffusivity in natural streams, 656.

"Mechanics of Streams with Movable Beds of Fine Sand," Norman H. Brooks (with discussion), 526.

"Plastics: Engineering Materials," C. Howard Adams, 511.

TENNESSEE VALLEY AUTHORITY

See RIVER VALLEY AUTHORITIES

TENSILE STRESS

See STRESS AND STRAIN

TENSIOMETERS

See GAGES, PRESSURE

TENSION

See STRESS AND STRAIN; STRUCTURES, THEORY OF

TERMINALS (structures and localities)

See FREIGHT TERMINALS; WATER TERMINALS; *see also* cross references under STATIONS

TERMINOLOGY (Arranged hereunder by specific or comprehensive subject word when possible)

Bridge profile variation terms need uniformity, 290.

Civil engineering as defined in 1828 by Institution of Civil Engineers (London), 511.

Dams. (Rock-fill dam defined), 792.

Highways and streets. (Limited access facility defined), 481.

Hydrodynamics. (Smooth, plane or flat as terms with special meanings), 551, 574.

Multi-purpose water project cost allocation defined, 85.

Rainfall. (Drought defined qualitatively. Quantitative definition regarded as lacking), 1208.

Rivers. (Perennial and ephemeral as natural stream classes), 106.

Soil compaction. (Coverage as a term), 9.

Soil stabilization term variations, 1.

Streams as defined by United States Supreme Court, 961.

Water flow. (Dominant discharge defined in several ways), 113.

Water flow. (Eddy diffusion or eddy transport defined), 648.

Water flow. (Transition profile, transition depth, transition discharge, and similar terms in relation to rapid and tranquil flow), 43, 45, 59, 63, 65.

Water surface profiles. (Paranormal depth used as a term in relation to water surface profiles), 50.

TESTING MACHINES

See under use, material, structure or structural part tested

TESTING STATIONS

See under relative subject

TESTS AND TESTING

See BUCKLING; ELASTICITY; FAILURES...; IMPACT; LABORATORIES (cross references thereunder); MODELS...; SHEAR; STRENGTH OF MATERIALS; STRESS AND STRAIN; STRUCTURES, THEORY OF—Materials and Tests; TORSION; WATER, FLOW OF ...; *also* under material, structure or structural part tested, e.g., SOILS—Tests and Testing

THEORIES

See cross references hereunder and under ANALYSIS OF DATA; *see also* STRUCTURES, THEORY OF; *see also* under relative subject or its relative science, e.g., HYDRAULICS; MATHEMATICS; WATER, FLOW OF, IN OPEN CHANNELS

THEORY OF BOUNDARY LAYER (fluid flow)

See BOUNDARY LAYER, THEORY OF (cross reference thereunder)

THEORY OF ELASTICITY

See ELASTICITY

THEORY OF FINITE DIFFERENCES

See MATHEMATICS

THEORY OF LIMIT DESIGN

See LIMIT DESIGN, THEORY OF

THEORY OF PLASTICITY

See PLASTICITY

THEORY OF SINGULAR POINTS

See WATER, FLOW OF, IN OPEN CHANNELS

THEORY OF STRUCTURES

See STRUCTURES, THEORY OF

THEORY OF ULTIMATE STRENGTH

See STRENGTH OF MATERIALS

THEORY, PLASTIC

See PLASTICITY

THEORY, REGIME

See REGIME THEORY (cross reference thereunder)

THERMOPLASTIC MATERIALS

See PLASTIC MATERIALS

THRUST

See under relative structure, structural part or material

THRUWAYS

See HIGHWAYS AND ROADS

TOOLS

See under general types of tools or usage; *also* cross references under APPARATUS; INSTRUMENTS; MACHINERY

TOPOGRAPHIC MAPS AND MAPPING

See MAPS AND MAPPING (General) (cross reference thereunder)

TORSION
"Load Distribution in Highway Bridge Decks," Arnold W. Hendry and Leslie G. Jaeger (with discussion), 1214.

TOWBOATS AND TOWING
See WATER TRANSPORTATION; *see also* BARGES

TRACKS
See under type of track, e.g., RAILROAD TRACKS (cross references thereunder)

TRADE WASTE
See INDUSTRIAL WASTE

TRAFFIC (General)
See also FREIGHT (cross reference thereunder); HIGHWAYS AND ROADS—Safety; TRANSPORTATION; WHEEL LOADS

TRAFFIC ACCIDENTS
See HIGHWAYS AND ROADS—Safety

TRAFFIC, BUS
See TRAFFIC, HIGHWAY AND ROAD; and other relative subject headings hereunder

TRAFFIC, EXPRESSWAY
See TRAFFIC, HIGHWAY AND ROAD

TRAFFIC, HIGHWAY AND ROAD
See also HIGHWAYS AND ROADS—Location; HIGHWAYS AND ROADS—Safety
"Control of Highway Access": A Symposium, Curtis J. Hooper, Bertram D. Tallamy, Dewitt C. Greer, Carl E. Vogelgesang, Hugo H. Winter, and Ralph A. Moyer, 465.

TRAFFIC, WATERWAY
Inland waterway traffic growth changes, 951.
Traffic in the Atlantic Ocean, 950.

TRAILERSHIPS
See SHIPS AND SHIPPING (cross reference thereunder)

TRAIN ...
See also RAIL ... (cross references thereunder)

TRAINING WALLS
See JETTIES; SPILLWAYS

TRAIN LOADS
See WHEEL LOADS

TRAINSHIPS
See SHIPS AND SHIPPING (cross reference thereunder)

TRAINS, RAILROAD
See RAILROAD TRAINS (cross reference thereunder)

TRANSITION PROFILES
See WATER, FLOW OF...

TRANSMISSION, PRESSURE, THROUGH SOLIDS
See EARTH PRESSURE

TRANSPORTATION
See also BRIDGES; CANALS; CONVEYERS AND CONVEYANCE; FREIGHT (cross reference thereunder); HIGHWAY TRANSPORTATION; PIPE LINES; RAIL... (cross references thereunder); RIVERS; SOIL TRANSPORTATION (cross references thereunder); TERMINALS (cross references thereunder); TRAFFIC; WATER TRANSPORTATION; WATERWAYS
Importance of integration and coordination recognized for all types of transportation, 377.
"Transportation Planning": A Symposium, Roger H. Gilman, Wilfred M. Post, Jr. and Burton W. Marsh, 359.

TRUCKING
See HIGHWAY TRANSPORTATION; STREET TRANSPORTATION (cross references thereunder)

TRUSSED STRUCTURES
See STRUCTURES, THEORY OF—Trusses and Trussed Structures; TRUSSES...

TRUSSES (General)
See STRESS AND STRAIN—Trusses and Trussed Structures; STRUCTURES, THEORY OF—Trusses and Trussed Structures

TRUSSES, PIN-CONNECTED
"Truss Analysis by Stiffness Considerations," Harold C. Martin, 1182.

TRUSSES, SIMPLE
Stiffness analysis of a simple truss, 1186.

TUBES
See CONDUITS; NOZZLES (cross reference thereunder); PIPE LINES; PIPES AND PIPING (cross references thereunder); SHELL STRUCTURES; STRUCTURES, THEORY OF—Shell Structures; TUNNELS, WATER; WATER, FLOW OF, IN PIPES

TUNNEL LININGS
"Friction Measurements in the Apalachia Tunnel," Rex A. Elder, 1249.

TUNNELS, PRESSURE

See TUNNELS, WATER

TUNNELS, WATER

See also INTAKES; WATER DIVERSION

"Friction Measurements in the Apalachia Tunnel," Rex A. Elder, 1249.

TURBULENCE (water agitation)

See also FRICTION...; WATER, FLOW OF...

"Mechanics of Streams with Movable Beds of Fine Sand," Norman H. Brooks (with discussion), 526.

"Mechanism of Reaeration in Natural Streams," Donald J. O'Connor and William E. Dobbins (with discussion), 641.

"Open-Channel Flow at Small Reynolds Numbers," Lorenz G. Straub, Edward Silberman and Herbert C. Nelson (with discussion), 685.

Bibliography

Re-aeration in natural streams, 663.

TWISTING

See TORSION; *also* under relative material, structure or structural part, e.g., BRIDGE FLOORS AND FLOORING

ULTIMATE STRENGTH, THEORY OF

See STRENGTH OF MATERIALS

UNDER-...

See also SUB-... (cross references thereunder)

UNDERFLOW

See SEEPAGE; WELLS

UNDERGROUND CORROSION

See CORROSION AND PROTECTION OF METALS

UNDERGROUND STRUCTURES

See under type of structure, e.g., CONDUITS; PIPES AND PIPING (cross references thereunder); SEWERS; *see also* under related subject, e.g., CORROSION AND PROTECTION OF...

UNDERGROUND WATER

See GROUND WATER

UNDERSEEPAGE

See SEEPAGE

UNDERWATER CONSTRUCTION

See under type of construction; *see also* EXCAVATION, HYDRAULIC (cross reference thereunder)

UNDERWATER FOUNDATIONS

See FOUNDATIONS, UNDERWATER

UNIT COSTS

See COSTS...

UNITED STATES WEATHER BUREAU

See under relative subject, e.g., EVAPORATION STATIONS

UPLIFT, HYDROSTATIC

See WATER PRESSURE

URBAN...

See CITIES

VALLEY AUTHORITIES

See RIVER VALLEY AUTHORITIES

VALLEYS (General)

See also DRAINAGE; *also* under related subject, e.g., RIVERS

VALLEYS (Geographical)

Apalachicola River Basin

"The Apalachicola River Basin Project," Clement P. Lindner, 973.

Mississippi River Valley

Sand strata permeability in the alluvial valley of the river, 869, 881, 882.

Missouri River Basin

"Experiences with Loess As Foundation Material," William A. Clevenger (with discussion), 151.

Tennessee River Valley. *See also* RIVER VALLEY AUTHORITIES

Eastern half of the Tennessee Valley, 421.

"Effects of Storage Impoundments on Water Quality," Milo A. Churchill, 419.

"Friction Measurements in the Apalachia Tunnel," Rex A. Elder, 1249.

VALUATION

See also COSTS

Land value changes as related to location of thruways, 465.

VEHICLES

See under general types of vehicles, e.g., MOTOR VEHICLES (cross references thereunder); *also* under specific type of vehicle, e.g., AUTOMOBILE (cross references thereunder); CONVEYERS AND CONVEYANCE

VEHICULAR TRAFFIC

See TRAFFIC...

VELOCITY

See also HYDRODYNAMICS; IMPACT; VIBRATION; WATER, FLOW OF...

VELOCITY (*Continued*)

"Earthquake Resistance of Rock-Fill Dams," Ray W. Clough and David Pirtz (with discussion), 792.

VELOCITY DISTRIBUTION

See WATER, FLOW OF...

VELOCITY METERS

See WATER VELOCITY METERS (cross reference thereunder)

VENTURI FLUMES

See FLUMES

VESSELS

See SHIP... (cross reference thereunder); *see also* BARGES

VIBRATION

See also EARTHQUAKES; IMPACT

"Dynamic Stresses in Continuous Plate-Girder Bridges," Roy C. Edgerton and Gordon W. Beecroft (with discussion), 266.

"Earthquake Resistance of Rock-Fill Dams," Ray W. Clough and David Pirtz (with discussion), 792.

VIBRATION RECORDING APPARATUS

Oscillograph types used for recording vibration of rock-fill dams, 801.

VISCOSITY

See also WATER, FLOW OF...

"Mechanism of Reaeration in Natural Streams," Donald J. O'Connor and William E. Dobbins (with discussion), 641.

"Open-Channel Flow at Small Reynolds Numbers," Lorenz G. Straub, Edward Silberman and Herbert C. Nelson (with discussion), 685.

"The Viscous Sublayer Along a Smooth Boundary," Hans A. Einstein and Huon Li, 293. *Discussion:* Edward Silberman; Nicols N. Ambraseys; and Hans A. Einstein and Huon Li, 314.

WALLS

See under relative structure or type of wall, e.g., BREAKWATERS; SEA WALLS

WAR AND ENGINEERING

See cross reference under MILITARY ENGINEERS AND ENGINEERING

WAREHOUSES

See BUILDINGS; FREIGHT TERMINALS

WASTE DISPOSAL

See INDUSTRIAL WASTE...; SANITATION (cross references thereunder); SEWAGE...; SEWER...; WATER POLLUTION

WASTE MATERIALS

See cross references under WASTE DISPOSAL

WASTE OF WATER

See WATER CONSERVATION (cross references thereunder); WATER, CONSUMPTIVE USE OF

WASTE WATER

See INDUSTRIAL WASTE; SEWAGE DISPOSAL; SEWERS; WATER POLLUTION

WASTE WATER, UTILIZATION OF

See WATER STORAGE

WATER...

See also AERATION; BACKWATER; CONDUITS; COSTS...; DAMS; DRAINAGE; DRAWDOWN (cross reference thereunder); DROP-DOWN (cross references thereunder); EROSION...; EVAPORATION; FILTERS AND FILTRATION, SEWAGE; FLOODS; FLUMES; FOUNDATIONS...; GOVERNMENT (cross references thereunder); GROUND WATER; HYDRAULIC...; HYDRO-...; IRRIGATION; LAND SUBMERGENCE (cross references thereunder); MARSHES; METERS AND METERING...; MODELS, HYDRAULIC; OCEAN...; OXYGENATION; PIPE LINES; PIPES AND PIPING (cross references thereunder); RAINFALL; RESERVOIRS; RIVER...; RUNOFF; SANITATION (cross references thereunder); SEEPAGE; SEWAGE DISPOSAL; SEWERS; SPILLWAYS; SPREADING BASINS; SPREADING, WATER; STORM WATER (cross references thereunder); TANKS...; TERMINOLOGY; TUNNELS, WATER; TURBULENCE; WAVES; WELLS

WATER ACCELERATION

See VELOCITY

WATER ANALYSIS

See BACTERIA; WATER TREATMENT

WATER CLARIFICATION

See WATER TREATMENT

WATER COLLECTION
See INFILTRATION (cross references thereunder); INTAKES; WELLS

WATER CONSERVATION
See RESERVOIRS (cross references thereunder); RIVER VALLEY AUTHORITIES; SPREADING, WATER; WATER STORAGE

WATER, CONSUMPTION OF (consumer usage)
"Surface Water Resources," Joseph V. B. Wells, 1206.

WATER, CONSUMPTIVE USE OF (in agriculture)
"Evaporation from Free Water Surfaces at High Altitudes," Harry F. Blaney (with discussion), 385.

WATER, DEMINERALIZATION OF
See WATER TREATMENT

WATER DISTRIBUTION
See CANALS; CONDUITS; IRRIGATION; PIPES AND PIPING (cross references thereunder)

WATER DIVERSION
See also COSTS, WATER DIVERSION; FLOOD ROUTING
Mississippi River diversion control plan structure requirements with comprehensive procedures to prevent change of course, 1129.
"Old River Diversion Control": A Symposium, John R. Hardin; Eugene A. Graves; Willard J. Turnbull and Woodland G. Shockley; and Norman R. Moore, 1129.

WATER, FLOW OF (General)
See also FLOODS; FLUMES; FRICTION; GROUND WATER; HYDRAULICS; MODELS, HYDRAULIC; SPILLWAYS; TURBULENCE; VISCOSITY; WATER GATES, MOVABLE WELLS

WATER, FLOW OF, FROM DRAINAGE BASINS
See DRAINAGE

WATER, FLOW OF, IN FLUMES
See WATER, FLOW OF, IN OPEN CHANNELS

WATER, FLOW OF, IN OPEN CHANNELS
See also BACKWATER; DROPDOWN (cross references thereunder); FLOODS; HYDROGRAPHS; STREAM FLOW; INTAKES; METERS AND METERING, CURRENT; SEDIMENT AND SEDIMENTATION; SILT AND SILTING, CHANNEL
"Design of Venturi Flumes in Circular Conduits," Edwin A. Wells, Jr. and Harold B. Gotaas (with discussion), 749.
"Mechanics of Streams with Movable Beds of Fine Sand," Norman H. Brooks (with discussion), 526.
"Mechanism of Reaeration in Natural Streams," Donald J. O'Connor and William E. Dobbins (with discussion), 641.
"Old River Diversion Control": A Symposium, John R. Hardin; Eugene A. Graves; Willard J. Turnbull and Woodland G. Shockley; and Norman R. Moore, 1129.
"Open-Channel Flow at Small Reynolds Numbers," Lorenz G. Straub, Edward Silberman and Herbert C. Nelson, 685. *Discussion:* Wallace M. Lansford and James M. Robertson; Chesley J. Posey; and Lorenz G. Straub, Edward Silberman and Herbert C. Nelson, 707.
Theory of singular points introduced in study of water surface profiles, 50.
"Transition Profiles in Nonuniform Channels," Francis F. Escoffier, 43. *Discussion:* Achille Lazard; Leon J. Tison; and Francis F. Escoffier, 57.
"The Viscous Sublayer Along a Smooth Boundary," Hans A. Einstein and Huon Li (with discussion), 293.

WATER, FLOW OF, IN PIPES
See also PIPE LINES; PIPES AND PIPING (cross references thereunder); PUMPS AND PUMPING; SEWERS; WATER PRESSURE
Comparison of open channel flow with pipe flow at small Reynolds numbers, 685.
"Friction Measurements in the Apalachia Tunnel," Rex A. Elder, 1249.
"The Viscous Sublayer Along a Smooth Boundary," Hans A. Einstein and Huon Li (with discussion), 293.

WATER, FLOW OF, IN TANKS
See WATER, FLOW OF, IN OPEN CHANNELS

WATER, FLOW OF, THROUGH CHUTES
See SPILLWAYS; WATER, FLOW OF, IN OPEN CHANNELS

WATER, FLOW OF, THROUGH ORIFICES
"A Flow Controller for Open or Closed Conduits," Victor L. Streeter, 883.

WATER, FLOW OF, THROUGH SAND

See GROUND WATER; SPREADING, WATER

WATER FRONT

See BREAKWATERS; BULKHEADS; DOCKS AND WHARVES; HARBORS; JETTIES; PILES AND PILE DRIVING; WATER TERMINALS; WATER TRANSPORTATION

WATER GATES, MOVABLE

See also COSTS, WATER GATE (MOVABLE); STRESS AND STRAIN—Water Gates, Movable

"Anchorages for Large Tainter Gates," Alexander H. Kenigsberg, 409. *Discussion:* Stephen Wearne, and Alexander H. Kenigsberg, 417.

Parallel arms tainter gates and their anchorages, 418.

"Trends in Hydraulic Gate Design," Dow A. Buzzell, 27. *Discussion:* Abraham Streiff; George R. Latham; William G. H. Holt; Kandaswami S. Chetty; William G. Weber; and Dow A. Buzzell, 33.

WATER, GROUND

See GROUND WATER

WATER IMPOUNDING

See RESERVOIRS, WATER STORAGE; WATER STORAGE

WATER INSTRUMENTS

See also GAGES . . . ; METERS AND METERING

"Soil Lysimeters in Waste Water Reclamation Studies," Gerald T. Orlob and Robert G. Butler (with discussion), 116.

WATER LAW

See also RIPARIAN RIGHTS; WATER RIGHTS; *see also* other LAW subject headings under WATER . . . , e.g., WATER SUPPLY LAW

"Effects of Storage Impoundments on Water Quality," Milo A. Churchill, 419.

"Surface Water Resources," Joseph V. B. Wells, 1206.

WATER MAINS

See PIPES AND PIPING (cross references thereunder); WATER SUPPLY

WATER METERS AND METERING (stream velocity)

See METERS AND METERING, CURRENT

WATER, OXYGEN SATURATION BY

See OXYGENATION

WATER PIPES AND PIPING

See PIPES AND PIPING (cross references thereunder)

WATER POLLUTION

See also AERATION; BACTERIA; INDUSTRIAL WASTE; OXYGENATION; SEWAGE DISPOSAL; WATER TREATMENT

Bibliography

Re-aeration in natural streams, 663.

Sewage Pollution

"Mechanism of Reaeration in Natural Streams," Donald J. O'Connor and William E. Dobbins (with discussion), 641.

WATER PONDAGE

See WATER STORAGE

WATER POWER

See DAMS; MULTI-PURPOSE PROJECTS; RIVER VALLEY AUTHORITIES

WATER PRESSURE

See also GAGES, PRESSURE; PIPE LINES; WAVES

"Anchorages for Large Tainter Gates," Alexander H. Kenigsberg (with discussion), 409.

"Friction Measurements in the Apalachia Tunnel," Rex A. Elder, 1249.

Hydrostatic pressure controlled by deep wellpoints, 719.

WATER PURIFICATION

See WATER TREATMENT

WATER RESISTANCE

See FRICTION; HYDRODYNAMICS; SOLIDS, FLOW OF (cross references thereunder); TURBULENCE

WATER RIGHTS

See also RIPARIAN RIGHTS; WATER LAW

Roswell Basin rights in New Mexico under scrutiny (1958), 781.

WATERSHEDS

See RAINFALL; RUNOFF

WATER SPREADING

See SPREADING, WATER

WATER STORAGE

See also RESERVOIRS . . . ; WATER CONSERVATION (cross references thereunder)

WATER STORAGE (*Continued*)
"Effects of Storage Impoundments on Water Quality," Milo A. Churchill, 419.
"Soil Lysimeters in Waste Water Reclamation Studies," Gerald T. Orlob and Robert G. Butler (with discussion), 116.

WATER, STORM
See SEWAGE DISPOSAL; SEWERS

WATER SUPPLY (General)
See also AERATION; DAMS; FLOOD . . . ; GAGES; GROUND WATER; IRRIGATION; METERS AND METERING; PIPE LINES; PIPES AND PIPING (cross references thereunder); PUMPS AND PUMPING; RAINFALL; RESERVOIRS; SILT AND SILTING; SPREADING . . . ; TANKS; TUNNELS, WATER; WATER . . . (related subject headings thereunder); WELLS

WATER SUPPLY (Geographical)
Eastern States
Problems of Miami, Florida, Long Island, New York, and Mississippi, 784, 785, 787, 790.
Southeastern States
"Effects of Storage Impoundments on Water Quality," Milo A. Churchill, 419.
United States
"Importance of Ground Water in the National Economy," Albert G. Fiedler, 776.
Total water requirements and ground water requirements compared for years 1900, 1955 and estimated for year 1975, 778.
Western States
Losses due to evaporation at high altitudes, 385.
Problem areas, including Roswell artesian basin in New Mexico, and midwestern area, 778, 786, 790.

WATER SUPPLY LAW
New Mexico ground water statute a model for similar laws in several states, 780, 781, 790.

WATER SURFACE EVAPORATION
See EVAPORATION

WATER SURFACE PROFILES
See BACKWATER; WATER, FLOW OF, IN OPEN CHANNELS

WATER TERMINALS (structures and localities)
"Shoreside Facilities for Special-Purpose Ships," Howard J. Marsden, 851.

WATER TRANSPORTATION (General)
See also BARGES; CANALS; CHANNELS; DOCKS AND WHARVES; FREIGHT . . . ; HARBORS; LOCKS; PIPE LINES; RIVERS; TRAFFIC, WATERWAY; TRANSPORTATION; WATER FRONT (cross references thereunder); WATER TERMINALS; WATERWAYS
"Bridge Clearances": A Symposium, Eugene W. Weber, Walter Kurylo, William E. Cleary, Nickles L. Caruthers, and Erhard E. Dittbrenner, 925.
Modification of vessels to meet needs of vertical clearance requirements, 944.
Towboat types in their relation to clearances, 952.

WATER TREATMENT
See also AERATION; AGGREGATES AND AGGREGATION; BACTERIA; WATER POLLUTION
"Effects of Storage Impoundments on Water Quality," Milo A. Churchill, 419.

WATER TUNNELS
See TUNNELS, WATER

WATER, UNDERGROUND
See GROUND WATER

WATER VELOCITY METERS
See METERS AND METERING, CURRENT

WATER, WASTE
See INDUSTRIAL WASTE; SEWAGE DISPOSAL; SEWERS; WATER POLLUTION

WATER, WASTE OF
See WATER CONSERVATION (cross references thereunder); WATER, CONSUMPTIVE USE OF

WATER, WASTE, UTILIZATION OF
See WATER STORAGE

WATER WAVES
See WAVES

WATERWAYS (General)
See also BRIDGES; CANALS; CHANNELS; COSTS . . . ;

WATERWAYS (General) (*Continued*)
DOCKS AND WHARVES; FREIGHT... ; HARBORS; LAKES; OCEAN... (cross references thereunder); RIPARIAN RIGHTS; RIVERS; SEA... ; TRAFFIC, WATERWAY; WATER DIVERSION; WATER RIGHTS; WATER TRANSPORTATION
"Bridge Clearances": A Symposium, Eugene W. Weber, Walter Kurylo, William E. Cleary, Nickles L. Caruthers, and Erhard E. Dittbrenner, 925.

WATERWAY TRAFFIC
See TRAFFIC, WATERWAY

WATERWAY TRANSPORTATION
See WATER TRANSPORTATION

WATERWORKS
See DAMS; INTAKES; PIPES AND PIPING (cross references thereunder); RESERVOIRS; TUNNELS, WATER; WATER STORAGE; WATER SUPPLY; WATER TREATMENT; WATER, WASTE OF (cross references thereunder); WELLS

WAVES
See also BREAKWATERS; EROSION, BEACH; JETTIES; RETAINING WALLS; SEA WALLS; SHORES AND SHORE PROTECTION; WATER PRESSURE
"Wave Run-Up on Shore Structures," Thorndike Saville, Jr., 139.

WEATHER
See FLOODS; HYDROLOGY; METEOROLOGY (cross references thereunder); RAINFALL; TEMPERATURE; WIND... (cross reference thereunder)

WEATHER BUREAU, UNITED STATES
See under relative subject, e.g., EVAPORATION STATIONS

WEBS
See CONNECTORS AND CONNECTIONS

WEIGHTS
See under relative subject, e.g., FRAMES

WELL DRILLING INDUSTRY
Economic importance of industry, 777.

WELLS
See also GROUND WATER
"Importance of Ground Water in the National Economy," Albert G. Fiedler, 776.
"Laboratory and In-Situ Permeability of Sand," Charles I. Mansur, 868.

WHARVES
See DOCKS AND WHARVES

WHEEL LOADS
"Dynamic Stresses in Continuous Plate-Girder Bridges," Roy C. Edgerton and Gordon W. Beecroft (with discussion), 266.

WIND...
See EROSION, BEACH

WOOD...
See PILES AND PILE DRIVING; TRUSSES... ; *also* under specific type of structure

WORK, COST OF
See COSTS

WORKS (industrial buildings and equipment)
See under general types of works, e.g., PUBLIC WORKS (cross references thereunder); *also* under specific type of works, e.g., SEWAGE WORKS (cross reference thereunder); WATERWORKS (cross references thereunder); *see also* cross references under HOUSES; PLANTS

WRECKS
See SAFETY (cross references thereunder)

AUTHOR INDEX

(including Memoirs of deceased members)

Adams, C. Howard
"Plastics: Engineering Materials," 511.

Adams, Elmer Ellsworth
Memoir of, 1286.

Ambraseys, Nicols N.
Rock-fill dams, 811.
Viscous sublayer, 315.

Au, Tung
Design simplification, 629.
Shells, 1014.

Banerjee, Santi P.
Shells, 1016.

Baron, Frank
"A Pressure-Line Concept for Inelastic Bending," 1039.

Barreto, Benjamin Franklin De Barros
Memoir of, 1286.

Barton, James R.
Sand beds, 550.

Beecroft, Gordon W.
"Dynamic Stresses in Continuous Plate-Girder Bridges," 266.

Bennett, Newcomb B., Jr.
"Cost Allocation for Multiple-Purpose Water Projects," 85.

Bisbort, Harold E.
"Mobile (Ala.) Harbor and Ship Channel," 1195.

Blaney, Harry F.
"Evaporation from Free Water Surfaces at High Altitudes," 385.

Blench, Thomas
Sand beds, 550.

Borland, Whitney M.
"Design of Stable Canals and Channels in Erodible Material," 101.

Brant, Austin E., Jr.
Ship chennels, 260.

Brooks, Norman H.
"Mechanics of Streams with Movable Beds of Fine Sand," 526.

Brooks, Raymond Wentworth
Memoir of, 1287.

Burdick, Roy Dayton
Memoir of, 1288.

Butler, Robert G.
"Soil Lysimeters in Waste Water Reclamation Studies," 116.

Buzzell, Dow A.
"Trends in Hydraulic Gate Design," 27.

Cain, Frank J.
Moment-distribution constants, 350.

Camp, Thomas R.
Re-aeration, 667.

Carlson, Enos J.
Sand beds, 563.

Caruthers, Nickles L.
"Bridge Clearances (symposium): The Operator's View," 946.

Cawley, William A.
"An Improved Dilution Method for Flow Measurements," 405.

Cedergren, Harry R.
Loess, 170.

Chaudhuri, Pranab Kumar
Load distribution, 1245.

Chesson, Eugene, Jr.
"Behavior of Riveted Truss-Type Connections," 1087.

Chetty, Kandaswami S.
Gate design, 36.

Chien, Ning
Sand beds, 553.

Chinn, James
Moments in flat slabs, 842.

Chipley, Chesley Allen
Memoir of, 1288.

Churchill, Milo A.
"Effects of Storage Impoundments on Water Quality," 419.

Cleary, William E.
"Bridge Clearances (symposium): Need for a Realistic Approach," 942.

Clevenger, William A.
"Experiences with Loess As Foundation Material," 151.

Clough, Ray W.
"Earthquake Resistance of Rock-Fill Dams," 792.

Cocks, Charles A.
Cost allocation, 96.

Coltrin, Gordon L.
"Morro Bay Steam Electric Plant," 207.

Crosby, Lothrop
Memoir of, 1289.

Davisson, Melvin T.
Soil modulus, 1065.

Diachishin, Alexander N.
Re-aeration, 672.

Dittbrenner, Erhard E.
"Bridge Clearances (symposium) : Problems in Northeastern United States," 949.

Dobbins, William E.
"Mechanism of Reaeration in Natural Streams," 641.

Edgerton, Roy C.
"Dynamic Stresses in Continuous Plate-Girder Bridges," 266.

Edmonston, Arthur Donald
Memoir of, 1290.

Einstein, Hans A.
Sand beds, 553.
"The Viscous Sublayer Along a Smooth Boundary," 293.

Elder, Rex A.
"Friction Measurements in the Apalachia Tunnel," 1249.

Ellingson, Olaf John Sverdrop
Memoir of, 1290.

Escoffier, Francis F.
"Harrison County (Mississippi) Artificial Beach," 817.
"Transition Profiles in Nonuniform Channels," 43.

Feld, Jacob
Steam plants, 233.

Ferguson, Phil M.
"Simplification of Design by Ultimate Strength Procedures," 602.

Fiedler, Albert G.
"Importance of Ground Water in the National Economy," 776.

Fleming, Julian R.
"Municipal Ordinances for Industrial Wastes," 595.

Focht, John A., Jr.
Compaction, 19.
"Soil Modulus for Laterally Loaded Piles," 1049.

Fok, Thomas D. Y.
Moment-distribution constants, 345.

Foster, Charles R.
"Stabilization of Materials by Compaction," 1.

Francis, Arthur J.
Riveted connections, 1087.

Fraschina, Keeno
Venturi flumes, 772.

Gannett, Farley
Memoir of, 1291.

Gilman, Roger H.
"Transportation Planning (symposium) : The Port a Focal Point," 361.

Given, Jay Ankeny
Memoir of, 1292.

Gizienski, Stanley F.
Pile tests, 744.

Gotaas, Harold B.
"Design of Venturi Flumes in Circular Conduits," 749.

Graves, Eugene A.
"Old River Diversion Control (symposium) : Hydraulic Requirements," 1142.

Greer, Dewitt C.
"Control of Highway Access (symposium) : Economic Effects of the Gulf Freeway," 472.

Hackett, Allen Storr
Memoir of, 1292.

Hansen, Vello
Soil modulus, 1065.

Hardin, John R.
"Old River Diversion Control (symposium) : The General Problem," 1131.

Harley, George Foster
Memoir of, 1293.

Heagy, Kenneth
"A Floodway for Houston, Tex.," 181.

Hendry, Arnold W.
"Load Distribution in Highway Bridge Decks," 1214.

Hinshaw, Carl
Memoir of, 1294.

Hirashima, Kazuo B.
Compaction, 23.

Holt, William G. H.
Gate design, 36.

Hooper, Curtis J.
"Control of Highway Access (symposium): Foreword," 466.

Hotes, Frederick L.
Cost allocation, 93.

Howson, Louis R.
"The Engineer's Place in the Second Century of Technology": Address at the Summer Convention, Portland, Ore., June 25, 1958, 1275.

Hribar, John A.
Inelastic bending, 1047.

Huggins, Mark W.
"Moments in Flat Slabs," 824.

Hunt, Horace Sinclair
Memoir of, 1294.

Ingersoll, Alfred C.
"Hydraulic Model Study of Hyperion Sewer Interchange," 908.

Ingerson, Irvin M.
Evaporation, 401.

Ireland, Herbert O.
Loess, 171.

Jaeger, Leslie G.
"Load Distribution in Highway Bridge Decks," 1214.

Johnson, Wendell E.
Cost allocation, 96.

Jones, Jonathan
"Bearing-Ratio Effect on Strength of Riveted Joints," 964.

Jordy, George L.
Steam plants, 235.

Kane, Joseph Maurice
Memoir of, 1295.

Kaufman, Robert I.
"Pile Tests, Low-Sill Structure, Old River, Louisiana," 715.

Kenigsberg, Alexander H.
"Anchorages for Large Tainter Gates," 409.

Ketter, Robert L.
Portal frames, 75.

Kiker, John E., Jr.
"Developments in Septic Tank Systems," 77.

Klyce, Paul Cage
Memoir of, 1296.

Knapp, Robert Talbot
Memoir of, 1296.

Kurylo, Walter
"Bridge Clearances (symposium): The Interest of the Bureau of Public Roads," 932.

Landman, Saul
Memoir of, 1297.

Lansford, Wallace M.
Open channel flow, 707.

Lantz, Ersel G.
Ship channels, 258.

Latham, George R.
Gate design, 35.

Laursen, Emmett M.
"Sediment-Transport Mechanics in Stable-Channel Design," 195.

Lawson, Thomas R.
Memoir of, 1298.

Lazard, Achille
Transition profiles, 57.

Leonards, Gerald A.
Compaction, 16.

Li, Huon
"The Viscous Sublayer Along a Smooth Boundary," 293.

Lin, Pin-Nam
Sand beds, 569.

Lin, Watone L.
"Moments in Flat Slabs," 824.

Lindner, Clement P.
"The Apalachicola River Basin Project," 973.

Liu, Hsin-Kuan
Sand beds, 565.

Luck, Gerald
Moment-distribution constants, 350.

Lundgren, Raymond
Soil modulus, 1069.

Mansur, Charles I.
"Laboratory and In-Situ Permeability of Sand," 868.
"Pile Tests, Low-Sill Structure, Old River, Louisiana," 715.

Marsden, Howard J.
"Shoreside Facilities for Special-Purpose Ships," 851.

Marsh, Burton W.
"Transportation Planning (symposium): Prospects for Coordination," 377.

Martin, Harold C.
"Truss Analysis by Stiffness Considerations," 1182.

Matlock, Hudson
Soil modulus, 1077.
McClelland, Bramlette
"Soil Modulus for Laterally Loaded Piles," 1049.
McDougle, Edwin Andrew
Memoir of, 1298.
Moore, Norman R.
"Old River Diversion Control (symposium): Structures Required," 1172.
Moore, William W.
Steam plants, 234.
Mostafa, M. Gamal
Sand beds, 563.
Moyer, Ralph A.
"Control of Highway Access (symposium): User Benefits in California," 499.
Munse, William H.
"Behavior of Riveted Truss-Type Connections," 1087.
Murphy, John C.
Moment distribution constants, 344.
Myers, Clyde
Memoir of, 1299.
Myers, Lloyd E., Jr.
"Relationship of Irrigation to Public Health," 846.
Nelson, Herbert C.
"Open-Channel Flow at Small Reynolds Numbers," 685.
Newnam, Frank H., Jr.
"Developing Port Facilities on Houston's Ship Channel," 239.
Nishida, Yoshichika
Pile tests, 744.
Noble, Charles M.
"Operation Problems on Controlled-Access Highways," 858.
O'Connor, Donald J.
"Mechanism of Reaeration in Natural Streams," 641.
Ohrt, Frederick
Memoir of, 1285.
Ondra, Otakar
"Moment-Distribution Constants from Models," 318.
Orlob, Gerald T.
"Soil Lysimeters in Waste Water Reclamation Studies," 116.
Pagon, W. Watters
Shells, 1015.
Parme, Alfred L.
"Shells of Double Curvature," 989.
Pasveer, Aale
Re-aeration, 677.
Peck, Ralph B.
Loess, 171.
Soil modulus, 1065.
Pirtz, David
"Earthquake Resistance of Rock-Fill Dams," 792.
Posey, Chesley J.
Open channel flow, 712.
Post, Wilfred M., Jr.
"Transportation Planning (symposium): The Airport, a National Facility," 370.
Prager, William
"Minimum-Weight Design of a Portal Frame," 66.
Pryor, William T.
"Bureau of Public Roads Experiences in Highway Surveys," 1026.
Rand, Myrton C.
Re-aeration, 679.
Reese, Lymon C.
Soil modulus, 1071.
Reuss, Ralph F.
Soil modulus, 1064.
Ripperger, Eugene A.
Soil modulus, 1074.
Roach, Alden Gallup
Memoir of, 1300.
Robertson, James M.
Open channel flow, 707.
Robinson, Delbert Walton
Memoir of, 1300.
Rogers, Harry Stanley
Memoir of, 1301.
Rohwer, Carl
Evaporation, 396.
Salvadori, Mario G.
Shells, 1018.
Saville, Thorndike, Jr.
"Wave Run-Up on Shore Structures," 139.
Schick, Herman S.
Design simplification, 630.
Schlick, William Japhia
Memoir of, 1302.
Shafer, Irven Clarence
Memoir of, 1302.

Shannon, John W.
Evaporation, 401.

Shifrin, Hymen
Memoir of, 1303.

Shockley, Woodland G.
"Old River Diversion Control (symposium): Foundation Design," 1160.

Shulits, Sam
Stable channel design, 204.

Silberman, Edward
"Open-Channel Flow at Small Reynolds Numbers, 685.
Viscous sublayer, 314.

Sobotka, Zdenek
Design simplification, 633.

Spanovich, Milan
Design simplification, 637.

Spielman, John V.
Rock-fill dams, 811.

Stewart, Lowell O.
Memoir of, 1304.

Stine, Walter Pearce
Memoir of, 1304.

Stone, Ralph
Soil lysimeters, 137.

Straub, Lorenz G.
"Open-Channel Flow at Small Reynolds Numbers, 685.

Streeter, Victor L.
"A Flow Controller for Open or Closed Conduits," 883.

Streiff, Abraham
Gate design, 33.

Stuenkel, Paul Lloyd
Memoir of, 1305.

Tallamy, Bertram D.
"Control of Highway Access (symposium): Experiences in New York," 467.

Tanaka, Hajime
"Hydraulic Model Study of Hyperion Sewer Interchange," 908.

Taylor, Donald Wood
Memoir of, 1306.

Terrell, Pete W.
"Design of Stable Canals and Channels in Erodible Material," 101.

Thon, J. George
"Morro Bay Steam Electric Plant," 207.

Tison, Leon J.
Transition profiles, 58.

Turnbull, Willard J.
"Old River Diversion Control (symposium): Foundation Design," 1160.
"Stabilization of Materials by Compaction," 1.

Turner, Kenneth M.
Evaporation, 397.

Ulicny, Sylvester M.
Design simplification, 637.

van der Goot, Herbert Albert
Memoir of, 1307.

Van London, William James
Memoir of, 1307.

Vogelgesang, Carl E.
"Control of Highway Access (symposium): Experiences in Indiana," 477.

Wearne, Stephen
Tainter gates, 417.

Weber, Eugene W.
"Bridge Clearances (symposium): Policies and Practice," 927.
Cost allocation, 95.

Weber, William G.
Gate design, 38.

Wells, Edwin A., Jr.
"Design of Venturi Flumes in Circular Conduits," 749.

Wells, Joseph V. B.
"Surface Water Resources," 1206.

Wen, Robert K. L.
Dynamic stresses, 287.

Williams, William Horace
Memoir of, 1308.

Wilson, William Munsey
Memoir of, 1309.

Winter, Hugo H.
"Control of Highway Access (symposium): Experiences in Los Angeles," 486.

Woodruff, Glenn B.
Steam plants, 233.

Woods, Jack W.
"An Improved Dilution Method for Flow Measurements," 405.

Worley, John Stephen
Memoir of, 1310.

Zegarra, Edmund J.
Compaction, 20.